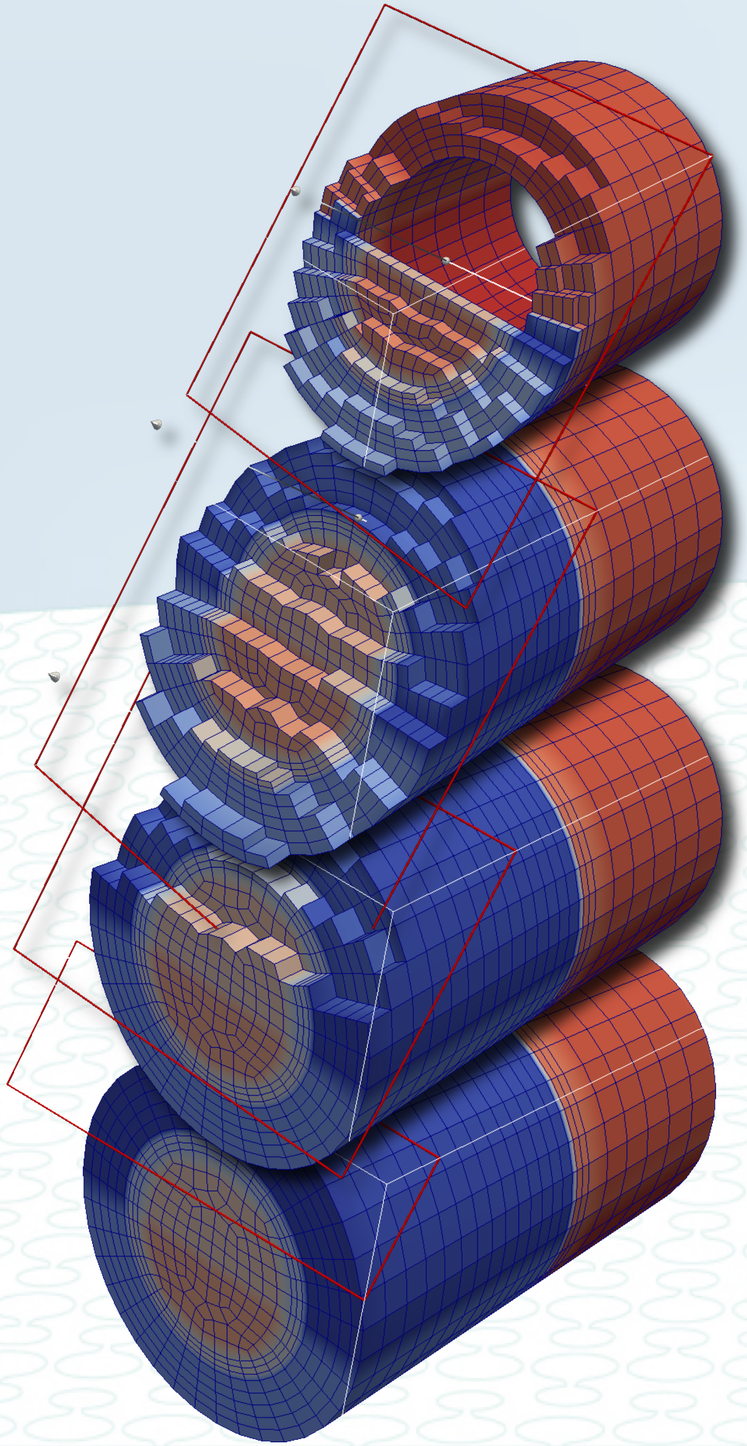


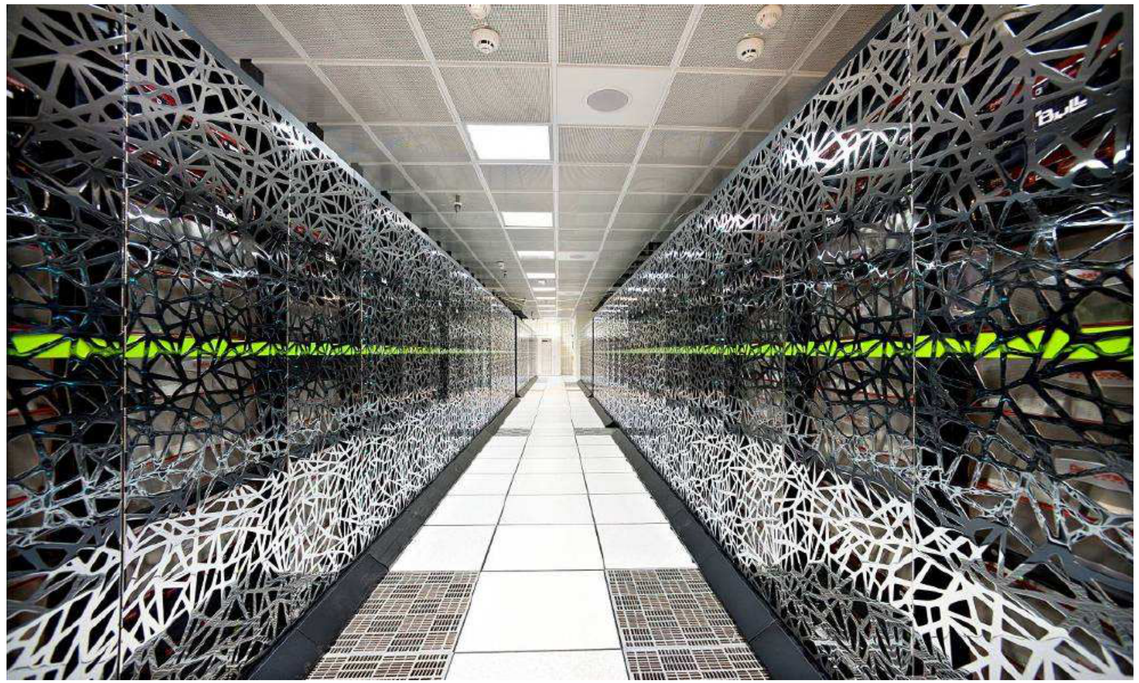
Domain Representations

M2S

J. Tierny

What?

What?



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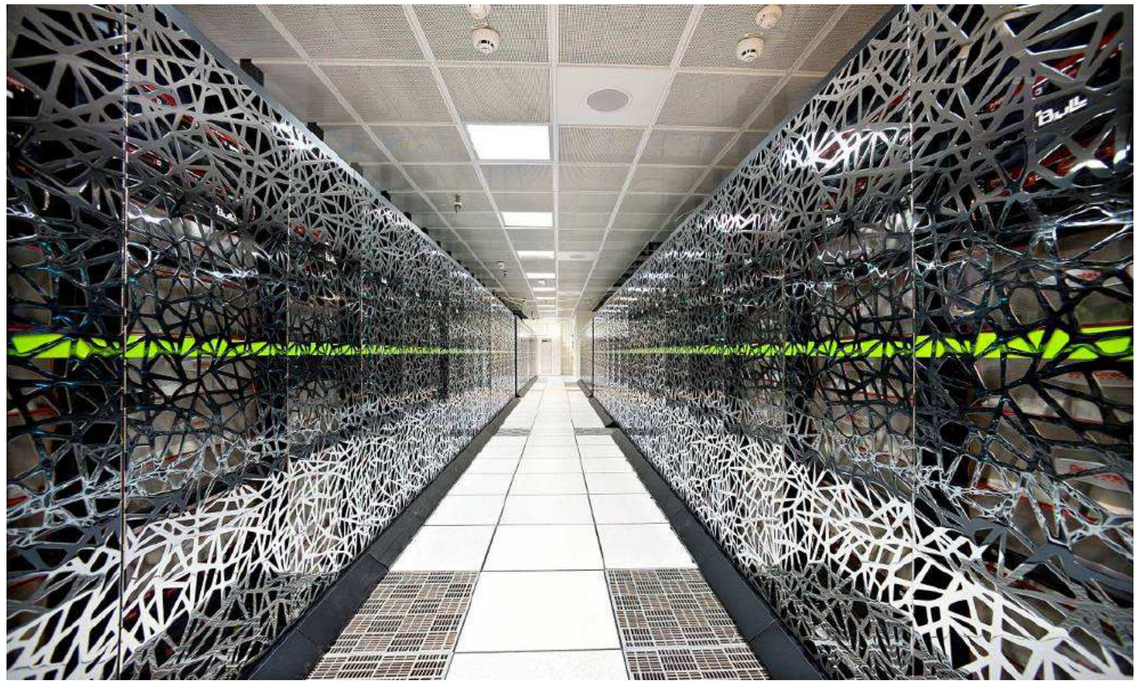

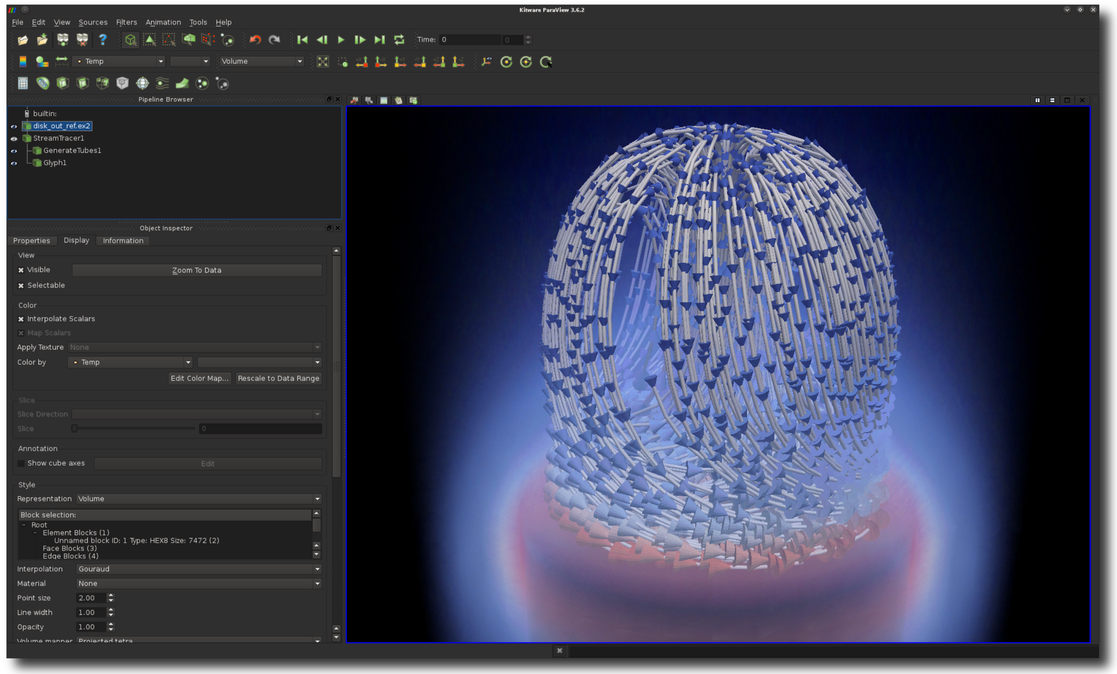
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Pipeline Browser
builtin:
disk_out_ref.ex2
StreamTracer1
GenerateTubes1
Glyph1
Object Inspector
Properties Display Information
View
Visible
Zoom To Data
Selectable
Color
Interpolate Scalars
Map Scalars
Apply Texture None
Color by Temp
Edit Color Map...
Rescale to Data Range
Slice
Slice Direction
Slice
Annotation
Show cube axes
Edit
Style
Representation Volume
Block selection:
Root
Element Blocks (1)
Unnamed block ID: 1 Type: HEX8 Size: 7472 (2)
Face Blocks (3)
Edge Blocks (4)
Interpolation Gouraud
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Point size 2.00
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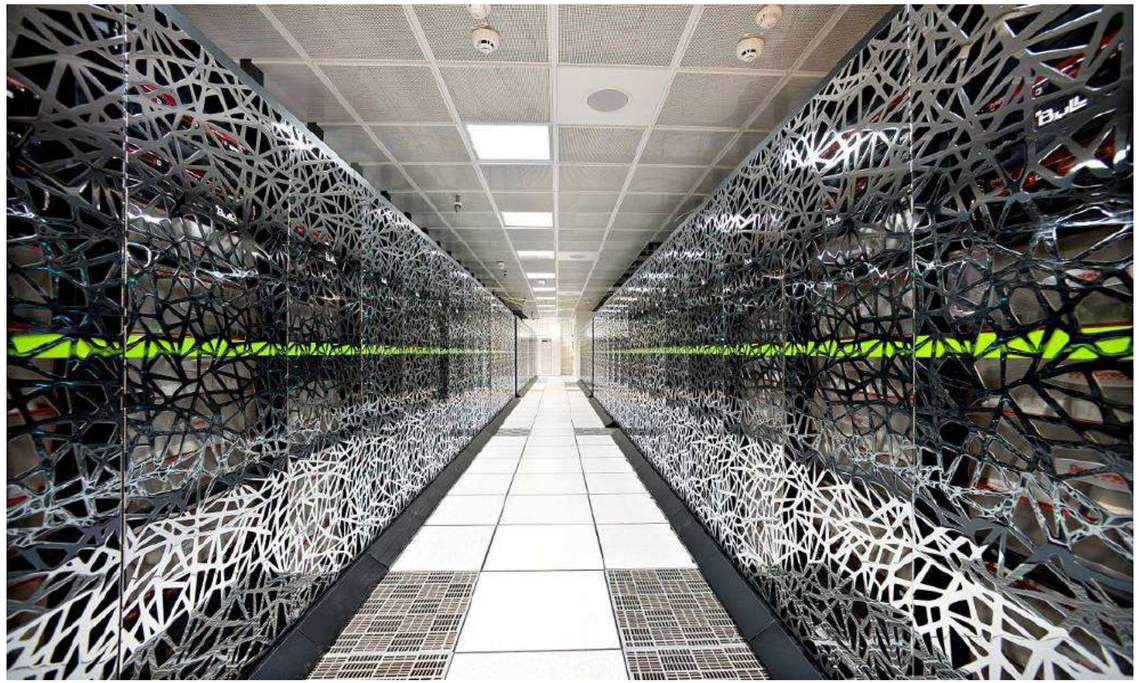
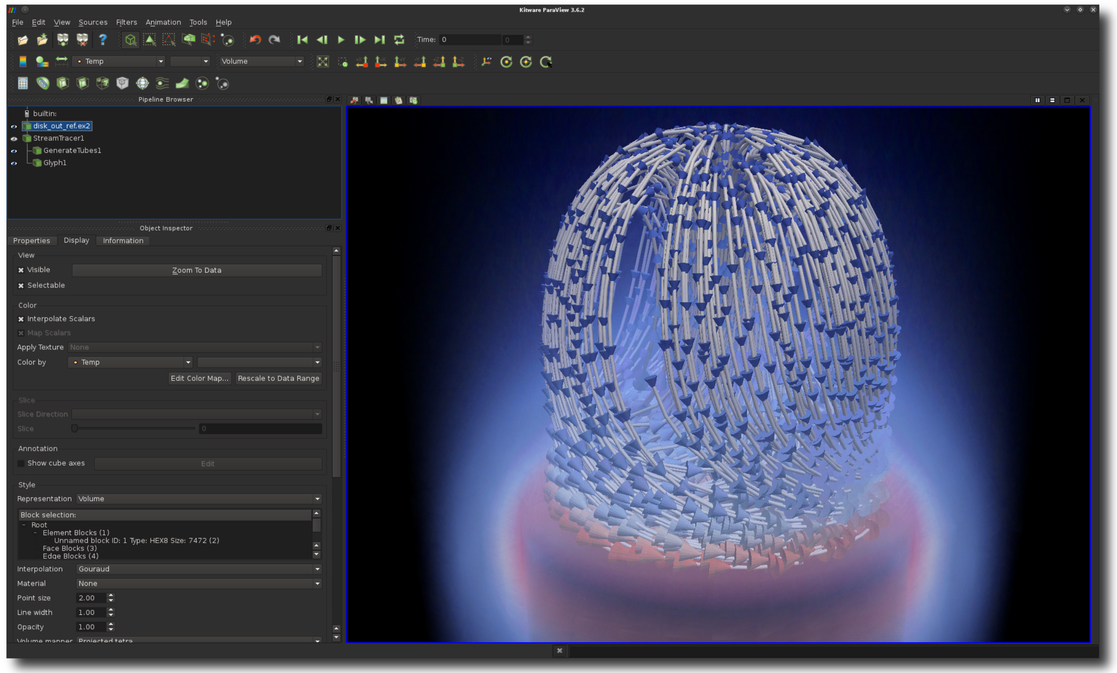
Opacity 1.00

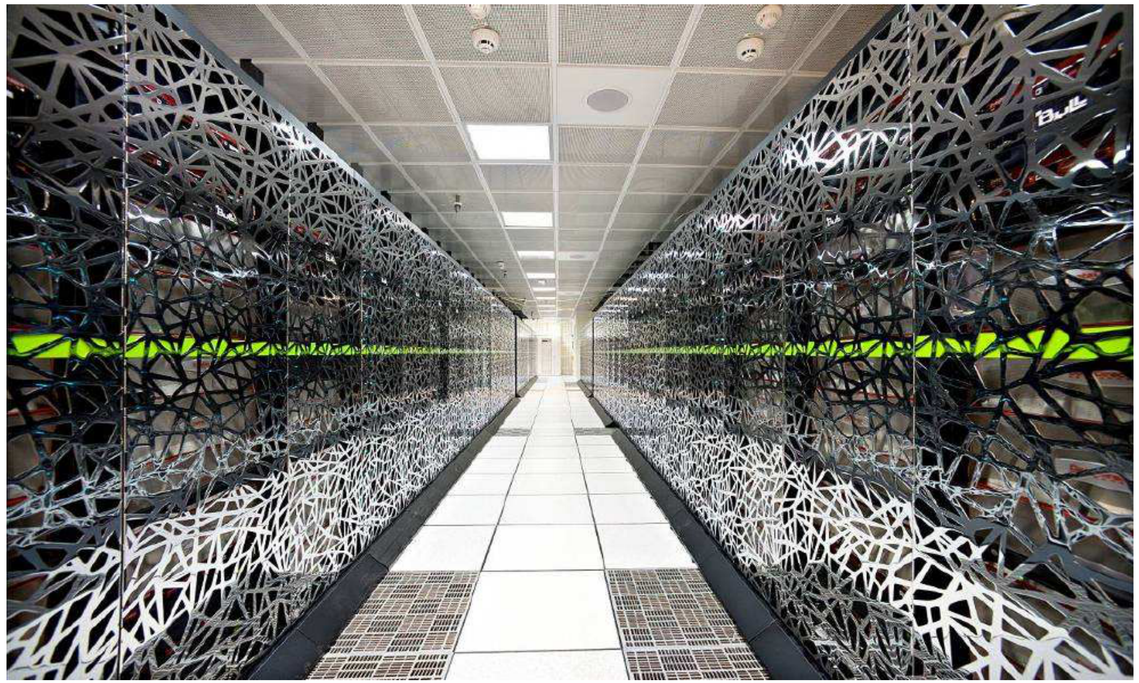
What?

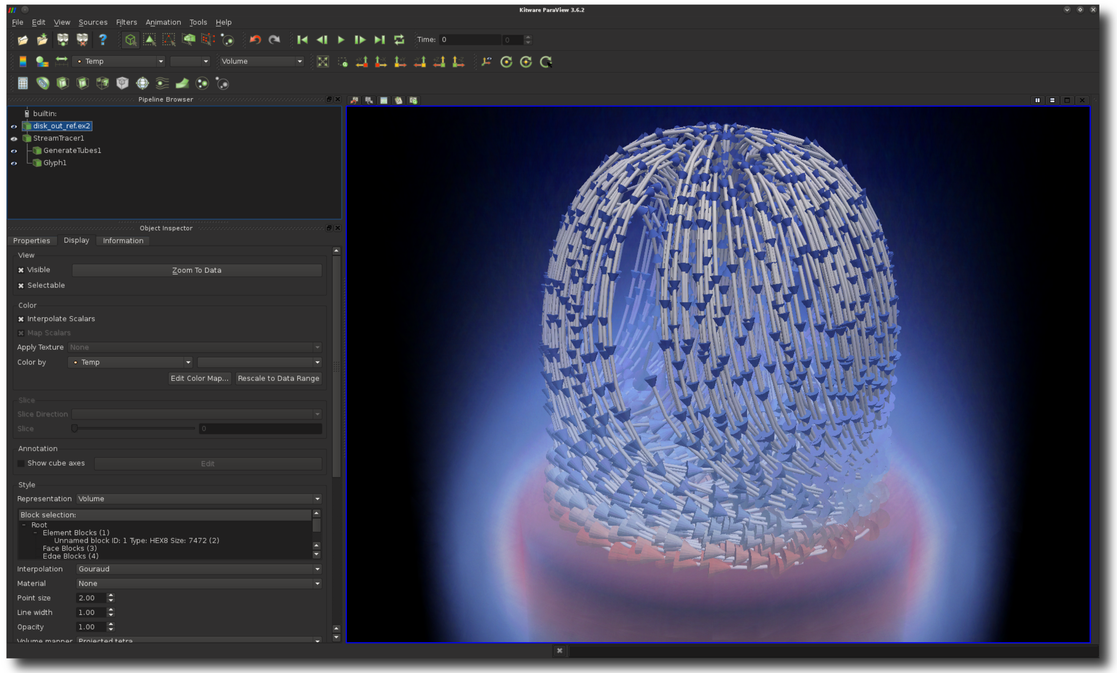




- What have we computed?

What?

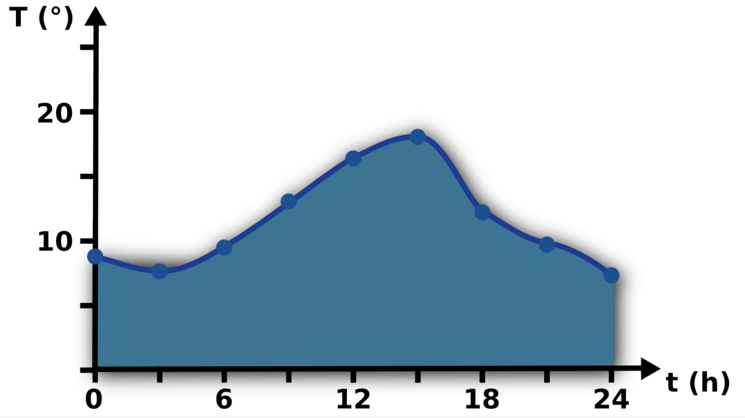




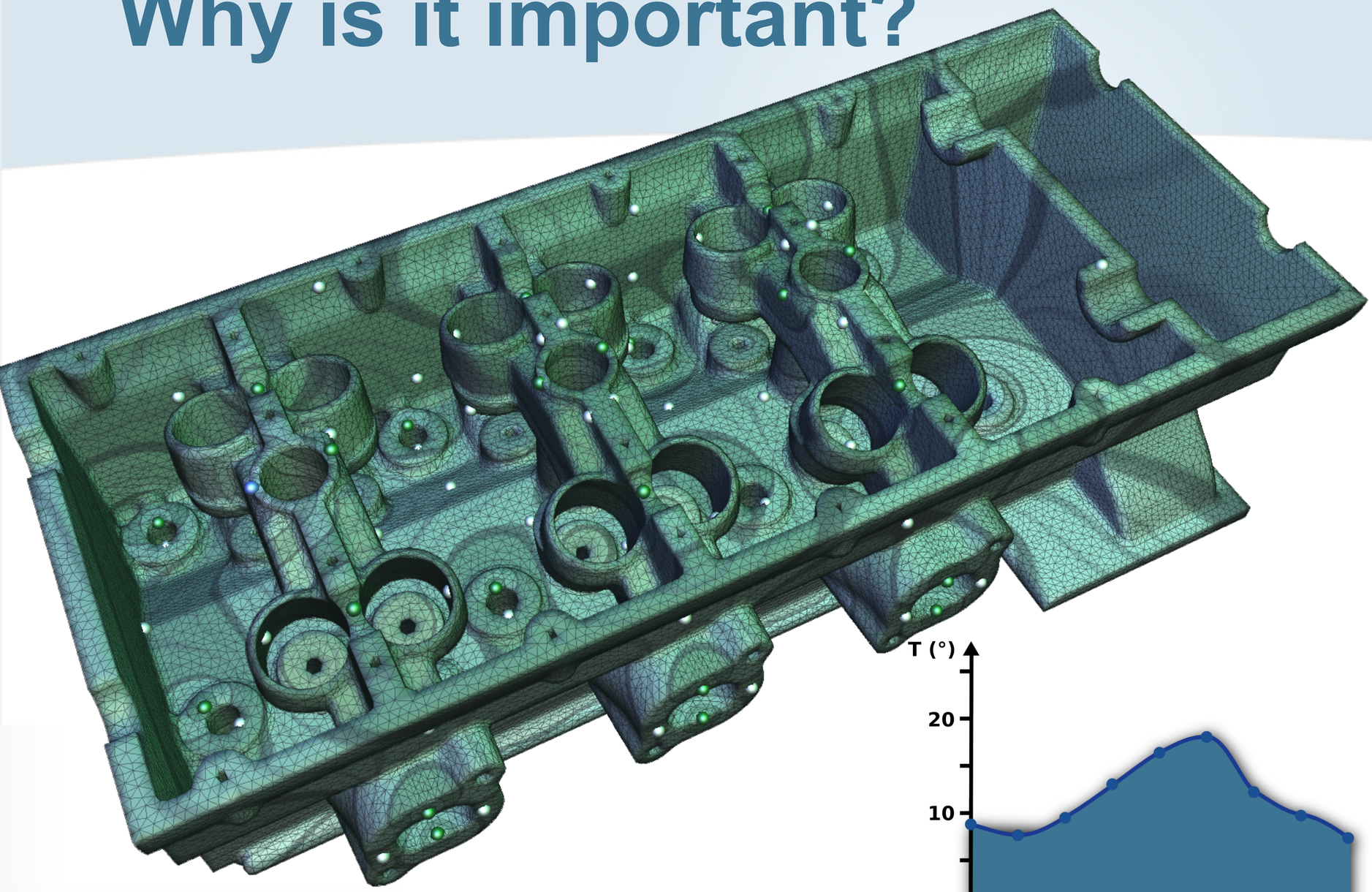
- What have we computed?
- On what kind of *discretized-space*?

Why is it important?

Why is it important?



Why is it important?



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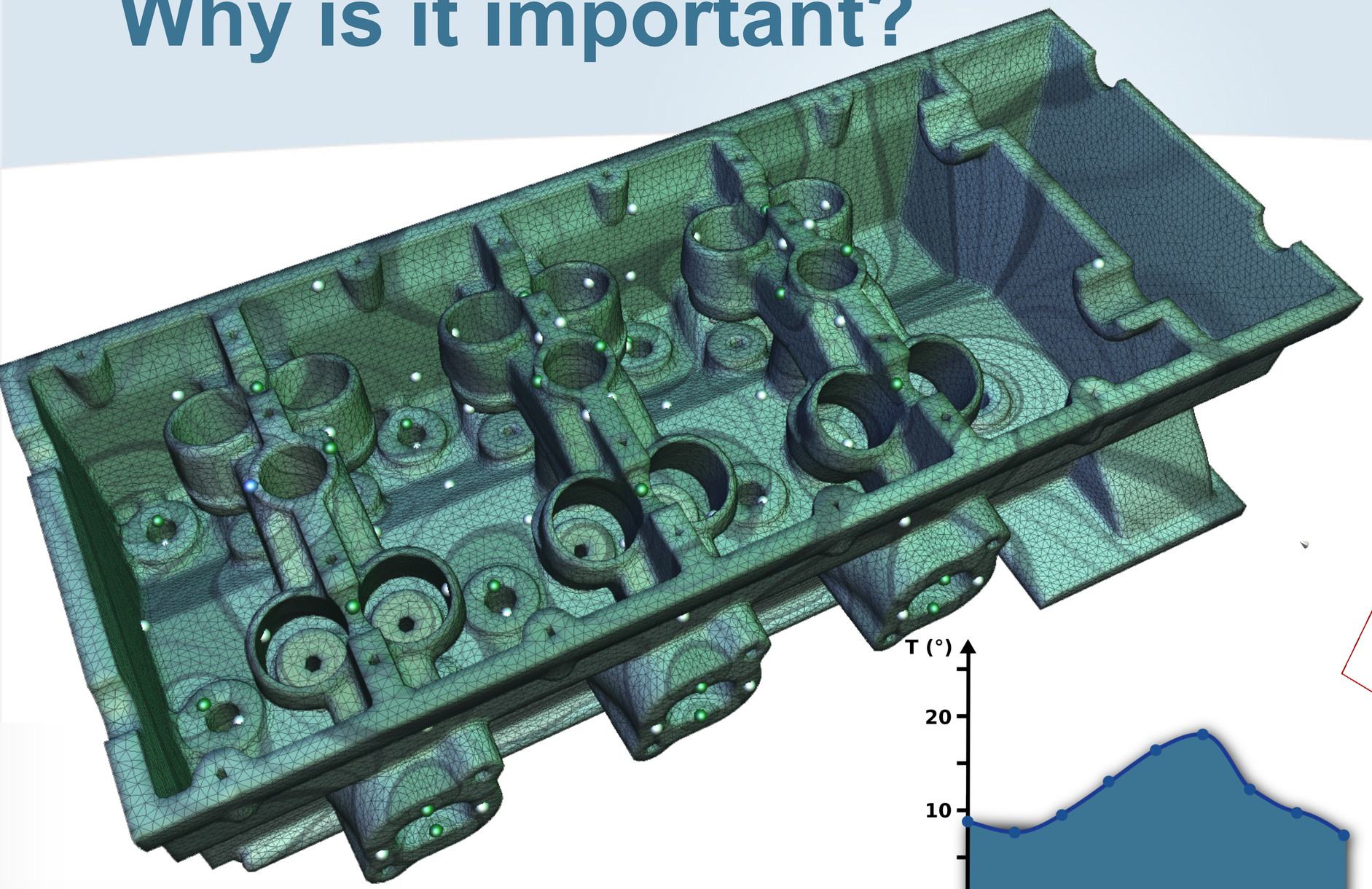
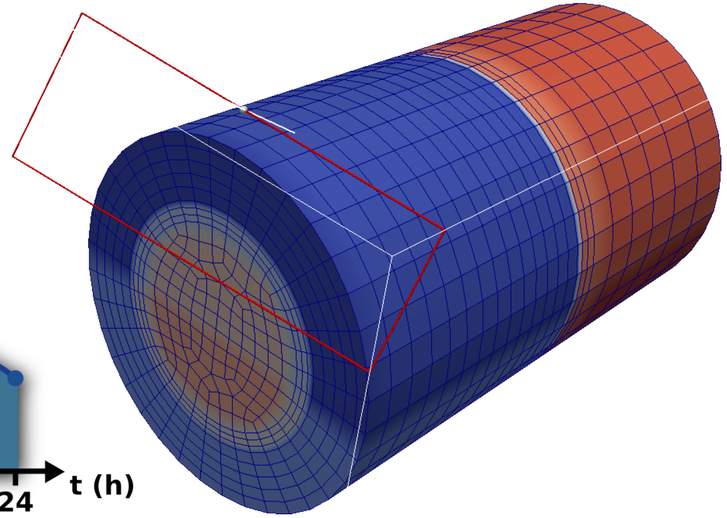
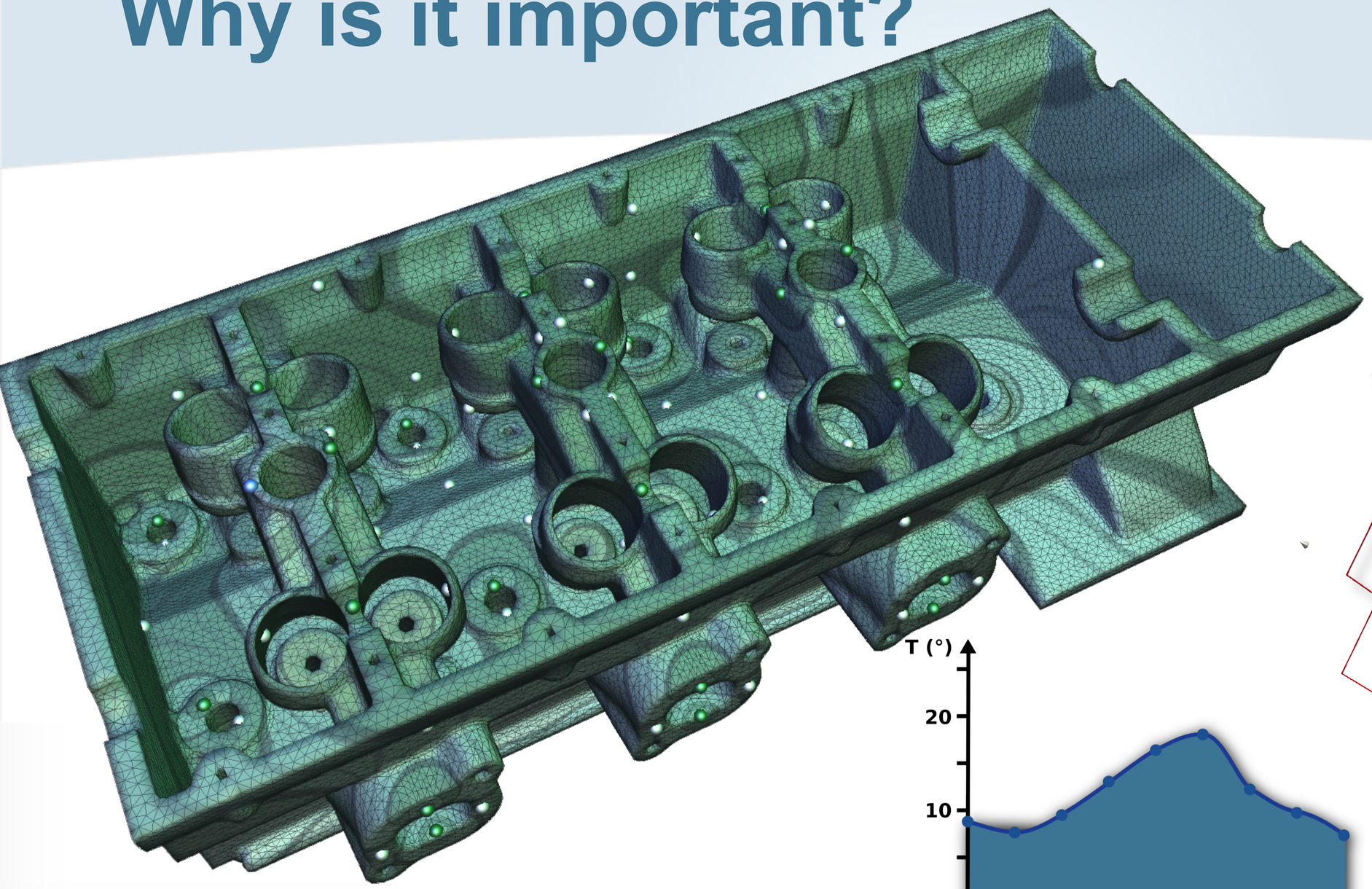
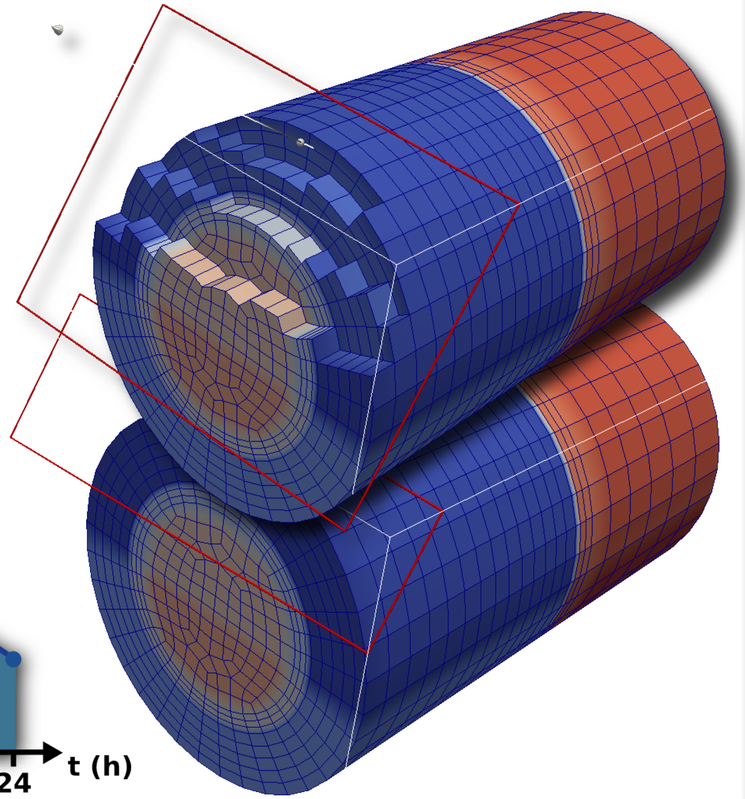


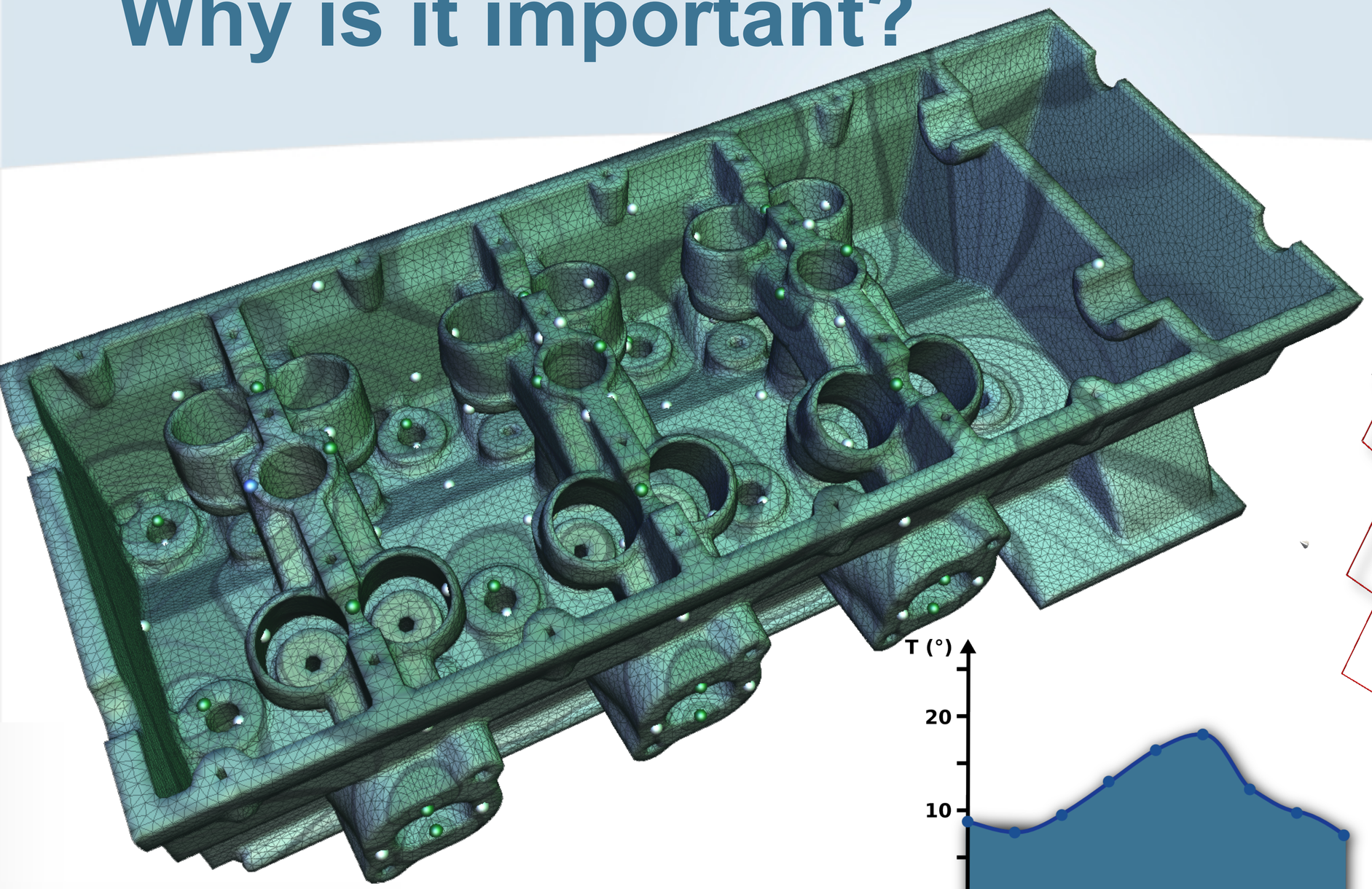


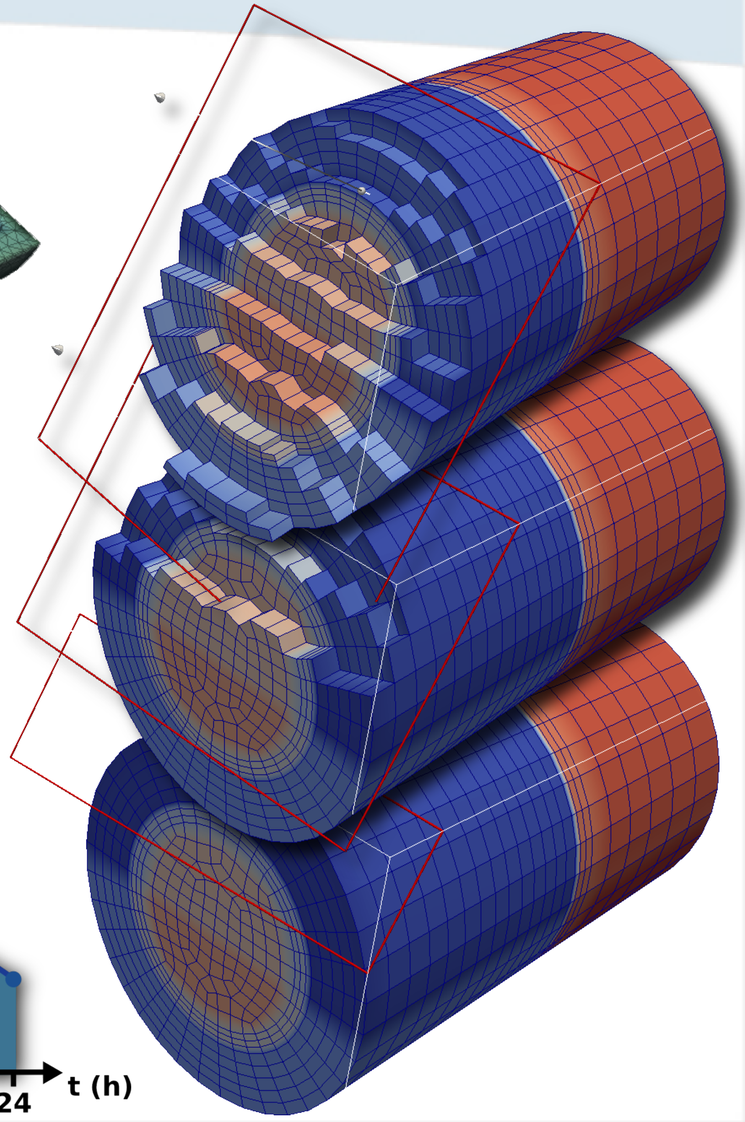
Why is it important?



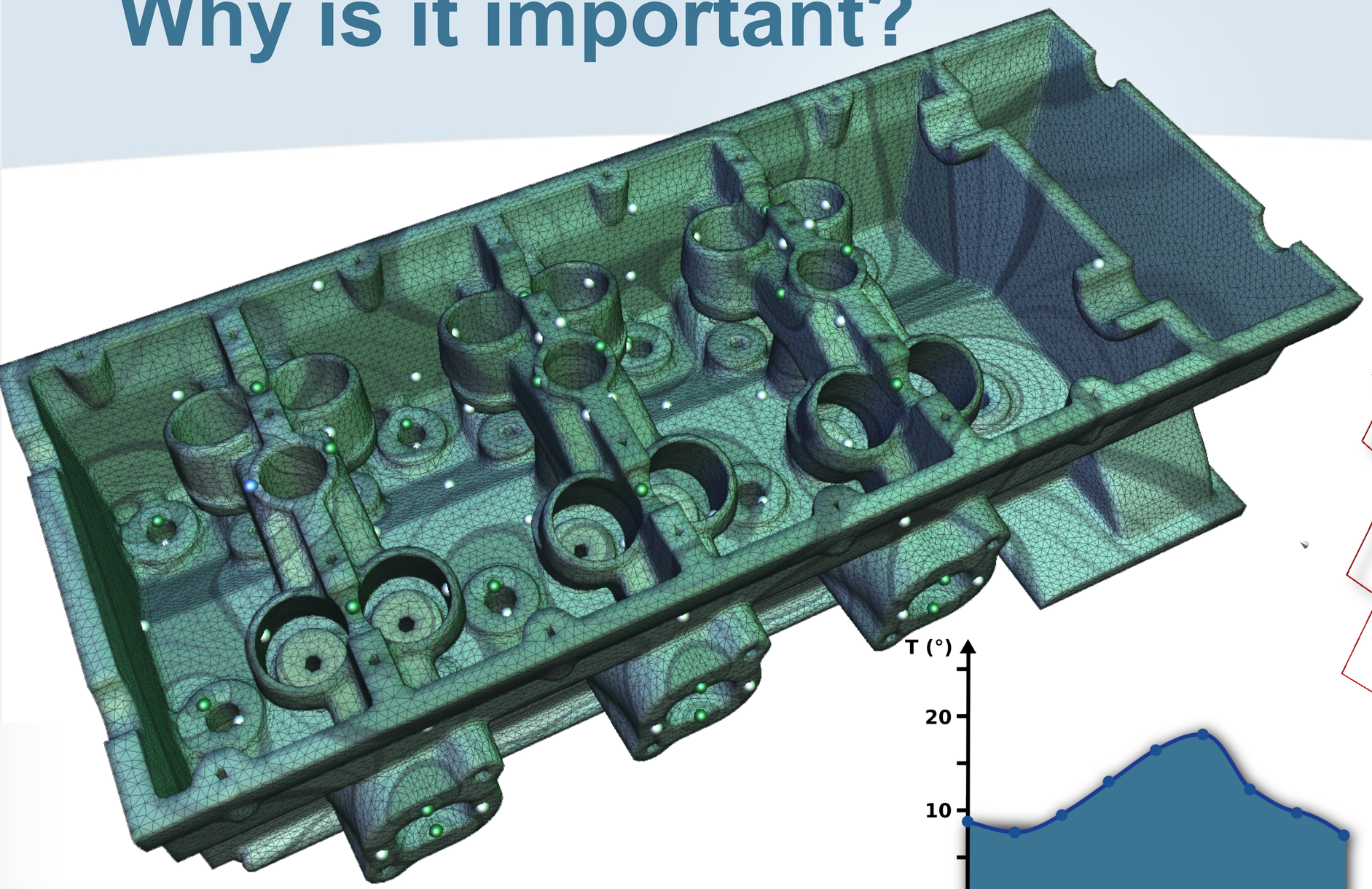


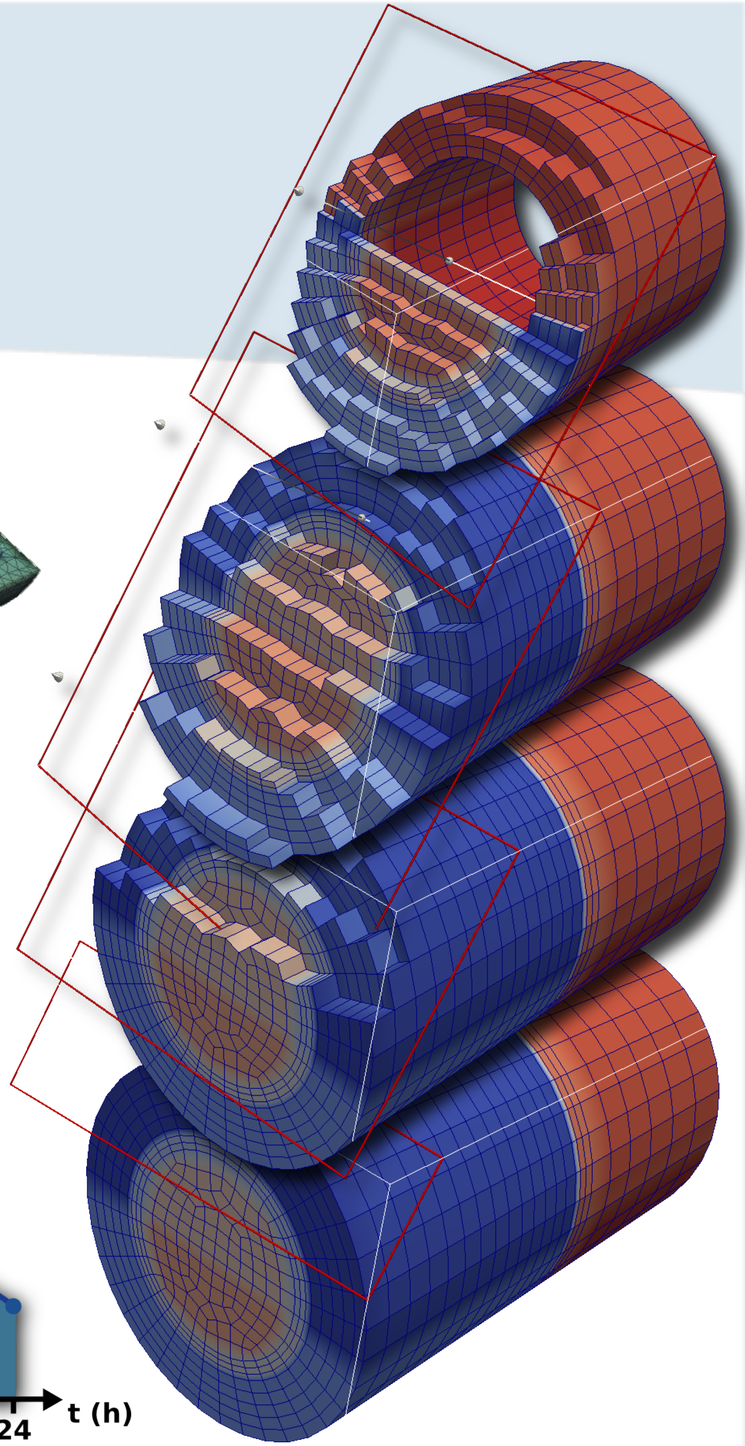
Why is it important?





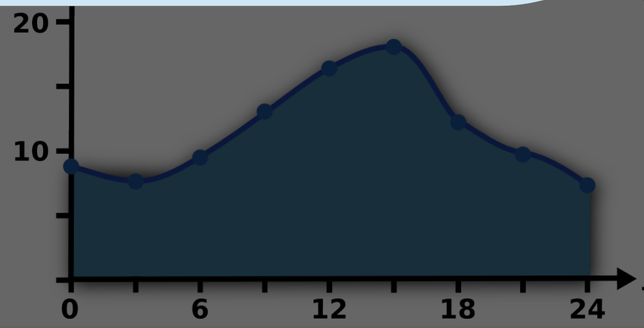
Why is it important?

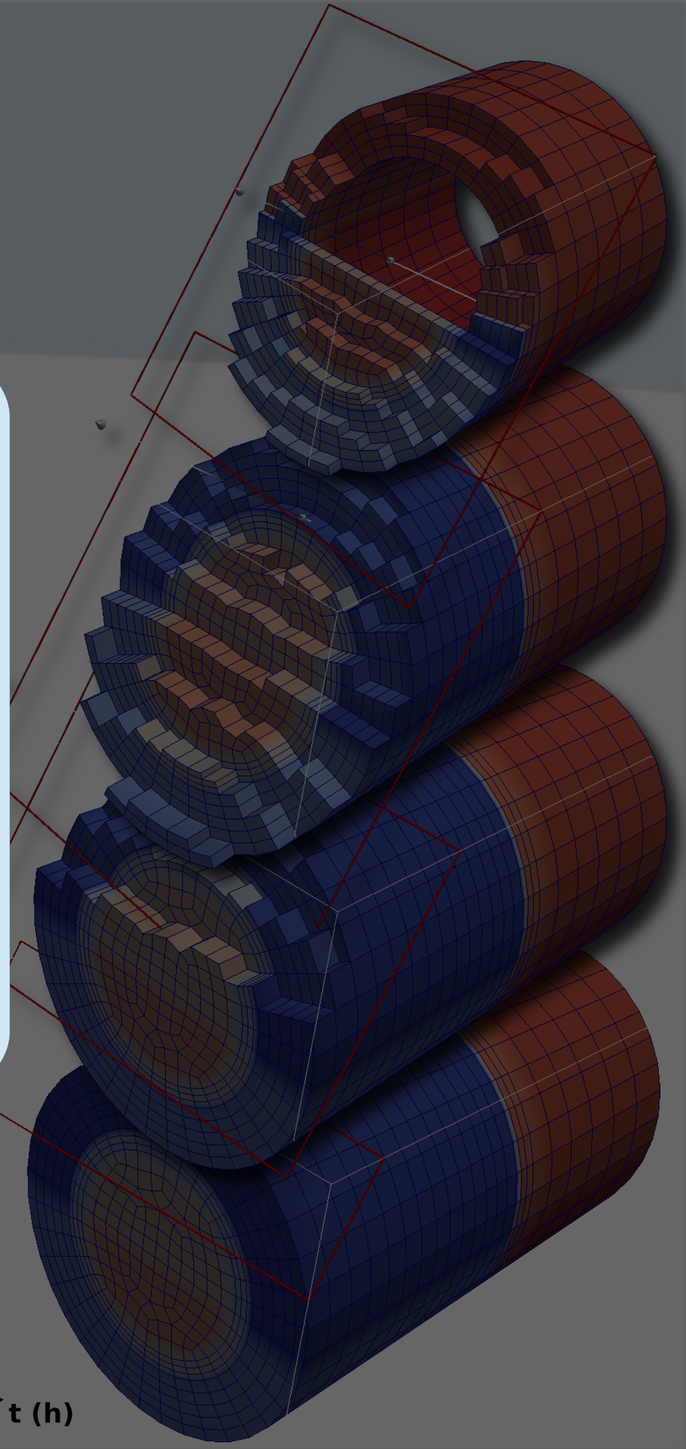




Why is it important?

- Dimension of the domain
- Topology of the domain
- Local topology of the discretization
- …





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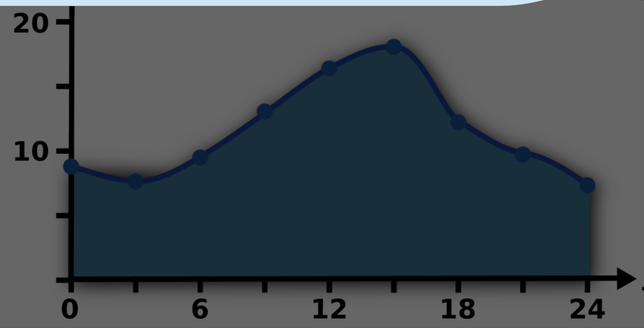
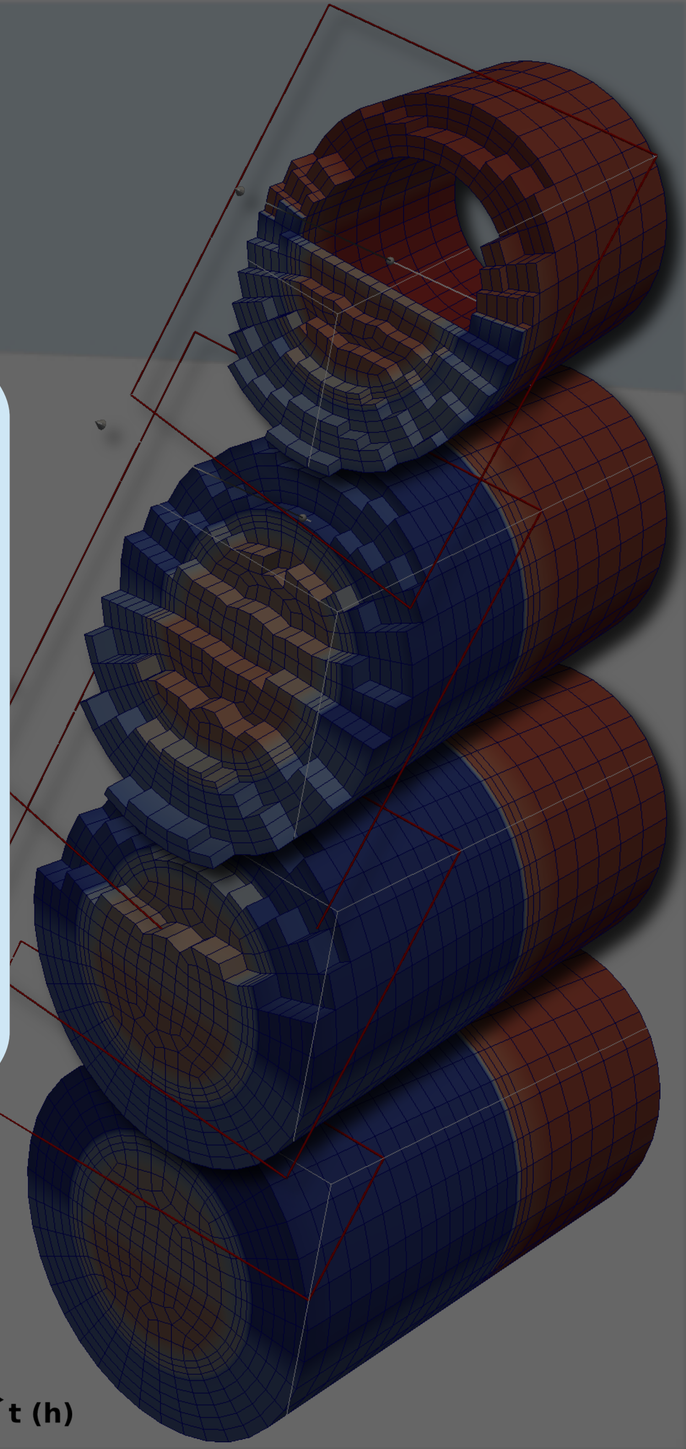
- …

→ **Adapted algorithms**





What do we mean by domain?

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$$f : \mathcal{D} \rightarrow \mathcal{A}$$

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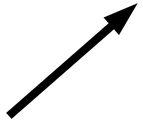
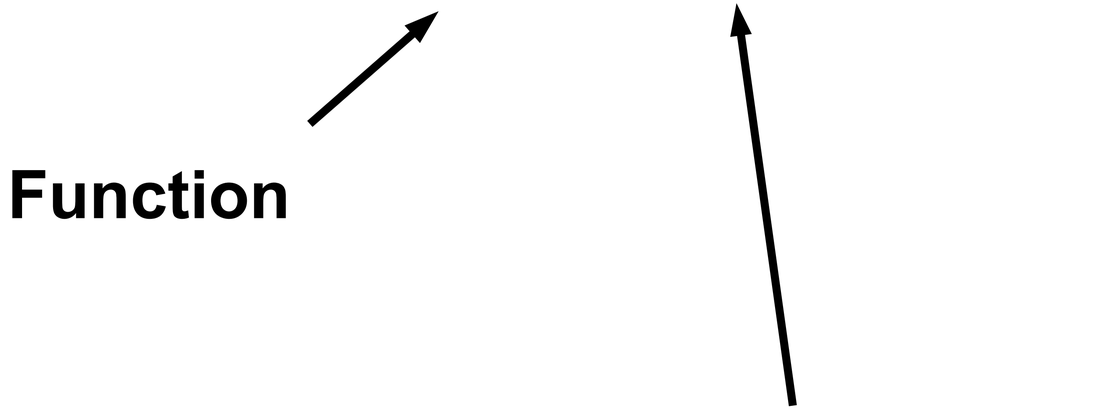


Function

What do we mean by domain?

$$f : \mathcal{D} \rightarrow \mathcal{A}$$



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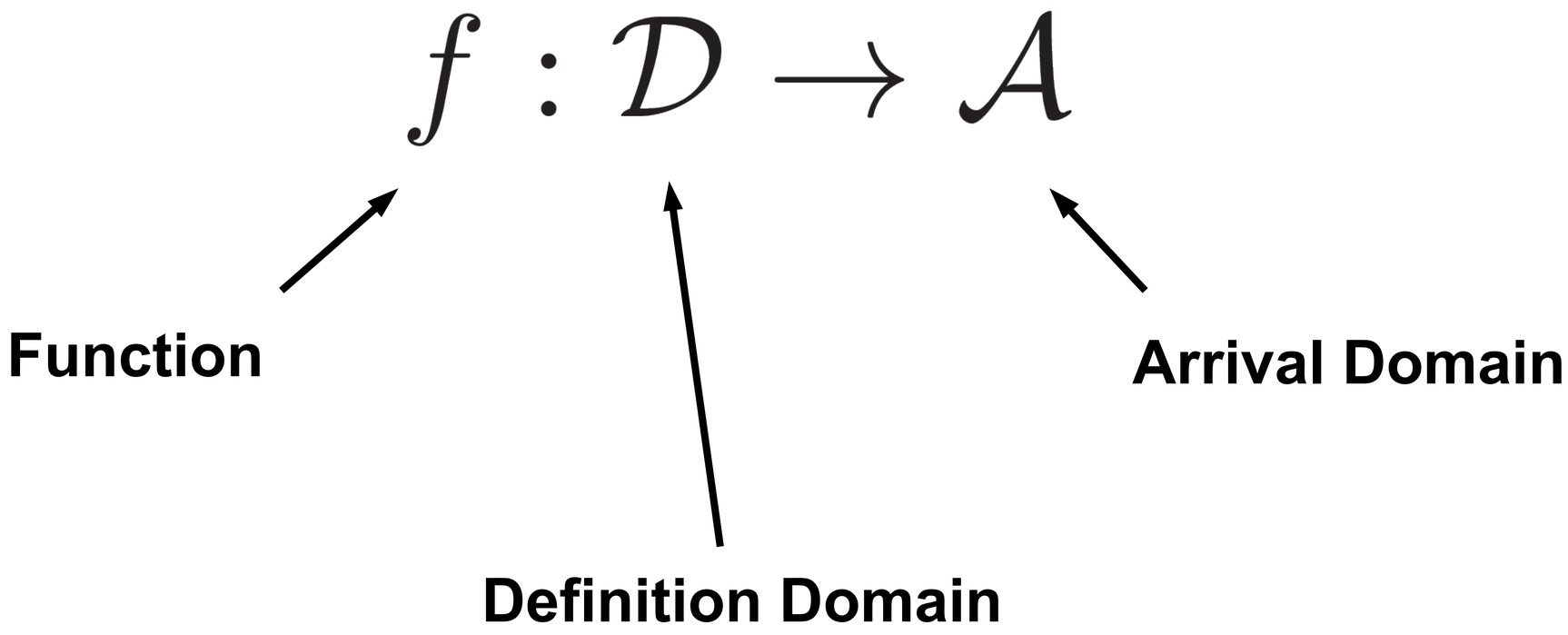

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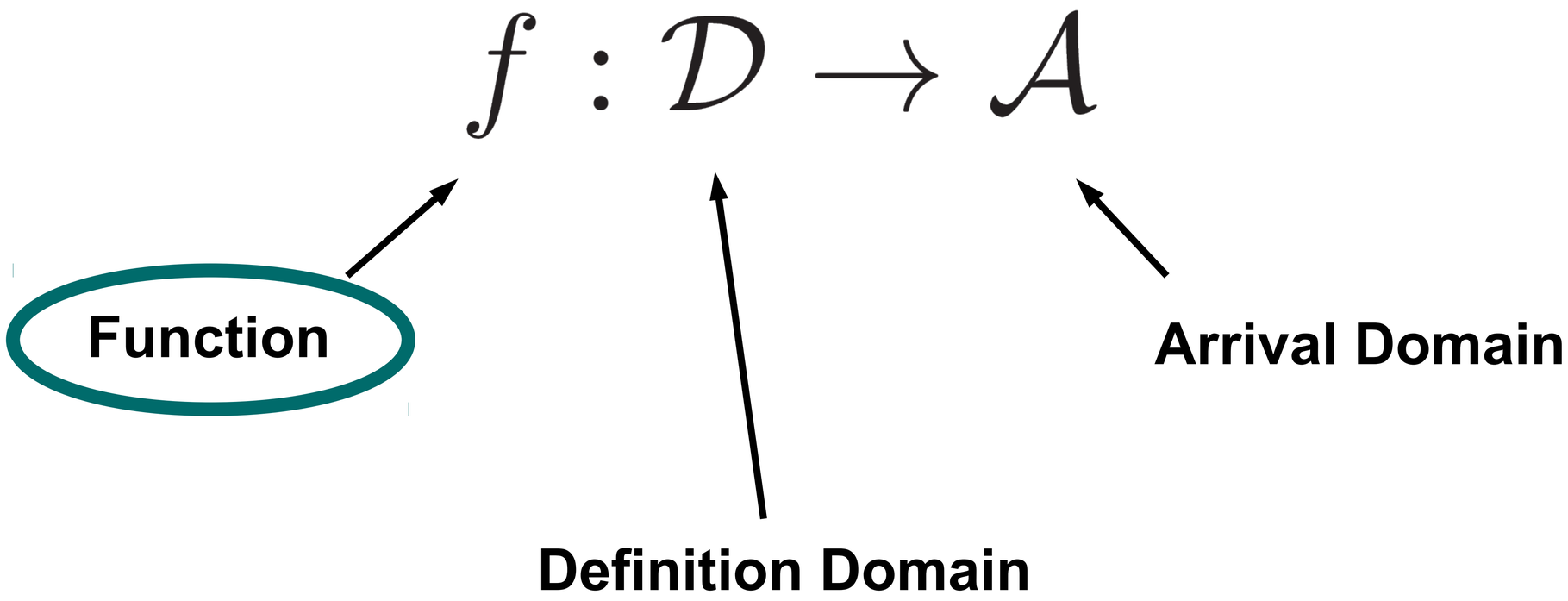

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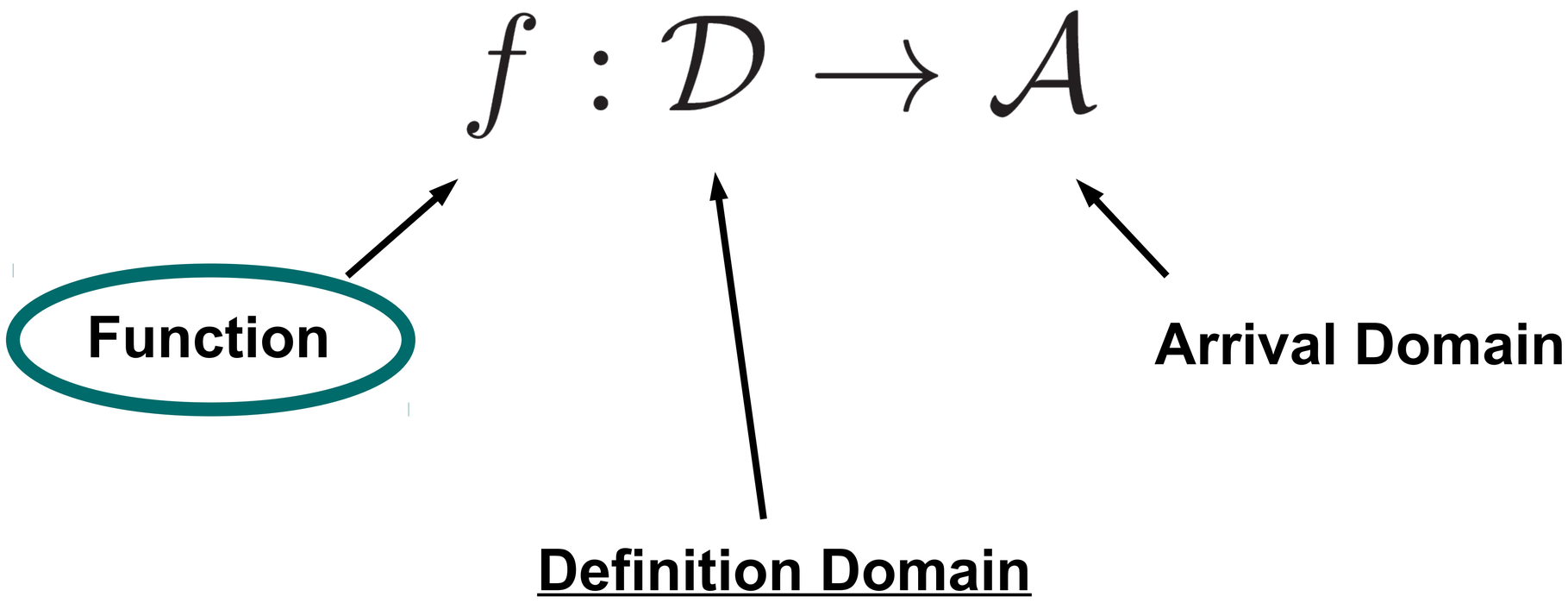

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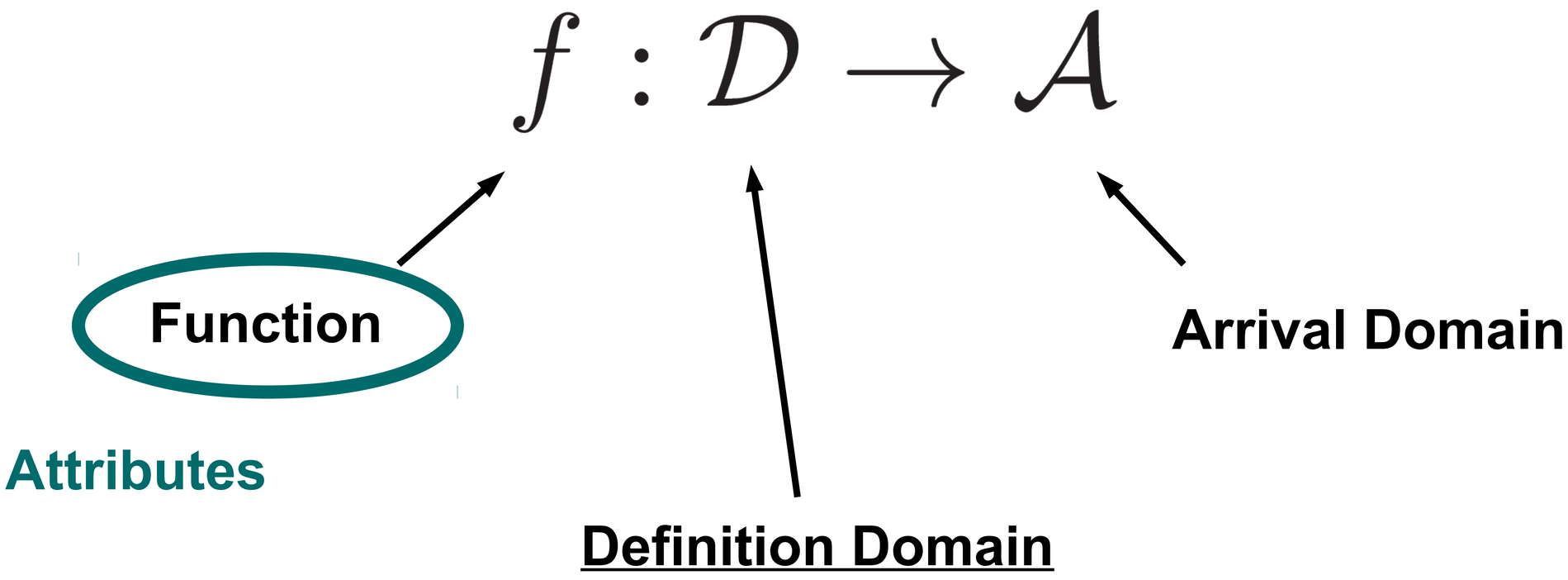

What do we mean by domain?

$$f : [0, 24] \to \mathbb{R}$$

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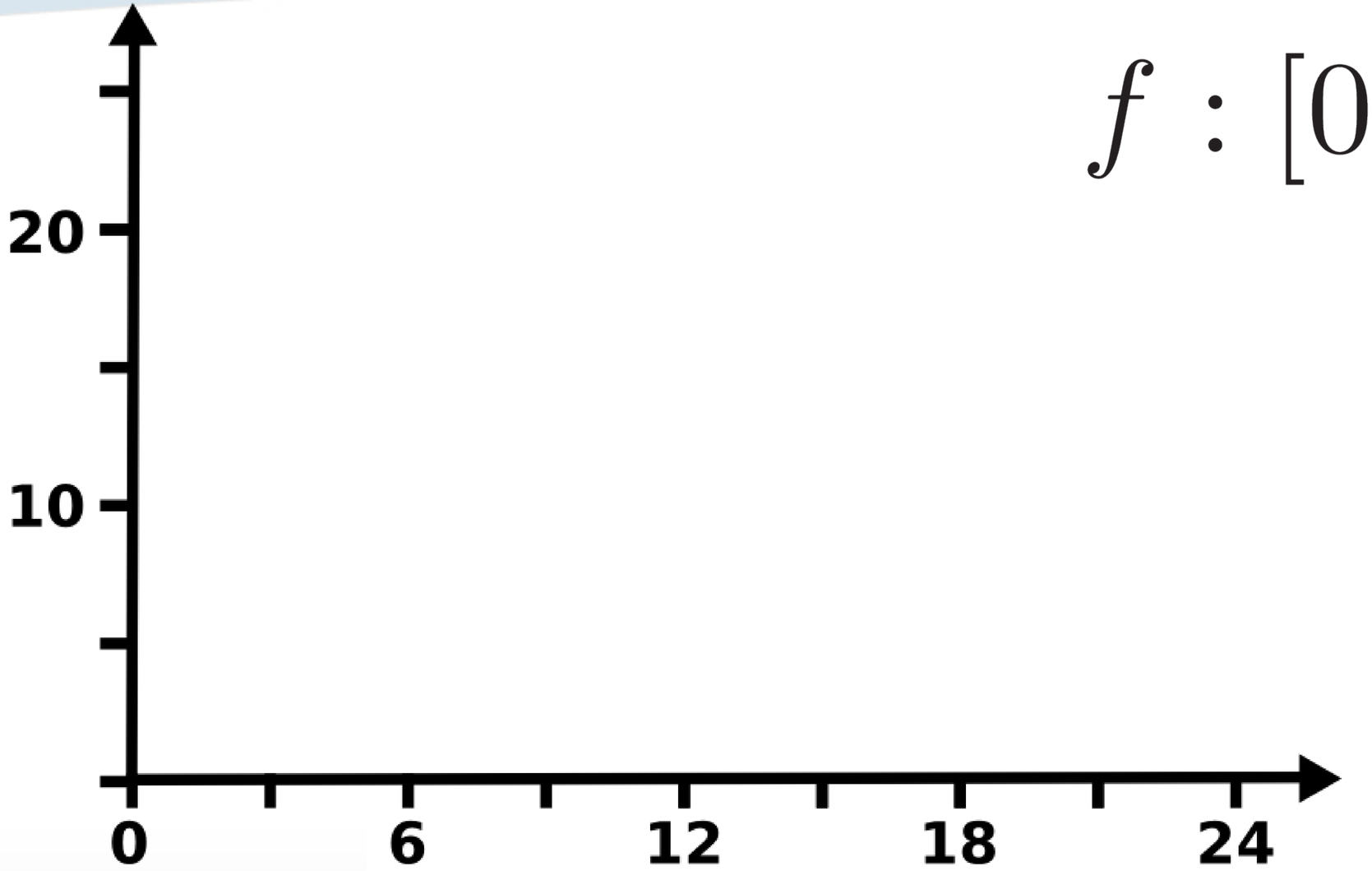

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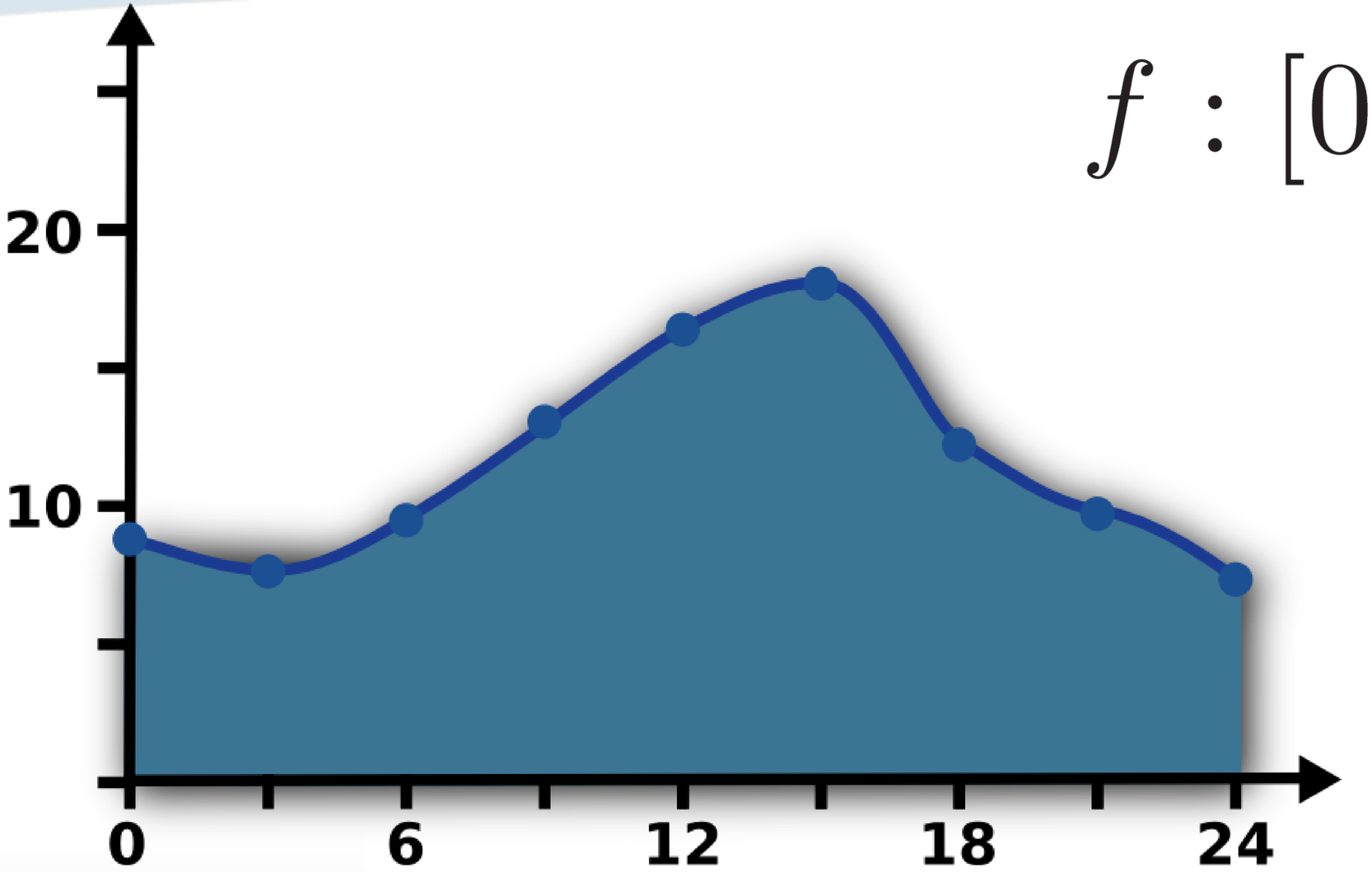
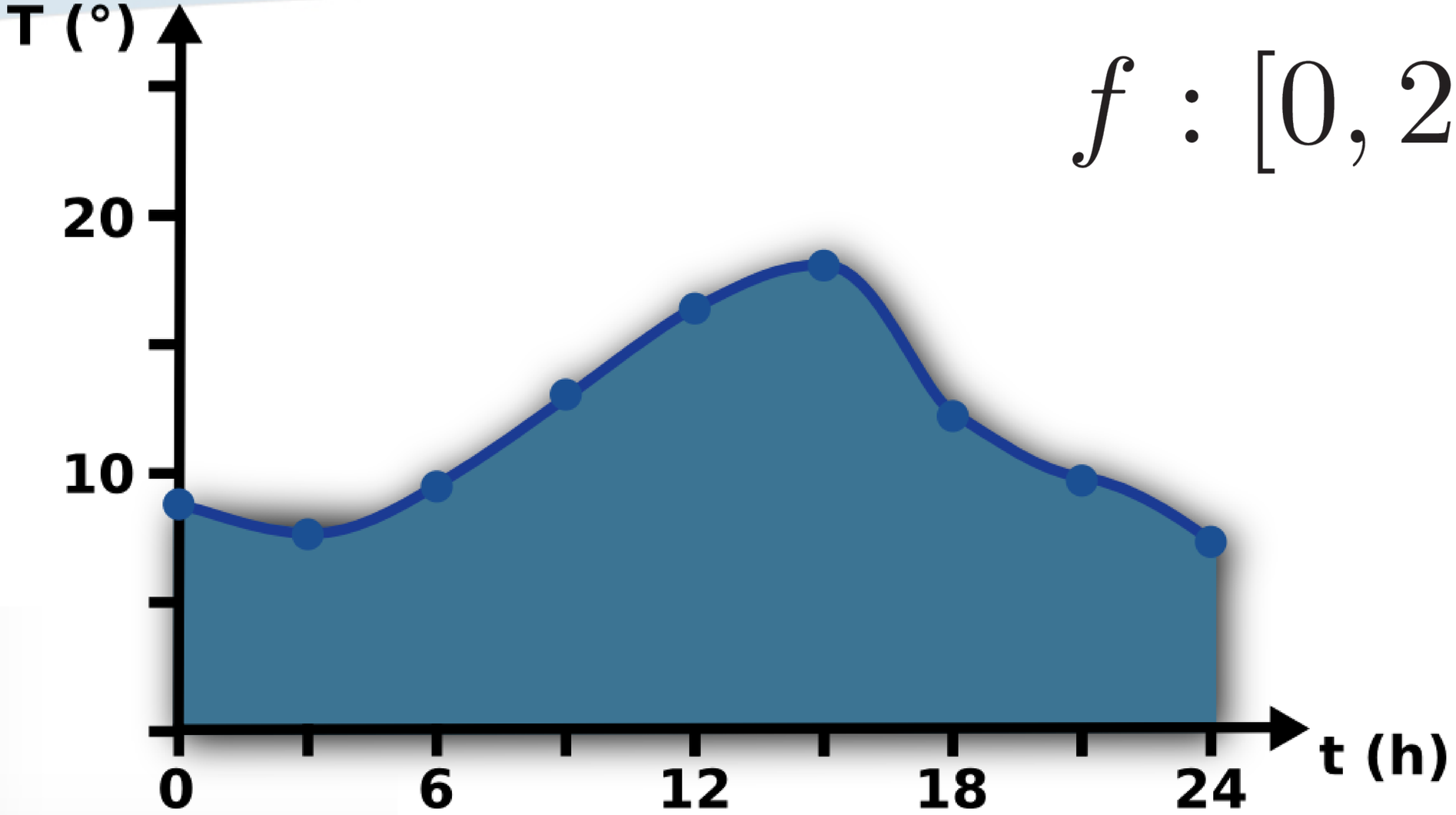


What do we mean by domain?



$$f : [0, 24] \to \mathbb{R}$$

What do we mean by domain?

