# Program Transformations as Abstract Interpretation

## MPRI — Cours 2.6 "Interprétation abstraite : application à la vérification et à l'analyse statique"

Xavier Rival

INRIA, ENS, CNRS

Jan, 21st. 2015

# Program transformations and static analysis

Previous lectures: **focus on static analysis techniques**, i.e.

1. take **one program as argument**
2. compute some **semantic properties** of the program
   e.g., compute an over-approximation of the reachable states
   e.g., verify the absence of runtime errors

**Today:** we consider **program transformations**

- functions that **compute a program from another program**
- thus, we will consider not a single program but **two**
- different set of **issues**
  - abstract interpretation to reason about and **verify the transformation**
  - static analysis to **enable the transformation**

# Compilation

- **Transforms programs in high level languages** (OCaml, C, Java) **into assembly**
- **Verifies** (e.g., types) and **Optimizes**

## Source code:

```
int f( int a, int b ){
  int x0 = a - b;
  if( x0 > 0 )
    return x0 * (a + b);
  else return 0;
}
```

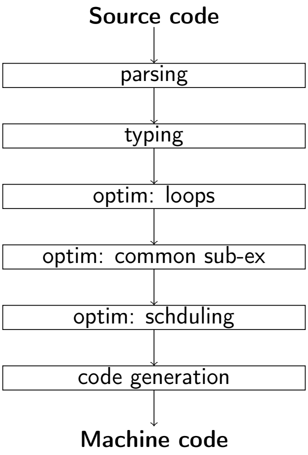
## Compiled code:

```
.file "foo.c"
.text
.globl f
.type f, @function
f:
.LFB0:
.cfi_startproc
pushl %ebp
.cfi_def_cfa_offset 8
.cfi_offset 5, -8
movl %esp, %ebp
.cfi_def_cfa_register 5
subl $16, %esp
```

```
movl 12(%ebp), %eax
movl 8(%ebp), %edx
movl %edx, %ecx
subl %eax, %ecx
movl %ecx, %eax
movl %eax, -4(%ebp)
cmpl $0, -4(%ebp)
jle .L2
movl 12(%ebp), %eax
movl 8(%ebp), %edx
addl %edx, %eax
imull -4(%ebp), %eax
jmp .L3
```

```
.L2:
movl $0, %eax
.L3:
leave
.cfi_restore 5
.cfi_def_cfa 4, 4
ret
.cfi_endproc
.LFE0:
.size f, .-f
.ident
"GCC: (Gentoo 4.7.3-r1 p1.4, pie-0.5
.section
.note.GNU-stack,"",@progbits
```

# Compilation phases




- **Parsing:** can be considered a static analysis
- **Typing:** static analysis
- **Optimizations:** enabled by static analysis
  e.g., code removed if proved dead
  e.g., expressions shared if common
- **Code generation:**
  by induction on syntax...

# Slicing

## Slice extraction

- a slice $\mathcal{S}$ is a **syntactic subset of a program** $\mathcal{P}$
- it is usually extracted following a **criterion** that describes **an observation of the program that is under study**
- there are **many** definitions of slicing criteria: a specific statement, a specific variable, the conjunction of both...

**Applications:**

- **program understanding:** you are given a program, and need to understand how it works...
- **program debugging:** a bug was identified, where `x` stores an unexpected value at line $N$...
- **program maintenance:** a legacy code needs to be extended; what will intended changes do ?

# Slicing

**Example:** slice to **understand the value of a at line 5**

| | |
|---|---|
| 1 : **input**(x); | 1 : **input**(x); |
| 2 : **input**(y); | 2 : **input**(y); |
| 3 : $a = 4 * x + 8$; | 3 : $a = 4 * x + 8$; |
| 4 : $b = 3 - 2 * y + a$; | 4 : $b = 3 - 2 * y + a$; |
| 5 : $c = a + b$; | 5 : $c = a + b$; |

$\rightarrow$

**Algorithm:**

1. **compute dependences:** usually, a dependence graph describes what x *immediately* depends on, at line $N$
2. **extract a set of slice dependences** from the slicing criterion
3. **collect the corresponding statements** and produce the slice

Effectively, 1 and 2 **are a static analysis**

# Partial evaluation

## Specialization and optimization of programs

- start from a **very general program**
- + possibly some **assumptions on the input values**
- compute a program that **behaves similarly on those programs that satisfy the inputs**
- **partial evaluation** of all program statements that can be, but may also involve unrolling of loop, duplication of functions...

**Applications:**

- practical:
  design a software for several products,
  and specialize it for each product
- theoretical: Futamura's projections
  **compilation** = specialization of an interpreter to a program

# Partial evaluation

```
while(c){
    if(b){
        x = 1;
    }else{
        x = f(x);
    }
    b = false;
}
```

hyp: b = **true**

$\longrightarrow$

```
if(c){
    x = 1;
    while(c){
        x = f(x);
    }
}
```

1. unfolding of the loop for a number of iterations
2. propagation of the value of b through the loop
3. simplification of conditions and removal of b

# Questions about program transformations

## Soundness:

- in **what sense** can we say a transformation is sound ?
- what properties should it preserves ?
  what properties should it modify ?
- **how to semantically specify a transformation** ?

## Use of semantic information:

- transformations often need **semantic properties of programs**, to decide what code to generate...
  e.g., for compiler optimizations, dependence information...
- in some cases the transformation itself may be potentially non terminating, and **require a widening** for convergence
  e.g., partial evaluation

# Example: semantics of C volatile variables

## From the ANSI C'99 / C'11 standards

For every read from or write to a volatile variable
that would be performed by a straightforward interpreter for C,
exactly one load or store from/to the memory location allocated to the variable should be performed.

In other words:

- volatile variables should be assumed to be **modifiable** by the external world **at any time** (this is a worst case assumption)
- multiple accesses to a single volatile variable **should never be optimized into a single read**
  (this is a very strong constraint on the optimizers)

**Do compilers follow this semantics ? NO...**

# Example: C compiler and volatile variables

**Study by E. Eide and J. Regher, "Volatiles are Miscompiled, and What to Do about it" (EMSOFT'2008)**

- **13 compilers tested**
- **none** of them is **exempt of volatile bugs**
- possible **consequences:**
  - **incorrect computations**
  - more serious crashes, such as **system hangs**
- one example on the next slide, more in the paper...

Since then, the **CompCert compiler** was tested free of volatile bugs using the same technique...

# Example: C compiler and volatile variables

## Compiler: LLVM GCC 2.2 (IA 32)

**Buggy optimization:**

```
const volatile int a;
void foo(void){
    int i;
    for(i = 0; i < 3; i++){
        a += 7;
    }
}
```

```
foo:
  movl   a, %eax
  leal   7(%eax), %ecx
  movl   %ecx, a
  leal   14(%eax), %ecx
  movl   %ecx, a
  addl   $21, %eax
  movl   %eax, a
  ret
```

**Only ONE store to a**

- loop **unrolled** three times
- **three stores** (correct), but only **one load** **(incorrect)**

# Main points of the lecture

**Formalize soundness of program transformations:**

- **compare the semantics of two programs**
- **select** the semantics to be compared by **abstraction**

**Consider some verification techniques:**

- **invariant verification** approach
- **local equivalence proof**...

These are **partly inspired from static analysis techniques**

# Outline

# Formalizing correctness: assumptions

**Source language: C like imperative language**

- very **simplified**: no procedure, library functions, etc

**Assembly language: RISC style** (similar to Power-PC)

- **registers:** diffentiated dep on types (floating-point, integers)
- **memory access:** direct, indirect, stack-based
- **condition register:**
  Tests and branchings are **separate** operations
  Conditional branching: tests the value of the condition register

**Compiler:**

- the lecture is not about showing a compiler...
- we first assume no optimization and consider optimizations later

# Transition systems

We assume a (source or compiled) program is a **transition system** $\mathcal{S} = (\mathbb{S}, \rightarrow, \mathbb{S}_{\mathcal{I}})$:

- $\mathbb{S} = \mathbb{L} \times \mathbb{M}$ is the set of **states**, where $\mathbb{M} = \mathbb{X} \rightarrow \mathbb{V}$
- $\rightarrow \subseteq \mathbb{S} \times \mathbb{S}$ is the **transition relation**
- $\mathbb{S}_{\mathcal{I}} \subseteq \mathbb{S}$ is the set of **initial states**

We consider their **finite traces semantics**:

- $[\![\mathcal{S}]\!] = \{\langle s_0, \ldots, s_n \rangle \in \mathbb{S}^\star \mid \forall i,\ s_i \rightarrow s_{i+1}\}$
- it can be defined as a **least fix-point**: $[\![\mathcal{S}]\!] = \mathbf{lfp}\, F$

$$
\begin{array}{rlcl}
F: & \mathcal{P}(\mathbb{S}^\star) & \longrightarrow & \mathcal{P}(\mathbb{S}^\star) \\
 & X & \longmapsto & \{\langle s_0 \rangle \mid s \in \mathbb{S}_{\mathcal{I}}\} \\
 & & & \cup \{\langle s_0, \ldots, s_n, s_{n+1} \rangle \\
 & & & \qquad \mid \langle s_0, \ldots, s_n \rangle \in X \wedge s_n \rightarrow s_{n+1}\}
\end{array}
$$

(exercise)

# A very minimal imperative language

$$
\begin{array}{rcll}
\mathtt{l} & ::= & \textbf{l-valules} & \\
& | & \mathtt{x} & (\mathtt{x} \in \mathbb{X}) \\
\mathtt{e} & ::= & \textbf{expressions} & \\
& | & c & (c \in \mathbb{V}) \\
& | & \mathtt{l} & (\mathit{lvalue}) \\
& | & \mathtt{e} \oplus \mathtt{e} & (\mathit{arithoperation}, \mathit{comparison}) \\
\mathtt{s} & ::= & \textbf{statements} & \\
& | & \mathtt{l} = \mathtt{e} & (\text{assignment}) \\
& | & \mathtt{s}; \ldots \mathtt{s}; & (\text{sequence}) \\
& | & \textbf{if}(\mathtt{e})\{\mathtt{s}\} & (\text{condition}) \\
& | & \textbf{while}(\mathtt{e})\{\mathtt{s}\} & (\text{loop})
\end{array}
$$

**Other extensions, not considered at this stage:**

- functions
- collection of arithmetic data types, structures, unions, pointers
- compilation units...

# A basic, PPC-like assembly language: principles

We now consider a (very simplified) **assembly language**

- machine integers: sequences of 32-bits (set: $\mathbb{B}^{32}$)
- instructions are encoded over 32-bits (set: $\mathbb{I}_{\mathrm{MIPS}}$)
  and stored into the same space as data (i.e., $\mathbb{I}_{\mathrm{MIPS}} \subseteq \mathbb{B}^{32}$)
- loads and store instructions, with relative addressing instructions
- conditional branching is indirect:
  comparison instruction sets condition register **cr** (comparison flag)
  conditional branching instruction reads **cr** and branches accordingly

## Memory locations

- **program counter** **pc** (current instruction address)
- **general purpose registers** $\mathbf{r}_0, \ldots, \mathbf{r}_{31}$
- **main memory** (RAM) $\mathbf{Addrs} \to \mathbb{B}^{32}$ where $\mathbf{Addrs} \subseteq \mathbb{B}^{32}$
- **condition register** **cr**

Then: $\mathbb{X}^c = \{\mathbf{pc}, \mathbf{cr}, \mathbf{r}_0, \ldots, \mathbf{r}_{31}\} \uplus \mathbf{Addrs}$

# A basic, PPC-like assembly language: instruction set

Instruction encoded into 32-bits words:

## Instruction set

$$v, dst, o \in \mathbb{B}^{32}, \quad \mathbf{cr} \in \{\mathrm{LT}, \mathrm{EQ}, \mathrm{GT}\}$$

$$\begin{array}{rcll}
i & ::= & (\in \mathbb{I}_{\mathrm{MIPS}}) & \\
 & \mid & \mathbf{li}\ \mathbf{r}_d, v & \text{load } v \in \mathbb{B}^{32} \\
 & \mid & \mathbf{add}\ \mathbf{r}_d, \mathbf{r}_{s0}, \mathbf{r}_{s1} & \text{addition} \\
 & \mid & \mathbf{addi}\ \mathbf{r}_d, \mathbf{r}_{s0}, v & \text{add. } v \in \mathbb{B}^{32} \\
 & \mid & \mathbf{sub}\ \mathbf{r}_d, \mathbf{r}_{s0}, \mathbf{r}_{s1} & \text{subtraction} \\
 & \mid & \mathbf{cmp}\ \mathbf{r}_{s0}, \mathbf{r}_{s1} & \text{comparison} \\
 & \mid & \mathbf{b}\ dst & \text{branch} \\
 & \mid & \mathbf{blt}\langle cr \rangle\ dst & \text{cond. branch} \\
 & \mid & \mathbf{ld}\ \mathbf{r}_d, o & \text{absolute load} \\
 & \mid & \mathbf{st}\ \mathbf{r}_d, o & \text{absolute store} \\
 & \mid & \mathbf{ldx}\ \mathbf{r}_d, o, \mathbf{r}_x & \text{relative load (aka indeXed load)} \\
 & \mid & \mathbf{stx}\ \mathbf{r}_d, o, \mathbf{r}_x & \text{relative store (aka indeXed store)}
\end{array}$$

# A basic, PPC-like assembly language: states

## Definition: state

A state is a tuple $s = (pc, \rho, cr, \mu)$ which comprises:

- a **program counter** value $pc \in \mathbb{B}^{32}$
- a function mapping each **general purpose register** to its value $\rho : \{0, \ldots, 31\} \to \mathbb{B}^{32}$
- a **condition register** value $cr \in \{\mathrm{LT}, \mathrm{EQ}, \mathrm{GT}\}$
- a function mapping each **memory cell** to its value $\mu$ : **Addrs** $\to \mathbb{B}^{32}$

**Equivalently, we can also write** $s = (\ell, m)$, where

- the **control state** $\ell$ is the **current** $pc$ **value**
- the **memory state** $m$ is the triple $(\rho, cr, \mu)$

(we use both notations in the following)

# A basic, PPC-like assembly language: instruction set

We assume a state $s = (pc, \rho, cr, \mu)$ and that $\mu(pc) = i$.

