Memory abstraction 1

MPRI — Cours 2.6 "Interprétation abstraite : application à la vérification et à l'analyse statique"

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Overview of the lecture

So far, we have shown **numerical abstract domains**

- non relational: intervals, congruences...
- relational: polyhedra, octagons, ellipsoids...

- **How to deal with non purely numerical states ?**
- **How to reason about complex data-structures ?**

$\Rightarrow$ **a very broad topic**, and two lectures:

This lecture

- **overview memory models** and **memory properties**
- abstraction of **pointer structures** and **separation logic based shape analysis**

Next lecture: arrays, shape/numerical abstraction, composition of shape abstractions

Outline

Assumptions for the two lectures on memory abstraction

Imperative programs viewed as **transition systems**:

- set of **control states:** $\mathbb{L}$ (program points)
- set of **variables:** $\mathbb{X}$ (all assumed globals)
- set of **values:** $\mathbb{V}$ **(so far: $\mathbb{V}$ consists of integers (or floats) only)**
- set of **memory states:** $\mathbb{M}$ **(so far: $\mathbb{M} = \mathbb{X} \rightarrow \mathbb{V}$)**
- **error state:** Ω
- **states:** $\mathbb{S}$

$$\begin{array}{rcl} \mathbb{S} & = & \mathbb{L} \times \mathbb{M} \\ \mathbb{S}_\Omega & = & \mathbb{S} \uplus \{\Omega\} \end{array}$$

- **transition relation:**

$$(\rightarrow) \subseteq \mathbb{S} \times \mathbb{S}_\Omega$$

Abstraction of sets of states

- **abstract domain** $\mathbb{D}^\sharp$
- **concretization** $\gamma : (\mathbb{D}^\sharp, \sqsubseteq^\sharp) \longrightarrow (\mathcal{P}(\mathbb{S}), \subseteq)$

Assumptions: syntax of programs

We start from the same language syntax and will extend l-values:

$$
\begin{array}{rcll}
\mathtt{l} & ::= & \textbf{l-values} & \\
 & | & \mathtt{x} & (\mathtt{x} \in \mathbb{X}) \\
 & | & \ldots & \textbf{we will add other kinds of l-values} \\
 & & & \textbf{pointers, array dereference...} \\
\mathtt{e} & ::= & \textbf{expressions} & \\
 & | & c & (c \in \mathbb{V}) \\
 & | & \mathtt{l} & \text{(lvalue)} \\
 & | & \mathtt{e} \oplus \mathtt{e} & \text{(arith operation, comparison)} \\
\mathtt{s} & ::= & \textbf{statements} & \\
 & | & \mathtt{l} = \mathtt{e} & \text{(assignment)} \\
 & | & \mathtt{s}; \ldots \mathtt{s}; & \text{(sequence)} \\
 & | & \textbf{if}(\mathtt{e})\{\mathtt{s}\} & \text{(condition)} \\
 & | & \textbf{while}(\mathtt{e})\{\mathtt{s}\} & \text{(loop)}
\end{array}
$$

Assumptions: semantics of programs

We assume **classical definitions for:**

- **l-values**: $[\![\mathtt{l}]\!] : \mathbb{M} \rightarrow \mathbb{X}$
- **expressions**: $[\![\mathtt{e}]\!] : \mathbb{M} \rightarrow \mathbb{V}$
- **programs and statements**:
  - we assume a label **before each statement**
  - each statement defines a **set of transitions** $(\rightarrow)$

In this course, we rely on the usual **reachable states semantics**

Reachable states semantics

The reachable states are computed as $[\![\mathcal{S}]\!]_{\mathcal{R}} = \mathbf{lfp} F$ where

$$\begin{array}{rrcl} F: & \mathcal{P}(\mathbb{S}) & \longrightarrow & \mathcal{P}(\mathbb{S}) \\ & X & \longmapsto & \mathbb{S}_{\mathcal{I}} \cup \{ s \in \mathbb{S} \mid \exists s' \in X,\ s' \rightarrow s \} \end{array}$$

and $\mathbb{S}_{\mathcal{I}}$ denotes the set of initial states.

Assumptions: general form of the abstraction

We assume an **abstraction for sets of memory states**:

- memory abstract domain $\mathbb{D}^\sharp_{\mathrm{mem}}$
- concretization function $\gamma_{\mathrm{mem}} : \mathbb{D}^\sharp_{\mathrm{mem}} \to \mathcal{P}(\mathbb{M})$

Reachable states abstraction

We construct $\mathbb{D}^\sharp = \mathbb{L} \to \mathbb{D}^\sharp_{\mathrm{mem}}$ and:

$$\begin{array}{rccl} \gamma : & \mathbb{D}^\sharp & \longrightarrow & \mathcal{P}(\mathbb{S}) \\ & X^\sharp & \longmapsto & \{(\ell, m) \in \mathbb{S} \mid m \in \gamma_{\mathrm{mem}}(X^\sharp(\ell))\} \end{array}$$

The whole question is how do we choose $\mathbb{D}^\sharp_{\mathrm{mem}}, \gamma_{\mathrm{mem}}$...

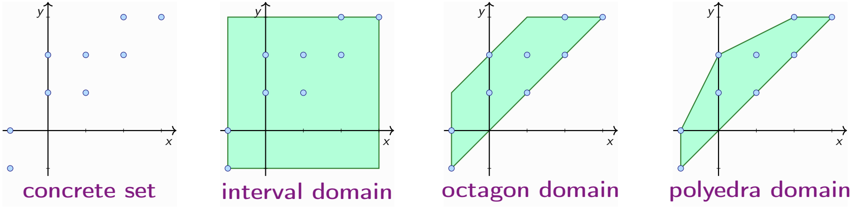
- previous lectures:
  $\mathbb{X}$ **is fixed and finite** and, $\mathbb{V}$ **is scalars** (integers or floats), thus, $\mathbb{M} \equiv \mathbb{V}^n$
- today:
  we will **extend the language** thus, also **need to extend** $\mathbb{D}^\sharp_{\mathrm{mem}}, \gamma_{\mathrm{mem}}$

Abstraction of purely numeric memory states

Purely numeric case

- $\mathbb{V}$ is a set of values of the same kind
- *e.g.*, integers ($\mathbb{Z}$), machine integers ($\mathbb{Z} \cap [-2^{63}, 2^{63} - 1]$)...
- If the set of variables is fixed, we can use **any abstraction for $\mathbb{V}^N$**

Example: $N = 2$, $\mathbb{X} = \{x, y\}$



Heterogeneous memory states

In real life languages, there are many kinds of values:

- **scalars** (integers of various sizes, boolean, floating-point values)...
- **pointers**, **arrays**...

Heterogeneous memory states and non relational abstraction

- **types** $t_0, t_1, \ldots$ and **values** $\mathbb{V} = \mathbb{V}_{t_0} \uplus \mathbb{V}_{t_1} \uplus \ldots$
- finitely many **variables**; each has a **fixed type:** $\mathbb{X} = \mathbb{X}_{t_0} \uplus \mathbb{X}_{t_1} \uplus \ldots$
- **memory states:** $\mathbb{M} = \mathbb{X}_{t_0} \to \mathbb{V}_{t_0} \times \mathbb{X}_{t_1} \to \mathbb{V}_{t_1} \ldots$

Principle: compose abstractions for sets of memory states of each type

Non relational abstraction of heterogeneous memory states

- $\mathbb{M} \equiv \mathbb{M}_0 \times \mathbb{M}_1 \times \ldots$ where $\mathbb{M}_i = \mathbb{X}_i \to \mathbb{V}_i$
- **Concretization function** (case with two types)

$$\begin{array}{rrcl} \gamma_{\mathrm{nr}} : & \mathcal{P}(\mathbb{M}_0) \times \mathcal{P}(\mathbb{M}_1) & \longrightarrow & \mathcal{P}(\mathbb{M}) \\ & (m_0^\sharp, m_1^\sharp) & \longmapsto & \{(m_0, m_1) \mid \forall i,\ m_i \in \gamma_i(m_i^\sharp)\} \end{array}$$

Memory structures

Common structures (non exhaustive list)

- **Structures, records, tuples**:
  sequences of cells accessed with fields
- **Arrays**:
  similar to structures; indexes are integers in $[0, n-1]$
- **Pointers**:
  numerical values corresponding to the address of a memory cell
- **Strings and buffers**:
  blocks with a sequence of elements and a terminating element (*e.g.*, 0x0)
- **Closures** (functional languages):
  pointer to function code and (partial) list of arguments)

To describe memories, the definition $\mathbb{M} = \mathbb{X} \to \mathbb{V}$ is **too restrictive**

Generally, non relational, heterogeneous abstraction cannot handle many such structures all at once: relations are needed!

Specific properties to verify

Memory safety

Absence of memory errors (crashes, or undefined behaviors)

Pointer errors:

- Dereference of a **null pointer** / of an **invalid pointer**

Access errors:

- **Out of bounds** array access, **buffer overruns** (often used for attacks)

Invariance properties

Data should not become corrupted (values or structures...)

Examples:

- **Preservation of structures**, *e.g.*, lists should remain connected
- **Preservation of invariants**, *e.g.*, of balanced trees

Properties to verify: examples

A program closing a list of file descriptors

```
//l points to a list
c = l;
while(c ≠ NULL){
  close(c → FD);
  c = c → next;
}
```

Correctness properties

1. memory safety
2. `l` is supposed to store all file descriptors at all times
   will its structure be preserved ?
   yes, no breakage of a `next` link
3. closure of all the descriptors

Examples of structure preservation properties

- Algorithms manipulating **trees**, **lists**...
- Libraries of algorithms on **balanced trees**
- **Not guaranteed by the language** !
  e.g., the balancing of `Maps` in the OCaml standard library was **incorrect** for years (performance bug)

A more realistic model

No one-to-one relation between memory cells and program variables

- a variable may indirectly reference **several cells** (structures...)
- **dynamically allocated cells** correspond to no variable at all...

Environment + Heap

- **Addresses** are values: $\mathbb{V}_{\rm addr} \subseteq \mathbb{V}$
- **Environments** $e \in \mathbb{E}$ map variables into their addresses
- **Heaps** ($h \in \mathbb{H}$) map addresses into values

$$\begin{array}{rcl} \mathbb{E} & = & \mathbb{X} \rightarrow \mathbb{V}_{\rm addr} \\ \mathbb{H} & = & \mathbb{V}_{\rm addr} \rightarrow \mathbb{V} \end{array}$$

  h is actually only a partial function
- **Memory states** (or **memories**): $\mathbb{M} = \mathbb{E} \times \mathbb{H}$

Note: **Avoid confusion between heap (function from addresses to values) and dynamic allocation space (often referred to as "heap")**

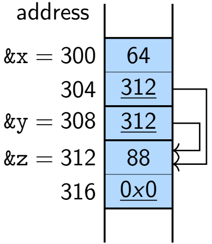
Example of a concrete memory state (variables)

Example setup:

- x and z are two list elements containing values 64 and 88, and where the former points to the latter
- y stores a pointer to z

Memory layout

(pointer values underlined)



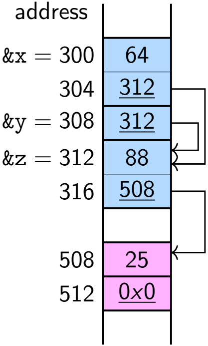
$$
\begin{array}{llll}
e: & \mathtt{x} & \mapsto & 300 \\
& \mathtt{y} & \mapsto & 308 \\
& \mathtt{z} & \mapsto & 312 \\
\\
h: & 300 & \mapsto & 64 \\
& 304 & \mapsto & 312 \\
& 308 & \mapsto & 312 \\
& 312 & \mapsto & 88 \\
& 316 & \mapsto & 0
\end{array}
$$

