MODELING AND ANALYSIS OF TIMED PETRI NETS USING HEAPS OF PIECES

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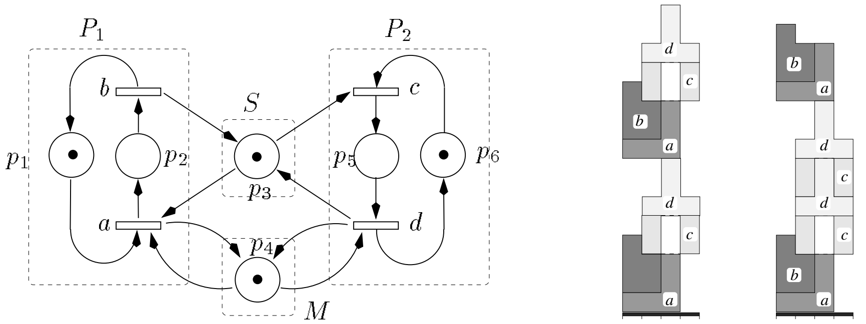
To the question: "what is the class of timed Petri nets which can be analysed using $(\max,+)$ algebra ?"; it is traditional to answer, informally: "timed event graphs are $(\max,+)$-linear dynamical systems". The purpose of this paper is to give a very different and essentially more general answer: *safe timed Petri nets can be modeled by Tetris-like heaps of pieces, whose height is computed by automata with multiplicities in the* $(\max,+)$ *semiring.*

Performance evaluation algorithms can be designed in terms of heaps and automata. They improve timed event graphs based algorithms, particularly if one is interested in the evaluation of a large number of schedules.

As an illustration, consider the timed Petri net below, which represents a manufacturing system with two part types P_1, P_2, requiring two heterogeneous resources S and M. We are interested in finding infinite periodic schedules $v^\omega = vvv\ldots$, where $v \in \{a,b,c,d\}^*$ is a finite pattern, that produces parts P_1 and P_2 in identical proportions.

For each special pattern v, the traditional approach builds a special timed event graph, whose size is proportional to the length of v. Here, we prove that the execution time (makespan) of any finite sequence w coincides with the height of the heap obtained by piling up pieces associated with letters in w, as shown in the figure (with $w = abcdabcd$ and $w = abcdcdab$). Extrapolating from the figure, it is clear that among patterns of length $4n$, $v = ab(cd)^n(ab)^{n-1}$ is optimal and yields a throughput of $n/(7n+1)$. Here, there is no periodic optimal schedule.



Holding and Firing Times:

a	b	c	d	p_1	p_2	p_3	p_4	p_5	p_6
1	2	2	1	1	0	0	2	0	0

Algebraically, the height is given by a product of matrices in the $(\max,+)$ semiring; the growth rate of infinite periodic heaps can be computed from the $(\max,+)$ eigenvalue of a small matrix (whose dimension is the width of the heap model).