Then:

- if $i = \mathbf{li}\ \mathbf{r}_d, v$, then:

$$s \rightarrow (pc + 4, \rho[d \mapsto v], cr, \mu)$$

- if $i = \mathbf{add}\ \mathbf{r}_d, \mathbf{r}_{s0}, \mathbf{r}_{s1}$, then:

$$s \rightarrow (pc + 4, \rho[d \mapsto (\rho(s0) + \rho(s1))], cr, \mu)$$

- if $i = \mathbf{addi}\ \mathbf{r}_d, \mathbf{r}_{s0}, v$, then:

$$s \rightarrow (pc + 4, \rho[d \mapsto (\rho(s0) + v)], cr, \mu)$$

- if $i = \mathbf{sub}\ \mathbf{r}_d, \mathbf{r}_{s0}, \mathbf{r}_{s1}$, then:

$$s \rightarrow (pc + 4, \rho[d \mapsto (\rho(s0) - \rho(s1))], cr, \mu)$$

# A basic, PPC-like assembly language: instruction set

We assume a state $s = (pc, \rho, cr, \mu)$ and that $\mu(pc) = i$.

Then:

- if $i = \mathbf{cmp}\ \mathbf{r}_{s0}, \mathbf{r}_{s1}$, then:

$$s \rightarrow \begin{cases} (pc + 4, \rho, \mathrm{LT}, \mu) & \text{if } \rho(s0) < \rho(s1) \\ (pc + 4, \rho, \mathrm{EQ}, \mu) & \text{if } \rho(s0) = \rho(s1) \\ (pc + 4, \rho, \mathrm{GT}, \mu) & \text{if } \rho(s0) > \rho(s1) \end{cases}$$

- if $i = \mathbf{blt}\langle cond \rangle\ dst$, then:

$$s \rightarrow \begin{cases} (dst, \rho, \mathbf{cr}, \mu) & \text{if } cr = cond \\ (pc + 4, \rho, \mathbf{cr}, \mu) & \text{otherwise} \end{cases}$$

- if $i = \mathbf{b}\ dst$, then:

$$s \rightarrow (dst, \rho, cr, \mu)$$

# A basic, PPC-like assembly language: instruction set

We assume a state $s = (pc, \rho, cr, \mu)$ and that $\mu(pc) = i$.

Then:

- if $i = \mathbf{ldx}\ \mathbf{r}_d, o, \mathbf{r}_x$, then:

$$s \to \begin{cases} (pc + 4, \rho[d \mapsto \mu(\rho(x) + o)], \mathbf{cr}, \mu) & \text{if } \mu(\rho(x) + o) \text{ is defined} \\ \Omega & \text{otherwise} \end{cases}$$

- if $i = \mathbf{ld}\ \mathbf{r}_d, o$, then:

$$s \to \begin{cases} (pc + 4, \rho[d \mapsto \mu(o)], \mathbf{cr}, \mu) & \text{if } \mu(o) \text{ is defined} \\ \Omega & \text{otherwise} \end{cases}$$

- if $i = \mathbf{stx}\ \mathbf{r}_d, o, \mathbf{r}_x$, then:

$$s \to \begin{cases} (pc + 4, \rho, \mathbf{cr}, \mu[\rho(x) + o) \mapsto \rho(d)]) & \text{if } \mu(\rho(x) + o) \text{ is defined} \\ \Omega & \text{otherwise} \end{cases}$$

- if $i = \mathbf{ld}\ \mathbf{r}_d, o$, then effect can be deduced from the above cases

# Output of a non optimizing compiler

## Assumptions and conventions:

- t is an array of integers initialized to $\mathtt{t} = \{0; 1; 4; -1\}$
- $\mathtt{i}, \mathtt{x}$ are integer variables
- in the assembly, $\underline{\mathtt{x}}$ denotes the address of x

| source code | | compiled code | |
|---|---|---|---|
| $l_0^s$ | $\mathtt{i} := \mathtt{i} + 1;$ | $l_0^c$ | $\mathsf{ld}\ \mathsf{r}_0, \underline{\mathtt{i}}$ |
| | | $l_1^c$ | $\mathsf{addi}\ \mathsf{r}_0, \mathsf{r}_0, 1$ |
| | | $l_2^c$ | $\mathsf{st}\ \mathsf{r}_0, \underline{\mathtt{i}}$ |
| $l_1^s$ | $\mathtt{x} := \mathtt{x} + \mathtt{t}[\mathtt{i}];$ | $l_3^c$ | $\mathsf{ld}\ \mathsf{r}_0, \underline{\mathtt{x}}$ |
| | | $l_4^c$ | $\mathsf{ld}\ \mathsf{r}_1, \underline{\mathtt{i}}$ |
| | | $l_5^c$ | $\mathsf{ldx}\ \mathsf{r}_2, \underline{\mathtt{t}}, \mathsf{r}_1$ |
| | | $l_6^c$ | $\mathsf{add}\ \mathsf{r}_0, \mathsf{r}_0, \mathsf{r}_2$ |
| | | $l_7^c$ | $\mathsf{st}\ \mathsf{r}_0, \underline{\mathtt{x}}$ |
| $l_2^s$ | ... | $l_8^c$ | ... |

**Is it sound ? What property does it preserve ?**

# A source level execution

$$\left\langle \begin{pmatrix} & \mathtt{i} \mapsto 1; \\ & \mathtt{x} \mapsto 1; \\ l_0^s, & \mathtt{t[0]} \mapsto 0; \\ & \mathtt{t[1]} \mapsto 1; \\ & \mathtt{t[2]} \mapsto 4; \\ & \mathtt{t[3]} \mapsto -1; \end{pmatrix}, \begin{pmatrix} & \mathtt{i} \mapsto 2; \\ & \mathtt{x} \mapsto 1; \\ l_1^s, & \mathtt{t[0]} \mapsto 0; \\ & \mathtt{t[1]} \mapsto 1; \\ & \mathtt{t[2]} \mapsto 4; \\ & \mathtt{t[3]} \mapsto -1; \end{pmatrix}, \begin{pmatrix} & \mathtt{i} \mapsto 2; \\ & \mathtt{x} \mapsto 5; \\ l_2^s, & \mathtt{t[0]} \mapsto 0; \\ & \mathtt{t[1]} \mapsto 1; \\ & \mathtt{t[2]} \mapsto 4; \\ & \mathtt{t[3]} \mapsto -1; \end{pmatrix}, \right\rangle$$

**Correctness of compilation:**

- we cannot find the **same** execution in the assembly: as memory locations are not the same at all
- thus, we expect a **"similar"** trace

# Corresponding assembly level execution

| | | | |
|---|---|---|---|
| $l_0^c$ | **ld** $\mathbf{r}_0, \underline{\mathtt{i}}$ | $l_4^c$ | **ld** $\mathbf{r}_1, \underline{\mathtt{i}}$ |
| $l_1^c$ | **addi** $\mathbf{r}_0, \mathbf{r}_0, 1$ | $l_5^c$ | **ldx** $\mathbf{r}_2, \underline{\mathtt{t}}, \mathbf{r}_1$ |
| $l_2^c$ | **st** $\mathbf{r}_0, \underline{\mathtt{i}}$ | $l_6^c$ | **add** $\mathbf{r}_0, \mathbf{r}_0, \mathbf{r}_2$ |
| $l_3^c$ | **ld** $\mathbf{r}_0, \underline{\mathtt{x}}$ | $l_7^c$ | **st** $\mathbf{r}_0, \underline{\mathtt{x}}$ |

We consider an **assembly level trace** starting from a **similar state**:

| state $s_i^c$ | $s_0^c$ | $s_1^c$ | $s_2^c$ | $s_3^c$ | $s_4^c$ | $s_5^c$ | $s_6^c$ | $s_7^c$ | $s_8^c$ |
|---|---|---|---|---|---|---|---|---|---|
| control state $pc_i$ | $l_0^c$ | $l_1^c$ | $l_2^c$ | $l_3^c$ | $l_4^c$ | $l_5^c$ | $l_6^c$ | $l_7^c$ | $l_8^c$ |
| register state $\rho_i(0)$ | 45 | 1 | 2 | 2 | 1 | 1 | 1 | 5 | 5 |
| register state $\rho_i(1)$ | $-5$ | $-5$ | $-5$ | $-5$ | $-5$ | 2 | 2 | 2 | 2 |
| register state $\rho_i(2)$ | 89 | 89 | 89 | 89 | 89 | 89 | 4 | 4 | 4 |
| memory state $\mu_i(\underline{\mathtt{i}})$ | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 |
| memory state $\mu_i(\underline{\mathtt{x}})$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 5 |
| memory state $\mu_i(\underline{\mathtt{t}}+0)$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| memory state $\mu_i(\underline{\mathtt{t}}+1)$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| memory state $\mu_i(\underline{\mathtt{t}}+2)$ | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| memory state $\mu_i(\underline{\mathtt{t}}+3)$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ |

# Source and assembly executions compared

| state $s_i^s$ | $s_0^s$ | $s_1^s$ | $s_2^s$ |
|---|---|---|---|
| control state $l_i^s$ | $l_0^s$ | $l_1^s$ | $l_2^s$ |
| memory state $m_i^s(\mathtt{i})$ | 1 | 2 | 2 |
| memory state $m_i^s(\mathtt{x})$ | 1 | 1 | 5 |
| memory state $m_i^s(\mathtt{t}[0])$ | 0 | 0 | 0 |
| memory state $m_i^s(\mathtt{t}[1])$ | 1 | 1 | 1 |
| memory state $m_i^s(\mathtt{t}[2])$ | 4 | 4 | 4 |
| memory state $m_i^s(\mathtt{t}[3])$ | $-1$ | $-1$ | $-1$ |

**Much more information in the assembly trace:**

- **registers** values
- more **control states**

| state $s_i^c$ | $s_0^c$ | $s_1^c$ | $s_2^c$ | $s_3^c$ | $s_4^c$ | $s_5^c$ | $s_6^c$ | $s_7^c$ | $s_8^c$ |
|---|---|---|---|---|---|---|---|---|---|
| control state $pc_i$ | $l_0^c$ | $l_1^c$ | $l_2^c$ | $l_3^c$ | $l_4^c$ | $l_5^c$ | $l_6^c$ | $l_7^c$ | $l_8^c$ |
| register state $\rho_i(0)$ | 45 | 1 | 2 | 2 | 1 | 1 | 1 | 5 | 5 |
| register state $\rho_i(1)$ | $-5$ | $-5$ | $-5$ | $-5$ | $-5$ | 2 | 2 | 2 | 2 |
| register state $\rho_i(2)$ | 89 | 89 | 89 | 89 | 89 | 89 | 4 | 4 | 4 |
| memory state $\mu_i(\underline{\mathtt{i}})$ | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 |
| memory state $\mu_i(\underline{\mathtt{x}})$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 5 |
| memory state $\mu_i(\underline{\mathtt{t}}+0)$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| memory state $\mu_i(\underline{\mathtt{t}}+1)$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| memory state $\mu_i(\underline{\mathtt{t}}+2)$ | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| memory state $\mu_i(\underline{\mathtt{t}}+3)$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ |

# An abstraction approach

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| state $s_i^s$ | $s_0^s$ | | | $s_1^s$ | | | | | $s_2^s$ |
| control state $l_i^s$ | $l_0^s$ | | | $l_1^s$ | | | | | $l_2^s$ |
| memory state $m_i^s(\mathtt{i})$ | 1 | | | 2 | | | | | 2 |
| memory state $m_i^s(\mathtt{x})$ | 1 | | | 1 | | | | | 5 |
| memory state $m_i^s(\mathtt{t}[0])$ | 0 | | | 0 | | | | | 0 |
| memory state $m_i^s(\mathtt{t}[1])$ | 1 | | | 1 | | | | | 1 |
| memory state $m_i^s(\mathtt{t}[2])$ | 4 | | | 4 | | | | | 4 |
| memory state $m_i^s(\mathtt{t}[3])$ | $-1$ | | | $-1$ | | | | | $-1$ |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| state $s_i^c$ | $s_0^c$ | $s_1^c$ | $s_2^c$ | $s_3^c$ | $s_4^c$ | $s_5^c$ | $s_6^c$ | $s_7^c$ | $s_8^c$ |
| control state $pc_i$ | $l_0^c$ | $l_1^c$ | $l_2^c$ | $l_3^c$ | $l_4^c$ | $l_5^c$ | $l_6^c$ | $l_7^c$ | $l_8^c$ |
| register state $\rho_i(0)$ | 45 | 1 | 2 | 2 | 1 | 1 | 1 | 5 | 5 |
| register state $\rho_i(1)$ | $-5$ | $-5$ | $-5$ | $-5$ | $-5$ | 2 | 2 | 2 | 2 |
| register state $\rho_i(2)$ | 89 | 89 | 89 | 89 | 89 | 89 | 4 | 4 | 4 |
| memory state $\mu_i(\underline{\mathtt{i}})$ | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 |
| memory state $\mu_i(\underline{\mathtt{x}})$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 5 |
| memory state $\mu_i(\underline{\mathtt{t}} + 0)$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| memory state $\mu_i(\underline{\mathtt{t}} + 1)$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| memory state $\mu_i(\underline{\mathtt{t}} + 2)$ | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| memory state $\mu_i(\underline{\mathtt{t}} + 3)$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ |

- We can **abstract away intermediate control states**

# An abstraction approach

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| state $s_i^s$ | $s_0^s$ | | | $s_1^s$ | | | | | $s_2^s$ |
| control state $l_i^s$ | $l_0^s$ | | | $l_1^s$ | | | | | $l_2^s$ |
| memory state $m_i^s(\mathtt{i})$ | 1 | | | 2 | | | | | 2 |
| memory state $m_i^s(\mathtt{x})$ | 1 | | | 1 | | | | | 5 |
| memory state $m_i^s(\mathtt{t}[0])$ | 0 | | | 0 | | | | | 0 |
| memory state $m_i^s(\mathtt{t}[1])$ | 1 | | | 1 | | | | | 1 |
| memory state $m_i^s(\mathtt{t}[2])$ | 4 | | | 4 | | | | | 4 |
| memory state $m_i^s(\mathtt{t}[3])$ | $-1$ | | | $-1$ | | | | | $-1$ |
| state $s_i^c$ | $s_0^c$ | $s_1^c$ | $s_2^c$ | $s_3^c$ | $s_4^c$ | $s_5^c$ | $s_6^c$ | $s_7^c$ | $s_8^c$ |
| control state $pc_i$ | $l_0^c$ | $l_1^c$ | $l_2^c$ | $l_3^c$ | $l_4^c$ | $l_5^c$ | $l_6^c$ | $l_7^c$ | $l_8^c$ |
| register state $\rho_i(0)$ | 45 | 1 | 2 | 2 | 1 | 1 | 1 | 5 | 5 |
| register state $\rho_i(1)$ | $-5$ | $-5$ | $-5$ | $-5$ | $-5$ | 2 | 2 | 2 | 2 |
| register state $\rho_i(2)$ | 89 | 89 | 89 | 89 | 89 | 89 | 4 | 4 | 4 |
| memory state $\mu_i(\underline{\mathtt{i}})$ | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 |
| memory state $\mu_i(\underline{\mathtt{x}})$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 5 |
| memory state $\mu_i(\underline{\mathtt{t}}+0)$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| memory state $\mu_i(\underline{\mathtt{t}}+1)$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| memory state $\mu_i(\underline{\mathtt{t}}+2)$ | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| memory state $\mu_i(\underline{\mathtt{t}}+3)$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ |

- Intermediate control states abstracted; we can **forget registers**

# An abstraction approach

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| state $s_i^s$ | $s_0^s$ | | | $s_1^s$ | | | | | $s_2^s$ |
| control state $l_i^s$ | $l_0^s$ | | | $l_1^s$ | | | | | $l_2^s$ |
| memory state $m_i^s(\mathtt{i})$ | 1 | | | 2 | | | | | 2 |
| memory state $m_i^s(\mathtt{x})$ | 1 | | | 1 | | | | | 5 |
| memory state $m_i^s(\mathtt{t}[0])$ | 0 | | | 0 | | | | | 0 |
| memory state $m_i^s(\mathtt{t}[1])$ | 1 | | | 1 | | | | | 1 |
| memory state $m_i^s(\mathtt{t}[2])$ | 4 | | | 4 | | | | | 4 |
| memory state $m_i^s(\mathtt{t}[3])$ | $-1$ | | | $-1$ | | | | | $-1$ |
| state $s_i^c$ | $s_0^c$ | $s_1^c$ | $s_2^c$ | $s_3^c$ | $s_4^c$ | $s_5^c$ | $s_6^c$ | $s_7^c$ | $s_8^c$ |
| control state $pc_i$ | $l_0^c$ | $l_1^c$ | $l_2^c$ | $l_3^c$ | $l_4^c$ | $l_5^c$ | $l_6^c$ | $l_7^c$ | $l_8^c$ |
| register state $\rho_i(0)$ | 45 | 1 | 2 | 2 | 1 | 1 | 1 | 5 | 5 |
| register state $\rho_i(1)$ | $-5$ | $-5$ | $-5$ | $-5$ | $-5$ | 2 | 2 | 2 | 2 |
| register state $\rho_i(2)$ | 89 | 89 | 89 | 89 | 89 | 89 | 4 | 4 | 4 |
| memory state $\mu_i(\underline{\mathtt{i}})$ | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 |
| memory state $\mu_i(\underline{\mathtt{x}})$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 5 |
| memory state $\mu_i(\underline{\mathtt{t}}+0)$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| memory state $\mu_i(\underline{\mathtt{t}}+1)$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| memory state $\mu_i(\underline{\mathtt{t}}+2)$ | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| memory state $\mu_i(\underline{\mathtt{t}}+3)$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ |

- Registers and intermediate control states removed
  **We get very similar traces !**

# Syntactic relations

**What we did remove:**

- **intermediate control states**
- **memory locations** associated to **registers**

**What we did preserve:**

- **control states in correspondence:**

$$\ell_0^s \leftrightarrow \ell_0^c \qquad \ell_1^s \leftrightarrow \ell_3^c \qquad \ell_2^s \leftrightarrow \ell_8^c$$

- **memory location in correspondence:**

$$\begin{array}{ccc} \mathtt{i} \leftrightarrow \underline{\mathtt{i}} & \mathtt{x} \leftrightarrow \underline{\mathtt{x}} & \mathtt{i} \leftrightarrow \underline{\mathtt{i}} \\ \mathtt{t}[0] \leftrightarrow \underline{\mathtt{t}} + 0 & \mathtt{t}[1] \leftrightarrow \underline{\mathtt{t}} + 1 & \mathtt{t}[2] \leftrightarrow \underline{\mathtt{t}} + 2 \\ \mathtt{t}[3] \leftrightarrow \underline{\mathtt{t}} + 3 & & \end{array}$$

**Intuitively, we did apply an abstraction (to a single trace)**

# Syntactic relations

## Definition

We define two **syntactic mappings**:

- **Between control points:** $\pi_l : \mathbb{L}'_s \to \mathbb{L}'_c$ (where $\mathbb{L}'_i \subseteq \mathbb{L}_i$)
- **Between memory locations:** $\pi_x : \mathbb{X}'_s \to \mathbb{X}'_c$ (where $\mathbb{X}'_i \subseteq \mathbb{X}_i$)

We consider only **subsets** $\mathbb{X}', \ldots$ of $\mathbb{X}, \ldots$. For instance:

- Some variables in the source code may be removed
- Registers in $P_c$ may not correspond to variables of $P_s$
- One statement in $P_s$ corresponds to several instructions in $P_c$

**In practice**, $\pi_l, \pi_x$ are **provided by the compiler**:

- **Linking information**
- **Line table**
- **Debugging information:** Stabs, COFF...

# Syntactic relations

## Definition

We define two **syntactic mappings**:

- **Between control points:** $\pi_l : \mathbb{L}'_s \to \mathbb{L}'_c$ (where $\mathbb{L}'_i \subseteq \mathbb{L}_i$)
- **Between memory locations:** $\pi_x : \mathbb{X}'_s \to \mathbb{X}'_c$ (where $\mathbb{X}'_i \subseteq \mathbb{X}_i$)

For our **example:**

- **Control points:**
  - $\mathbb{L}'_s = \{l^s_0, l^s_1, l^s_2\}$ and $\mathbb{L}'_c = \{l^c_0, l^c_3, l^c_7\}$
  - $\pi_l : l^s_0 \mapsto l^c_0;\ l^s_1 \mapsto l^c_3;\ l^s_2 \mapsto l^c_7$
- **Memory locations:**
  - $\mathbb{X}'_s = \{\mathtt{i}, \mathtt{x}, \mathtt{t}[0], \mathtt{t}[1], \mathtt{t}[2], \mathtt{t}[3]\}$ and $\mathbb{X}'_c = \{\underline{\mathtt{i}}, \underline{\mathtt{x}}, \underline{\mathtt{t}}, \underline{t} + 1, \underline{\mathtt{t}} + 2, \underline{t} + 3\}$
  - $\pi_x : \begin{cases} \mathtt{i} & \mapsto & \underline{i} \\ \mathtt{x} & \mapsto & \underline{x} \\ \mathtt{t}[n] & \mapsto & \underline{t} + n \end{cases}$

# State observational abstraction

We now formalize the process to **project out irrelevant behaviors**:

- in **states**
- in **traces**
- in **the semantics**

We consider the assembly level first:

## Definition: state abstraction

We let the **compiled code-level memory state abstraction** $\pi_c^{\mathbf{m}}$ be defined by:

$$\begin{array}{rrcl} \pi_c^{\mathbf{m}} : & (\mathbb{X}_c \to \mathbb{V}) & \longrightarrow & (\mathbb{X}'_c \to \mathbb{V}) \\ & m & \longmapsto & \lambda(x \in \mathbb{X}'_c) \cdot m(x) \end{array}$$

Similar definition at the source level...
(though no variable needs to be abstracted at this point, we will make use of that possibility further in this course)

# State observational abstraction: example

We recall that

$$\begin{array}{rcl} \mathbb{X}'_s & = & \{\mathtt{i}, \mathtt{x}, \mathtt{t}[0], \mathtt{t}[1], \mathtt{t}[2], \mathtt{t}[3]\} \\ \mathbb{X}'_c & = & \{\underline{\mathtt{i}}, \underline{\mathtt{x}}, \underline{\mathtt{t}}, \underline{t} + 1, \underline{\mathtt{t}} + 2, \underline{t} + 3 \end{array}$$

**Then** $\pi_c^{\mathbf{m}} : (pc, \rho, \mathbf{cr}, \mu) \longmapsto \mu$

So, in particular:

$$\pi_c^{\mathbf{m}} : \left( \begin{array}{llcl} pc & & \mapsto & l_0^c \\ \rho : & 0 & \mapsto & 45 \\ & 1 & \mapsto & -5 \\ & 2 & \mapsto & 4 \\ \mu : & \underline{\mathtt{i}} & \mapsto & 1 \\ & \underline{\mathtt{x}} & \mapsto & 1 \\ & \underline{\mathtt{t}} + 0 & \mapsto & 0 \\ & \underline{\mathtt{t}} + 1 & \mapsto & 1 \\ & \underline{\mathtt{t}} + 2 & \mapsto & 4 \\ & \underline{\mathtt{t}} + 3 & \mapsto & -1 \end{array} \right) \longmapsto \left( \begin{array}{llcl} \mu : & \underline{\mathtt{i}} & \mapsto & 1 \\ & \underline{\mathtt{x}} & \mapsto & 1 \\ & \underline{\mathtt{t}} + 0 & \mapsto & 0 \\ & \underline{\mathtt{t}} + 1 & \mapsto & 1 \\ & \underline{\mathtt{t}} + 2 & \mapsto & 4 \\ & \underline{\mathtt{t}} + 3 & \mapsto & -1 \end{array} \right)$$

# Trace observational abstraction

We can now lift the same abstraction principle to traces:

## Definition: trace abstraction

We let the **compiled code-level trace abstraction** $\pi_c^{\mathbf{tr}}$ be defined by:

$$\begin{array}{rrcl} \pi_c^{\mathbf{tr}}: & (\mathbb{L}_c \times (\mathbb{X}_c \to \mathbb{V}))^\star & \longrightarrow & (\mathbb{L}'_c \times (\mathbb{X}'_c \to \mathbb{V}))^\star \\ & \langle (l_0, m_0), \ldots, (l_n, m_n) \rangle & \longmapsto & \langle (l_{k_0}, \pi_c^{\mathbf{m}}(m_{k_0})), \ldots, (l_{k_m}, \pi_c^{\mathbf{m}}(m_{k_m})) \rangle \end{array}$$

$$\text{where: } \left\{ \begin{array}{l} \{k_0, \ldots, k_m\} = \{k \mid 0 \leq k \leq n \wedge l_k \in \mathbb{L}'_c\} \\ k_0 < \ldots < k_m \end{array} \right.$$

Similar definition at the source level...
(though no control state / variable needs to be abstracted at this point, we will make use of that possibility further in this course)

# Trace observational abstraction: example

$\pi^{\mathbf{tr}}$ :

| control state $pc_i$ | $l_0^c$ | $l_1^c$ | $l_2^c$ | $l_3^c$ | $l_4^c$ | $l_5^c$ | $l_6^c$ | $l_7^c$ | $l_8^c$ |
|---|---|---|---|---|---|---|---|---|---|
| register state $\rho_i(0)$ | 45 | 1 | 2 | 2 | 1 | 1 | 1 | 5 | 5 |
| register state $\rho_i(1)$ | $-5$ | $-5$ | $-5$ | $-5$ | $-5$ | 2 | 2 | 2 | 2 |
| register state $\rho_i(2)$ | 89 | 89 | 89 | 89 | 89 | 89 | 4 | 4 | 4 |
| memory state $\mu_i(\underline{\mathtt{i}})$ | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 |
| memory state $\mu_i(\underline{\mathtt{x}})$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 5 |
| memory state $\mu_i(\underline{\mathtt{t}}+0)$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| memory state $\mu_i(\underline{\mathtt{t}}+1)$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| memory state $\mu_i(\underline{\mathtt{t}}+2)$ | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| memory state $\mu_i(\underline{\mathtt{t}}+3)$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ |

$\longmapsto$

| control state $pc_i$ | $l_0^c$ | $l_3^c$ | $l_8^c$ |
|---|---|---|---|
| memory state $\mu_i(\underline{\mathtt{i}})$ | 1 | 2 | 2 |
| memory state $\mu_i(\underline{\mathtt{x}})$ | 1 | 1 | 5 |
| memory state $\mu_i(\underline{\mathtt{t}}+0)$ | 0 | 0 | 0 |
| memory state $\mu_i(\underline{\mathtt{t}}+1)$ | 1 | 1 | 1 |
| memory state $\mu_i(\underline{\mathtt{t}}+2)$ | 4 | 4 | 4 |
| memory state $\mu_i(\underline{\mathtt{t}}+3)$ | $-1$ | $-1$ | $-1$ |

# Observable behaviors inclusions

Applying this systematically to all traces results into **an abstraction**:

## Result: compiled code observational abstraction

We let $\alpha_c^r$ be the **compiled code observational abstraction**:

$$\begin{array}{rlcl} \alpha_c^r : & \mathcal{P}((\mathbb{L}_c \times (\mathbb{X}_c \to \mathbb{V}))^\star) & \longrightarrow & \mathcal{P}((\mathbb{L}_c' \times (\mathbb{X}_c' \to \mathbb{V}))^\star) \\ & \mathcal{E} & \longmapsto & \{\pi_c^{\mathbf{tr}}(\sigma) \mid \sigma \in \mathcal{E}\} \end{array}$$