Example of a concrete memory state (variables + dyn. cell)

Example setup:

- same configuration
- + second field of z points to a dynamically allocated list element (in purple)

Memory layout



$$
\begin{array}{llll}
e: & \mathtt{x} & \mapsto & 300 \\
 & \mathtt{y} & \mapsto & 308 \\
 & \mathtt{z} & \mapsto & 312 \\
 & & & \\
\hbar: & 300 & \mapsto & 64 \\
 & 304 & \mapsto & 312 \\
 & 308 & \mapsto & 312 \\
 & 312 & \mapsto & 88 \\
 & 316 & \mapsto & 508 \\
 & 508 & \mapsto & 25 \\
 & 512 & \mapsto & 0
\end{array}
$$

Extending the semantic domains

Some slight modifications to the semantics of the initial language:

- **Addresses are values:** $\mathbb{V}_{\text{addr}} \subseteq \mathbb{V}$
- **L-values evaluate into addresses:** $[\![\mathtt{l}]\!] : \mathbb{M} \to \mathbb{V}_{\text{addr}}$

$$[\![\mathtt{x}]\!](e, h) \quad = \quad e(\mathtt{x})$$

- **Semantics of expressions** $[\![\mathtt{e}]\!] : \mathbb{M} \to \mathbb{V}$, mostly unchanged

$$[\![\mathtt{l}]\!](e, h) \quad = \quad h([\![\mathtt{l}]\!](e, h))$$

- **Semantics of assignment** $\ell_0 : \mathtt{l} := \mathtt{e}; \ell_1 : \ldots$:

$$(\ell_0, e, h_0) \longrightarrow (\ell_1, e, h_1)$$

  where

$$h_1 = h_0[[\![\mathtt{l}]\!](e, h_0) \leftarrow [\![\mathtt{e}]\!](e, h_0)]$$

Realistic definitions of memory states

Our model is still not very accurate for most languages

- Memory cells do not all have the same **size**
- **Memory management algorithms** usually do not treat cells one by one, *e.g.*, **malloc** returns a pointer to a **block**
  applying **free** to that pointer will dispose the *whole block*

Other refined models

- **Partition of the memory** in **blocks** with a **base address** and a **size**
- **Partition of blocks** into **cells** with a **size**
- Description of **fields** with an **offset**
- Description of **pointer values** with a **base address** and an **offset**...

For a **very formal** description of such concrete memory states:
see **CompCert** project source files (Coq formalization)

Language semantics: program crash

In an abnormal situation, we assume that **the program will crash**

- advantage: **very clear semantics**
- disadvantage (for the compiler designer): **dynamic checks** are required

Error state

- Ω denotes an **error configuration**
- Ω is a **blocking**: $(\rightarrow) \subseteq \mathbb{S} \times (\{\Omega\} \uplus \mathbb{S})$

OCaml:

- out-of-bound array access:
  `Exception: Invalid_argument "index out of bounds".`
- no notion of a null pointer

Java:

- exception in case of out-of-bound array access, null dereference:
  `java.lang.ArrayIndexOutOfBoundsException`

Language semantics: undefined behaviors

Alternate choice: leave the behavior of the program **unspecified** when an abnormal situation is encountered

- advantage: **easy implementation** (often architecture driven)
- disadvantage: **unintuitive semantics**, errors hard to reproduce
  different compilers may make different choices...
  or in fact, make no choice at all (= let the program evaluate even when performing invalid actions)

Modeling of undefined behavior

- Very hard to capture what a program operation may modify
- Abnormal situation at (l_0, m_0) such that $\forall m_1 \in \mathbb{M},\ (l_0, m_0) \rightarrow (l_1, m_1)$

- **In C**:
  array out-of-bound accesses and dangling pointer dereferences lead to undefined behavior (and potentially, memory corruption) whereas a null-pointer dereference always result into a crash

Composite objects

How are contiguous blocks of information organized ?

Java objects, OCaml struct types

- sets of fields
- each field has a type
- **no assumption** on physical storage, **no pointer arithmetics**

C composite structures and unions

- **physical mapping** defined by the norm
- each field has a specified **size** and a specified **alignment**
- **union types / casts**:
  implementations may allow several views

Pointers and records / structures / objects

Many languages provide **pointers** or **references** and allow to manipulate **addresses**, but with different levels of expressiveness

What kind of objects can be referred to by a pointer ?

Pointers only to records / structures / objects

- **Java**: only pointers to objects
- **OCaml**: only pointers to records, structures...

Pointers to fields

- **C**: pointers to any valid cell...

$$\textbf{struct}\ \{\textbf{int}\ \mathtt{a};\ \textbf{int}\ \mathtt{b}\}\ \mathtt{x};$$
$$\textbf{int} * \mathtt{y} = \&(\mathtt{x} \cdot \mathtt{b});$$

Pointer arithmetics

What kind of operations can be performed on a pointer ?

Classical pointer operations

- Pointer **dereference**:
  `*p` returns the contents of the cell of address `p`
- **"Address of"** operator: `&x` returns the address of variable `x`
- Can be analyzed with **a rather coarse pointer model**
  e.g., symbolic base + symbolic field

Arithmetics on pointers, requiring a more precise model

- **Addition of a numeric constant**:
  `p` + n: address contained in `p` + n times the size of the type of `p`
  Interaction with pointer casts...
- **Pointer subtraction**: returns a numeric offset

Manual memory management

Allocation of unbounded memory space

- How are new memory blocks **created** by the program ?
- How do old memory blocks get **freed** ?

OCaml memory management

- **implicit allocation**
  when declaring a new object
- **garbage collection**: purely automatic process, that frees unreachable blocks

C memory management

- **manual allocation**: **malloc**
  operation returns a pointer to a new block
- **manual de-allocation**: **free**
  operation (block base address)

Manual memory management is not safe:

- **memory leaks**: growing unreachable memory region; memory exhaustion
- **dangling pointers** if freeing a block that is still referred to

Summary on the memory model

Language dependent items

- **Clear error cases** or **undefined behaviors**
  for analysis, a semantics with clear error cases is preferable
- **Composite objects**: structure fully exposed or not
- **Pointers to object fields**: allowed or not
- **Pointer arithmetic**: allowed or not
  i.e., are pointer values symbolic values or numeric values
- **Memory management**: automatic or manual

In this course, we start with a simple model, and study specific features one by one and in isolation from the others

Rest of the course

Structures for which we introduce abstractions:

- **pointers** and **dynamically allocated pointer structures** (today)
- **arrays** (in a few weeks)
- **combinations of structures** (in a few weeks)

Abstract operations:

- post-condition for the **reading** of a cell defined by an l-value
  e.g., $\mathtt{x} = \mathtt{a}[\mathtt{i}]$ or $\mathtt{x} = *\mathtt{p}$
- post-condition for the **writing of a heap cell**
  e.g., $\mathtt{a}[\mathtt{i}] = \mathtt{p}$ or $\mathtt{p} \texttt{->} \mathtt{f} = \mathtt{x}$
- **abstract join**, that approximates unions of concrete states

Outline

Programs with pointers: syntax

Syntax extension: we add pointer operations

$$
\begin{array}{rcll}
\mathtt{l} & ::= & \textbf{l-values} & \\
& | & \mathtt{x} & (\mathtt{x} \in \mathbb{X}) \\
& | & \ldots & \\
& | & *\mathtt{e} & \text{pointer dereference} \\
& | & \mathtt{l} \cdot \mathtt{f} & \text{field read} \\
\mathtt{e} & ::= & \textbf{expressions} & \\
& | & \mathtt{l} & \\
& | & \ldots & \\
& | & \&\mathtt{l} & \text{"address of" operator} \\
\mathtt{s} & ::= & \textbf{statements} & \\
& | & \ldots & \\
& | & \mathtt{x} = \textbf{malloc}(c) & \text{allocation of } c \text{ bytes} \\
& | & \textbf{free}(\mathtt{x}) & \text{deallocation of the block pointed to by } \mathtt{x}
\end{array}
$$

We do not consider **pointer arithmetics here**

Programs with pointers: semantics

Case of l-values:

$$\begin{array}{rcl} [\![\mathtt{x}]\!](e, \hbar) & = & e(\mathtt{x}) \\ [\![*\mathtt{e}]\!](e, \hbar) & = & \begin{cases} \hbar([\![\mathtt{e}]\!](e, \hbar)) & \text{if } [\![\mathtt{e}]\!](e, \hbar) \neq 0 \wedge [\![\mathtt{e}]\!](e, \hbar) \in \mathbf{Dom}(\hbar) \\ \Omega & \text{otherwise} \end{cases} \\ [\![\mathtt{l} \cdot \mathtt{f}]\!](e, \hbar) & = & [\![\mathtt{l}]\!](e, \hbar) + \mathbf{offset}(\mathtt{f}) \text{ (numeric offset)} \end{array}$$

Case of expressions:

$$\begin{array}{rcll} [\![\mathtt{l}]\!](e, \hbar) & = & \hbar([\![\mathtt{l}]\!](e, \hbar)) & \text{(evaluates into the contents)} \\ [\![\&\mathtt{l}]\!](e, \hbar) & = & [\![\mathtt{l}]\!](e, \hbar) & \text{(evaluates into the address)} \end{array}$$

Case of statements:

- **memory allocation** $\mathtt{x} = \mathbf{malloc}(c)$: $(e, \hbar) \rightarrow (e, \hbar')$ where $\hbar' = \hbar[e(\mathtt{x}) \leftarrow k] \uplus \{k \mapsto v_k, k+1 \mapsto v_{k+1}, \ldots, k+c-1 \mapsto v_{k+c-1}\}$ and $k, \ldots, k+c-1$ are fresh and unused in $\hbar$
- **memory deallocation** $\mathbf{free}(\mathtt{x})$: $(e, \hbar) \rightarrow (e, \hbar')$ where $k = e(\mathtt{x})$ and $\hbar = \hbar' \uplus \{k \mapsto v_k, k+1 \mapsto v_{k+1}, \ldots, k+c-1 \mapsto v_{k+c-1}\}$

Pointer non relational abstractions

We rely on the **non relational abstraction of heterogeneous states** that was introduced earlier, with a few changes:

- we let $\mathbb{V} = \mathbb{V}_{\text{addr}} \uplus \mathbb{V}_{\text{int}}$ and $\mathbb{X} = \mathbb{X}_{\text{addr}} \uplus \mathbb{X}_{\text{int}}$
- **concrete memory cells** now include **structure fields**, and fields of **dynamically allocated regions**
- **abstract cells** $\mathbb{C}^\sharp$ finitely summarize concrete cells
- we apply a **non relational abstraction**:

Non relational pointer abstraction

- Set of **pointer abstract values** $\mathbb{D}^\sharp_{\text{ptr}}$
- **Concretization** $\gamma_{\text{ptr}} : \mathbb{D}^\sharp_{\text{ptr}} \to \mathcal{P}(\mathbb{V}_{\text{addr}})$ into pointer sets

We will see **several instances** of this kind of abstraction

Pointer non relational abstraction: null pointers

The dereference of a null pointer will cause a crash

To establish **safety**: compute **which pointers may be null**

Null pointer analysis

Abstract domain for addresses:

- $\gamma_{\mathrm{ptr}}(\bot) = \emptyset$
- $\gamma_{\mathrm{ptr}}(\top) = \mathbb{V}_{\mathrm{addr}}$
- $\gamma_{\mathrm{ptr}}(\neq \texttt{NULL}) = \mathbb{V}_{\mathrm{addr}} \setminus \{0\}$

$$\begin{array}{c} \top \\ | \\ \neq \texttt{NULL} \\ | \\ \bot \end{array}$$

- we may also use a lattice with a fourth element $= \texttt{NULL}$
  exercise: what do we gain using this lattice ?
- very **lightweight**, can typically resolve rather trivial cases
- useful for **C**, but also for **Java**

Pointer non relational abstraction: dangling pointers

The dereferece of a null pointer will cause a crash

To establish **safety**: compute **which pointers may be dangling**

Null pointer analysis

Abstract domain for addresses:

- $\gamma_{\mathrm{ptr}}(\bot) = \emptyset$
- $\gamma_{\mathrm{ptr}}(\top) = \mathbb{V}_{\mathrm{addr}} \times \mathbb{H}$
- $\gamma_{\mathrm{ptr}}(\text{Not dangling}) = \{(v, \hbar) \mid \hbar \in \mathbb{H} \wedge v \in \textbf{Dom}(\hbar)\}$

$$\top \;-\; \text{Not dangling} \;-\; \bot$$

- very **lightweight**, can typically resolve rather trivial cases
- useful for **C**, useless for Java (initialization requirement + GC)

Pointer non relational abstraction: points-to sets

Determine where a pointer may store a reference to

```
1 :  int x, y;
2 :  int * p;
3 :  y = 9;
4 :  p = &x;
5 :  *p = 0;
```

- what is the final value for x ?
  0, since **it is modified at line 5**...
- what is the final value for y ?
  9, since **it is not modified at line 5**...