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- Continuous discrete representation of a space $\mathbb{X}$

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- *Continuous discrete* representation of a space $\mathbb{X}$

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- *Continuous discrete* representation of a space $\mathbb{X}$
  - Finite set of samples

What do we mean by domain?

- *Continuous discrete* representation of a space $\mathbb{X}$
  - Finite set of samples
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- *Continuous discrete* representation of a space $\mathbb{X}$
  - Finite set of samples
    - Positional information in an embedding space $\mathbb{E}$
    - Attributes
  - Connectivity of the samples

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- *Continuous discrete* representation of a space $\mathbb{X}$
  - Finite set of samples
    - Positional information in an embedding space $\mathbb{E}$
    - Attributes
  - Connectivity of the samples
    - Continuity of the domain

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- *Continuous discrete* representation of a space $\mathbb{X}$
  - Finite set of samples
    - Positional information in an embedding space $\mathbb{E}$
    - Attributes
  - Connectivity of the samples
    - Continuity of the domain
    - Cellular elements (dimension of $\mathbb{X}$)

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- *Continuous discrete* representation of a space $\mathbb{X}$
  - Finite set of samples
    - Positional information in an embedding space $\mathbb{E}$
    - Attributes
  - Connectivity of the samples
    - Continuity of the domain
    - Cellular elements (dimension of $\mathbb{X}$)
  - Cell interpolation scheme

What do we mean by domain?

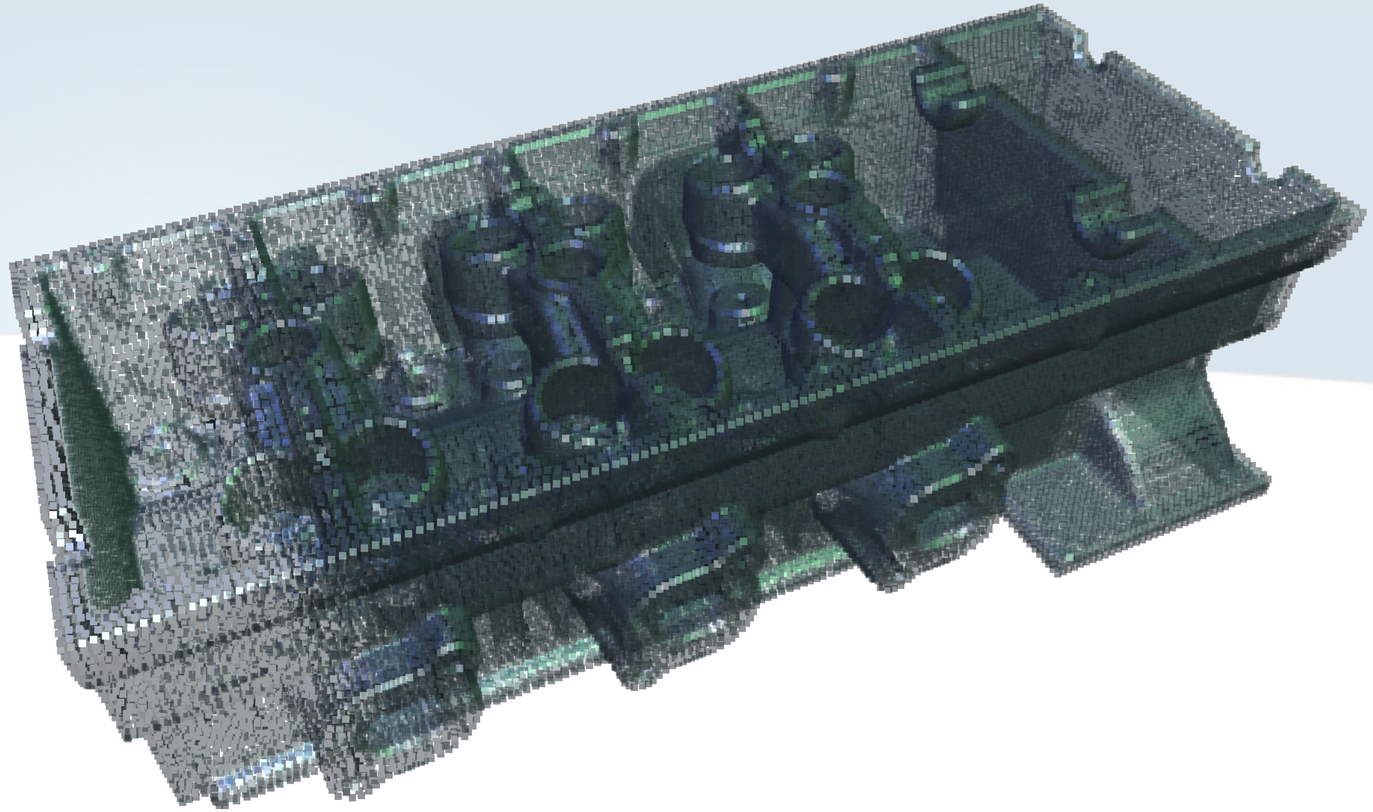
- *Continuous discrete* representation of a space $\mathbb{X}$
  - Finite set of samples
    - Positional information in an embedding space $\mathbb{E}$
    - Attributes
  - Connectivity of the samples
    - Continuity of the domain
    - Cellular elements (dimension of $\mathbb{X}$)
  - Cell interpolation scheme
    - Continuity of the attributes

What do we mean by domain?

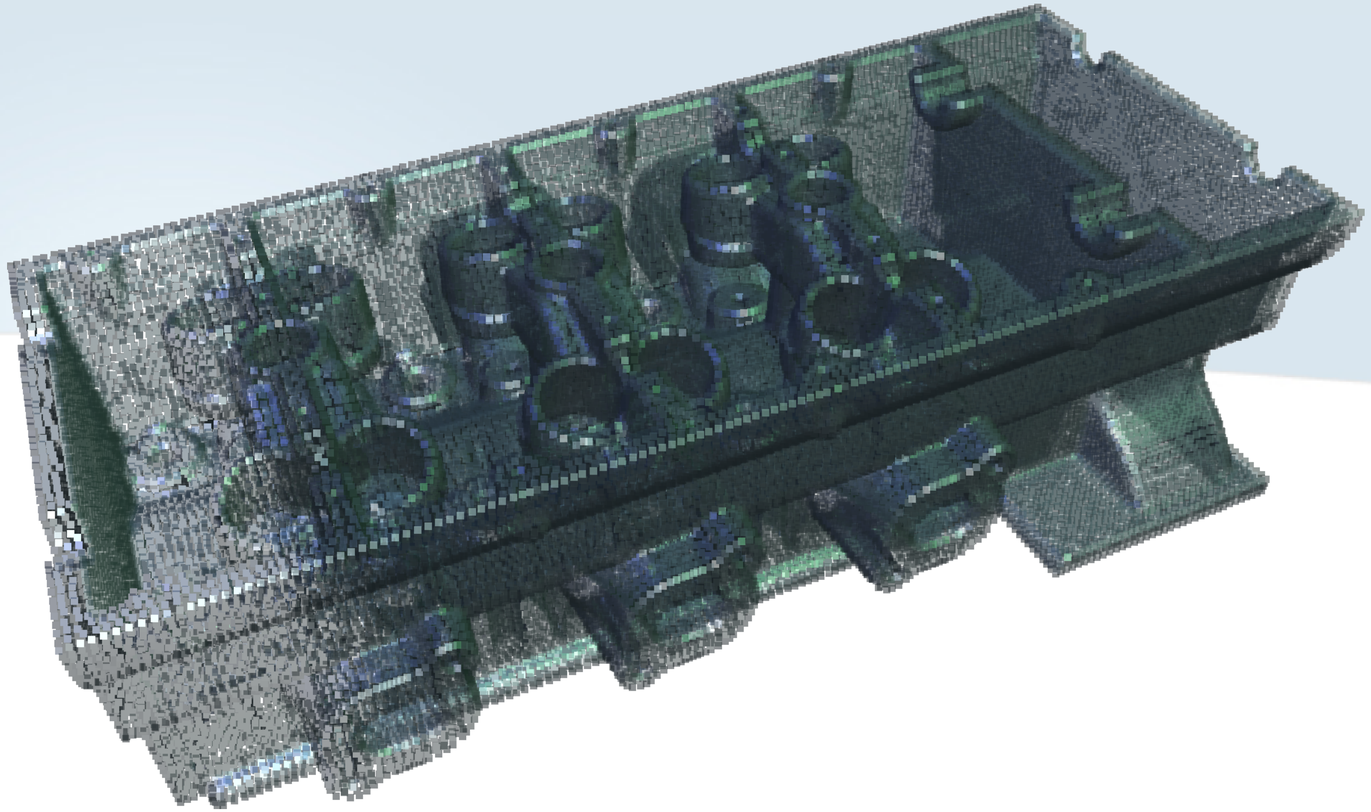
- *Continuous discrete* representation of a space $\mathbb{X}$
  - **Finite set of samples**
    - Positional information in an embedding space $\mathbb{E}$
    - Attributes
  - **Connectivity of the samples**
    - Continuity of the domain
    - Cellular elements (dimension of $\mathbb{X}$)
  - **Cell interpolation scheme**
    - Continuity of the attributes

Example

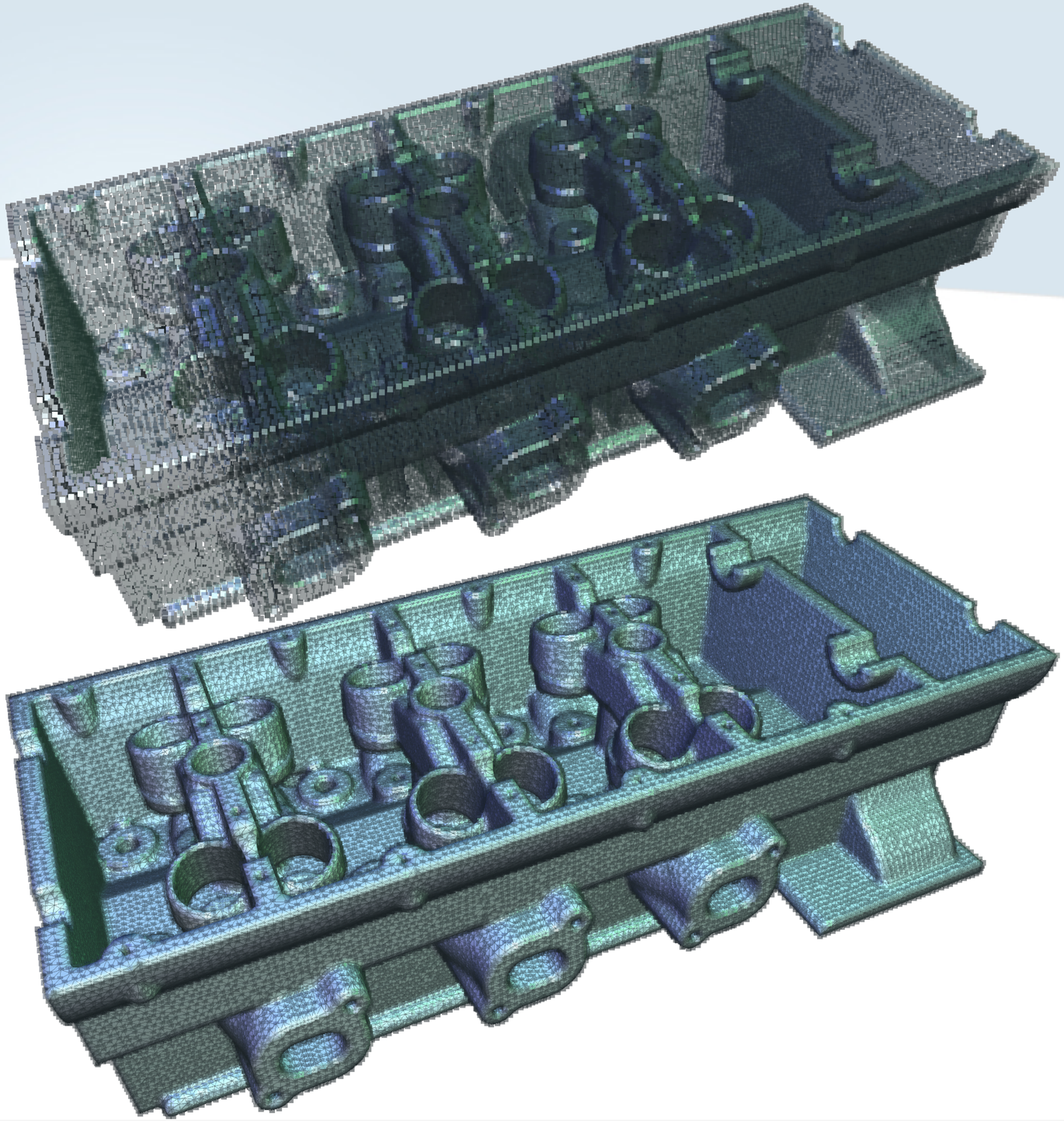
Example



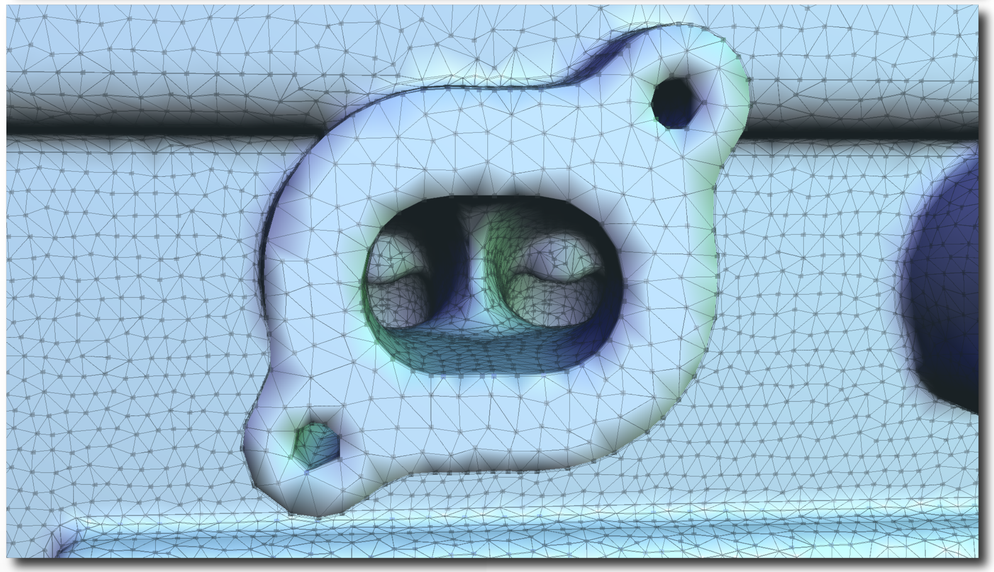
Example

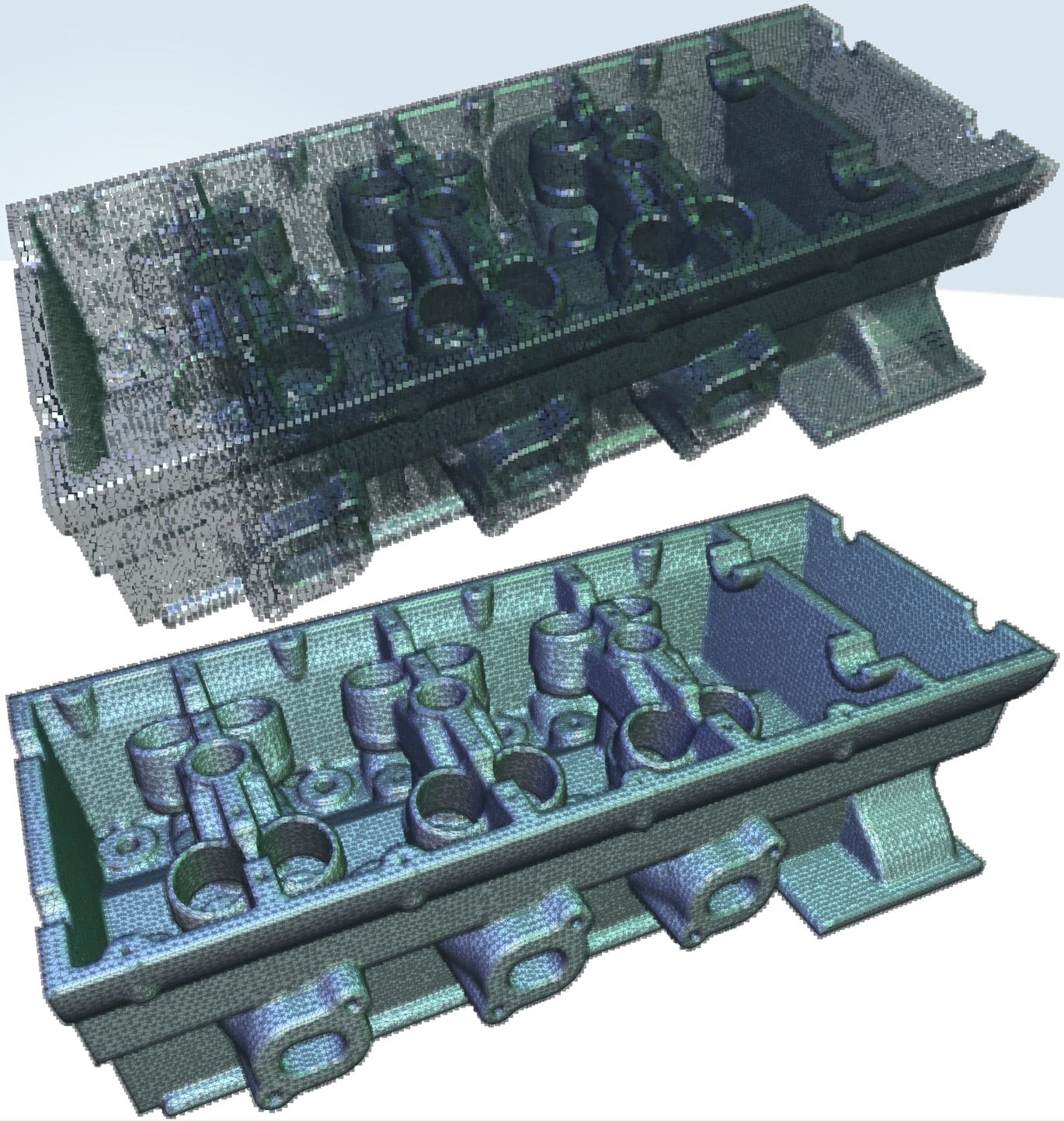


Example

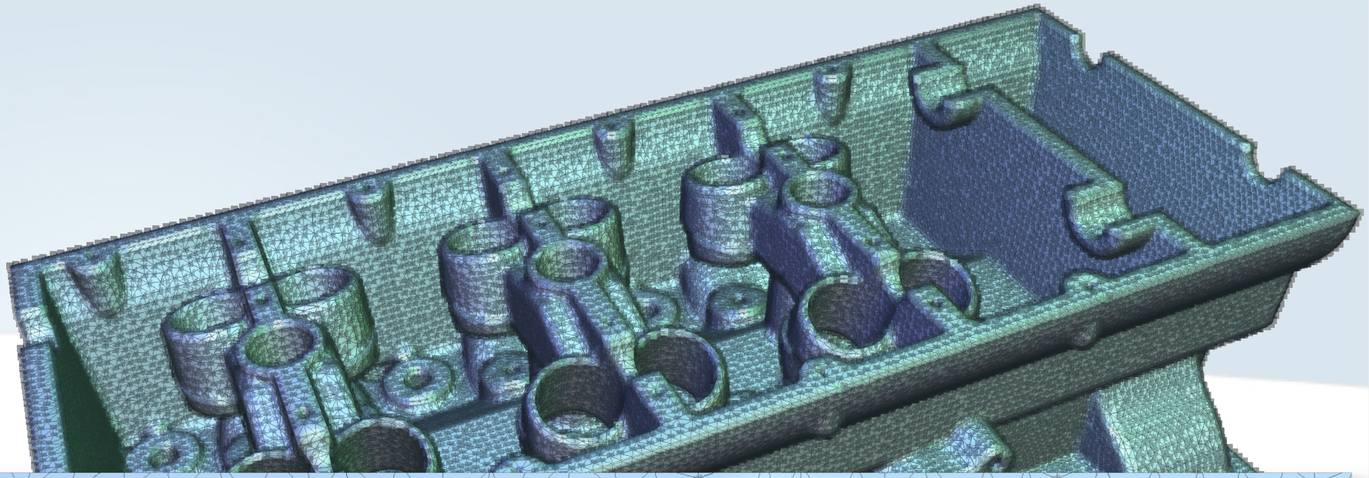


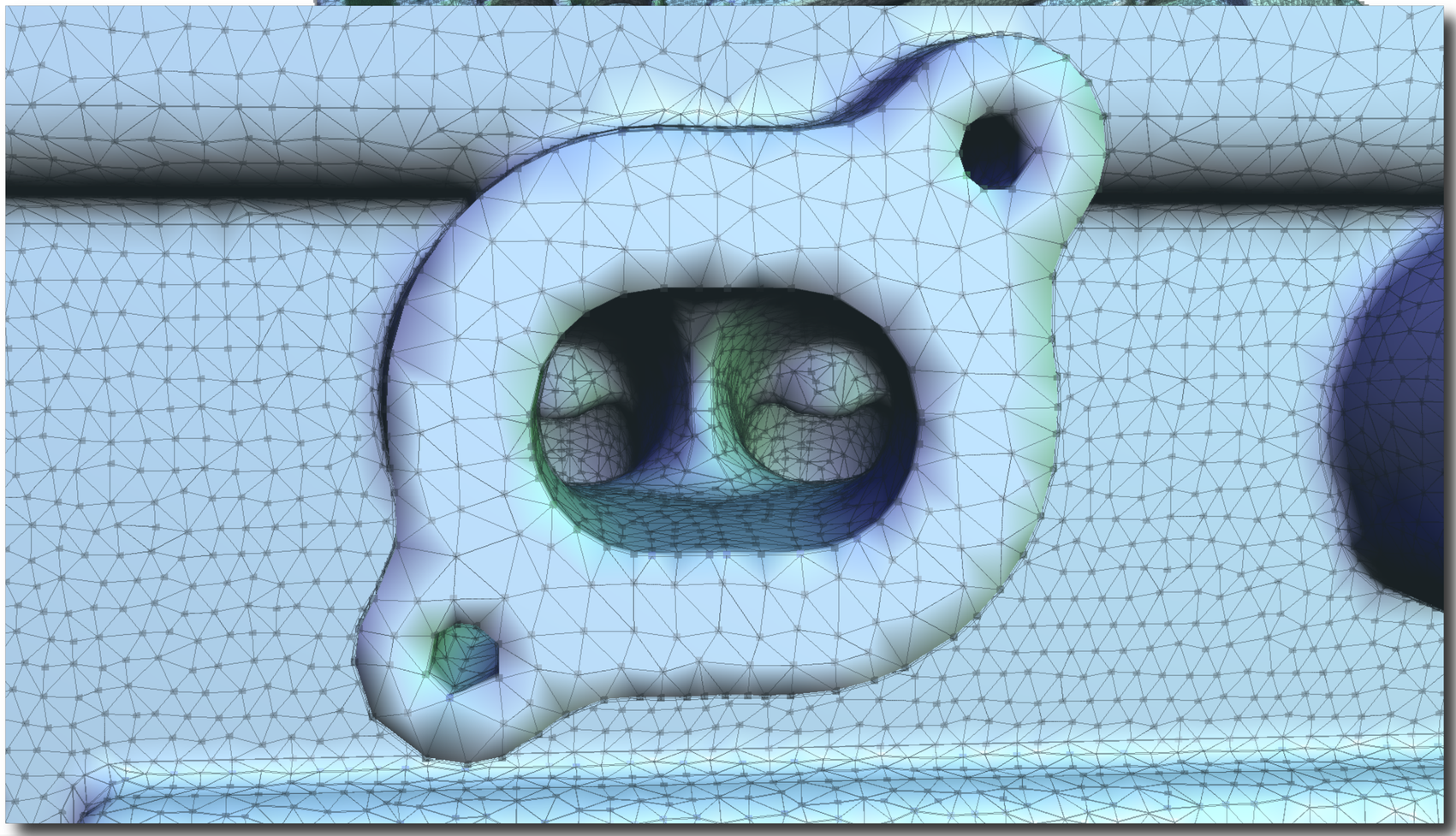
Example



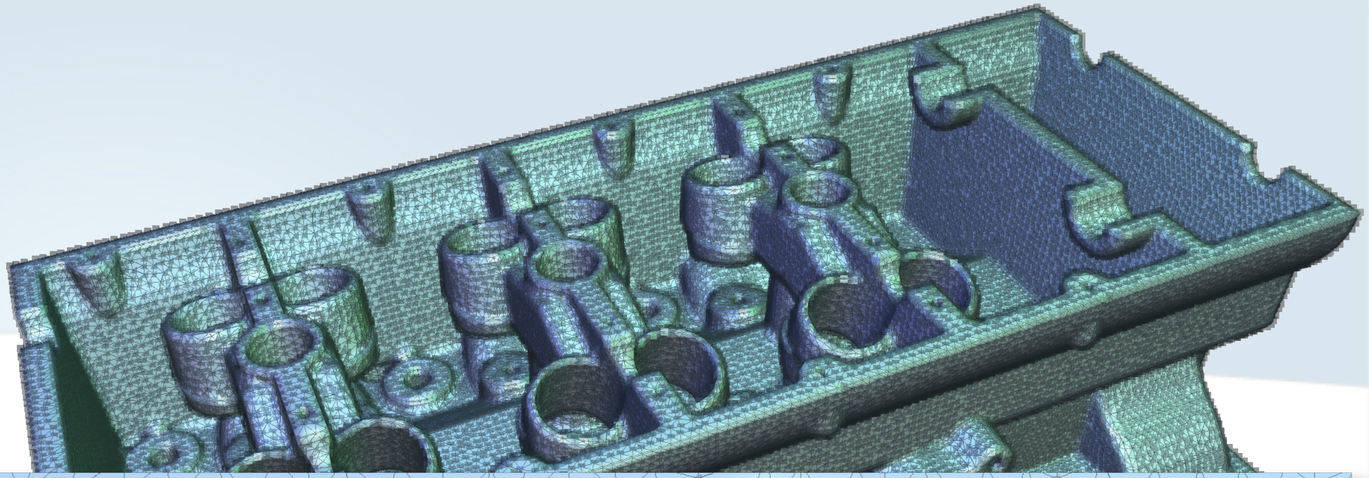


Example

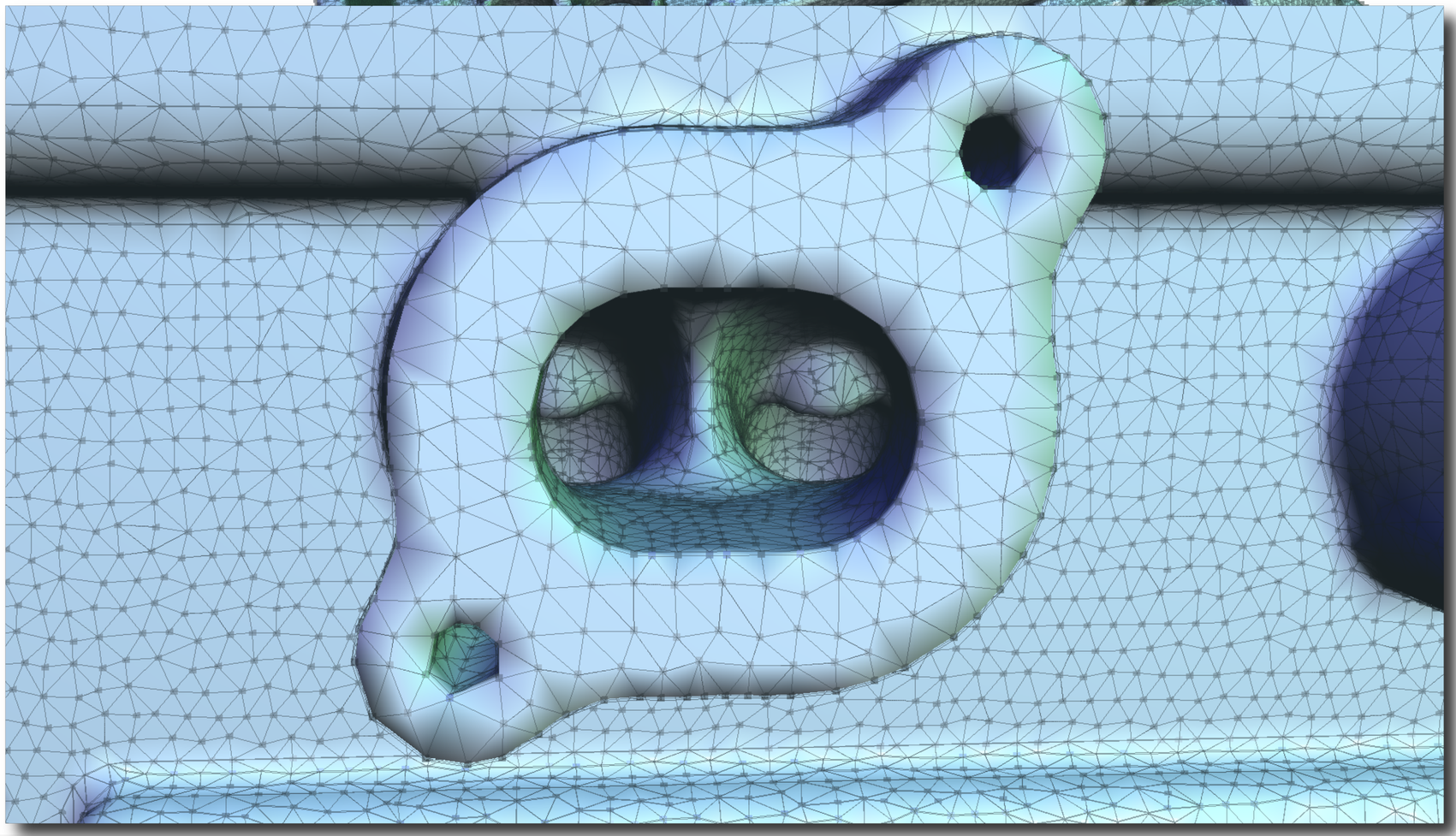




Example



- Domain
- Embedding space
- Cellular elements



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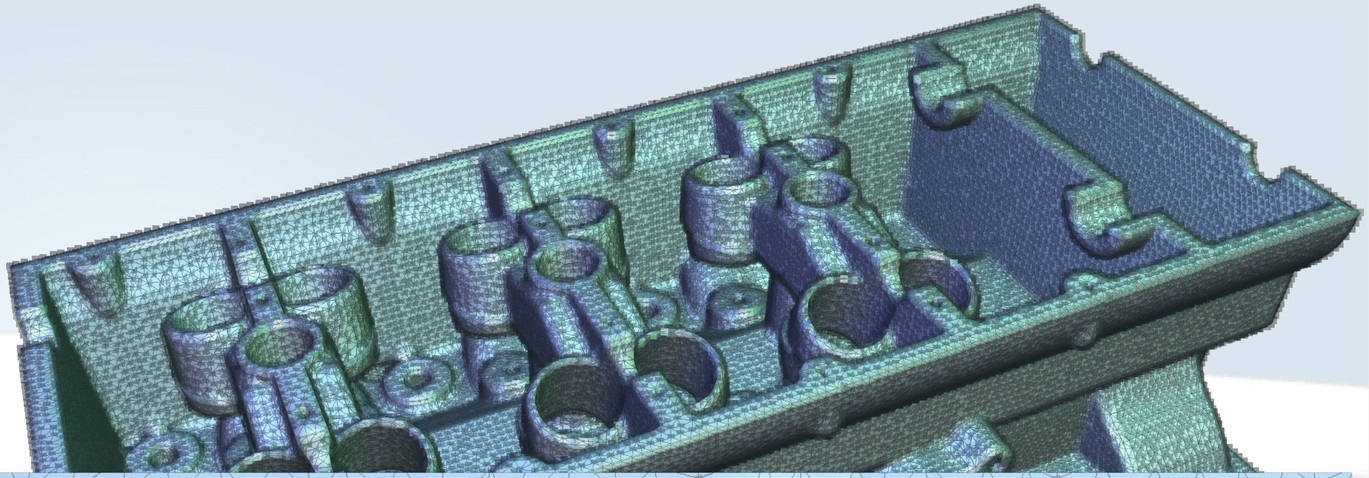
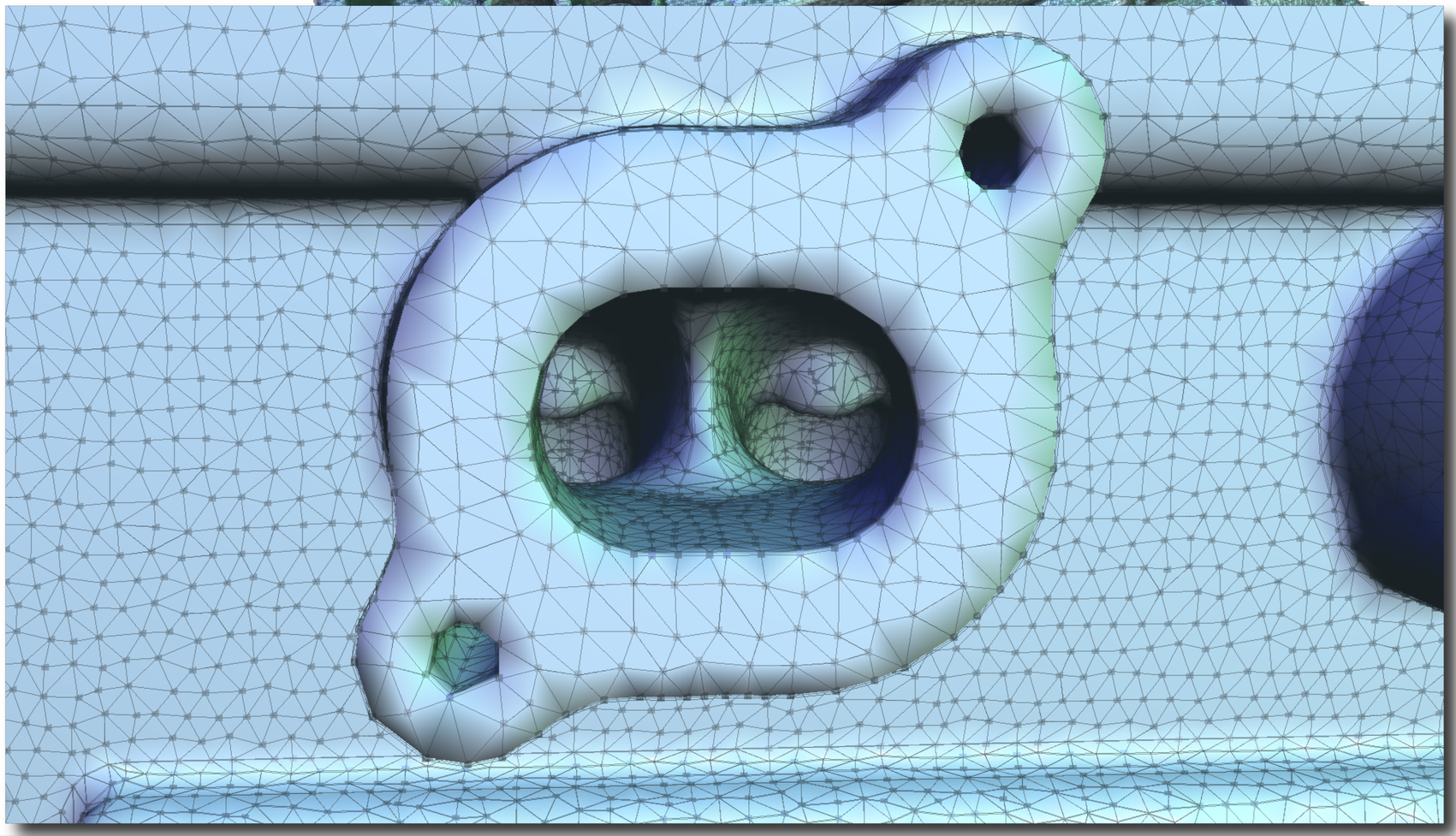


- Domain
  - 2-manifold $\mathcal{S}$
- Embedding space

- Cellular elements



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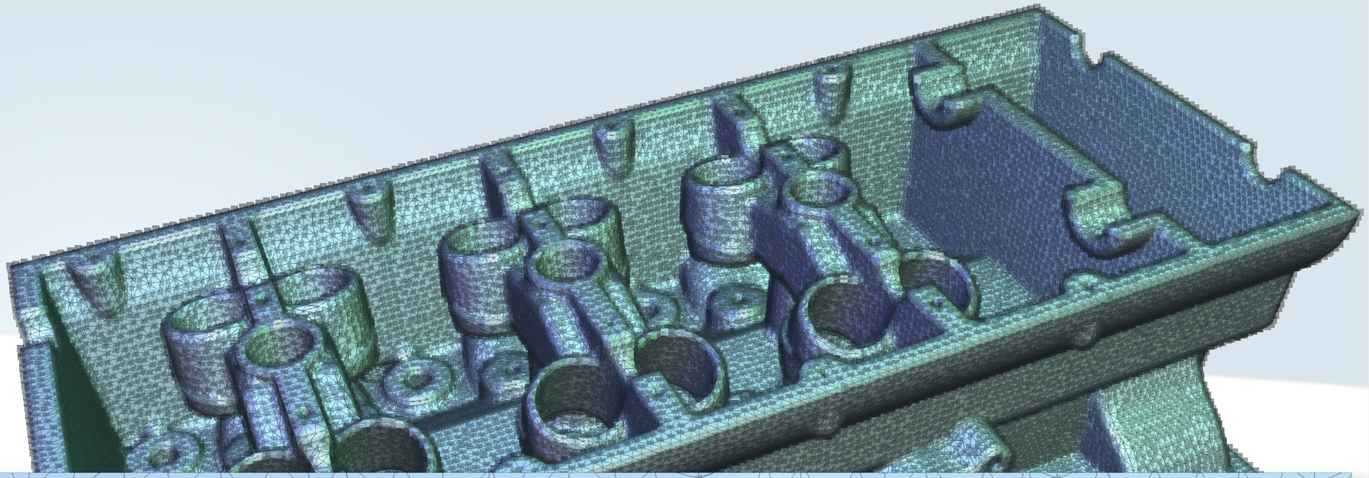
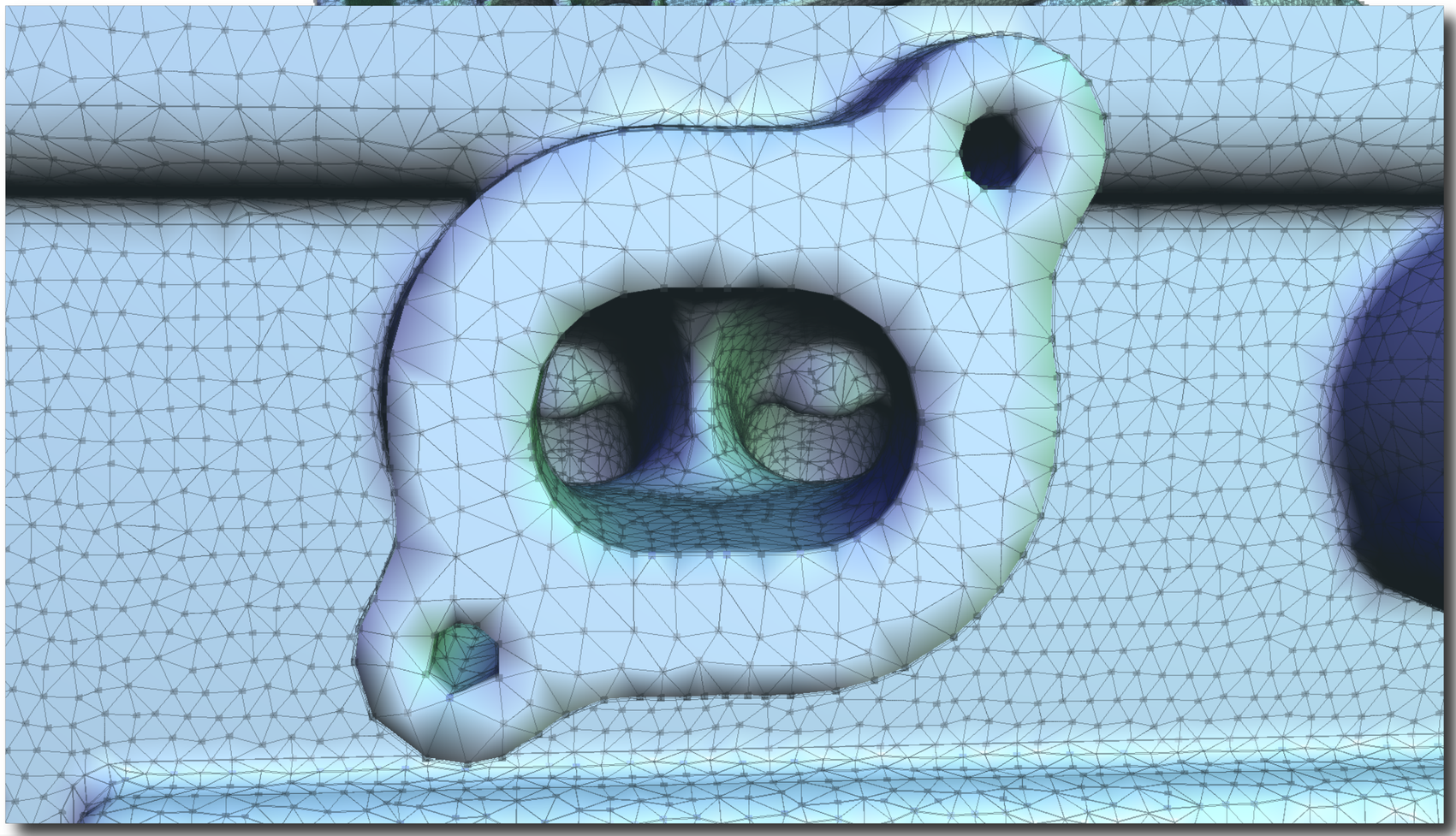
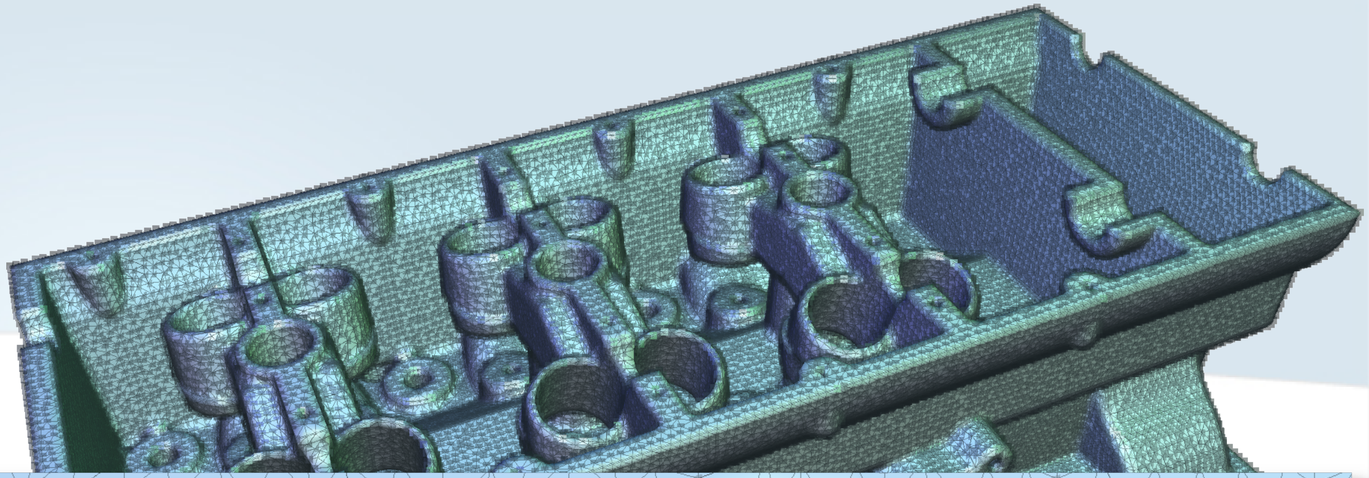


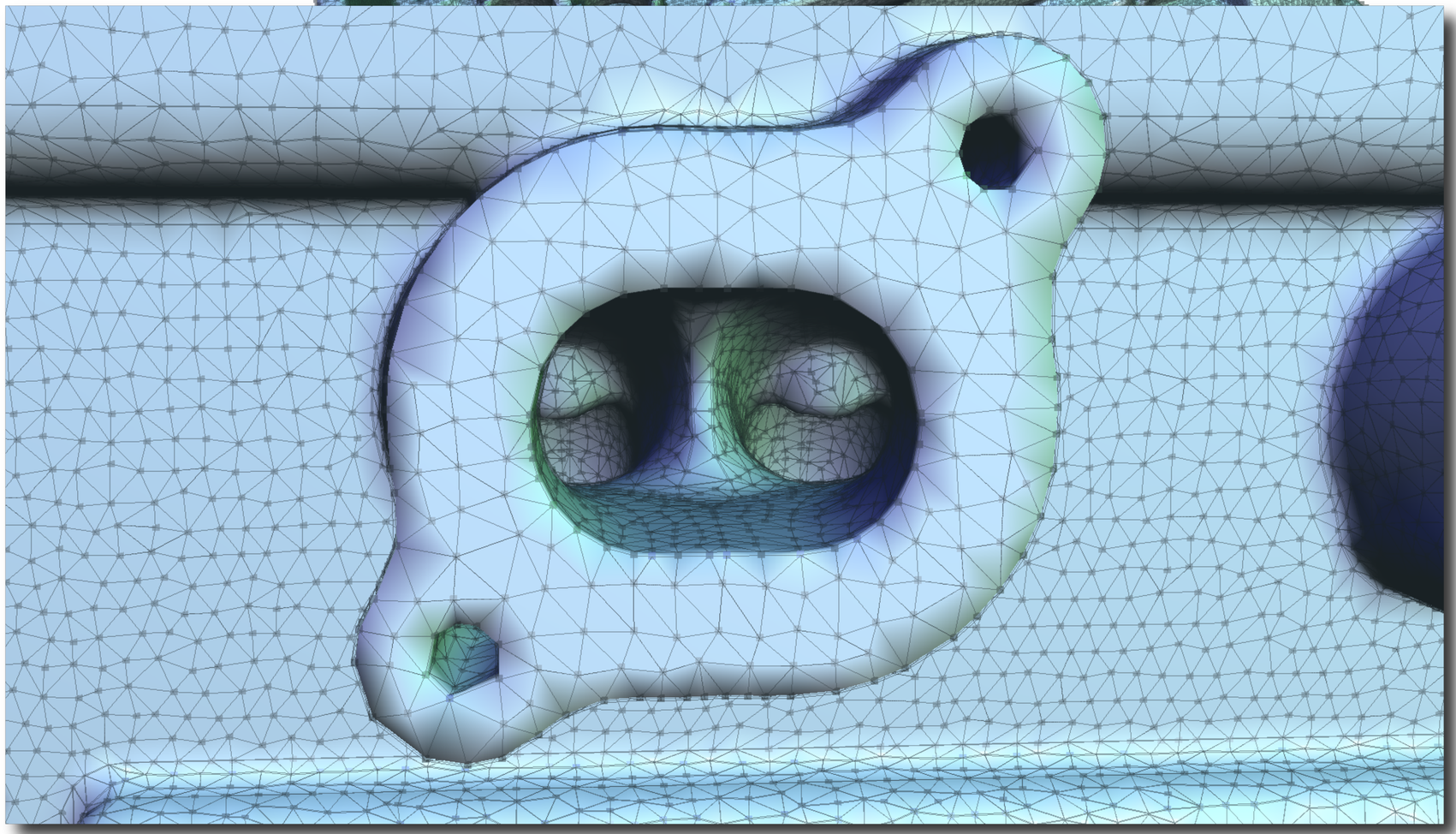
- Domain
  - 2-manifold $\mathcal{S}$
- Embedding space
  - $\mathbb{R}^3$
- Cellular elements



Example



- Domain
  - 2-manifold $\mathcal{S}$
- Embedding space
  - $\mathbb{R}^3$
- Cellular elements
  - Triangles



Domains of interest: Manifolds

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- d-manifold

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  - Topological space such that:

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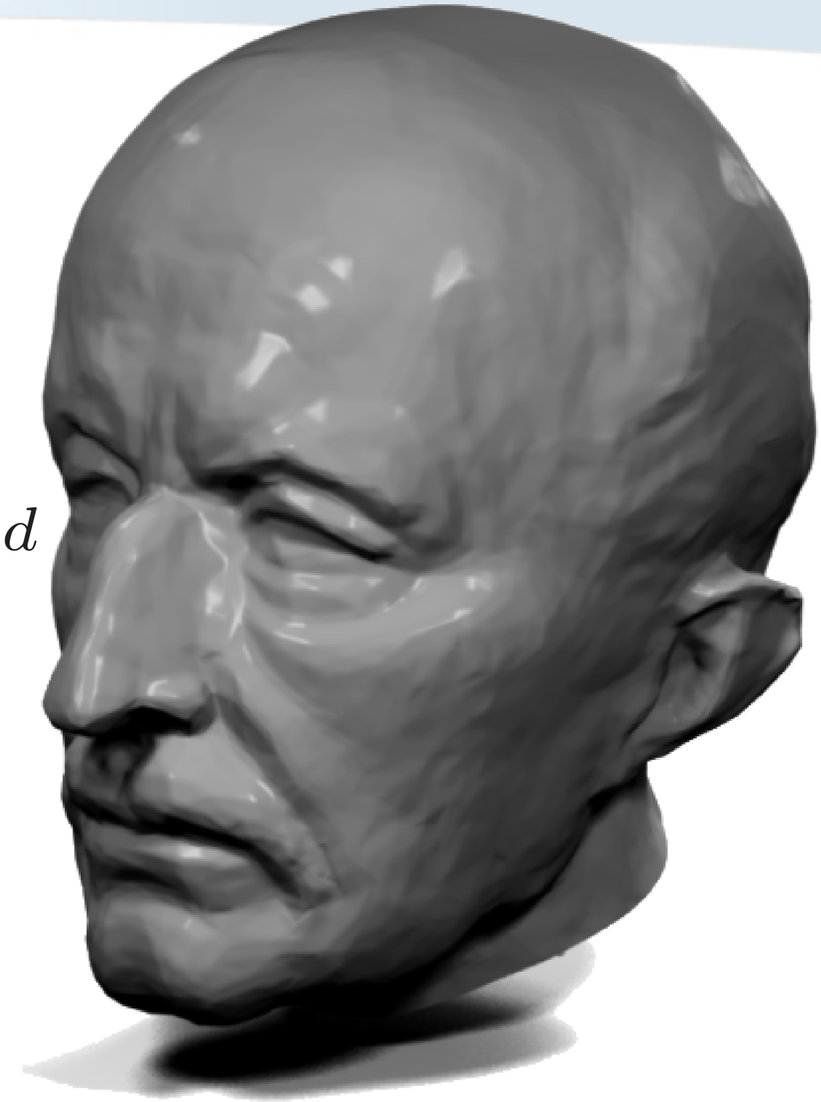
    - Any point has a neighborhood homeomorphic to a neighborhood of $\mathbb{R}^d$



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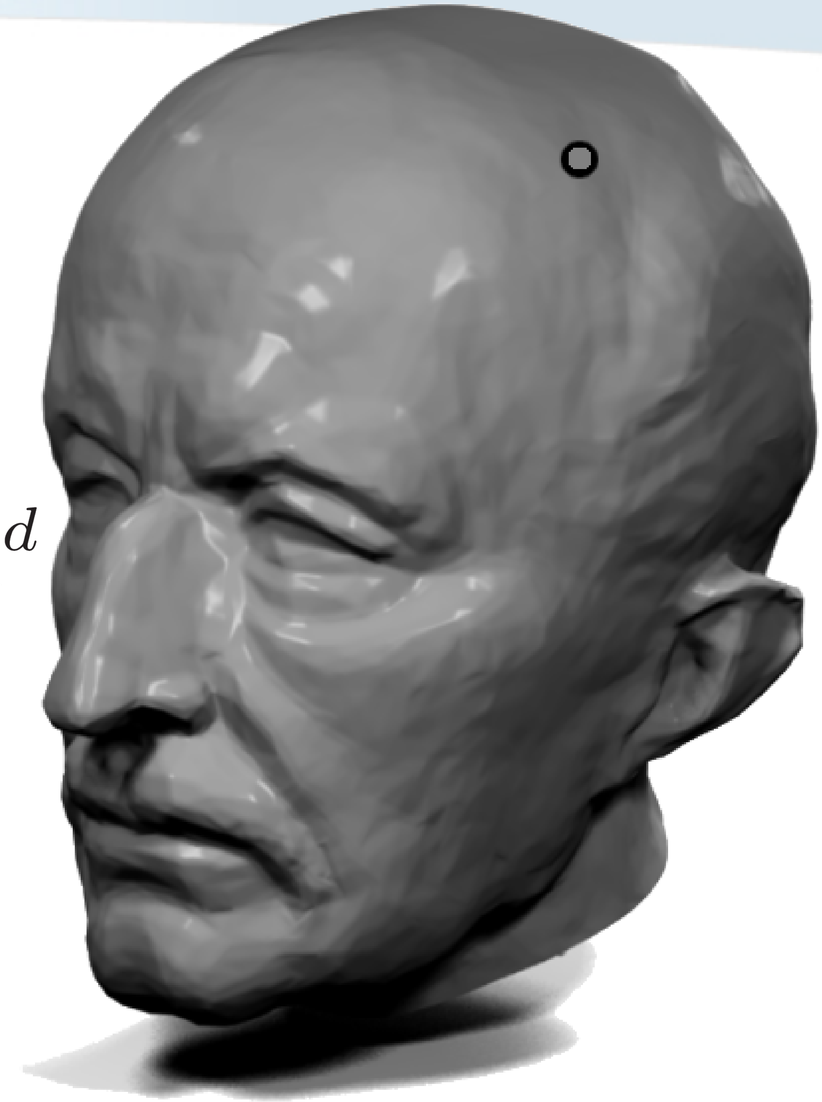

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- d-manifold with boundary
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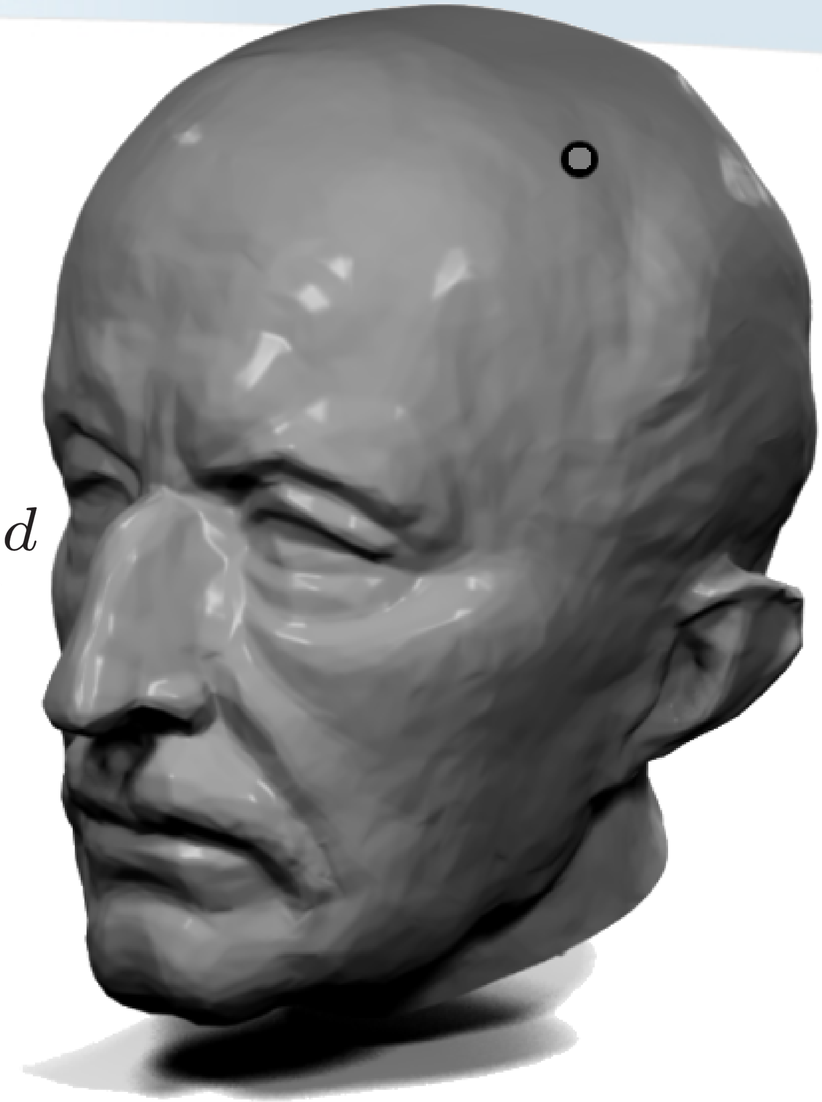

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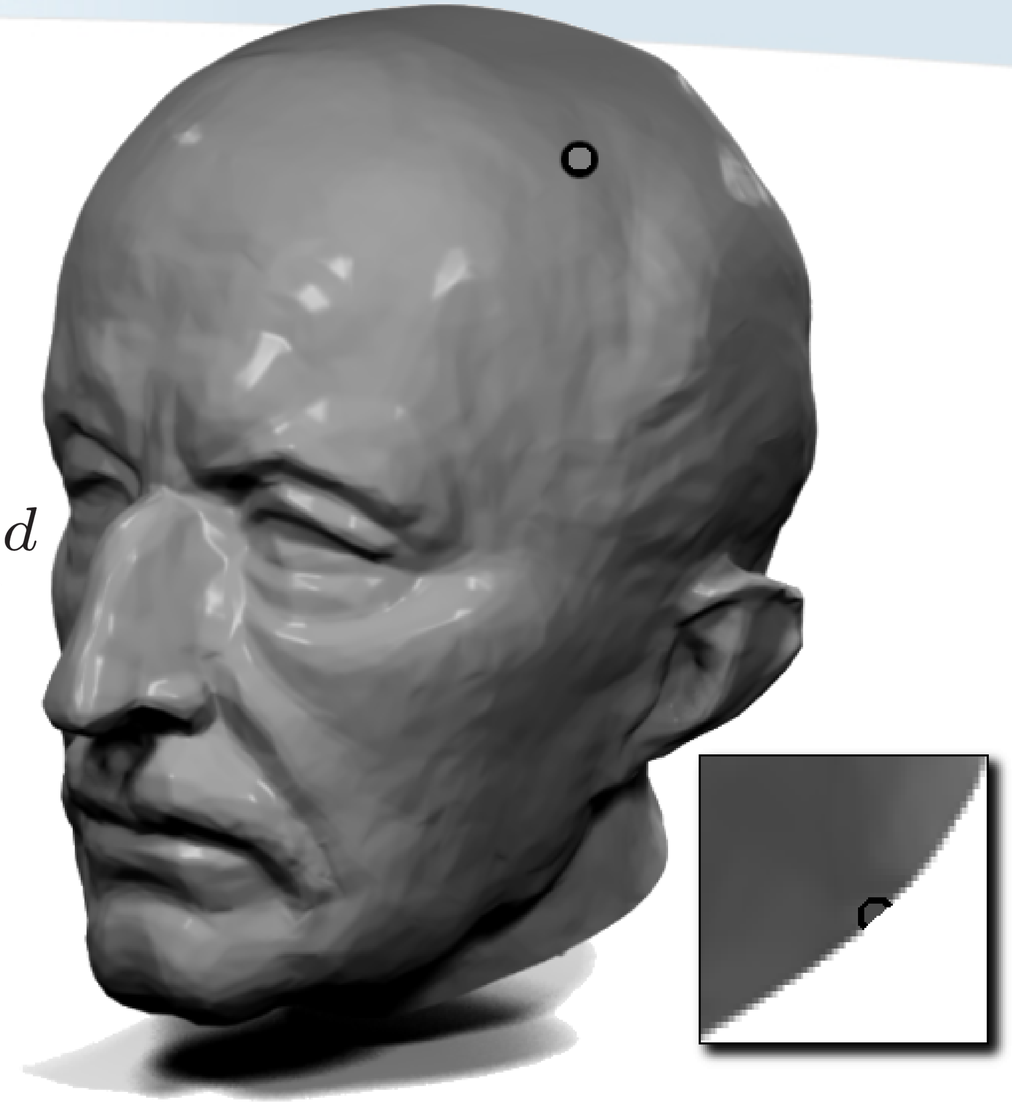

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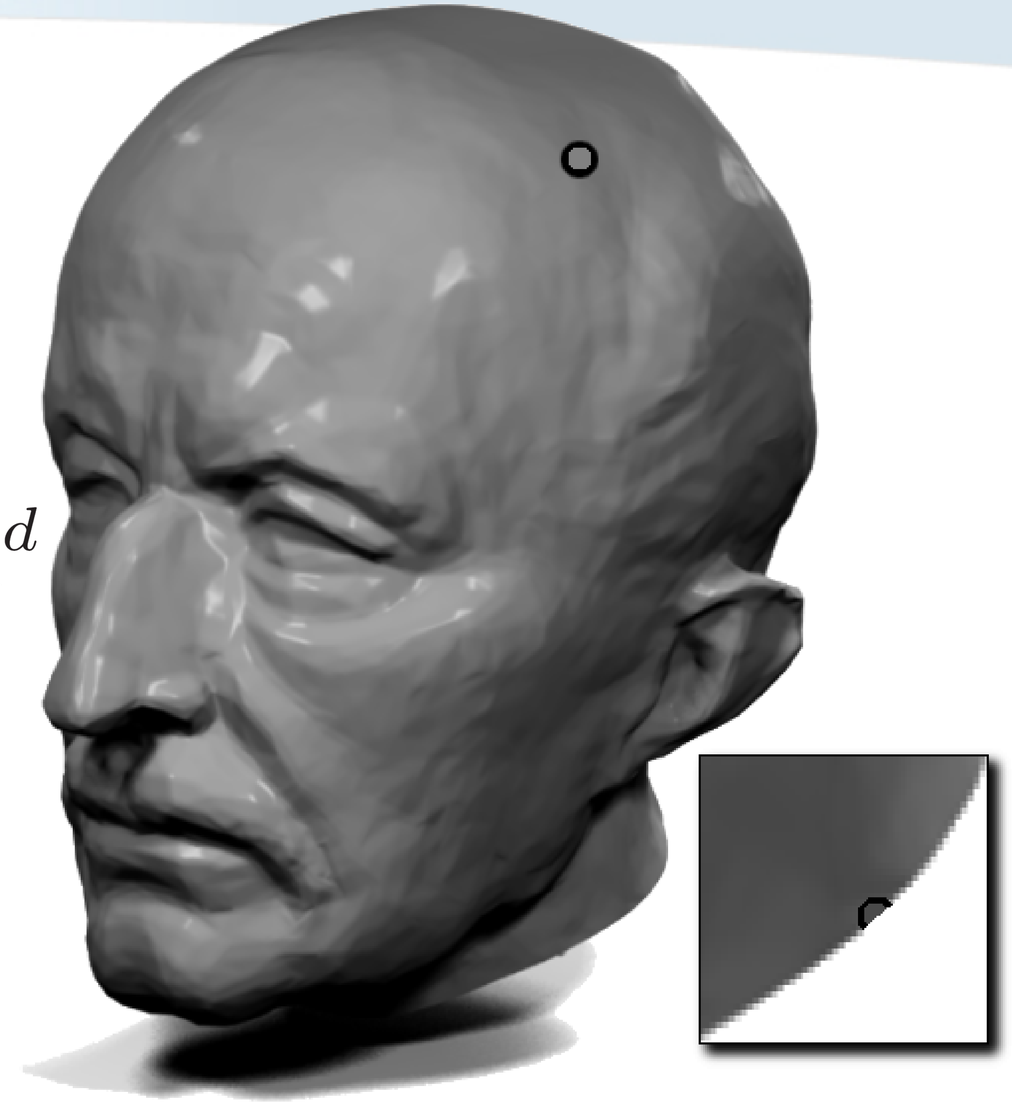
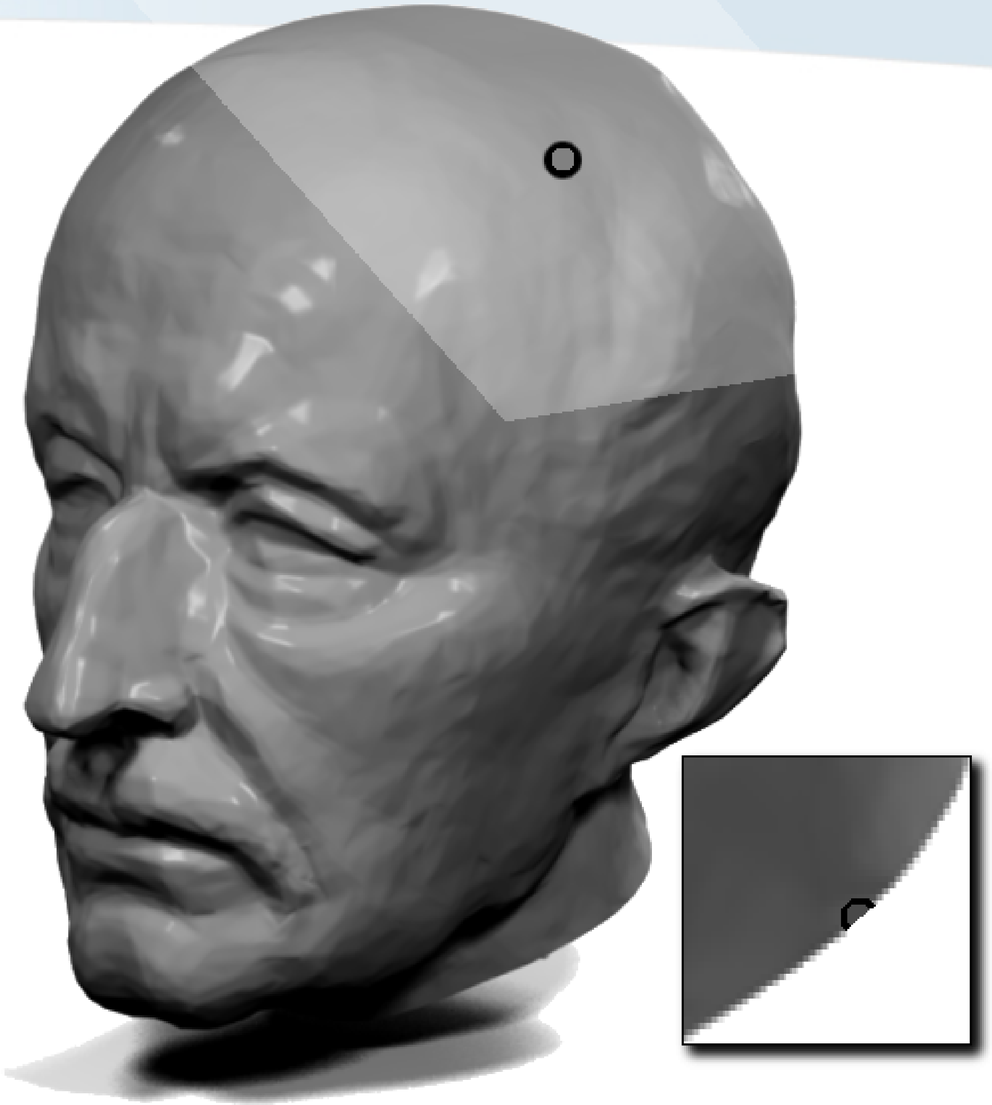
- Boundary: closed (d-1)-manifold