It defines a **Galois connection** with an adjoint **concretization** $\gamma_c^r$:

$$(\mathcal{P}((\mathbb{L}_c \times (\mathbb{X}_c \to \mathbb{V}))^\star), \subseteq) \underset{\alpha_c^r}{\overset{\gamma_c^r}{\leftrightarrows}} (\mathcal{P}((\mathbb{L}_c' \times (\mathbb{X}_c' \to \mathbb{V}))^\star), \subseteq)$$

- $\alpha_c^r$ is monotone and the concrete domain is a complete lattice; the concretization function follows and is defined by $\gamma_c^r(\mathcal{E}') = \bigcup_{\mathcal{E}} \{\mathcal{E} \mid \alpha_c^r(\mathcal{E}) \subseteq \mathcal{E}'\} = \{\sigma \mid \pi^{\mathbf{tr}}(\sigma) \in \mathcal{E}'\}$
- The **observational semantics is defined by:** $[\![P_c]\!]_{\mathbf{obs}} = \alpha_c^r([\![P_c]\!])$

# Correctness by semantic equivalence

- The same construction holds at the source level
- The resulting traces are **very similar**, up-to a **basic renaming**
- To define it, we assume the **syntactic mappings** $\pi_l, \pi_x$ are **bijective**

## Memory state renaming

We let the **memory state renaming function** be defined by:

$$\begin{array}{rccc} \pi_m : & (\mathbb{X}'_s \to \mathbb{V}) & \longrightarrow & (\mathbb{X}'_c \to \mathbb{V}) \\ & m & \longmapsto & m \circ \pi_x^{-1} \end{array}$$

## Trace renaming

We let the **trace renaming function** be defined by:

$$\begin{array}{rccc} \pi_t : & \mathbb{L}'_s \times (\mathbb{X}'_s \to \mathbb{V}) & \longrightarrow & \mathbb{L}'_c \times (\mathbb{X}'_c \to \mathbb{V}) \\ & \langle (l_0, m_0), \ldots, (l_n, m_n) \rangle & \longmapsto & \langle (\pi_l(l_0), \pi_m(m_0)), \ldots, (\pi_l(l_n), \pi_m(m_n)) \rangle \end{array}$$

# Correctness by semantic equivalence

We can now state the **compilation correctness definition**

## Definition: compilation correctness

**Compilation of $P_s$ into $P_c$ is correct *with respect to* $\pi_l, \pi_x$ if and only if $\pi_t$ establishes a bijection between $\alpha^r_s(\llbracket P_s \rrbracket)$ and $\alpha^r_c(\llbracket P_c \rrbracket)$.**

This definition can be illustrated by the diagram:

$$
\begin{tikzcd}
\alpha^r_s(\llbracket P_s \rrbracket) \arrow[rr, equal, "\pi_t"] & & \alpha^r_c(\llbracket P_c \rrbracket) \\
\llbracket P_s \rrbracket \arrow[u, "\text{observation}"] & & \llbracket P_c \rrbracket \arrow[u, "\text{observation}"'] \\
P_s \arrow[u, "\text{semantics}"] \arrow[rr, "\text{compilation}"] & & P_c \arrow[u, "\text{semantics}"']
\end{tikzcd}
$$

# Correctness by semantic equivalence

**This approach generalizes to other program transformations**

This definition can be illustrated by the diagram:

$$
\begin{tikzcd}
\alpha_s^r(\llbracket P_i \rrbracket) \arrow[r, equal, "\textit{semantic transformation}"] & \alpha_c^r(\llbracket P_o \rrbracket) \\
\llbracket P_i \rrbracket \arrow[u, "\text{observation}"] & \llbracket P_o \rrbracket \arrow[u, "\text{observation}"'] \\
P_i \arrow[u, "\text{semantics}"] \arrow[r, "\text{transformation}"] & P_o \arrow[u, "\text{semantics}"']
\end{tikzcd}
$$

# Choice of another concrete semantics: consequences

**New compilation correctness definition**

$$\forall \rho \in \mathbb{M},\ [\![P_c]\!]_{\mathrm{rel}} \equiv [\![P_s]\!]_{\mathrm{rel}} \textbf{ modulo } \pi_l, \pi_x$$

**This new definition is much weaker**:

- **Correctness** assumes **no relation** about
  - intermediate control states
  - non terminating executions
- **More compilers** are considered correct
- **Weaker relation** between source and compiled programs
  This new definition really **misses something**, and impedes verification

Ways to **circumvent the limitation:**

1. Include **the whole trace into the final state!**
   Back to the previous definition, hard to formalize,
   says nothing about $\infty$...
2. Better way: **get it right first** and choose the right semantics!

# Choice of another concrete semantics

We have built our definition of compilation correctness upon **operational (trace) semantics**.
What if we abstracted into **another observational semantics** ?

**Alternate choice:** let us consider a **more abstract semantics**

For instance, **relational semantics** (equivalent to denotational semantics)

- Notation fornitial (resp. **final**) control states: $\ell_\vdash$ (resp. $\ell_\dashv$)
- Notation for **non-termination** written $\infty$;
- **Observational semantics:** relations between $\mathbb{M}$ and $\mathbb{M} \cup \{\infty\}$
- **Observational abstraction** defined by collecting for all traces:

$$\begin{array}{rcc} \langle(\ell_\vdash, \rho), \ldots, (\ell_\dashv, \rho')\rangle & \mapsto & (\rho, \rho') \\ \sigma = \langle(\ell_\vdash, \rho), \ldots\rangle & \mapsto & (\rho, \infty) \text{ if } \sigma \text{ infinite} \end{array}$$

- **Denotational semantics** defined by:

$$[\![P]\!]_{\mathrm{rel}} = \{(\rho, \rho') \mid \ldots\} \uplus \{(\rho, \infty) \mid \ldots\}$$

# Outline

# Optimizations

**Until now** we focused on **non-optimizing compilation**

**In practice,** compilers perform various **optimizations**

- **Elimination:** dead code, dead variables...
- **Instruction scheduling:** Instruction-Level-Parallelism...
- **Global transformations:** Propagation of common expressions...
- **Structural transformations:** Loop unrolling...

**Consequences:** $\pi_l$, $\pi_x$, $\mathbb{L}'_i$, $\mathbb{X}'_i$ **may not be defined**

**Framework extension:**

- **Redefine** the "**most precise observation preserved by compilation**"
- Would be **more difficult** with **bissimulations**
- **Next slides:** consider a few optimizations...

# Dead-code elimination definition

## Principle

**Do not compile statements of the source program that provably never are executed**

- This **saves space** as smaller executables get generated
- It also **improves runtime** as some tests may be removed (when they always produce the same result)

**Example:**

| source code | | compiled code | |
|---|---|---|---|
| $l_0^s$ | $\mathrm{x} := 4;$ | $l_0^c$ | $\mathbf{li}\ \mathbf{r}_0, 4$ |
| | | $l_1^c$ | $\mathbf{st}\ \mathbf{r}_0, \underline{\mathrm{x}}$ |
| $l_1^s$ | $\mathbf{if}(\mathrm{x} < 0)\{$ | | %% no code generated |
| $l_2^s$ | $\quad \mathrm{x} = -\mathrm{x};$ | | %% no code generated |
| $l_3^s$ | $\}$ | | %% no code generated |
| $l_4^s$ | $\mathrm{x} = \mathrm{x} + 1$ | $l_2^c$ | $\mathbf{ld}\ \mathbf{r}_1, \underline{\mathrm{x}}$ |

# Dead-code elimination correctness

**How to set up a formal definition of compilation, that considers dead-code elimination correct ?**

- we have to abstract away **all labels removed by the optimizations**
- this is **trivial**:
  we should simply **not include them in $\mathbb{L}'_s$**
- thus, our previous definition of compilation correctness **already accomodates dead-code eliminiation**

**Compilation correctness in presence of dead-code elimination**

**Same definition as before**

# Dead-variable elimination definition

## Principle

**Discard entirely the variables that are never used anymore**
(the compiler may reuse cells of dead local variables as well)

- This obviously both **saves space** and **improves runtime**
- **There is a caveat though: this may change the error semantics** indeed, expressions may be optimized away, so a program that normally fails (e.g., on a division by zero) may not fail after optimization

```
...
x := y;
while(i < 10){
    x := x + 1;
    y := y - x - 1;
    i := i + 1;
}
use(x);
```

- x **read after the loop, but not** y
- thus, y **can be removed** with no observable change
- the purple statement disappears
- but y **does not disappear everywhere**

# Dead-variable elimination correctness

**How to set up a formal definition of compilation, that considers dead-variable elimination correct ?**

- variables may need be removed **at certain program points**
- it is not possible to simply remove the dead variables from $\mathbb{X}_s$ alltogether: in the example, this would not be correct, as $\mathtt{y}$ would be completely lost
- thus, $\pi_x$ should be relational

**Compilation correctness in presence of variable-code elimination**

**Similar definition as before, but with $\pi_x : \mathbb{L}_s \times \mathbb{X}_s \to \mathbb{X}_c$ instead.**

**Exercise:** formalize the new definition, inspired from the previous one, and with $\pi_x : \mathbb{L}_s \times \mathbb{X}_s \to \mathbb{X}_c$ instead

# Path modifying optimizations

Some optimization **deeply modify the control flow paths:**

- **loop unrolling**
- **loop exchange**
- **loop tiling**
- **loop interchange**
- flattening of **conditions**

**Gains:**

- more efficient code, due to **fewer conditions** (unrolling, tiling)
- enabling of **other optimizations**, e.g., vectorization (tiling, interchange...)

In the next few slides, we consider the case of **loop unrolling**

# Loop unrolling example

**Assumption:** a for loop run an even number of times
(loop unrolling may also apply to loops run a non statically known number of times, but it is more complex in that case)

source code

$$\begin{array}{ll}
l_0^s & \mathtt{i} := 0; \\
l_1^s & \textbf{while}(\mathtt{i} < 1000) \\
l_2^s & \qquad \mathtt{x} := \mathtt{x} * \mathtt{y}; \\
l_3^s & \qquad \mathtt{y} := \mathtt{y} - 1; \\
l_4^s & \qquad \mathtt{i} := \mathtt{i} + 1; \\
l_5^s & \}
\end{array}$$

optimized code

$$\begin{array}{ll}
l_0^o & \mathtt{i} := 0; \\
l_1^o & \textbf{while}(\mathtt{i} < 500) \\
l_2^o & \qquad \mathtt{x} := \mathtt{x} * \mathtt{y}; \\
l_3^o & \qquad \mathtt{y} := \mathtt{y} - 1; \\
l_4^s & \qquad \mathtt{x} := \mathtt{x} * \mathtt{y}; \\
l_5^s & \qquad \mathtt{y} := \mathtt{y} - 1; \\
l_6^s & \qquad \mathtt{i} := \mathtt{i} + 1; \\
l_7^s & \}
\end{array}$$

**Control state correspondance $\pi_l$ is clearly broken:**

$$\pi_l : \left\{ \begin{array}{ccc} l_2^s & \leftrightarrow & l_2^o \\ l_2^s & \leftrightarrow & l_4^o \end{array} \right.$$

# Loop unrolling source and assembly traces

We consider executions in the source and the optimized code, and only display **control states at the assignment to** $\mathtt{x}$ and the **values of** $\mathtt{i}, \mathtt{y}$:

- At the **source code level:**

| control state | $l_2^s$ | $l_2^s$ | $l_2^s$ | $l_2^s$ |
|---|---|---|---|---|
| value of `i` | 0 | 1 | 2 | 3 |
| value of `y` | 1200 | 1199 | 1198 | 1197 |

- At the **compiled code level:**

| control state | $l_2^o$ | $l_4^o$ | $l_2^o$ | $l_4^o$ |
|---|---|---|---|---|
| value of `i` | 0 | 1 | 2 | 3 |
| value of `y` | 1200 | 1199 | 1198 | 1197 |

As expected:

- **the correlation** between the values of `i` and the other variables is **lost**
- the real correspondence is between **values of other variables** and **iterations even-ness**

# Loop unrolling observational abstractions

**How to set up a formal definition of compilation, that accpets loop unrolling as correct ?**

- the loop counter variable `i` should be **excluded from $\mathbb{X}_s, \mathbb{X}_o$**
- each control state in the **source loop** should be **divided into a pair of labels**, that carry an **even-ness tab**:

$$\begin{array}{rcl} l_2^s & \mapsto & l_2^{s,e}, l_2^{s,o} \\ l_3^s & \mapsto & l_3^{s,e}, l_3^{s,o} \\ \ldots & \mapsto & \ldots \end{array}$$

- the trace abstraction function $\pi_s^{\mathbf{tr}}$ should map each loop body state into a state with a **consistent iteration even-ness**

**This amounts to doing an even-ness based trace partitioning**

# Loop unrolling observational abstractions

We can consider the traces again:

| | | | | | |
|---|---|---|---|---|---|
| source code | control state | $l_2^s$ | $l_2^s$ | $l_2^s$ | $l_2^s$ |
| | value of `i` | 0 | 1 | 2 | 3 |
| | value of `y` | 1200 | 1199 | 1198 | 1197 |
| source code, abstract | control state | $l_2^{s,e}$ | $l_2^{s,o}$ | $l_2^{s,e}$ | $l_2^{s,o}$ |
| | value of `i` | 0 | 1 | 2 | 3 |
| | value of `y` | 1200 | 1199 | 1198 | 1197 |
| optimized code | control state | $l_2^o$ | $l_4^o$ | $l_2^o$ | $l_4^o$ |
| | value of `i` | 0 | 1 | 2 | 3 |
| | value of `y` | 1200 | 1199 | 1198 | 1197 |

We observe the **following control state correspondance:**

$$\pi_l : \begin{array}{ccc} l_2^{s,e} & \longmapsto & l_2^o \\ l_4^{s,o} & \longmapsto & l_4^o \end{array}$$

# Loop unrolling correctness

Then, the definition follows a **very similar form** as before:

### Compilation correctness in presence of loop unrolling

**Similar definition as before, but with:**

- **trace partitioning $\alpha_s^r$ abstraction**
- a mapping $\pi_l$ that **preserves even-ness**

# Instruction scheduling: instruction level parallelism

We now consider optimizations that modify the code **locally**, and take **instruction scheduling** as an example.