Basic pointer abstraction

- We assume a set of **abstract memory locations** $\mathbb{A}^\sharp$ is fixed:
$$\mathbb{A}^\sharp = \{\&\mathrm{x}, \&\mathrm{y}, \ldots, \&\mathrm{t}, a_0, a_1, \ldots, a_N\}$$
- **Concrete addresses** are **abstracted into** $\mathbb{A}^\sharp$ by $\phi_{\mathbb{A}} : \mathbb{A} \to \mathbb{A}^\sharp \uplus \{\top\}$
- A pointer value is abstracted by the abstraction of the addresses it may point to, *i.e.*, $\mathbb{D}^\sharp_{\mathrm{ptr}} = \mathcal{P}(\mathbb{A}^\sharp)$
  and $\gamma_{\mathrm{ptr}}(a^\sharp) = \{a \in \mathbb{A} \mid \phi_{\mathbb{A}}(a) = a^\sharp\}$

- **example:** p may point to $\{\&\mathrm{x}\}$

Points-to sets computation example

Example code:

```
1 :   int x, y;
2 :   int * p;
3 :   y = 9;
4 :   p = &x;
5 :   *p = 0;
6 :   ...
```

Abstract locations: $\{\&x, \&y, \&p\}$

Analysis results:

	&x	&y	&p
1	$\top$	$\top$	$\top$
2	$\top$	$\top$	$\top$
3	$\top$	$\top$	$\top$
4	$\top$	$[9, 9]$	$\top$
5	$\top$	$[9, 9]$	$\{\&x\}$
6	$[0, 0]$	$[9, 9]$	$\{\&x\}$

Points-to sets computation and imprecision

```
   x ∈ [−10, −5]; y ∈ [5, 10]
1 : int ∗ p;
2 : if(?){
3 :         p = &x;
4 : } else {
5 :         p = &y;
6 : }
7 : ∗p = 0;
8 : ...
```

	&x	&y	&p
1	$[-10, -5]$	$[5, 10]$	$\top$
2	$[-10, -5]$	$[5, 10]$	$\top$
3	$[-10, -5]$	$[5, 10]$	$\top$
4	$[-10, -5]$	$[5, 10]$	$\{\&x\}$
5	$[-10, -5]$	$[5, 10]$	$\top$
6	$[-10, -5]$	$[5, 10]$	$\{\&y\}$
7	$[-10, -5]$	$[5, 10]$	$\{\&x, \&y\}$
8	$[-10, 0]$	$[0, 10]$	$\{\&x, \&y\}$

- What is the final range for x ?
- What is the final range for y ?

Abstract locations: $\{\&x, \&y, \&p\}$

Imprecise results

- The abstract information about both x and y are weakened
- The fact that $x \neq y$ is lost

Weak-updates

We can formalize this imprecision a bit more:

Weak updates

- **The modified concrete cell cannot be uniquely mapped into a well identified abstract cell that describes only it**
- The resulting abstract information is obtained by **joining the new value and the old information**

Effect in pointer analysis, in the case of an **assignment**:

- if the points-to set contains **exactly one element**, the analysis can perform a **strong update**
  as in the first example: $\mathrm{p} \Mapsto \{\&\mathrm{x}\}$
- if the points-to set may contain **more than one element**, the analysis needs to perform a **weak-update**
  as in the second example: $\mathrm{p} \Mapsto \{\&\mathrm{x}, \&\mathrm{y}\}$

Pointer aliasing based on equivalence on access paths

Aliasing relation

Given $m = (e, h)$, pointers p and q are **aliases** iff $h(e(\mathrm{p})) = h(e(\mathrm{q}))$

Abstraction to infer pointer aliasing properties

- An **access path** describes a sequence of dereferences to resolve an l-value (*i.e.*, an address); *e.g.*:

$$a ::= \mathtt{x} \mid a \cdot f \mid * a$$

- An **abstraction for aliasing** is an over-approximation for **equivalence relations** over access paths

Examples of aliasing abstractions:

- **set abstractions**: map from access paths to their equivalence class (**ex:** $\{\{\mathtt{p}_0, \mathtt{p}_1, \&\mathtt{x}\}, \{\mathtt{p}_2, \mathtt{p}_3\}, \ldots\}$)
- **numerical relations**, to describe aliasing among paths of the form $\mathtt{x(\text{->}n)}^{\mathtt{k}}$ (**ex:** $\{\{\mathtt{x(\text{->}n)}^{\mathtt{k}}, \&(\mathtt{x(\text{->}n)}^{\mathtt{k+1}}) \mid k \in \mathbb{N}\}$)

Limitation of basic pointer analyses seen so far

Weak updates:

- **imprecision in updates** that spread out as soon as points-to set contain several elements
- impact **client analyses** severely (*e.g.*, low precision on numerical)

Unsatisfactory abstraction of unbounded memory:

- common assumption that $\mathbb{C}^\sharp$ **be finite**
- programs using **dynamic allocations** often perform **unbounded** numbers of **malloc** calls (*e.g.*, allocation of a list)

Unable to express well structural invariants:

- for instance, that a structure should be a **list**, a **tree**...
- **very indirect** abstraction in numeric / path equivalence abstration

A common solution:
shape abstraction

Outline

Separation logic principle: avoid weak updates

How to deal with weak updates ?

Avoid them !

- Always materialize exactly the cell that needs be modified
- Can be very costly to achieve, and not always feasible

- Notion of property that holds **over a memory region**:
  special separating conjunction operator $*$
- **Local reasoning**:
  powerful principle, which allows to consider only part of the memory
- Separation logic has been used in **many contexts**, including **manual verification**, **static analysis**, etc...

Separation logic

Two kinds of formulas:

- **pure formulas** behave like formulas in first-order logic
  i.e., are not attached to a memory region
- **spatial formulas** describe properties attached to a memory region

Pure formulas denote value properties

$$
\begin{array}{rclll}
\mathtt{e} & ::= & n & (n \in \mathbb{N}) & \text{constants} \\
& | & \mathtt{l} & & \text{l-value} \\
& | & \mathtt{e}_0 + \mathtt{e}_1 & & \text{binary operations} \\
& | & \ldots & & \\
\mathtt{P} & ::= & \mathtt{e}_0 = \mathtt{e}_1 \mid \mathtt{P}' \vee \mathtt{P}'' \mid \mathtt{P}' \wedge \mathtt{P}'' \ldots & & \text{pure predicates}
\end{array}
$$

Pure formulas semantics: $\gamma(\mathrm{P}) \subseteq \mathbb{E} \times \mathbb{M}$

Separation logic: points-to predicates

The next slides introduce the main **separation logic formulas** $\mathtt{F} ::= \ldots$

We start with the most basic predicate, that **describes a single cell**:

Points-to predicate

- **Predicate:**

$$\mathtt{F} ::= \ldots \mid a \mapsto \mathtt{v} \qquad \text{where } a \text{ is an address and } \mathtt{v} \text{ is a value}$$

- **Concretization:**

$$(e, h) \in \gamma(\mathtt{l} \mapsto \mathtt{v}) \quad \text{if and only if} \quad h = [[\![\mathtt{l}]\!](e, h) \mapsto \mathtt{v}]$$

- **Example:**

$$\mathtt{F} = \&\mathtt{x} \mapsto 18 \qquad \&\mathtt{x} = 308 \; \boxed{18}$$

- We also note $\mathtt{l} \mapsto \mathtt{e}$, as an l-value $\mathtt{l}$ denotes an address

Separation logic: separating conjunction

Merge of concrete heaps: let $h_0, h_1 \in (\mathbb{V}_{\mathrm{addr}} \rightarrow \mathbb{V})$, such that $\mathbf{dom}(h_0) \cap \mathbf{dom}(h_1) = \emptyset$; then, we let $h_0 \circledast h_1$ be defined by:

$$h_0 \circledast h_1 : \begin{array}{lcl} \mathbf{dom}(h_0) \cup \mathbf{dom}(h_1) & \longrightarrow & \mathbb{V} \\ x \in \mathbf{dom}(h_0) & \longmapsto & h_0(x) \\ x \in \mathbf{dom}(h_1) & \longmapsto & h_1(x) \end{array}$$

Separating conjunction

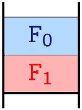
- **Predicate:**

$$\mathtt{F} ::= \ldots \mid \mathtt{F}_0 * \mathtt{F}_1$$

- **Concretization:**

$$\gamma(\mathtt{F}_0 * \mathtt{F}_1) = \{(e, h_0 \circledast h_1) \mid (e, h_0) \in \gamma(\mathtt{F}_0) \wedge (e, h_1) \in \gamma(\mathtt{F}_1)\}$$

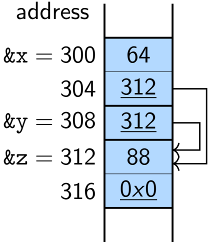
$\mathtt{F}_0 * \mathtt{F}_1$



An example

Concrete memory layout

(pointer values underlined)



$$
\begin{array}{llcl}
e: & \mathtt{x} & \mapsto & 300 \\
 & \mathtt{y} & \mapsto & 308 \\
 & \mathtt{z} & \mapsto & 312 \\
 & & & \\
\hbar: & 300 & \mapsto & 64 \\
 & 304 & \mapsto & 312 \\
 & 308 & \mapsto & 312 \\
 & 312 & \mapsto & 88 \\
 & 316 & \mapsto & 0
\end{array}
$$

A formula that abstracts away the addresses:

$$\&x \mapsto \langle 64, \&z \rangle * \&y \mapsto \&z * \&z \mapsto \langle 88, 0 \rangle$$

Separation logic: non separating conjunction

We can also add the **conventional conjunction operator**, with its **usual concretization**:

Non separating conjunction

- **Predicate:**

$$\mathtt{F} ::= \ldots \mid \mathtt{F}_0 \wedge \mathtt{F}_1$$

- **Concretization:**

$$\gamma(\mathtt{F}_0 \wedge \mathtt{F}_1) = \gamma(\mathtt{F}_0) \cap \gamma(\mathtt{F}_1)$$

Exercise: **describe** and **compare** the concretizations of

- $\&\mathtt{a} \mapsto \&\mathtt{b} \wedge \&\mathtt{b} \mapsto \&\mathtt{a}$
- $\&\mathtt{a} \mapsto \&\mathtt{b} * \&\mathtt{b} \mapsto \&\mathtt{a}$