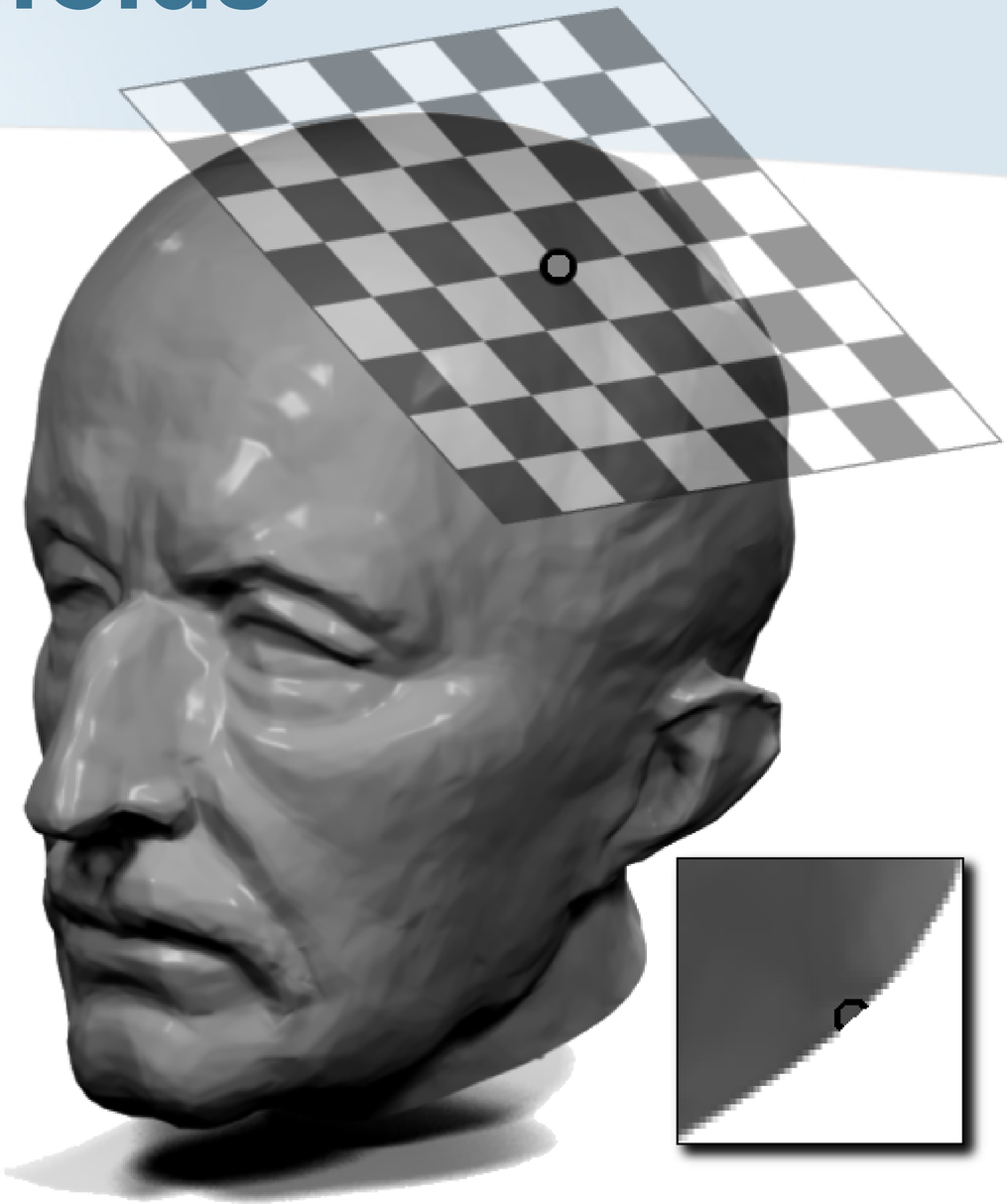
Domains of interest: Manifolds

- Tangent space
  - Localized Euclidean approximation



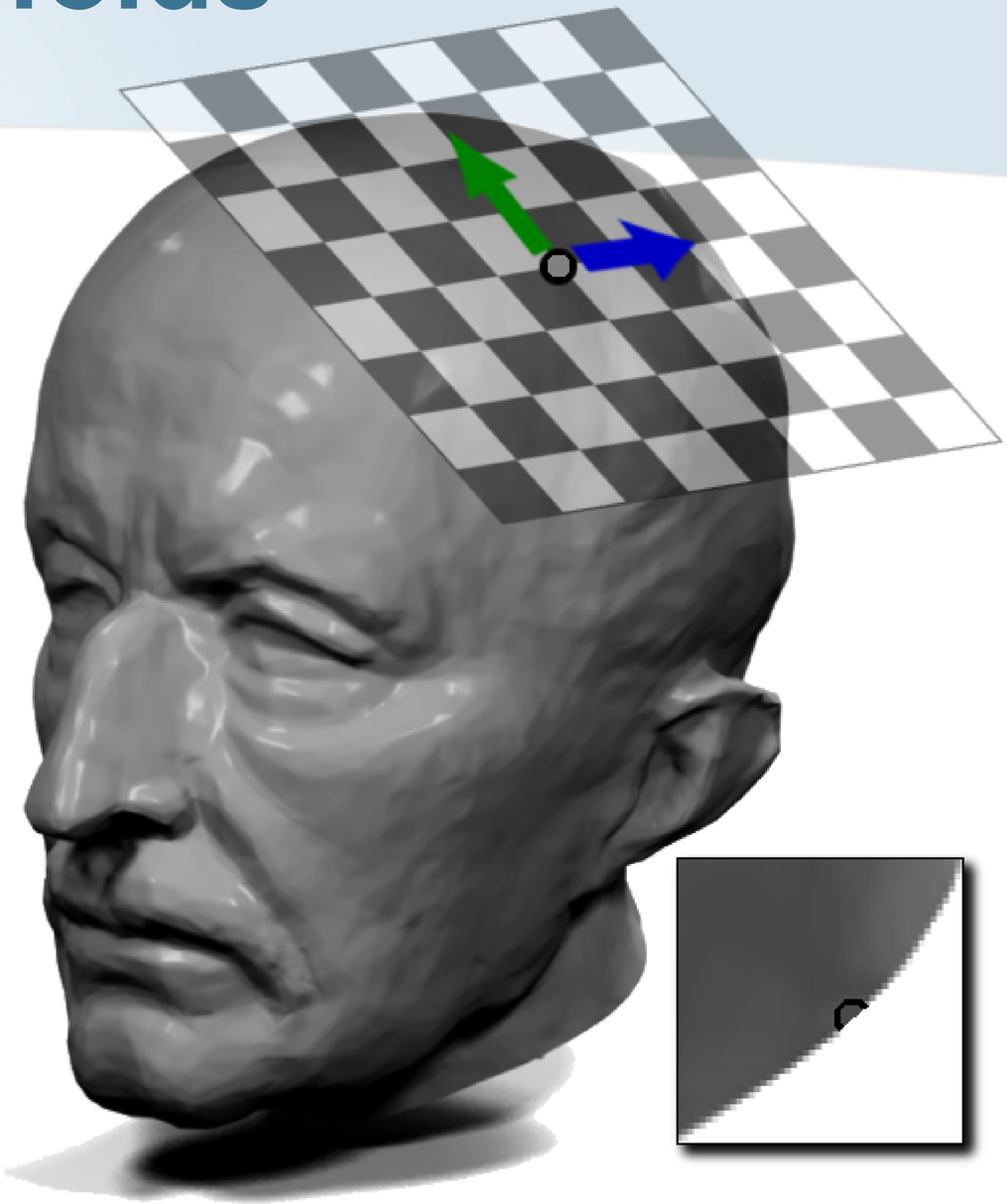
Domains of interest: Manifolds

- Tangent space
  - Localized Euclidean approximation
    - Localized Euclidean geometry



Domains of interest: Manifolds

- Tangent space
  - Localized Euclidean approximation
    - Localized Euclidean geometry
      - Inner products



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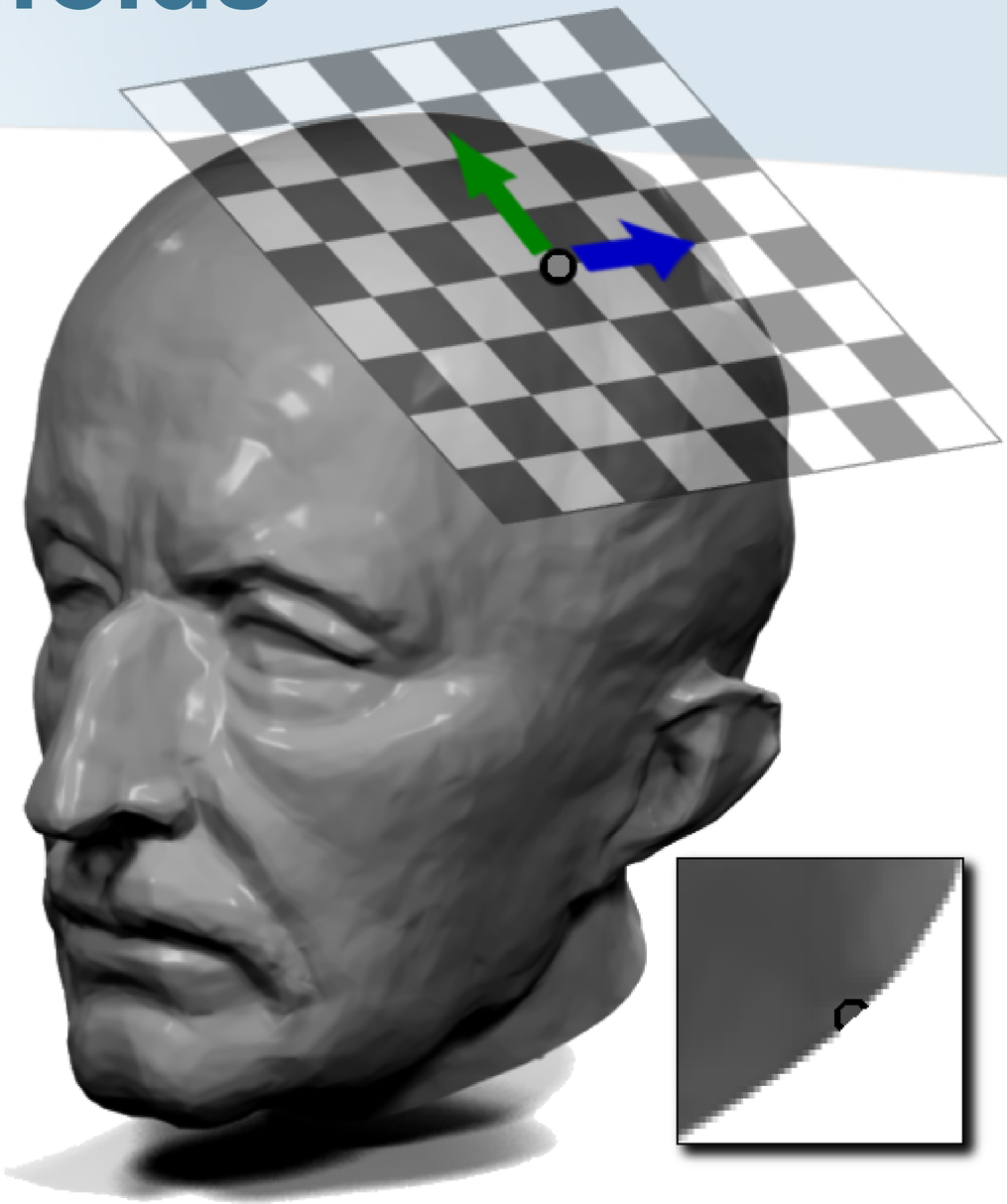
    - Localized Euclidean geometry
      - Inner products
      - Distances, angles, areas, …



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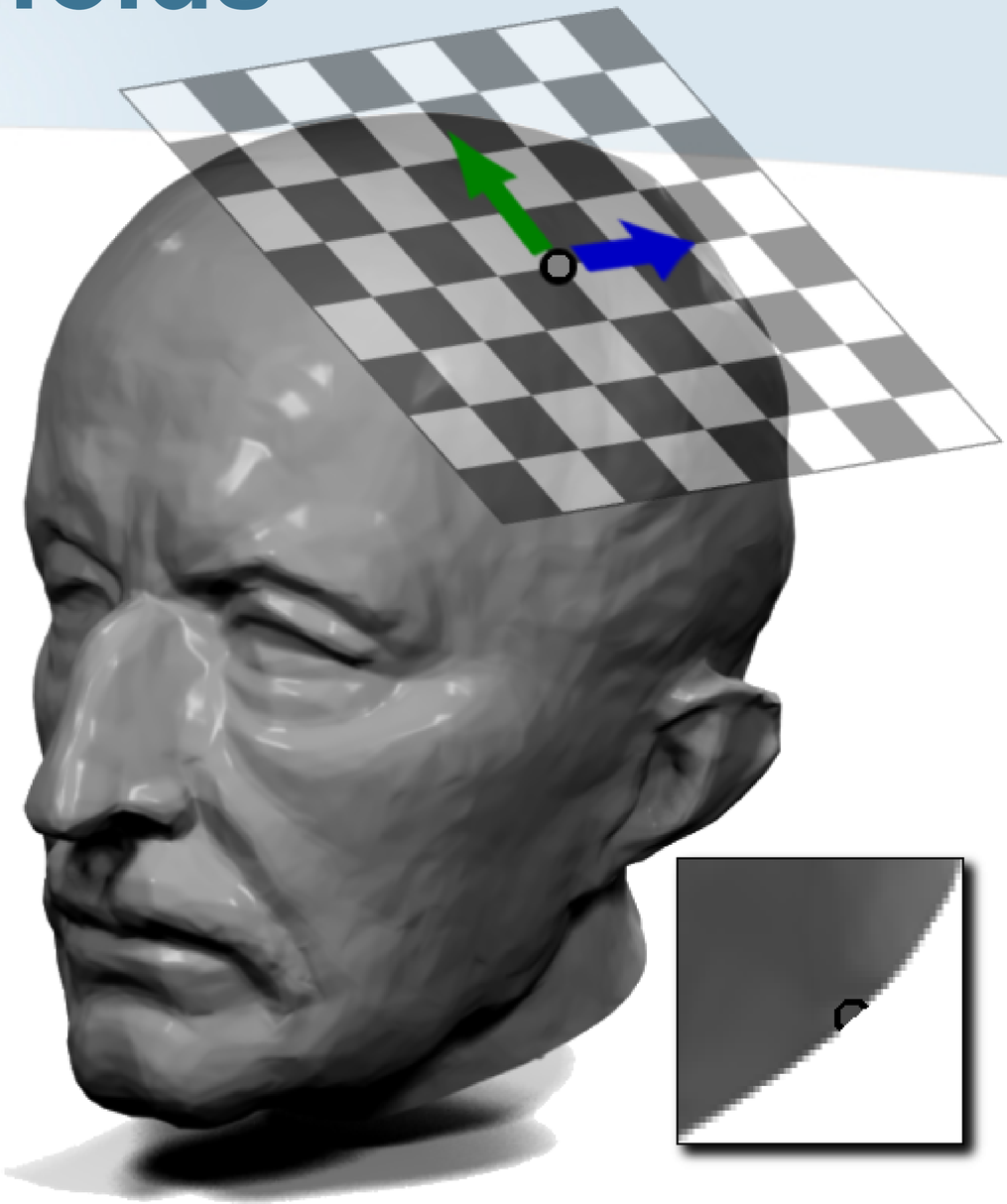

Domains of interest: Manifolds

- Tangent space
  - Localized Euclidean approximation
    - Localized Euclidean geometry
      - Inner products
      - Distances, angles, areas, …
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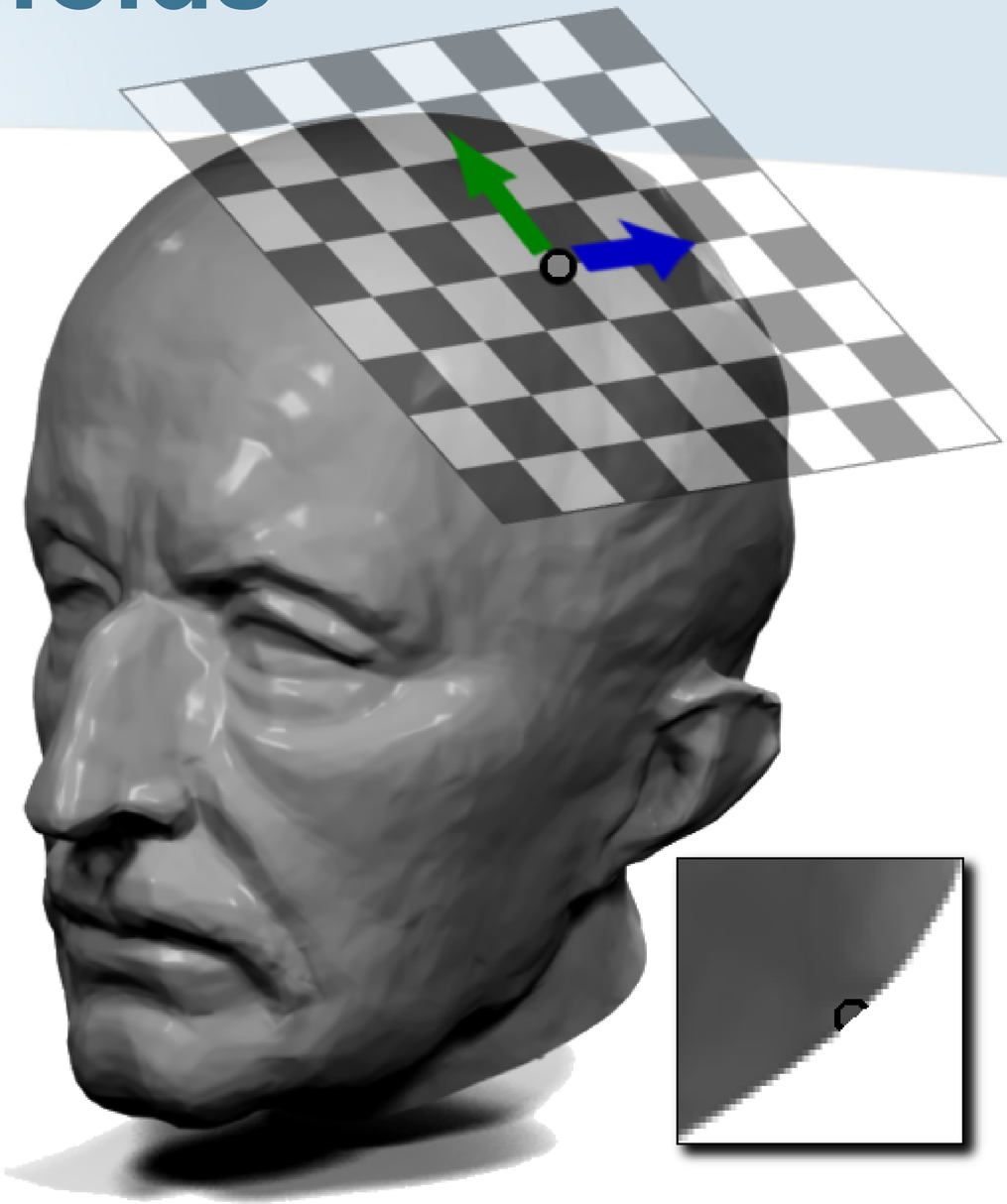
- Discrete surface in $\mathbb{R}^3$
  - Triangular (planar) cells



Manifold examples

Manifold examples

- Dimension?
- Embedding space?
- Boundary?

Manifold examples

$\mathbb{R}$

- Dimension?
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Manifold examples

$\mathbb{R}$

$[0, 1]$

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$$\mathbb{R}$$

$$[0, 1]$$

$$\{p \in \mathbb{R}^2 \ / \ \sqrt{p_x^2 + p_y^2} = r\}$$

Manifold examples

$$\mathbb{R}^2$$

$$[0, 1]$$

$$\{p \in \mathbb{R}^2 \ / \ \sqrt{p_x^2 + p_y^2} = r\}$$

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Manifold examples

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Manifold examples

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Manifold examples

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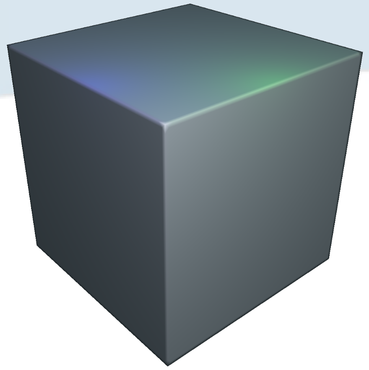
Manifold examples



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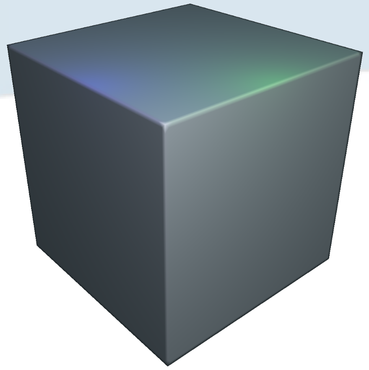

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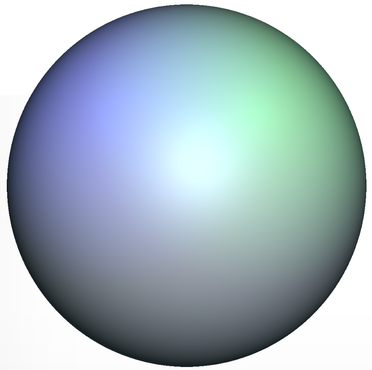

$$\{p \in \mathbb{R}^3 \ / \ \sqrt{p_x^2 + p_y^2 + p_z^2} \leq r\}$$

Summary

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- Euclidean representations

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- Euclidean representations
- Non-euclidean representations

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- Dimension-specific data-structures

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Euclidean spaces

Euclidean spaces

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  - 325 BC, 265 BC

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    - 5 axioms (first tome)

Euclidean spaces on a computer

Euclidean spaces on a computer

- Given an origin

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  - $o = (0, 0, \ldots, 0) \in \mathbb{R}^n$

Euclidean spaces on a computer

- Given an origin
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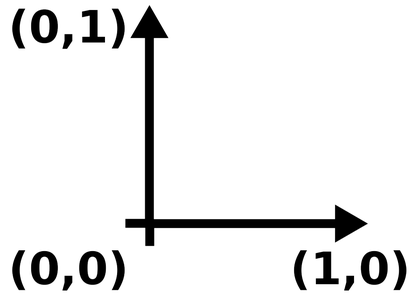

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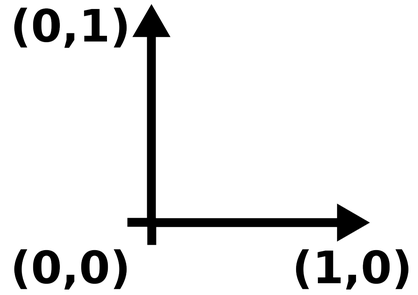
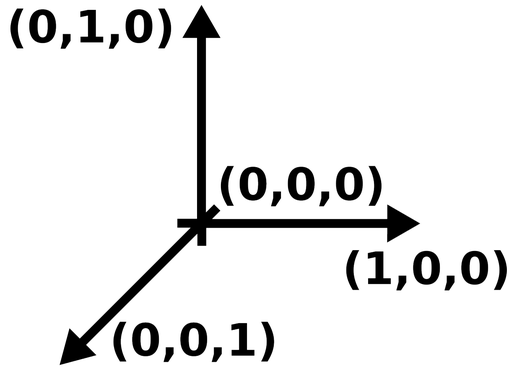
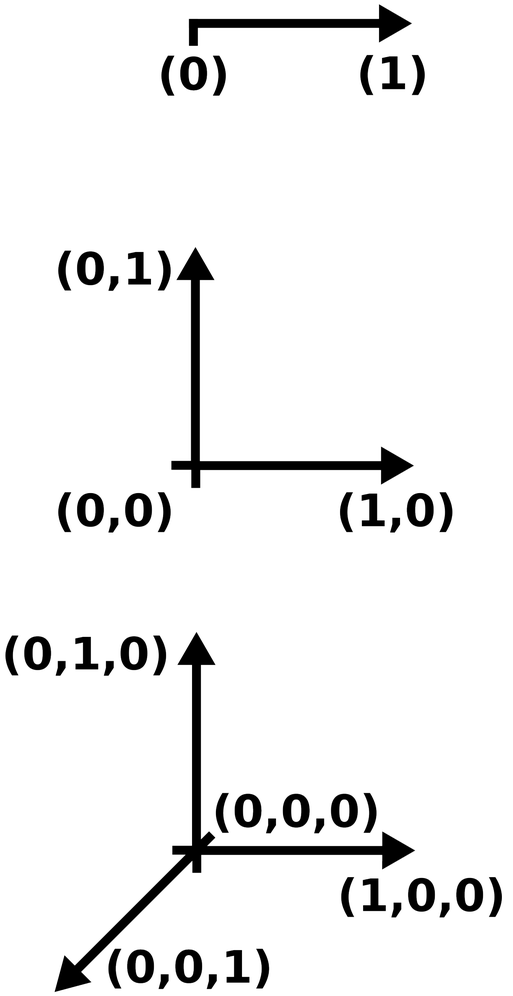




Euclidean spaces on a computer

- Direct product of closed unit intervals



Euclidean spaces on a computer

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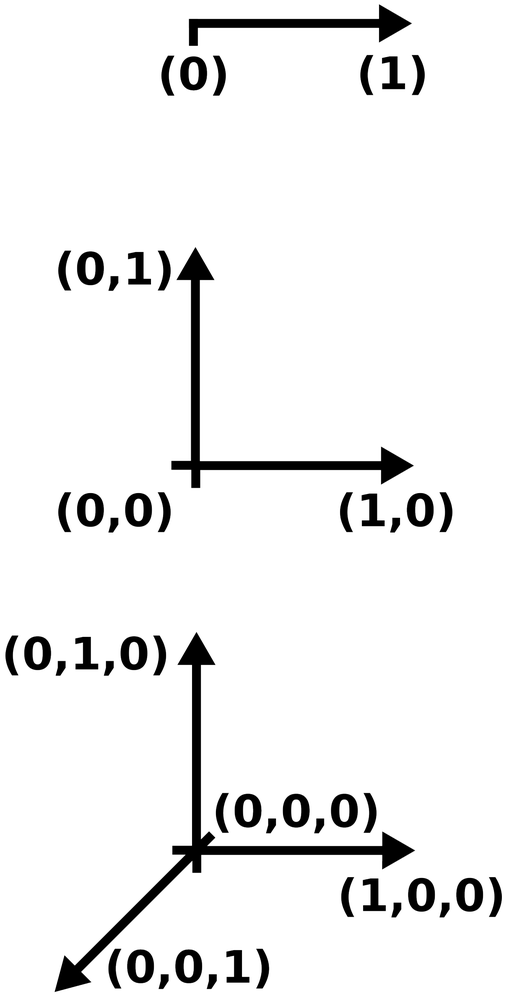
  - $[0,1] \times [0,1] \times \cdots \times [0,1]$



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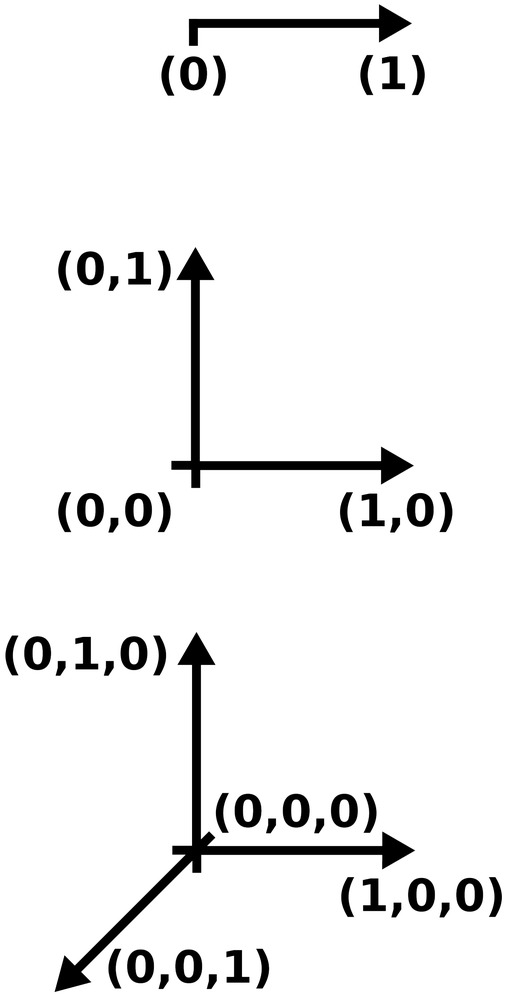

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- Direct product of closed unit intervals
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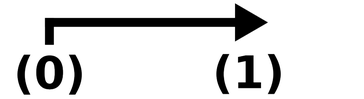
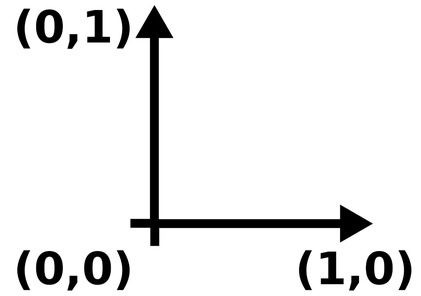
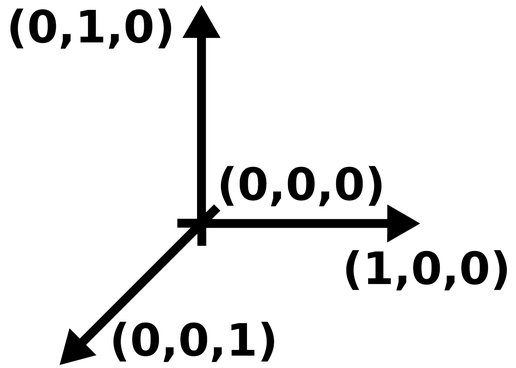
    - Unit translations along the vectors of the orthonormal basis

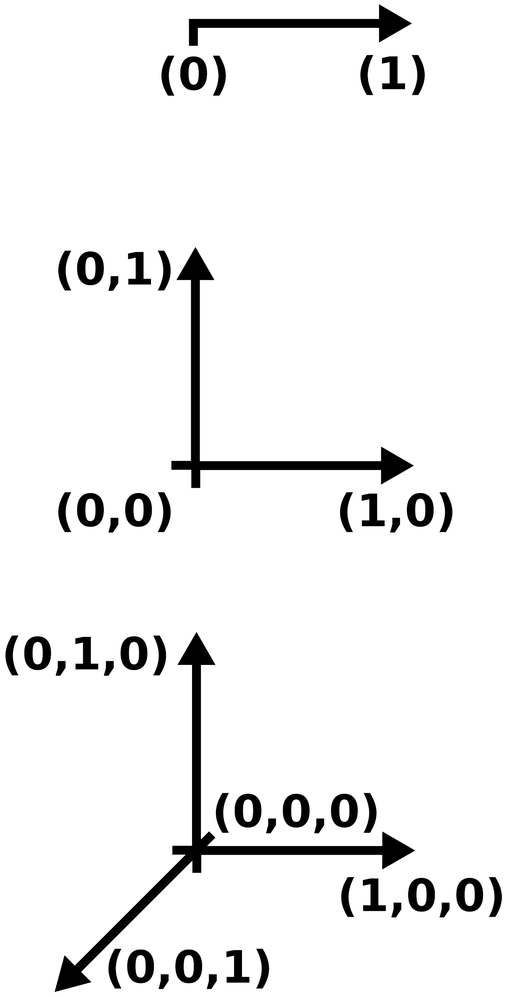






Euclidean spaces on a computer

- Direct product of closed unit intervals
  - $[0,1] \times [0,1] \times \cdots \times [0,1]$
  - By construction
    - Unit translations along the vectors of the orthonormal basis
    - Covering a bounded, compact region of $\mathbb{R}^n$



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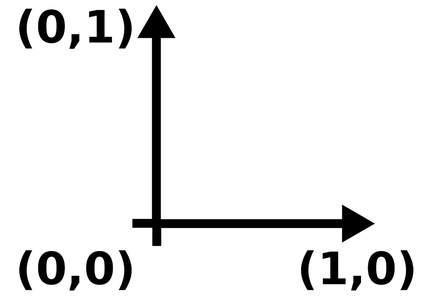
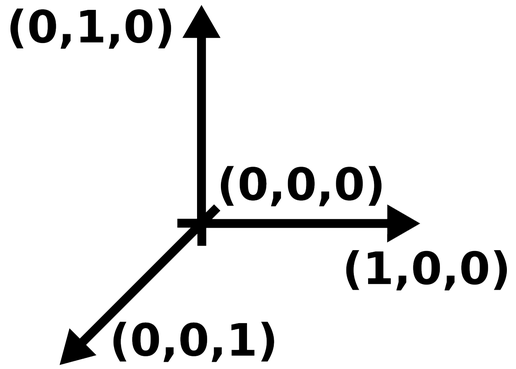
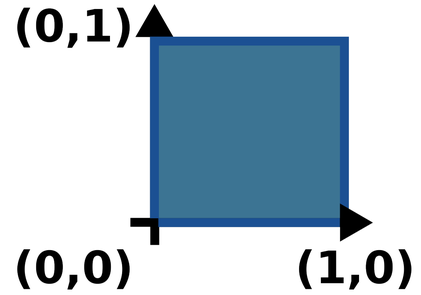
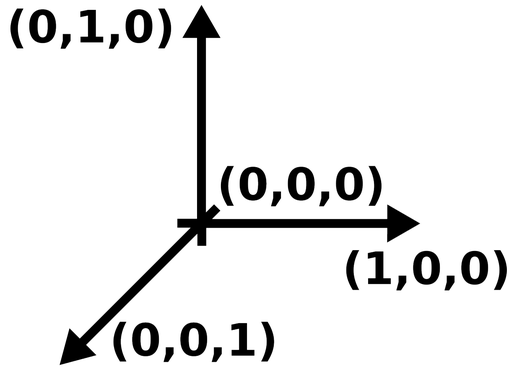




Euclidean spaces on a computer

- Direct product of closed unit intervals
  - $[0,1] \times [0,1] \times \cdots \times [0,1]$
  - By construction
    - Unit translations along the vectors of the orthonormal basis
    - Covering a bounded, compact region of $\mathbb{R}^n$





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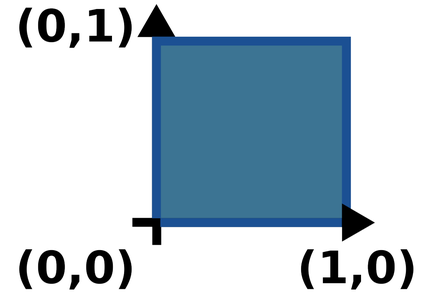

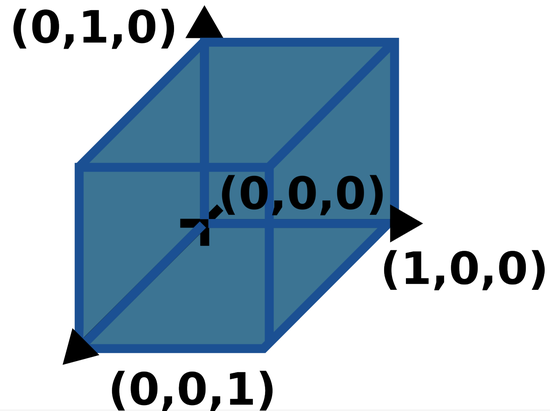

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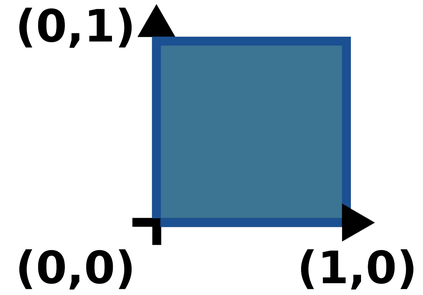
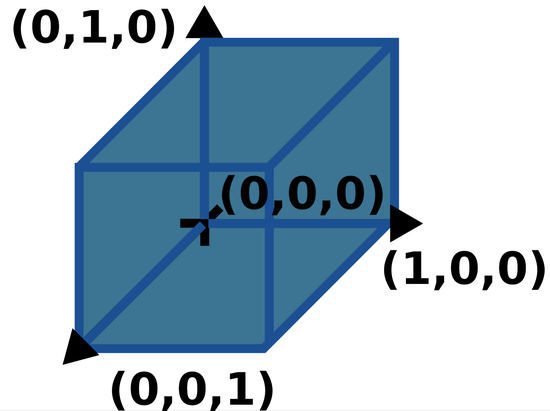
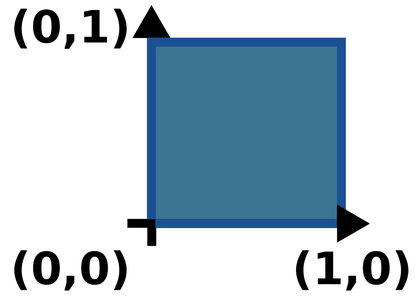


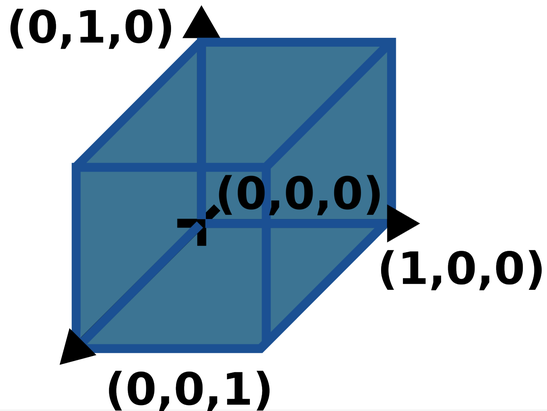


- **Unit cell**

Euclidean spaces on a computer

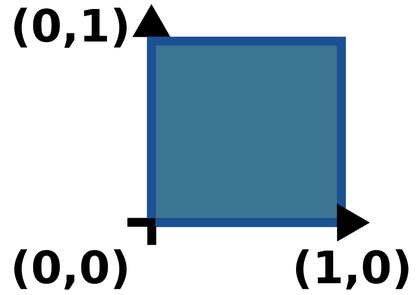
- Given a unit cell of $\mathbb{R}^n$

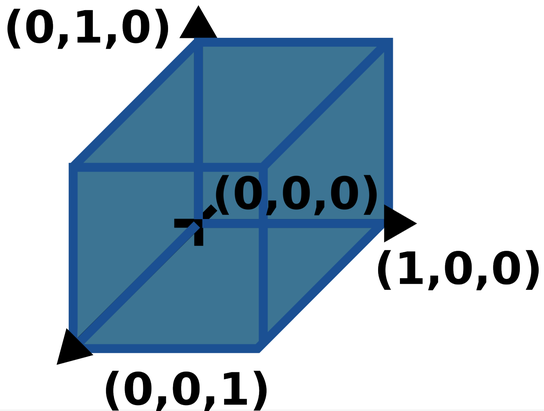




Euclidean spaces on a computer

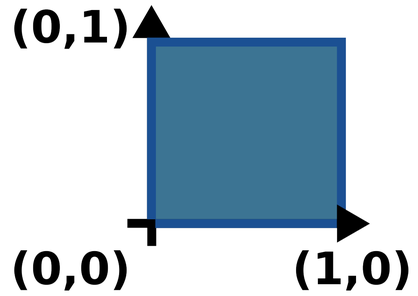
- Given a unit cell of $\mathbb{R}^n$
  - What's the minimum number of samples necessary to describe its convex hull?

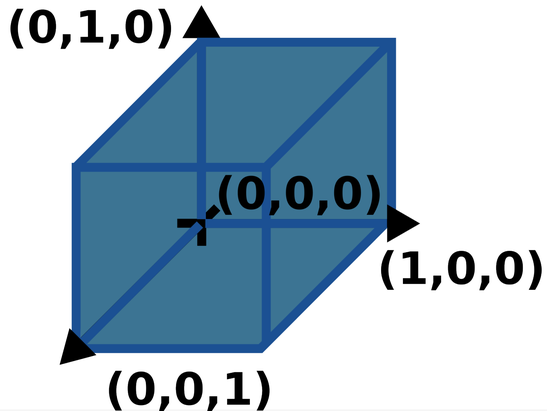




Euclidean spaces on a computer

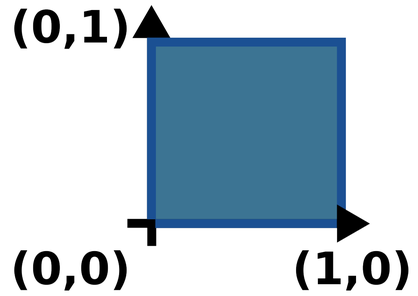
- Given a unit cell of $\mathbb{R}^n$
  - What's the minimum number of samples necessary to describe its convex hull?
  - 2^n

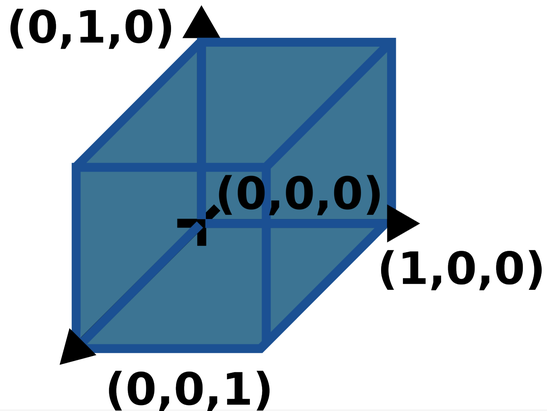




Euclidean spaces on a computer

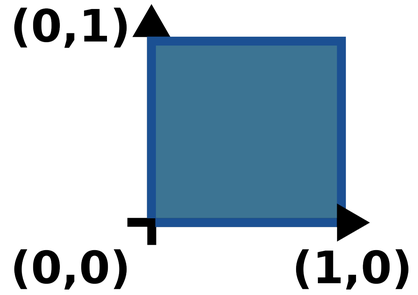
- Given a unit cell of $\mathbb{R}^n$
  - What's the minimum number of samples necessary to describe its convex hull?
  - 2^n
- Arbitrary closed and bounded subsets of $\mathbb{R}^n$

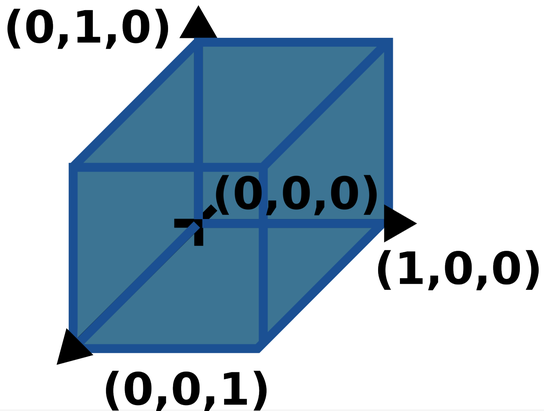




Euclidean spaces on a computer

- Given a unit cell of $\mathbb{R}^n$
  - What's the minimum number of samples necessary to describe its convex hull?
  - 2^n
- Arbitrary closed and bounded subsets of $\mathbb{R}^n$
  - Collection of unit cells





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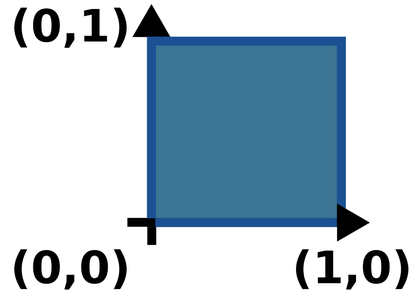

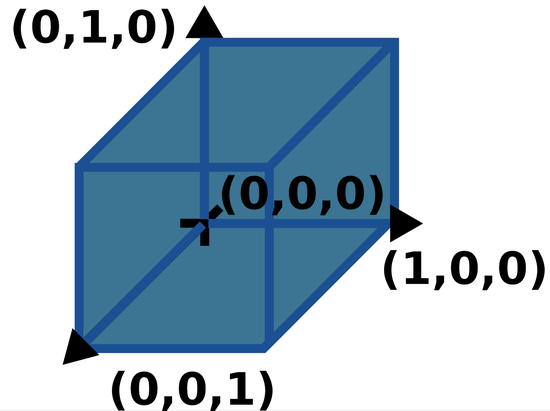

Euclidean spaces on a computer

- Notion of **regular grid** $\mathcal{G}$
  - Finite collection of unit cells
  - Entirely covering the direct product of closed intervals, such that
    - $[0, k_1] \times [0, k_2] \times \cdots \times [0, k_n]$
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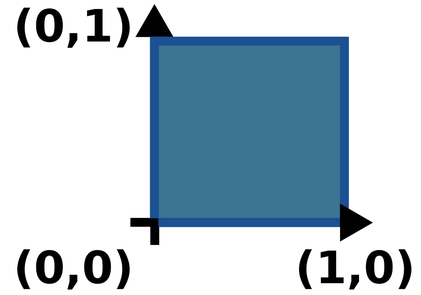

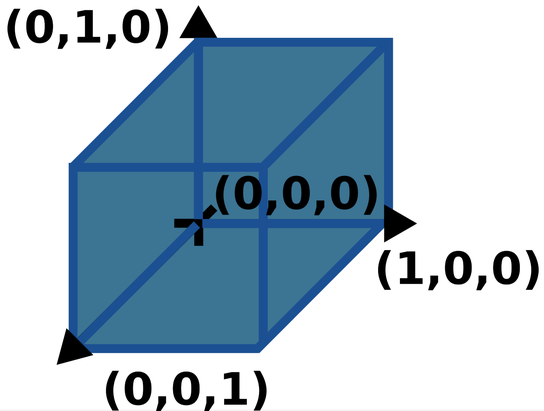

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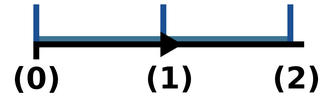

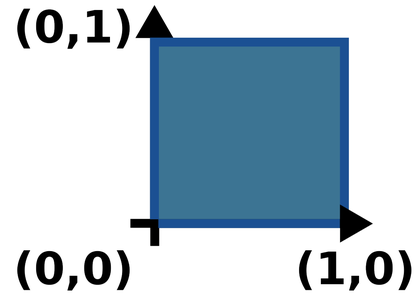

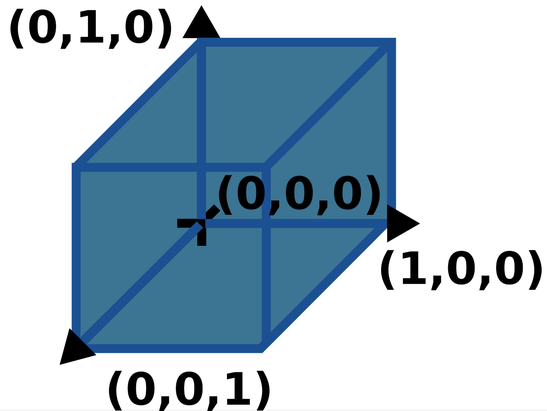

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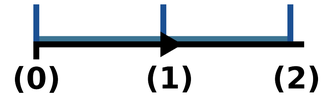

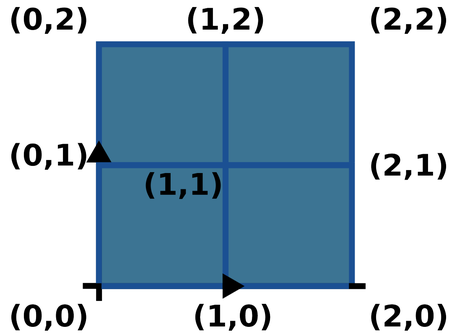

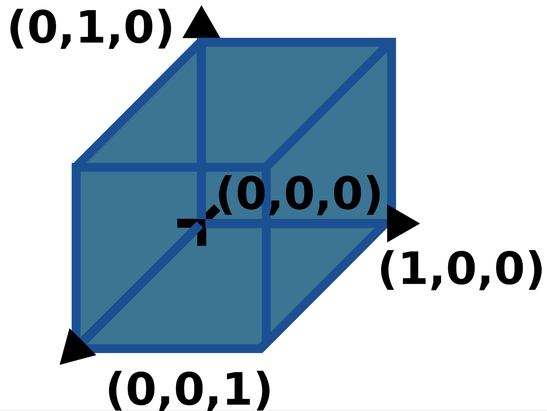

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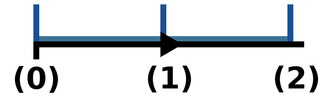

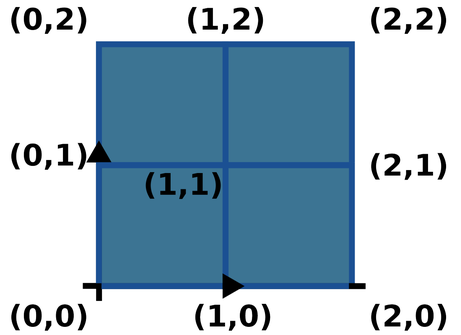

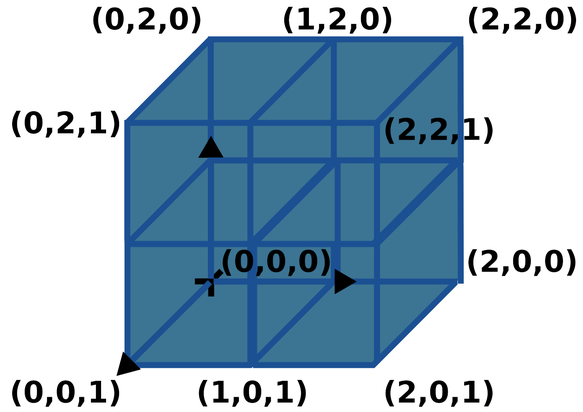

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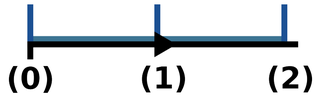
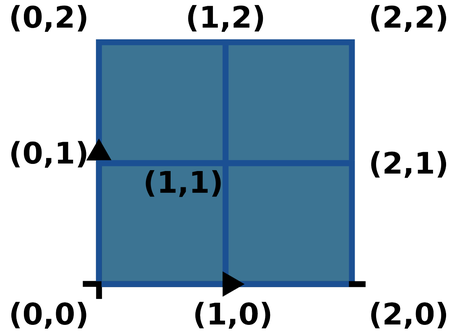
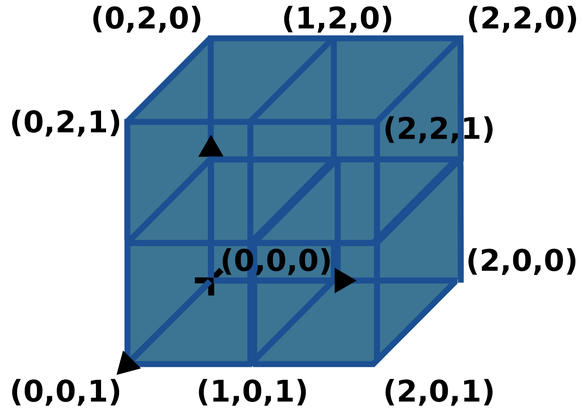
  - How many unit cells in $[0, k_1] \times [0, k_2] \times [0, k_3]$?







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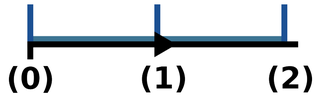
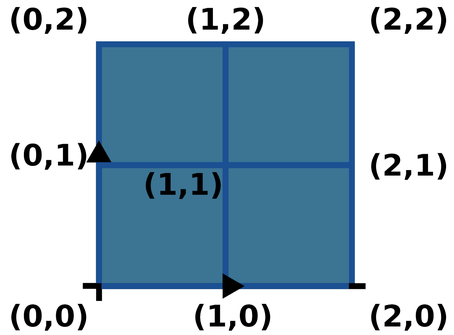
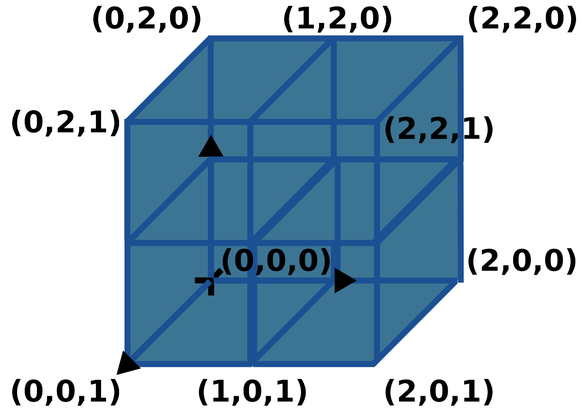
    - $k_1.k_2.k_3$







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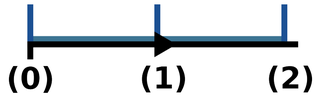
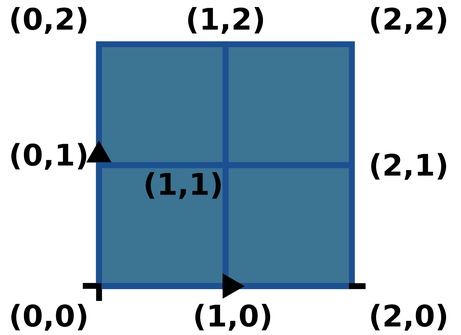
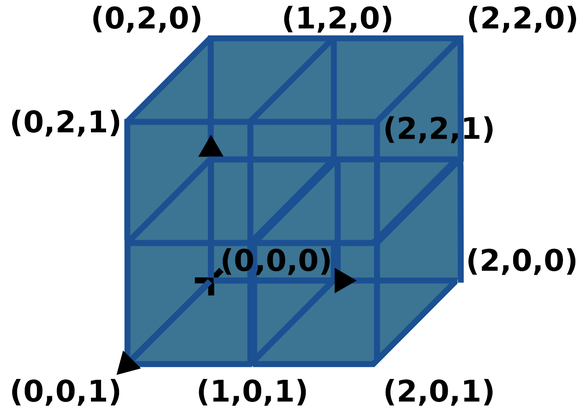
    - $k_i \in \mathbb{N}$
  - How many unit cells in $[0, k_1] \times [0, k_2] \times [0, k_3]$?
    - $k_1.k_2.k_3$
  - How many samples?







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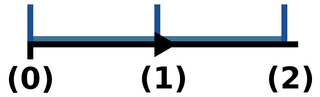
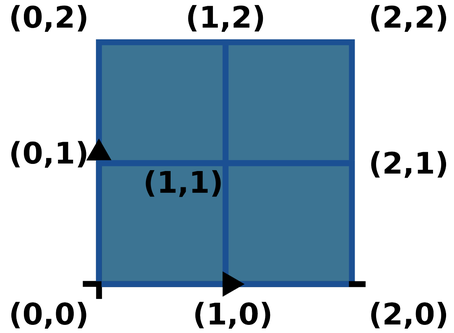
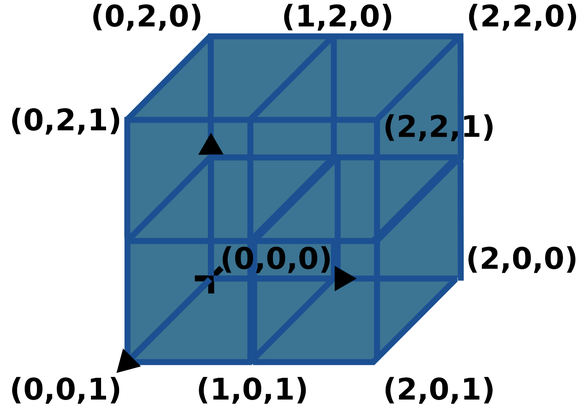
  - How many unit cells in $[0, k_1] \times [0, k_2] \times [0, k_3]$?
    - $k_1.k_2.k_3$
  - How many samples? $(k_1 + 1).(k_2 + 1).(k_3 + 1)$







Regular grids of $\mathbb{R}$

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$$f : [0, 24] \to \mathbb{R}$$

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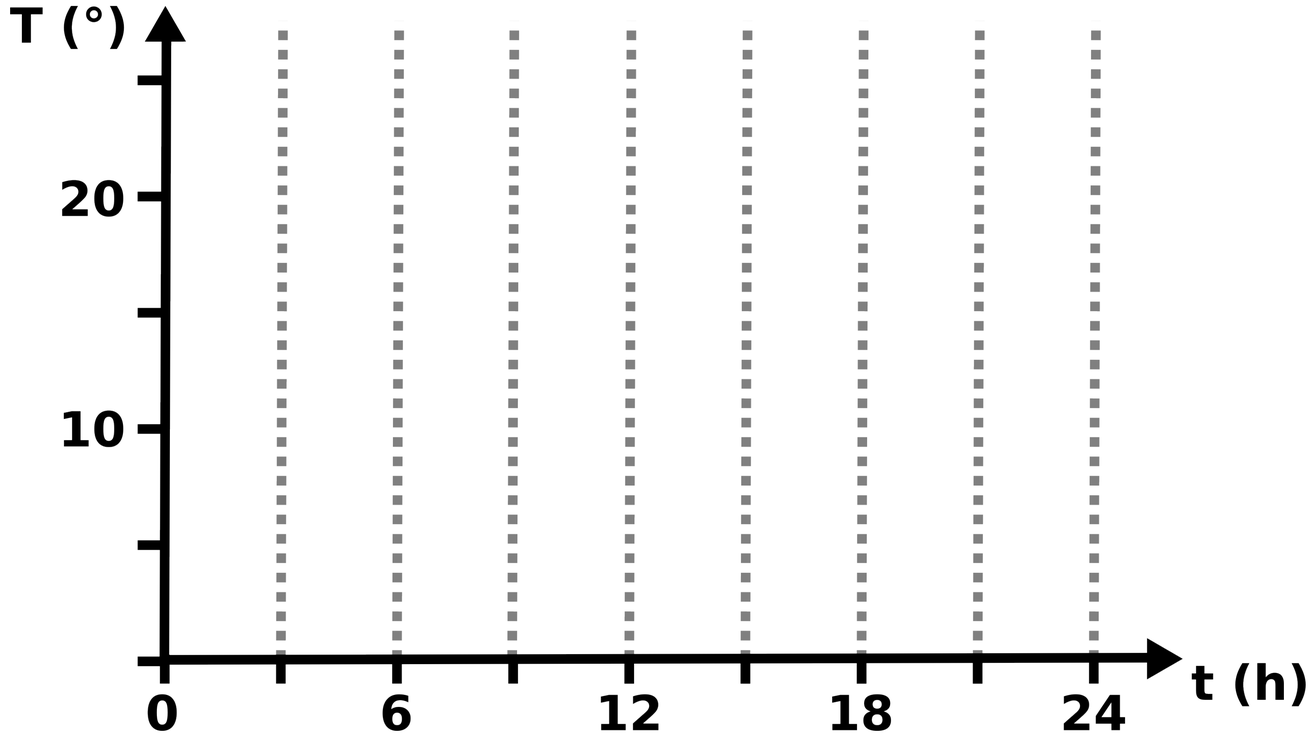

Regular grids of $\mathbb{R}$

- We need
  - A one-dimensional structure

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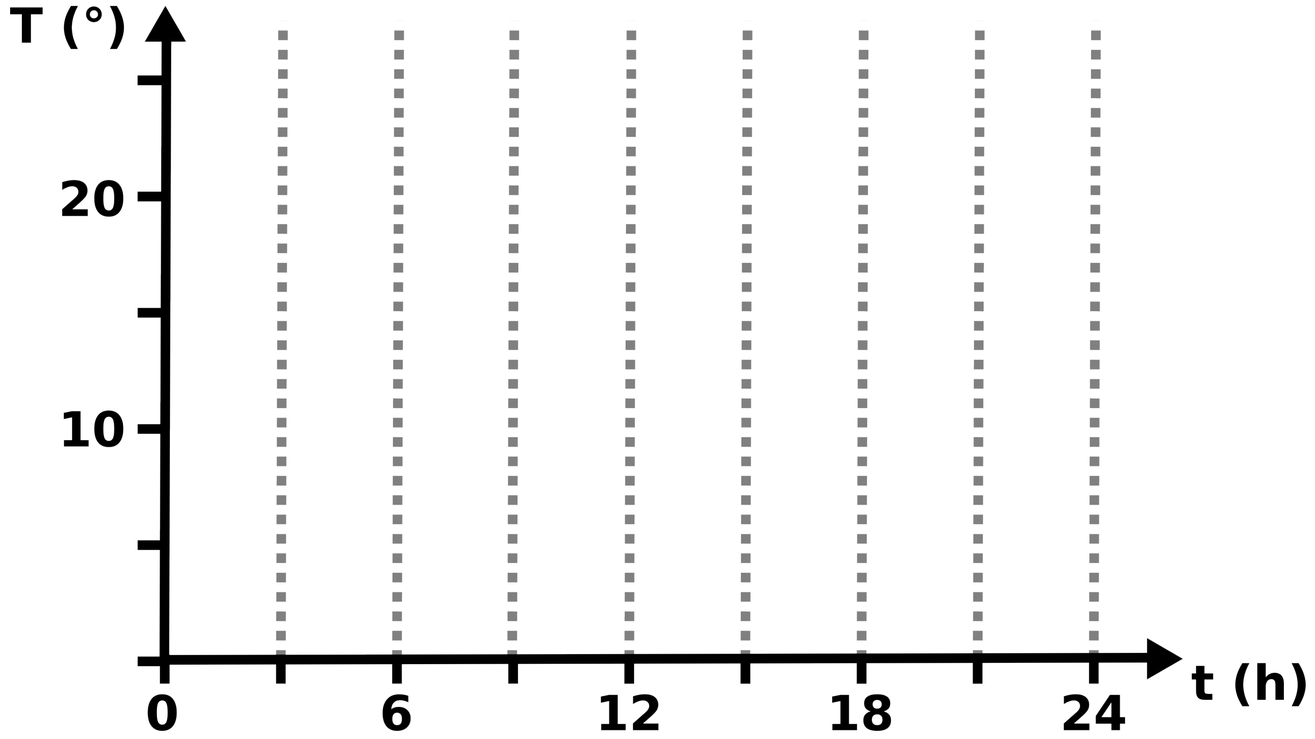

Regular grids of $\mathbb{R}$

- We need
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  - To store values
    - Scalars, vectors, tensors...