**Instruction-level parallelism** is a feature of modern processors:

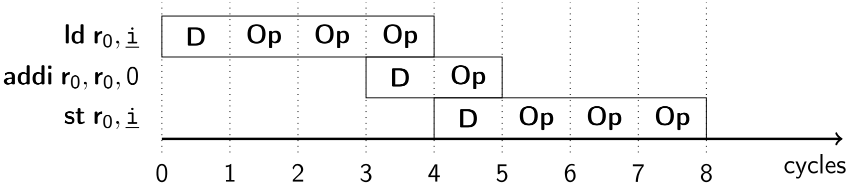
- **one** instruction = **one or several** cycles
  - memory typically slow: **load, store** take several cycles
    speed depends on the content of cache (hit/miss); can be 100 cycles!
  - arithmetic operations are usually faster
- **Pipeline:** run several instructions in parallel
- Some instructions **cannot be evaluated in parallel** due to dependences
- **Scheduling: re-ordering of instructions**
  so as to limit the number of *stall* cycles

# Instruction level parallelism example

**Assumptions:**

- **arith. instructions**: 1 cycle instruction decoding, 1 cycle op.
- **load/store instructions**: 1 cycle instruction decoding, 3 cycle op.

We consider the code below:




Then, we observe **a two cycles stall after the load**

**Consequence of this observation: instruction scheduling**

**More efficient code is generated if there are more instructions between load/store instruction and uses of the values loaded/stored**

# Instruction scheduling example

| source code | | non optimized code | | optimized code | |
|---|---|---|---|---|---|
| $l_0^s$ | $\mathtt{i} := \mathtt{i} + 1;$ | $l_0^a$ | ld $r_0, \underline{\mathtt{i}}$ | $l_0^o$ | ld $r_0, \underline{\mathtt{i}}$ |
| | | $l_1^a$ | addi $r_0, r_0, 1$ | $l_1^o$ | ld $r_1, \underline{\mathtt{x}}$ |
| | | $l_2^a$ | st $r_0, \underline{\mathtt{i}}$ | $l_2^o$ | addi $r_0, r_0, 1$ |
| $l_1^s$ | $\mathtt{x} := \mathtt{x} + \mathtt{t}[\mathtt{i}];$ | $l_3^a$ | ld $r_1, \underline{\mathtt{x}}$ | $l_3^o$ | ldx $r_2, \underline{\mathtt{t}}, r_0$ |
| | | $l_4^a$ | ldx $r_2, \underline{\mathtt{t}}, r_0$ | $l_4^o$ | st $r_0, \underline{\mathtt{i}}$ |
| | | $l_5^a$ | add $r_1, r_1, r_2$ | $l_5^o$ | add $r_1, r_1, r_2$ |
| | | $l_6^a$ | st $r_1, \underline{\mathtt{x}}$ | $l_6^o$ | st $r_1, \underline{\mathtt{x}}$ |
| $l_2^s$ | $\ldots$ | $l_7^a$ | $\ldots$ | $l_7^o$ | $\ldots$ |

**Without optimization:**
**4 stall cycles, 14 cycles total**

$$\begin{array}{ccc} l_0^s & \leftrightarrow & l_0^a \\ l_1^s & \leftrightarrow & l_3^a \\ l_2^s & \leftrightarrow & l_7^a \end{array}$$

**Without optimization:**
**2 stall cycles, 12 cycles total**

$$\begin{array}{ccc} l_0^s & \leftrightarrow & l_0^o \\ l_1^s & \leftrightarrow & ??? \\ l_2^s & \leftrightarrow & l_7^o \end{array}$$

# Instruction scheduling observational abstractions

## Issues to fix our definition:

- **Instructions execution order modified:**
  $l_1^a \to l_2^a$ and $l_2^a \to l_3^a$ are postponed
- **Mapping $\pi_l$ is broken:**
  - The intermediate state $l_1^s$ **has no clear counterpart in the assembly**
  - For `i`, it corresponds to $l_5^o$
  - For `x`, it corresponds to $l_1^o$
  - *In general:* this happens for all control points!
    (except for initial points, final points)

Thus, we need a **relational mapping** $(\pi_l, \pi_x)$,
i.e., a single function taking care of both variables and control states:

### Relational syntactic mapping

A **relational syntactic mapping** is defined by an injective function

$$\pi_{\mathbb{X}\times\mathbb{X}} : (\mathbb{L}'_s \times \mathbb{X}'_s) \longrightarrow (\mathbb{L}_c \times \mathbb{X}_c)$$

# Instruction scheduling observational abstractions

## Intuition

A source control state $\ell^s$ corresponds to a **fictitious control state** where values of corresponding locations are gathered at different points in the execution of the optimized, compiled code

**source code**

$\ell_0^s$ $\mathtt{i} := \mathtt{i} + 1;$

$\ell_1^s$ $\mathtt{x} := \mathtt{x} + \mathtt{t}[\mathtt{i}];$

$\ell_2^s$ $\ldots$

**optimized code**

$\ell_0^o$ **ld** $\mathbf{r}_0, \underline{\mathtt{i}}$
$\ell_1^o$ **ld** $\mathbf{r}_1, \underline{\mathtt{x}}$
$\ell_2^o$ **addi** $\mathbf{r}_0, \mathbf{r}_0, 1$
$\ell_3^o$ **ldx** $\mathbf{r}_2, \underline{\mathtt{t}}, \mathbf{r}_0$
$\ell_4^o$ **st** $\mathbf{r}_0, \underline{\mathtt{i}}$
$\ell_5^o$ **add** $\mathbf{r}_1, \mathbf{r}_1, \mathbf{r}_2$
$\ell_6^o$ **st** $\mathbf{r}_1, \underline{\mathtt{x}}$
$\ell_7^o$ $\ldots$

We then have:

$$\pi_{\mathbb{X}\times\mathbb{X}} : \begin{array}{rcl} (\ell_0^s, \mathtt{i}) & \mapsto & (\ell_0^o, \underline{\mathtt{i}}) \\ (\ell_0^s, \mathtt{x}) & \mapsto & (\ell_0^o, \underline{\mathtt{x}}) \\ (\ell_1^s, \mathtt{i}) & \mapsto & (\ell_5^o, \underline{\mathtt{i}}) \\ (\ell_1^s, \mathtt{x}) & \mapsto & (\ell_1^o, \underline{\mathtt{x}}) \\ (\ell_2^s, \mathtt{i}) & \mapsto & (\ell_7^o, \underline{\mathtt{i}}) \\ (\ell_2^s, \mathtt{x}) & \mapsto & (\ell_7^o, \underline{\mathtt{x}}) \end{array}$$

# Instruction scheduling correctness

The source level observational abstraction is unchanged.

### Optimized level observational abstraction

Optimized code observational abstraction $\alpha_s^r$ abstracts traces into **sequences of states observed at fictitious points**

We now obtain:

### Compilation correctness in presence of instruction scheduling

**Similar definition as before, but with:**

- **optimized code observational abstraction** $\alpha_s^r$ derived from $\pi_{\mathbb{X}\times\mathbb{X}}$
- **semantic mapping** $\pi_t$ derived from $\pi_{\mathbb{X}\times\mathbb{X}}$

# Compilation correctness

## Definition: compilation correctness

**Compilation of $P_s$ into $P_c$ is correct *with respect to* $\pi_I, \pi_x$ (resp., $\pi_{\mathbb{X}\times\mathbb{X}}$) if and only if $\pi_t$ establishes a bijection between $\alpha_s^r(\llbracket P_s \rrbracket)$ and $\alpha_c^r(\llbracket P_c \rrbracket)$.**

$$\begin{tikzcd}
\alpha_s^r(\llbracket P_s \rrbracket) \arrow[r, equal, "\pi_t"] & \alpha_c^r(\llbracket P_c \rrbracket) \\
\llbracket P_s \rrbracket \arrow[u, "\text{observation}"] & \llbracket P_c \rrbracket \arrow[u, "\text{observation}"'] \\
P_s \arrow[u, "\text{semantics}"] \arrow[r, "\text{compilation}"] & P_c \arrow[u, "\text{semantics}"']
\end{tikzcd}$$

**Main idea: optimizations handled as standard compilation, but with more complex mappings, and observational abstractions**

# On the formalization of program transformations

## Methodology:

1. Set up the **standard semantics**
2. Define the **observation preserved by the transformation**
3. Derive the corresponding **abstractions**
4. Establish the correctness at the abstract level

## Advantages of this approach:

- The framework **can be extended** (e.g., with more complex abstractions)
- Abstract Interpretation theorems apply (e.g., fix-point transfers)

## Other extensions:

- **Define** the transformation **at the semantic level**
- **Derive** an implementation of the transformation, from the definition

# Outline

# Verifying compiled code

**Kinds of properties:**

- **safety** (no runtime errors, no overflows, no NaN...)
- **security** (no undesired information flow, in the sence of non-interference)

**Two benefits:**

- of course, verifying the generated code...
- but also, that **the compiler does not turn a correct (already verified) program into an incorrect assembly one**...

In the following, we consider **safety properties and invariants**

# The invariant translation approach

## Process

1. **Analyze the source** program $P_s$ and compute an invariant $\mathcal{I}_s$
2. **Translate** $\mathcal{I}_s$ into assembly level candidate invariant $\mathcal{I}_t$
3. Perform an **assembly level check** of $\mathcal{I}_t$

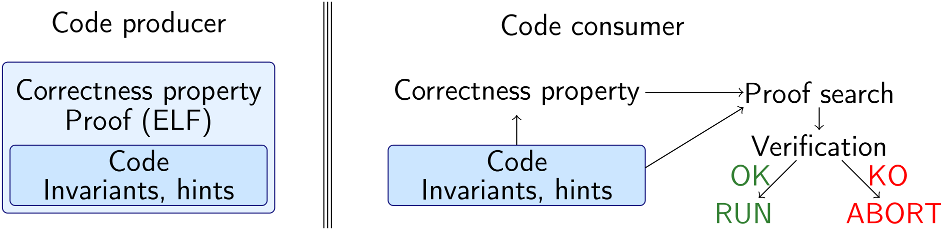
**Motivation:**

- inferring invariants is **hard** in general...
- and **even more so at the assembly level**
  due to an important loss of structure at compile time
  (data-structures flattened, control flow more complex, additional steps to perform an arithmetic assignment –with separate load and store– or a test –with separate test and branching instructions)

# Example 1: Proof Carrying Codes (PCC)

**Principle:**

- **"Code producer":** provides code and **proof annotations** in binaries (i.e., proof of correctness),
- **"Code consumer":** checks the safety of the code
  1. consistance of annotations: very quick proof search, from invariants
  2. annotations $\Rightarrow$ the safety property we wish to enforce




**Context:** execution of **non-trusted** code downloaded in the Internet
e.g., it could contain a **security bug** (information leak, buffer overflow)

# Examples: TAL, compiled code certification by abstract interpretation

**Typed and type safe assembly language:**

- **Java bytecode:** interpreted (rather slow at runtime)
- **TALx86:** annotations for an assembly language closed to Intel **80x86**
- Removing types $\Rightarrow$ executable code
- A specific compiler **translate** source level types

**Advantages:**

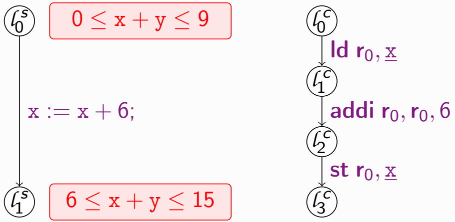
- Ensure the safety of **linkage** thanks to types
  Linkage of object files usually *not* sound
- Improve the reliability of **optimizations**
  Constraint: they should *preserve* types!
- Compilation of type-safe versions of C (CCured, CClone)

**Certification of assembly code**

Principle similar to PCC and TAL

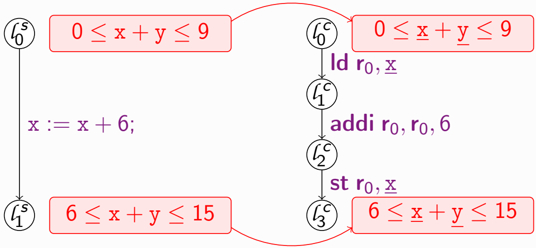
**but computation of invariants by abstract interpretation**