Separating conjunction vs non separating conjunction

- **Classical conjunction**: properties for the same memory region
- **Separating conjunction**: properties for **disjoint** memory regions

$\&a \mapsto \&b \wedge \&b \mapsto \&a$

- the same heap verifies $\&a \mapsto \&b$ and $\&b \mapsto \&a$
- there can be only **one cell**
- thus $a = b$

$\&a \mapsto \&b * \&b \mapsto \&a$

- two **separate** sub-heaps respectively satisfy $\&a \mapsto \&b$ and $\&b \mapsto \&a$
- thus $a \neq b$

- Separating conjunction and non-separating conjunction have **very different properties**
- Both **express very different properties**
  e.g., no ambiguity on weak / strong updates

Separating and non separating conjunction

Logic rules of the two conjunction operators of SL:

- **Separating conjunction:**

$$\frac{(e, h_0) \in \gamma(\mathtt{F}_0) \qquad (e, h_1) \in \gamma(\mathtt{F}_1)}{(e, h_0 \circledast h_1) \in \gamma(\mathtt{F}_0 * \mathtt{F}_1)}$$

- **Non separating conjunction:**

$$\frac{(e, h) \in \gamma(\mathtt{F}_0) \qquad (e, h) \in \gamma(\mathtt{F}_1)}{(e, h) \in \gamma(\mathtt{F}_0 \wedge \mathtt{F}_1)}$$

Reminiscent of Linear Logic [Girard87]:
resource aware / non resource aware conjunction operators

Separation logic: empty store

Empty store

- **Predicate:**

$$\mathtt{F} ::= \ldots \mid \mathbf{emp}$$

- **Concretization:**

$$\gamma(\mathbf{emp}) = \{(e, []) \mid e \in \mathbb{E}\} = \mathbb{E} \times \{[]\}$$

where [] denotes the empty store

- **emp** is the **neutral element for** $*$ (monoid structure induced by $*$)
- by contrast the **neutral element for** $\wedge$ is TRUE, with concretization:

$$\gamma(\mathtt{TRUE}) = \mathbb{E} \times \mathbb{H}$$

Separation logic: other connectors

Disjunction:

- $\mathtt{F} ::= \ldots \mid \mathtt{F}_0 \vee \mathtt{F}_1$
- concretization:

$$\gamma(\mathtt{F}_0 \vee \mathtt{F}_1) = \gamma(\mathtt{F}_0) \cup \gamma(\mathtt{F}_1)$$

Spatial implication (*aka*, magic wand):

- $\mathtt{F} ::= \ldots \mid \mathtt{F}_0 \mathrel{-\!\!*} \mathtt{F}_1$
- concretization:

$$\gamma(\mathtt{F}_0 \mathrel{-\!\!*} \mathtt{F}_1) = \\ \{(e, h) \mid \forall h_0 \in \mathbb{H},\ (e, h_0) \in \gamma(\mathtt{F}_0) \Longrightarrow (e, h \circledast h_0) \in \gamma(\mathtt{F}_1)\}$$

- very powerful connector to describe **structure segments**, used in complex SL proofs

Separation logic

Summary of the main separation logic constructions seen so far:

Separation logic main connectors

$$
\begin{array}{rcl}
\gamma(\mathbf{emp}) & = & \mathbb{E} \times \{[]\} \\
\gamma(\mathtt{TRUE}) & = & \mathbb{E} \times \mathbb{H} \\
\gamma(\mathtt{l} \mapsto \mathtt{v}) & = & \{(e, [[\![\mathtt{l}]\!](e, h) \mapsto \mathtt{v}]) \mid e \in \mathbb{E}\} \\
\gamma(\mathtt{F}_0 * \mathtt{F}_1) & = & \{(e, h_0 \circledast h_1) \mid (e, h_0) \in \gamma(\mathtt{F}_0) \wedge (e, h_1) \in \gamma(\mathtt{F}_1)\} \\
\gamma(\mathtt{F}_0 \wedge \mathtt{F}_1) & = & \gamma(\mathtt{F}_0) \cap \gamma(\mathtt{F}_1) \\
\gamma(\mathtt{F}_0 \mathrel{-\!\!*} \mathtt{F}_1) & = & \{(e, h) \mid \forall h_0 \in \mathbb{H},\, (e, h_0) \in \gamma(\mathtt{F}_0) \Longrightarrow (e, h \circledast h_0) \in \gamma(\mathtt{F}_1)\}
\end{array}
$$

Concretization of pure formulas is standard

How does this help for program reasoning ?

Separation logic triple

Program proofs based on Hoare triples

- **Notation**: $\{\mathtt{F}\}p\{\mathtt{F}'\}$ if and only if:
$$\forall s, s' \in \mathbb{S},\ s \in \gamma(\mathtt{F}) \wedge s' \in [\![p]\!](s) \Longrightarrow s' \in \gamma(\mathtt{F}')$$
- Application: **formalize proofs of programs**

A few rules (straightforward proofs):

$$\frac{\mathtt{F}_0 \Longrightarrow \mathtt{F}'_0 \qquad \{\mathtt{F}'_0\}\mathtt{b}\{\mathtt{F}'_1\} \qquad \mathtt{F}'_1 \Longrightarrow \mathtt{F}_1}{\{\mathtt{F}_0\}\mathtt{b}\{\mathtt{F}_1\}}\ \text{consequence}$$

$$\frac{}{\{\&\mathtt{x} \mapsto ?\}\mathtt{x} := \mathtt{e}\{\&\mathtt{x} \mapsto \mathtt{e}\}}\ \text{mutation}$$

$$\frac{\mathtt{x}\ \text{does not appear in}\ \mathtt{F}}{\{\&\mathtt{x} \mapsto ? * \mathtt{F}\}\mathtt{x} := \mathtt{e}\{\&\mathtt{x} \mapsto \mathtt{e} * \mathtt{F}\}}\ \text{mutation-2}$$

(we assume that e does not allocate memory)

The frame rule

What about the resemblance between rules "mutation" and "mutation-2" ?

Theorem: the frame rule

$$\frac{\{\mathtt{F_0}\}\mathtt{b}\{\mathtt{F_1}\} \qquad \mathtt{freevar}(\mathtt{F}) \cap \mathtt{write}(\mathtt{b}) = \emptyset}{\{\mathtt{F_0} * \mathtt{F}\}\mathtt{b}\{\mathtt{F_1} * \mathtt{F}\}} \text{ frame}$$

- Proof by induction on the logical rules on program statements, *i.e.*, essentially a large case analysis
  (see biblio for a more complete set of rules)
- Rules are proved by case analysis on the program syntax

The frame rule allows to reason locally about programs

Application of the frame rule

A program with intermittent invariants, derived using **the frame rule**, since each step **impacts a disjoint region**:

$$
\begin{array}{l}
\quad \textbf{int}\ \mathtt{i};\\
\quad \textbf{int} * \mathtt{x};\\
\quad \textbf{int} * \mathtt{y};\\
\{\&\mathtt{i} \mapsto ? * \&\mathtt{x} \mapsto ? * \&\mathtt{y} \mapsto ?\}\\
\quad \mathtt{x} = \&\mathtt{i};\\
\{\&\mathtt{i} \mapsto ? * \&\mathtt{x} \mapsto \&\mathtt{i} * \&\mathtt{y} \mapsto ?\}\\
\quad \mathtt{y} = \&\mathtt{i};\\
\{\&\mathtt{i} \mapsto ? * \&\mathtt{x} \mapsto \&\mathtt{i} * \&\mathtt{y} \mapsto \&\mathtt{i}\}\\
\quad * \mathtt{x} = 42;\\
\{\&\mathtt{i} \mapsto 42 * \&\mathtt{x} \mapsto \&\mathtt{i} * \&\mathtt{y} \mapsto \&\mathtt{i}\}
\end{array}
$$

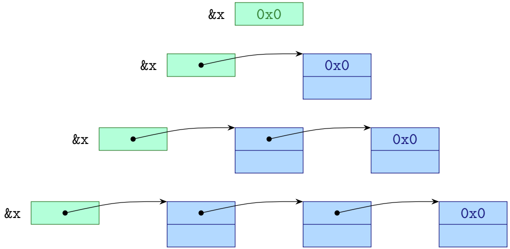
Many other program proofs done using separation logic
e.g., verification of the Deutsch-Shorr-Waite algorithm (biblio)

Summarization and inductive definitions

What do we still miss ?

So far, formulas denote **fixed sets of cells**
Thus, no summarization of unbounded regions...

- **Example** all lists pointed to by x, such as:



- How to precisely abstract these stores with **a single formula**
  i.e., no infinite disjunction ?

Inductive definitions in separation logic

List definition

$$\begin{array}{rcl} \alpha \cdot \textbf{list} & := & \alpha = 0 \wedge \textbf{emp} \\ & \vee & \alpha \neq 0 \wedge \alpha \cdot \mathtt{next} \mapsto \delta * \alpha \cdot \mathtt{data} \mapsto \beta * \delta \cdot \textbf{list} \end{array}$$

- Formula abstracting our set of structures:

$$\&\mathtt{x} \mapsto \alpha * \alpha \cdot \textbf{list}$$

- **Summarization**:
  this formula is finite and describe infinitely many heaps
- **Concretization**: next slide...

Practical implementation in verification/analysis tools

- **Verification:** hand-written definitions
- **Analysis:** either built-in or user-supplied, or partly inferred

Concretization by unfolding

Intuitive semantics of inductive predicates

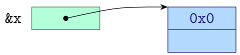
- Inductive predicates can be **unfolded**, by **unrolling their definitions**
  Syntactic unfolding is noted $\xrightarrow{\mathcal{U}}$
- A formula $\mathtt{F}$ with inductive predicates describes all stores described by all formulas $\mathtt{F}'$ such that $\mathtt{F} \xrightarrow{\mathcal{U}} \mathtt{F}'$

Example:

- Let us start with $\mathtt{x} \mapsto \alpha_0 * \alpha_0 \cdot \mathbf{list}$; we can unfold it as follows:

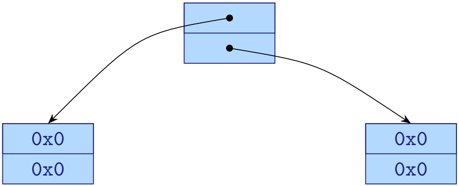
$$\begin{array}{ll} \multicolumn{2}{l}{\&\mathtt{x} \mapsto \alpha_0 * \alpha_0 \cdot \mathbf{list}} \\ \xrightarrow{\mathcal{U}} & \&\mathtt{x} \mapsto \alpha_0 * \alpha_0 \cdot \mathtt{next} \mapsto \alpha_1 * \alpha_0 \cdot \mathtt{data} \mapsto \beta_1 * \alpha_1 \cdot \mathbf{list} \\ \xrightarrow{\mathcal{U}} & \&\mathtt{x} \mapsto \alpha_0 * \alpha_0 \cdot \mathtt{next} \mapsto \alpha_1 * \alpha_0 \cdot \mathtt{data} \mapsto \beta_1 * \mathbf{emp} \wedge \alpha_1 = \mathbf{0x0} \end{array}$$

- We get the concrete state below:



Example: tree

- **Example:**



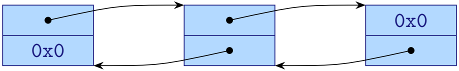
Inductive definition

- **Two recursive calls** instead of one:

$$
\begin{array}{rcll}
\alpha \cdot \textbf{tree} & := & & \alpha = 0 \land \textbf{emp} \\
& & \lor & \alpha \neq 0 \land \alpha \cdot \texttt{left} \mapsto \beta * \alpha \cdot \texttt{right} \mapsto \delta \\
& & & * \, \beta \cdot \textbf{tree} * \delta \cdot \textbf{tree}
\end{array}
$$