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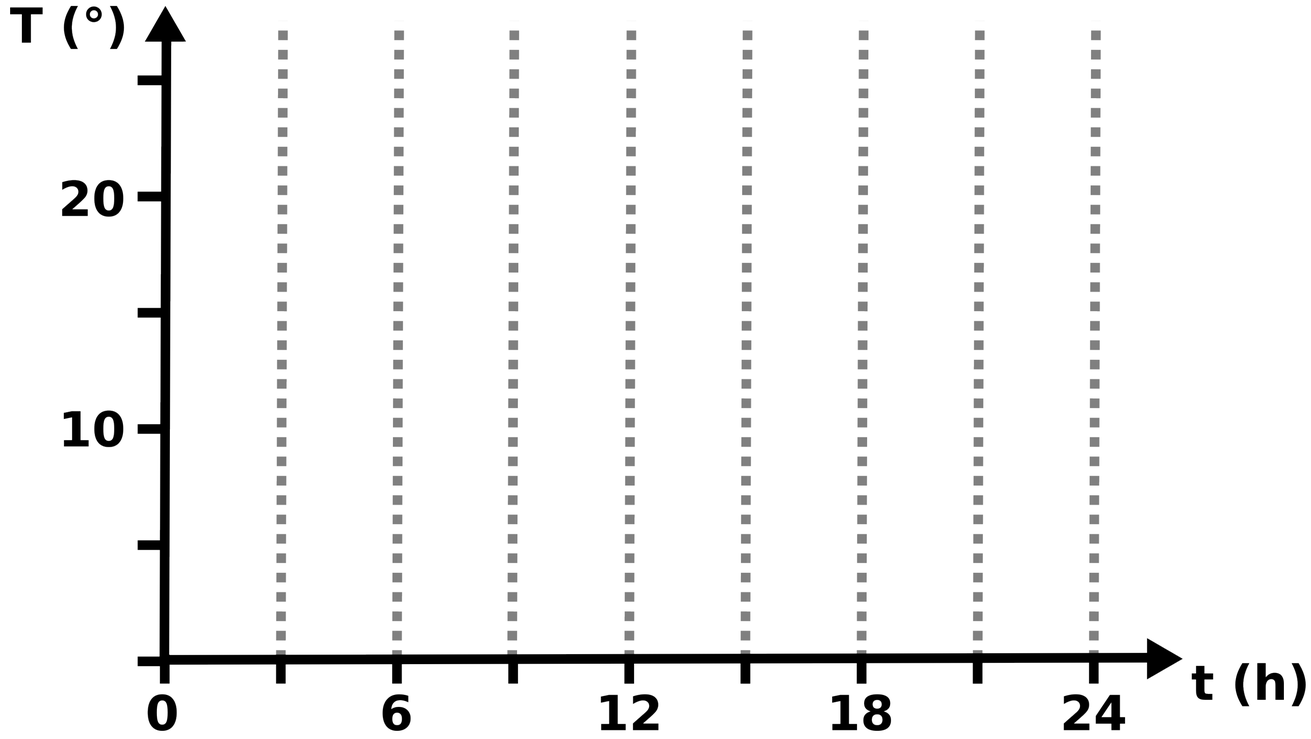

Regular grids of $\mathbb{R}$

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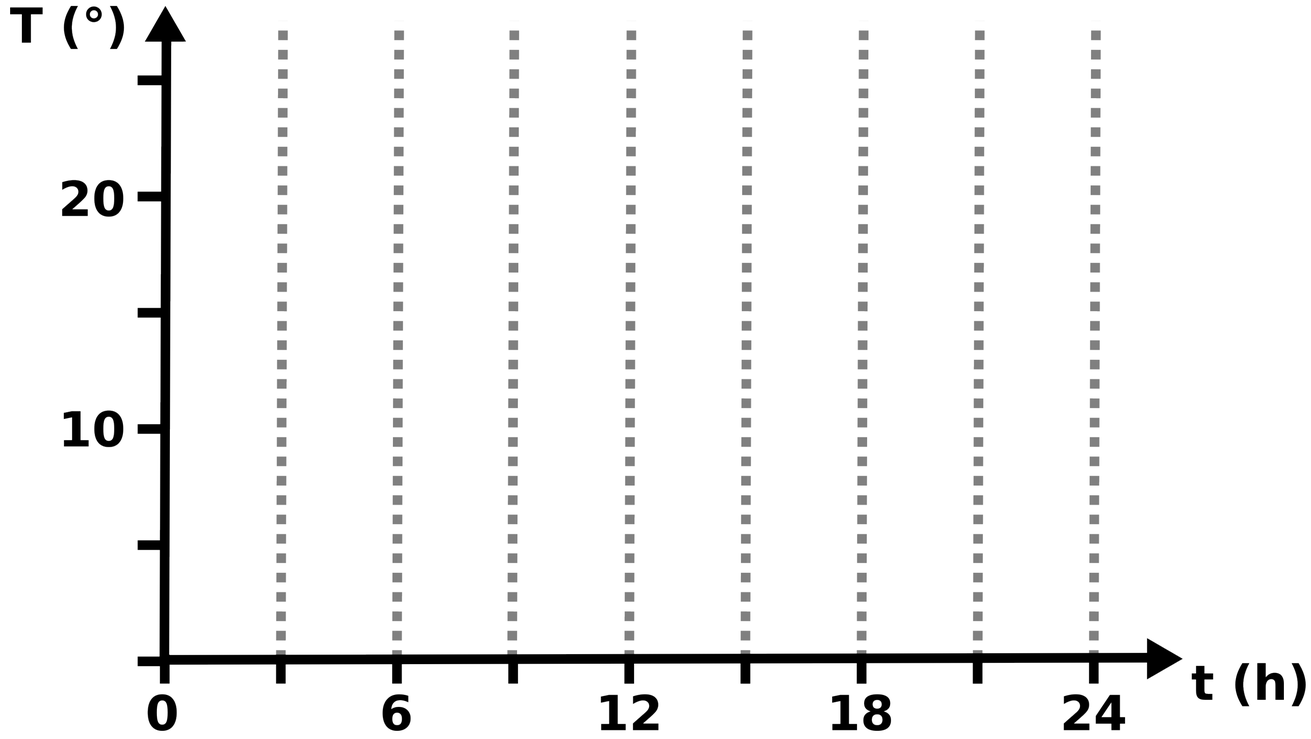

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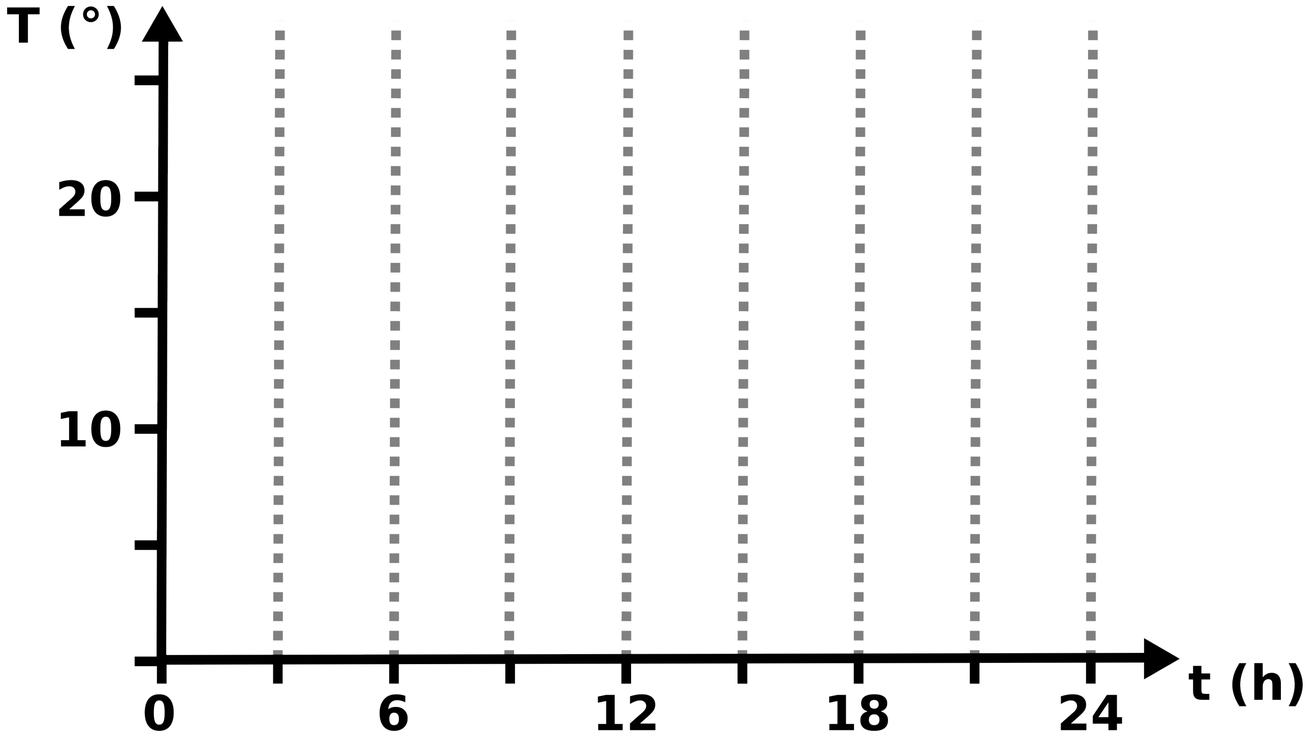

Regular grids of $\mathbb{R}$

- We need
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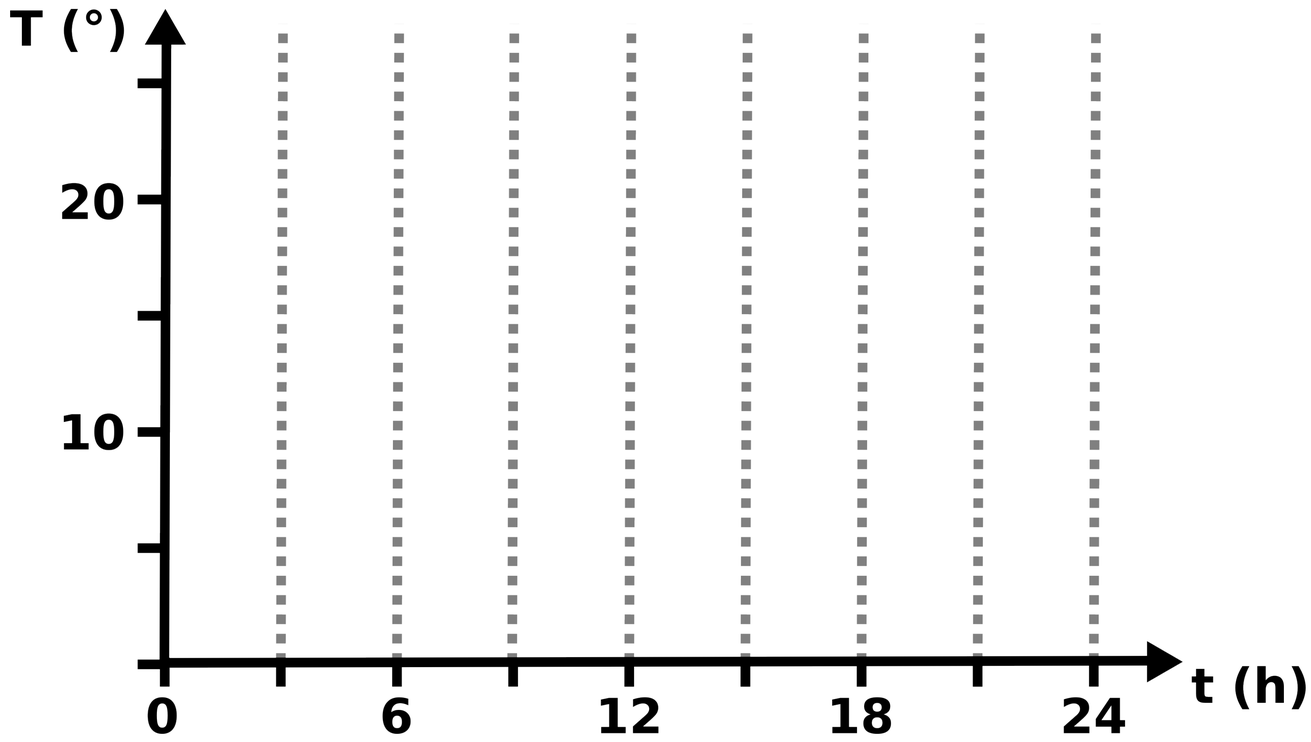
    - Scalars, vectors, tensors...
- Arrays :)

9	7	10	12	16	18	11	10	7

$$f : [0, 24] \to \mathbb{R}$$



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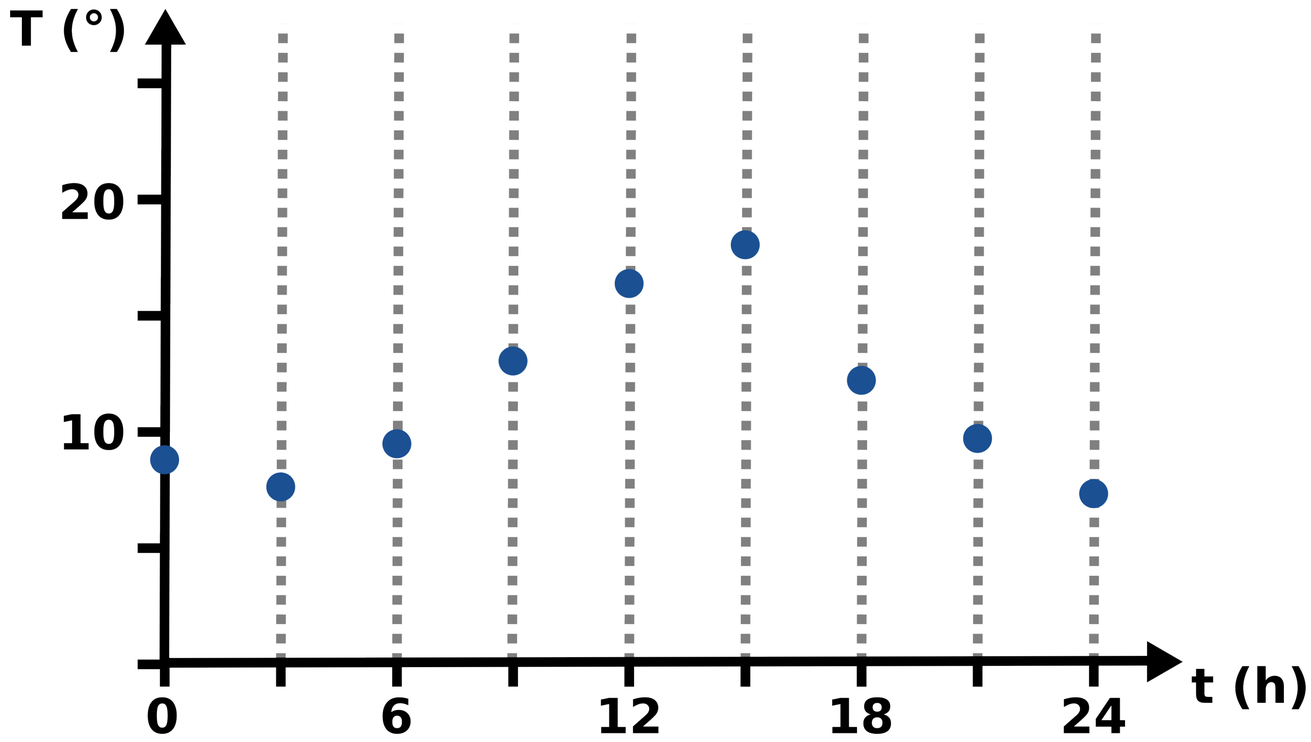

Regular grids of $\mathbb{R}^2$

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$$[0,3] \times [0,3]$$

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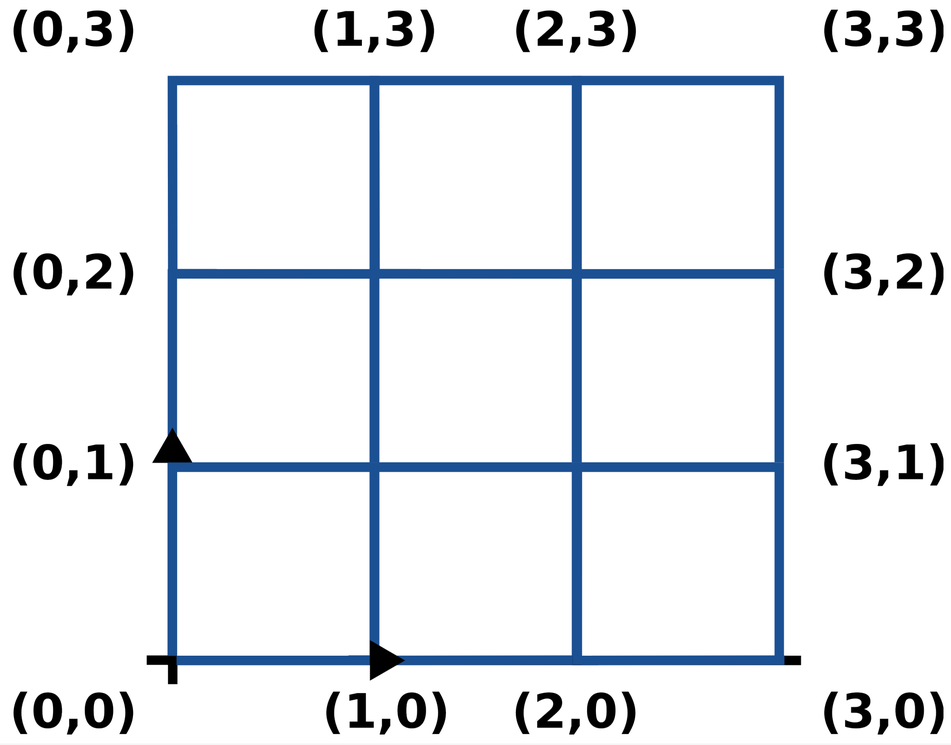

Regular grids of $\mathbb{R}^2$

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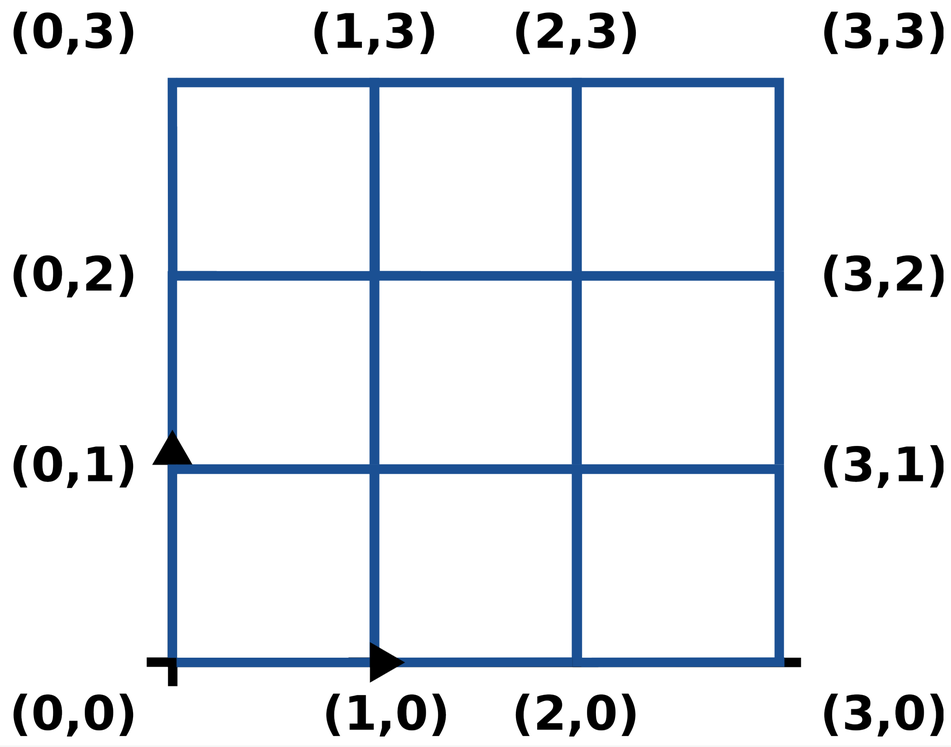
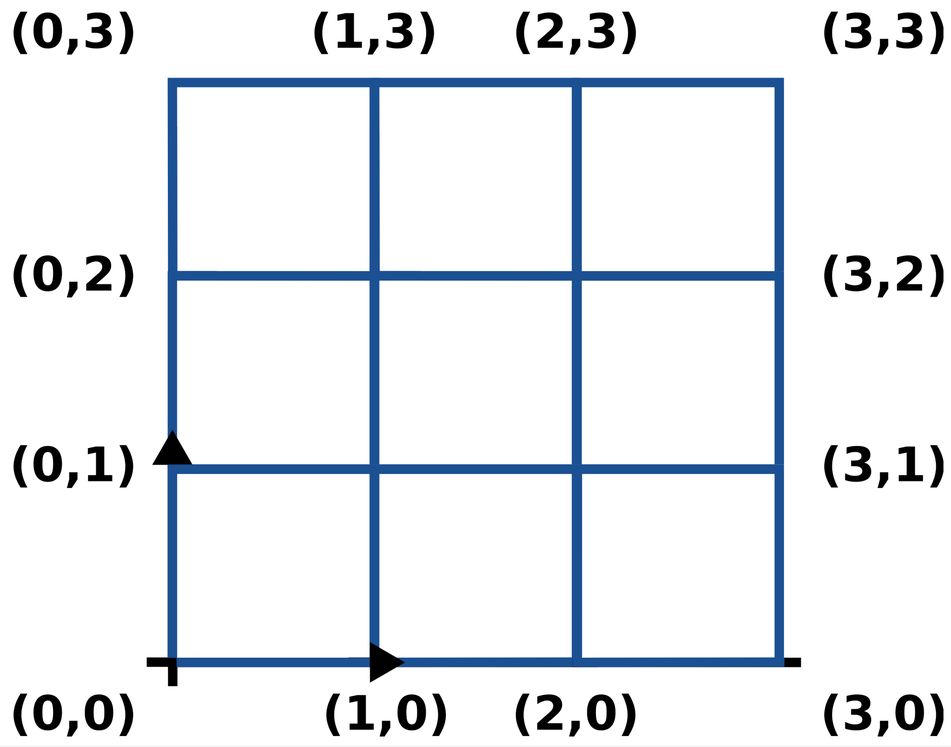


Regular grids of $\mathbb{R}^2$

$$f(p) = \sqrt{p_x^2 + p_y^2}$$

$$f : [0,3] \times [0,3] \rightarrow \mathbb{R}$$



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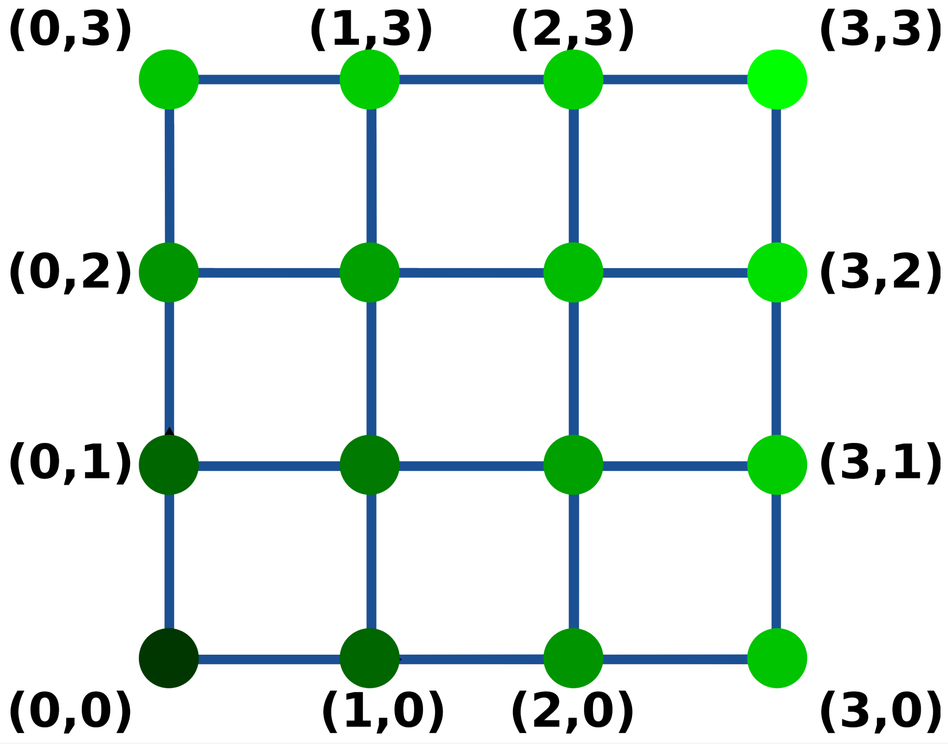

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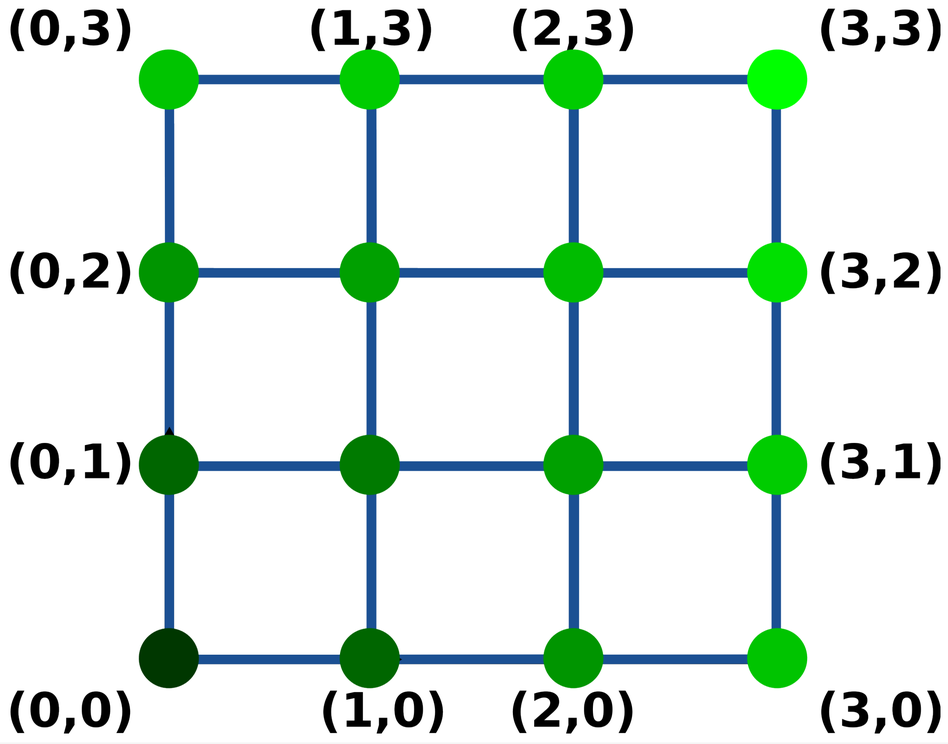

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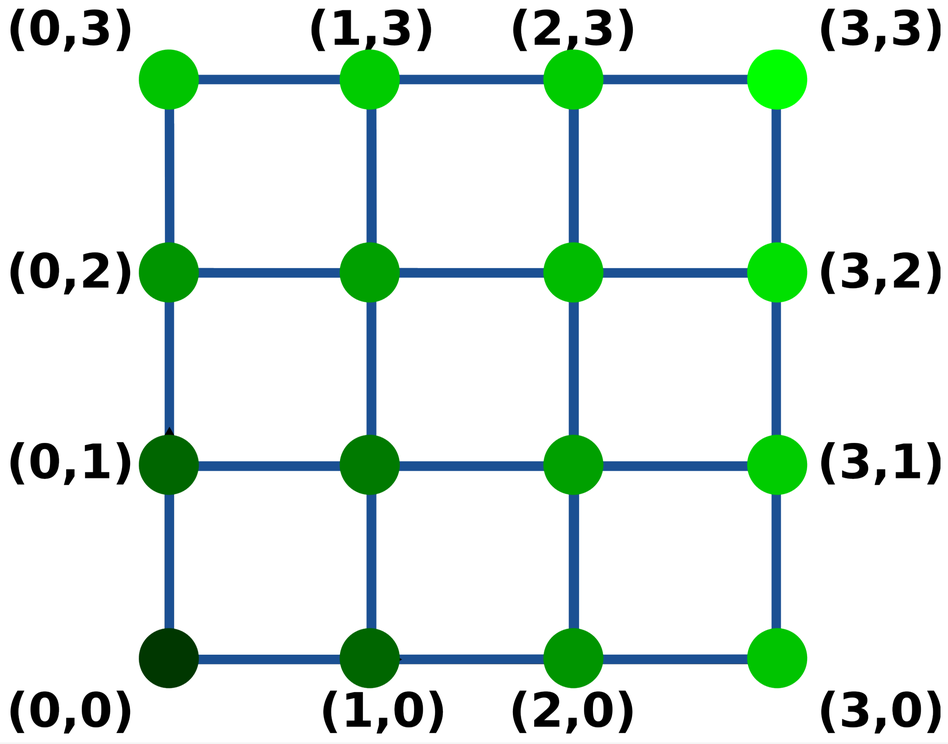

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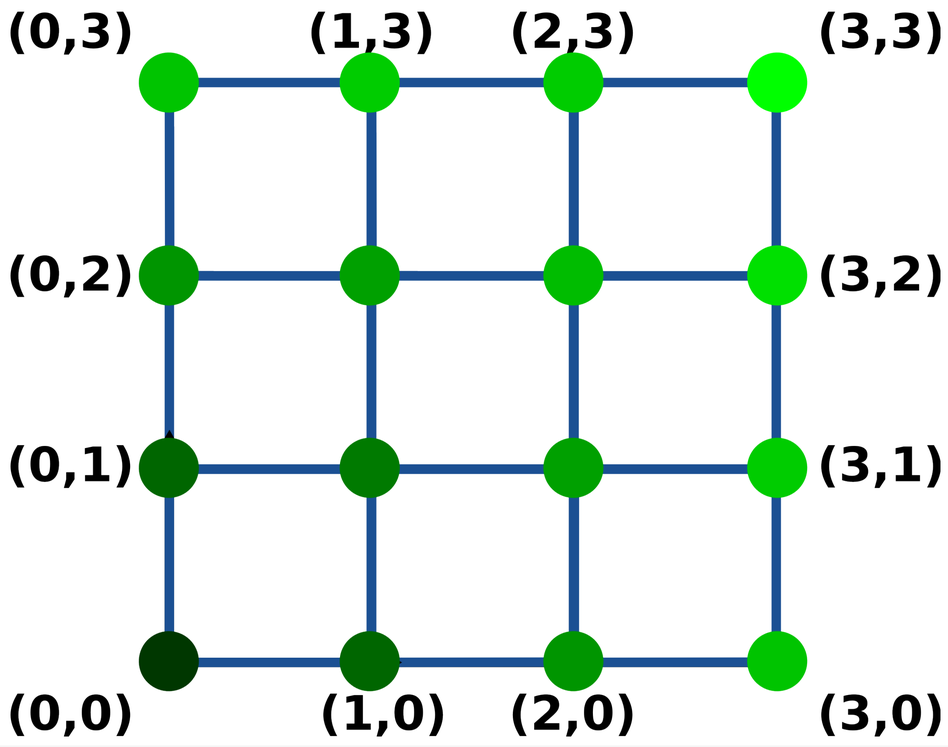

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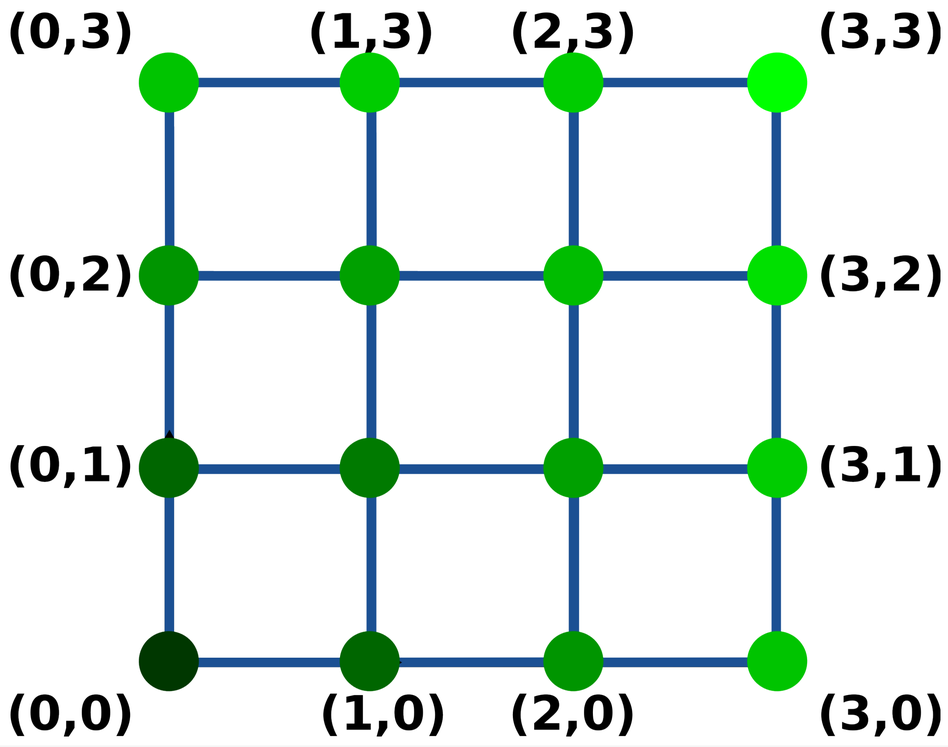

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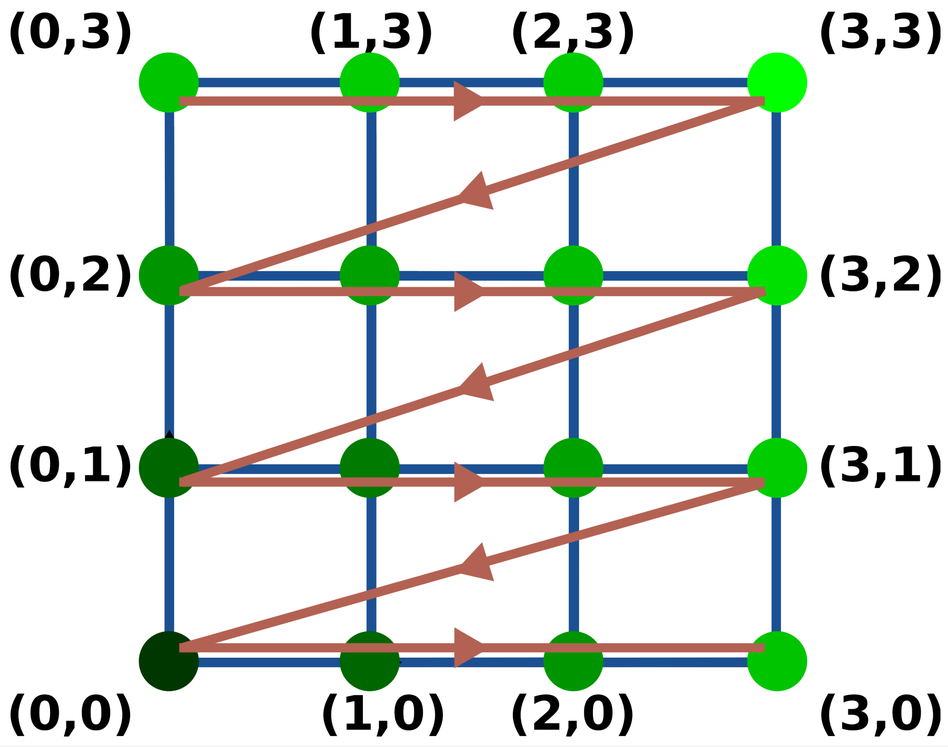

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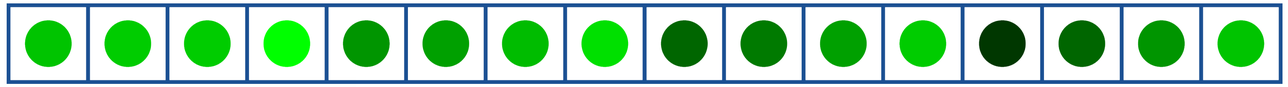

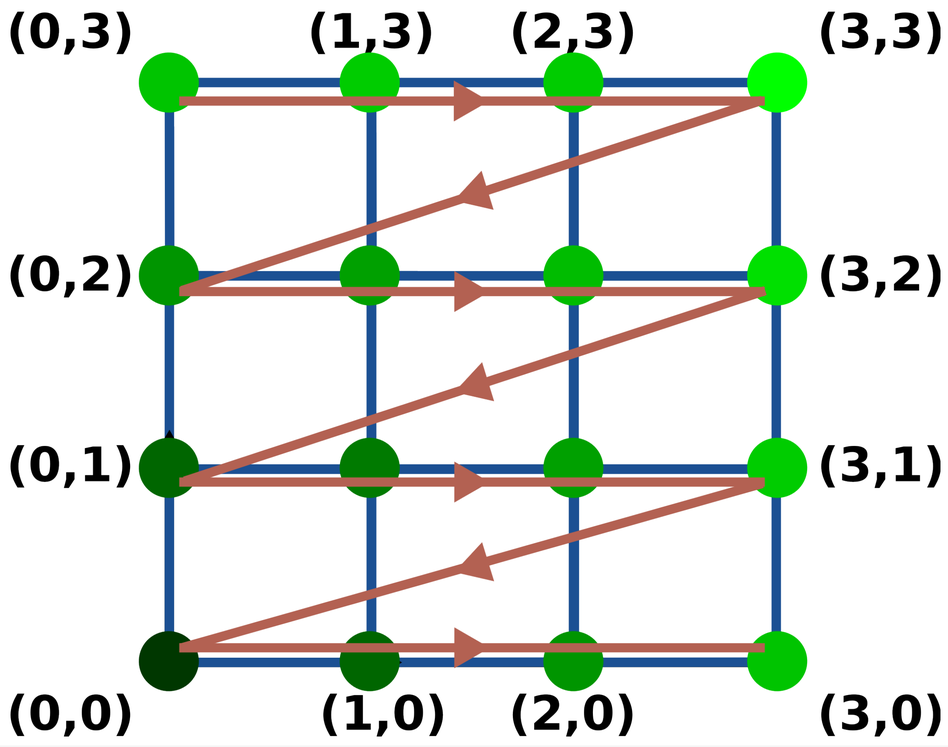

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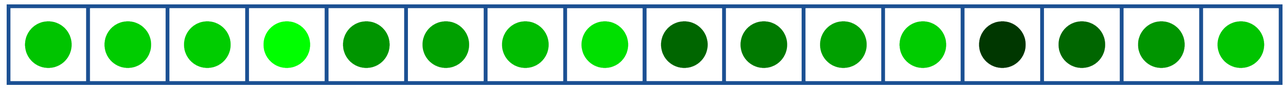

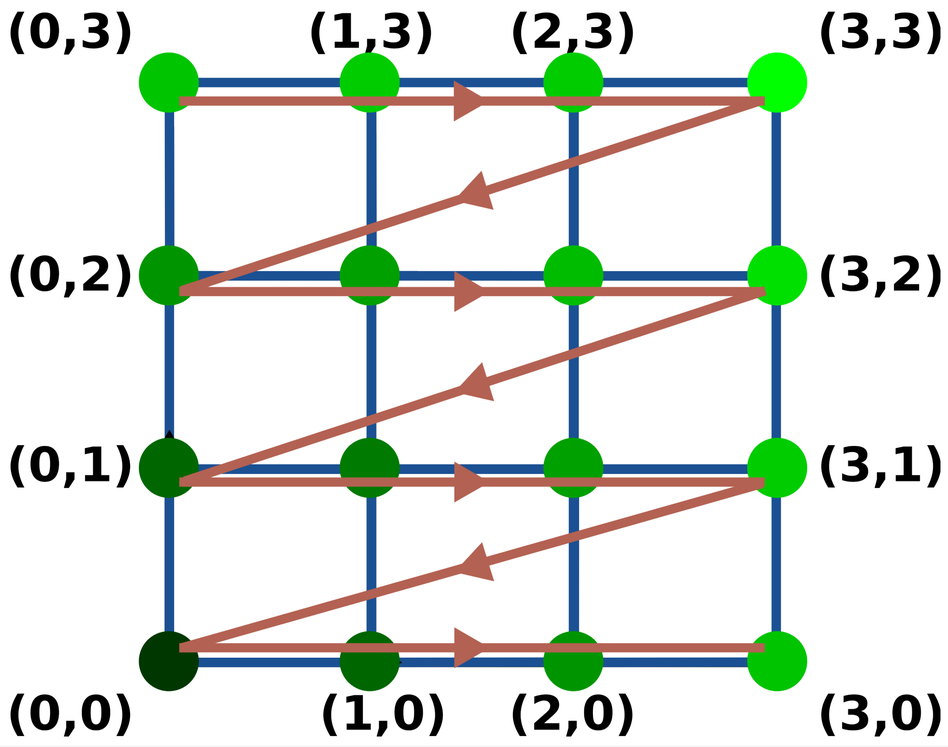

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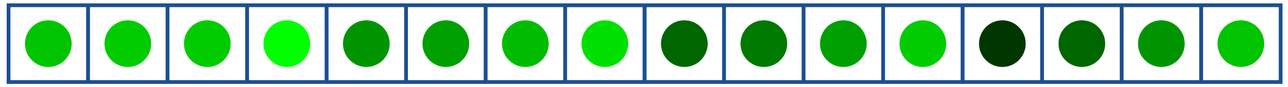

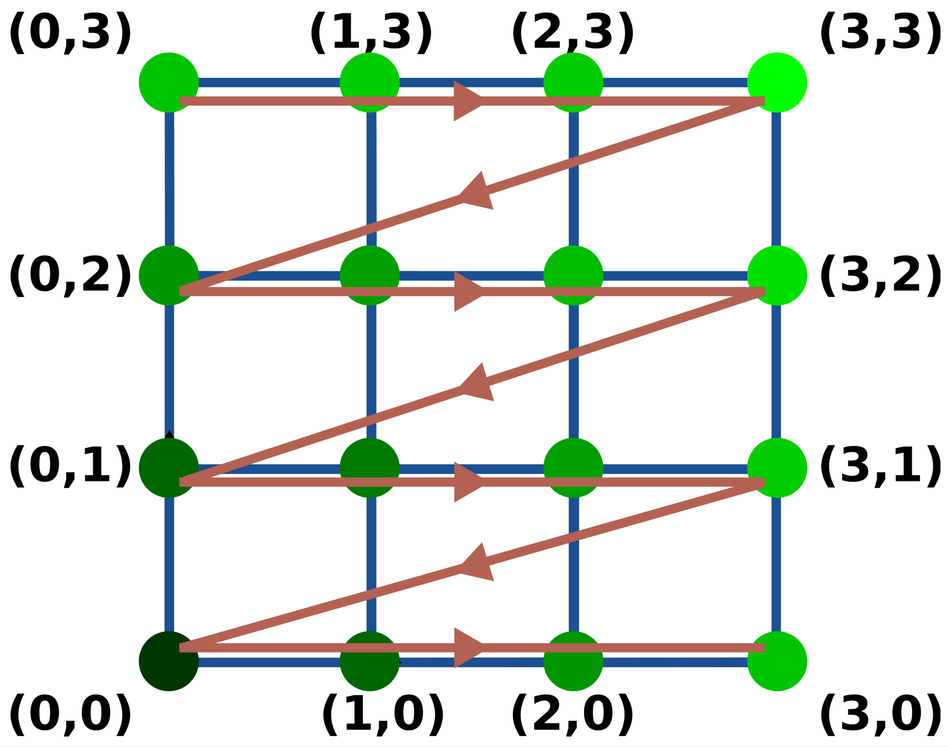

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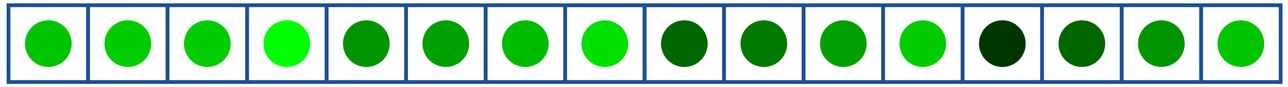

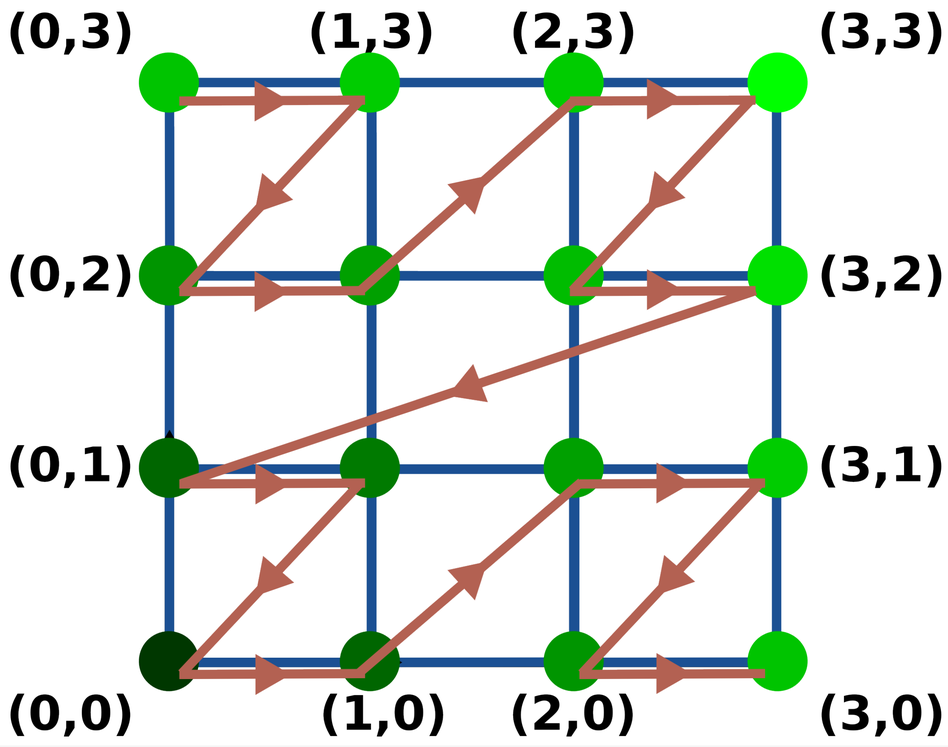

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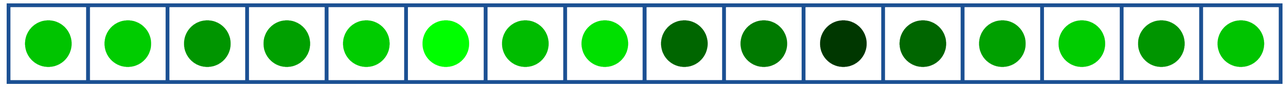

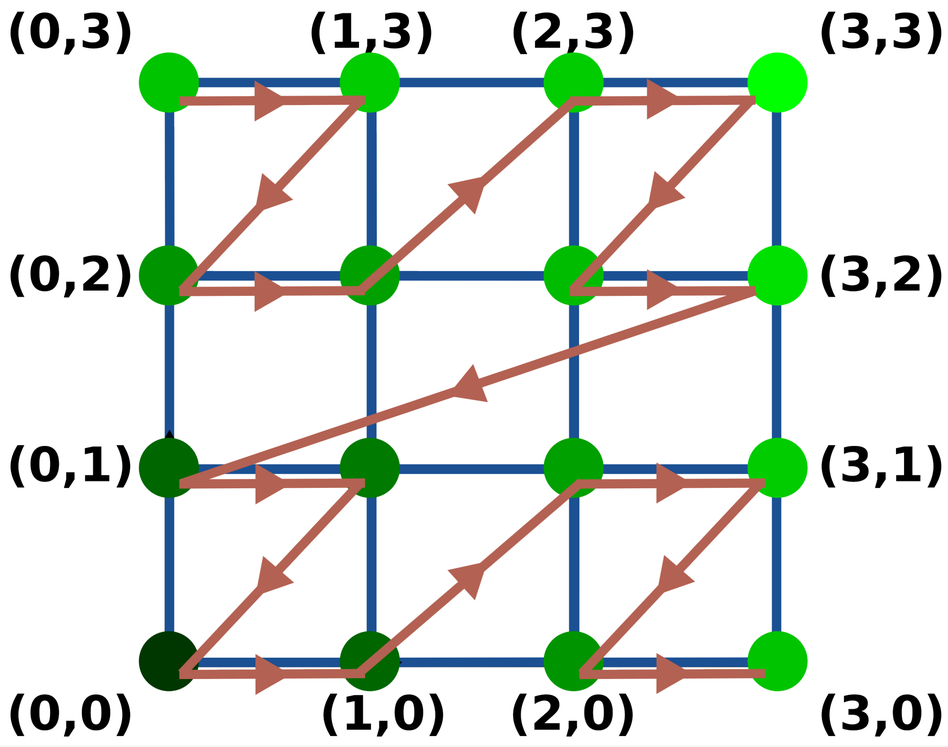

Regular grids of $\mathbb{R}^3$

[Wikipedia]

Regular grids of $\mathbb{R}^3$

- Same problem
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[Wikipedia]

Regular grids of $\mathbb{R}^3$

- Same problem
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[Wikipedia]

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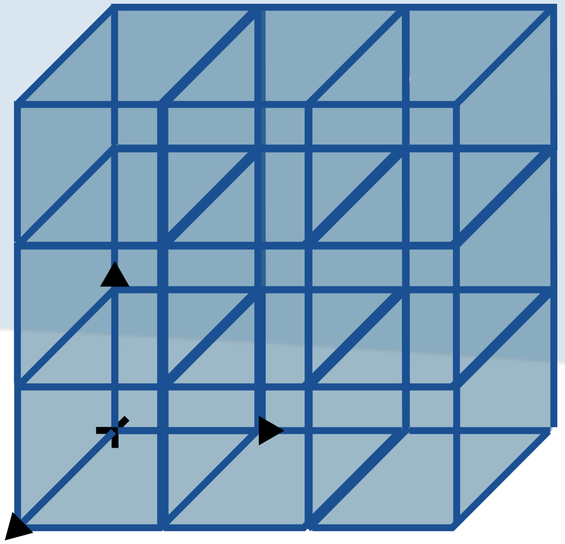

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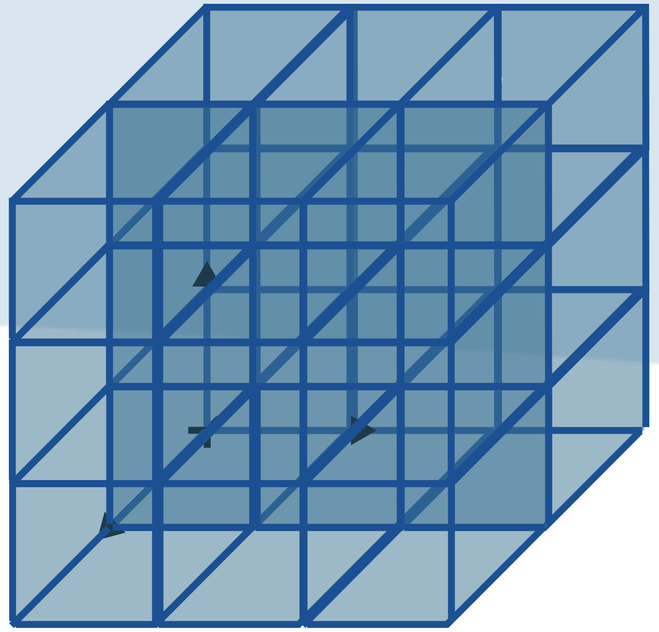
[Wikipedia]

Regular grids of $\mathbb{R}^3$



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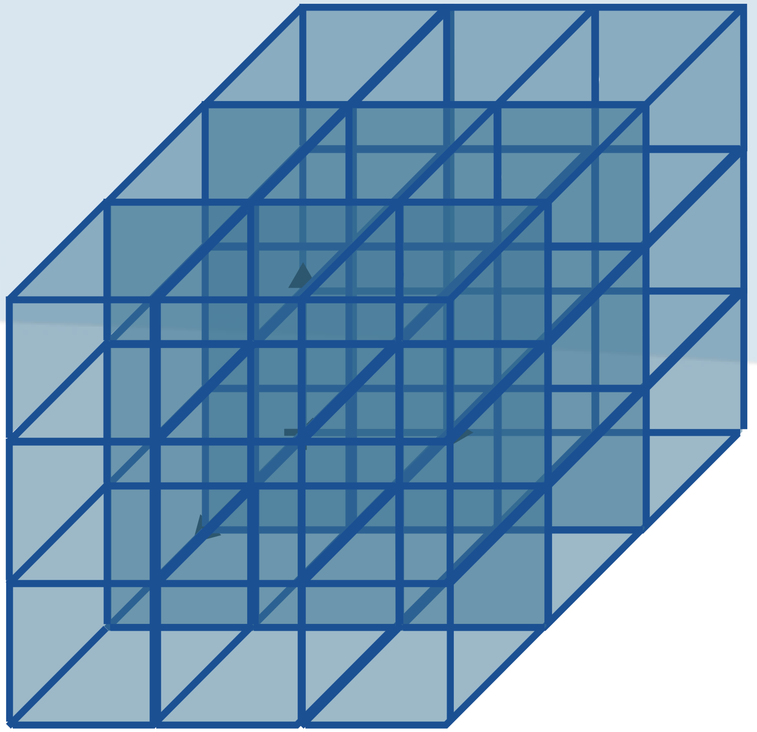
[Wikipedia]

Regular grids of $\mathbb{R}^3$



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[Wikipedia]

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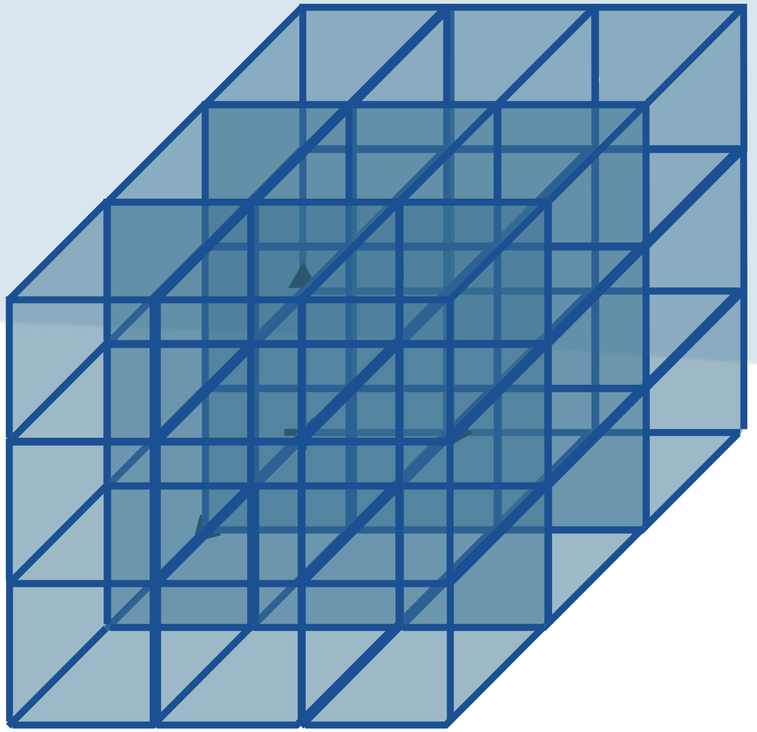

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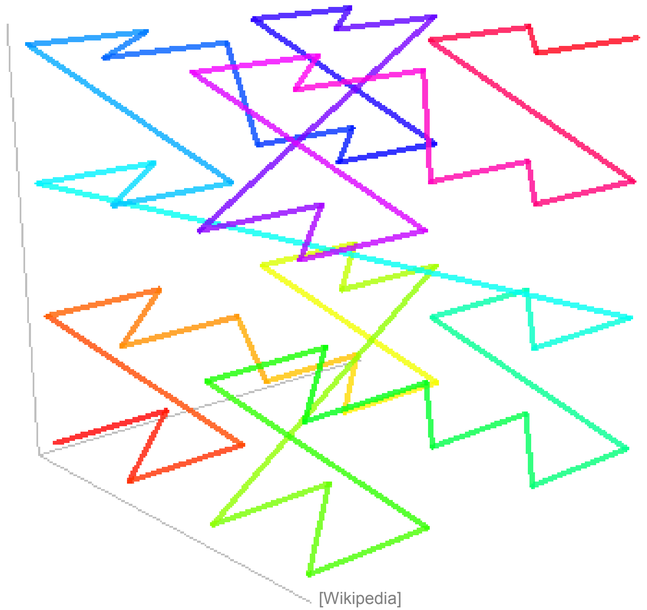
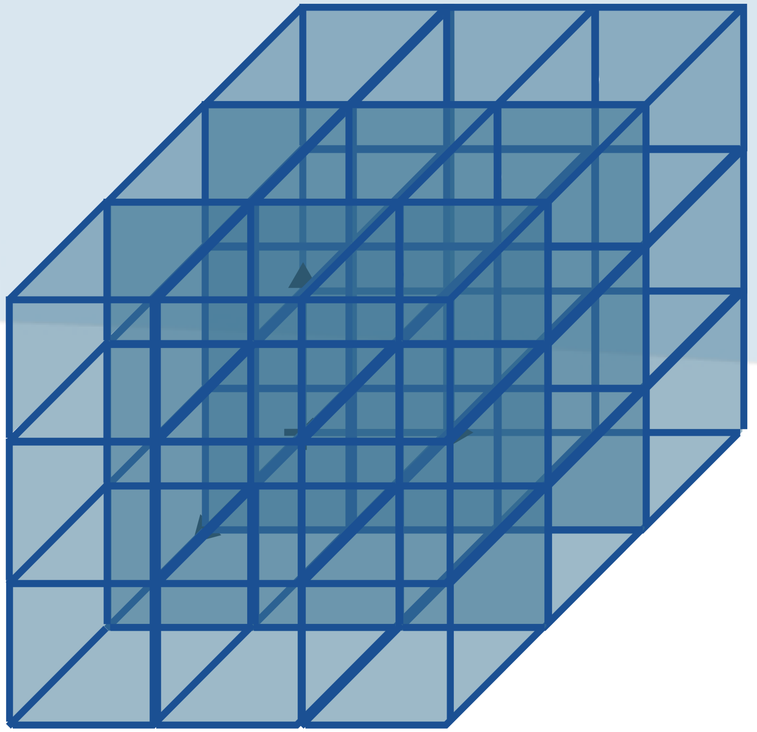
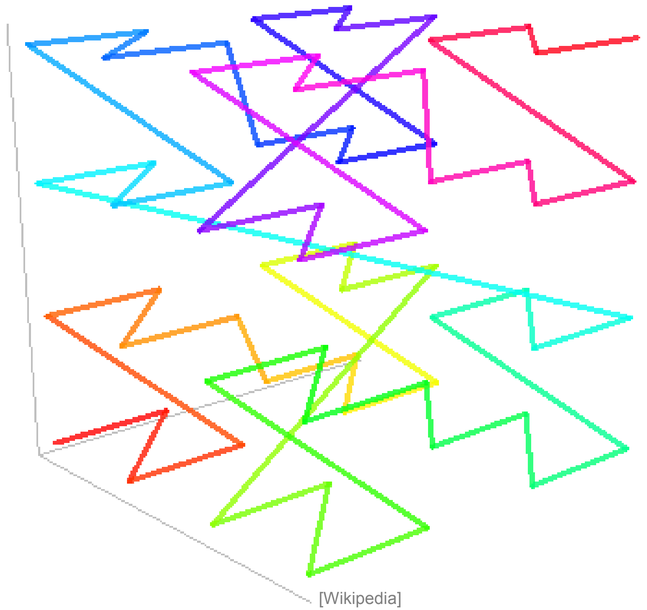


[Wikipedia]

Regular grids of $\mathbb{R}^n$

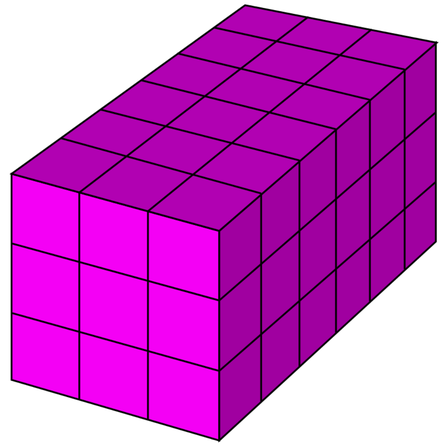


- Same problem
  - Computer memory is one-dimensional!
  - Same solutions
    - One dimension at a time
    - Peano's curves
    - Hilbert's curves, ...
    - Lebesgue's curves (z-order)
- Generalizes to higher dimensions



[Wikipedia]

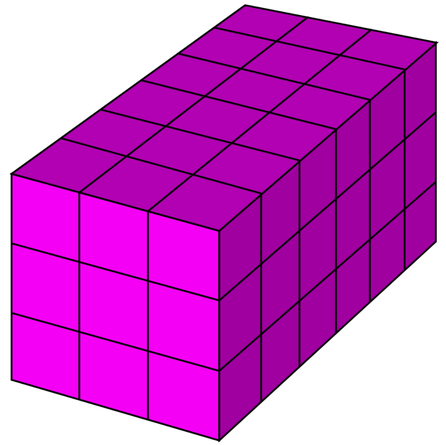
Embedding functions



[Wikipedia]

Embedding functions

- Problem
  - What if we want to adapt the sampling to the geometry of the data?



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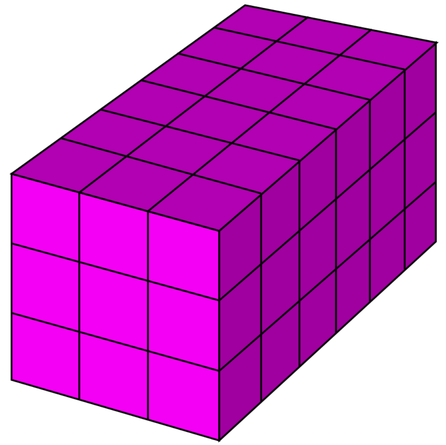
Embedding functions

- Problem
  - What if we want to adapt the sampling to the geometry of the data?

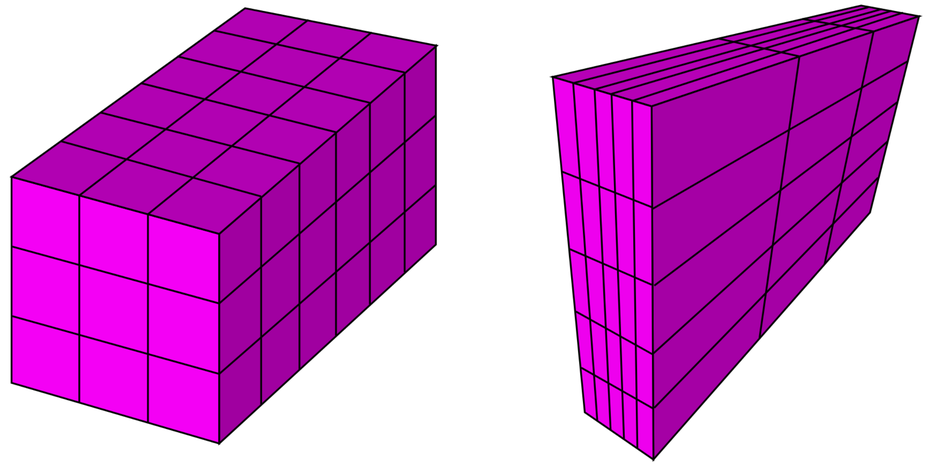


- Embedding functions
  - $e : \mathcal{G} \rightarrow \mathbb{R}^n$

[Wikipedia]

Embedding functions

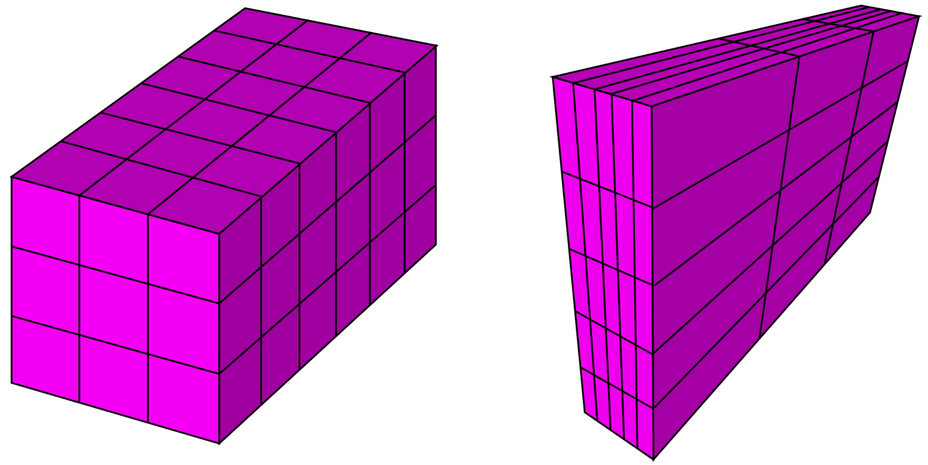
- Problem
  - What if we want to adapt the sampling to the geometry of the data?
- Embedding functions
  - $e : \mathcal{G} \rightarrow \mathbb{R}^n$



[Wikipedia]

Embedding functions

- Problem
  - What if we want to adapt the sampling to the geometry of the data?

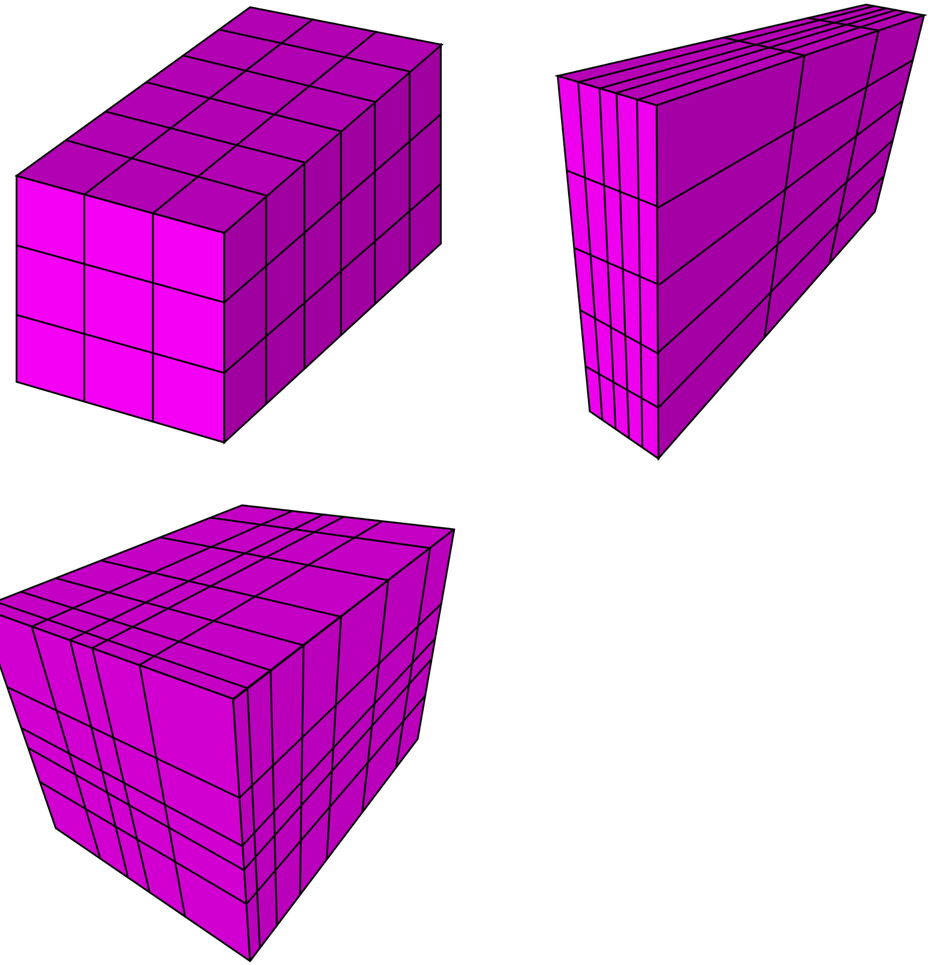


- Embedding functions
  - $e : \mathcal{G} \rightarrow \mathbb{R}^n$
  - Attributes attached to the grid

[Wikipedia]

Embedding functions

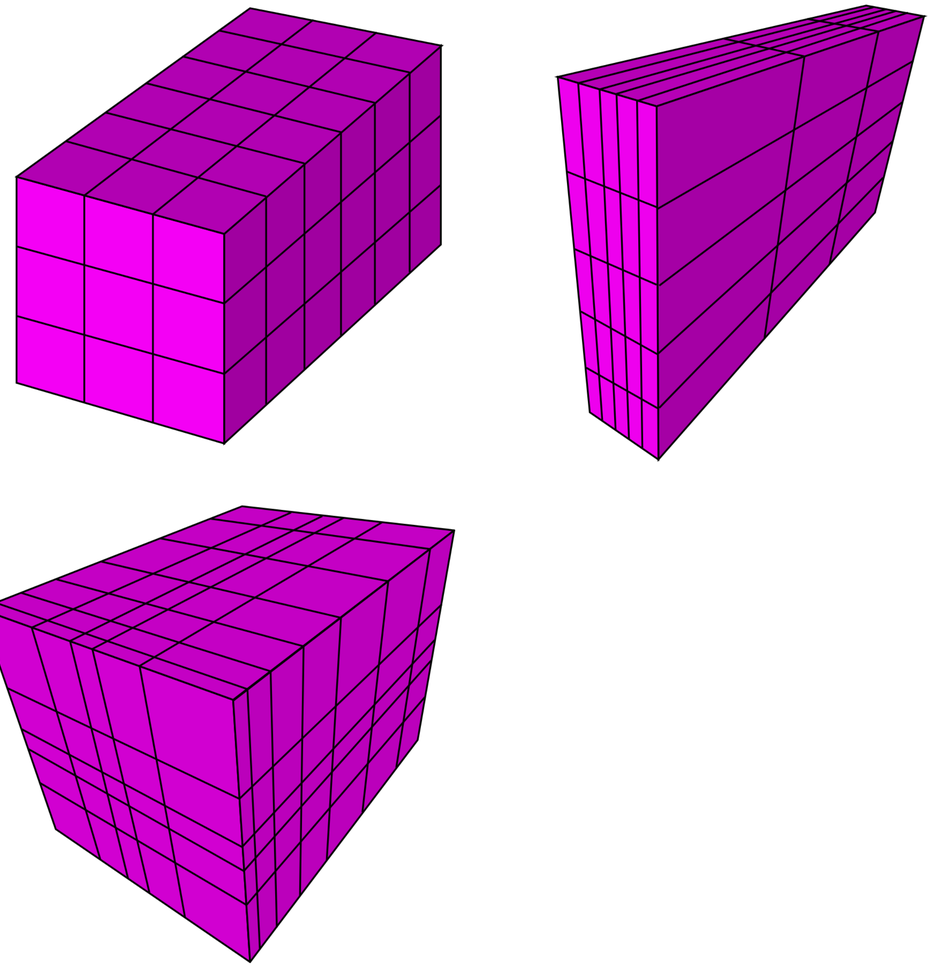
- Problem
  - What if we want to adapt the sampling to the geometry of the data?
- Embedding functions
  - $e : \mathcal{G} \rightarrow \mathbb{R}^n$
  - Attributes attached to the grid
- Non uniform sampling



[Wikipedia]

Embedding functions

- Problem
  - What if we want to adapt the sampling to the geometry of the data?
- Embedding functions
  - $e : \mathcal{G} \rightarrow \mathbb{R}^n$
  - Attributes attached to the grid
- Non uniform sampling
  - *Rectilinear* grid



[Wikipedia]

Embedding functions

- Problem
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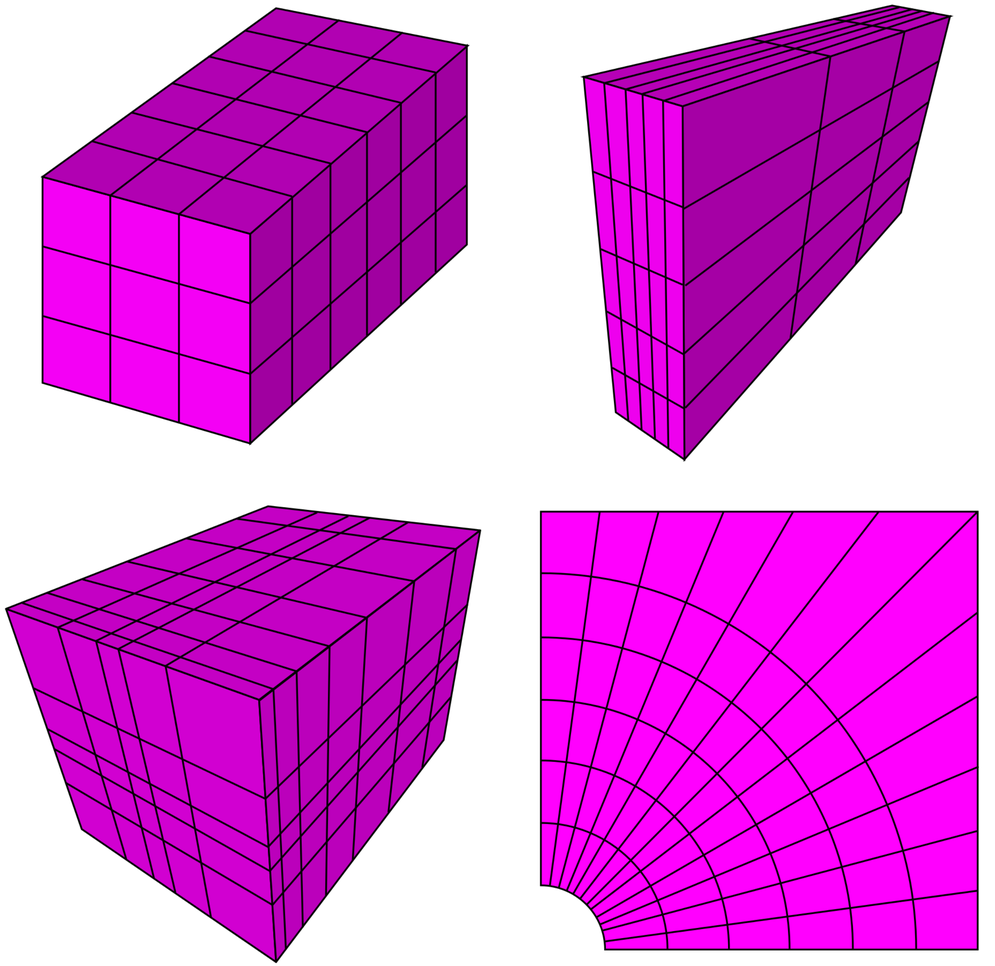
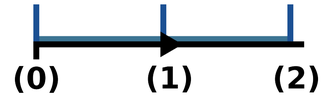
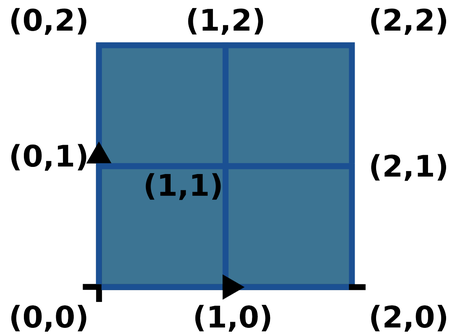


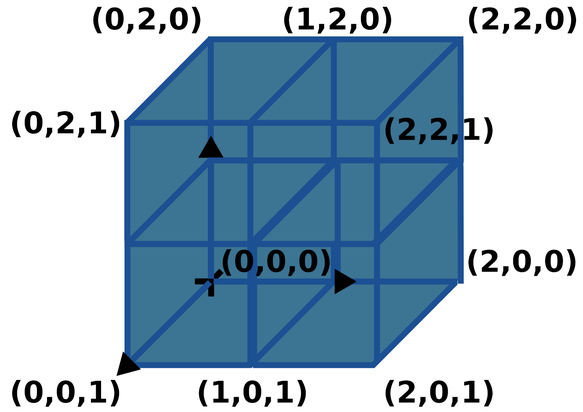
[Wikipedia]

Euclidean spaces on a computer

- Notion of **regular grid** $\mathcal{G}$







Euclidean spaces on a computer

- Notion of **regular grid** $\mathcal{G}$
  - Set of **vertices** (samples) in $\mathbb{R}^n$
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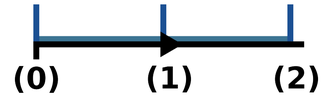
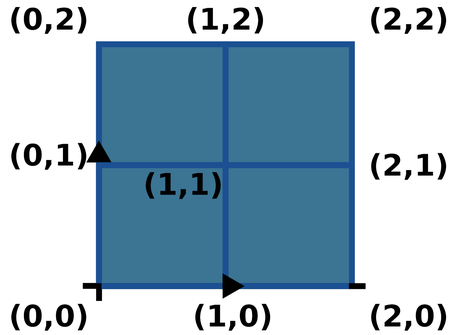
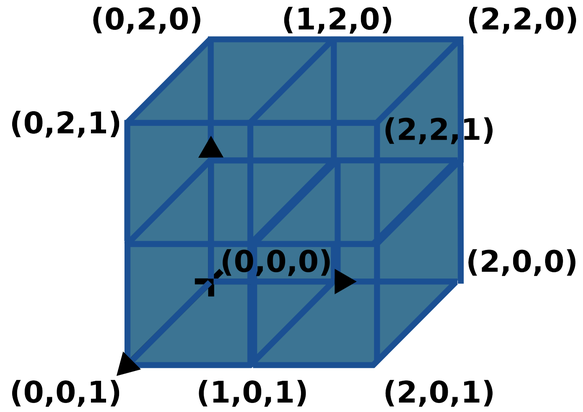
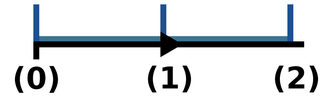


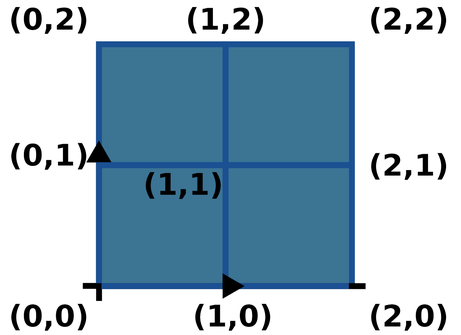


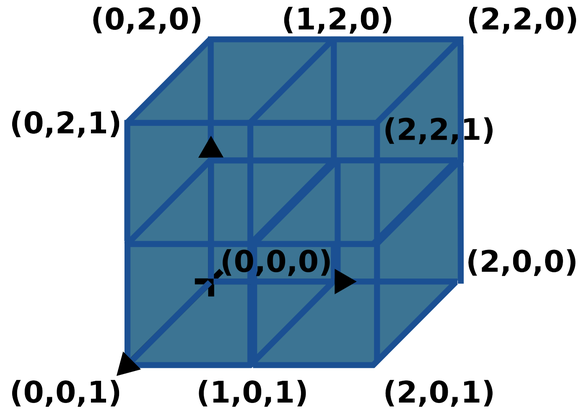


Euclidean spaces on a computer

- Notion of **regular grid** $\mathcal{G}$
  - Set of **vertices** (samples) in $\mathbb{R}^n$
    - With embedding functions to $\mathbb{R}^n$
  - **Connectivity**







Euclidean spaces on a computer

- Notion of **regular grid** $\mathcal{G}$
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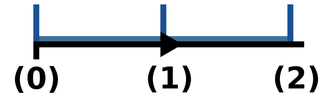
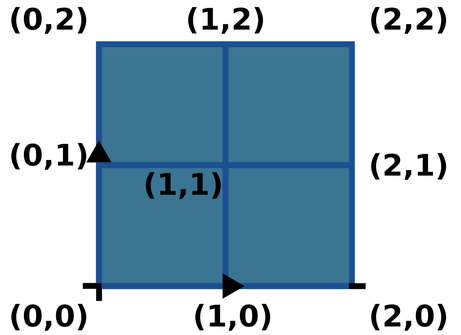
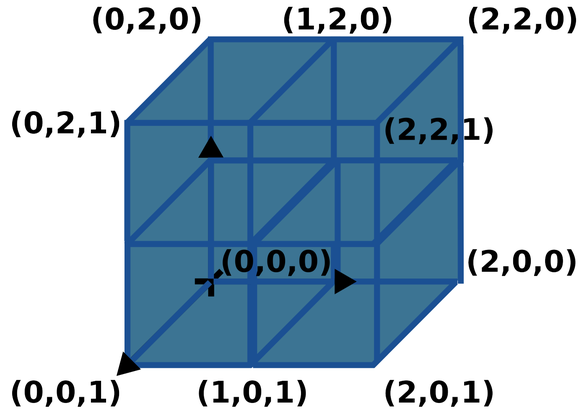
    - Unit cells (pixels, voxels)







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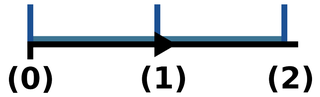
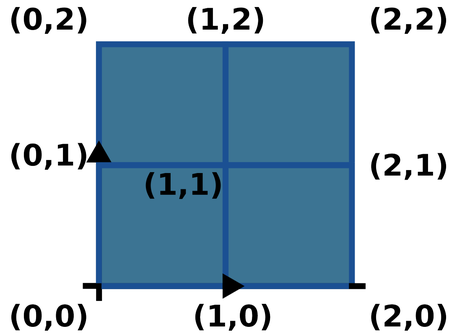
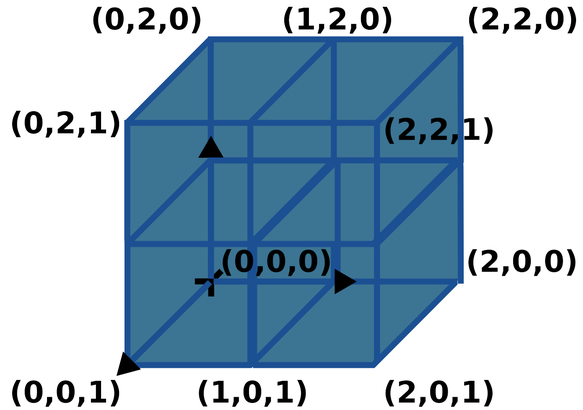
    - Unit cells (pixels, voxels)
    - Implicitly encoded by the samples (space fill)







Euclidean spaces on a computer

- Notion of **regular grid** $\mathcal{G}$
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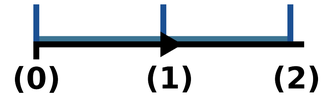
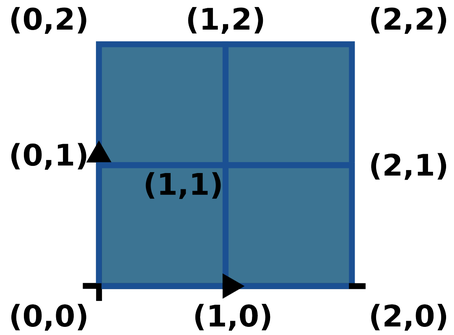
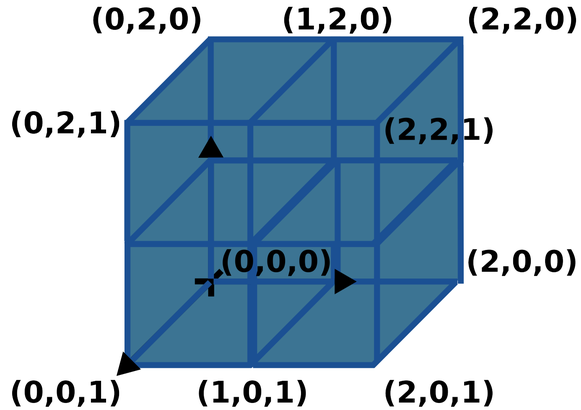
  - **Connectivity**
    - Unit cells (pixels, voxels)
    - Implicitly encoded by the samples (space fill)
  - What's missing?