# Example



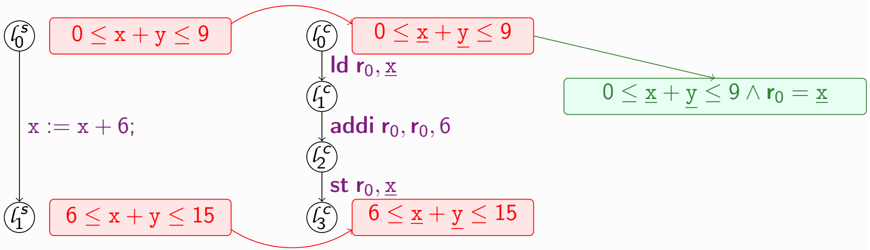

- Start with **invariants on the source code**

# Example



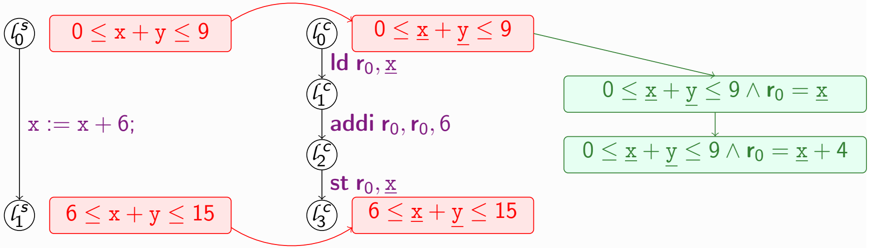

- **Translates those invariants**
  but not all control states are decorated

# Example



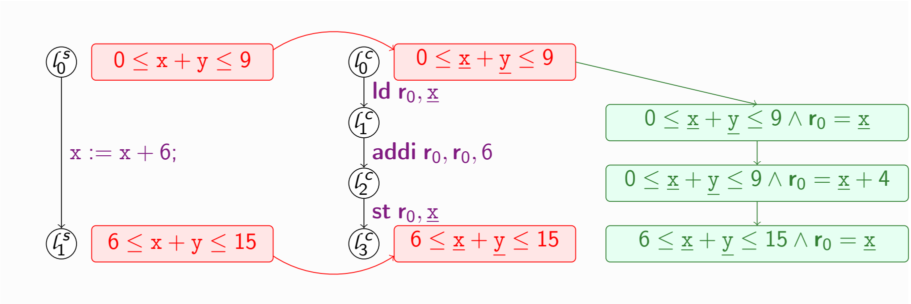

- **Propagates** the invariants and **computes refined local invariants**

# Example



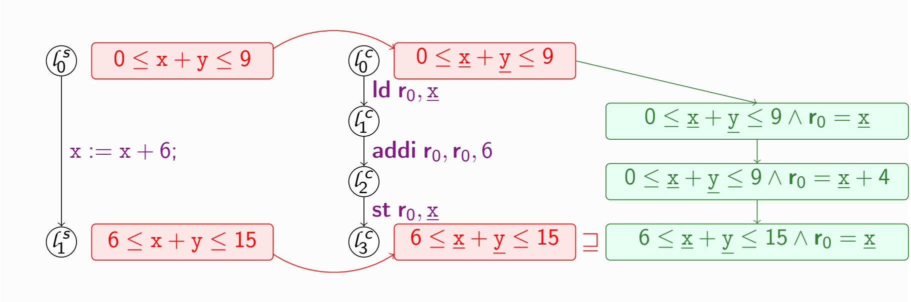

- **Propagates** the invariants and **computes refined local invariants**

# Example




- **Propagates** the invariants and **computes refined local invariants**

# Example




- **Checks invariance** at the end of the computation

# Source static analysis: assumptions

- We assume an **abstraction of sets of stores** defined by **an abstraction function for sets of stores**

$$\alpha_{\mathrm{num}} : (\mathcal{P}(\mathbb{M}_s), \subseteq) \to (\mathbb{D}^{\sharp}_{\mathrm{num}}, \sqsubseteq)$$

- We derive an **abstraction for sets of executions**:

$$\begin{array}{rccl} \alpha_{i,s} : & \mathcal{P}(\mathbb{S}_P{}^{\star}) & \longrightarrow & \mathbb{L}_s \to \mathbb{D}^{\sharp}_{\mathrm{num}} \\ & X & \longmapsto & (\ell \in \mathbb{L}_s) \mapsto \alpha_{\mathrm{num}}(\{m \mid \langle \ldots, (\ell, m), \ldots \rangle \in X\}) \end{array}$$

- We assume also a **source code static analysis**, that computes a sound over-approximation of the behaviors of the program:

$$\alpha_{i,s}([\![P_s]\!]) \sqsubseteq [\![P_s]\!]^{\sharp}_i$$

# Abstract invariant translation

**Two abstractions** have been defined:

- Abstraction for **static analysis** of $P_s$
- Abstraction for **defining compilation correcntess**

$$
\begin{tikzcd}
\llbracket P_s \rrbracket^\sharp_i & \alpha^r_s(\llbracket P_s \rrbracket) \arrow[r, equal, "\pi_t"] & \alpha^r_c(\llbracket P_c \rrbracket) \\
& \llbracket P_s \rrbracket \arrow[ul, "\text{analysis}"'] \arrow[u] & \llbracket P_s \rrbracket \arrow[u] \\
& P_s \arrow[u] \arrow[r, "\text{compilation}"] & P_c \arrow[u]
\end{tikzcd}
$$

**Those abstractions are in general not comparable**

# Abstract invariant translation

We can derive **another abstraction**, more abstract than both $\alpha_s^r$ and $\alpha_{l,s}$:

- **theoretical result**: Galois-connections of a concrete domain form a **lattice**
- in practice, this common abstraction should **abstract away all the elements that are not in $\mathbb{L}'_s, \mathbb{X}'_s$**:
  e.g., all dead variables, all unreachable control states...
  e.g., in case of loop unrolling, it should perform the same trace partitioning

Moreover, $\pi_l, \pi_x$ induce a **safe abstract invariant translation function** $\pi^\sharp : (\mathbb{L}'_s \to \mathbb{D}^\sharp_{\mathrm{num}}) \to (\mathbb{L}'_c \to \mathbb{D}^\sharp_{\mathrm{num}})$

- for each pair of control points in correspondance in $\pi_l$
- it maps numerical invariants among variables of $P_s$ into numerical invariants among variables of $P_c$

# Abstract invariant translation

## Invariant translation process:

1. **Apply** $\pi^\sharp$ to an abstract invariant $\llbracket P_s \rrbracket_i^\sharp$ computed for $P_s$
2. Result: **a candidate invariant** $\pi^\sharp(\llbracket P_s \rrbracket_i^\sharp)$ **for** $P_c$

$$
\begin{tikzcd}
 & (\alpha_s^r)^\sharp(\llbracket P_s \rrbracket_i^\sharp) \arrow[rr] & & \pi^\sharp \circ (\alpha_s^r)^\sharp(\llbracket P_s \rrbracket_i^\sharp) \\
\llbracket P_s \rrbracket_i^\sharp \arrow[ur, "\text{abs. observable}"] & \alpha_s^r(\llbracket P_s \rrbracket) \arrow[u, "\sqsubseteq"'] \arrow[rr, equal, "\pi_t"] & & \alpha_c^r(\llbracket P_c \rrbracket) \\
 & \llbracket P_s \rrbracket \arrow[ul, "\text{analysis}"] \arrow[u] & & \llbracket P_s \rrbracket \arrow[u] \\
 & P_s \arrow[u] \arrow[rr, "\text{compilation}"] & & P_c \arrow[u]
\end{tikzcd}
$$

# Invariant translation: soundness

## Soundness lemma

If:

- the **compilation $P_s \rightarrow P_c$ is sound** with respect to $\pi_I, \pi_x$;
- the **analysis of $P_s$ computes a sound $[\![P_s]\!]^\sharp_i$** $\alpha_{i,s}([\![P_s]\!]) \sqsubseteq [\![P_s]\!]^\sharp_i$

Then, $\pi^\sharp((\alpha^r_s)^\sharp([\![P_s]\!]^\sharp_i))$ **is a sound approximation of $[\![P_c]\!]$:**

$$\alpha_{i,r,c}([\![P_c]\!]) \sqsubseteq \pi^\sharp((\alpha^r_s)^\sharp([\![P_s]\!]^\sharp_i))$$

**Consequence of the choice of another observational semantics** for compilation correctness:
If $\alpha^r_s([\![P_s]\!])$, $\alpha^r_c([\![P_c]\!])$ are **weakened**, then the invariants that can be translated **are also weakened**

# Invariant translation: soundness

## Proof summarized:

$$
\begin{tikzcd}
 & (\alpha_s^r)^\sharp(\llbracket P_s \rrbracket_i^\sharp) \arrow[r] & \pi^\sharp \circ (\alpha_s^r)^\sharp(\llbracket P_s \rrbracket_i^\sharp) \\
\llbracket P_s \rrbracket_i^\sharp \arrow[ur, "\text{abs. observable}"] & \alpha_s^r(\llbracket P_s \rrbracket) \arrow[u, "\sqsubseteq"'] \arrow[r, equal, "\pi_t"] & \alpha_c^r(\llbracket P_c \rrbracket) \arrow[u, "\sqsubseteq"'] \\
 & \llbracket P_s \rrbracket \arrow[ul, "\text{analysis}"] \arrow[u] & \llbracket P_s \rrbracket \arrow[u] \\
 & P_s \arrow[u] \arrow[r, "\text{compilation}"] & P_c \arrow[u]
\end{tikzcd}
$$

## Assumptions are very strong:

compilation, analysis, translation need to be correct

**We need an independent verification of translated invariants**

# Independent verification of translated invariants

**Principle of invariant checking: post-fixpoint checking**

**Theorem: invariant verification**

Using a concretization function $\gamma$,

- **The *concrete* function $F$ is montone**,
- $F \circ \gamma \subseteq \gamma \circ F^{\sharp}$,
- $F^{\sharp}(x) \sqsubseteq x$,

Then, **lfp** $F \sqsubseteq \gamma(x)$

Proof left as exercise

- **Only the verifier needs to be sound** even if the assumptions of the translation soundness lemma are not met
  i.e., we can have an incorrect compiler, translate an incorrect invariant, and still obtain and check a correct translated invariant !
- In turn, **invariant checking is incomplete**

# Independent verification of translated invariants

**Principle of invariant checking: post-fixpoint checking**

**Theorem: invariant verification**

Using a concretization function $\gamma$,

- **The *concrete* function $F$ is montone**,
- $F \circ \gamma \subseteq \gamma \circ F^\sharp$,
- $F^\sharp(x) \sqsubseteq x$,

Then, **lfp** $F \sqsubseteq \gamma(x)$

**Invariant checking refines abstract predicates:**
this phase also produces more precise abstract properties about:

- **memory locations** in $\mathbb{X}_c \setminus \mathbb{X}'_c$
- **program points** in $\mathbb{L}_c \setminus \mathbb{L}'_c$

In practice, **every cycle of the compiled code control flow graph should contain an element of $\mathbb{X}_s$**

# Invariant checking and difficulties

We consider the verification of invariants **around a condition test**

**Assumptions:**

- $\mathrm{x} \in [0, 12]$ at the entry point;
- we wish to **verify the assert in the compiled code**;
- we use a **non relational abstract domain: intervals**

**Source code:**

$$
\begin{array}{l}
\textbf{if}(\mathrm{x} \leq 5)\{ \\
\quad \textbf{assert}(\mathrm{x} \leq 5); \\
\quad \ldots \\
\}\textbf{else}\{ \\
\quad \ldots \\
\}
\end{array}
$$

**Compiled code:**

$$
\begin{array}{ll}
0 & \textbf{ld } \mathbf{r}_0, \underline{\mathrm{x}} \\
4 & \textbf{li } \mathbf{r}_1, 5 \\
8 & \textbf{cmp } \mathbf{r}_0, \mathbf{r}_1 \\
12 & \textbf{blt}\langle \mathrm{GT} \rangle\ \ell \quad \texttt{\# (jump point)} \\
16 & \quad \ldots \texttt{\# true branch contents} \\
\ell: & \texttt{\# false branch contents}
\end{array}
$$

# Invariant checking and difficulties

0 : $\underline{\mathtt{x}} \in [0, 12]$

**ld** $\mathbf{r}_0, \underline{\mathtt{x}}$

4 :

**li** $\mathbf{r}_1, 5$

8 :

**cmp** $\mathbf{r}_0, \mathbf{r}_1$

12 :

**blt**$\langle \mathrm{GT} \rangle\ \ell$ `# (jump point)`

16 :

# Invariant checking and difficulties

$0:\quad \underline{\mathtt{x}} \in [0, 12]$

$\quad\textbf{ld}\ \mathbf{r}_0, \underline{\mathtt{x}}$

$4:\quad \underline{\mathtt{x}} \in [0, 12] \wedge \mathbf{r}_0 \in [0, 12]$

$\quad\textbf{li}\ \mathbf{r}_1, 5$

$8:$

$\quad\textbf{cmp}\ \mathbf{r}_0, \mathbf{r}_1$

$12:$

$\quad\textbf{blt}\langle\mathrm{GT}\rangle\ \ell$ `# (jump point)`

$16:$

# Invariant checking and difficulties

$0:\quad \underline{\mathtt{x}} \in [0,12]$

$\quad\quad \mathbf{ld}\ \mathbf{r}_0, \underline{\mathtt{x}}$

$4:\quad \underline{\mathtt{x}} \in [0,12] \wedge \mathbf{r}_0 \in [0,12]$

$\quad\quad \mathbf{li}\ \mathbf{r}_1, 5$

$8:\quad \underline{\mathtt{x}} \in [0,12] \wedge \mathbf{r}_0 \in [0,12] \wedge \mathbf{r}_1 \in [5,5]$

$\quad\quad \mathbf{cmp}\ \mathbf{r}_0, \mathbf{r}_1$

$12:$

$\quad\quad \mathbf{blt}\langle \mathrm{GT} \rangle\ \ell$ `# (jump point)`

$16:$

# Invariant checking and difficulties

$0:\quad \underline{\mathtt{x}} \in [0, 12]$

**ld** $\mathbf{r}_0, \underline{\mathtt{x}}$

$4:\quad \underline{\mathtt{x}} \in [0, 12] \wedge \mathbf{r}_0 \in [0, 12]$

**li** $\mathbf{r}_1, 5$

$8:\quad \underline{\mathtt{x}} \in [0, 12] \wedge \mathbf{r}_0 \in [0, 12] \wedge \mathbf{r}_1 \in [5, 5]$

**cmp** $\mathbf{r}_0, \mathbf{r}_1$

$12:\quad \underline{\mathtt{x}} \in [0, 12] \wedge \mathbf{r}_0 \in [0, 12] \wedge \mathbf{r}_1 \in [5, 5] \wedge \mathbf{cr} \in \{\mathrm{LT}, \mathrm{EQ}, \mathrm{GT}\}$

**blt**$\langle \mathrm{GT} \rangle\ \ell$ `# (jump point)`

$16:$

# Invariant checking and difficulties

0 : $\underline{\mathtt{x}} \in [0, 12]$

**ld** $\mathbf{r}_0, \underline{\mathtt{x}}$

4 : $\underline{\mathtt{x}} \in [0, 12] \wedge \mathbf{r}_0 \in [0, 12]$

**li** $\mathbf{r}_1, 5$

8 : $\underline{\mathtt{x}} \in [0, 12] \wedge \mathbf{r}_0 \in [0, 12] \wedge \mathbf{r}_1 \in [5, 5]$

**cmp** $\mathbf{r}_0, \mathbf{r}_1$

12 : $\underline{\mathtt{x}} \in [0, 12] \wedge \mathbf{r}_0 \in [0, 12] \wedge \mathbf{r}_1 \in [5, 5] \wedge \mathbf{cr} \in \{\mathrm{LT}, \mathrm{EQ}, \mathrm{GT}\}$