Example: doubly linked list

- **Example:**



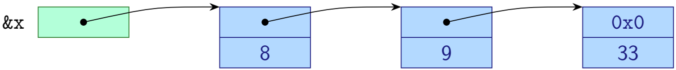
Inductive definition

- We need to propagate the `prev` pointer as an additional parameter:

$$\begin{array}{rcll} \alpha \cdot \mathbf{dll}(\delta) & := & & \alpha = 0 \wedge \mathbf{emp} \\ & & \vee & \alpha \neq 0 \wedge \alpha \cdot \mathtt{next} \mapsto \beta * \alpha \cdot \mathtt{prev} \mapsto \delta \\ & & & * \, \beta \cdot \mathbf{dll}(\alpha) \end{array}$$

Example: sortedness

- **Example:** sorted list



Inductive definition

- Each element should be greater than the previous one
- The first element simply needs be greater than $-\infty$...
- We need to propagate **the lower bound**, using a scalar parameter

$$
\begin{array}{rcl}
\alpha \cdot \mathbf{lsort}_{\mathrm{aux}}(n) & := & \quad \alpha = 0 \, \wedge \, \mathbf{emp} \\
 & \vee & \quad \alpha \neq 0 \, \wedge \, n \leq \beta \, \wedge \, \alpha \cdot \mathtt{next} \mapsto \delta \\
 & & \quad * \, \alpha \cdot \mathtt{data} \mapsto \beta * \delta \cdot \mathbf{lsort}_{\mathrm{aux}}(\beta)
\end{array}
$$

$$
\alpha \cdot \mathbf{lsort}() \quad := \quad \alpha \cdot \mathbf{lsort}_{\mathrm{aux}}(-\infty)
$$

Outline

Design of an abstract domain

A lot of things are missing to turn SL into an abstract domain

Set of logical predicates:

- separation logic formulas are **very expressive**
  e.g., arbitrary **alternations** of $\wedge$ and $*$
- such expressiveness is not necessarily required in static analysis

Representation:

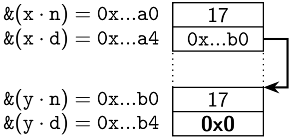
- unstructured formulas can be represented as **ASTs**,
  but this representation is **not easy to manipulate efficiently**
- intuition over memory states typically involves **graphs**

Analysis algorithms:

- inference of "optimal" invariants in SL, with numerical predicates obviously **not computable**

Basic abstraction: structures and their contents (1/2)

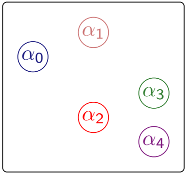
- **Concrete memory states**
  - very **low level** description
    numeric offsets / field names
  - pointers, numeric values:
    raw sequences of bits

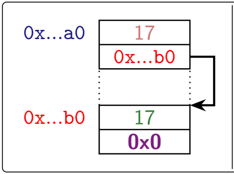


Basic abstraction: structures and their contents (1/2)

- **Concrete memory states**

- **Abstraction of values into symbolic variables** (nodes)



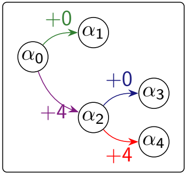


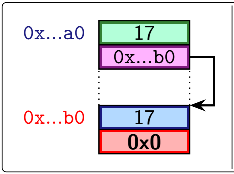
$$\begin{aligned}\nu(\alpha_0) &= \texttt{0x...a0}\\ \nu(\alpha_1) &= 17\\ \nu(\alpha_2) &= \texttt{0x...b0}\\ \nu(\alpha_3) &= 17\\ \nu(\alpha_4) &= \mathbf{0x0}\end{aligned}$$

  - characterized by **valuation** ν
  - ν maps **symbolic variables** into **concrete addresses**

Basic abstraction: structures and their contents (1/2)

- **Concrete memory states**
- **Abstraction of values into symbolic variables** / nodes
- **Abstraction of regions into points-to edges**

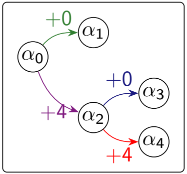


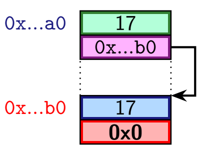


$\nu(\alpha_0) = \texttt{0x...a0}$
$\nu(\alpha_1) = 17$
$\nu(\alpha_2) = \texttt{0x...b0}$
$\nu(\alpha_3) = 17$
$\nu(\alpha_4) = \mathbf{0x0}$

Basic abstraction: structures and their contents (1/2)

- **Concrete memory states**
- **Abstraction of values into symbolic variables** / nodes
- **Abstraction of regions into points-to edges**





$$\nu(\alpha_0) = \texttt{0x...a0}$$
$$\nu(\alpha_1) = 17$$
$$\nu(\alpha_2) = \texttt{0x...b0}$$
$$\nu(\alpha_3) = 17$$
$$\nu(\alpha_4) = \mathbf{0x0}$$

- **Shape graph concretization**

$$\gamma_{\mathsf{sh}}(G) = \{(h, \nu) \mid \ldots\}$$

valuation ν plays an important role to combine abstraction...

Structure of shape graphs

Valuations bridge the gap between nodes and values

Symbolic variables / **nodes** and intuitively abstract concrete values:

Symbolic variables

We let $\mathbb{V}^\sharp$ denote a countable set of **symbolic variables**; we usually let them be denoted by Greek letters in the following: $\mathbb{V}^\sharp = \{\alpha, \beta, \delta, \ldots\}$

When concretizing a shape graph, we need to **characterize how the concrete instance evaluates each symbolic variable**, which is the purpose of the **valuation functions**:

Valuations

A **valuation** is a function from **symbolic variables** into **concrete values** (and is often denoted by ν): $\textbf{Val} = \mathbb{V}^\sharp \longrightarrow \mathbb{V}$

Note that valuations treat **in the same way addresses** and **raw values**

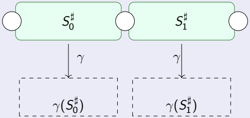
Structure of shape graphs

Distinct edges describe separate regions

In particular, if we **split** a graph into **two parts**:

Separating conjunction

$$\gamma_{\mathbf{sh}}(S_0^\sharp * S_1^\sharp) \quad = \quad \{(\hbar_0 \circledast \hbar_1, \nu) \mid (\hbar_0, \nu) \in \gamma_{\mathbf{sh}}(S_0^\sharp) \wedge (\hbar_1, \nu) \in \gamma_{\mathbf{sh}}(S_1^\sharp)\}$$



Similarly, when considering the **empty set of edges**, we get the empty heap (where $\mathbb{V}^\sharp$ is the set of nodes):

$$\gamma_{\mathbf{sh}}(\mathbf{emp}) = \{(\emptyset, \nu) \mid \nu : \mathbb{V}^\sharp \to \mathbb{V}\}$$

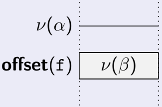
Abstraction of contiguous regions

A single points-to edge represents one heap cell

A **points-to edge** encodes **basic points to predicate in separation logic**:

Points-to edges

- **Syntax**

Graph edge	Separation logic formula	Concrete view
$\alpha \xrightarrow{\mathtt{f}} \beta$	$\alpha \cdot \mathtt{f} \mapsto \beta$	

- **Concretization**:

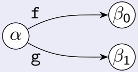
$$\gamma_{\mathsf{sh}}(\alpha \cdot \mathtt{f} \mapsto \beta) = \{([\nu(\alpha) + \mathbf{offset}(\mathtt{f}) \mapsto \nu(\beta)], \nu) \mid \nu : \{\alpha, \beta, \ldots\} \to \mathbb{N}\}$$

Abstraction of contiguous regions

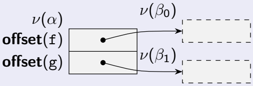
Contiguous regions are described by adjacent points-to edges

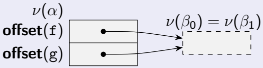
To describe **blocks** containing series of **cells** (*e.g.*, in a **C structure**), shape graphs utilize several outgoing edges from the node representing the base address of the block

Field splitting model

- Separation impacts edges / fields, *not pointers*
- Shape graph 

accounts for both abstract states below:





In other words, in a field splitting model, separation:

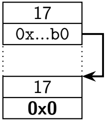
- asserts addresses are distinct
- says nothing about contents

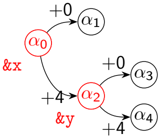
Abstraction of the environment

Environments bind variables to their (concrete / abstract) address

$\&\mathtt{x} = \&(\mathtt{x}\cdot\mathtt{n}) = \mathtt{0x...a0}$
$\&(\mathtt{x}\cdot\mathtt{d}) = \mathtt{0x...a4}$

$\&\mathtt{y} = \&(\mathtt{y}\cdot\mathtt{n}) = \mathtt{0x...b0}$
$\&(\mathtt{y}\cdot\mathtt{d}) = \mathtt{0x...b4}$





$$\begin{array}{llll} \nu: & \alpha_0 & \mapsto & \mathtt{0x...a0} \\ & \alpha_2 & \mapsto & \mathtt{0x...b0} \\ & \ldots & \mapsto & \ldots \end{array}$$

$$\begin{array}{lll} e^\sharp: & \mathtt{x} \mapsto \alpha_0 & (\overset{\nu}{\mapsto} \mathtt{0x...a0}) \\ & \mathtt{y} \mapsto \alpha_2 & (\overset{\nu}{\mapsto} \mathtt{0x...b0}) \end{array}$$

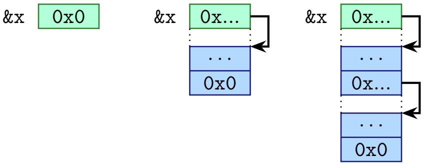
Abstract environments

- An **abstract environment** is a function $e^\sharp$ from **variables** to **symbolic nodes**
- The **concretization** extends as follows:

$$\gamma_{\mathbf{mem}}(e^\sharp, S^\sharp) = \{(e, h, \nu) \mid (h, \nu) \in \gamma_{\mathbf{sh}}(S^\sharp) \land e = \nu \circ e^\sharp\}$$

Basic abstraction: summarization

Set of all lists of any length:

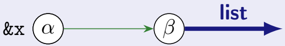


Well-founded list inductive def.

$$\begin{array}{rl} \alpha \cdot \textbf{list} := & \\ & (\textbf{emp} \wedge \alpha = \textbf{0x0}) \\ \vee & (\alpha \cdot \mathtt{d} \mapsto \beta_0 * \alpha \cdot \mathtt{n} \mapsto \beta_1 \\ & \quad * \beta_1 \cdot \textbf{list} \wedge \alpha \neq \textbf{0x0}) \end{array}$$

well-founded predicate

Inductive summary predicates



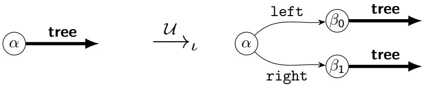
Concretization based on **unfolding** and **least-fixpoint**:

- $\xrightarrow{\mathcal{U}}$ replaces **an $\alpha \cdot$ list predicate** with **one of its premises**
- $\gamma(S^\sharp, \mathtt{F}) = \bigcup \{\gamma(S^\sharp_u, \mathtt{F}_u) \mid (S^\sharp, \mathtt{F}) \xrightarrow{\mathcal{U}} (S^\sharp_u, \mathtt{F}_u)\}$

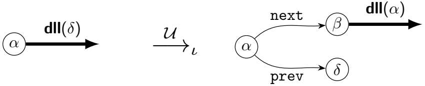
Inductive structures: a few instances

As before, **many interesting inductive predicates** encode nicely into graph inductive definitions:

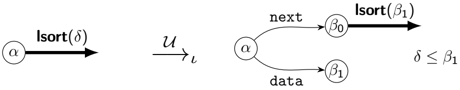
- **More complex shapes**: **trees**



- **Relations among pointers**: **doubly-linked lists**

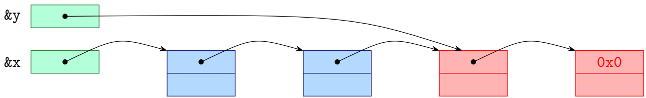


- **Relations between pointers and numerical**: **sorted lists**



Inductive segments

A frequent pattern:



A first attempt:

- $\mathtt{x}$ points to a list, so $\&\mathtt{x} \mapsto \alpha * \alpha \cdot \textbf{list}$ holds
- $\mathtt{y}$ points to a list, so $\&\mathtt{y} \mapsto \beta * \beta \cdot \textbf{list}$ holds

However, the following **does not hold**

$$\&\mathtt{x} \mapsto \alpha * \alpha \cdot \textbf{list} * \&\mathtt{y} \mapsto \beta * \beta \cdot \textbf{list}$$

Why ? **violation of separation!**

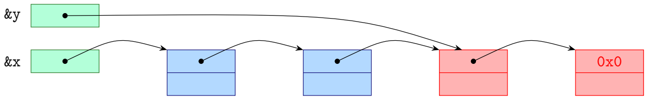
A second attempt:

$$(\&\mathtt{x} \mapsto \alpha * \alpha \cdot \textbf{list} * \mathtt{TRUE}) \wedge (\&\mathtt{y} \mapsto \beta * \beta \cdot \textbf{list} * \mathtt{TRUE})$$

Why is it still not all that good ? **relation lost!**

Inductive segments

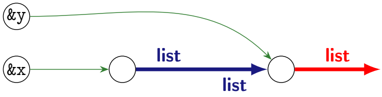
A frequent pattern:



Could be **expressed directly** as an inductive with a parameter:

$$\begin{array}{rcl} \alpha \cdot \textbf{list\_endp}(\pi) & ::= & (\textbf{emp}, \alpha = \pi) \\ & | & (\alpha \cdot \texttt{next} \mapsto \beta_0 * \alpha \cdot \texttt{data} \mapsto \beta_1 \\ & & \quad * \beta_0 \cdot \textbf{list\_endp}(\pi), \alpha \neq 0) \end{array}$$

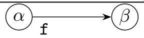
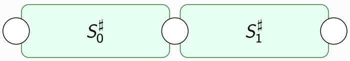
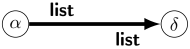
This definition **straightforwardly derives** from **list**
Thus, we make **segments** part of the **fundamental predicates of the domain**



Multi-segments: possible, but harder for analysis

Shape graphs and separation logic

Semantic preserving translation Π of graphs into separation logic formulas:

Graph $S^\sharp \in \mathbb{D}^\sharp_{\mathbf{sh}}$	Translated formula $\Pi(S^\sharp)$
	$\alpha \cdot \mathtt{f} \mapsto \beta$
	$\Pi(S^\sharp_0) * \Pi(S^\sharp_1)$
	$\alpha \cdot \mathbf{list}$
	$\alpha \cdot \mathbf{list\_endp}(\delta)$
other inductives and segments	similar

Note that:

- **shape graphs can be encoded into separation logic formula**
- **the opposite is usually not true**

Value information:

- discussed in the next course
- intuitively, assume we maintain numerical information next to shape graphs

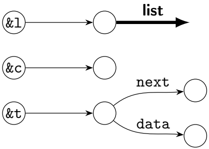
Outline

Static analysis overview

A list insertion function:

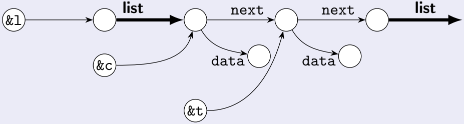
```
list * l assumed to point to a list
list * t assumed to point to a list element
list * c = l;
while(c != NULL && c -> next != NULL && (...)){
    c = c -> next;
}
t -> next = c -> next;
c -> next = t;
```

- **list** inductive structure def.
- Abstract precondition:



Result of the (interprocedural) analysis

- **Over-approximations** of reachable concrete states
  e.g., after the insertion:



Transfer functions

Abstract interpreter design

- **Follows the semantics** of the language under consideration
- The abstract domain should provide **sound transfer functions**

Transfer functions:

- **Assignment:** $\mathrm{x} \to f = \mathrm{y} \to g$ or $\mathrm{x} \to f = e_{\mathrm{arith}}$
- **Test:** analysis of conditions (if, while)
- Variable **creation** and **removal**
- **Memory management:** **malloc**, **free**

Abstract operators:

- **Join** and **widening**: over-approximation
- **Inclusion checking**: check stabilization of abstract iterates

Should be **sound** *i.e.*, not forget any concrete behavior

Abstract operations

Denotational style abstract interpreter

- Concrete **denotational semantics** $[\![\mathtt{b}]\!] : \mathbb{S} \longrightarrow \mathcal{P}(\mathbb{S})$
- **Abstract post-condition** $[\![\mathtt{b}]\!]^\sharp(\mathbf{S})$, computed by the analysis:

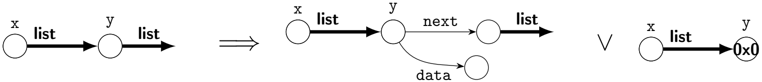
$$s \in \gamma(\mathbf{S}) \Longrightarrow [\![\mathtt{b}]\!](s) \subseteq \gamma([\![\mathtt{b}]\!]^\sharp(\mathbf{S}))$$

Analysis by induction on the syntax using **domain operators**

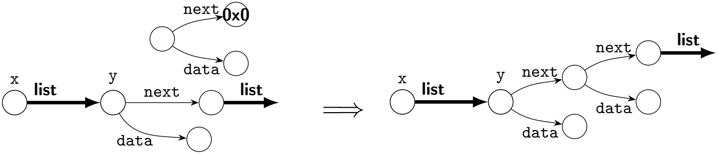
$$\begin{array}{rcl}
[\![\mathtt{b}_0;\mathtt{b}_1]\!]^\sharp(\mathbf{S}) & = & [\![\mathtt{b}_1]\!]^\sharp \circ [\![\mathtt{b}_0]\!]^\sharp(\mathbf{S}) \\
[\![\mathtt{l} = \mathtt{e}]\!]^\sharp(\mathbf{S}) & = & \mathit{assign}(\mathtt{l}, \mathtt{e}, \mathbf{S}) \\
[\![\mathtt{l} = \mathbf{malloc}(n)]\!]^\sharp(\mathbf{S}) & = & \mathit{alloc}(\mathtt{l}, n, \mathbf{S}) \\
[\![\mathbf{free}(\mathtt{l})]\!]^\sharp(\mathbf{S}) & = & \mathit{free}(\mathtt{l}, n, \mathbf{S}) \\
[\![\mathbf{if}(\mathtt{e})\ \mathtt{b}_t\ \mathbf{else}\ \mathtt{b}_f]\!]^\sharp(\mathbf{S}) & = & \left\{ \begin{array}{l} \mathit{join}([\![\mathtt{b}_t]\!]^\sharp(\mathit{test}(\mathtt{e}, \mathbf{S})), \\ \qquad\qquad [\![\mathtt{b}_f]\!]^\sharp(\mathit{test}(\mathtt{e} = \mathbf{false}, \mathbf{S}))) \end{array} \right. \\
[\![\mathbf{while}(\mathtt{e})\mathtt{b}]\!]^\sharp(\mathbf{S}) & = & \mathit{test}(\mathtt{e} = \mathbf{false}, \mathbf{lfp}^\sharp{}_{\mathbf{S}} F^\sharp) \\
\multicolumn{3}{c}{\text{where, } F^\sharp : \mathbf{S}_0 \mapsto [\![\mathtt{b}]\!]^\sharp(\mathit{test}(\mathtt{e}, \mathbf{S}_0))}
\end{array}$$

The algorithms underlying the transfer functions

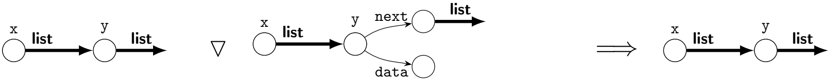
- **Unfolding**: **cases analysis on summaries**



- **Abstract postconditions**, on **"exact"** regions, *e.g.* **insertion**



- **Widening**: **builds summaries** and ensures **termination**



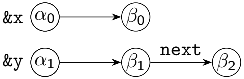
Outline

Analysis of an assignment in the graph domain

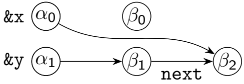
Steps for analyzing $\mathtt{x} = \mathtt{y} \mathbin{\texttt{->}} \mathtt{next}$ (local reasoning)

1. Evaluate **l-value** `x` into **points-to edge** $\alpha \mapsto \beta$
2. Evaluate **r-value** `y -> next` into **node** β'
3. Replace points-to edge $\alpha \mapsto \beta$ with **points-to edge** $\alpha \mapsto \beta'$

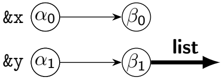
With pre-condition:



- Step 1 produces $\alpha_0 \mapsto \beta_0$
- Step 2 produces β_2
- End result:



With pre-condition:



- Step 1 produces $\alpha_0 \mapsto \beta_0$
- **Step 2 fails**

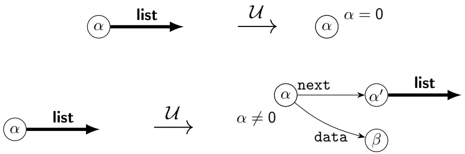
- Abstract state **too abstract**
- We need to **refine it**

Unfolding as a local case analysis

Unfolding principle

- **Case analysis**, based on the inductive definition
- Generates **symbolic disjunctions** (analysis performed in a **disjunction domain**, *e.g.*, trace partitioning)

- Example, for lists:



- **Numeric predicates:** next course on shape + value abstraction

Soundness: by definition of the concretization of inductive structures

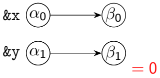
$$\gamma_{\mathsf{sh}}(S^\sharp) \subseteq \bigcup \{ \gamma_{\mathsf{sh}}(S^\sharp_0) \mid S^\sharp \xrightarrow{\mathcal{U}} S^\sharp_0 \}$$

Analysis of an assignment, with unfolding

Principle

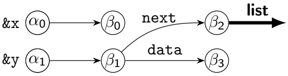
- We have $\gamma_{\mathbf{sh}}(\alpha \cdot \iota) = \bigcup \{\gamma_{\mathbf{sh}}(S^\sharp) \mid \alpha \cdot \iota \xrightarrow{\mathcal{U}} S^\sharp\}$
- Replace $\alpha \cdot \iota$ with a finite number of disjuncts and continue

Disjunct 1:

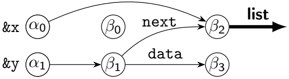


- Step 1 produces $\alpha_0 \mapsto \beta_0$
- **Step 2 fails: Null pointer !**
- In a **correct** program, would be ruled out by a **condition** $\mathtt{y} \neq 0$ *i.e.*, $\beta_1 \neq 0$ in $\mathbb{D}^\sharp_{\mathbf{num}}$

Disjunct 2:

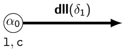


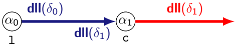
- Step 1 produces $\alpha_0 \mapsto \beta_0$
- Step 2 produces β_2
- **End result**:



Unfolding and degenerated cases

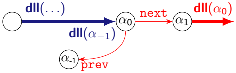
assume(`l points to a dll`)
$\texttt{c} = \texttt{l};$
① **while**($\texttt{c} \neq \texttt{NULL}$ && condition)
    $\texttt{c} = \texttt{c} \texttt{ -> } \texttt{next};$
② **if**($\texttt{c} \neq 0$ && $\texttt{c} \texttt{ -> } \texttt{prev} \neq 0$)
    $\texttt{c} = \texttt{c} \texttt{ -> } \texttt{prev} \rightarrow \texttt{prev};$

- at ①: 

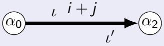
- at ②: 

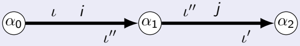
$\Rightarrow$ **non trivial unfolding**

- **Materialization of** `c -> prev`:

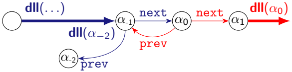


Segment splitting lemma: basis for segment unfolding



describes the same set of stores as


- **Materialization of** `c -> prev -> prev`:



- **Implementation issue**: discover **which inductive edge** to unfold
very hard !