Euclidean spaces on a computer

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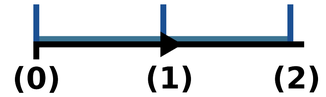
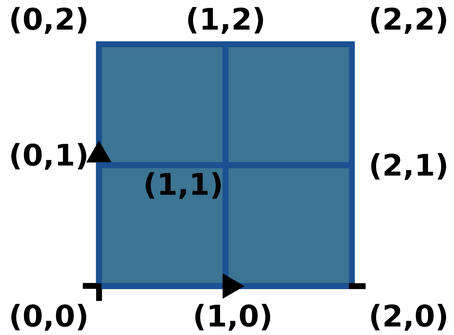
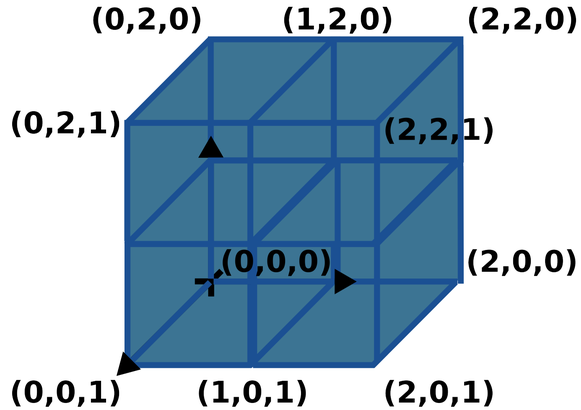
  - **Connectivity**
    - Unit cells (pixels, voxels)
    - Implicitly encoded by the samples (space fill)
  - What's missing?
    - The **interpolation** scheme

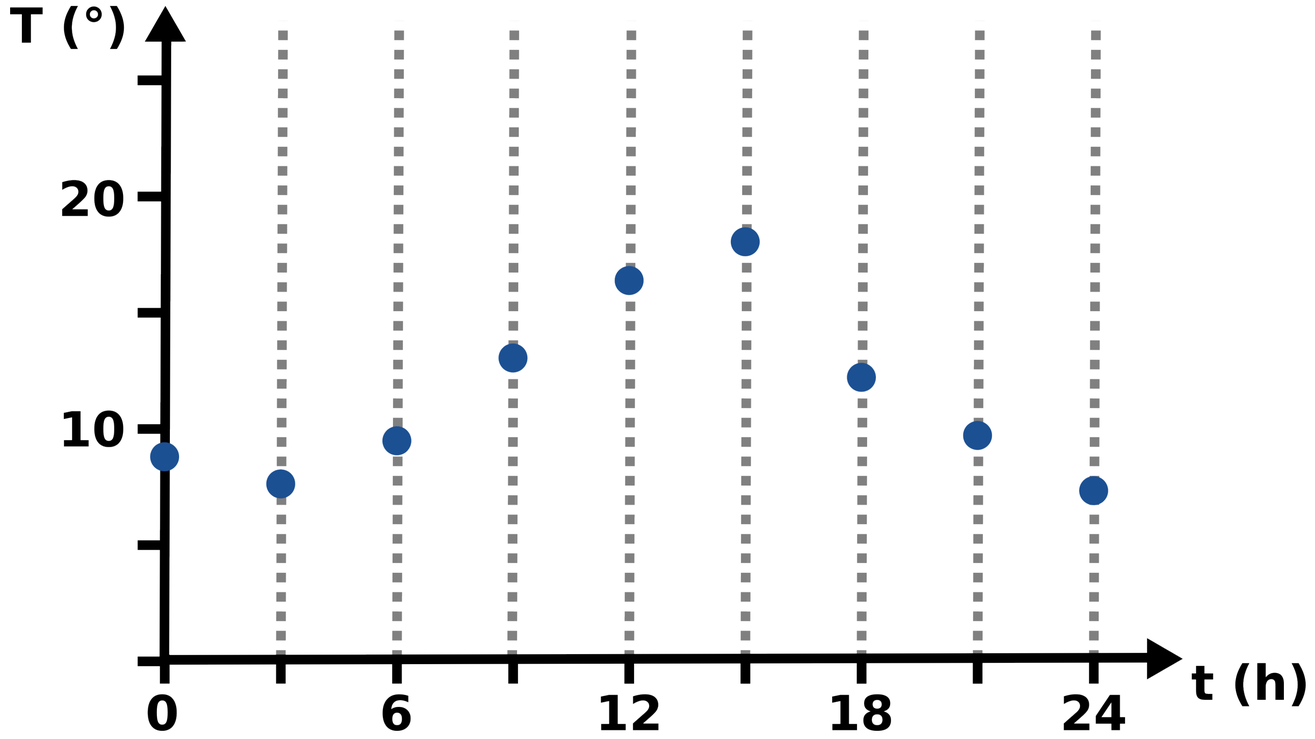






Interpolants for regular grids

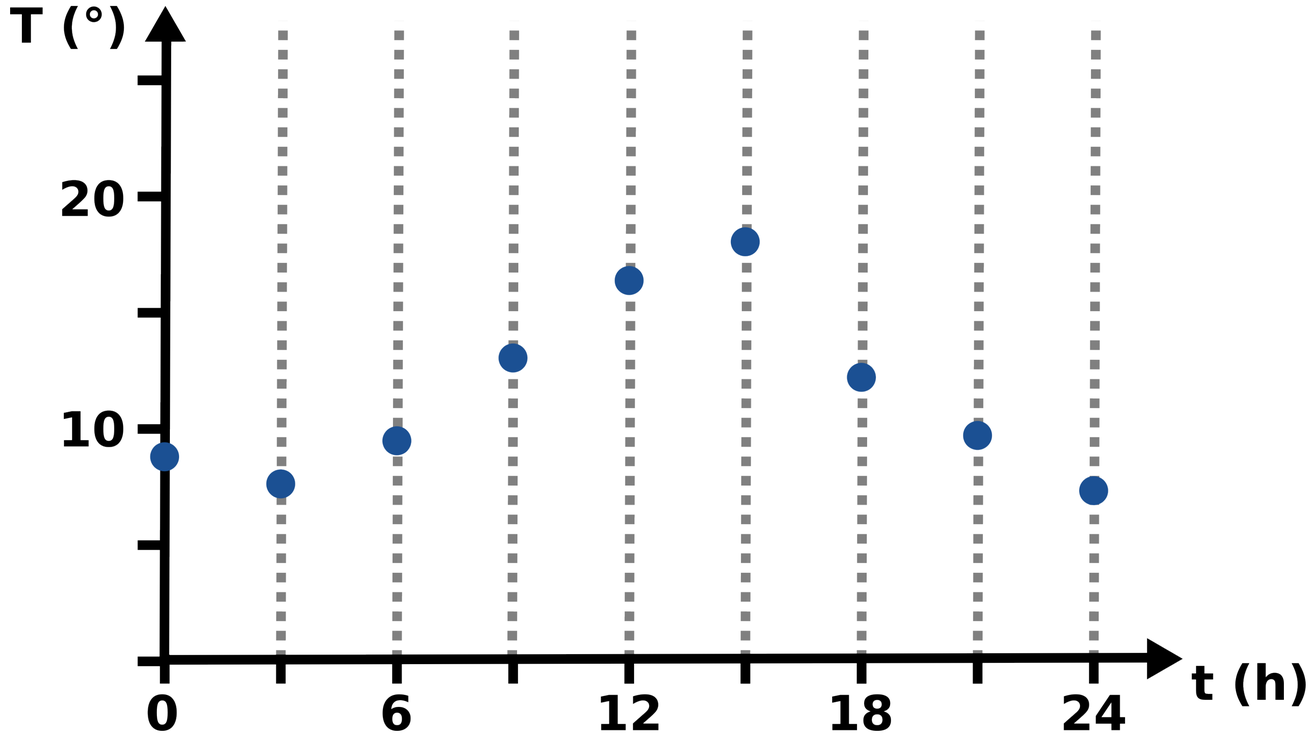
$$f : [0, 24] \to \mathbb{R}$$



Interpolants for regular grids

- Given values on the vertices

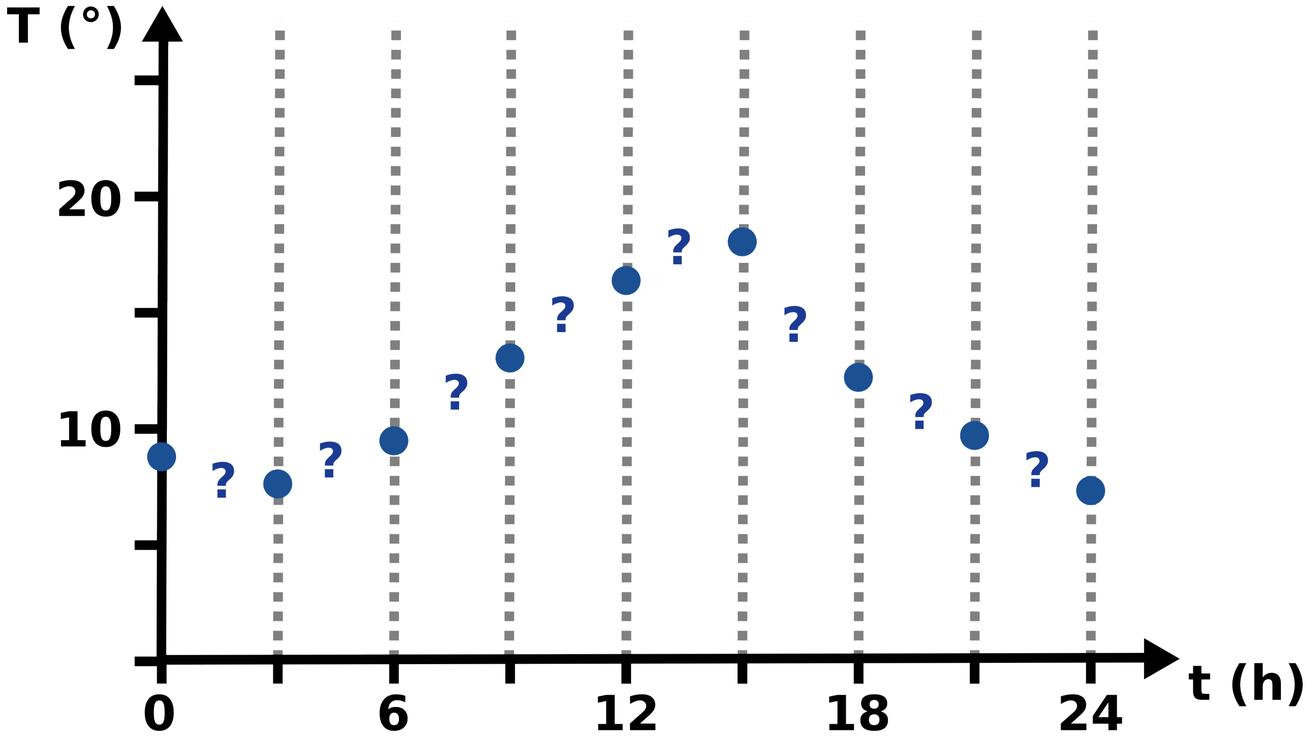
$$f : [0, 24] \to \mathbb{R}$$



Interpolants for regular grids

- Given values on the vertices
  - How can we deduce the values within the unit cells?

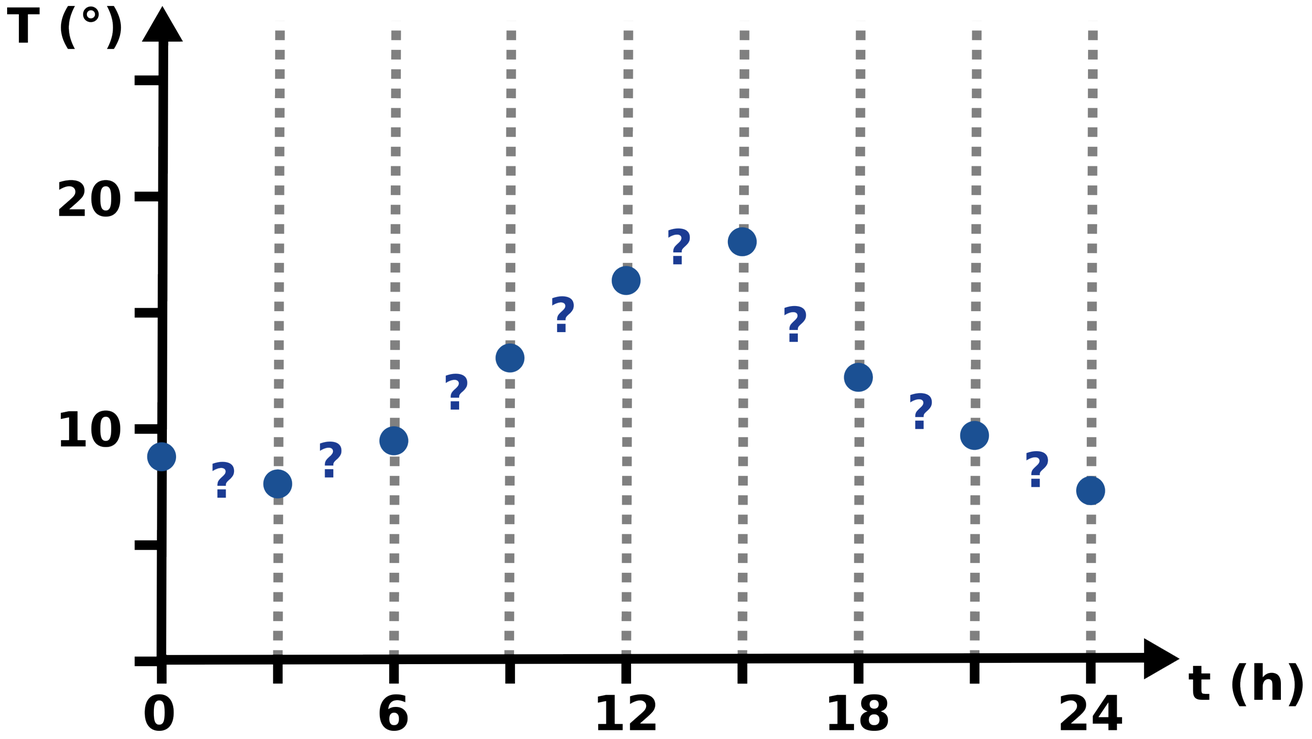
$$f : [0, 24] \to \mathbb{R}$$



Interpolants for regular grids

- Given values on the vertices
  - How can we deduce the values within the unit cells?
- For regular grids of $\mathbb{R}$
  - $\mathbb{I} : \ ]0,1[\ \rightarrow \ ]0,1[$

$$f : [0, 24] \rightarrow \mathbb{R}$$



Interpolants for regular grids

- Given values on the vertices
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  - $\mathbb{I} : \ ]0,1[\ \rightarrow \ ]0,1[$
  - $lim_{x \rightarrow 0} \ \mathbb{I}(x) = 0$
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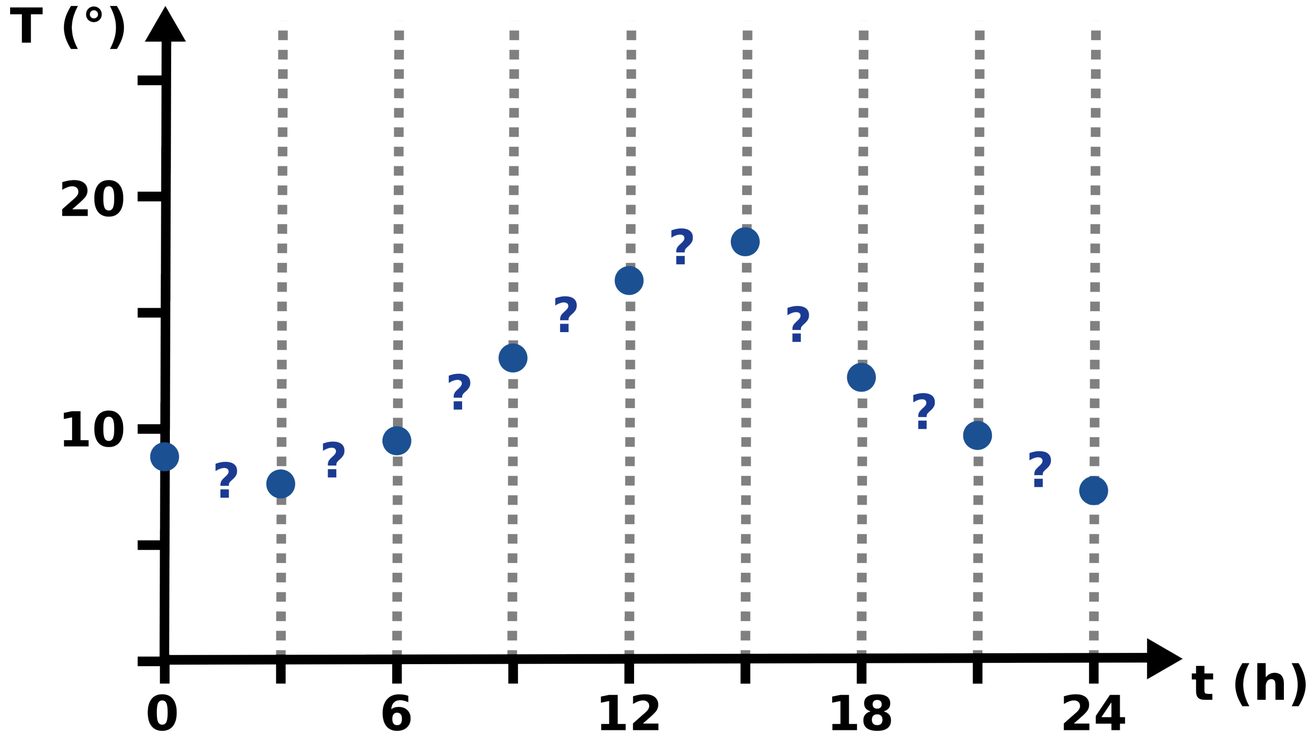
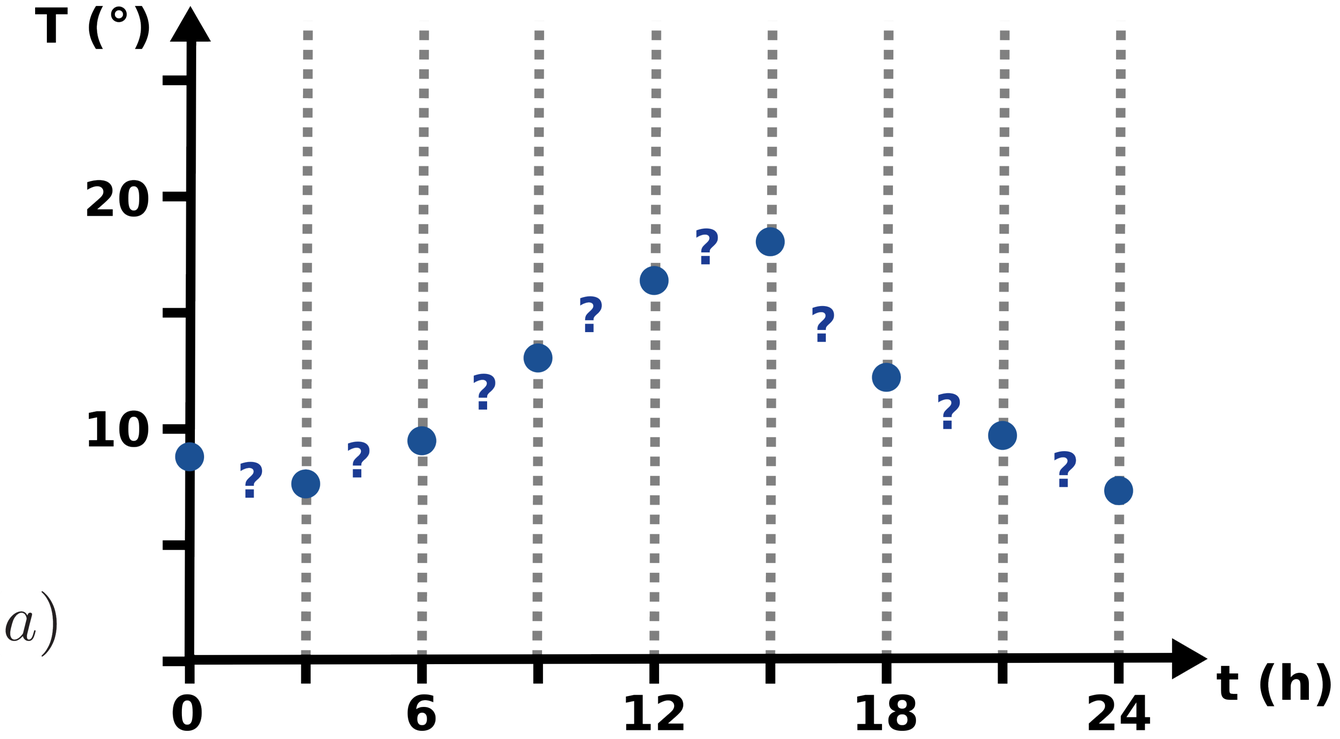
$$f : [0, 24] \rightarrow \mathbb{R}$$



Interpolants for regular grids

- Given values on the vertices
  - How can we deduce the values within the unit cells?
- For regular grids of $\mathbb{R}$
  - $\mathbb{I} : \ ]0,1[\ \rightarrow \ ]0,1[$
  - $lim_{x \rightarrow 0} \ \mathbb{I}(x) = 0$
  - $lim_{x \rightarrow 1} \ \mathbb{I}(x) = 1$
  - $\forall x \in [a,b]$
    - $f(x) = \mathbb{I}(\frac{x-a}{b-a}) \, (f(b) - f(a)) + f(a)$

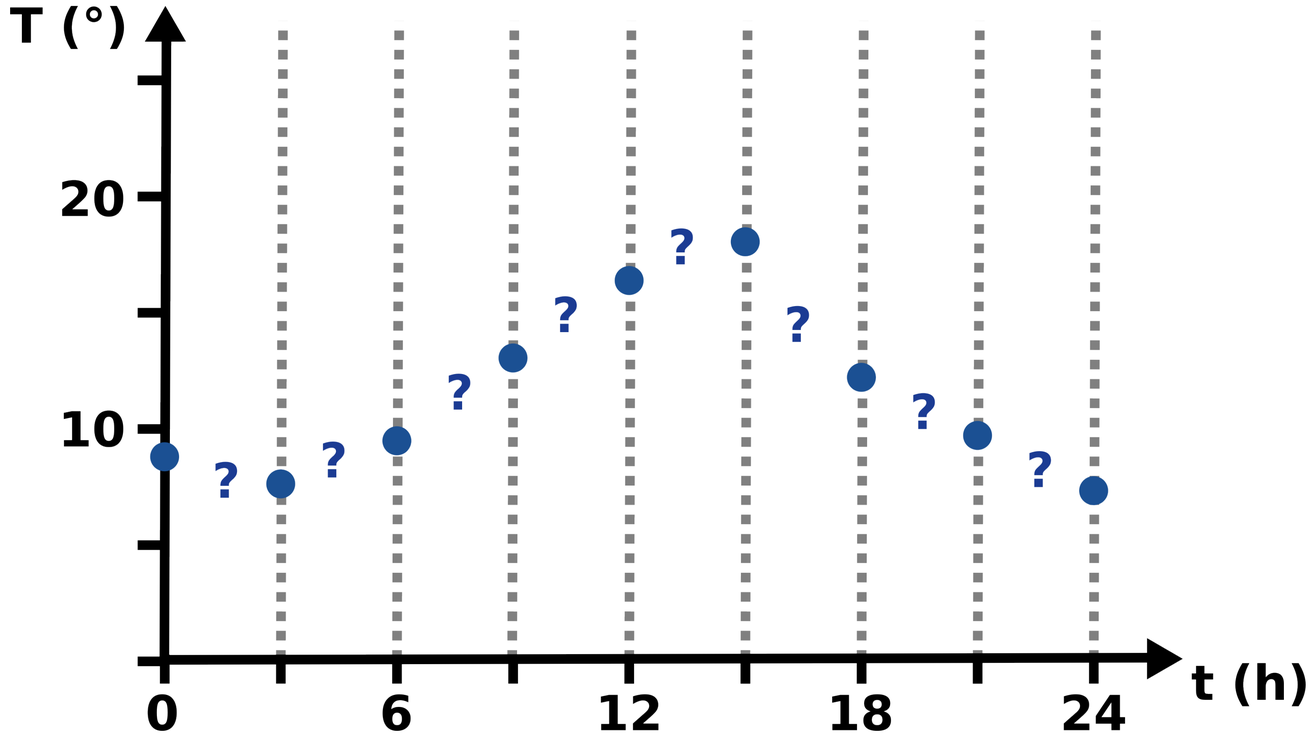
$$f : [0,24] \rightarrow \mathbb{R}$$



Interpolants for regular grids

- Interpolation function

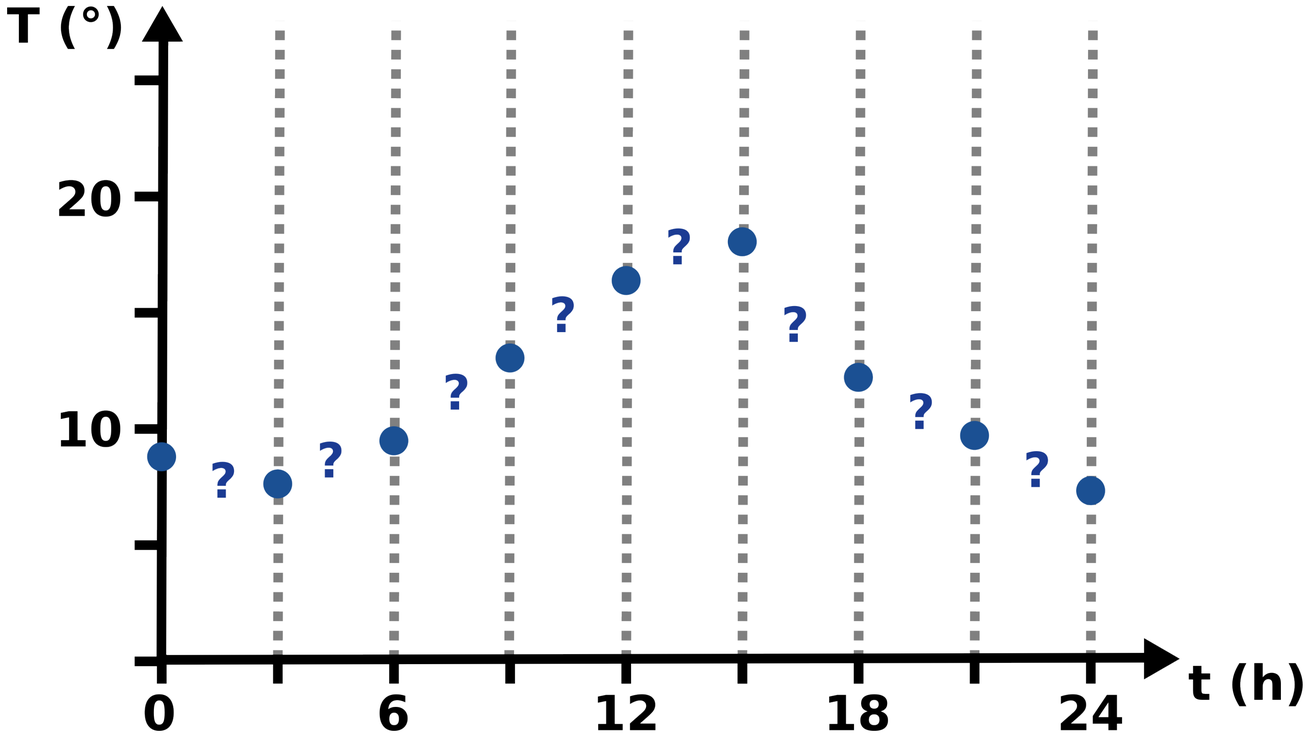
$$f : [0, 24] \to \mathbb{R}$$



Interpolants for regular grids

- Interpolation function
  - Given with the data

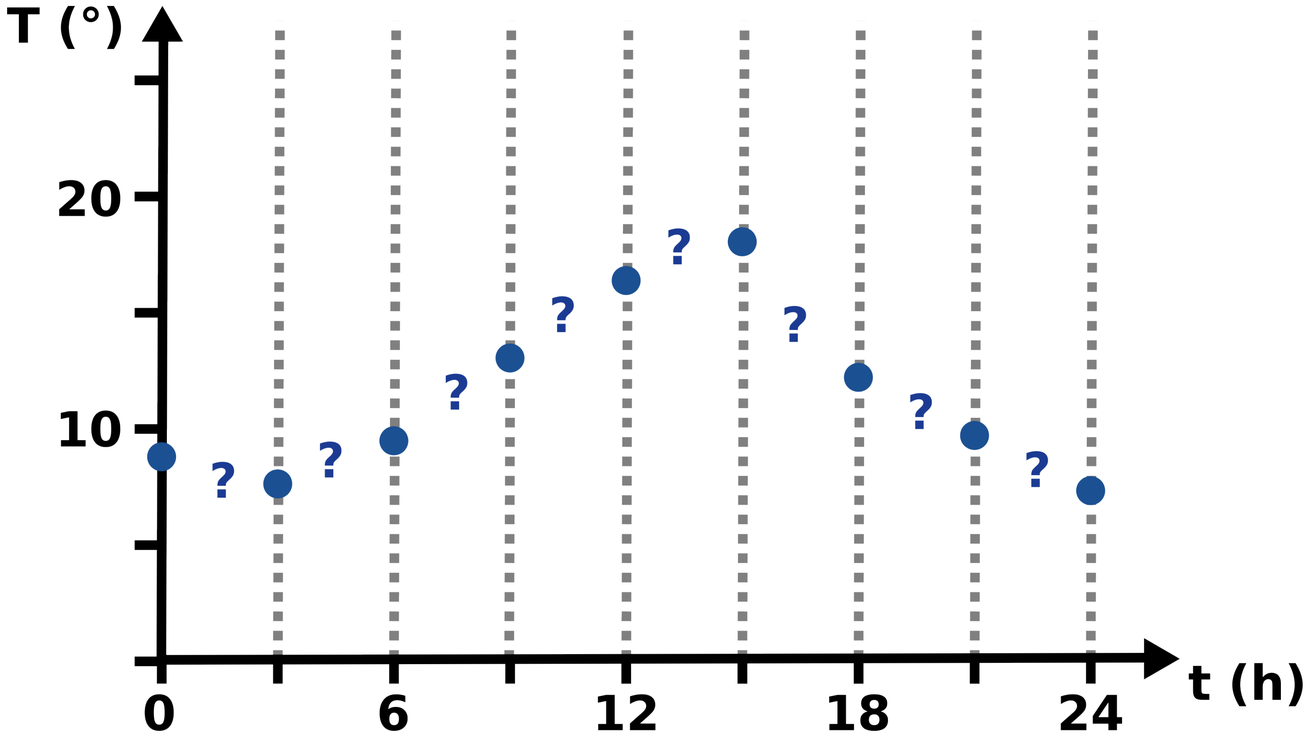
$$f : [0, 24] \to \mathbb{R}$$



Interpolants for regular grids

- Interpolation function
  - Given with the data
  - Can be defined per unit cell

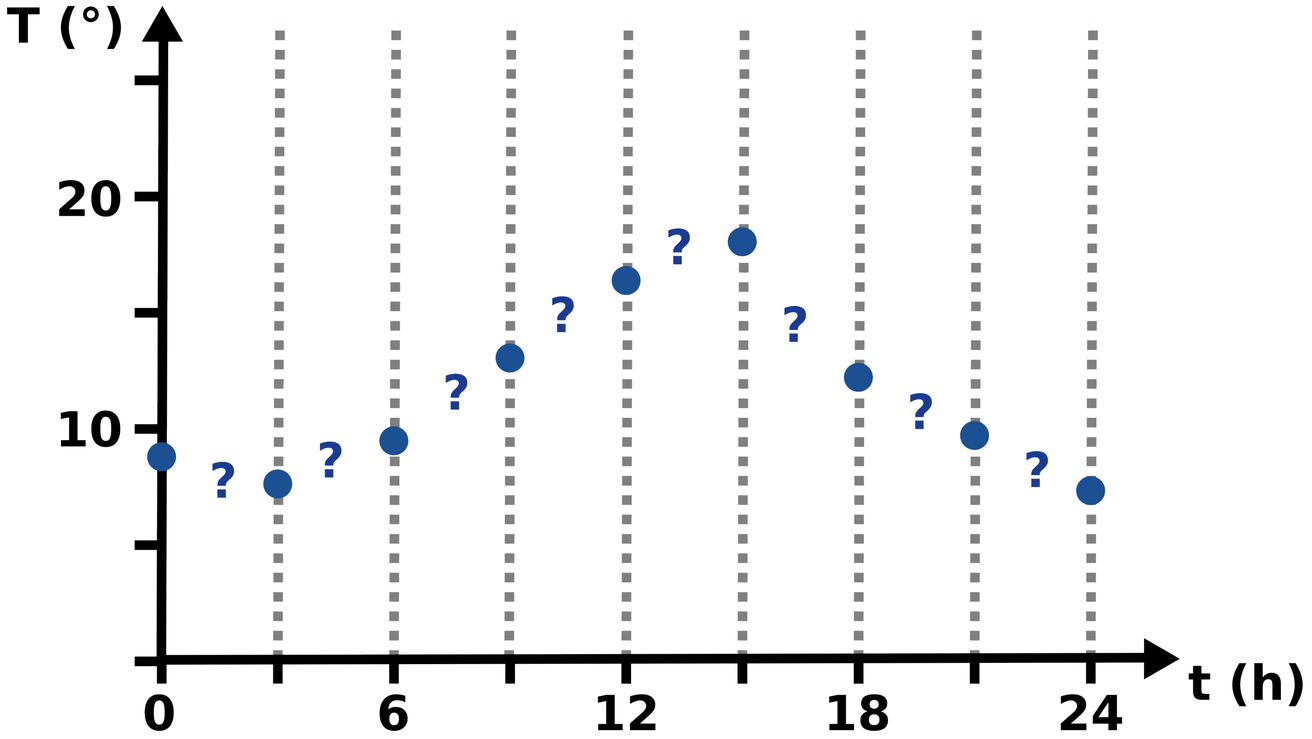
$$f : [0, 24] \to \mathbb{R}$$



Interpolants for regular grids

- Interpolation function
  - Given with the data
  - Can be defined per unit cell
  - Computed on demand

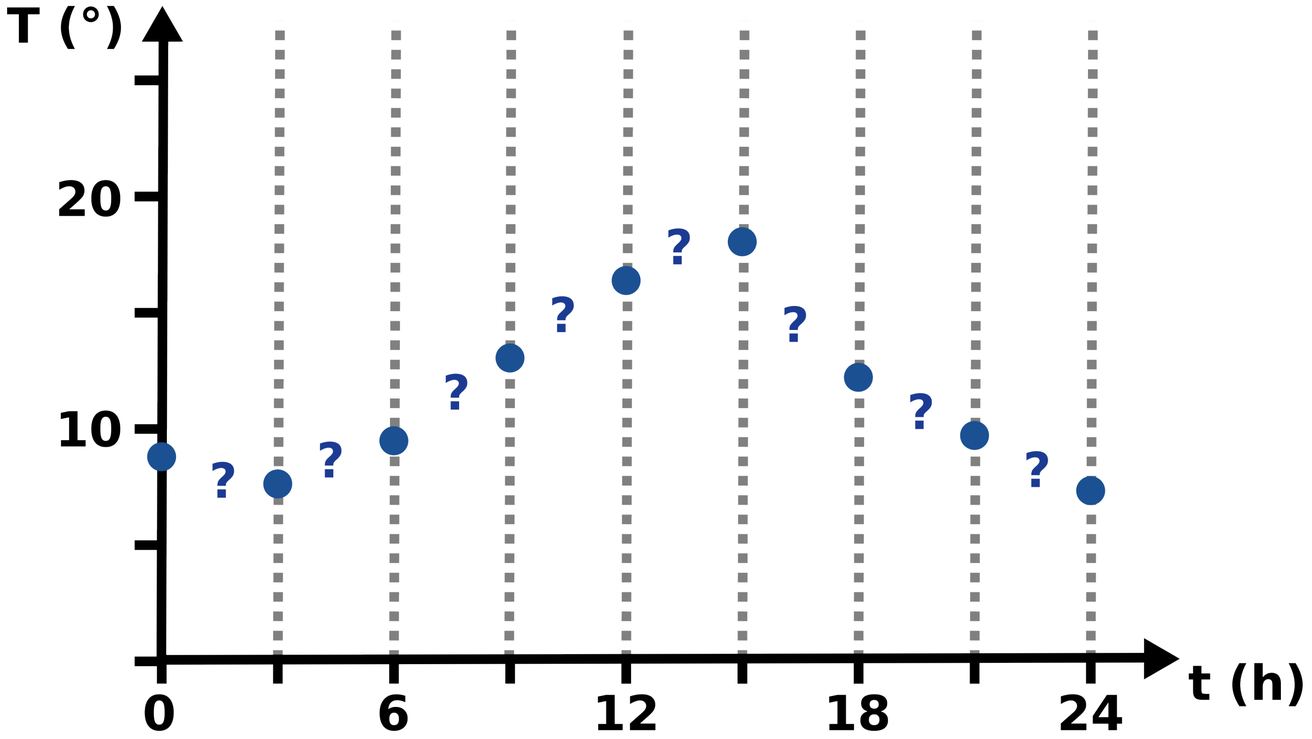
$$f : [0, 24] \rightarrow \mathbb{R}$$



Interpolants for regular grids

- Interpolation function
  - Given with the data
  - Can be defined per unit cell
  - Computed on demand
- Examples $\mathbb{I}: \ ]0,1[\ \rightarrow \ ]0,1[$

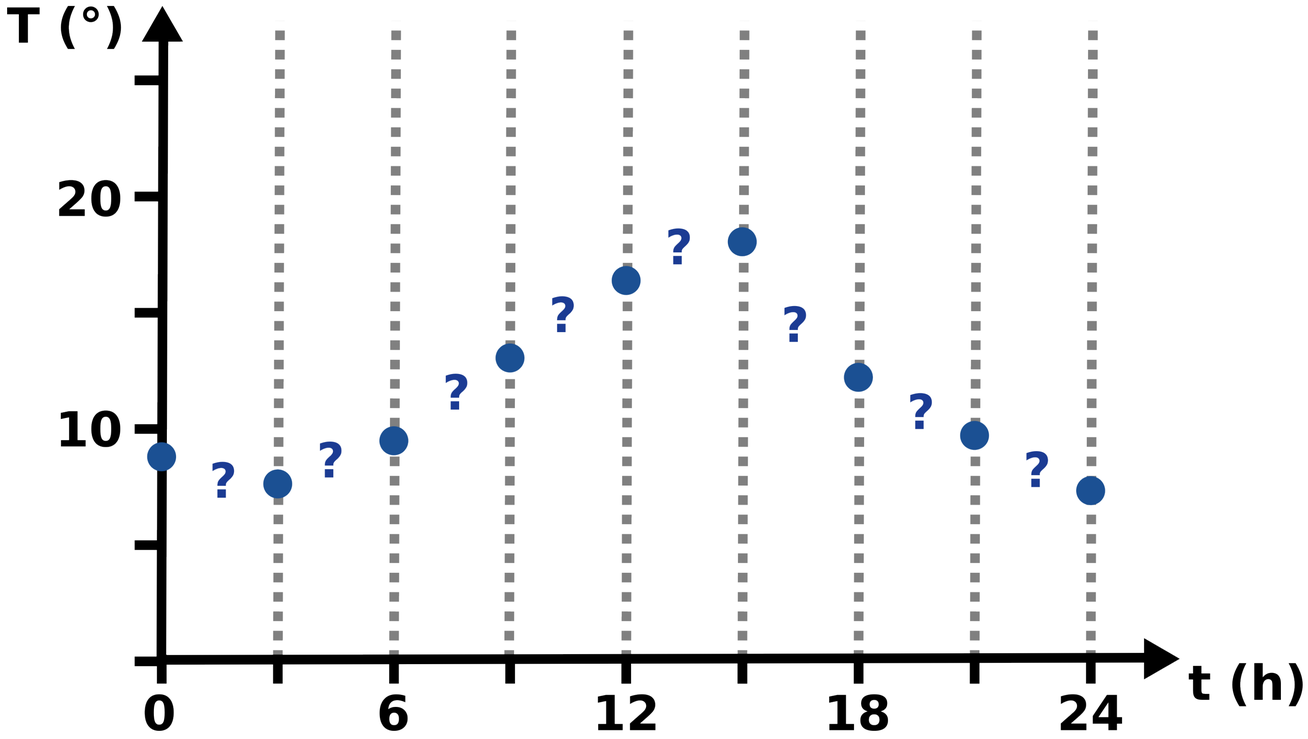
$$f : [0, 24] \rightarrow \mathbb{R}$$



Interpolants for regular grids

- Interpolation function
  - Given with the data
  - Can be defined per unit cell
  - Computed on demand
- Examples $\mathbb{I} : \ ]0,1[\ \rightarrow \ ]0,1[$
  - Piecewise constant
    - $\mathbb{I}(x) = \delta(x-1)$

$$f : [0, 24] \rightarrow \mathbb{R}$$



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  - Piecewise constant
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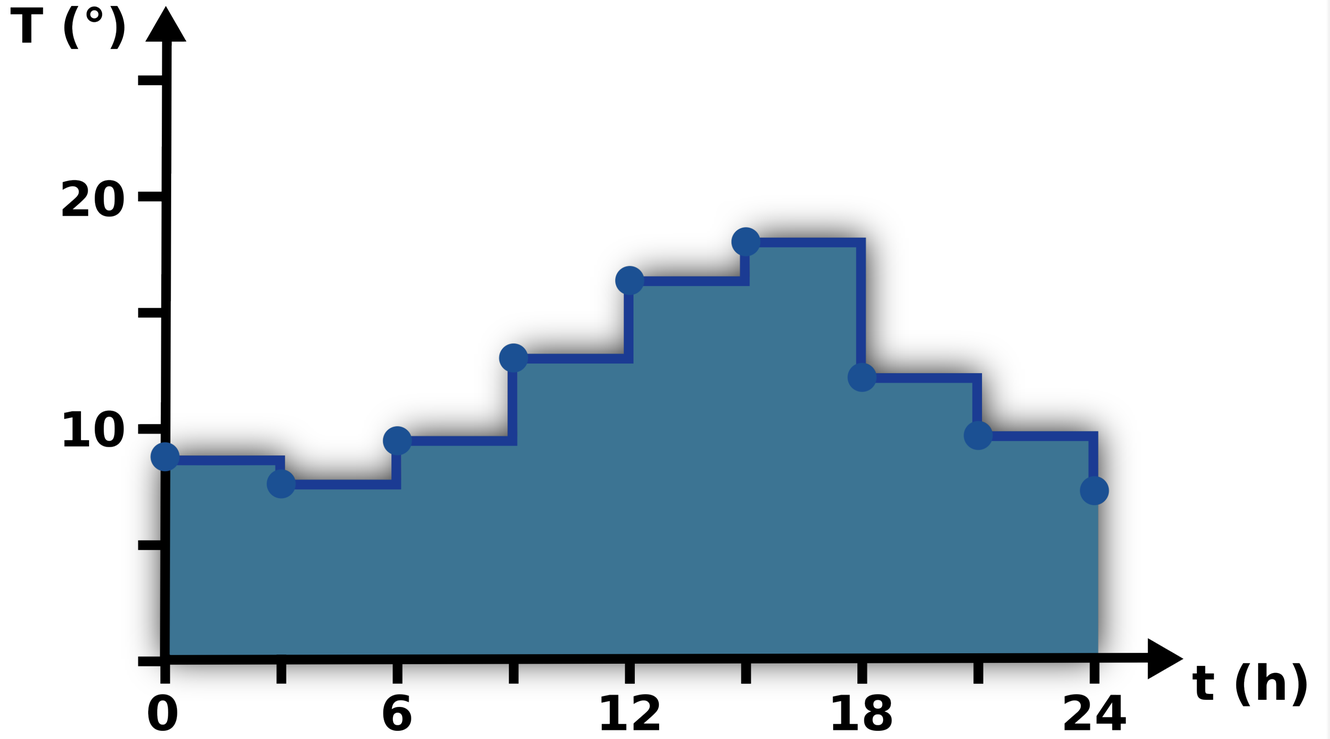
$$f : [0, 24] \rightarrow \mathbb{R}$$



Interpolants for regular grids

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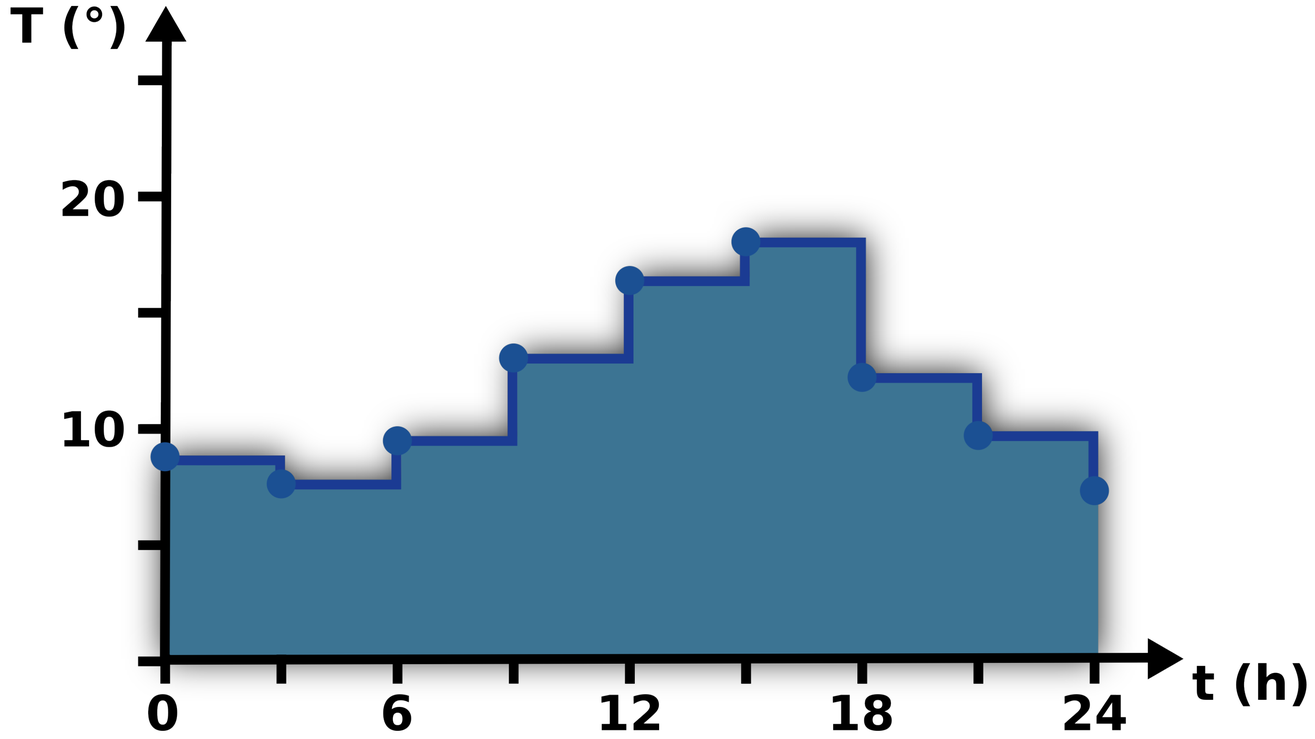

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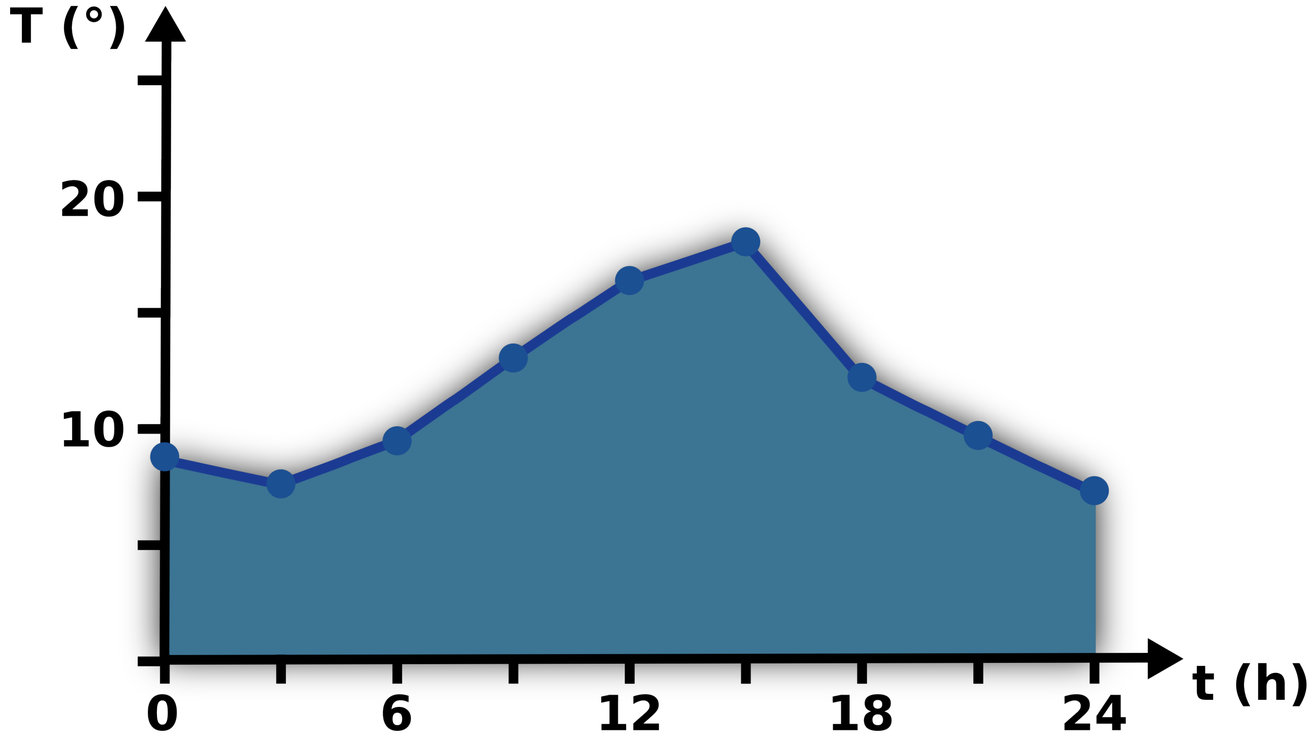

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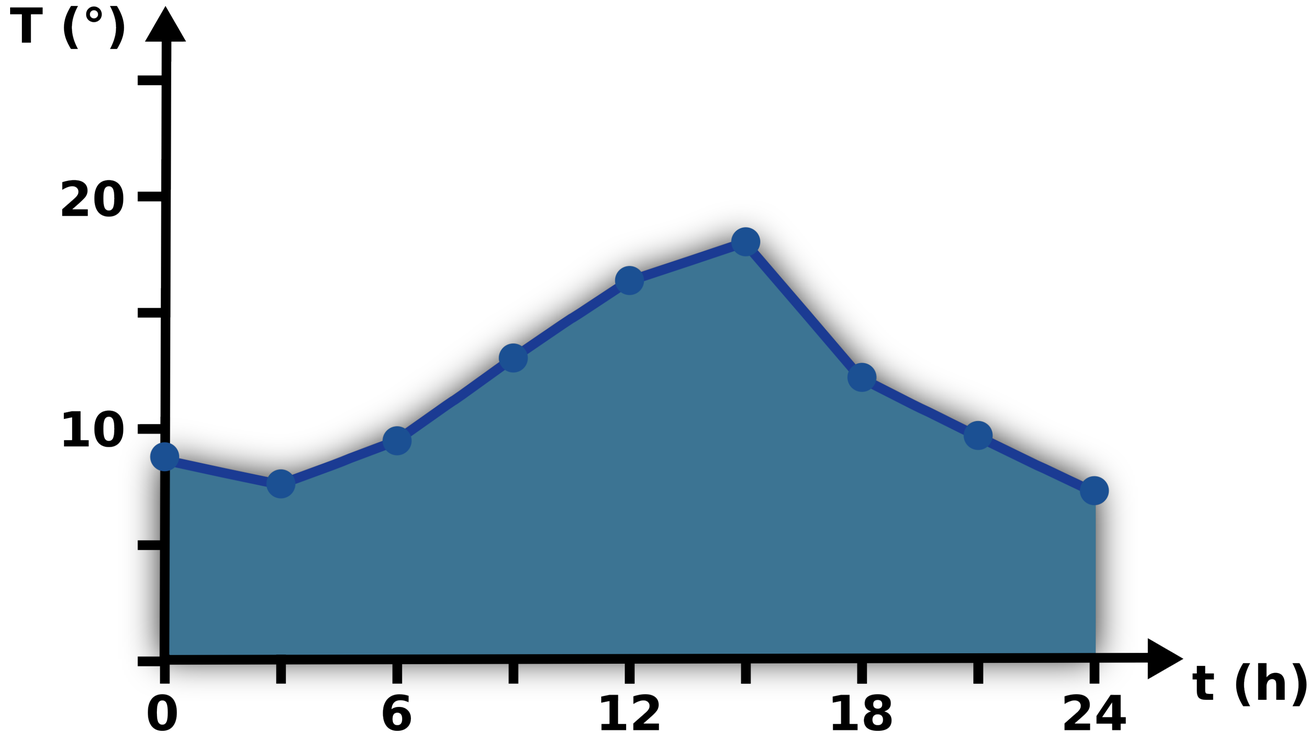

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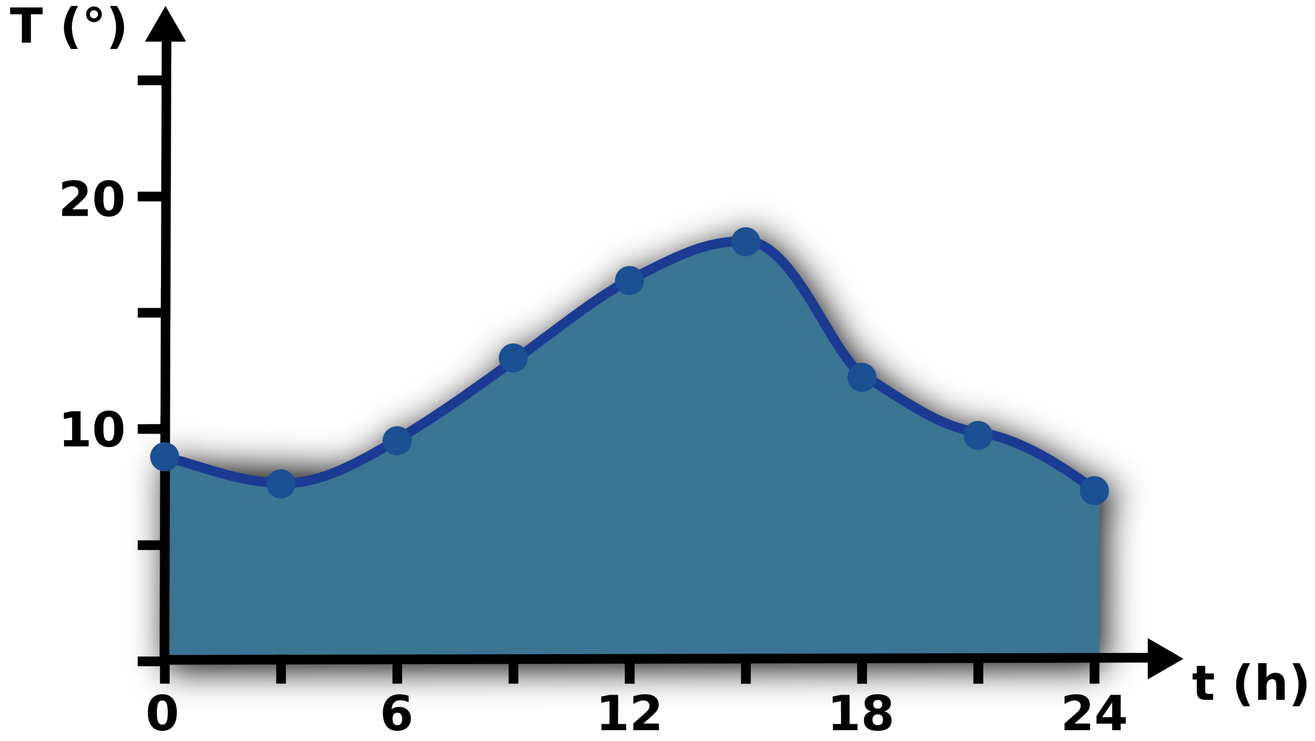

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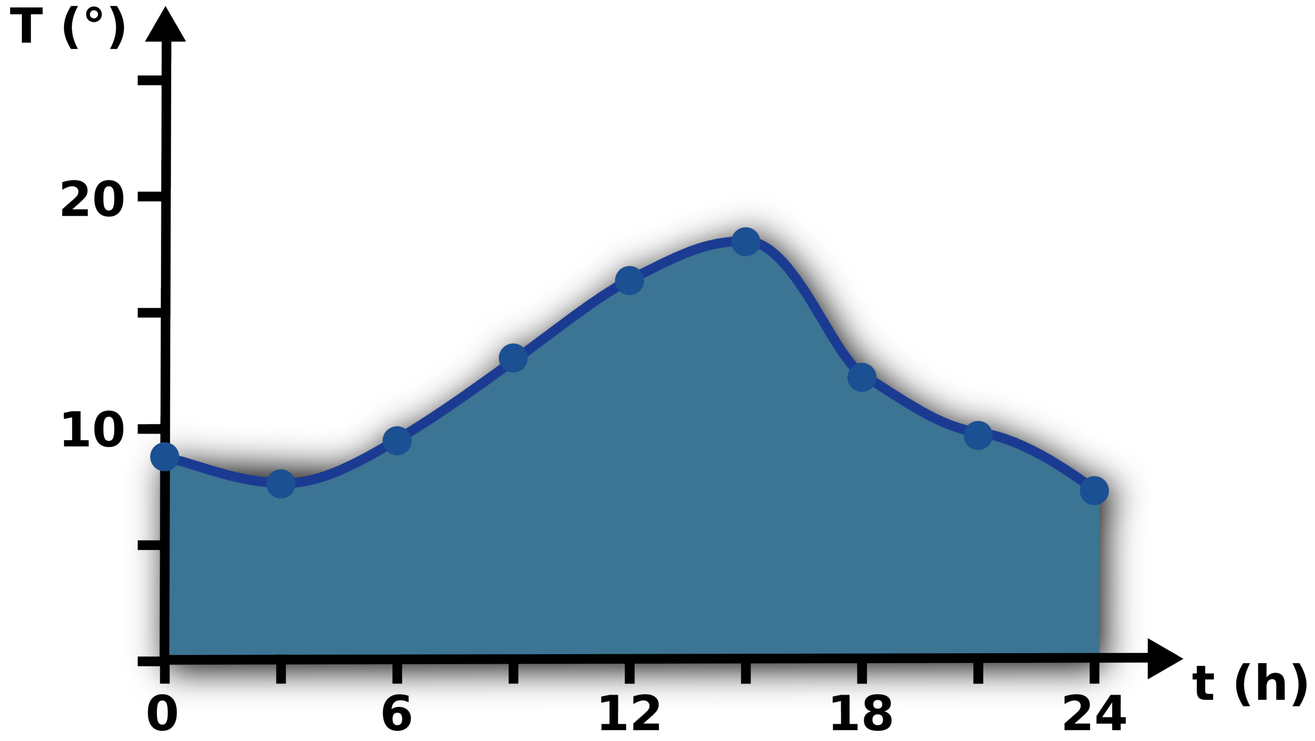

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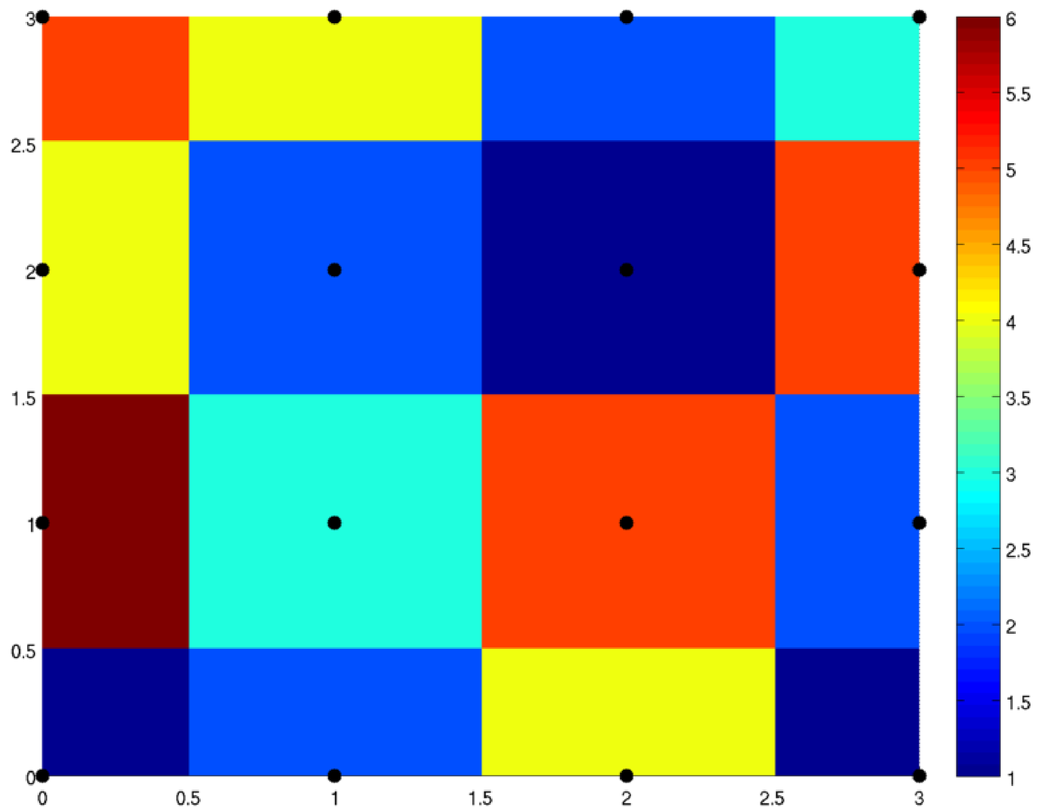

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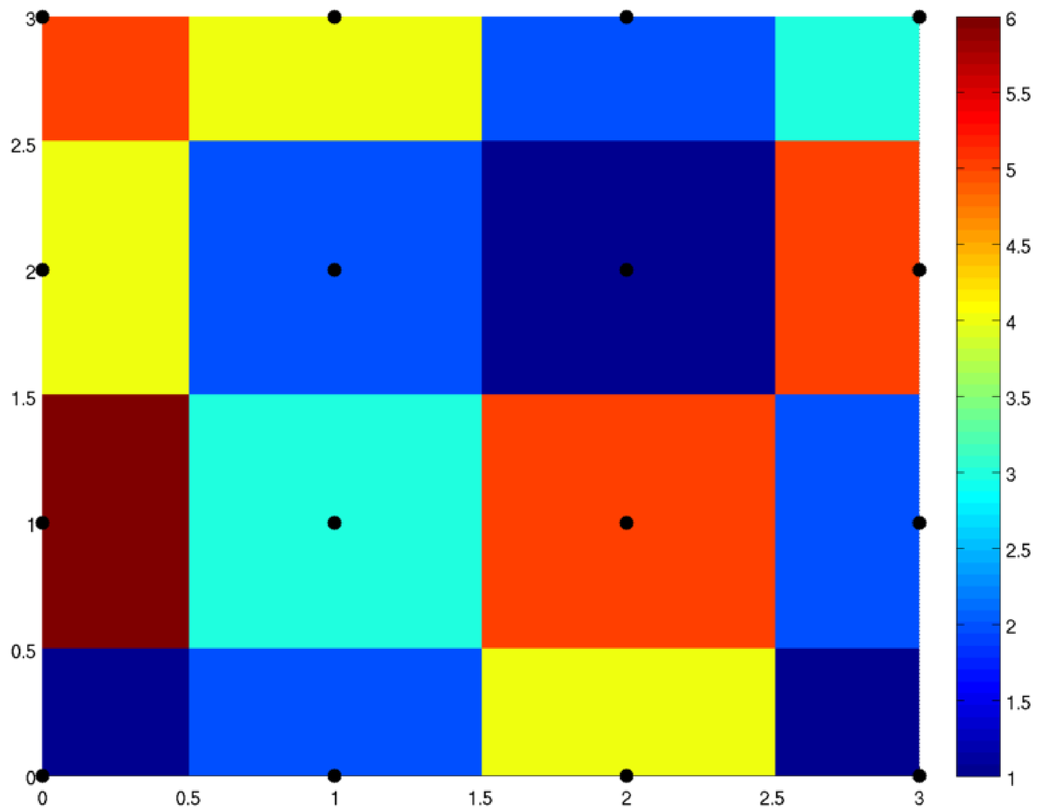

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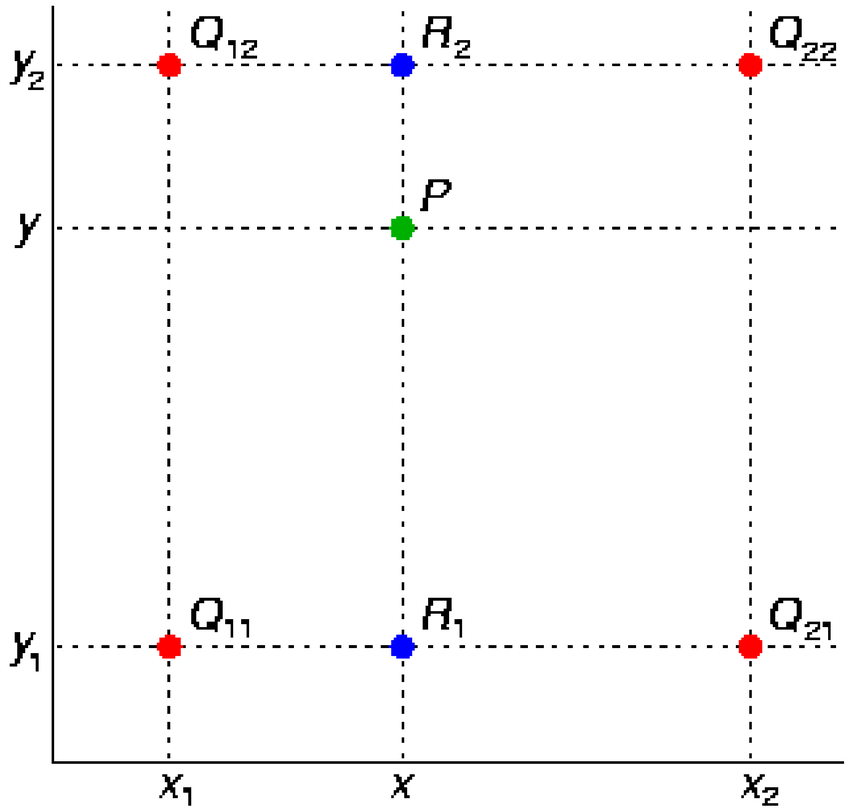

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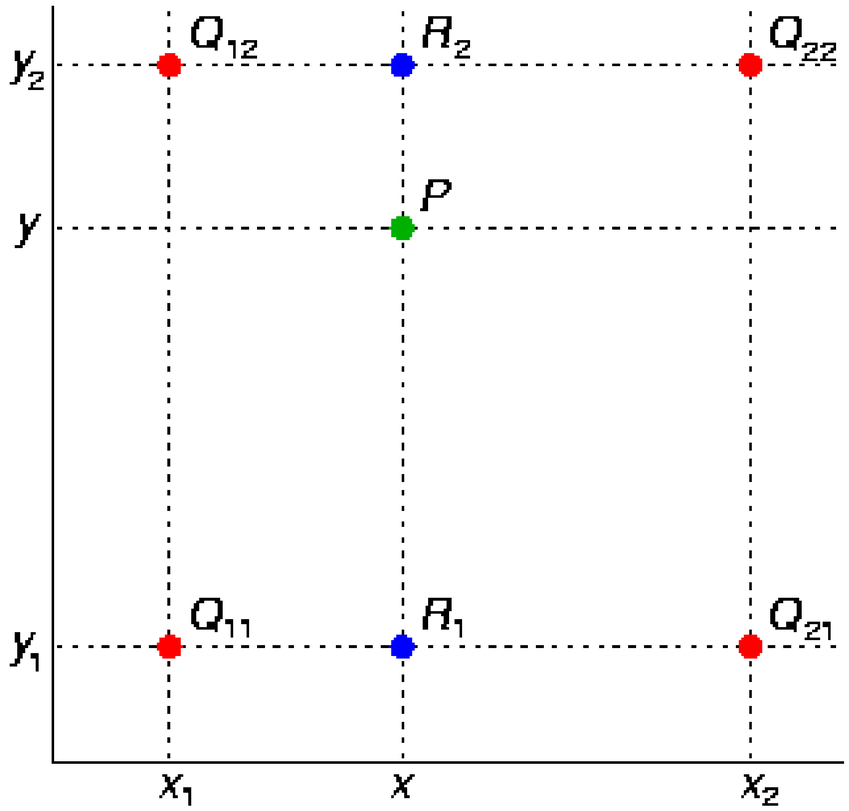

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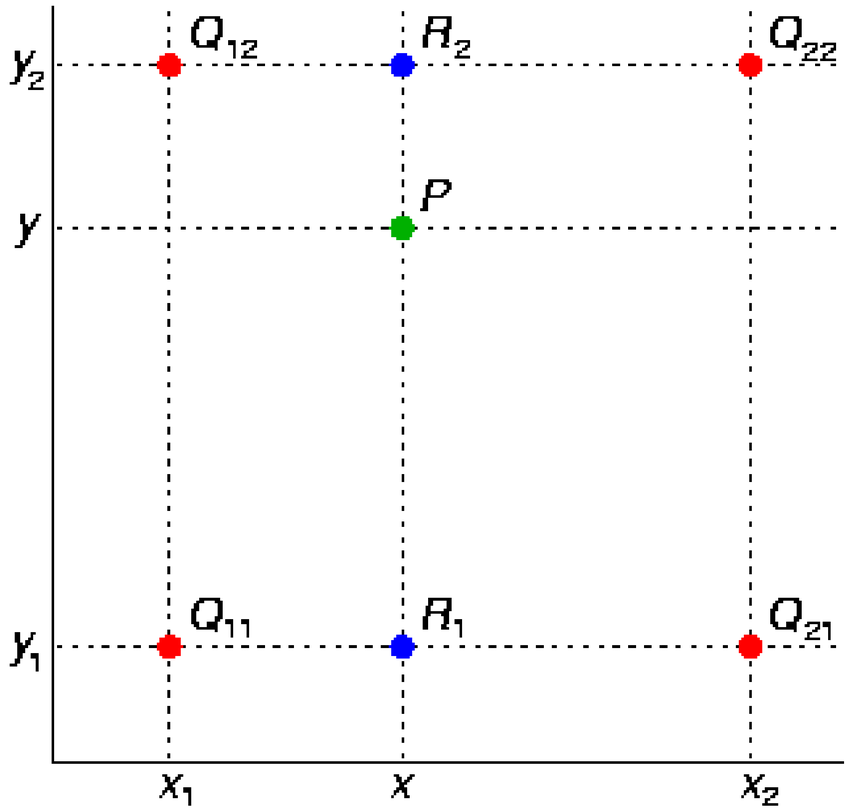