**blt**$\langle \mathrm{GT} \rangle$ $\ell$ `# (jump point)`

16 : $\underline{\mathtt{x}} \in [0, 12] \wedge \mathbf{r}_0 \in [0, 12] \wedge \mathbf{r}_1 \in [5, 5] \wedge \mathbf{cr} \in \{\mathrm{LT}, \mathrm{EQ}\}$

# Invariant checking and difficulties

0 : $\underline{\mathrm{x}} \in [0, 12]$

**ld** $\mathbf{r}_0, \underline{\mathrm{x}}$

4 : $\underline{\mathrm{x}} \in [0, 12] \wedge \mathbf{r}_0 \in [0, 12]$

**li** $\mathbf{r}_1, 5$

8 : $\underline{\mathrm{x}} \in [0, 12] \wedge \mathbf{r}_0 \in [0, 12] \wedge \mathbf{r}_1 \in [5, 5]$

**cmp** $\mathbf{r}_0, \mathbf{r}_1$

12 : $\underline{\mathrm{x}} \in [0, 12] \wedge \mathbf{r}_0 \in [0, 12] \wedge \mathbf{r}_1 \in [5, 5] \wedge \mathbf{cr} \in \{\mathrm{LT}, \mathrm{EQ}, \mathrm{GT}\}$

**blt**$\langle \mathrm{GT} \rangle$ $\ell$ `# (jump point)`

16 : $\underline{\mathrm{x}} \in [0, 12] \wedge \mathbf{r}_0 \in [0, 12] \wedge \mathbf{r}_1 \in [5, 5] \wedge \mathbf{cr} \in \{\mathrm{LT}, \mathrm{EQ}\}$

## The condition at the branch point is not precise

**The range of $\mathrm{x}$ was not refined by the test:**

- the test and branching are **independent**
  relations between test results and values need be tracked
- the test is made on **a copy of** $\mathrm{x}$
  equalities between copies need be tracked by the verifier

# Refinement of the verifier

**Relation between test and branching:**

- each value in $\{\mathrm{LT}, \mathrm{EQ}, \mathrm{GT}\}$ should be bound to the ranges of the other location
- this is obtained by a **value partitioning**, based on the value of **cr**:

$$\begin{array}{rlcl} \gamma: & (\{\mathrm{LT}, \mathrm{EQ}, \mathrm{GT}\} \to \mathbb{D}^\sharp_{\mathrm{num}}) & \longrightarrow & \mathcal{P}(\mathbb{M}) \\ & \phi^\sharp & \longmapsto & \{m \mid m \in \gamma_{\mathrm{num}} \circ \phi^\sharp \circ m(\mathbf{cr})\} \end{array}$$

**Equalities between copies**, e.g., of $\underline{\mathrm{x}}$ and $\mathbf{r}_0$:

- an **equality abstraction** abstracts partitions of $\mathbb{X}_c$
- replacement of $\mathbb{D}^\sharp_{\mathrm{num}}$ with **a reduced product of $\mathbb{D}^\sharp_{\mathrm{num}}$ and an equality abstraction**

# Invariant checking: fixed

0 : $\underline{\mathtt{x}} \in [0, 12]$

**ld** $\mathbf{r}_0, \underline{\mathtt{x}}$

4 :

**li** $\mathbf{r}_1, 5$

8 :

**cmp** $\mathbf{r}_0, \mathbf{r}_1$

12 :

**blt**$\langle \mathrm{GT} \rangle$ $\ell$ `# (jump point)`

16 :

**In general, invariant checking is incomplete...**
**It may require some refinement in the verifier**

# Invariant checking: fixed

0 : $\underline{\mathtt{x}} \in [0, 12]$

$\mathbf{ld}\ \mathbf{r}_0, \underline{\mathtt{x}}$

4 : $\underline{\mathtt{x}} \in [0, 12] \wedge \mathbf{r}_0 \in [0, 12] \wedge \underline{\mathtt{x}} = \mathbf{r}_0$

$\mathbf{li}\ \mathbf{r}_1, 5$

8 :

$\mathbf{cmp}\ \mathbf{r}_0, \mathbf{r}_1$

12 :

$\mathbf{blt}\langle \mathrm{GT} \rangle\ \ell$ `# (jump point)`

16 :

**In general, invariant checking is incomplete...**
**It may require some refinement in the verifier**

# Invariant checking: fixed

0 : $\underline{x} \in [0, 12]$

**ld** $\mathbf{r}_0, \underline{x}$

4 : $\underline{x} \in [0, 12] \wedge \mathbf{r}_0 \in [0, 12] \wedge \underline{x} = \mathbf{r}_0$

**li** $\mathbf{r}_1, 5$

8 : $\underline{x} \in [0, 12] \wedge \mathbf{r}_0 \in [0, 12] \wedge \mathbf{r}_1 \in [5, 5] \wedge \underline{x} = \mathbf{r}_0$

**cmp** $\mathbf{r}_0, \mathbf{r}_1$

12 :

**blt**$\langle \mathrm{GT} \rangle$ $\ell$ `# (jump point)`

16 :

**In general, invariant checking is incomplete...**
**It may require some refinement in the verifier**

# Invariant checking: fixed

0 : $\underline{\mathrm{x}} \in [0, 12]$

**ld** $\mathbf{r}_0, \underline{\mathrm{x}}$

4 : $\underline{\mathrm{x}} \in [0, 12] \wedge \mathbf{r}_0 \in [0, 12] \wedge \underline{\mathrm{x}} = \mathbf{r}_0$

**li** $\mathbf{r}_1, 5$

8 : $\underline{\mathrm{x}} \in [0, 12] \wedge \mathbf{r}_0 \in [0, 12] \wedge \mathbf{r}_1 \in [5, 5] \wedge \underline{\mathrm{x}} = \mathbf{r}_0$

**cmp** $\mathbf{r}_0, \mathbf{r}_1$

12 : $\begin{cases} \mathbf{cr} = \mathrm{LT} \implies \underline{\mathrm{x}} \in [0, 4] \wedge \mathbf{r}_0 \in [0, 4] \wedge \underline{\mathrm{x}} = \mathbf{r}_0 \wedge \mathbf{r}_1 \in [5, 5] \\ \mathbf{cr} = \mathrm{EQ} \implies \underline{\mathrm{x}} \in [5, 5] \wedge \mathbf{r}_0 \in [5, 5] \wedge \underline{\mathrm{x}} = \mathbf{r}_0 \wedge \mathbf{r}_1 \in [5, 5] \\ \mathbf{cr} = \mathrm{GT} \implies \underline{\mathrm{x}} \in [6, 12] \wedge \mathbf{r}_0 \in [6, 12] \wedge \underline{\mathrm{x}} = \mathbf{r}_0 \wedge \mathbf{r}_1 \in [5, 5] \end{cases}$

**blt**$\langle \mathrm{GT} \rangle$ $\ell$ `# (jump point)`

16 :

**In general, invariant checking is incomplete...**
**It may require some refinement in the verifier**

# Invariant checking: fixed

$$
\begin{array}{ll}
0: & \underline{x} \in [0,12] \\
& \textbf{ld } \mathbf{r}_0, \underline{x} \\
4: & \underline{x} \in [0,12] \wedge \mathbf{r}_0 \in [0,12] \wedge \underline{x} = \mathbf{r}_0 \\
& \textbf{li } \mathbf{r}_1, 5 \\
8: & \underline{x} \in [0,12] \wedge \mathbf{r}_0 \in [0,12] \wedge \mathbf{r}_1 \in [5,5] \wedge \underline{x} = \mathbf{r}_0 \\
& \textbf{cmp } \mathbf{r}_0, \mathbf{r}_1 \\
12: & \left\{ \begin{array}{lcl}
\mathbf{cr} = \mathrm{LT} & \Longrightarrow & \underline{x} \in [0,4] \wedge \mathbf{r}_0 \in [0,4] \wedge \underline{x} = \mathbf{r}_0 \wedge \mathbf{r}_1 \in [5,5] \\
\mathbf{cr} = \mathrm{EQ} & \Longrightarrow & \underline{x} \in [5,5] \wedge \mathbf{r}_0 \in [5,5] \wedge \underline{x} = \mathbf{r}_0 \wedge \mathbf{r}_1 \in [5,5] \\
\mathbf{cr} = \mathrm{GT} & \Longrightarrow & \underline{x} \in [6,12] \wedge \mathbf{r}_0 \in [6,12] \wedge \underline{x} = \mathbf{r}_0 \wedge \mathbf{r}_1 \in [5,5]
\end{array} \right. \\
& \textbf{blt}\langle \mathrm{GT} \rangle\ \ell \qquad \texttt{\# (jump point)} \\
16: & \left\{ \begin{array}{lcl}
\mathbf{cr} = \mathrm{LT} & \Longrightarrow & \underline{x} \in [0,4] \wedge \mathbf{r}_0 \in [0,4] \wedge \underline{x} = \mathbf{r}_0 \wedge \mathbf{r}_1 \in [5,5] \\
\mathbf{cr} = \mathrm{EQ} & \Longrightarrow & \underline{x} \in [5,5] \wedge \mathbf{r}_0 \in [5,5] \wedge \underline{x} = \mathbf{r}_0 \wedge \mathbf{r}_1 \in [5,5] \\
\mathbf{cr} = \mathrm{EQ} & \Longrightarrow & \perp
\end{array} \right.
\end{array}
$$

**In general, invariant checking is incomplete...**
**It may require some refinement in the verifier**

# Outline

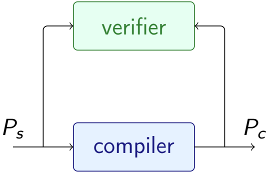
# Verifying a compiler result

**Principle: verify the semantic equivalence between source and compiled programs**

## Verification process: translation validation

1. **Establish mappings** $\pi_l, \pi_x$ between source and compiled programs
2. **Prove (with a specialized prover) the semantic equivalence of each basic block**

**Process:**




$$\begin{tikzcd}
\alpha_s^r([\![P_s]\!]) \arrow[r, equal, "\pi_t"] & \alpha_c^r([\![P_c]\!]) \\
{[\![P_s]\!]} \arrow[u, "\text{observation}"] & {[\![P_c]\!]} \arrow[u, "\text{observation}"'] \\
P_s \arrow[u, "\text{semantics}"] \arrow[r, "\text{compilation}"] & P_c \arrow[u, "\text{semantics}"']
\end{tikzcd}$$

# A technique based on fixpoint transfer

**Foundation: fixpoint transfer**

**Theorem**

Let $F_s : \mathcal{P}(\mathbb{S}_s{}^\star) \rightarrow \mathcal{P}(\mathbb{S}_s{}^\star)$ and $F_c : \mathcal{P}(\mathbb{S}_c{}^\star) \rightarrow \mathcal{P}(\mathbb{S}_c{}^\star)$ and $\pi_t : \mathbb{S}_s{}^\star \rightarrow \mathbb{S}_c{}^\star$ (complete for join), such that:

- $F_s$, $F_c$ **are monotone**
- $\pi_t(\emptyset) = \emptyset$ ($\emptyset$ least element);
- $\pi_t \circ F_s = F_c \circ \pi_t$

**then both functions have a least fixpoint and:**

$$\mathbf{lfp}\, F_c = \pi_t(\mathbf{lfp}\, F_s)$$

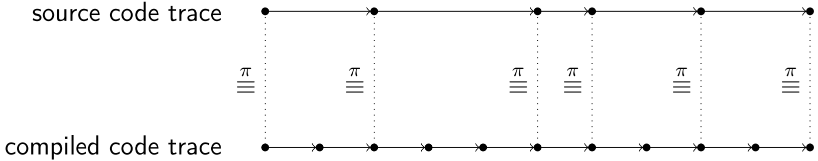
**Proof:** exercise

But the theorem does not apply directly:

**source and compiled executions are not correlated step-by-step**

# A technique based on fixpoint transfer

## Equivalence of source and assembly traces:




- **standard semantics** $[\![P_s]\!]$ and $[\![P_c]\!]$ are expressed as least fixpoints, but not directly correlated by $\pi_X, \pi_l$
- **observational semantics** $\alpha_s^r([\![P_s]\!])$ and $\alpha_c^r([\![P_c]\!])$ are directly correlated by not expressed as least fixpoint

**We need fixpoint definitions for $\alpha_s^r([\![P_s]\!]), \alpha_c^r([\![P_c]\!])$ (e.g., each basic block in the assembly code should be one computation step)**

# Symbolic transfer functions: definition

## A language to describe the effect of a basic block

- basic blocks usually contain **series of assignment**:
  we **flatten sequences of assignments into parallel assignments**
- a basic block may branch to **several points** (often two)
- **no loop**: each cycle in the compiled code control flow graph is associated to at least one control state in the source

### Symbolic transfer functions

**Symbolic transfer functions** are defined by the grammar:

$$
\begin{array}{rcll}
\delta(\in \mathbb{T}) & ::= & \Box & \text{no transition (dead branch, error)} \\
 & \mid & \lfloor \overrightarrow{x} \leftarrow \overrightarrow{e} \rfloor & \text{parallel assignment} \\
 & \mid & \lfloor c \;?\; \delta_0 \mid \delta_1 \rfloor & \text{conditionnal}
\end{array}
$$

Intuitively, a symbolic transfer function is a **store transformer**

# Symbolic transfer functions: semantics

## Semantic domain:

- $\perp$ corresponds to the absence of behavior (error, blocking)
- $[\![\delta]\!] \in \mathbb{M} \to \mathbb{M} \cup \{\perp\}$

