Université de Paris

Sémantique
Feuille n° 7 : Sémantique opérationnelle

Exercice 1 : Sémantiques à petits pas et à grands pas

$$\frac{t \rightsquigarrow_v t'}{\langle t, u\rangle \rightsquigarrow_v \langle t', u\rangle} \qquad \frac{u \rightsquigarrow_v u'}{\langle V, u\rangle \rightsquigarrow_v \langle V, u'\rangle}$$

$$\frac{u \rightsquigarrow_v u'}{\texttt{let } x = u \texttt{ in } r \rightsquigarrow_v \texttt{let } x = u' \texttt{ in } r} \qquad \frac{}{\texttt{let } x = V \texttt{ in } r \rightsquigarrow_v r\{x/V\}}$$

$$\frac{t \rightsquigarrow_v t'}{t\, u \rightsquigarrow_v t'\, u} \qquad \frac{u \rightsquigarrow_v u'}{V\, u \rightsquigarrow_v V\, u'}$$

$$\frac{}{(\lambda x.t)\, V \rightsquigarrow_v t\{x \backslash V\}}$$

$$\frac{}{\texttt{fst}\ \langle V_1, V_2\rangle \rightsquigarrow_v V_1} \qquad \frac{}{\texttt{snd}\ \langle V_1, V_2\rangle \rightsquigarrow_v V_2}$$

$$\frac{}{\texttt{ifthenelse}\langle \texttt{true}, \langle V_1, V_2\rangle\rangle \rightsquigarrow_v V_1} \qquad \frac{}{\texttt{ifthenelse}\langle \texttt{false}, \langle V_1, V_2\rangle\rangle \rightsquigarrow_v V_2}$$

$$(\textbf{Values})\ V, W ::= \mathsf{c} \mid \langle V, V\rangle \mid \lambda x.t$$

Meaningless expressions such as $(\langle 1, 1\rangle\ 3)$ or $(\texttt{true}\ 3)$ are not considered as values.

$$\frac{V \text{ is a value}}{V \Downarrow_v V} \qquad \frac{t_1 \Downarrow_v V_1 \quad t_2 \Downarrow_v V_2}{\langle t_1, t_2\rangle \Downarrow_v \langle V_1, V_2\rangle} \qquad \frac{u \Downarrow_v V \quad r\{x/V\} \Downarrow_v W}{\texttt{let } x = u \texttt{ in } r \Downarrow_v W}$$

$$\frac{t \Downarrow_v \lambda x.r \quad u \Downarrow_v W \quad r\{x/W\} \Downarrow_v V}{t\, u \Downarrow_v V}$$

$$\frac{t \Downarrow_v \texttt{fst} \quad u \Downarrow_v \langle V_1, V_2\rangle}{t\, u \Downarrow_v V_1} \qquad \frac{t \Downarrow_v \texttt{snd} \quad u \Downarrow_v \langle V_1, V_2\rangle}{t\, u \Downarrow_v V_2}$$

$$\frac{t \Downarrow_v \texttt{ifthenelse} \quad u \Downarrow_v \langle \texttt{true}, \langle V_1, V_2\rangle\rangle}{tu \Downarrow_v V_1} \qquad \frac{t \Downarrow_v \texttt{ifthenelse} \quad u \Downarrow_v \langle \texttt{true}, \langle V_1, V_2\rangle\rangle}{tu \Downarrow_v V_2}$$

$$(\textbf{Lazy Forms})\ P ::= \mathsf{c} \mid \langle t, u\rangle \mid \lambda x.t$$

$$\frac{P \text{ is a lazy form}}{P \Downarrow_n P} \qquad \frac{r\{x/u\} \Downarrow_n P}{\texttt{let } x = u \texttt{ in } r \Downarrow_n P}$$

$$\frac{t \Downarrow_n \lambda x.r \quad r\{x/u\} \Downarrow_n P}{t\, u \Downarrow_n P}$$

$$\frac{t \Downarrow_n \texttt{fst} \quad u \Downarrow_n \langle t_1, t_2\rangle \quad t_1 \Downarrow_n P}{tu \Downarrow_n P} \qquad \frac{t \Downarrow_n \texttt{snd} \quad u \Downarrow_n \langle t_1, t_2\rangle \quad t_2 \Downarrow_n P}{tu \Downarrow_n P}$$

$$\frac{t \Downarrow_n \texttt{ifthenelse} \quad u \Downarrow_n \langle u_1, u_2\rangle \quad u_1 \Downarrow_n \texttt{true} \quad u_2 \Downarrow_n \langle m_1, m_2\rangle \quad m_1 \Downarrow_n P}{tu \Downarrow_n P}$$

$$\frac{t \Downarrow_n \texttt{ifthenelse} \quad u \Downarrow_n \langle u_1, u_2\rangle \quad u_1 \Downarrow_n \texttt{false} \quad u_2 \Downarrow_n \langle m_1, m_2\rangle \quad m_2 \Downarrow_n P}{tu \Downarrow_n P}$$

$$\frac{}{\mathtt{let}\ x = t\ \mathtt{in}\ r \rightsquigarrow_n r\{x/t\}}$$

$$\frac{t \rightsquigarrow_n t'}{t\,u \rightsquigarrow_n t'\,u} \qquad \frac{}{(\lambda x.t)\,u \rightsquigarrow_n t\{x\backslash u\}}$$

$$\frac{t \rightsquigarrow_n t'}{\mathtt{fst}\ t \rightsquigarrow_n \mathtt{fst}\ t'} \qquad \frac{}{\mathtt{fst}\ \langle t, u\rangle \rightsquigarrow_n t}$$

$$\frac{t \rightsquigarrow_n t'}{\mathsf{snd}\ t \rightsquigarrow_n \mathsf{snd}\ t'} \qquad \frac{}{\mathsf{snd}\ \langle t, u\rangle \rightsquigarrow_n u}$$

Montrer que

- Si $t \rightsquigarrow_n t'$ et $t' \Downarrow_n u$ ($t \rightsquigarrow_v t'$ et $t' \Downarrow_v u$) et u est une forme paresseuse (u est une valeur), alors $t \Downarrow_n u$ ($t \Downarrow_v u$)
- En déduire que : si $t \rightsquigarrow_n^* u$ ($t \rightsquigarrow_v^* u$) et u est une forme paresseuse (u est une valeur), alors $t \Downarrow_n u$ ($t \Downarrow_v u$).

Exercice 2 : Call-by-need

Réduire les termes suivants :

- $(\lambda x.xx)(\lambda y.y)$
- $(\lambda x.\lambda y.yxx)(\lambda z.z)(\lambda w.w)$
- $(\lambda x.x)((\lambda y.y)\Omega)$ où $\Omega = (\lambda x.xx)(\lambda x.xx)$

Exercice 3 : Machine abstraite

Main ingredients:

- An environment is a list of elements of the form $[x\backslash c]$, where c is a closure.
- A closure is a pair term and environment.
- A state of the KAM is a 3-uple **Term** | **Environment** | **Stack**.

The transitions between states:

$$\begin{array}{lcll} x \mid e \mid \pi & \mapsto & t \mid e' \mid \pi & \text{where } e(x) = (t, e') \\ \lambda x.t \mid e \mid c :: \pi & \mapsto & t \mid [x\backslash c] :: e \mid \pi & \\ tu \mid e \mid \pi & \mapsto & t \mid e \mid (u, e) :: \pi & \end{array}$$

Implémenter la machine de Krivine et la tester sur des exemples.