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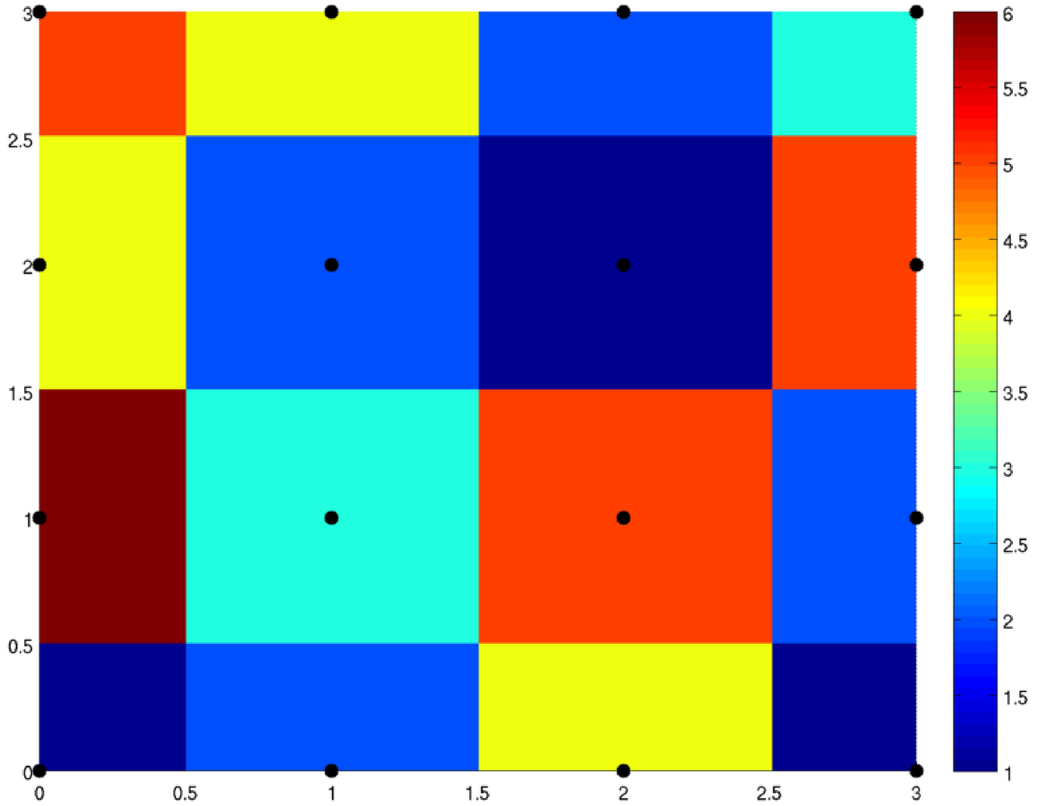

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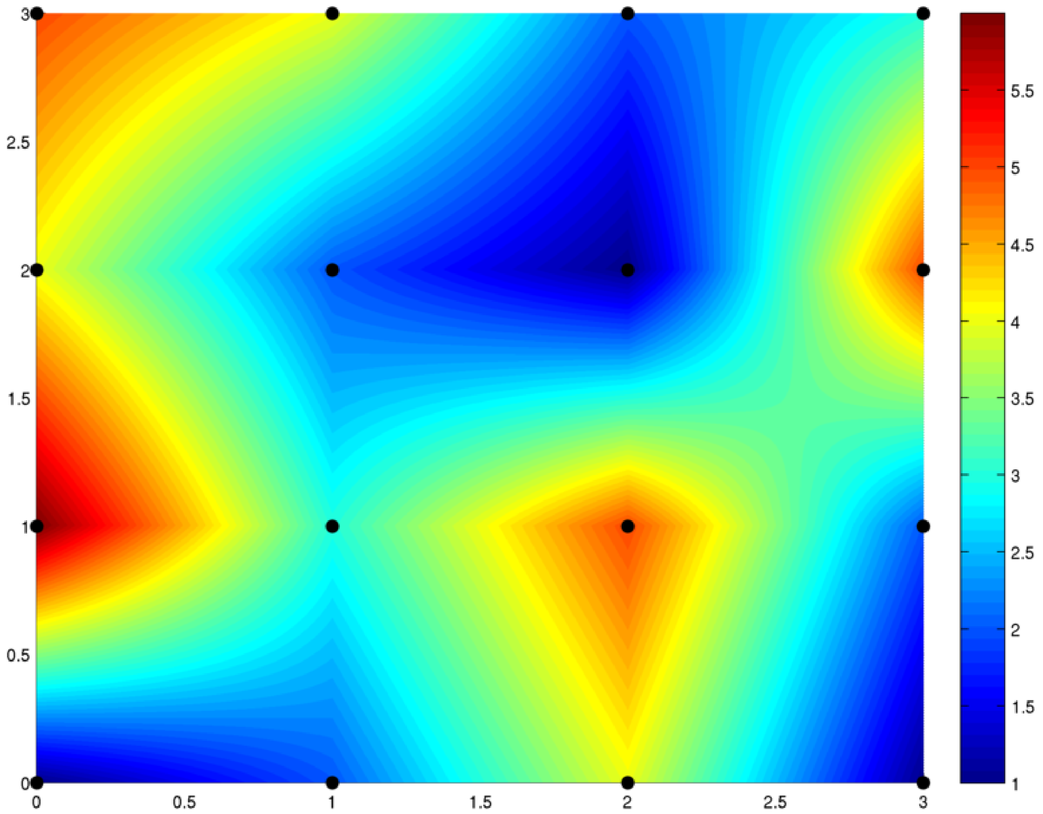

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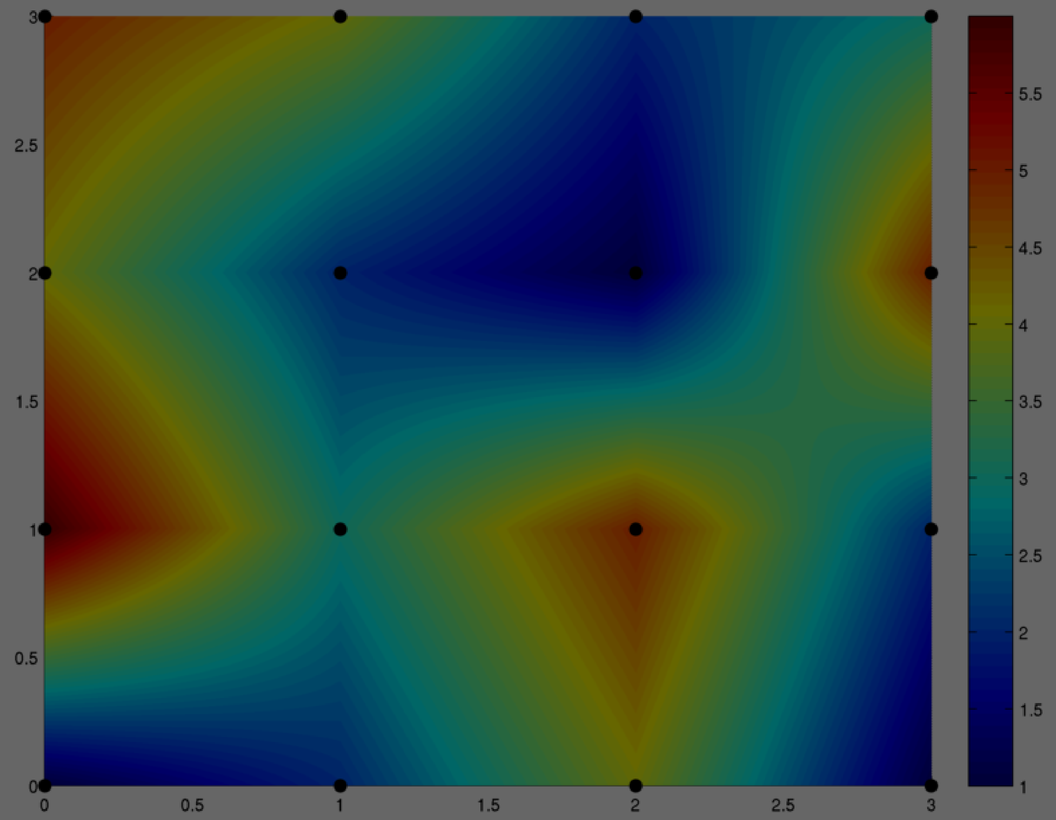

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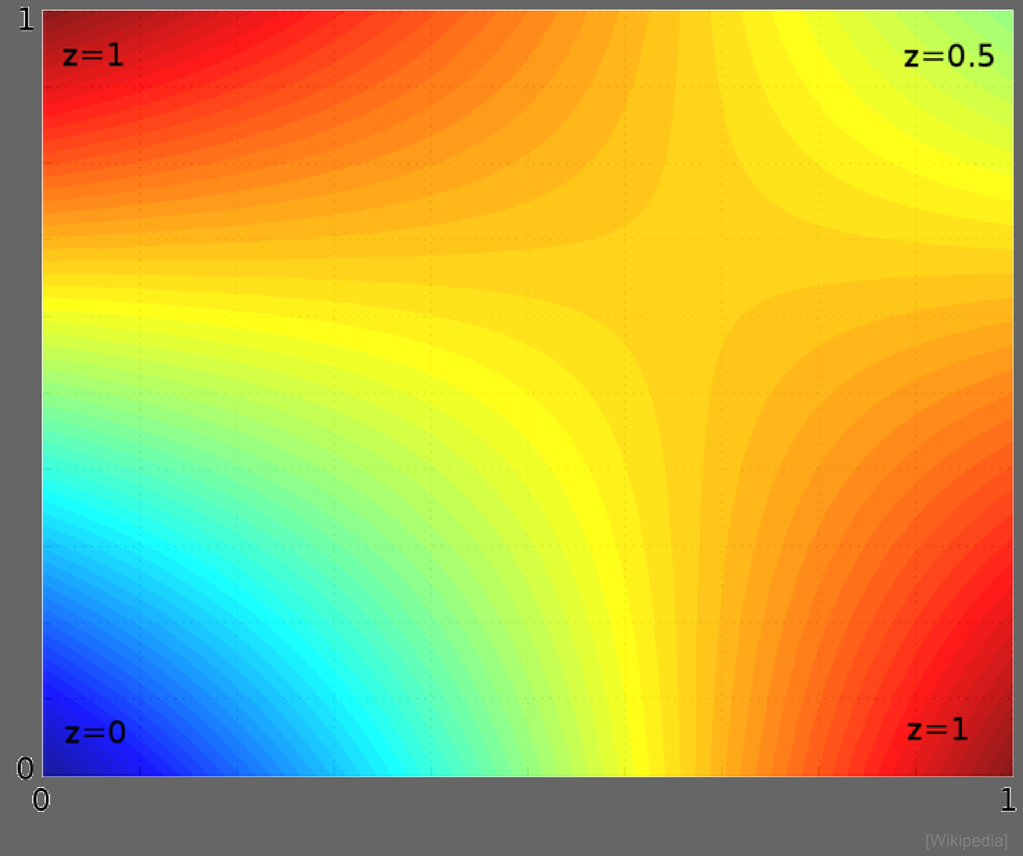

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Interpolants for regular grids

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  - Problem

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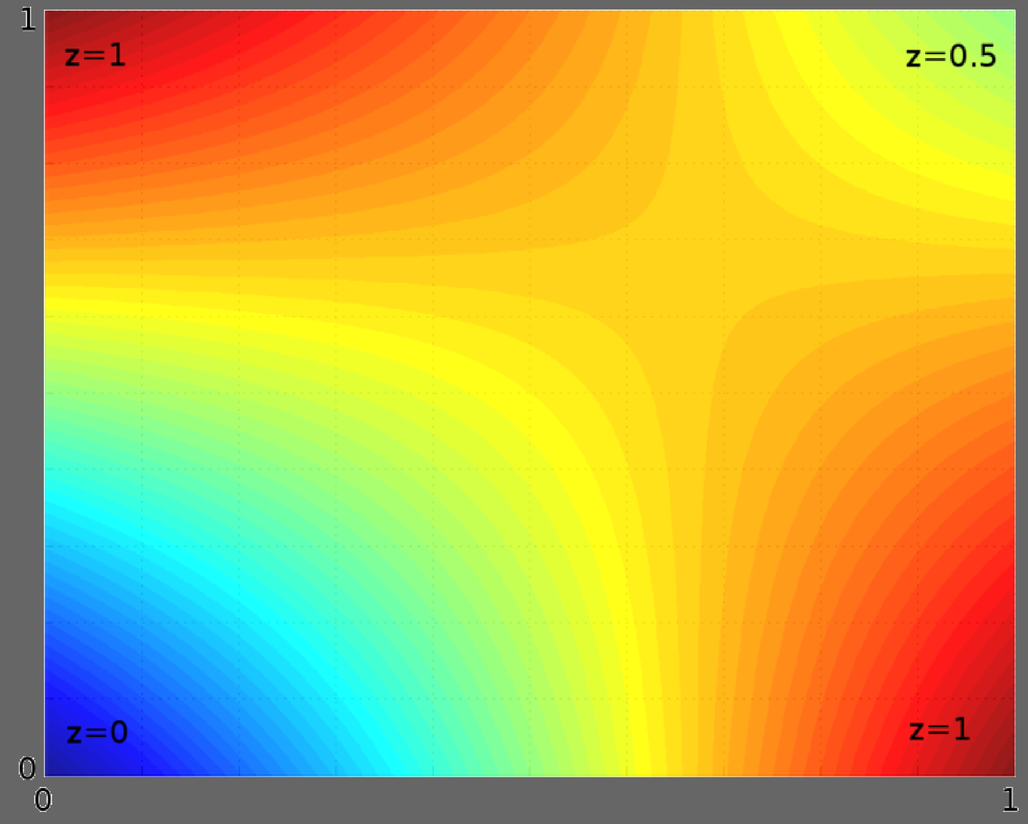

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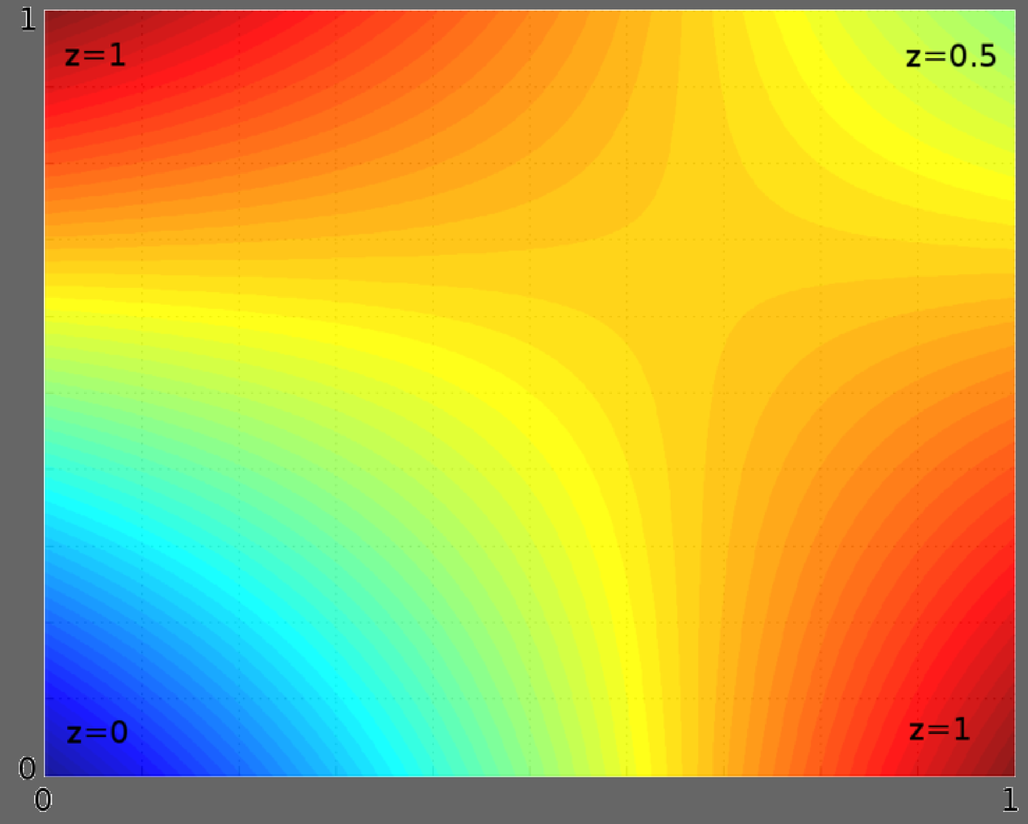

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Interpolants for regular grids

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    - Source of many ambiguities in visualization

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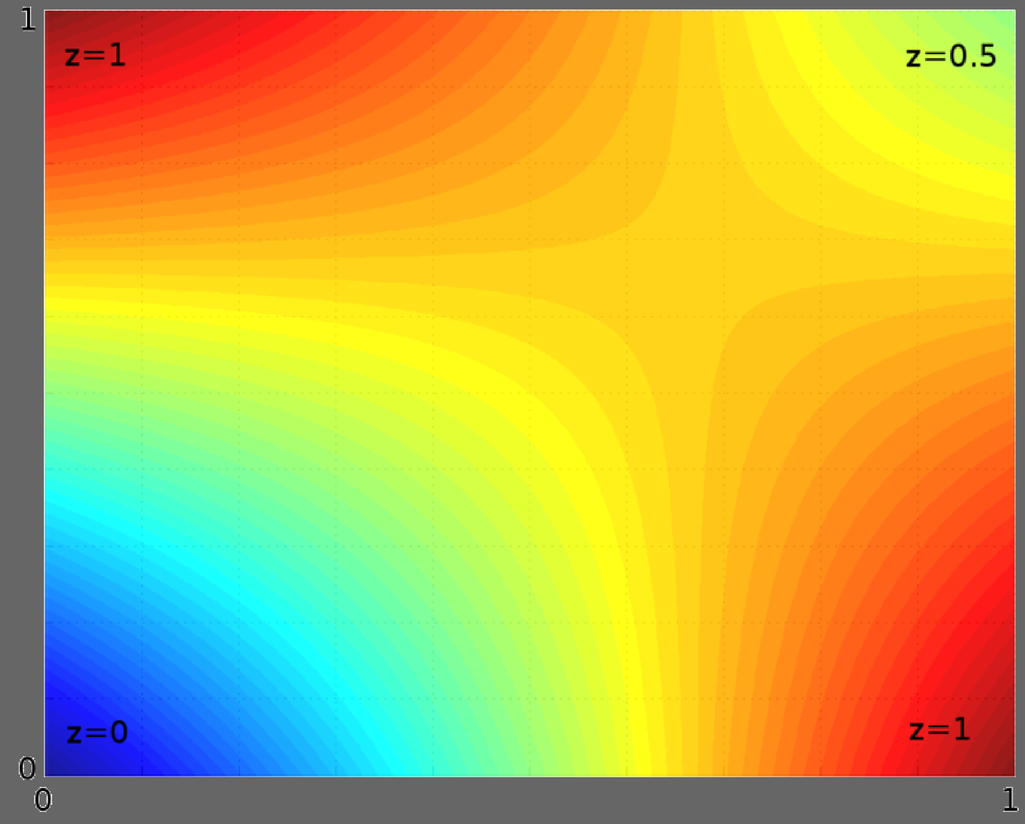

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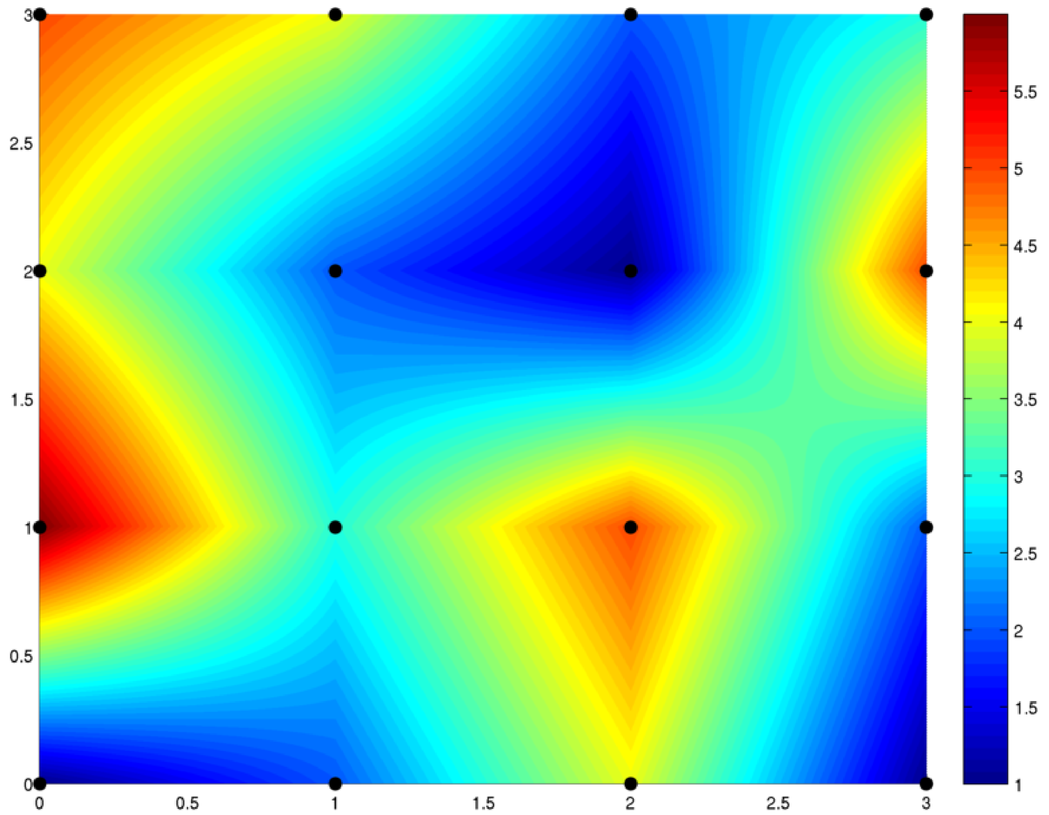

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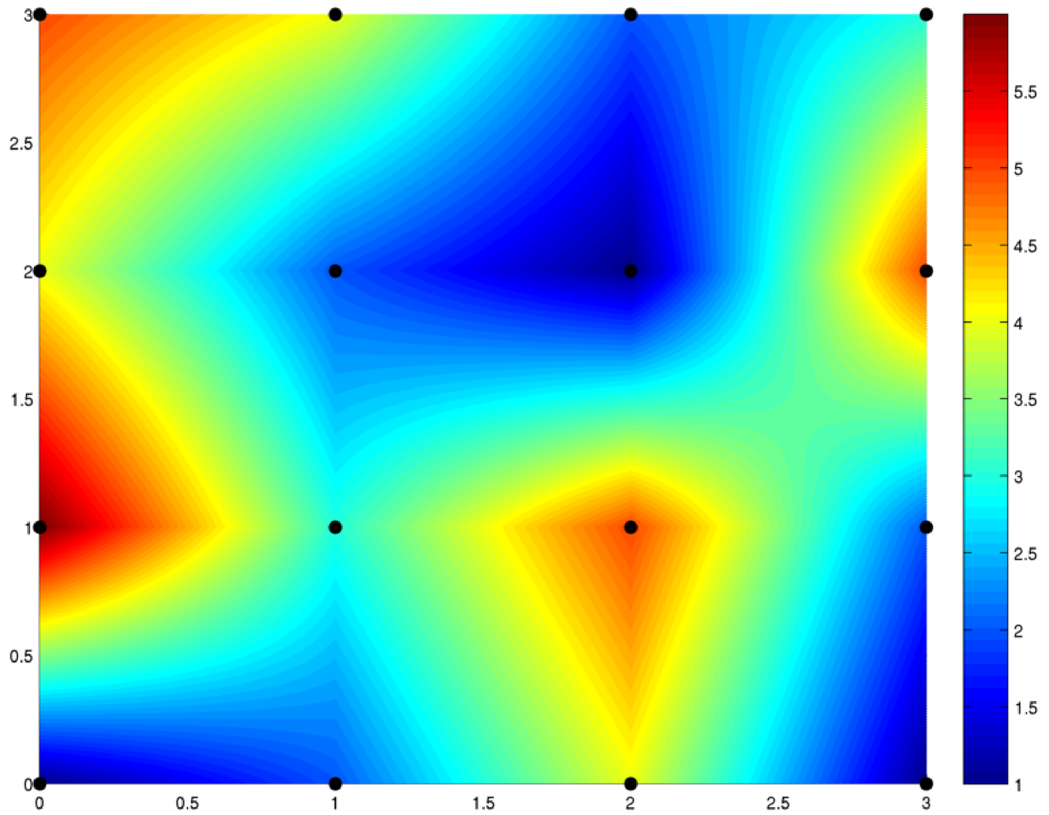

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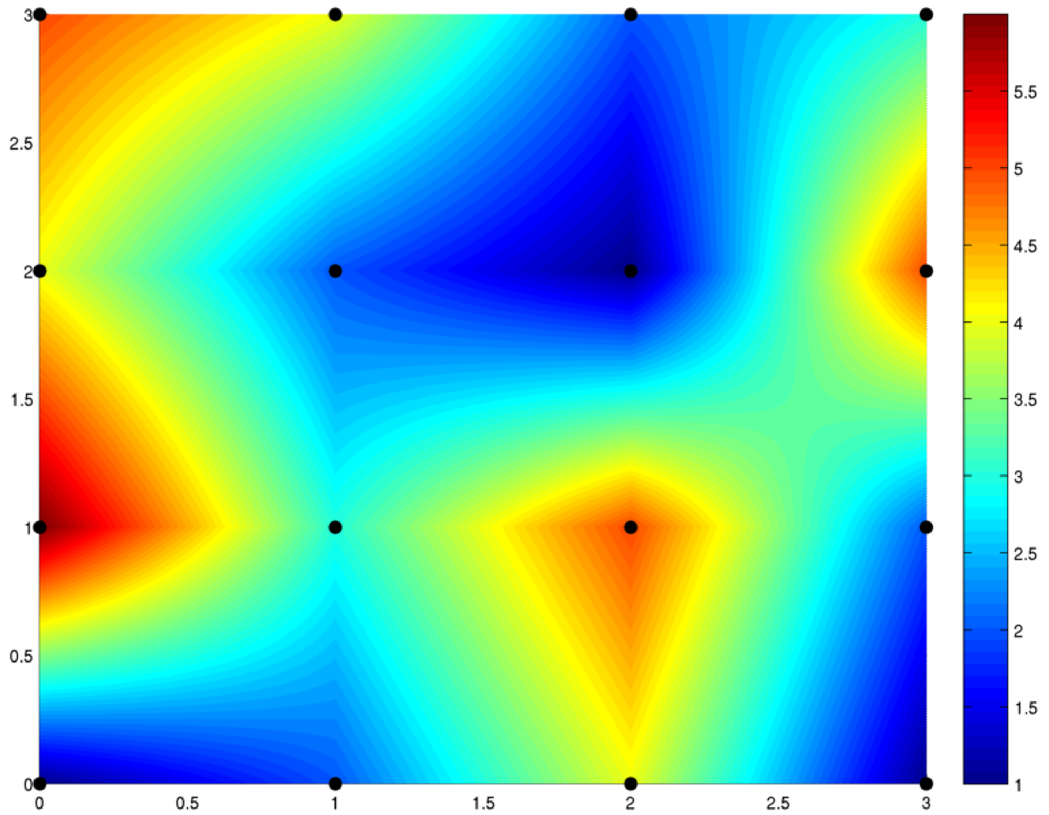

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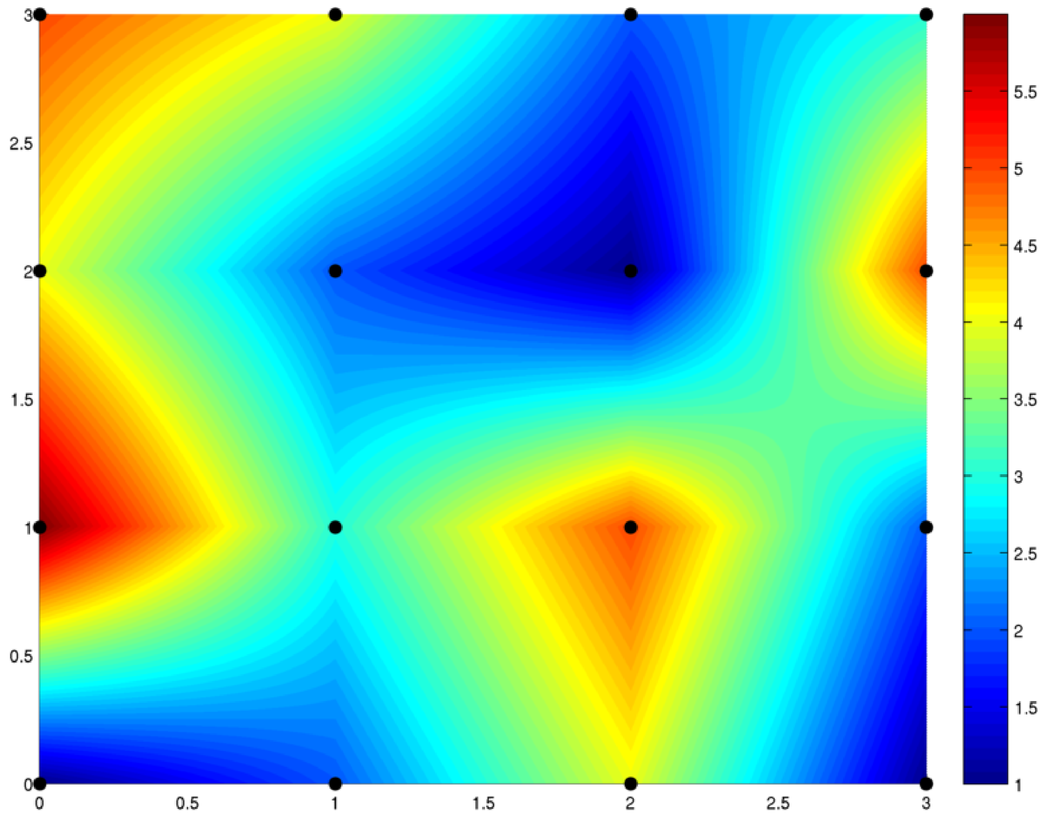

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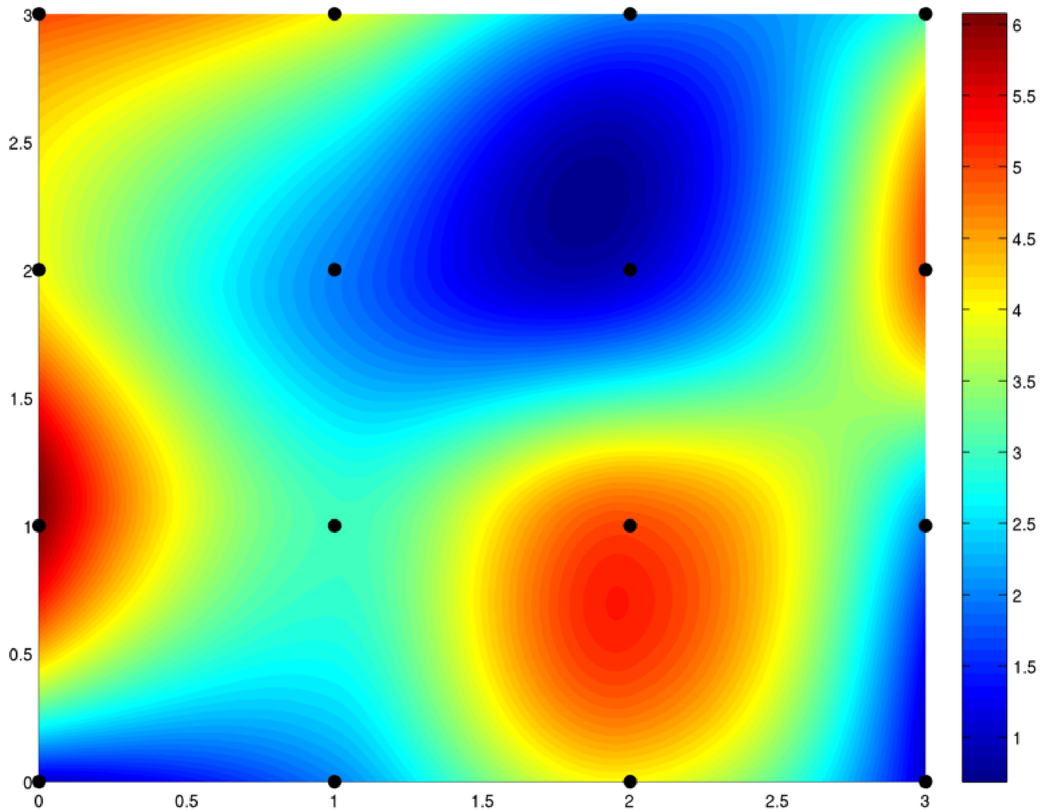

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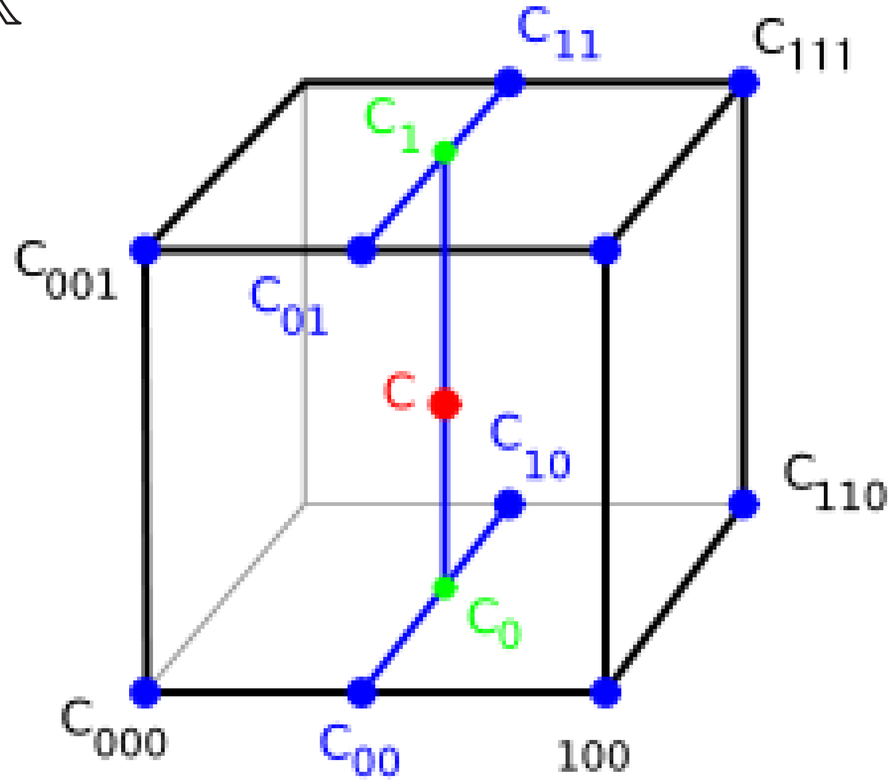

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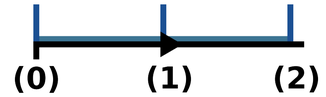
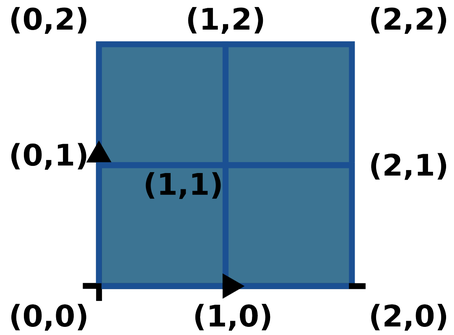
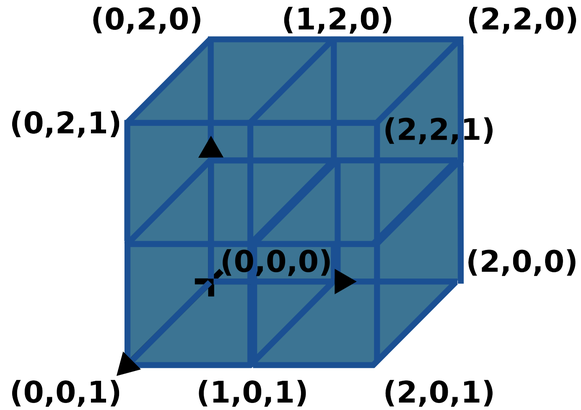
Euclidean spaces on a computer

- Notion of **regular grid** $\mathcal{G}$
  - Set of **vertices** (samples) in $\mathbb{R}^n$
    - With embedding functions to $\mathbb{R}^n$
  - **Connectivity**
    - Unit cells (pixels, voxels)
    - Implicitly encoded by the samples (space fill)
  - **Interpolant**

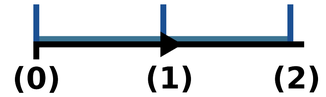


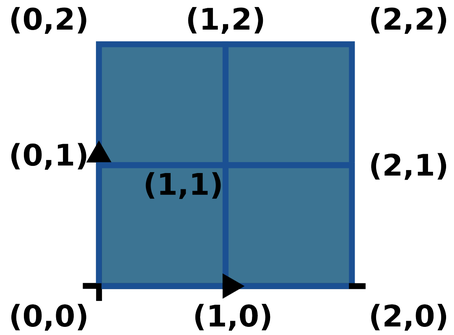


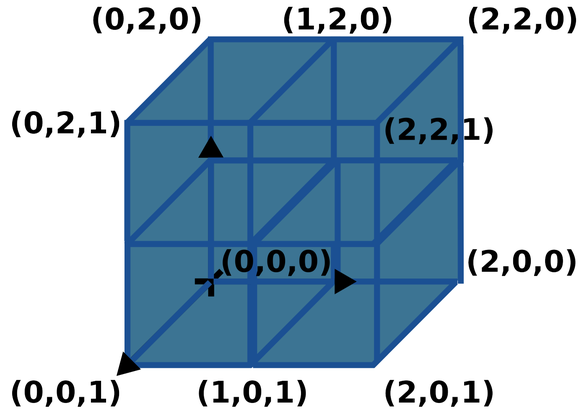


Euclidean spaces on a computer

- Notion of **regular grid** $\mathcal{G}$
  - Set of **vertices** (samples) in $\mathbb{R}^n$
    - With embedding functions to $\mathbb{R}^n$
  - **Connectivity**
    - Unit cells (pixels, voxels)
    - Implicitly encoded by the samples (space fill)
  - **Interpolant**
    - Computed on demand per cell







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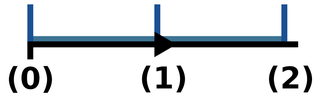
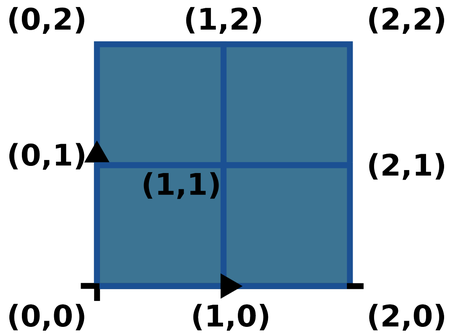
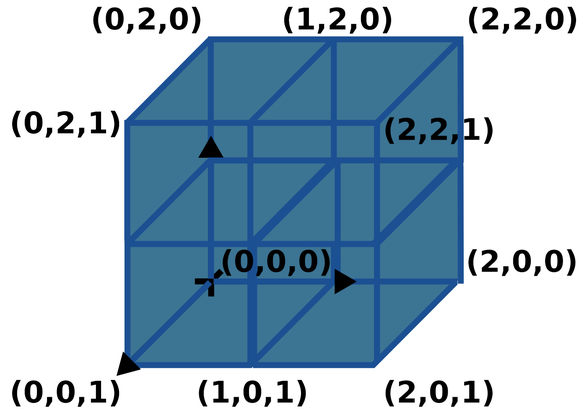
    - Usually linear functions



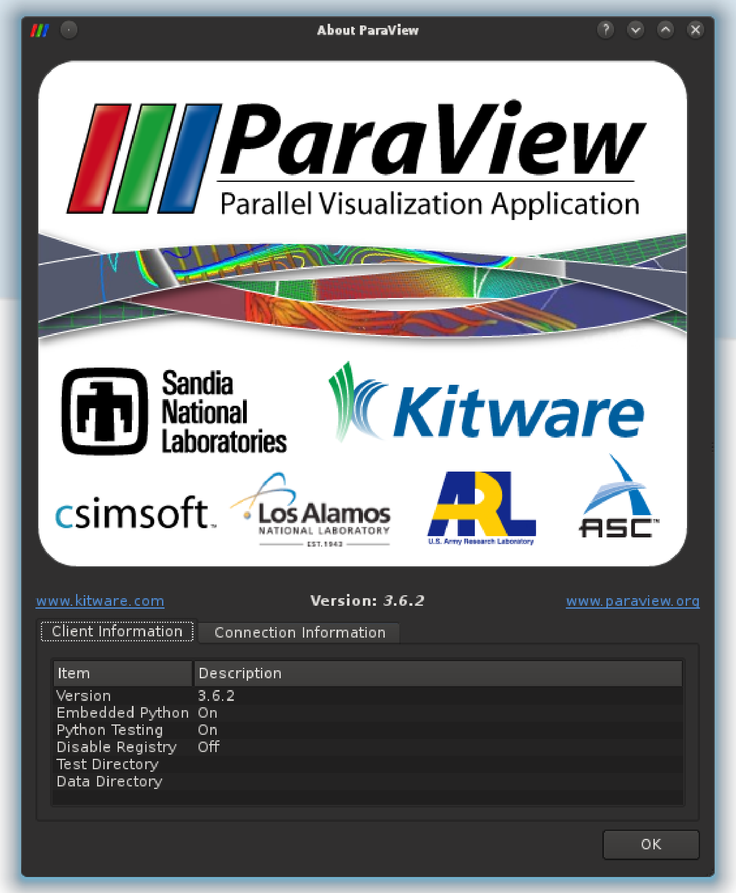




In practice

In practice

- Paraview
  - Open-source, interactive visualization platform



In practice

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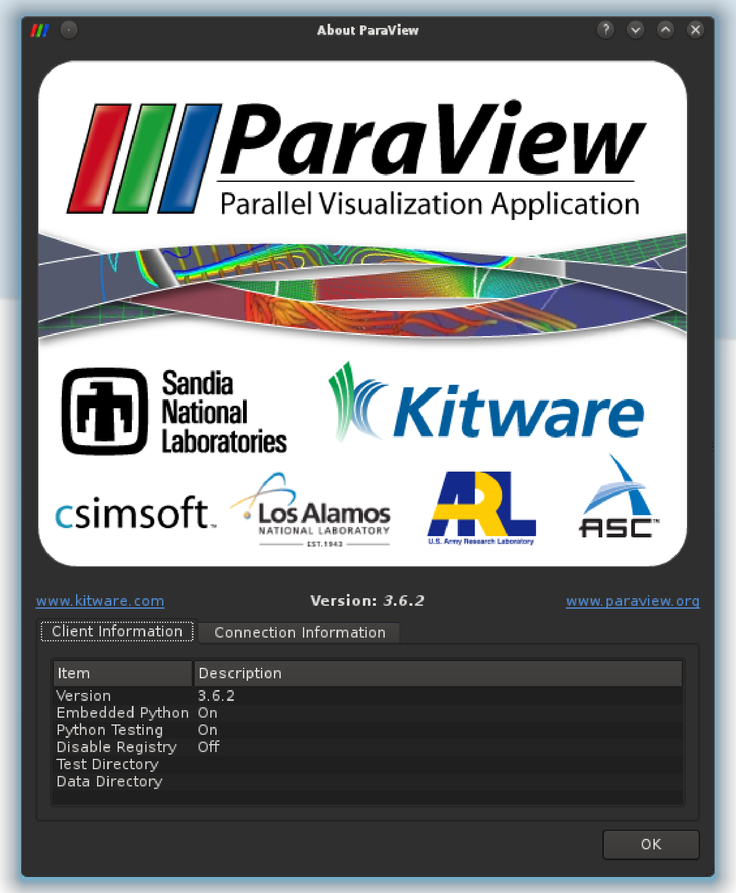
  - User interface front-end for the Visualization Tool Kit (VTK) C++ library (open-source)



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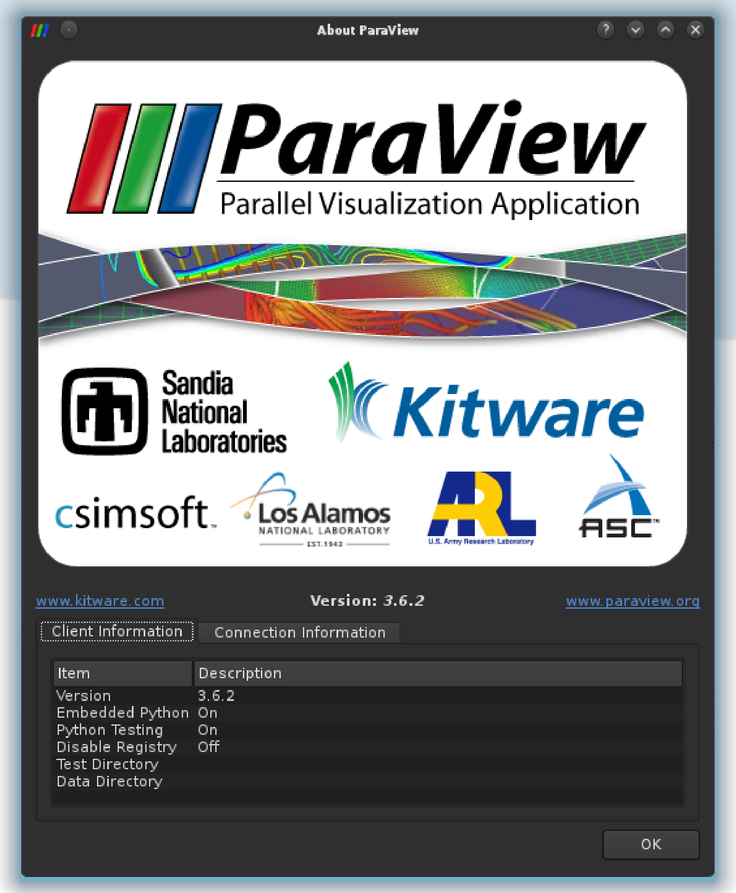
  - Started in 1993
    - Initially General Electric, then Kitware (1998)



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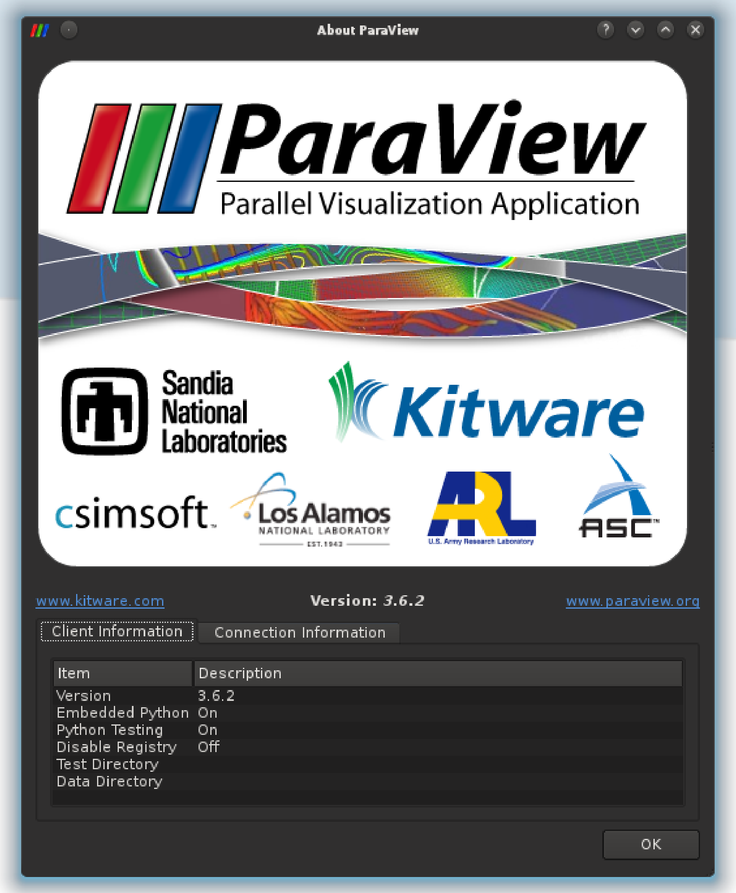

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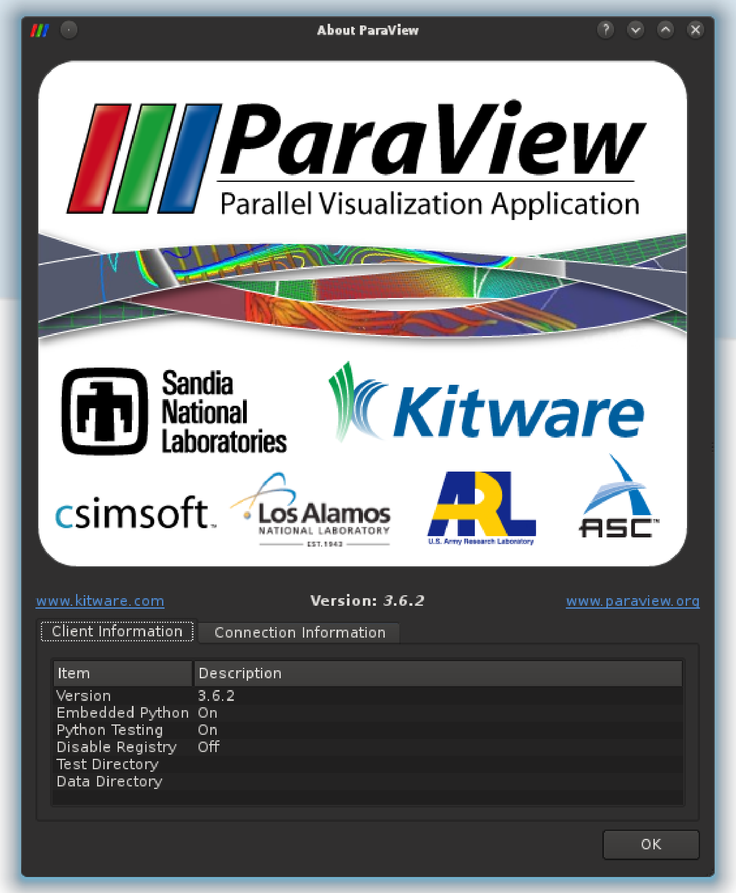
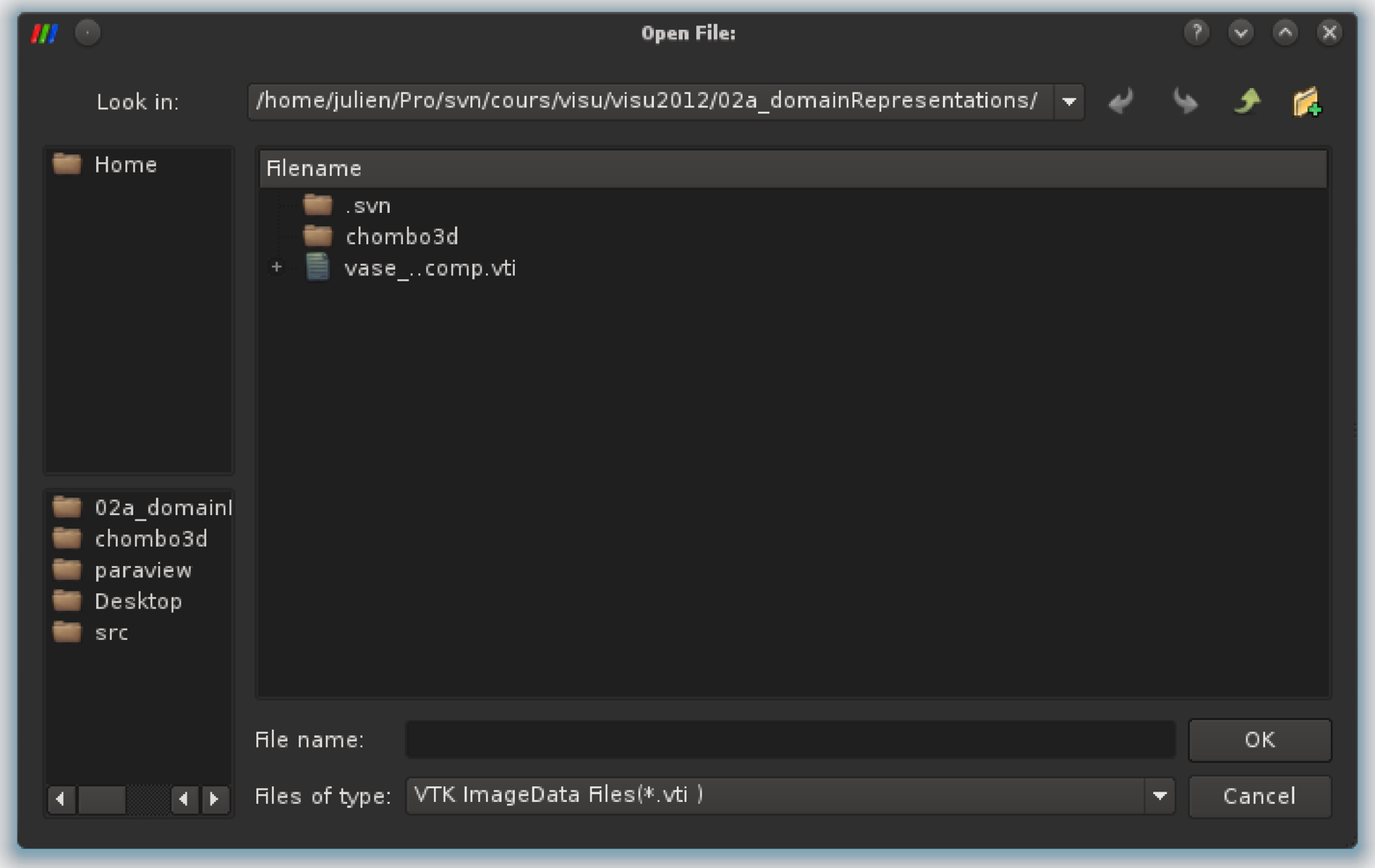
    - Initially General Electric, then Kitware (1998)
    - Many institutional contributors (companies, national labs, universities)
    - Over a million lines of code
    - Tens of thousands of users worldwide



In practice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Home
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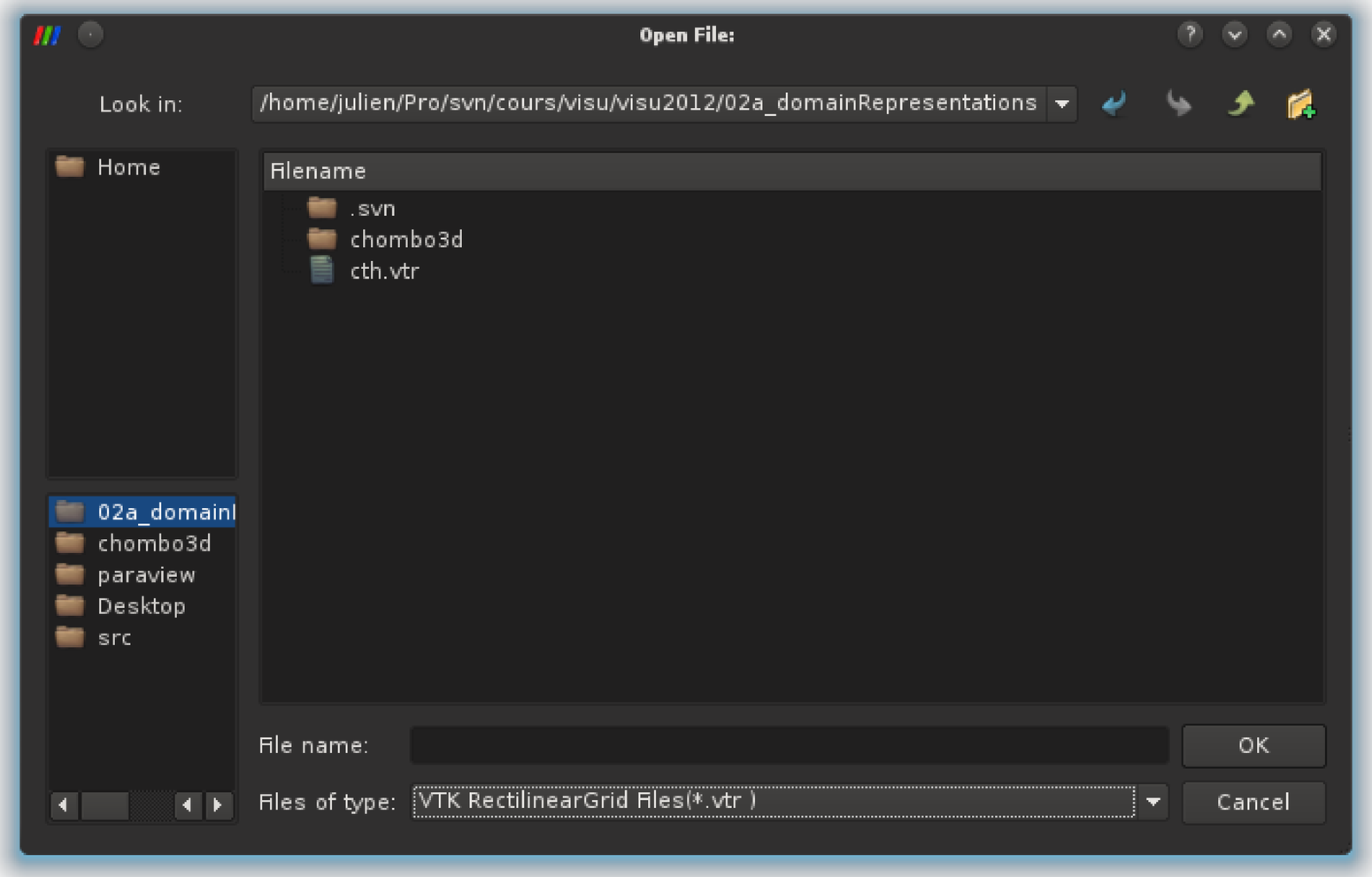
Desktop
src
File name:
OK
Files of type:
VTK ImageData Files(*.vti)
Cancel

In practice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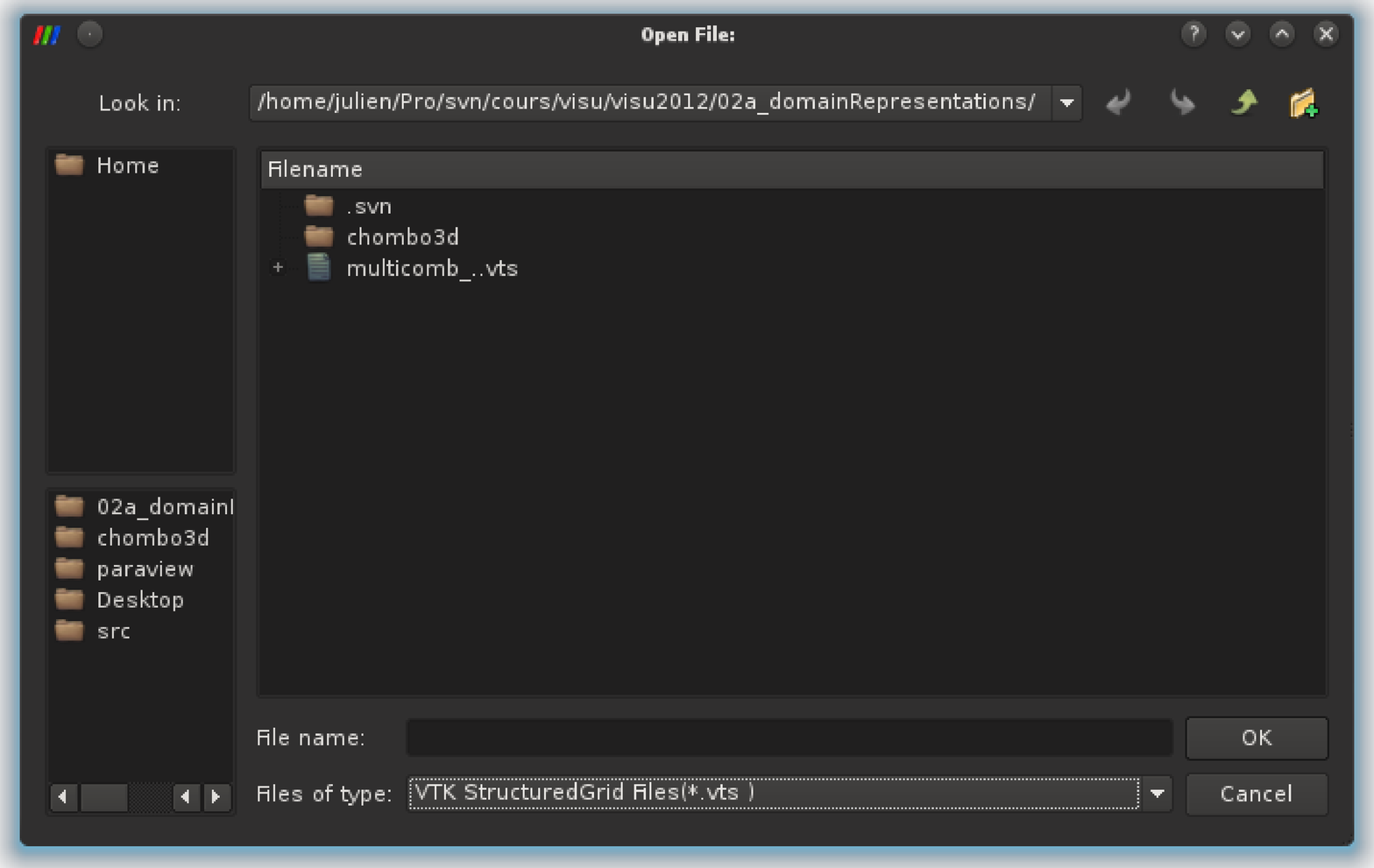
VTK RectilinearGrid Files(*.vtr)
Cancel

In practice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Desktop
src
File name:
OK
Files of type:
VTK StructuredGrid Files(*.vts)
Cancel

Kitware ParaView 3.6.2

File Edit View Sources Filters Animation Tools Help

Time: 0 0

SLCImage | Surface With Edges

Edit Color Map

Pipeline Browser

- builtin:
  - vase_*
    - ExtractCellsByRegion1
  - vase_*
  - vase_*

Object Inspector

Properties | Display | Information

Map Scalars

Apply Texture: None

Color by: SLCImage

Edit Color Map... | Rescale to Data Range

Slice

Slice Direction: XY Plane

Slice: 0

Annotation

Show cube axes | Edit

Style

Representation: Surface With Edges

Interpolation: Gouraud

Material: None

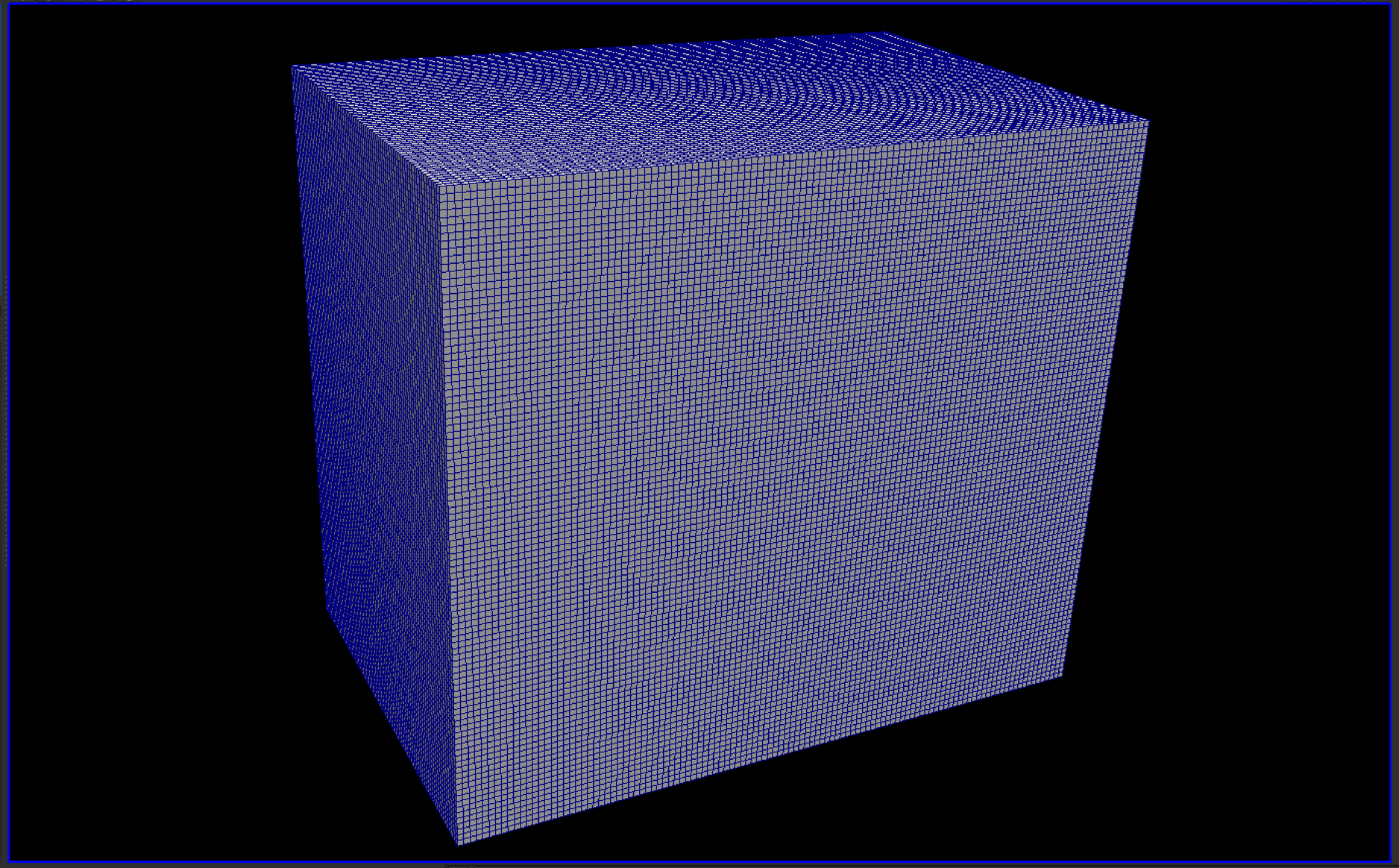
Point size: 2.00

Line width: 1.00

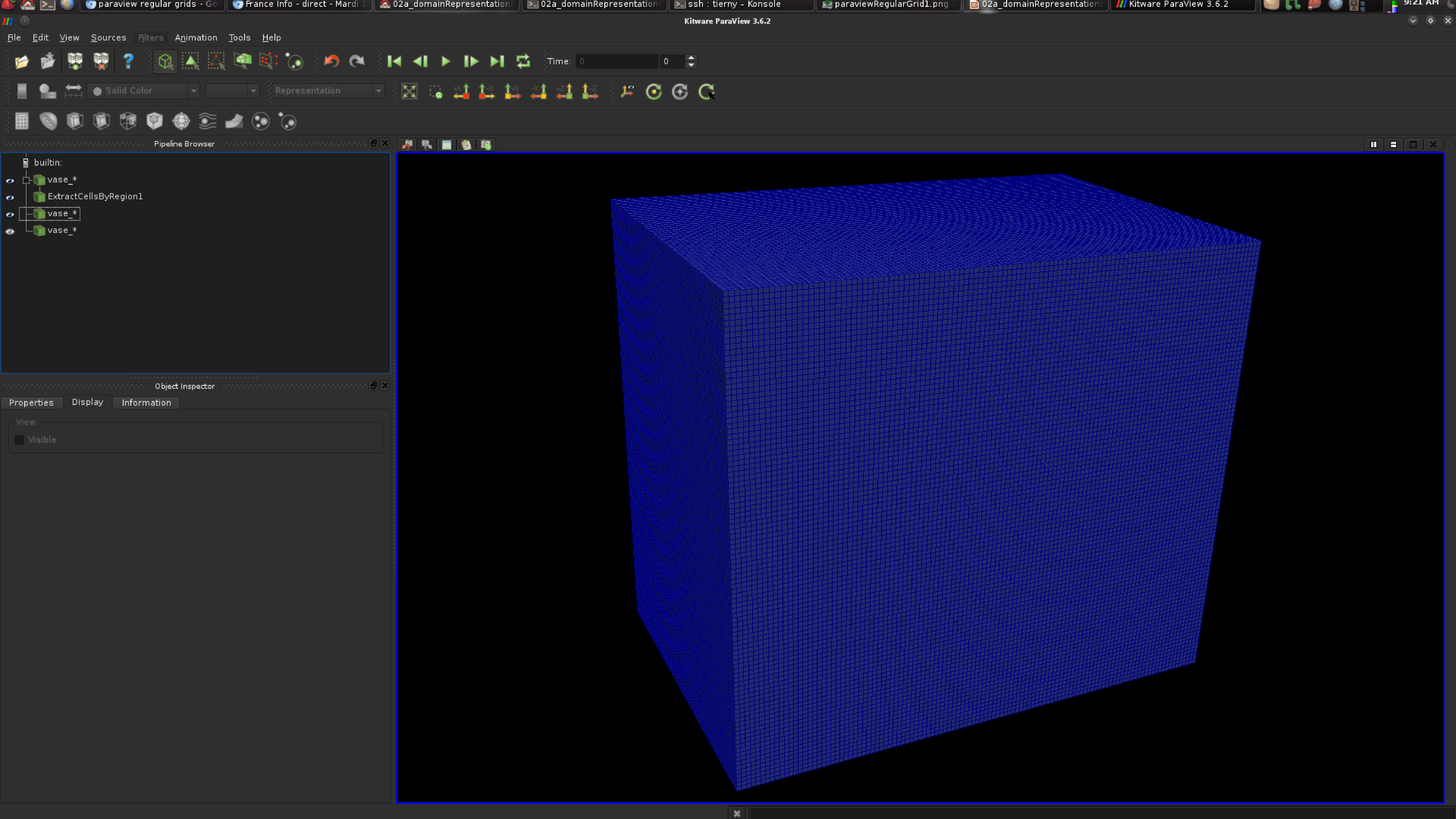
Opacity: 1.00

Volume mapper

Edge Style



Edit Color Map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ExtractCellsByRegion1
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Object Inspector
Properties Display Information
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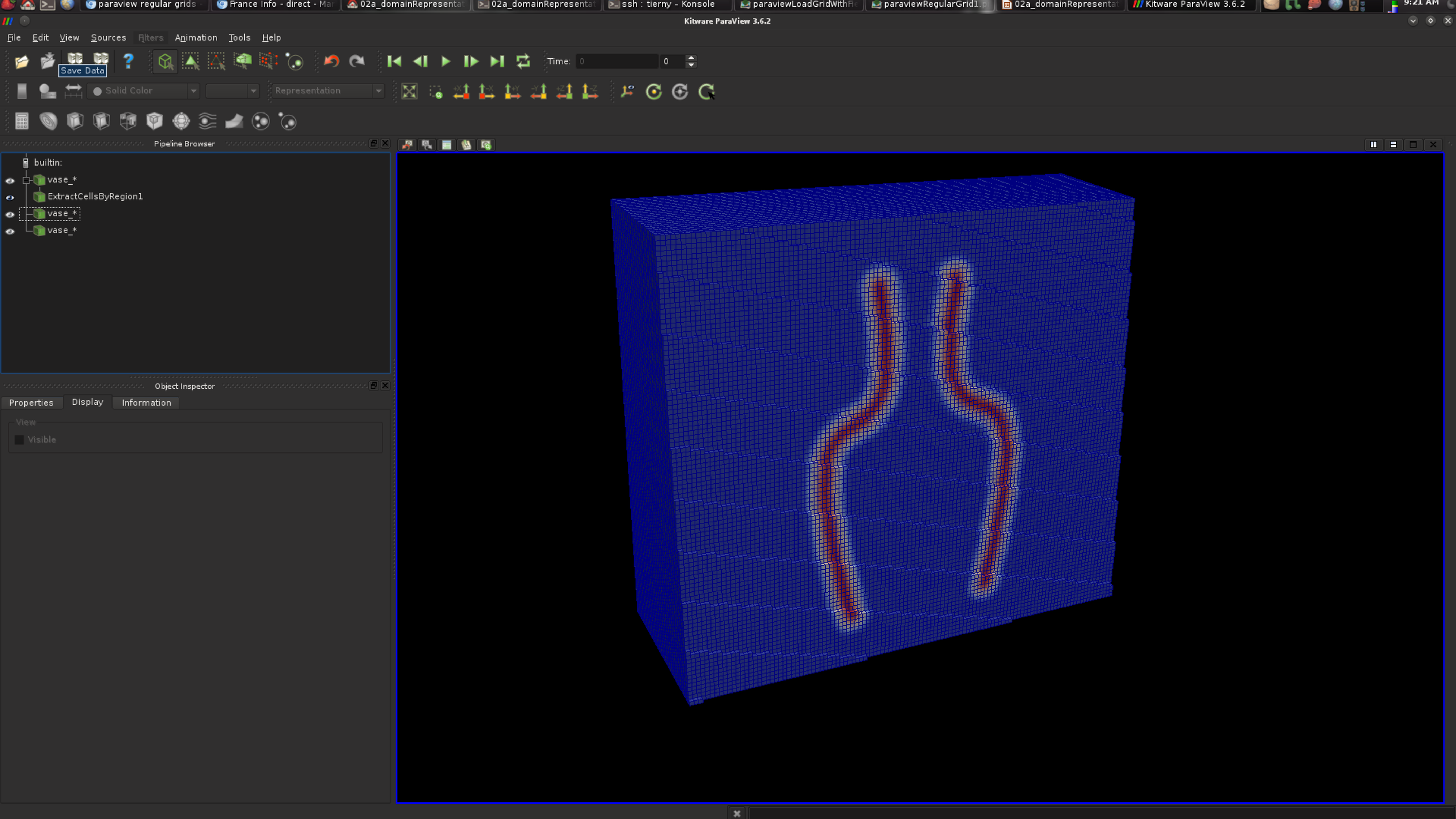
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File Edit View Sources Filters Animation Tools Help
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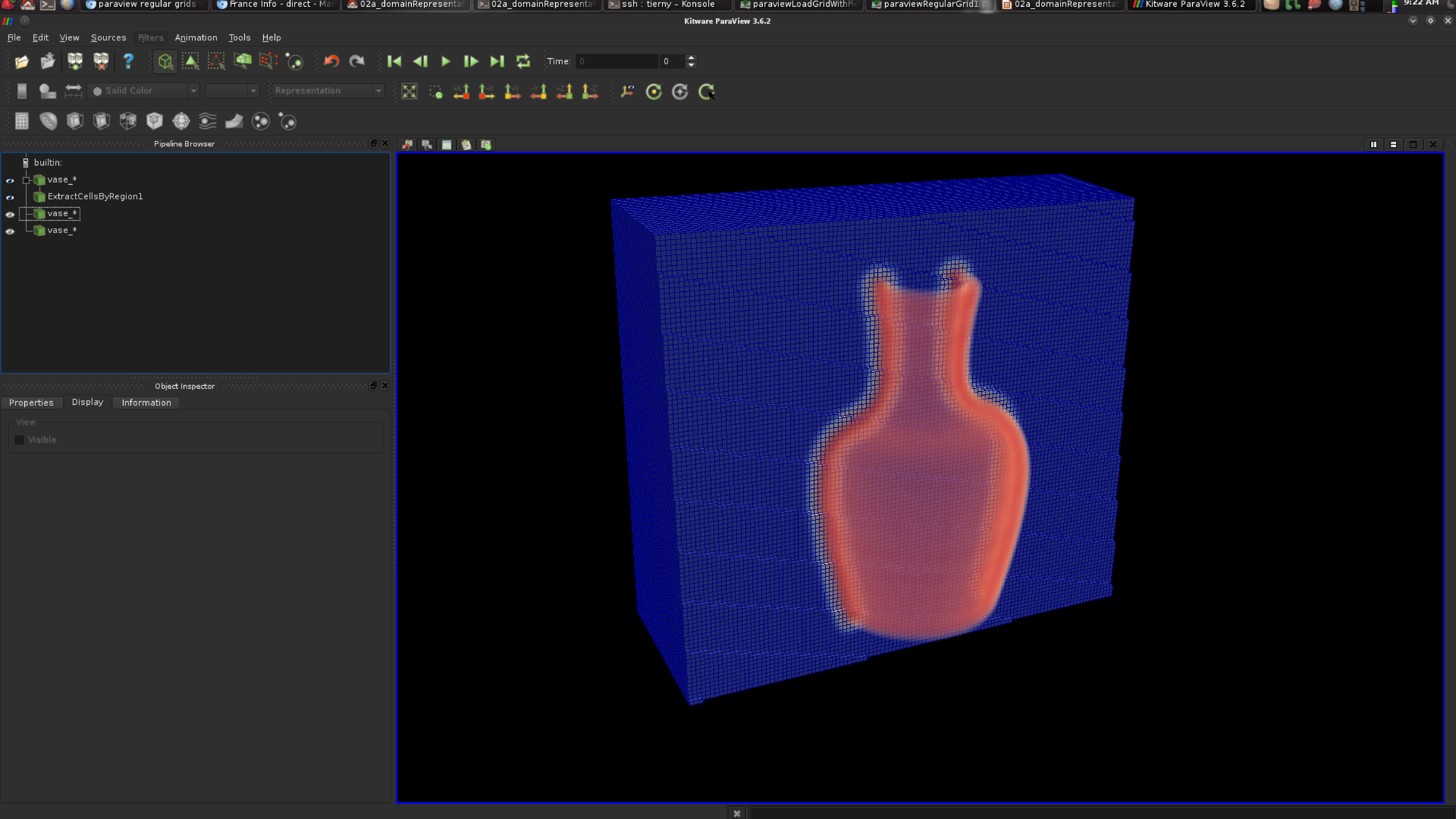
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Object Inspector
Properties Display Information
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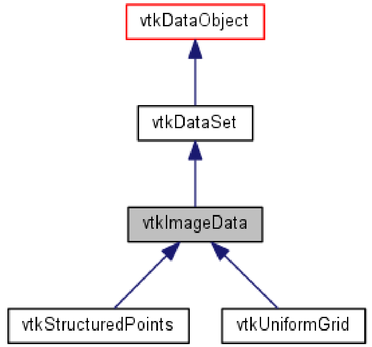
VTK

vtkImageData Class Reference

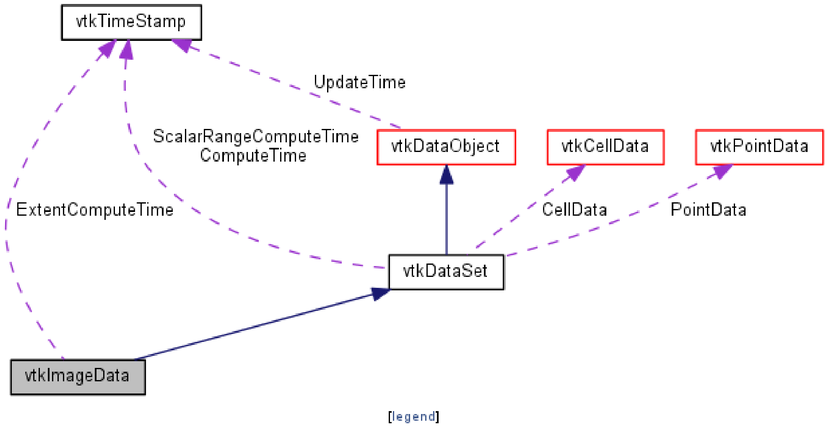
topologically and geometrically regular array of data More...