Outline

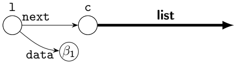
Need for a folding operation

Back to the **list traversal** example:

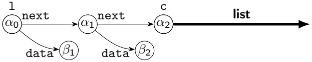
First iterates in the loop:

- at **iteration 0** (before entering the loop):

- at **iteration 1**:



- at **iteration 2**:




```
assume(l points to a list)
c = l;
while(c ≠ NULL){
  c = c → next;
}
```

The analysis **unfolds**, but **never folds**:

$$\begin{tikzcd} S_0 \arrow[d, "\text{unfold}"'] & & & \\ S_{0,u} \arrow[r, "f"] & S_1 \arrow[d, "\text{unfold}"'] & & \\ & S_{1,u} \arrow[r, "f"] & S_2 & \cdots \end{tikzcd}$$

- How to guarantee **termination** of the analysis ?
- How to **introduce segment edges** / perform **abstraction** ?

Widening

- The lattice of shape abstract values has **infinite height**
- Thus iteration sequences **may not terminate**

Definition of a widening operator ∇

- **Over-approximates join:**

$$\left\{ \begin{array}{rcl} \gamma(X^\sharp) & \subseteq & \gamma(X^\sharp \nabla Y^\sharp) \\ \gamma(Y^\sharp) & \subseteq & \gamma(X^\sharp \nabla Y^\sharp) \end{array} \right.$$

- **Enforces termination:** for all sequence $(X_n^\sharp)_{n\in\mathbb{N}}$, the **sequence $(Y_n^\sharp)_{n\in\mathbb{N}}$ defined below is ultimately stationary**

$$\left\{ \begin{array}{rcl} Y_0^\sharp & = & X_0^\sharp \\ \forall n \in \mathbb{N},\ Y_{n+1}^\sharp & = & Y_n^\sharp \nabla X_{n+1}^\sharp \end{array} \right.$$

Canonicalization

Upper closure operator

$\rho : \mathbb{D}^\sharp \longrightarrow \mathbb{D}^\sharp_{\mathbf{can}} \subseteq \mathbb{D}^\sharp$ is an **upper closure operator** (uco) iff it is monotone, extensive and idempotent.

Canonicalization

- **Disjunctive completion:** $\mathbb{D}^\sharp_\vee$ = finite disjunctions over $\mathbb{D}^\sharp$
- **Canonicalization operator** $\rho_\vee$ defined by $\rho_\vee : \mathbb{D}^\sharp_\vee \longrightarrow \mathbb{D}^\sharp_{\mathbf{can}\vee}$ and $\rho_\vee(X^\sharp) = \{\rho(x^\sharp) \mid x^\sharp \in X^\sharp\}$ where ρ is an uco and $\mathbb{D}^\sharp_{\mathbf{can}}$ is finite

- Canonicalization is used in **many shape analysis tools**
- **Easier to compute** but **less powerful** than widening: does not exploit history of computation

Weakening: definition

To design **inclusion test**, **join** and **widening** algorithms, we first study a more general notion of **weakening**:

Weakening

We say that $S_0^\sharp$ **can be weakened into** $S_1^\sharp$ if and only if

$$\forall(\hbar, \nu) \in \gamma_{\mathsf{sh}}(S_0^\sharp),\ \exists \nu' \in \mathbf{Val},\ (\hbar, \nu') \in \gamma_{\mathsf{sh}}(S_1^\sharp)$$

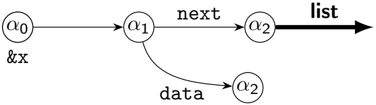
We then note $S_0^\sharp \preccurlyeq S_1^\sharp$

Applications:

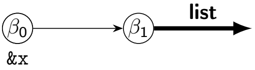
- **inclusion test** (comparison) inputs $S_0^\sharp, S_1^\sharp$; if returns true $S_0^\sharp \preccurlyeq S_1^\sharp$
- **canonicalization** (unary weakening) inputs $S_0^\sharp$ and returns $\rho(S_0^\sharp)$ such that $S_0^\sharp \preccurlyeq \rho(S_0^\sharp)$
- **widening** / **join** (binary weakening ensuring termination or not) inputs $S_0^\sharp, S_1^\sharp$ and returns $S_{\mathrm{up}}^\sharp$ such that $S_i^\sharp \preccurlyeq S_{\mathrm{up}}^\sharp$

Weakening: example

We consider $S_0^\sharp$ defined by:



and $S_1^\sharp$ defined by:



Then, we have **the weakening $S_0^\sharp \preccurlyeq S_1^\sharp$ up-to a renaming in $S_1^\sharp$**:

$$\Psi : \begin{array}{ccc} \beta_0 & \longmapsto & \alpha_0 \\ \beta_1 & \longmapsto & \alpha_1 \end{array}$$

- weakening **up-to renaming** is generally required as graphs do not have the same name space
- formalized a bit later...

Local weakening: separating conjunction rule

We can apply the local reasoning principle to weakening

If $S_0^\sharp \preccurlyeq S_{0,\mathbf{weak}}^\sharp$ and $S_1^\sharp \preccurlyeq S_{1,\mathbf{weak}}^\sharp$ then:

Separating conjunction rule ($\preccurlyeq_*$)

Let us assume that

- $S_0^\sharp$ and $S_1^\sharp$ have distinct set of **source nodes**
- we can weaken $S_0^\sharp$ into $S_{0,\mathbf{weak}}^\sharp$
- we can weaken $S_1^\sharp$ into $S_{1,\mathbf{weak}}^\sharp$

then:

we can weaken $S_0^\sharp * S_1^\sharp$ into $S_{0,\mathbf{weak}}^\sharp * S_{1,\mathbf{weak}}^\sharp$

Local weakening: unfolding rule, identity rule

Weakening unfolded region ($\preccurlyeq_{\mathcal{U}}$)

Let us assume that $S_0^\sharp \xrightarrow{\mathcal{U}} S_1^\sharp$. Then, by definition of the concretization of unfolding

we can weaken $S_1^\sharp$ into $S_0^\sharp$

- the proof follows from the definition of unfolding
- it can be applied locally, on graph regions that differ due to unfolding of inductive definitions

Identity weakening ($\preccurlyeq_{\mathbf{Id}}$)

we can weaken $S^\sharp$ into $S^\sharp$

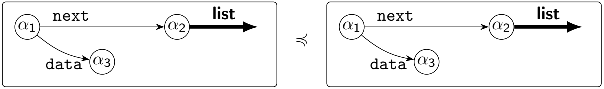
- the proof is trivial:

$$\gamma_{\mathbf{sh}}(S^\sharp) \subseteq \gamma_{\mathbf{sh}}(S^\sharp)$$

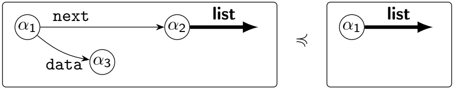
- on itself, this principle is not very useful, but it can be applied locally, and combined with ($\preccurlyeq_{\mathcal{U}}$) on graph regions that are not equal

Local weakening: example

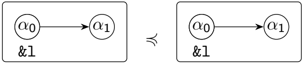
By **rule** ($\preccurlyeq_{\mathbf{Id}}$):



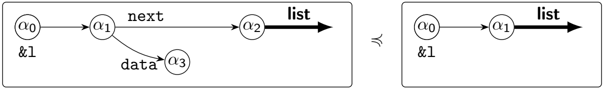
Thus, by **rule** ($\preccurlyeq_{\mathcal{U}}$):



Additionally, by **rule** ($\preccurlyeq_{\mathbf{Id}}$):



Thus, by **rule** ($\preccurlyeq_{*}$):



Inclusion checking rules in the shape domain

Graphs to compare have distinct sets of nodes, thus inclusion check should carry out a **valuation transformer** $\Psi : \mathbb{V}^\sharp(S_1^\sharp) \longrightarrow \mathbb{V}^\sharp(S_0^\sharp)$ (important when dealing also with content values)

Using (and extending) the weakening principles, we obtain the following rules (considering only inductive definition **list**, though these rules would extend to other definitions straightforwardly):

- **Identity rules:**

$$\begin{array}{rcrcl}
\forall i,\ \Psi(\beta_i) = \alpha_i & \Longrightarrow & \alpha_0 \cdot \mathtt{f} \mapsto \alpha_1 & \sqsubseteq^\sharp{}_\Psi & \beta_0 \cdot \mathtt{f} \mapsto \beta_1 \\
\Psi(\beta) = \alpha & \Longrightarrow & \alpha \cdot \mathbf{list} & \sqsubseteq^\sharp{}_\Psi & \beta \cdot \mathbf{list} \\
\forall i,\ \Psi(\beta_i) = \alpha_i & \Longrightarrow & \alpha_0 \cdot \mathbf{list_endp}(\alpha_1) & \sqsubseteq^\sharp{}_\Psi & \beta_0 \cdot \mathbf{list_endp}(\beta_1)
\end{array}$$

- **Rules on inductives:**

$$\begin{array}{rcrcl}
\forall i,\ \Psi(\beta_i) = \alpha & \Longrightarrow & \mathbf{emp} & \sqsubseteq^\sharp{}_\Psi & \beta_0 \cdot \mathbf{list_endp}(\beta_1) \\
S_0^\sharp \sqsubseteq^\sharp{}_\Psi S_1^\sharp \wedge \beta \cdot \iota \xrightarrow{\mathcal{U}} S_1^\sharp & \Longrightarrow & S_0^\sharp & \sqsubseteq^\sharp{}_\Psi & \beta \cdot \iota \\
\multicolumn{5}{l}{\text{if } \beta_1 \text{ fresh },\Psi' = \Psi[\beta_1 \mapsto \alpha_1] \text{ and } \Psi(\beta_0) = \alpha_0 \text{ then,}} \\
S_0^\sharp \sqsubseteq^\sharp{}_{\Psi'} \beta_1 \cdot \mathbf{list} & \Longrightarrow & \alpha_0 \cdot \mathbf{list_endp}(\alpha_1) * S_0^\sharp & \sqsubseteq^\sharp{}_\Psi & \beta_0 \cdot \iota
\end{array}$$

Inclusion checking algorithm

Comparison of $(e_0^\sharp, S_0^\sharp)$ and $(e_1^\sharp, S_1^\sharp)$

1. start with Ψ defined by $\Psi(\beta) = \alpha$ if and only if there exists a variable $\mathtt{x}$ such that $e_0^\sharp(\mathtt{x}) = \alpha \wedge e_1^\sharp(\mathtt{x}) = \beta$
2. iteratively **apply local rules**, and extend Ψ when needed
3. return **true** when both shape graphs become empty

- the first step ensures both environments are consistent

This algorithm is sound:

Soundness

$$(e_0^\sharp, S_0^\sharp) \sqsubseteq^\sharp (e_1^\sharp, S_1^\sharp) \Longrightarrow \gamma(e_0^\sharp, S_0^\sharp) \subseteq \gamma(e_1^\sharp, S_1^\sharp)$$

Over-approximation of union

The principle of join and widening algorithm **is similar to that of $\sqsubseteq^\sharp$:**

- It can be computed **region by region**, as for weakening in general:
  If $\forall i \in \{0, 1\},\ \forall s \in \{\mathrm{lft}, \mathrm{rgh}\},\ S^\sharp_{i,s} \preccurlyeq S^\sharp_s$,

  The partitioning of inputs / different nodes sets requires a **node correspondence function**

$$\Psi : \mathbb{V}^\sharp(S^\sharp_{\mathrm{lft}}) \times \mathbb{V}^\sharp(S^\sharp_{\mathrm{rgh}}) \longrightarrow \mathbb{V}^\sharp(S^\sharp)$$

- The computation of the shape join progresses by the application of **local join rules**, that produce a **new (output) shape graph, that weakens both inputs**

Over-approximation of union: syntactic identity rules

In the next few slides, we focus on $\triangledown$
though the abstract union would be defined similarly in the shape domain

Several rules derive **from** ($\preccurlyeq_{\mathbf{Id}}$):

- If $S^\sharp_{\mathrm{lft}} = \alpha_0 \cdot \mathtt{f} \mapsto \alpha_1$
  and $S^\sharp_{\mathrm{lft}} = \beta_0 \cdot \mathtt{f} \mapsto \beta_1$
  and $\Psi(\alpha_0, \beta_0) = \delta_0$, $\Psi(\alpha_1, \beta_1) = \delta_1$, then:

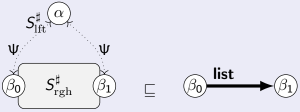
$$S^\sharp_{\mathrm{lft}} \triangledown S^\sharp_{\mathrm{rgh}} = \delta_0 \cdot \mathtt{f} \mapsto \delta_1$$

- If $S^\sharp_{\mathrm{lft}} = \alpha_0 \cdot \mathbf{list}$
  and $S^\sharp_{\mathrm{lft}} = \beta_0 \cdot \mathbf{list}_1$
  and $\Psi(\alpha_0, \beta_0) = \delta_0$, then:

$$S^\sharp_{\mathrm{lft}} \triangledown S^\sharp_{\mathrm{rgh}} = \delta_0 \cdot \mathbf{list}$$

Over-approximation of union: segment introduction rule

Rule

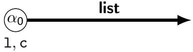
if 

then

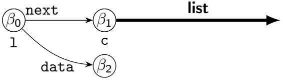
$$\begin{cases} S^\sharp_{\mathrm{lft}} \triangledown S^\sharp_{\mathrm{rgh}} = \delta_0 \xrightarrow{\textsf{list}} \delta_1 \\ (\alpha, \beta_0) \stackrel{\Psi}{\longleftrightarrow} \delta_0 \\ (\alpha, \beta_1) \stackrel{\Psi}{\longleftrightarrow} \delta_1 \end{cases}$$

Application to list traversal, at the end of iteration 1:

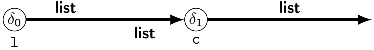
- **before iteration 0:**



- **end of iteration 0:**



- **join, before iteration 1:**

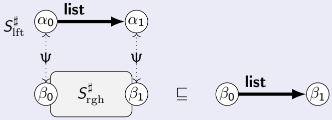


$$\begin{cases} \Psi(\alpha_0, \beta_0) = \delta_0 \\ \Psi(\alpha_0, \beta_1) = \delta_1 \end{cases}$$

Over-approximation of union: segment extension rule

Rule

if



then

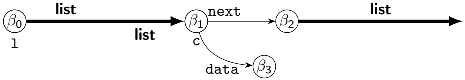
$$\left\{ \begin{array}{l} S^\sharp_{\mathrm{lft}} \triangledown S^\sharp_{\mathrm{rgh}} = \end{array} \right.$$

$$(\alpha_0, \beta_0) \xleftrightarrow{\Psi} \delta_0$$
$$(\alpha_1, \beta_1) \xleftrightarrow{\Psi} \delta_1$$

Application to list traversal, at the end of iteration 1:

- **previous invariant before iteration 1:**

- **end of iteration 1:**



- **join, before iteration 1:**

$$\left\{ \begin{array}{lcl} \Psi(\alpha_0, \beta_0) & = & \delta_0 \\ \Psi(\alpha_1, \beta_2) & = & \delta_1 \end{array} \right.$$

Over-approximation of union: rewrite system properties

- Comparison, canonicalization and widening algorithms can be considered **rewriting systems over tuples of graphs**
- **Success configuration**: weakening applies on all components, *i.e.*, the inputs are fully **"consumed"** in the weakening process
- **Failure configuration**: some components **cannot be weakened** *i.e.*, the algorithm should return the conservative answer (*i.e.*, $\top$)

Termination

- The systems are **terminating**
- This ensures comparison, canonicalization, widening are **computable**

Non confluence !

- The results depends on the order of application of the rules
- Implementation requires the choice of an **adequate strategy**

Over-approximation of union in the combined domain

Widening of $(e_0^\sharp, S_0^\sharp)$ and $(e_1^\sharp, S_1^\sharp)$

1. define Ψ, e by $\Psi(\alpha, \beta) = e(\mathrm{x}) = \delta$ (where δ is a fresh node) if and only if $e_0^\sharp(\mathrm{x}) = \alpha \wedge e_1^\sharp(\mathrm{x}) = \beta$
2. iteratively **apply join local rules**, and extend Ψ when new relations are inferred (for instance for points-to edges)
3. return the result obtained when all regions of both inputs are approximated in the output graph

This algorithm is sound:

Soundness

$$\gamma(e_0^\sharp, S_0^\sharp) \cup \gamma(e_1^\sharp, S_1^\sharp) \subseteq \gamma(e^\sharp, S^\sharp)$$

Widening also enforces **termination** (it only introduces segments, and the growth induced by the introduction of segments is bounded)

Outline

Assumptions

What assumptions do we make ?
How do we prove soundness of the analysis of a loop ?

- **Assumptions in the concrete level**, and for block $\mathtt{b}$:

$$\begin{array}{rl} (\mathcal{P}(\mathbb{M}), \subseteq) & \text{is a complete lattice, hence a CPO} \\ F : \mathcal{P}(\mathbb{M}) \to \mathcal{P}(\mathbb{M}) & \text{is the concrete semantic ("post") function of } \mathtt{b} \end{array}$$

  thus, the concrete semantics writes down as $[\![\mathtt{b}]\!] = \mathbf{lfp}_{\emptyset} F$

- **Assumptions in the abstract level:**

$$\begin{array}{rl} \mathbb{M}^\sharp & \text{set of abstract elements, no order a priori} \\ & m^\sharp ::= (e^\sharp, S^\sharp) \\ \gamma_{\mathrm{mem}} : \mathbb{M}^\sharp \to \mathcal{P}(\mathbb{M}) & \text{concretization} \\ F^\sharp : \mathbb{M}^\sharp \to \mathbb{M}^\sharp & \text{sound abstract semantic function} \\ & \textit{i.e.}\text{, such that } F \circ \gamma_{\mathrm{mem}} \subseteq \gamma_{\mathrm{mem}} \circ F^\sharp \\ \triangledown : \mathbb{M}^\sharp \times \mathbb{M}^\sharp \to \mathbb{M}^\sharp & \text{widening operator, terminates, and such that} \\ & \gamma_{\mathrm{mem}}(m_0^\sharp) \cup \gamma_{\mathrm{mem}}(m_1^\sharp) \subseteq \gamma_{\mathrm{mem}}(m_0^\sharp \triangledown m_1^\sharp) \end{array}$$

Computing a loop abstract post-condition

Loop abstract semantics

The abstract semantics of loop **while**(**rand**()){b} is calculated as the limit of the sequence of abstract iterates below:

$$\begin{cases} m_0^\sharp & = & \bot \\ m_{n+1}^\sharp & = & m_n^\sharp \,\triangledown\, F^\sharp(m_n^\sharp) \end{cases}$$

Soundness proof:

- by induction over n, $\bigcup_{k \leq n} F^k(\emptyset) \subseteq \gamma_{\mathrm{mem}}(m_n^\sharp)$
- by the property of widening, the abstract sequence converges at a rank N: $\forall k \geq N,\ m_k^\sharp = m_N^\sharp$, thus

$$\mathbf{lfp}_\emptyset F = \bigcup_k F^k(\emptyset) \subseteq \gamma_{\mathrm{mem}}(m_N^\sharp)$$

Discussion on the abstract ordering

How about the abstract ordering ? We assumed *NONE* so far...

- **Logical ordering**, induced by concretization, used for **proofs**

$$m_0^\sharp \sqsubseteq m_1^\sharp \quad ::= \quad \text{"}\gamma_{\mathrm{mem}}(m_0^\sharp) \subseteq \gamma_{\mathrm{mem}}(m_1^\sharp)\text{"}$$

- **Approximation of the logical ordering**, **implemented** as a function $\textbf{is\_le} : \mathbb{M}^\sharp \times \mathbb{M}^\sharp \to \{\textbf{true}, \top\}$, used to **test the convergence of abstract iterates**

$$\textbf{is\_le}(m_0^\sharp, m_1^\sharp) = \textbf{true} \quad \Longrightarrow \quad \gamma_{\mathrm{mem}}(m_0^\sharp) \subseteq \gamma_{\mathrm{mem}}(m_1^\sharp)$$

Abstract semantics is not assumed (and is actually most likely NOT) monotone with respect to either of these orders...

- Also, **computational ordering** would be used for **proving widening termination**

Outline

Updates and summarization

Weak updates cause significant precision loss...
Separation logic makes updates strong

Separation logic

Separating conjunction combines properties on disjoint stores

- Fundamental idea: $*$ **forces to identify what is modified**
- Before an **update** (or a **read**) takes place, memory cells need to be **materialized**
- **Local reasoning**: properties on unmodified cells pertain

Summaries

Inductive predicates describe unbounded memory regions

- Last lecture: **array segments** and **transitive closure** (TVLA)

Bibliography

- **[JR]**: **Separation Logic: A Logic for Shared Mutable Data Structures.**
  John C. Reynolds.
  In LICS'02, pages 55–74, 2002.

- **[DHY]**: **A Local Shape Analysis Based on Separation Logic.**
  Dino Distefano, Peter W. O'Hearn and Hongseok Yang.
  In TACAS'06, pages 287–302.

- **[CR]**: **Relational inductive shape analysis.**
  Bor-Yuh Evan Chang and Xavier Rival.
  In POPL'08, pages 247–260, 2008.

Assignment and paper reading

The Frame rule:

- formalize the Hoare logic rules for a language with pointer assignments and condition tests
- prove the Frame rule by induction over the syntax of programs

Reading:

Separation Logic: A Logic for Shared Mutable Data Structures.
John C. Reynolds.
In LICS'02, pages 55–74, 2002.

Formalizes the Frame rule, among others

Outline

Internships on memory abstraction

Summarization based on universal quantification:

- memory abstractions use **summaries**
  today, we consider inductive linked structures; we will also see arrays...
- **another form of summarization** based on an **unbounded set** E

$$\ast\{P(x) \mid x \in E\}$$

  requires the definition of fold / unfold, analysis operations...
- towards a parametric abstract domain:
  - generic dictionary abstraction
  - arrays (generalization of existing)
  - union finds and DAGs

Other topics:

application to the verification of Operating System components