MPRI

Some notions of information flow

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Syntax

Let $\mathcal{V} \stackrel{\Delta}{=} \{V, V_1, V_2, \ldots\}$ be a finite set of variables.
Let $\mathbb{Z} \stackrel{\Delta}{=} \{z, \ldots\}$ be the set of relative numbers.
Expressions are polynomial of variables $\mathcal{V}$.

$$E ::= z \mid V \mid E + E \mid E \times E$$

Programs are given by the following grammar:

$$\begin{array}{rcl} P & :== & \text{skip} \\ & \mid & P;P \\ & \mid & V := E \\ & \mid & \text{if } (V \geq 0)\ \{P\} \text{ else } \{P\} \\ & \mid & \text{while } (V \geq 0)\ \{P\} \end{array}$$

Semantics

We define the semantics $[\![P]\!] \in \mathcal{F}((\mathcal{V} \to \mathbb{Z}) \cup \Omega)$ of a program P:

- $[\![\text{skip}]\!](\rho) = \rho$,
- $[\![P_1;P_2]\!](\rho) = \begin{cases} \Omega & \text{if } [\![P_1]\!](\rho) = \Omega \\ [\![P_2]\!]([\![P_1]\!](\rho)) & \text{otherwise} \end{cases}$
- $[\![V := E]\!](\rho) = \begin{cases} \Omega & \text{if } \rho = \Omega \\ \rho\,[V \mapsto \overline{\rho}(E)] & \text{otherwise} \end{cases}$
- $[\![\text{if } (V \geq 0)\ \{P_1\} \text{ else } \{P_2\}]\!](\rho) = \begin{cases} \Omega & \text{if } \rho = \Omega \\ [\![P_1]\!](\rho) & \text{if } \rho(V) \geq 0 \\ [\![P_2]\!](\rho) & \text{otherwise} \end{cases}$
- $[\![\text{while } (V \geq 0)\ \{P\}]\!](\rho) = \begin{cases} \Omega & \text{if } \rho = \Omega \\ \rho' & \text{if } \{\rho'\} = \{\rho' \in \mathit{Inv} \mid \rho'(V) < 0\} \\ \Omega & \text{otherwise} \end{cases}$

  where $\mathit{Inv} = \mathit{lfp}\,(X \mapsto \{\rho\} \cup \{\rho'' \mid \exists \rho' \in X,\ \rho'(V) \geq 0 \text{ and } \rho'' \in [\![P]\!](\rho')\})$.

Flow of information

Given a program P, we say that the variable V_1 flows into the variable V_2 if, and only if, the final value of V_2 depends on the initial value of V_1, which is written $V_1 \Rightarrow_P V_2$.

More formally,
$V_1 \Rightarrow_P V_2$ if and only if there exists $\rho \in \mathcal{V} \to \mathbb{Z}$, $z, z' \in \mathbb{Z}$ such that one of the following three assertions is satisfied:

1. $[\![P]\!](\rho[V_1 \mapsto z]) \neq \Omega$, $[\![P]\!](\rho[V_1 \mapsto z']) \neq \Omega$,
   and $[\![P]\!](\rho[V_1 \mapsto z])(V_2) \neq [\![P]\!](\rho[V_1 \mapsto z'])(V_2)$;
2. $[\![P]\!](\rho[V_1 \mapsto z]) = \Omega$ and $[\![P]\!](\rho[V_1 \mapsto z']) \neq \Omega$;
3. $[\![P]\!](\rho[V_1 \mapsto z]) \neq \Omega$ and $[\![P]\!](\rho[V_1 \mapsto z']) = \Omega$.

Syntactic approximation (tentative)

Let P be a program.

We define the following binary relation $\rightarrow_P$ among variables in $\mathcal{V}$:
$V_1 \rightarrow_P V_2$ if and only if there is an assignement in P of the form $V_2 := E$ such that V_1 occurs in E.

Does $V_1 \Rightarrow_P V_2$ imply that $V_1 \rightarrow_P^* V_2$?

Counter-example

We consider the following progrem P:

$$\begin{array}{rl} P ::= & \text{if } (V_1 \geq 0) \\ & \quad \{V_2 := 0\} \\ & \text{else} \\ & \quad \{V_2 := 1\} \end{array}$$

For any $\rho \in \mathcal{V} \to \mathbb{Z}$,
we have $[\![P]\!](\rho[V_1 \mapsto 0])(V_2) = 0$;
but, $[\![P]\!](\rho[V_1 \mapsto 1])(V_2) = 1$;
so $V_1 \Rightarrow_P V_2$;
But $V_1 \not\rightarrow^*_P V_2$.

Syntactic approximation (tentative)

For each program point p in P,
we denote by *test*(p) the set of variables which occur in the guards of tests and while loops the scope of which contains the program point p.

We define the following binary relation $\rightarrow$ among variables in $\mathcal{V}$:
$V_1 \rightarrow_P V_2$ if and only if there is an assignement in P of the form $V_2 := E$ at program point p such that:

1. either V_1 occurs in E;
2. or $V_1 \in$ *test*(p).

Does $V_1 \Rightarrow_P V_2$ imply that $V_1 \rightarrow_P^* V_2$?

Counter-example

We consider the following progrem P:

$$P ::= \quad \text{while } (V_1 \geq 0) \ \{\text{skip}\}$$

For any $\rho \in \mathcal{V} \to \mathbb{Z}$,
we have $[\![P]\!](\rho[V_1 \mapsto -1]) \neq \Omega$;
but, $[\![P]\!](\rho[V_1 \mapsto 0]) = \Omega$;
so $V_1 \Rightarrow_P V_2$;
But $V_1 \not\to_P^* V_2$.

Approximation of the information flow

So as to get a sound approximation of the information flow, we have to consider that a variable that is tested in the guard of a loop may flow in any variable.

We define the following binary relation $\rightarrow_P$ among variables in $\mathcal{V}$: $V_1 \rightarrow V_2$ if and only if there is an assignement in P of the form $V_2 := E$ at program point p such that:

1. either V_1 occurs in E;
2. or V_1 is tested in the guard of a loop;
3. or $V_1 \in \mathit{test}(p)$.

Theorem 1 If $V_1 \Rightarrow_P V_2$, then $V_1 \rightarrow_P^* V_2$!

Limitations

The approximation is highly syntax-oriented.

- It is context-insensitive;
- It is very rough in the case of while loop,

  $\Longrightarrow$ we could show statically that some loops always terminate to avoid fictitious dependencies;
- we could detect some invariants to avoid fictitious dependencies.

Other forms of attacks could be modeled in the semantics: an attacker could observe:

- computation time;
- memory assumption;
- heating.

(attacks cannot be exhaustively specified).