`#include <vtkImageData.h>`

Inheritance diagram for vtkImageData:



Collaboration diagram for vtkImageData:



Public Types

typedef **vtkDataSet** **Superclass**

Public Member Functions

virtual int	**IsA** (const char *type)
vtkImageData *	**NewInstance** () const
void	**PrintSelf** (ostream &os, **vtkIndent** indent)
virtual void	**CopyStructure** (**vtkDataSet** *ds)
virtual int	**GetDataObjectType** ()
virtual void	**Initialize** ()
virtual void	**SetDimensions** (**int** i, **int** j, **int** k)
virtual void	**SetDimensions** (const **int** dims[3])
virtual int	**GetDataDimension** ()
virtual void	**AllocateScalars** (**int** dataType, **int** numComponents)
virtual void	**AllocateScalars** (**vtkInformation** *pipeline_info)
virtual void	**Crop** (const **int** *updateExtent)
virtual unsigned long	**GetActualMemorySize** ()
int	**GetScalarType** ()
const char *	**GetScalarTypeAsString** ()
virtual void	**CopyInformationFromPipeline** (**vtkInformation** *information)
virtual void	**PrepareForNewData** ()
void	**GetArrayIncrements** (**vtkDataArray** *array, **vtkIdType** increments[3])
void	**ComputeInternalExtent** (**int** *intExt, **int** *tgtExt, **int** *bnds)
virtual int	**GetExtentType** ()
virtual vtkIdType	**GetNumberOfCells** ()
virtual vtkIdType	**GetNumberOfPoints** ()
virtual double *	**GetPoint** (**vtkIdType** ptId)
virtual void	**GetPoint** (**vtkIdType** id, **double** x[3])
virtual vtkCell *	**GetCell** (**vtkIdType** cellId)
virtual void	**GetCell** (**vtkIdType** cellId, **vtkGenericCell** *cell)
virtual void	**GetCellBounds** (**vtkIdType** cellId, **double** bounds[6])
virtual vtkIdType	**FindPoint** (**double** x, **double** y, **double** z)
virtual vtkIdType	**FindPoint** (**double** x[3])
virtual vtkIdType	**FindCell** (**double** x[3], **vtkCell** *cell, **vtkIdType** cellId, **double** tol2, **int** &subId, **double** pcoords[3], **double** *weights)
virtual vtkIdType	**FindCell** (**double** x[3], **vtkCell** *cell, **vtkGenericCell** *gencell, **vtkIdType** cellId, **double** tol2, **int** &subId, **double** pcoords[3], **double** *weights)
virtual vtkCell *	**FindAndGetCell** (**double** x[3], **vtkCell** *cell, **vtkIdType** cellId, **double** tol2, **int** &subId, **double** pcoords[3], **double** *weights)
virtual int	**GetCellType** (**vtkIdType** cellId)
virtual void	**GetCellPoints** (**vtkIdType** cellId, **vtkIdList** *ptIds)
virtual void	**GetPointCells** (**vtkIdType** ptId, **vtkIdList** *cellIds)
virtual void	**ComputeBounds** ()
virtual int	**GetMaxCellSize** ()
virtual int *	**GetDimensions** ()
virtual void	**GetDimensions** (**int** dims[3])
virtual int	**ComputeStructuredCoordinates** (const **double** x[3], **int** ijk[3], **double** pcoords[3])

Recap

- Euclidean domains

Recap

- Euclidean domains
  - Easily represented with regular grids

Recap

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- What about other spaces?

Recap

- Euclidean domains
  - Easily represented with regular grids

- What about other spaces?
  - 5th axiom of the *Elements*:
    - *Through a point not on a given straight line, at most one line can be drawn that never meets the given line*

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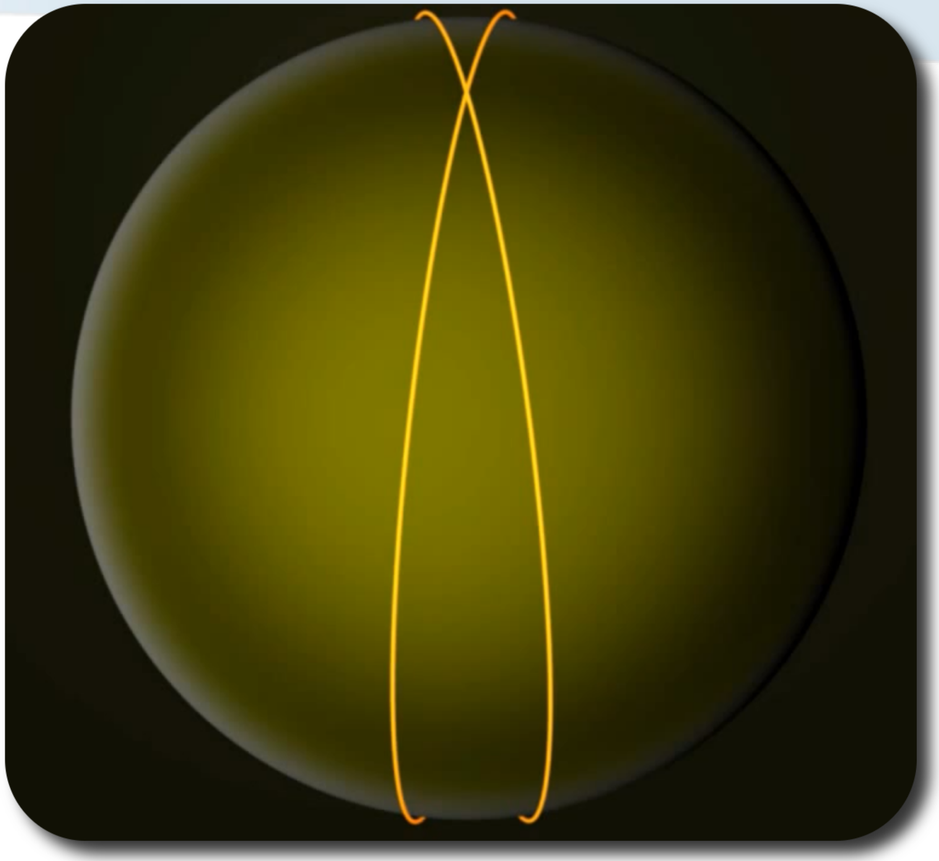

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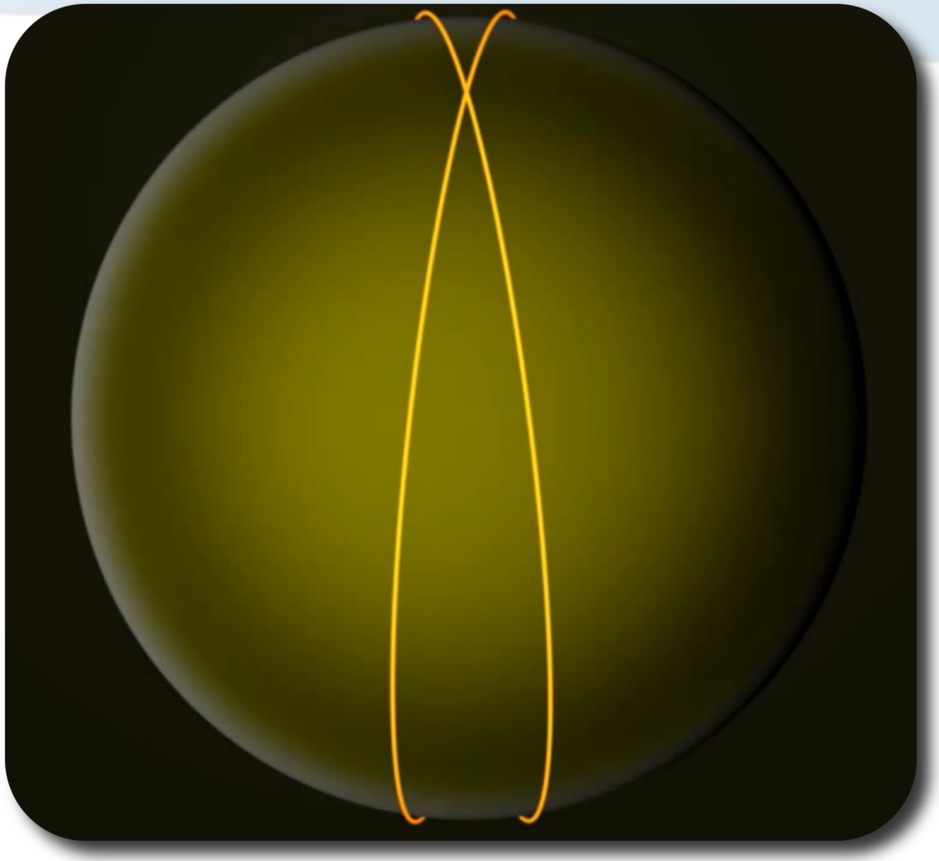
  - What if this axiom was not true?



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- What about other spaces?
  - 5th axiom of the *Elements*:
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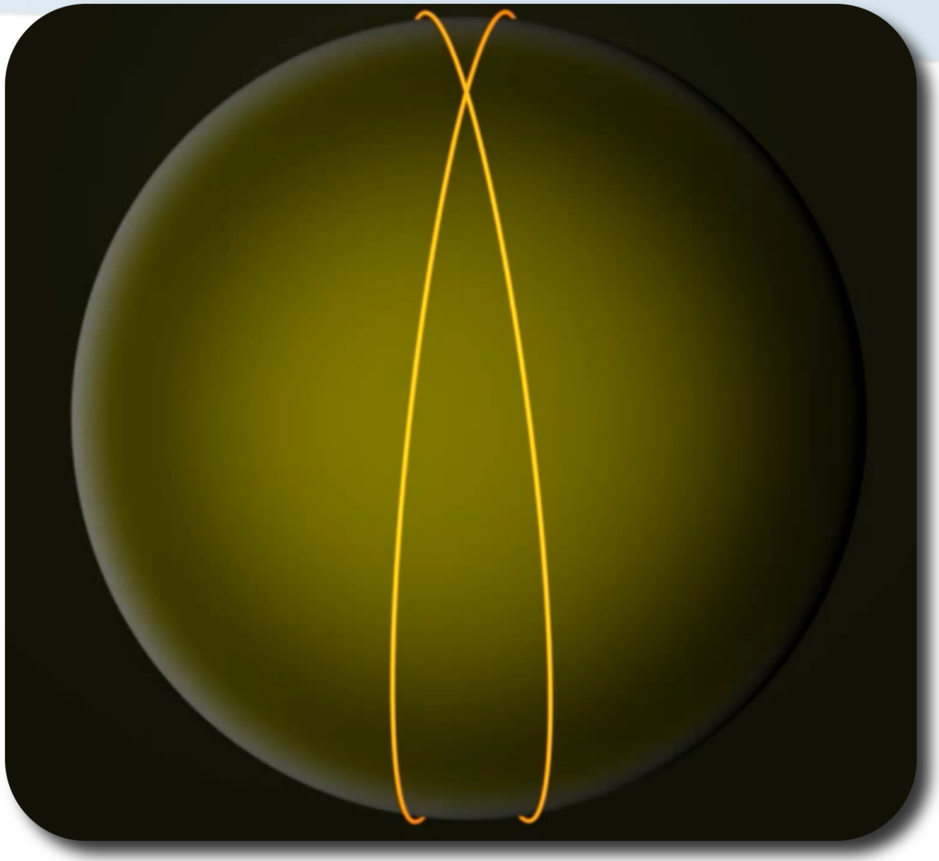
  - What if this axiom was not true?
  - What would geometry look like?



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- Euclidean domains
  - Easily represented with regular grids
- What about other spaces?
  - Riemannian geometry

Recap

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  - Easily represented with regular grids
- What about other spaces?
  - Riemannian geometry
  - 19th century

Recap

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- What about other spaces?
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  - Gauss, Lobachevsky, Poincaré, Beltrami, etc...

Recap

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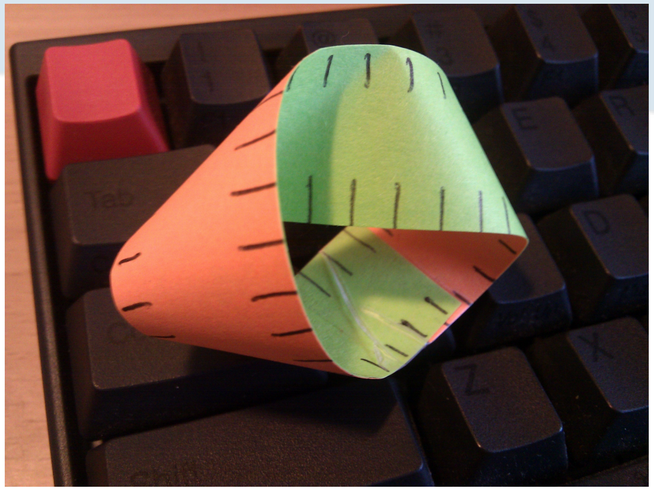
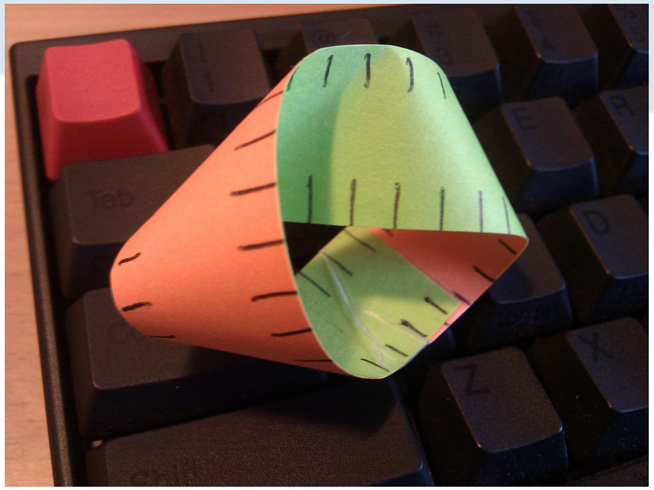
- What about other spaces?
  - Riemannian geometry
  - 19th century
  - Gauss, Lobachevsky, Poincaré, Beltrami, etc...
  - General notion of manifold

Manifolds on a computer



- Notion of simplex
  - A d-simplex σ is the convex hull of (d + 1) affinely independent points of an euclidean space $\mathbb{R}^n$
  - $0 \leq d \leq n$
  - d is the dimension of σ

Manifolds on a computer



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  - $0 \leq d \leq n$
  - d is the dimension of σ
  - Smallest combinatorial construction to represent a cell of dimension d

Manifolds on a computer

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Manifolds on a computer

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  - $0 \leq d \leq n$
  - 0-simplex: *vertex*

Manifolds on a computer

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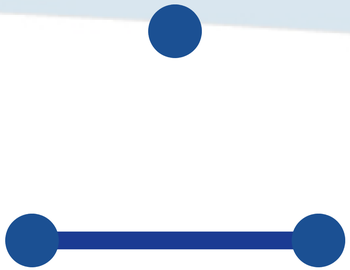
  - $0 \leq d \leq n$
  - 0-simplex: *vertex*
  - 1-simplex: *edge*



Manifolds on a computer

- Notion of simplex
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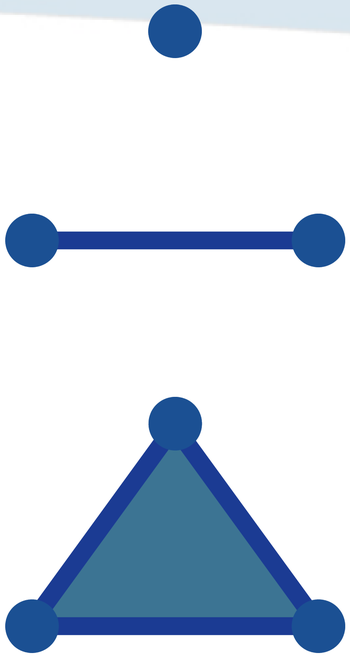
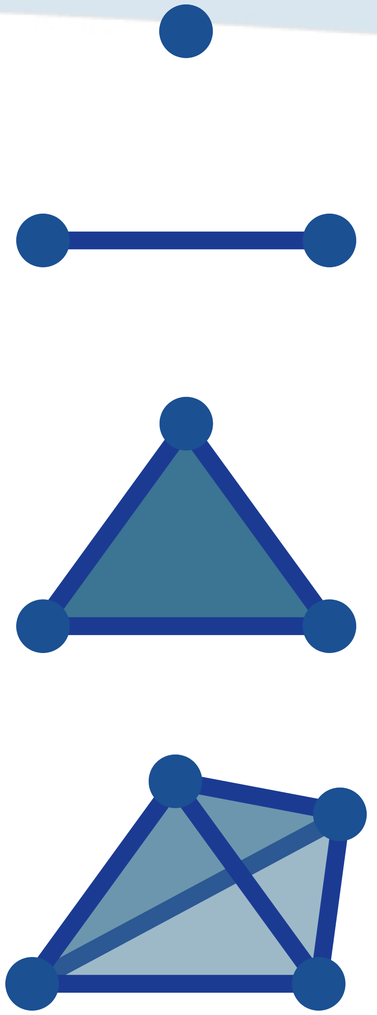
  - $0 \leq d \leq n$
  - 0-simplex: *vertex*
  - 1-simplex: *edge*
  - 2-simplex: *triangle*



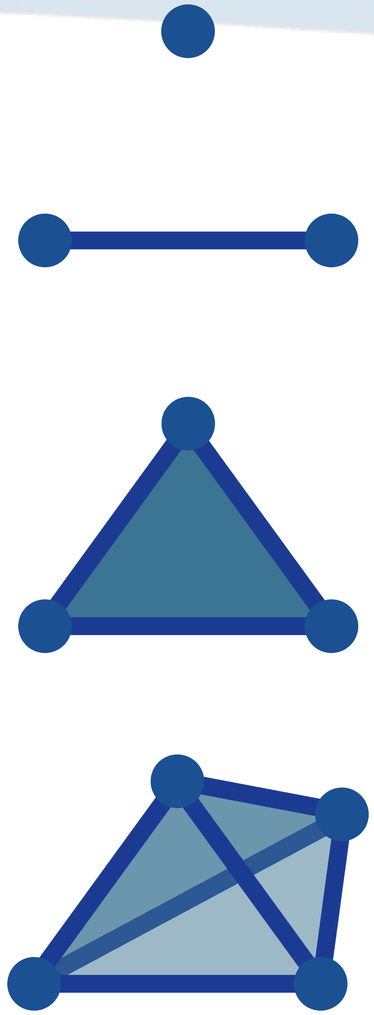
Manifolds on a computer

- Notion of simplex
  - A d-simplex σ is the convex hull of (d + 1) affinely independent points of an euclidean space $\mathbb{R}^n$
  - $0 \leq d \leq n$
  - 0-simplex: *vertex*
  - 1-simplex: *edge*
  - 2-simplex: *triangle*
  - 3-simplex: *tetrahedron*



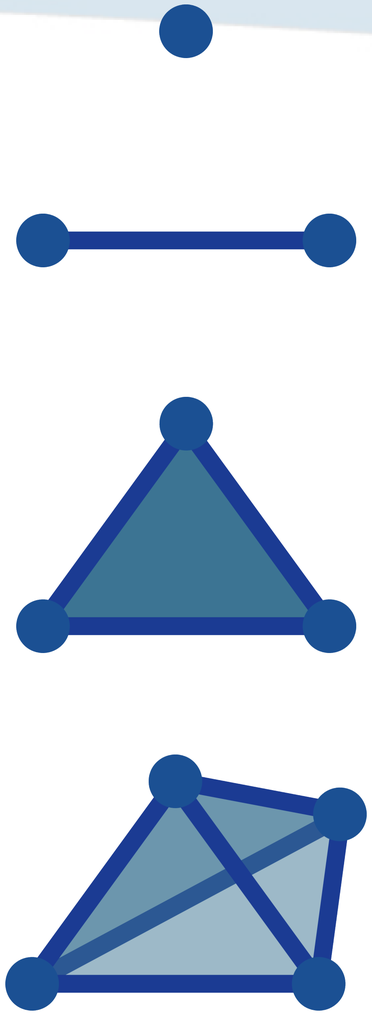
Manifolds on a computer

- Notion of face



Manifolds on a computer

- Notion of face
  - A face τ of a d-simplex σ is the simplex defined by a non-empty subset of the d+1 points of σ
  - Noted $\tau \leq \sigma$



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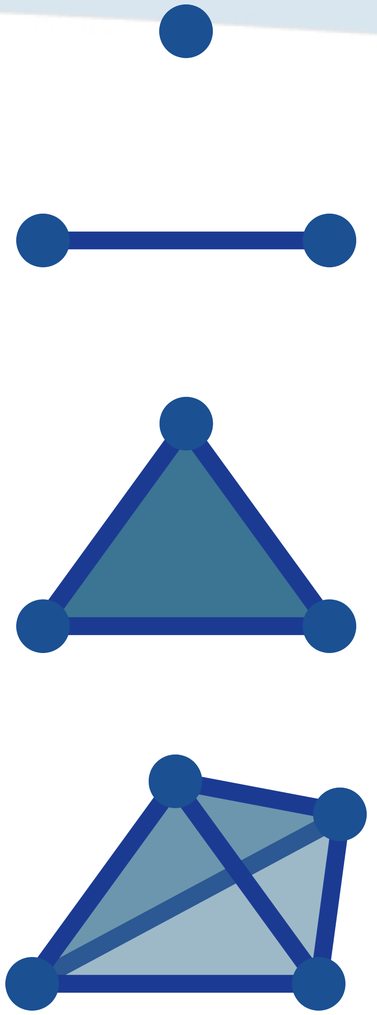

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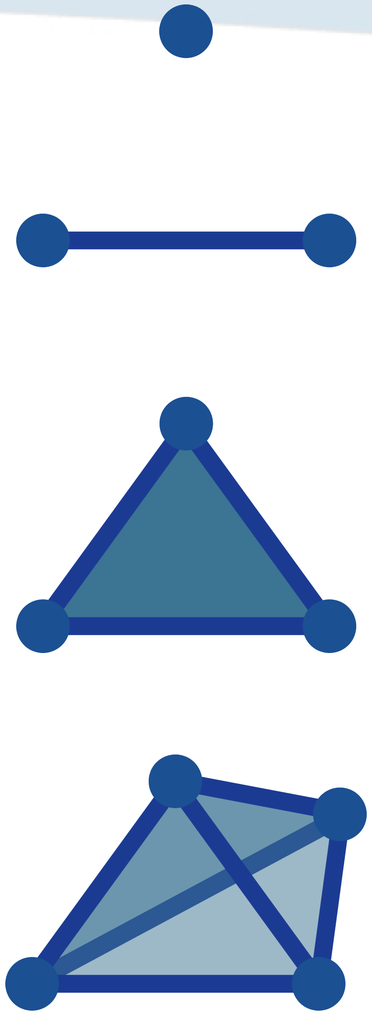

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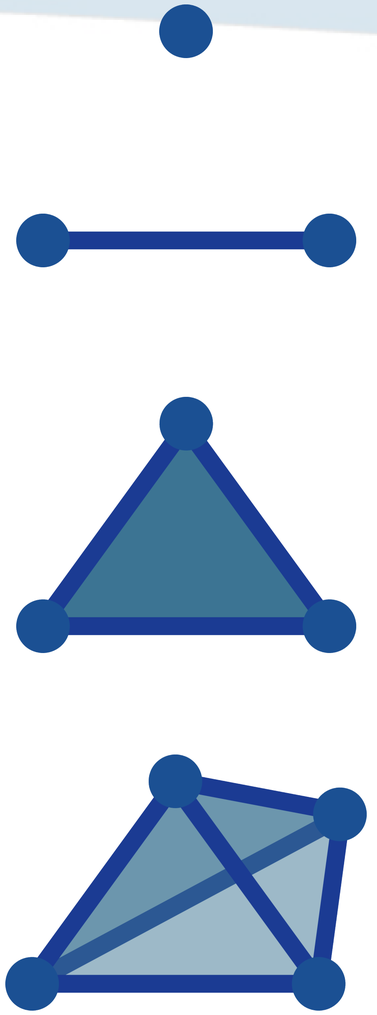

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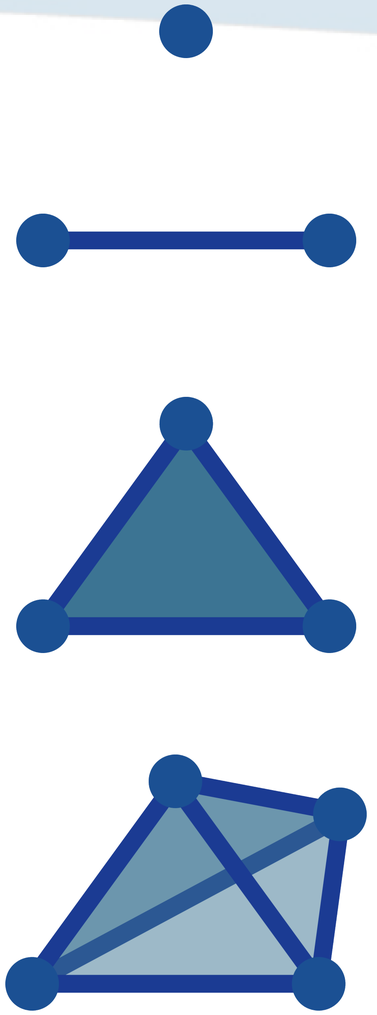

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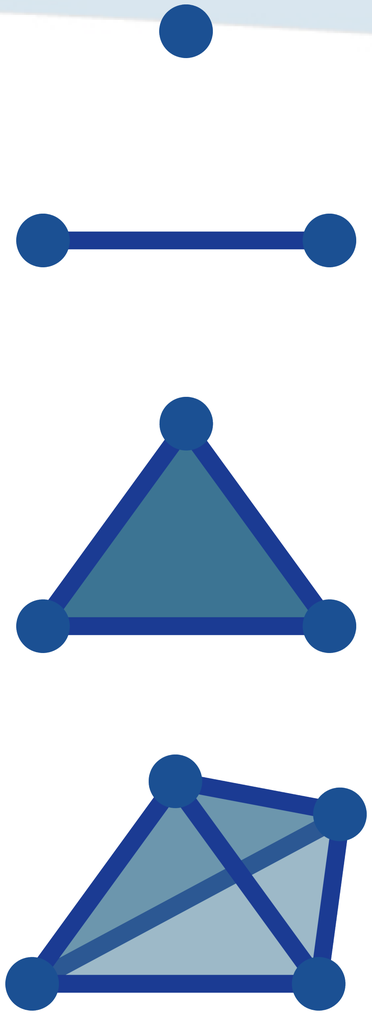

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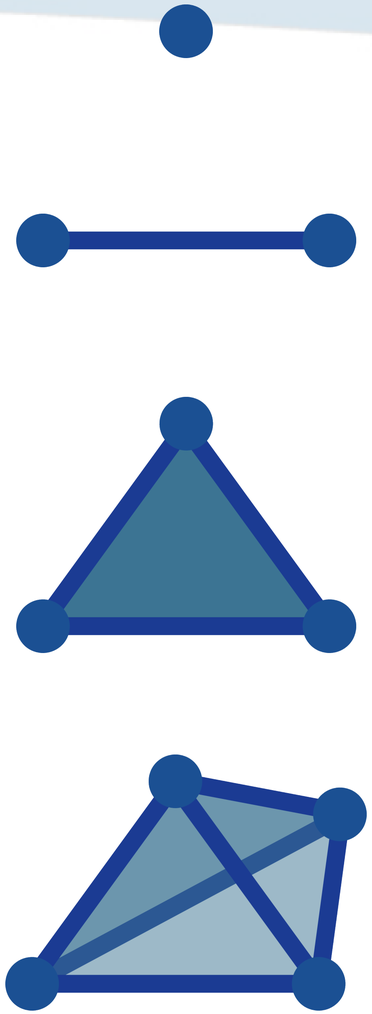
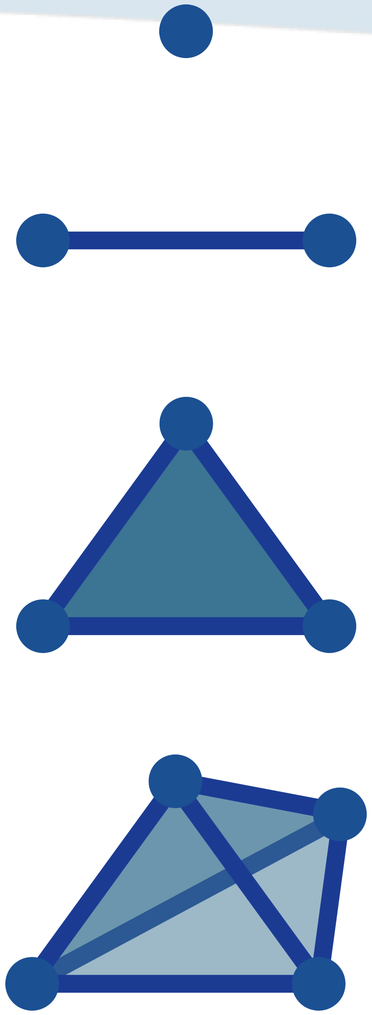


Manifolds on a computer

- Notion of **simplicial complex**



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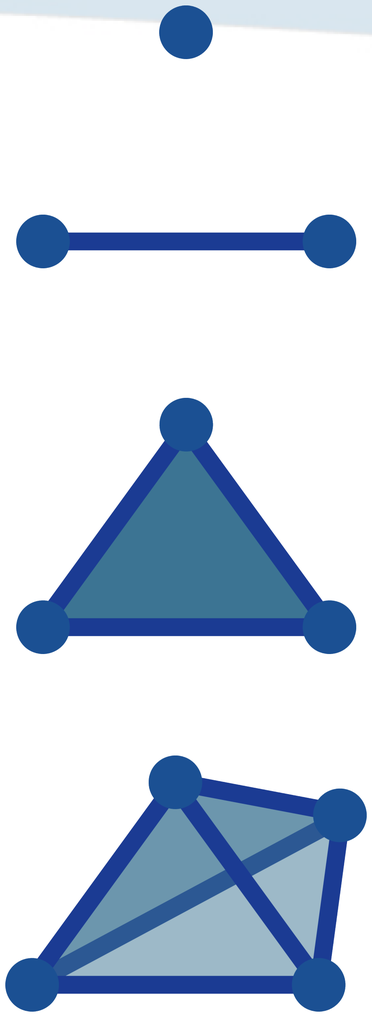

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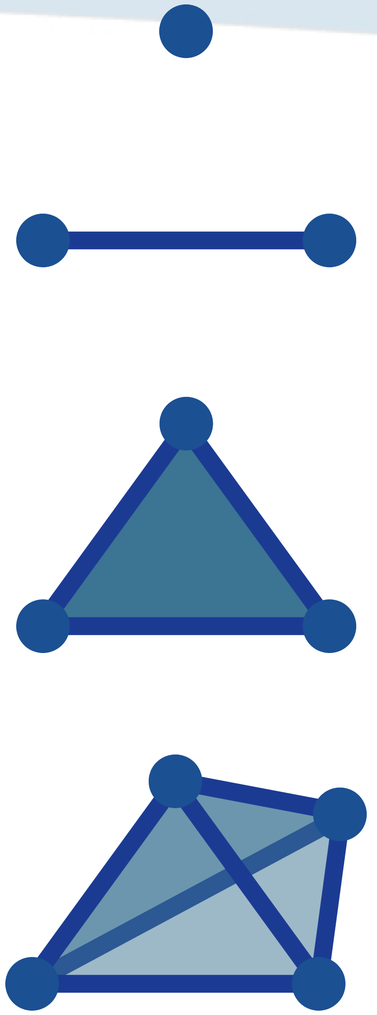

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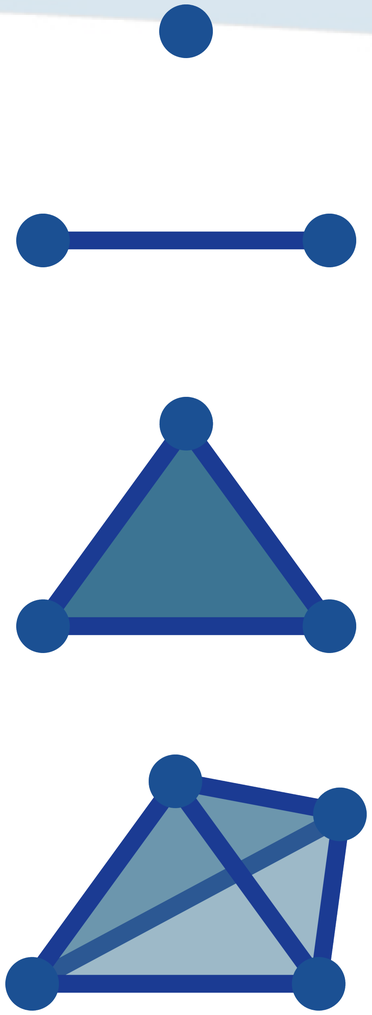

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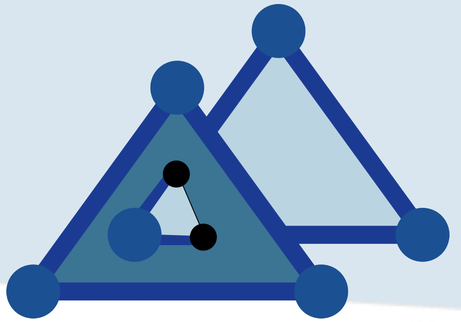

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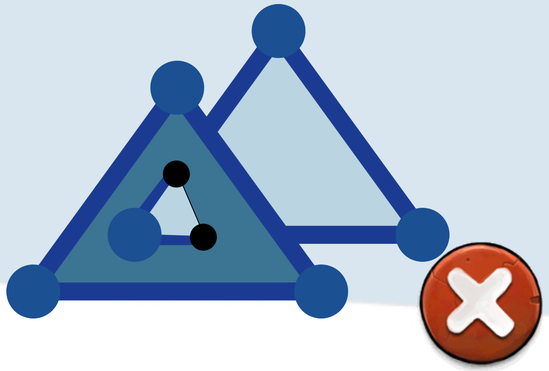

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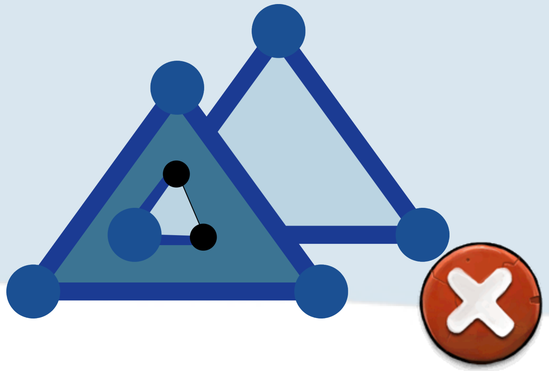

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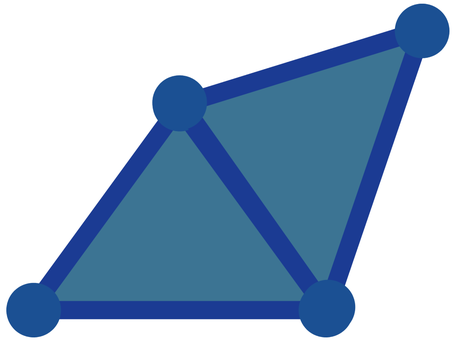

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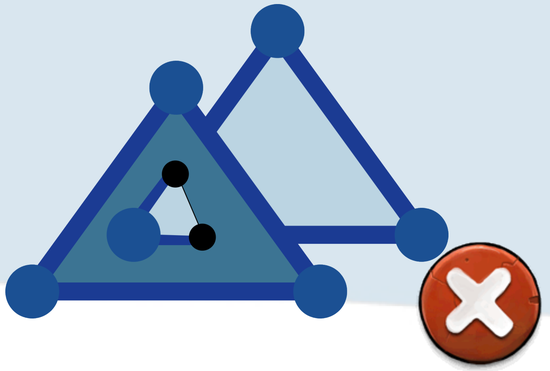

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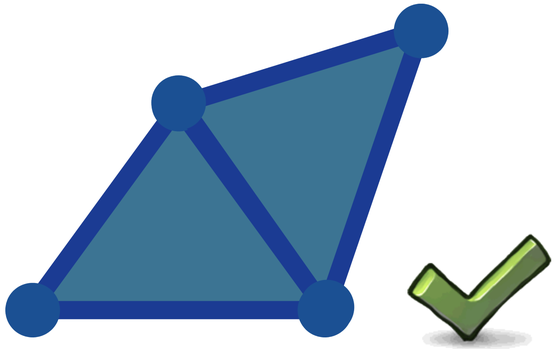

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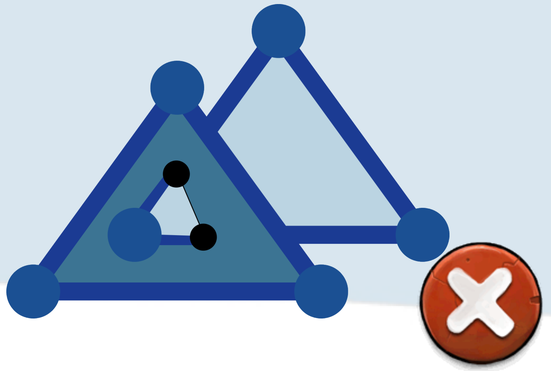

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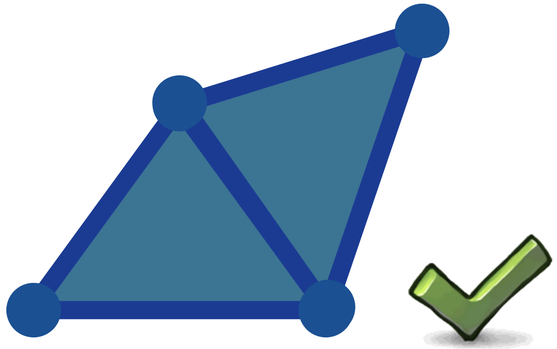

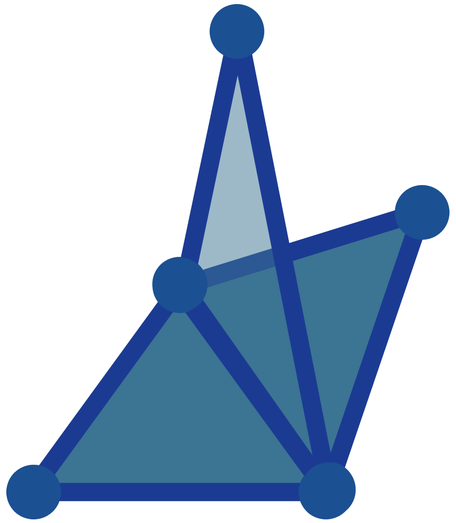

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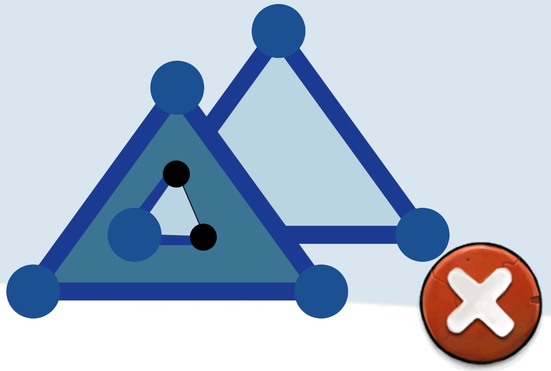

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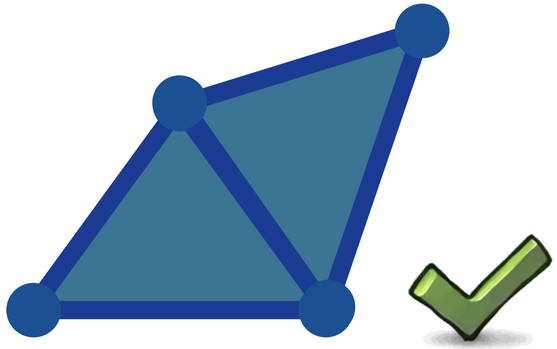

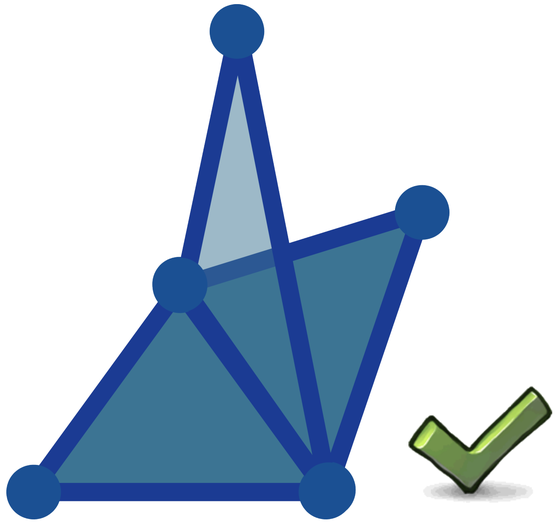

Manifolds on a computer

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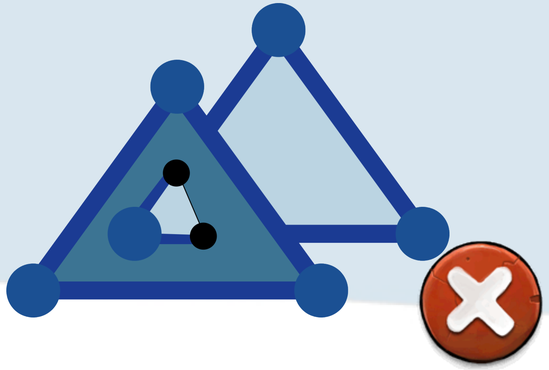

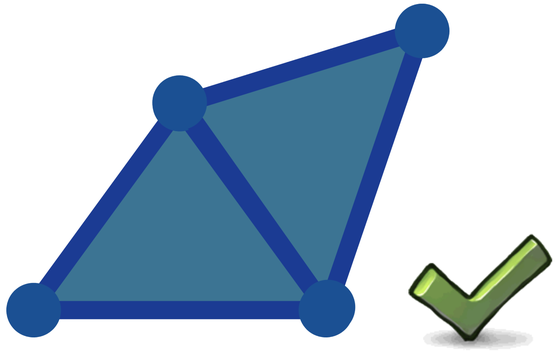

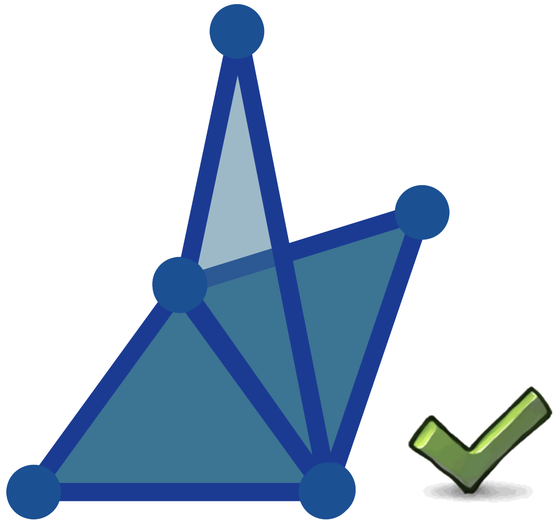

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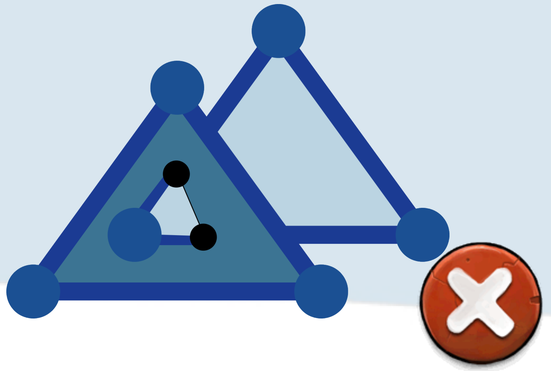

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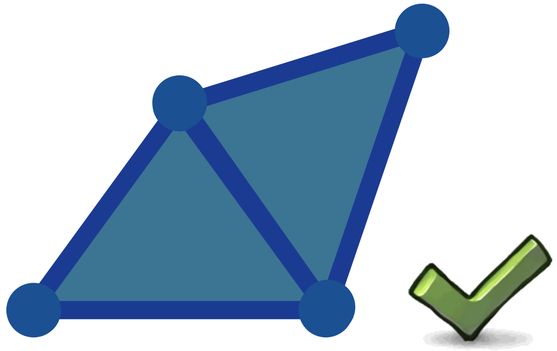

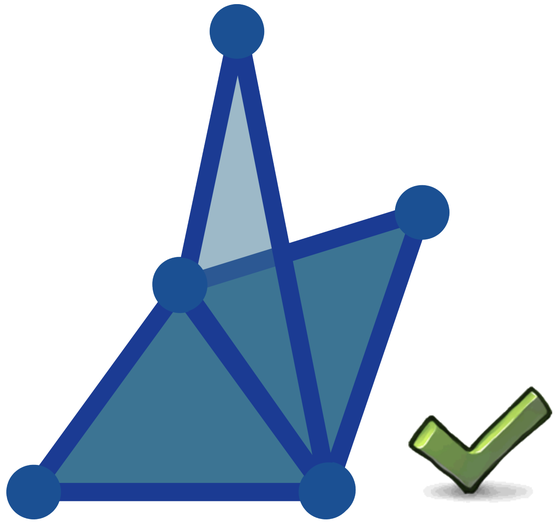

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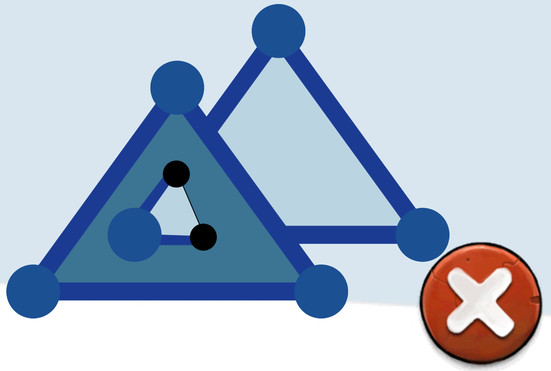

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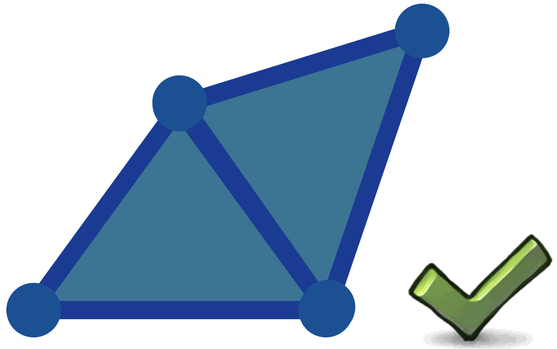

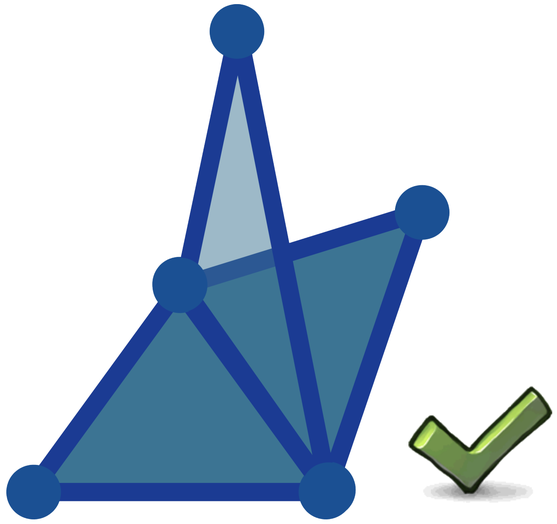

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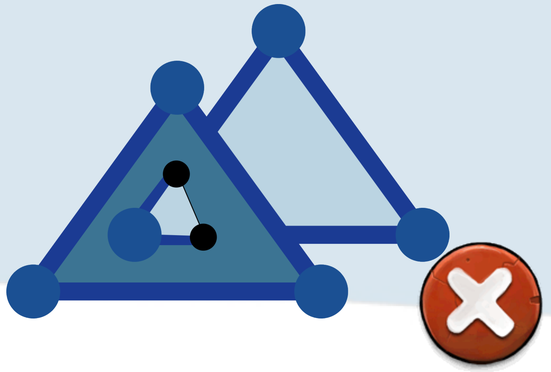

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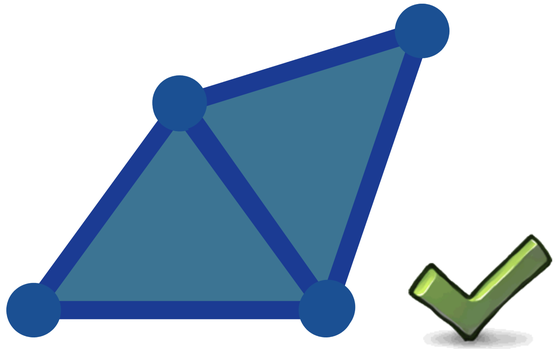

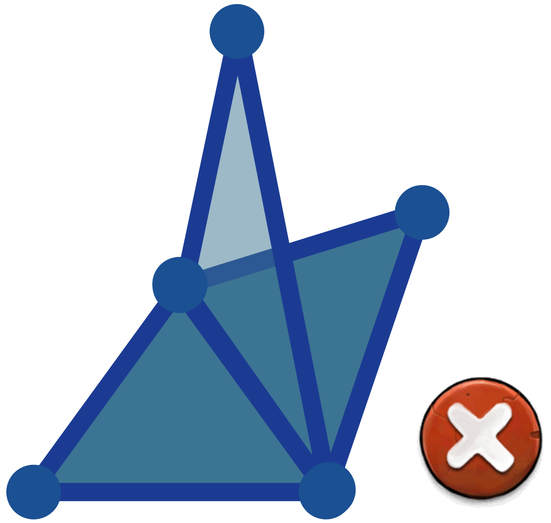

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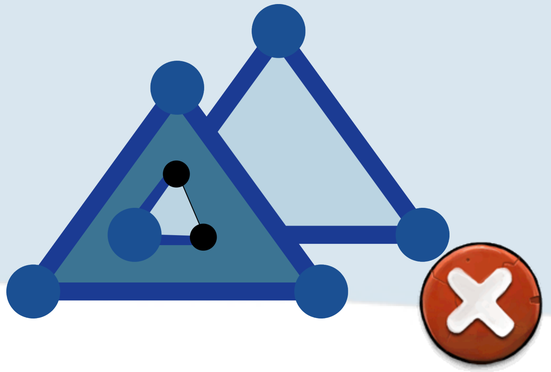

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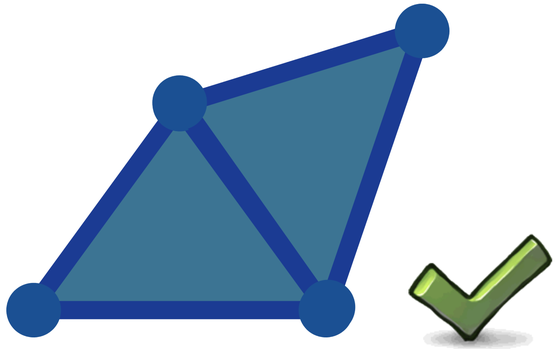
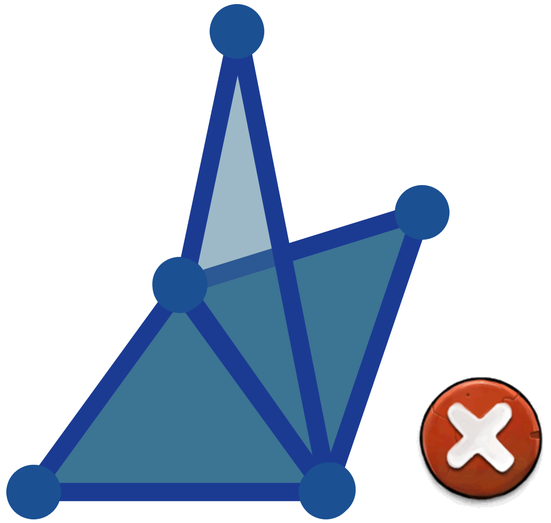
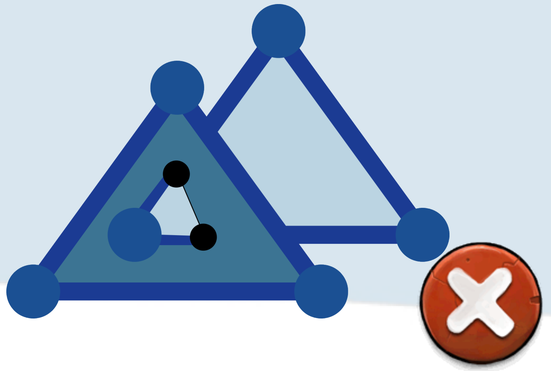
    - The union $|\mathcal{K}| = \cup_{\sigma \in \mathcal{K}} \sigma$ of the simplices of $\mathcal{K}$ is homeomorphic to $\mathbb{M}$
  - Any open set of $|\mathcal{K}|$ is homeomorphic to $\mathbb{R}^d$

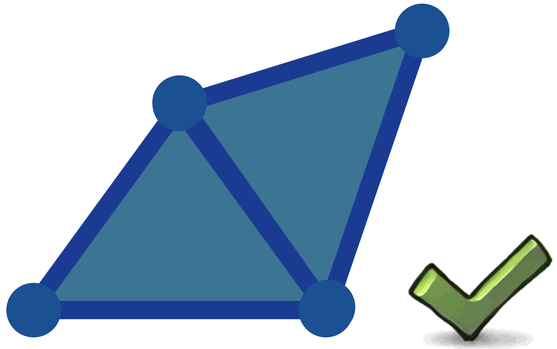




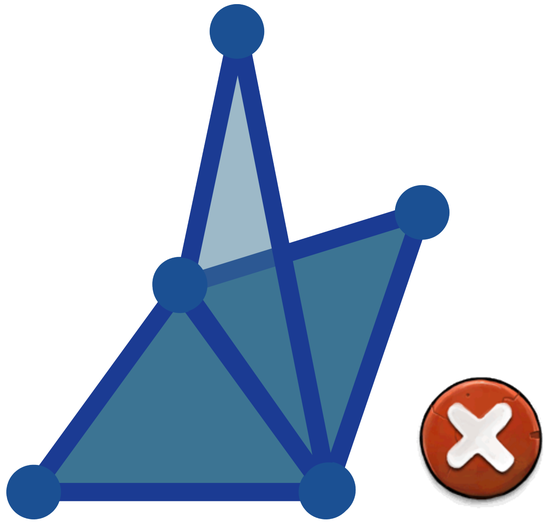
Manifolds on a computer



- Notion of **triangulation**
  - The triangulation of a d-manifold $\mathbb{M}$ is a simplicial complex $\mathcal{K}$ such that
    - The union $|\mathcal{K}| = \cup_{\sigma \in \mathcal{K}} \sigma$ of the simplices of $\mathcal{K}$ is homeomorphic to $\mathbb{M}$
  - Any open set of $|\mathcal{K}|$ is homeomorphic to $\mathbb{R}^d$

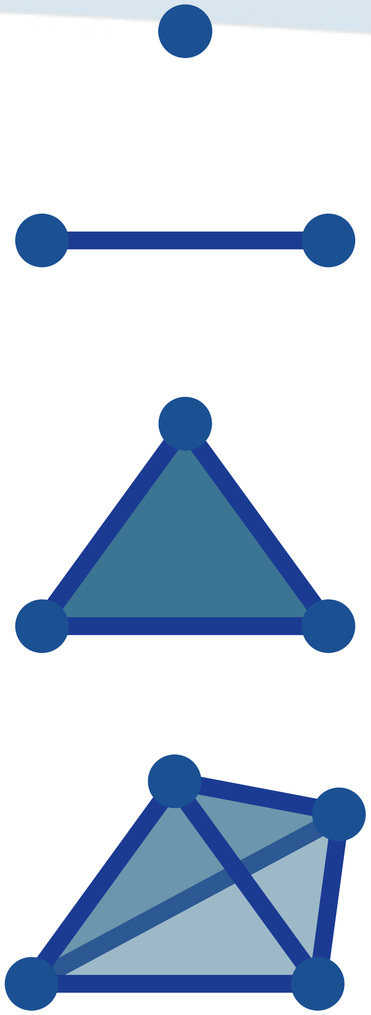


- 2-triangulation: *triangle mesh*
- 3-triangulation: *tetrahedral mesh*



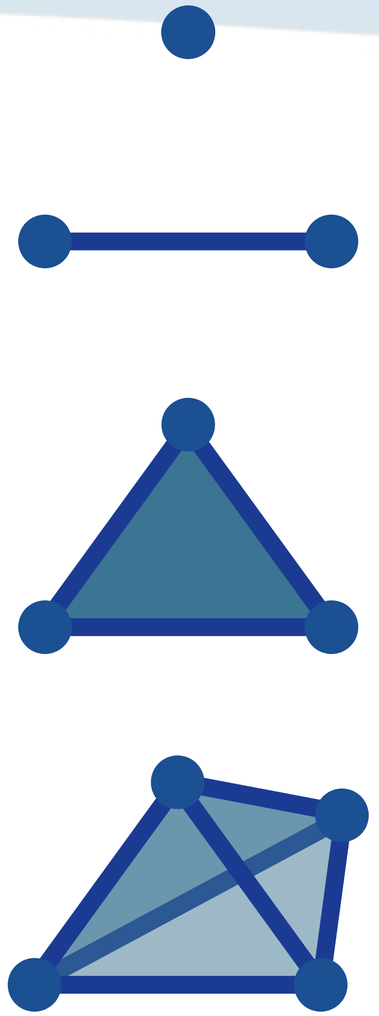
Manifolds on a computer

- How to represent that in memory?