### Denotational Semantics:

- $[\![\Box]\!](\rho) = \perp$
- $[\![\lfloor x \leftarrow e \rfloor]\!](\rho) = \rho[\forall i,\ [\![x_i]\!](\rho) \leftarrow [\![e_i]\!](\rho)]$
  if $\forall i,\ [\![x_i]\!](\rho) \neq \mathrm{error}$ and $\forall i,\ [\![e_i]\!](\rho) \neq \mathrm{error}$
  $[\![\lfloor x \leftarrow e \rfloor]\!](\rho) = \perp$ otherwise
- $$[\![\lfloor e\ ?\ \delta_0 \mid \delta_1 \rfloor]\!](\rho) = \begin{cases} [\![\delta_0]\!](\rho) & \text{if } [\![e]\!](\rho) = \mathbf{true} \\ [\![\delta_1]\!](\rho) & \text{if } [\![e]\!](\rho) = \mathbf{false} \\ \perp & \text{if } [\![e]\!](\rho) = \mathrm{error} \end{cases}$$

Note: observe the identity is described by $\iota = \lfloor \cdot \leftarrow \cdot \rfloor$ (parallel assignment, with empty support)

# Symbolic transfer functions: example

## Encoding of a few instructions:

- **"Addition"** $l_0$ : **addi** $\mathbf{r}_1, \mathbf{r}_1, v$; $l_1$ : ...:

$$\delta_{l_0,l_1} = \lfloor \mathbf{r}_0 \leftarrow \mathbf{r}_1 + v \rfloor$$

- **"Comparison"** $l_0$ : **cmp** $\mathbf{r}_0, \mathbf{r}_1$; $l_1$ : ...:

$$\begin{aligned}\delta_{l_0,l_1} = \lfloor \mathbf{r}_0 < \mathbf{r}_1 \ ? & \\ & \lfloor \mathbf{cr} \leftarrow \mathrm{LT} \rfloor \\ & \mid \lfloor \mathbf{r}_0 = \mathbf{r}_1 \ ? \ \lfloor \mathbf{cr} \leftarrow \mathrm{EQ} \rfloor \mid \lfloor \mathbf{cr} \leftarrow \mathrm{GT} \rfloor \rfloor \rfloor\end{aligned}$$

- **"Conditional branching"** $l_0$ : **blt**$\langle \mathrm{LT} \rangle$ $l_1$; $l_2$ : ...:

$$\begin{aligned}\delta_{l_0,l_1} &= \lfloor \mathbf{cr} = \mathrm{LT} \ ? \ \iota \mid \square \rfloor \\ \delta_{l_0,l_2} &= \lfloor \mathbf{cr} = \mathrm{LT} \ ? \ \square \mid \iota \rfloor\end{aligned}$$

# Symbolic transfer functions: example

**Encoding of a few instructions:**

- **"Load"** $l_0$ : **ldx** $\mathbf{r}_d, o, \mathbf{r}_x$; $l_1$ : . . .:

$$\delta_{l_0,l_1} = \lfloor \mathbf{r}_d \leftarrow \mu(o + \mathbf{r}_x) \rfloor$$

- **"Load"** $l_0$ : **ld** $\mathbf{r}_d, o$; $l_1$ : . . .:

$$\delta_{l_0,l_1} = \lfloor \mathbf{r}_d \leftarrow \mu(o) \rfloor$$

- **"Store"** $l_0$ : **stx** $\mathbf{r}_d, o, \mathbf{r}_x$; $l_1$ : . . .:

$$\delta_{l_0,l_1} = \lfloor \mu(o + \mathbf{r}_x) \leftarrow \mathbf{r}_d \rfloor$$

The encoding of **the source semantics** is **straightforward**

# Symbolic transfer functions: composition operation

## Theorem

We can define a **fully syntactic composition operation** $\otimes : \mathbb{T} \times \mathbb{T} \to \mathbb{T}$ such that:

$$[\![\delta_0 \otimes \delta_1]\!] \simeq [\![\delta_0]\!] \circ [\![\delta_1]\!]$$

Full proof left as exercise; we consider a few cases:

- $\Box \otimes \delta = \Box$
- $\delta \otimes \Box = \Box$
- $\delta \otimes \lfloor c \;?\; \delta_0 \mid \delta_1 \rfloor = \lfloor c \;?\; \delta \otimes \delta_0 \mid \delta \otimes \delta_1 \rfloor$
- $\lfloor x_0 \leftarrow e_0 \rfloor \otimes \lfloor x_1 \leftarrow e_1 \rfloor = \begin{cases} \lfloor x_0 \leftarrow e_0[x_1 \leftarrow e_1] \rfloor & \text{if } x_0 = x_1 \\ \left\lfloor \begin{array}{lcl} x_0 & \leftarrow & e_0[x_1 \leftarrow e_1] \\ x_1 & \leftarrow & e_1 \end{array} \right\rfloor & \text{otherwise} \end{cases}$

  (note aliases must be treated with care)

# Symbolic transfer functions: composition operation

**Example:**

- no aliasing between $x, y, z$
  (i.e., locations $x, y, z$ are disjoint pairwise)
- $\delta_0 = \left\lfloor \begin{array}{lll} x & \leftarrow & y + 4 \\ y & \leftarrow & 3 \end{array} \right\rfloor$
- $\delta_1 = \lfloor y \leftarrow z + 1 \rfloor$
-

Then:

$$\delta_0 \otimes \delta_1 = \left\lfloor \begin{array}{lll} x & \leftarrow & z + 5 \\ y & \leftarrow & 3 \end{array} \right\rfloor$$

Note that $y$ is overwritten, and the expression written into $x$ takes into account that assignment

# Translation validation with symbolic transfer functions

Application of symbolic transfer functions:

Definition of **a new program (labeled transition system)** $P'_c$

## Program Reduction

- **States:** $L'_c$
- $\rightarrow$ **is defined by a table of symbolic transfer functions:**
  $(l, \rho) \rightarrow (l', \rho') \iff$
  $\begin{cases} \exists l_0, \ldots, l_n \in \mathbb{L}_c \setminus \mathbb{L}'_c, \\ \rho' = [\![\delta_{l_n, l'} \otimes \ldots \otimes \delta_{l_i, l_{i+1}} \otimes \delta_{l_{i-1}, l_i} \otimes \ldots \otimes \delta_{l, l_0}]\!](\rho) \end{cases}$

## Symbolic semantic abstraction

- **Semantics:** $[\![P'_c]\!] = \mathbf{lfp}\, F'_c$ where $F^r_c$ is derived from $P'_c$
- **Soundness property:** $\alpha^r_c([\![P_c]\!]) = [\![P'_c]\!] = \mathbf{lfp}\, F'_c$
  **Proof:** by induction on the length of the traces of $P'_c$

# Translation validation: example (condition test)

## Source code:

$$\begin{array}{l}\textbf{if}(\mathrm{x} \leq 5)\{ \\ \quad \textbf{assert}(\mathrm{x} \leq 5); \\ \quad \ldots \\ \}\textbf{else}\{ \\ \quad \ldots \\ \}\end{array}$$

## STF to the true branch:

$\delta^s = \lfloor \mathrm{x} \leq 5 \; ? \; \iota \mid \square \rfloor$

## Compiled code:

$$\begin{array}{ll} 0 & \textbf{ld } \mathbf{r}_0, \underline{\mathrm{x}} \\ 4 & \textbf{li } \mathbf{r}_1, 5 \\ 8 & \textbf{cmp } \mathbf{r}_0, \mathbf{r}_1 \\ 12 & \textbf{blt}\langle \mathrm{GT} \rangle \; \ell \qquad \texttt{\# (jump point)} \\ 16 & \quad \ldots \texttt{\# true branch contents} \\ \ell : & \texttt{\# false branch contents} \end{array}$$

## STF to $\ell$:

$$\delta^c_\ell = \lfloor \underline{\mathrm{x}} < 5 \; ? \left[ \begin{array}{lll} \mathbf{r}_0 & \leftarrow & \mu(\underline{\mathrm{x}}) \\ \mathbf{r}_1 & \leftarrow & 5 \\ \mathbf{cr} & \leftarrow & \mathrm{LT} \end{array} \right] \mid \ldots \rfloor$$

## STF in $P'_c$:

$\delta^c_\ell = \lfloor \underline{\mathrm{x}} < 5 \; ? \; \iota \mid \lfloor \underline{\mathrm{x}} = 5 \; ? \; \iota \mid \square \rfloor \rfloor$

# Translation validation and optimization: instruction scheduling

**source code**

$\ell_0^s$ $\mathtt{i} := \mathtt{i} + 1;$

$\ell_1^s$ $\mathtt{x} := \mathtt{x} + \mathtt{t}[\mathtt{i}];$

$\ell_2^s$ $\ldots$

**optimized code**

$\ell_0^o$ **ld** $\mathbf{r}_0, \underline{\mathtt{i}}$
$\ell_1^o$ **ld** $\mathbf{r}_1, \underline{\mathtt{x}}$
$\ell_2^o$ **addi** $\mathbf{r}_0, \mathbf{r}_0, 1$
$\ell_3^o$ **ldx** $\mathbf{r}_2, \underline{\mathtt{t}}, \mathbf{r}_0$
$\ell_4^o$ **st** $\mathbf{r}_0, \underline{\mathtt{i}}$
$\ell_5^o$ **add** $\mathbf{r}_1, \mathbf{r}_1, \mathbf{r}_2$
$\ell_6^o$ **st** $\mathbf{r}_1, \underline{\mathtt{x}}$
$\ell_7^o$ $\ldots$

Syntactic mappings:

$$\pi_{\mathbb{X}\times\mathbb{X}} : \begin{array}{rcl} (\ell_0^s, \mathtt{i}) & \mapsto & (\ell_0^o, \underline{\mathtt{i}}) \\ (\ell_0^s, \mathtt{x}) & \mapsto & (\ell_0^o, \underline{\mathtt{x}}) \\ (\ell_1^s, \mathtt{i}) & \mapsto & (\ell_5^o, \underline{\mathtt{i}}) \\ (\ell_1^s, \mathtt{x}) & \mapsto & (\ell_1^o, \underline{\mathtt{x}}) \\ (\ell_2^s, \mathtt{i}) & \mapsto & (\ell_7^o, \underline{\mathtt{i}}) \\ (\ell_2^s, \mathtt{x}) & \mapsto & (\ell_7^o, \underline{\mathtt{x}}) \end{array}$$

Thus, $\ell_f^o = \mathtt{i}@\ell_5^o; \mathtt{x}@\ell_1^o$

- **Source level transfer functions:**

$$\delta_{\ell_0^s, \ell_1^s} = \lfloor \mathtt{i} \leftarrow \mathtt{i} + 1 \rfloor \qquad \delta_{\ell_1^s, \ell_2^s} = \lfloor \mathtt{x} \leftarrow \mathtt{x} + \mathtt{t}[\mathtt{i}] \rfloor$$

- **Optimized level transfer functions** (registered not displayed):

$$\delta_{\ell_0^o, \ell_f^o} = \lfloor \mu(\mathtt{i}) \leftarrow \mu(\mathtt{i}) + 1 \rfloor \qquad \delta_{\ell_f^o, \ell_7^o} = \lfloor \mu(\underline{\mathtt{x}}) \leftarrow \mu(\underline{\mathtt{x}}) + \mu(\underline{\mathtt{t}} + \mu(\underline{\mathtt{i}})) \rfloor$$

# Translation validation and optimizations

**Program reduction:**

- produces a **set of symbolic transfer functions** that encode the transition relation of the program **up-to observational abstraction**
- **abstracts the effect of optimizations**
  as in the instruction scheduling example
  loop unrolling would result into unrolling at the source level (partitioning)

**Translation validation:**

- based on a **specialized prover**, to establish **equivalence of transfer functions**

# Outline

# Conclusion

**Formalization of Compilation:**

- At the **concrete level**: independant from analysis
- Very **broad**; works as well for
  - other architectures
  - optimizations (use of other abstractions)

**Algorithms for certified compilation** described in **the abstract interpretation frameworks**:

- **Invariant translation**
- **Invariant checking**
- **Translation validation**
- **Compiler formal certification**

**Symbolic transfer functions** and use in **static analysis** and **program transformations**.

**This approach applies to other program transformations**

# Homework

1. Formalize the dead variable elimination correctness (P. 46)
2. **Read:**

   **P. Cousot and R. Cousot.**
   **Systematic design of program transformation frameworks by abstract interpretation.**
   In *Conference Record of the 29th Symposium on Principles of Programming Languages (POPL'02)*, pages 178–190, Portland, Oregon, January 2002.

# Semantics


- **Program transformations:** **P. Cousot and R. Cousot.**

  **Systematic design of program transformation frameworks by abstract interpretation.**

  In *Conference Record of the 29th Symposium on Principles of Programming Languages (POPL'02)*, pages 178–190, Portland, Oregon, January 2002.

- **Relation between types and static analysis:**

  **P. Cousot**,

  **Types as Abstract Interpretations.**

  In *POPL'97*, pages 316–331, Paris, january 1997.

- **Symbolic transfer functions:**

  **C. Colby and P. Lee.**

  **Trace-based program analysis.**

  In *23rd POPL*, pages 195–207, St. Petersburg Beach, (Florida USA), 1996.

# Bibliography: Certified Compilation


- **Proof Carrying Codes: G. C. Necula.**
  **Proof-Carrying Code.**
  In *24th POPL*, pages 106–119, 1997.
- **Typed Assembly languages:**
  **G. Morrisett, D. Tarditi, P. Cheng, C. Stone, R. Harper, and P. Lee.**
  **The TIL/ML Compiler: Performance and Safety Through Types.**
  In *WCSSS*, 1996.
- **Abstract invariant translation (after compilation):**
  **X. Rival.**
  **Abstract Interpretation-based Certification of Assembly Code.**
  In *4th VMCAI*, New York (USA), 2003.

# Bibliography: Certified Compilation


- **Translation validation: A. Pnueli, O. Shtrichman, and M. Siegel.**
  **Translation Validation for Synchronous Languages.**
  In *ICALP'98*, pages 235–246. Springer-Verlag, 1998.
- **Formal proof:**
  **X. Leroy.**
  **Formal certification of a compiler back-end, or: programming a compiler with a proof assistant.**
  In *POPL'06*, Charleston, january 2006.
- **A generic frameork:**
  **X. Rival.**
  **Symbolic-Transfer Function-Based Approaches to Compilation Certification**
  In *POPL'04*, Venice, january 2004.