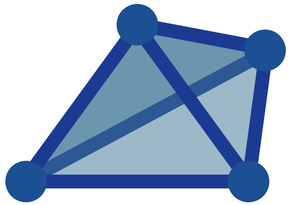
Manifolds on a computer

- How to represent that in memory?
  - Much trickier (arbitrary valence of the vertices)
  - The connectivity cannot be inferred by the geometry
  - It has to be explicitly stored



Manifolds on a computer

- General d-triangulation
  - List of d-simplices, (d-1)-simplices, … 0-simplices



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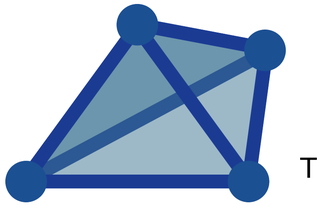
T



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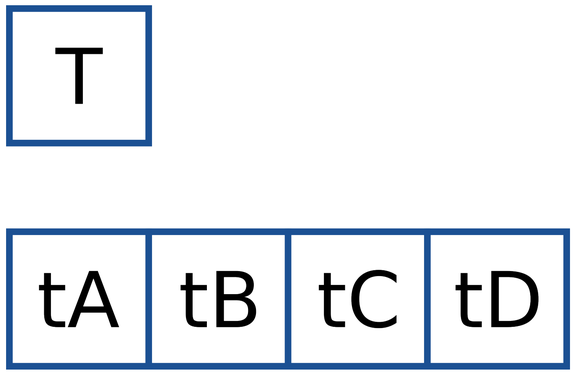

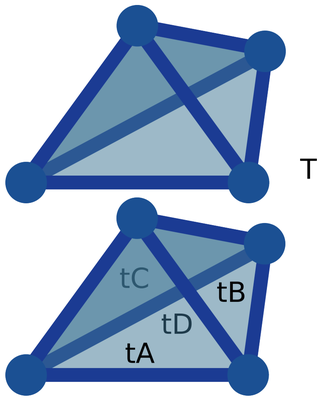

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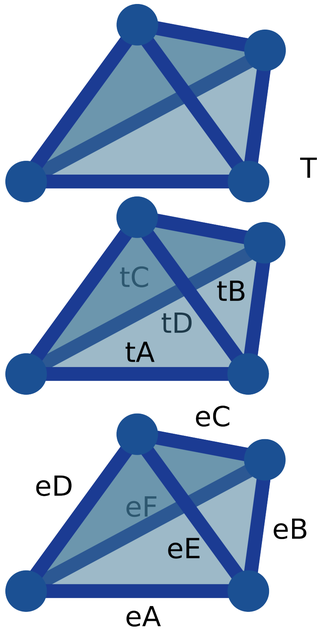
tA	tB	tC	tD

eA	eB	eC	eD	eE	eF



Manifolds on a computer

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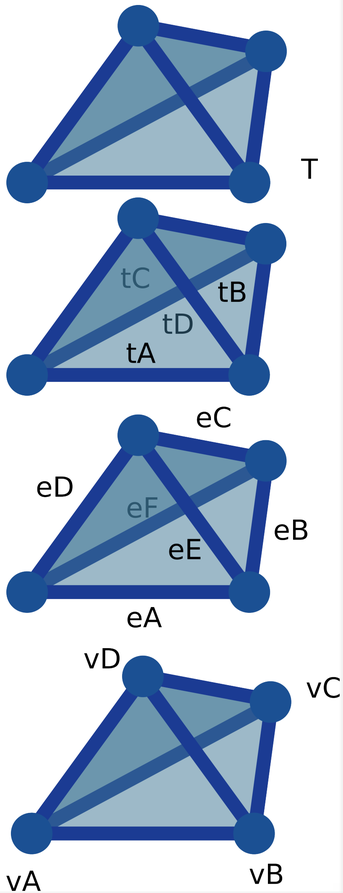
eA	eB	eC	eD	eE	eF

vA	vB	vC	vD



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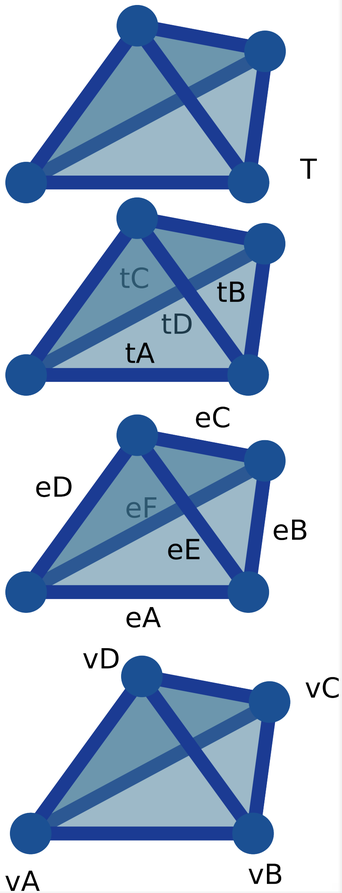

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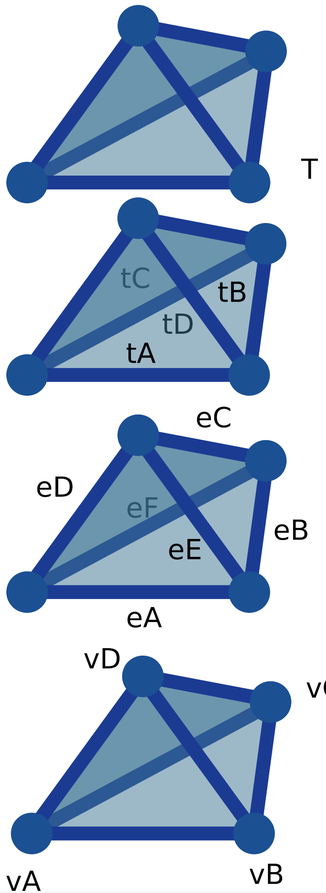
eA	eB	eC	eD	eE	eF

vA	vB	vC	vD



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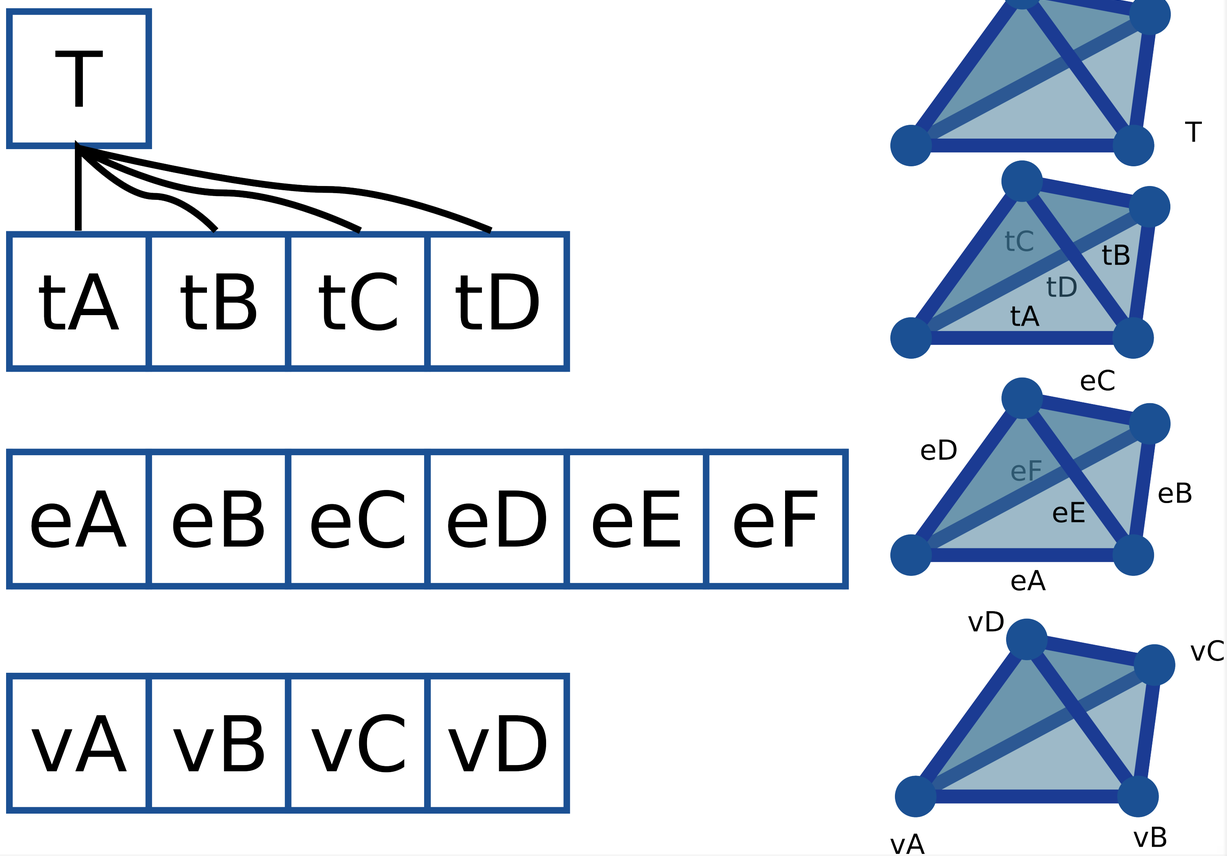

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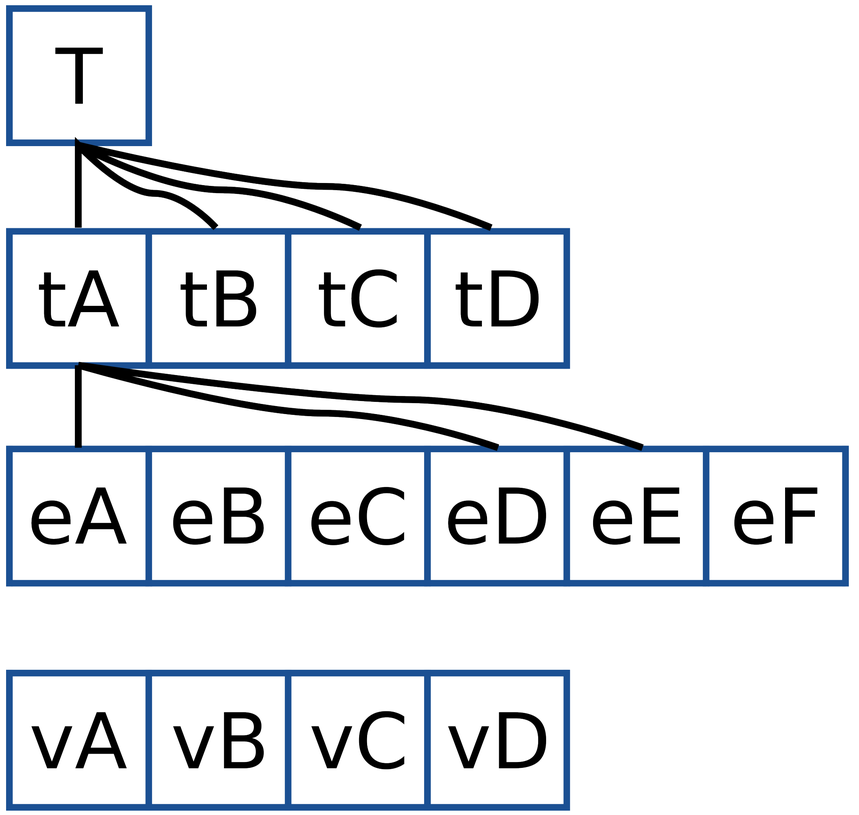

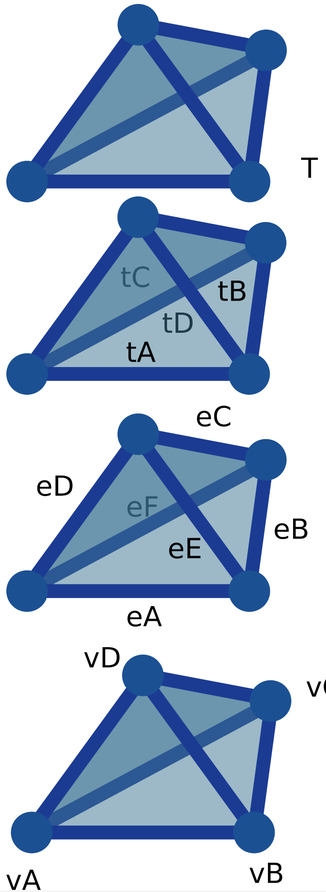

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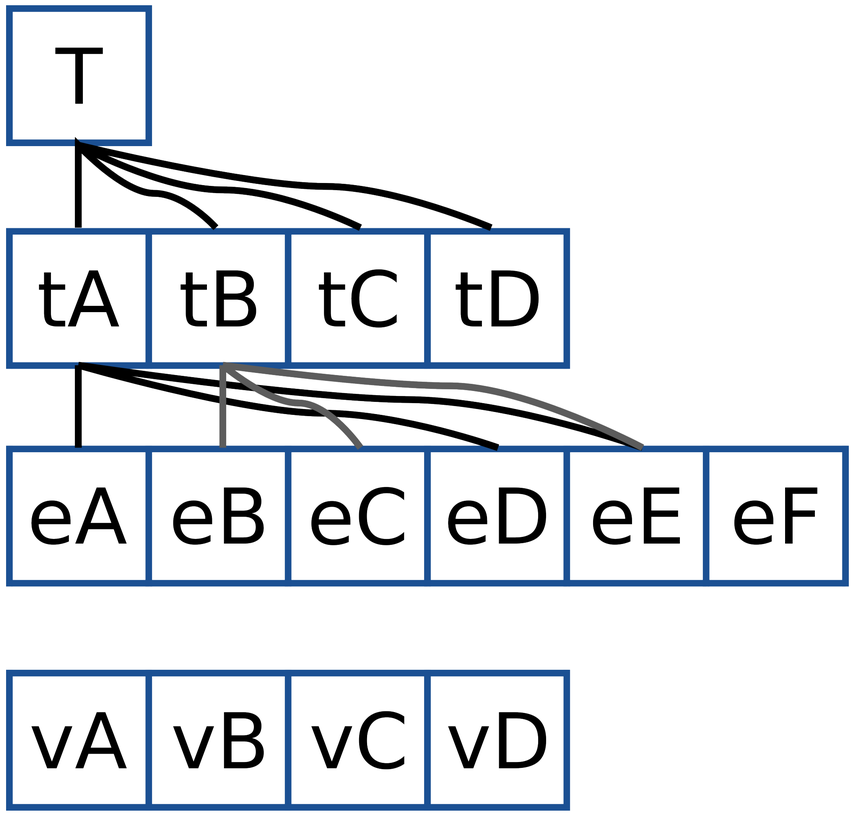

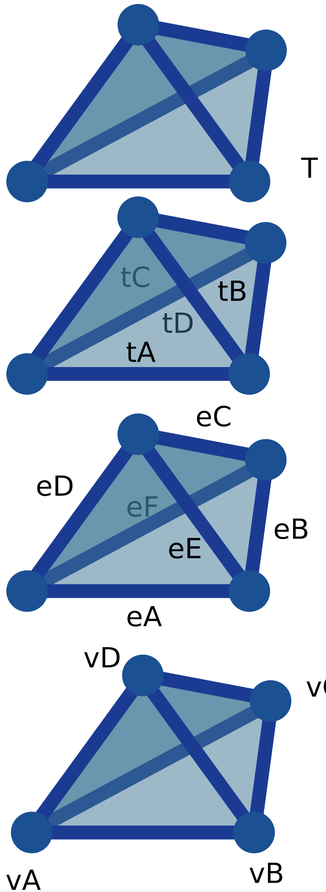

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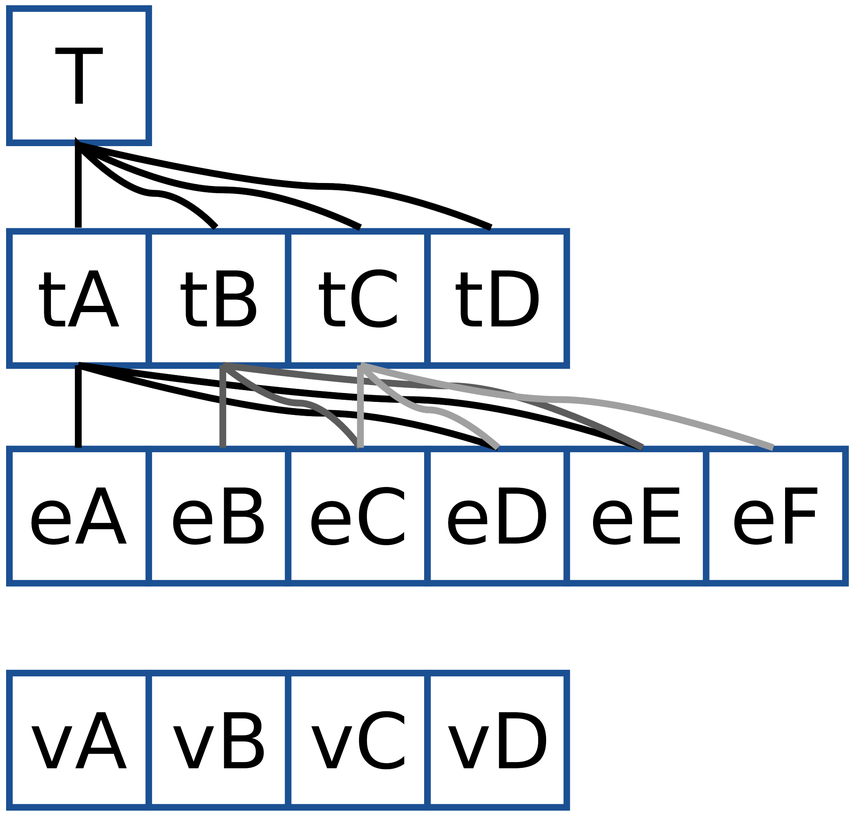

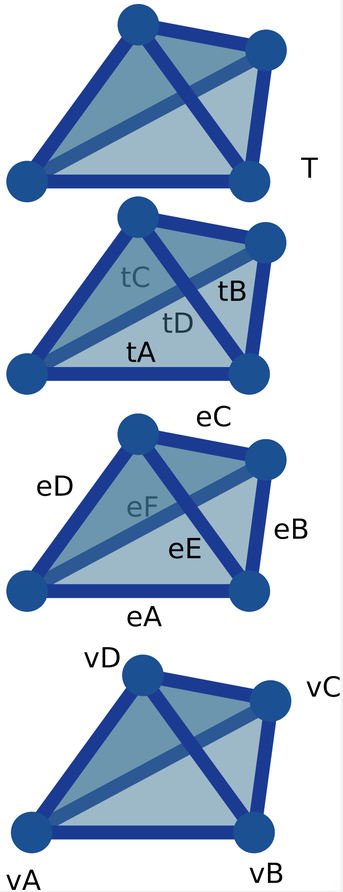

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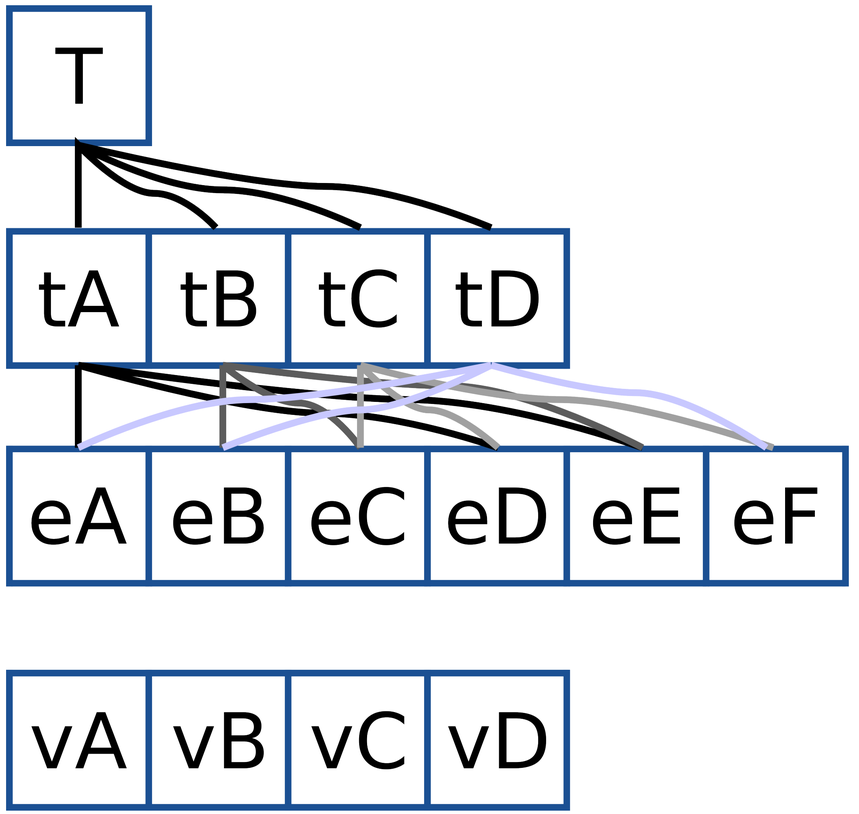

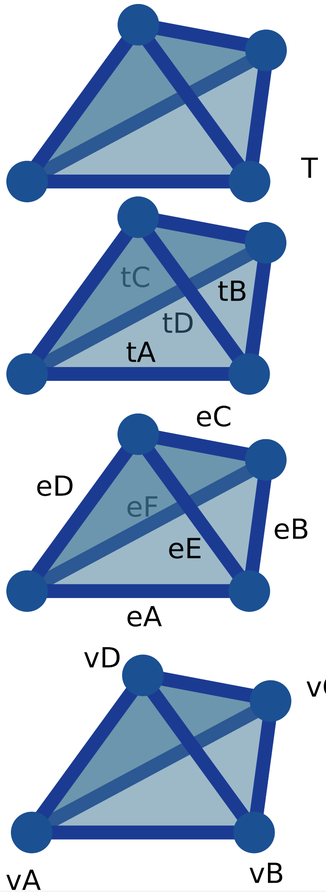

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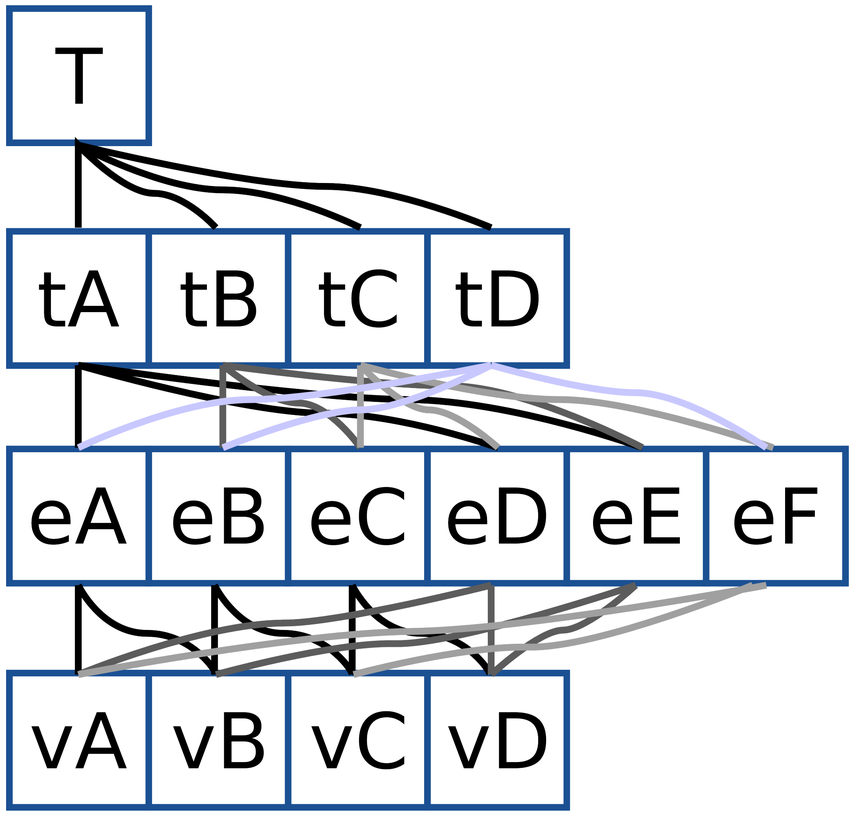
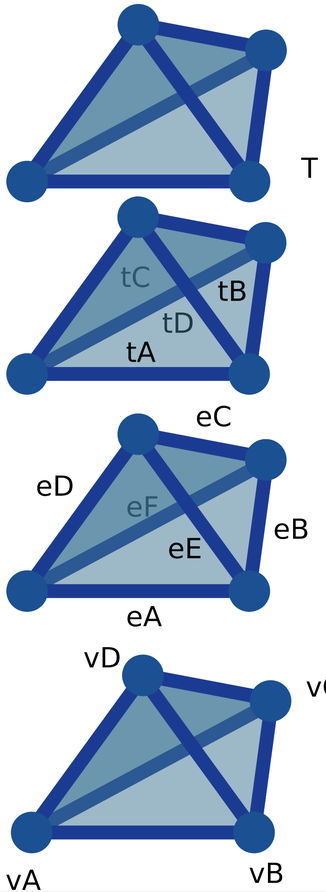
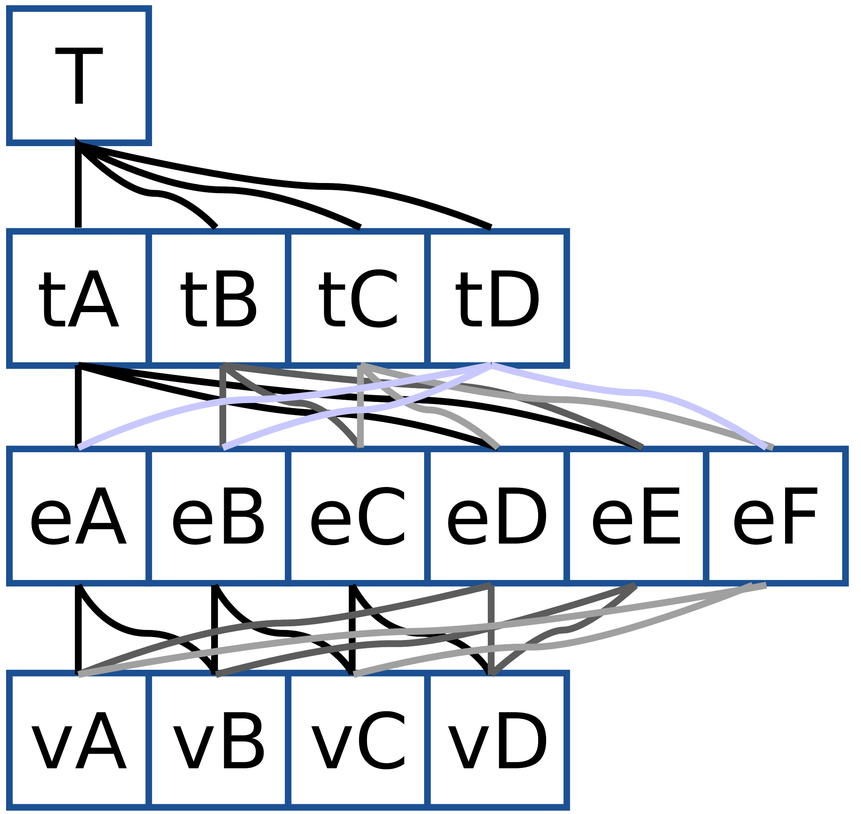
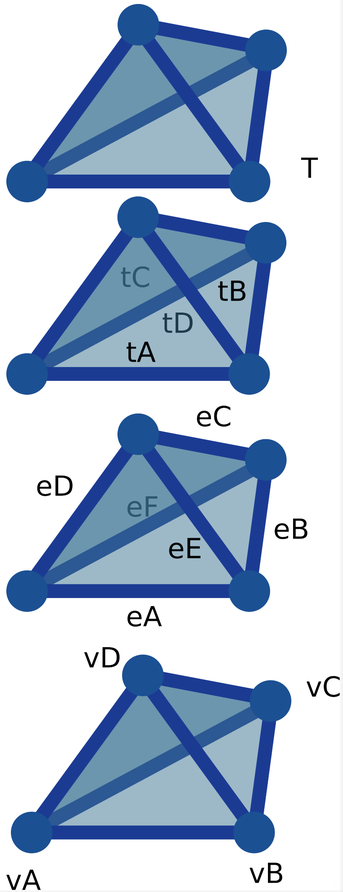




Manifolds on a computer

- General d-triangulation
  - List of d-simplices, (d-1)-simplices, … 0-simplices
  - Attributes:
    - Assigned to the vertices
  - Each d-simplex stores pointers to its (d+1) faces of dimension (d-1)
  - Navigation with bi-directional pointers





Consistency check

- There's a lot of room for bugs....

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$$\chi = \sum_{i=0}^{d} (-1)^i |\{\sigma^i\}|$$

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$$\chi = \sum_{i=0}^{d} (-1)^i |\{\sigma^i\}| = \sum_{i=0}^{d} (-1)^i \beta_i$$

Consistency check

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$$\chi = \#_{vertices} - \#_{edges} + \#_{triangles} - \#_{tetrahedra} = \sum_{i=0}^{d} (-1)^i \beta_i$$

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$$\chi = \#_{vertices} - \#_{edges} + \#_{triangles} - \#_{tetrahedra} = \sum_{i=0}^{d} (-1)^i \beta_i$$

$$\chi = \#_{connected\_components} - \#_{cycles} + \#_{voids}$$

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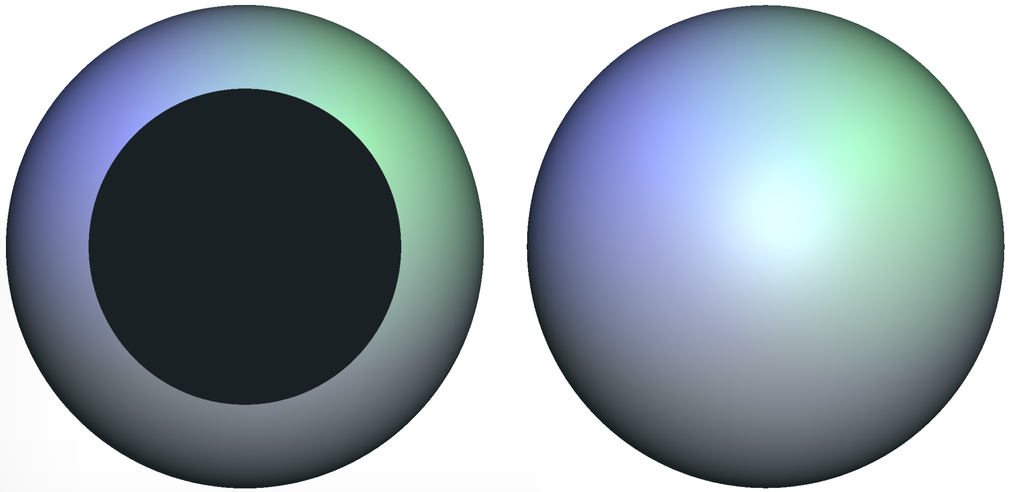

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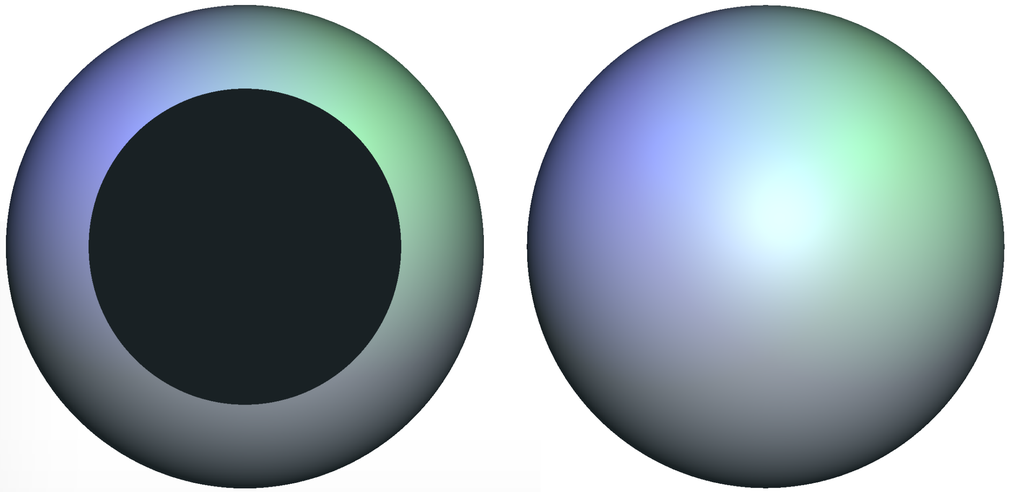

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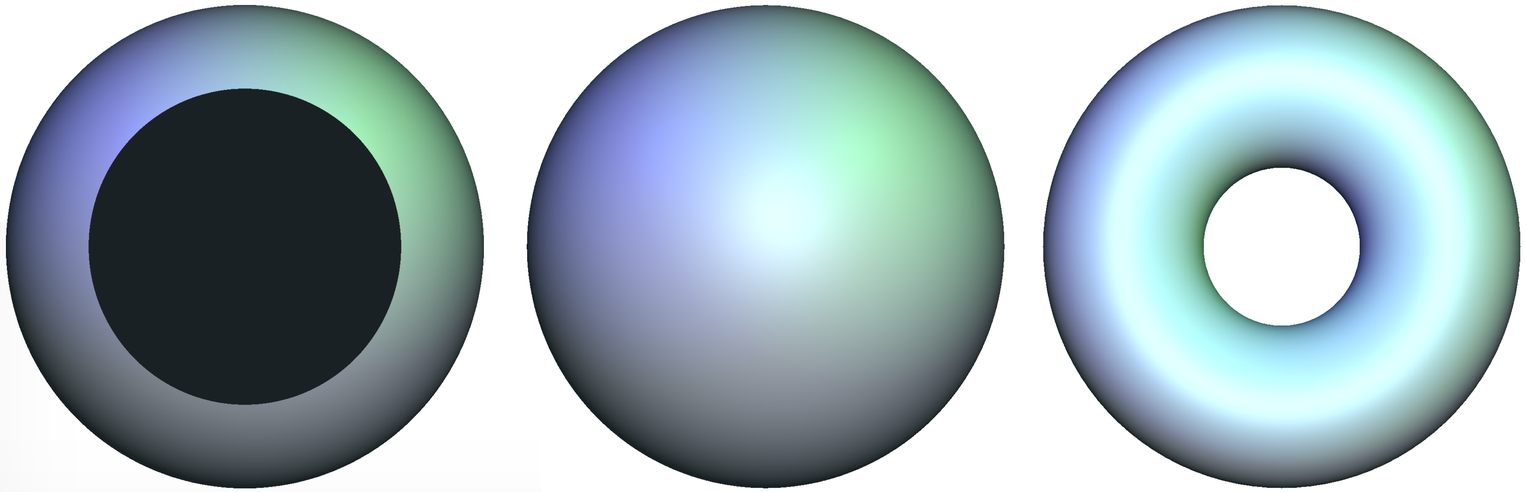

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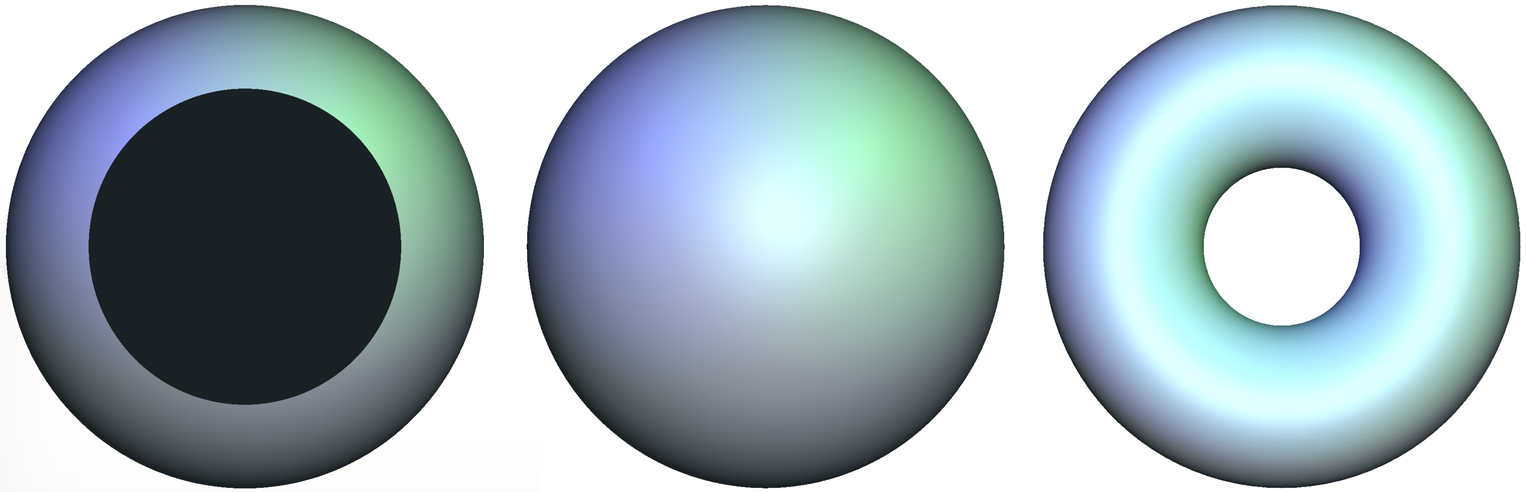

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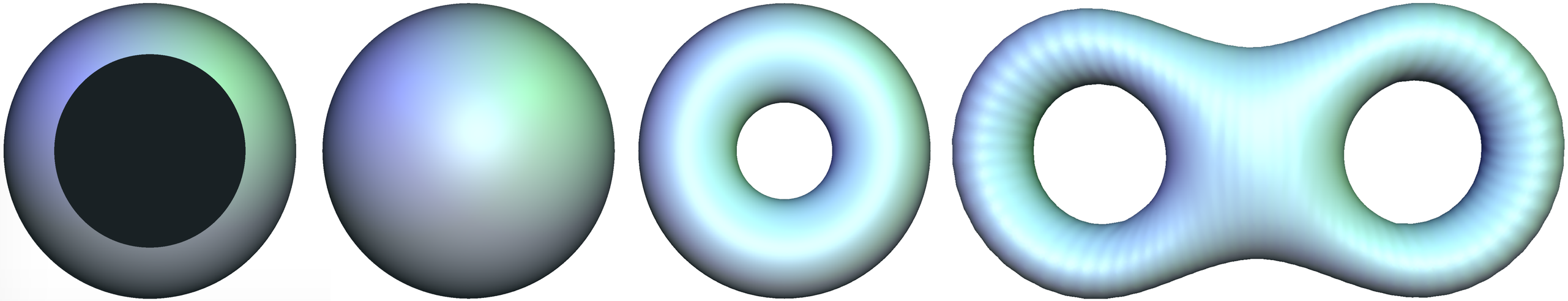

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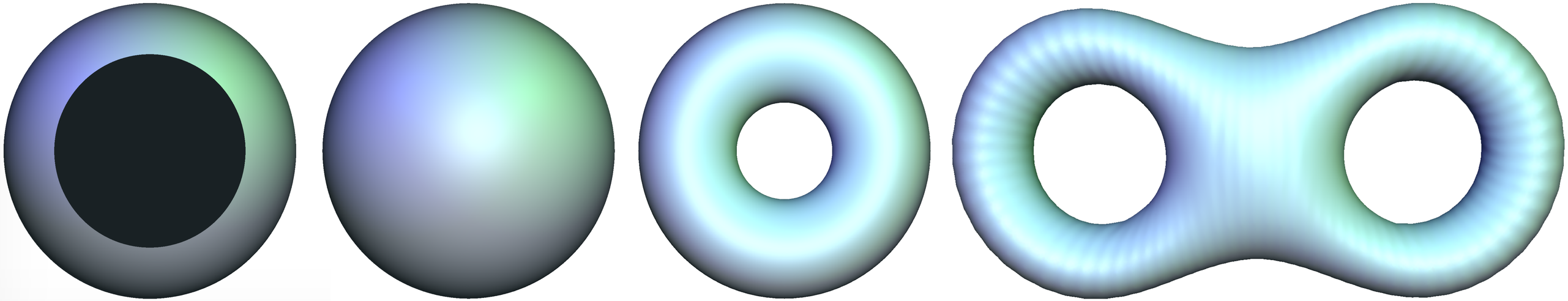
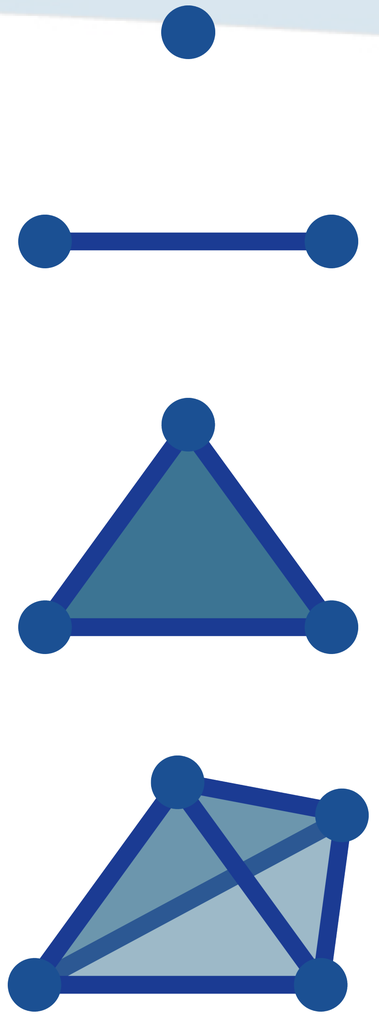


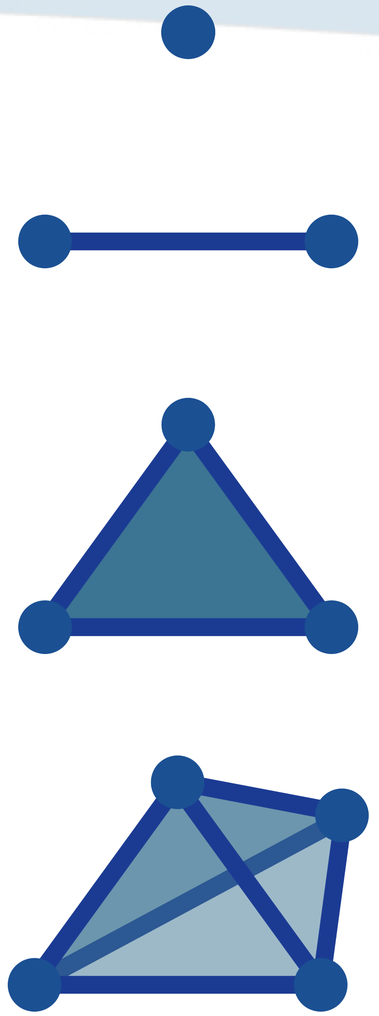
Manifold spaces on a computer

- Notion of **triangulation** $\mathcal{T}$



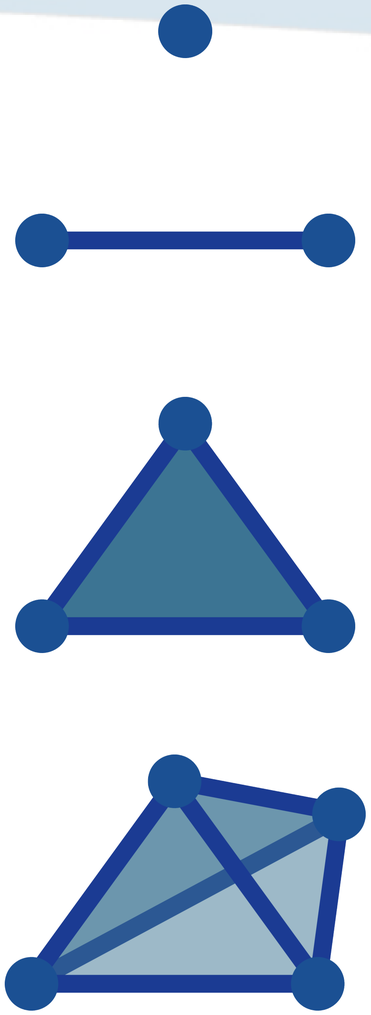
Manifold spaces on a computer

- Notion of **triangulation** $\mathcal{T}$
  - Set of **vertices** (samples)
    - With embedding functions to $\mathbb{R}^n$



Manifold spaces on a computer

- Notion of **triangulation** $\mathcal{T}$
  - Set of **vertices** (samples)
    - With embedding functions to $\mathbb{R}^n$
  - **Connectivity**
    - **Explicitly** represented
    - Enumeration of simplices and their faces



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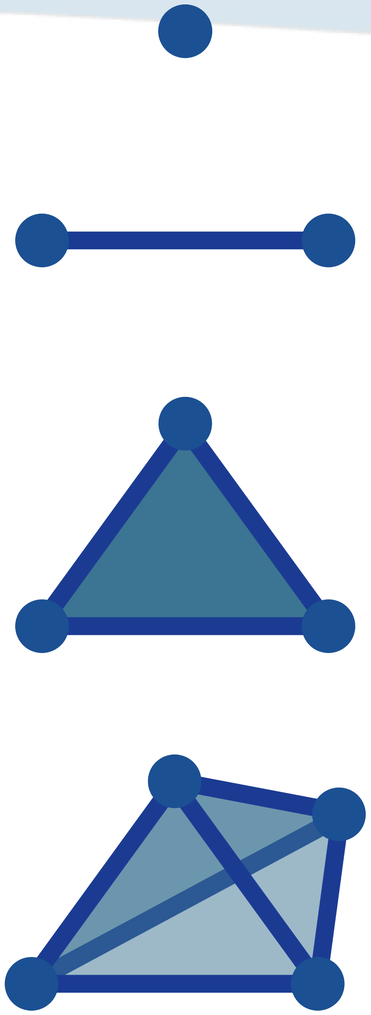

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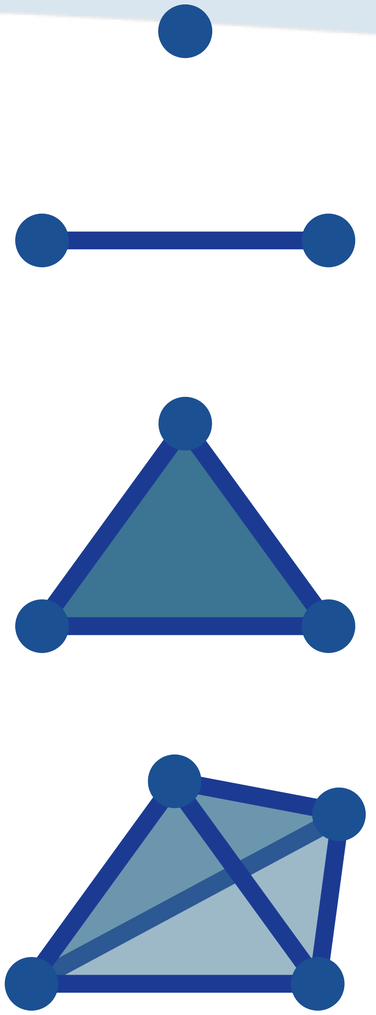
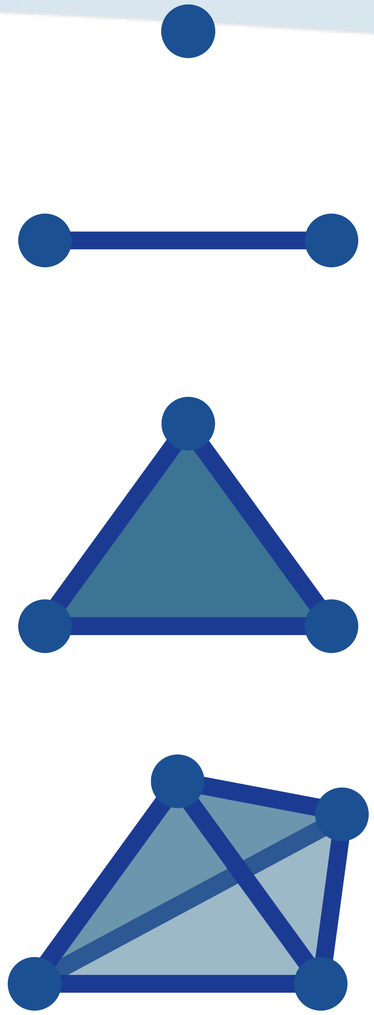
  - **Connectivity**
    - **Explicitly** represented
    - Enumeration of simplices and their faces
  - What's missing?
    - The **interpolation** scheme



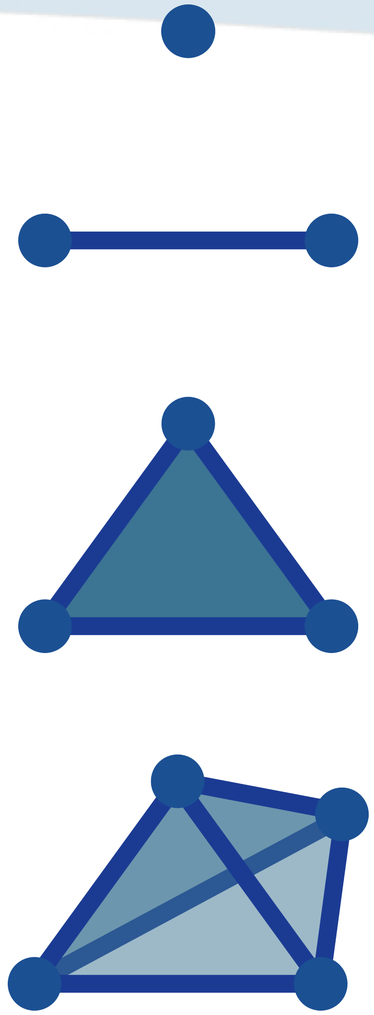
Interpolants for triangulations

- Like regular grids, several options



Interpolants for triangulations

- Like regular grids, several options
  - Piecewise constant
  - Piecewise linear
  - Piecewise polynomials,
  - etc.



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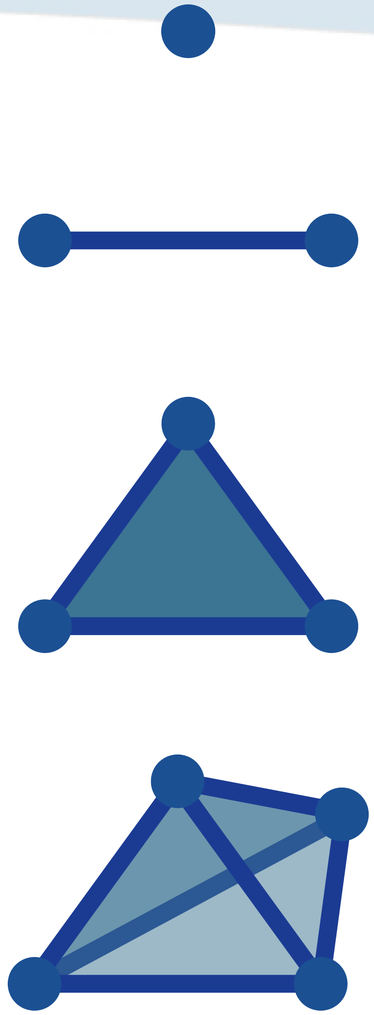
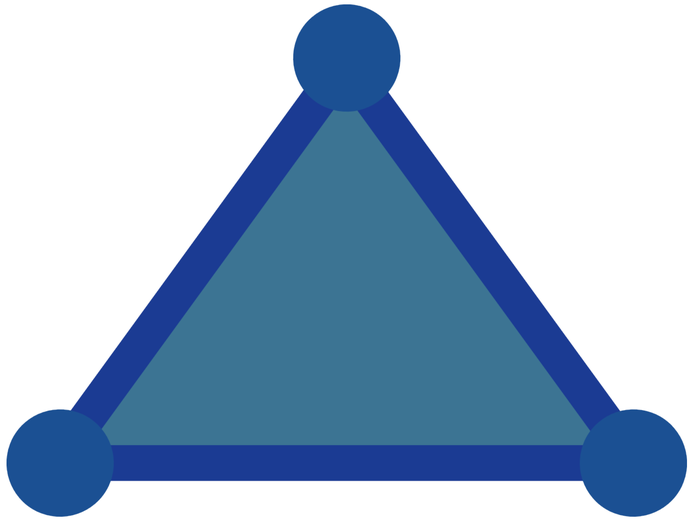


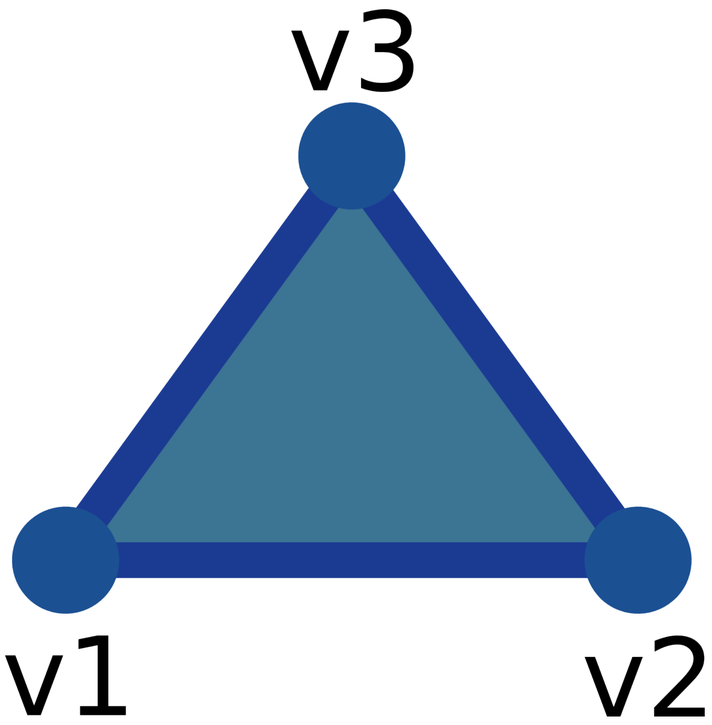
Piecewise linear interpolant on triangulations

- Interpolation as a boundary value problem



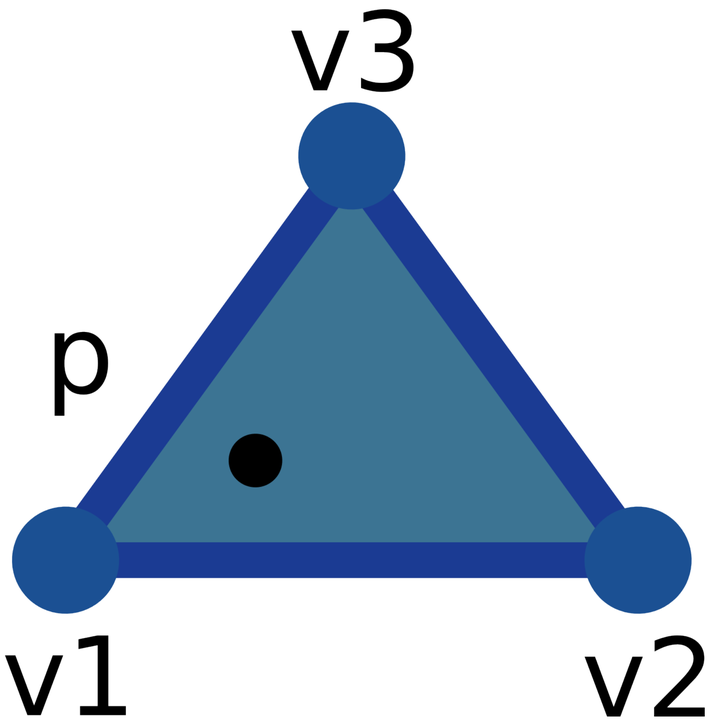
Piecewise linear interpolant on triangulations

- Interpolation as a boundary value problem
  - Given a simplex and its vertices



Piecewise linear interpolant on triangulations

- Interpolation as a boundary value problem
  - Given a simplex and its vertices
  - Express the value of any point



Piecewise linear interpolant on triangulations

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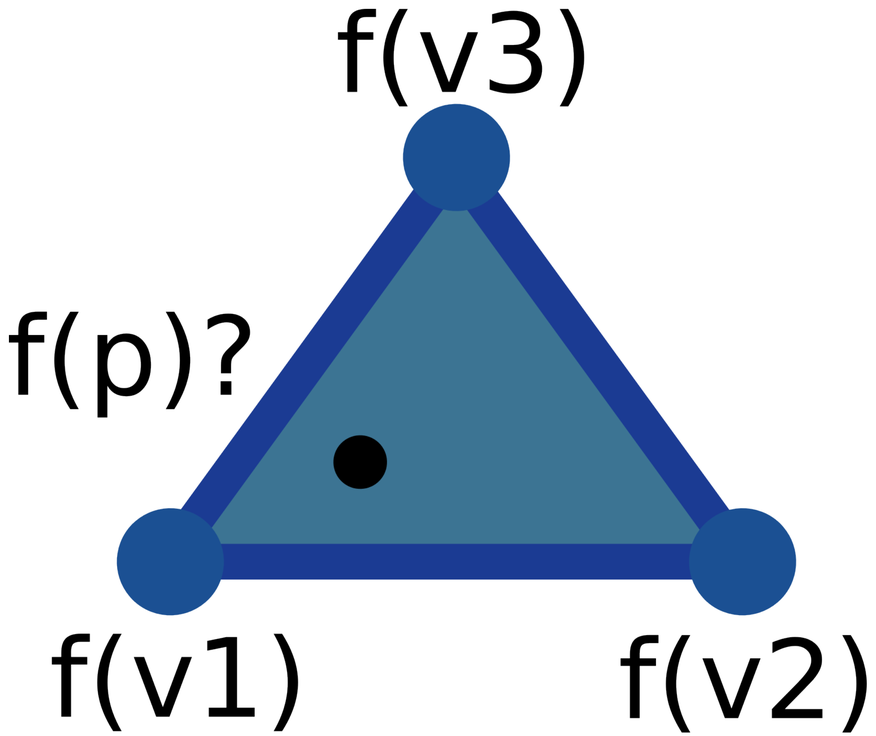
  - As a linear combination of those of the vertices



Piecewise linear interpolant on triangulations

- Interpolation as a boundary value problem
  - Given a simplex and its vertices
  - Express the value of any point
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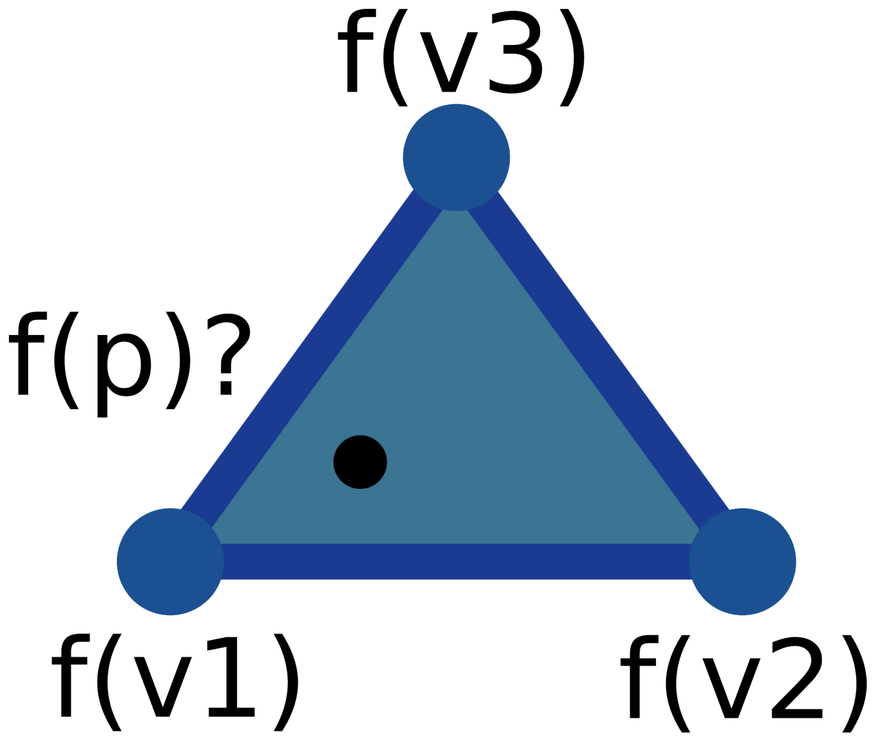
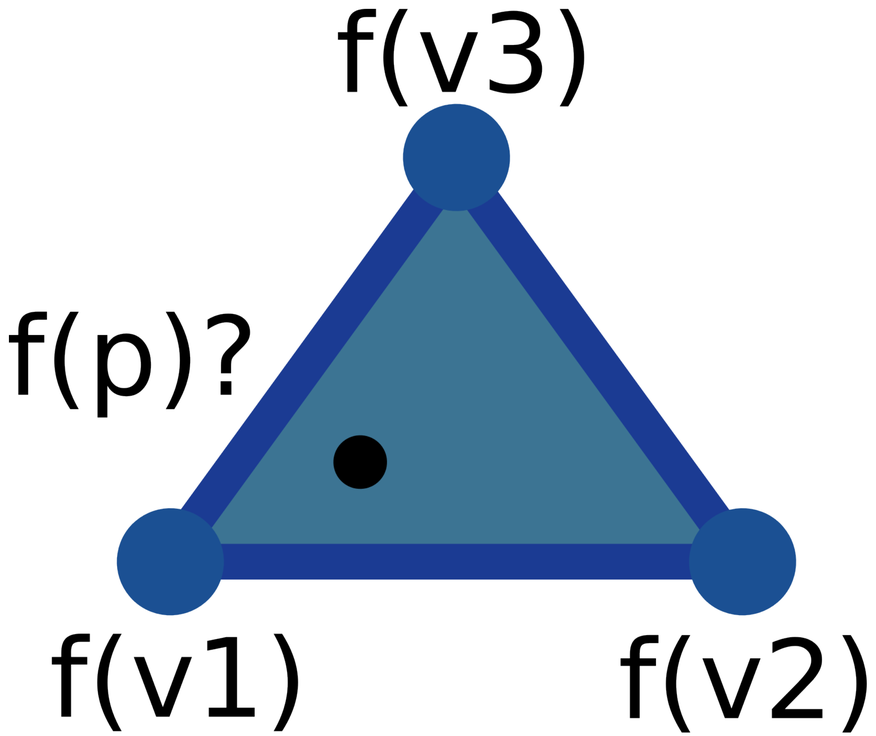
- Notion of barycentric coordinates:
  - $f(p) = \alpha_1 f(v_1) + \alpha_2 f(v_2) + \alpha_3 f(v_3)$



Barycentric coordinates

- There exists many forms
  - With specific properties



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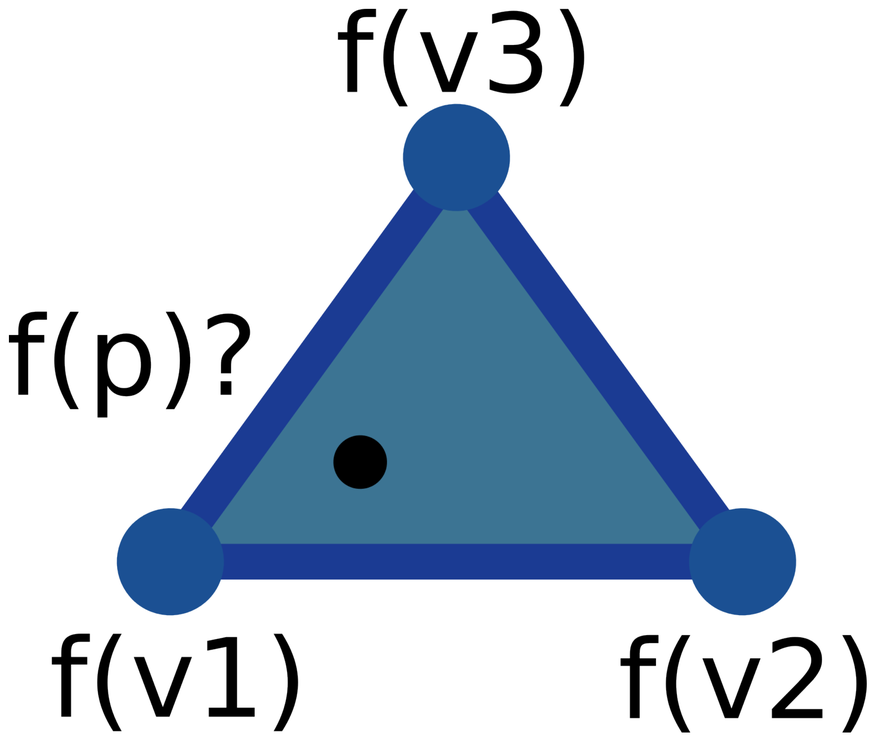

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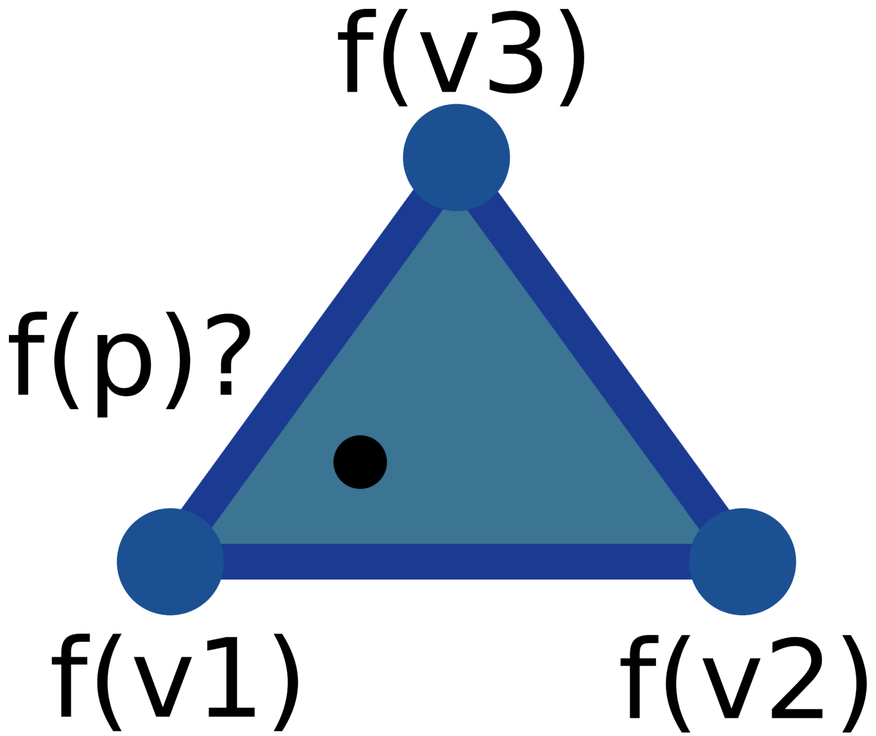
  - $\alpha_1 + \alpha_2 + \alpha_3 = 1$



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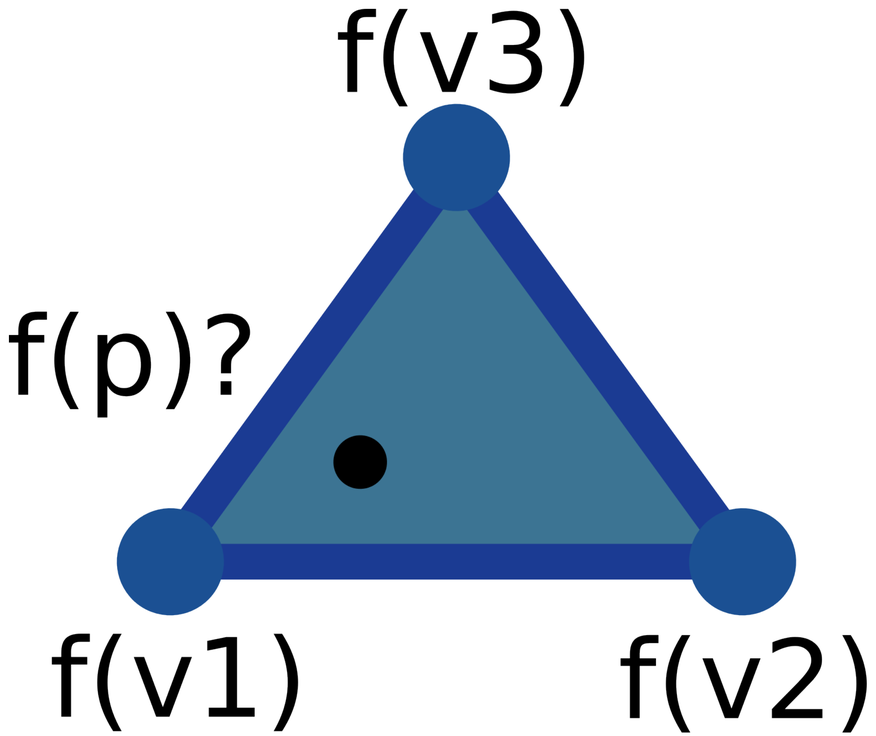

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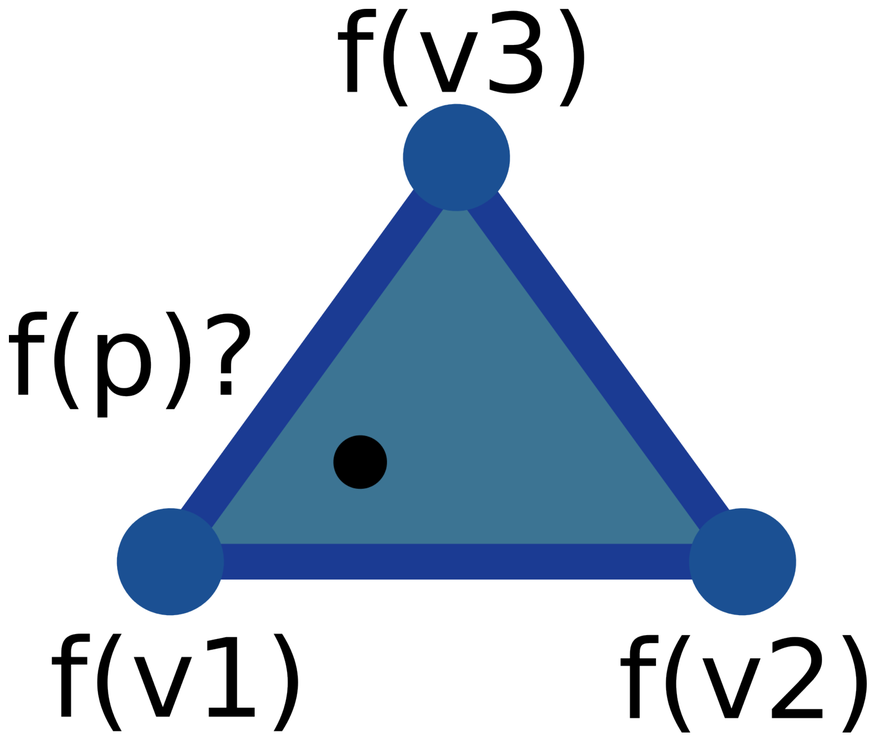
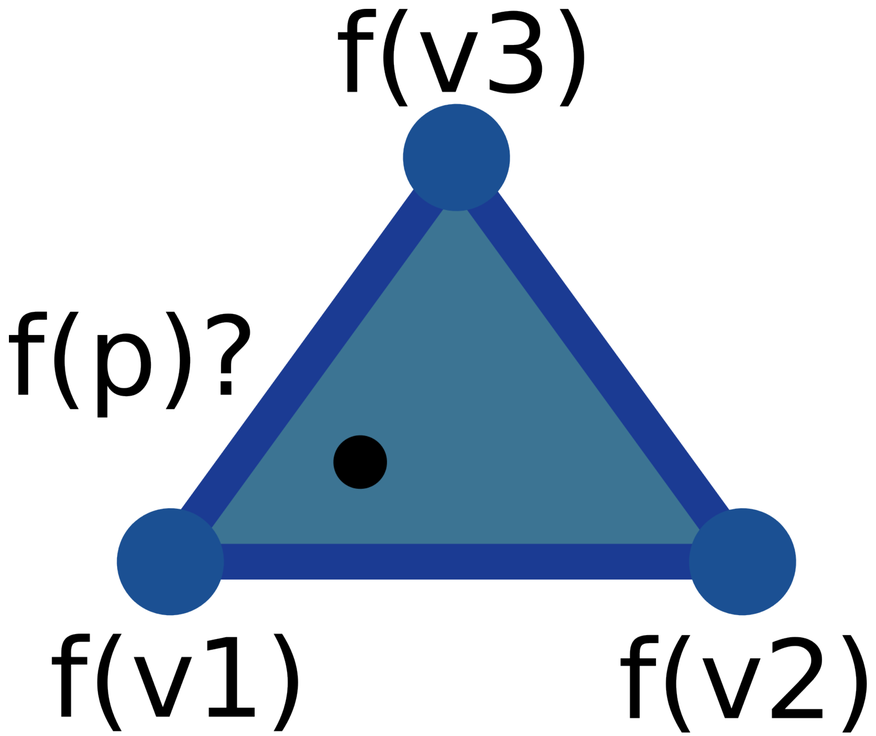
  - $f(p) = \alpha_1 f(v_1) + \alpha_2 f(v_2) + \alpha_3 f(v_3)$
  - Must hold for any $f : \mathcal{T} \to \mathbb{R}$



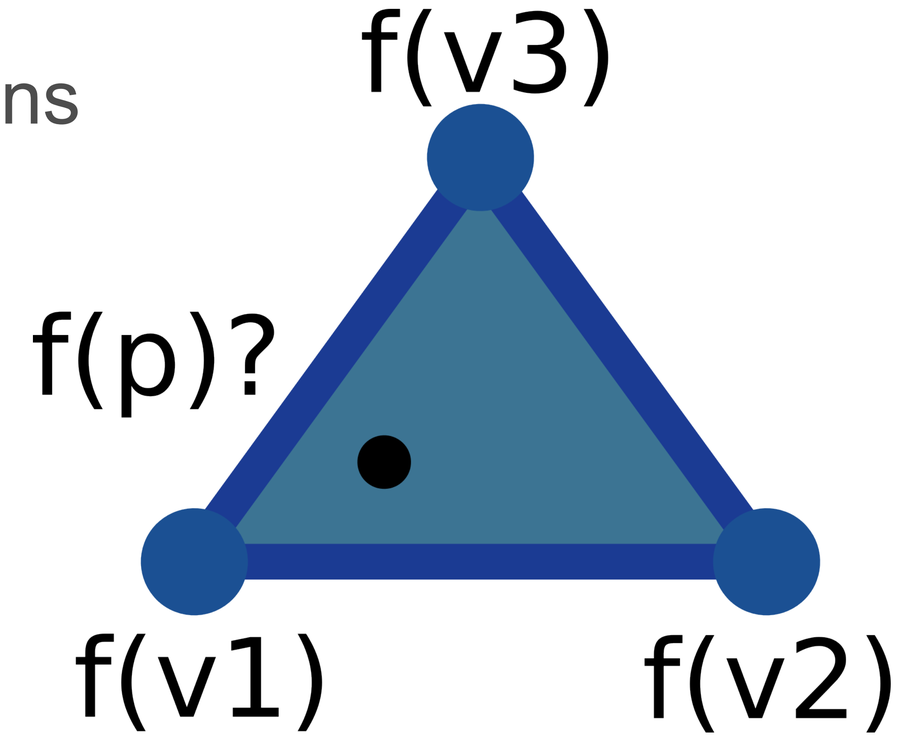
Barycentric coordinates

- In particular



Barycentric coordinates

- In particular
  - It must also hold for the embedding functions



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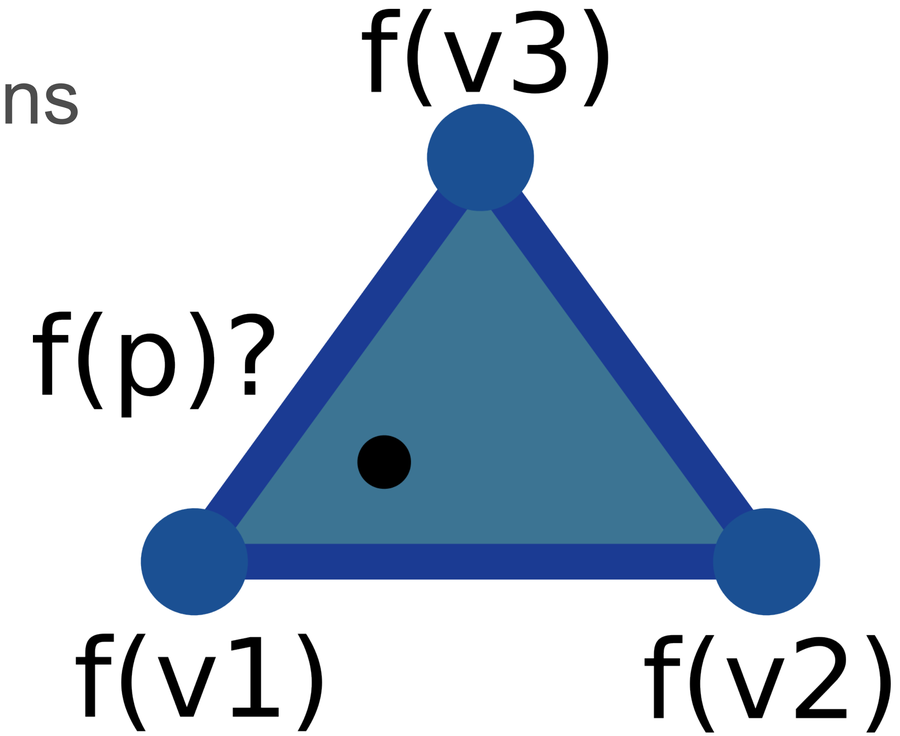

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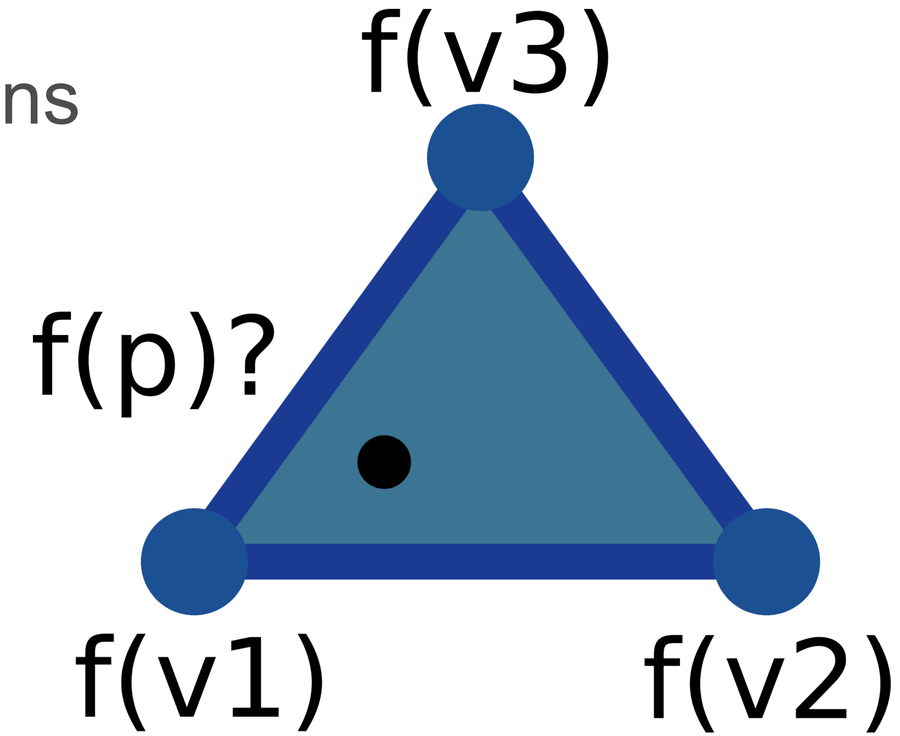
- 3 linear equations with 3 unknowns



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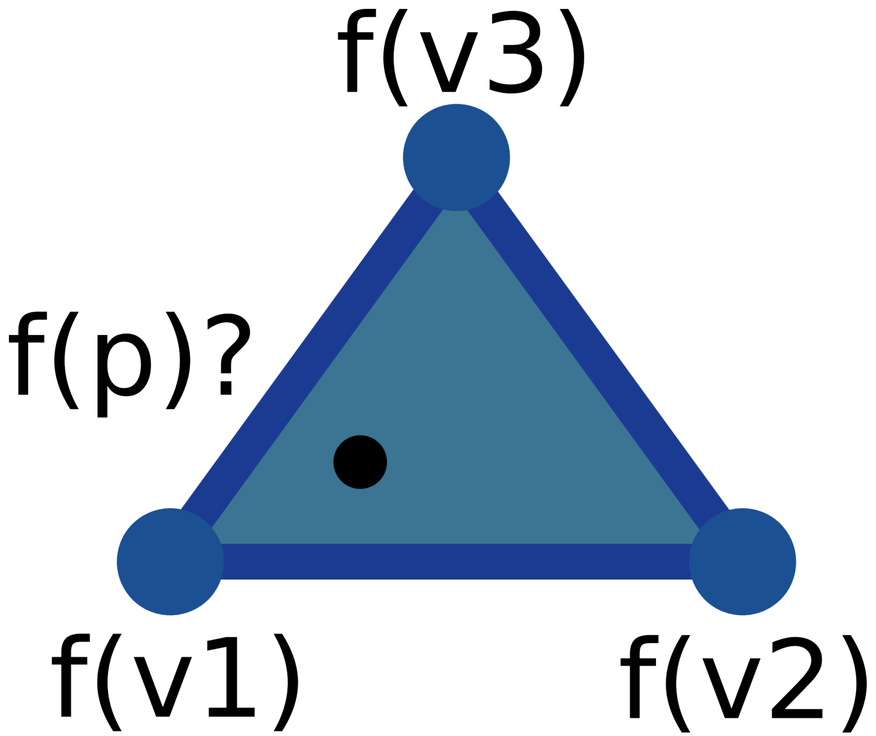

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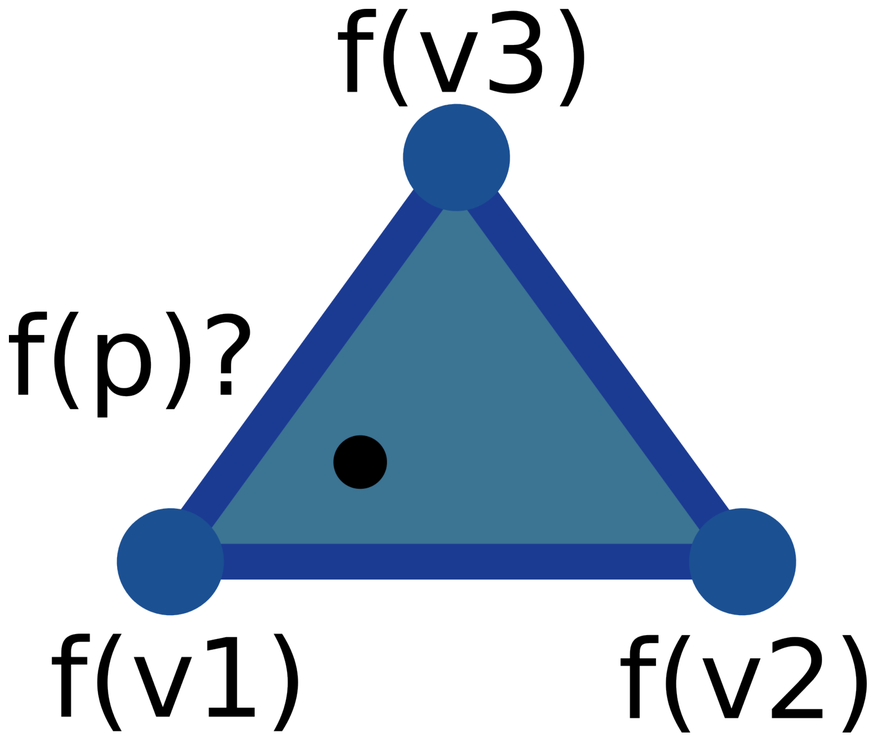
- Automatically interpolates on the edges



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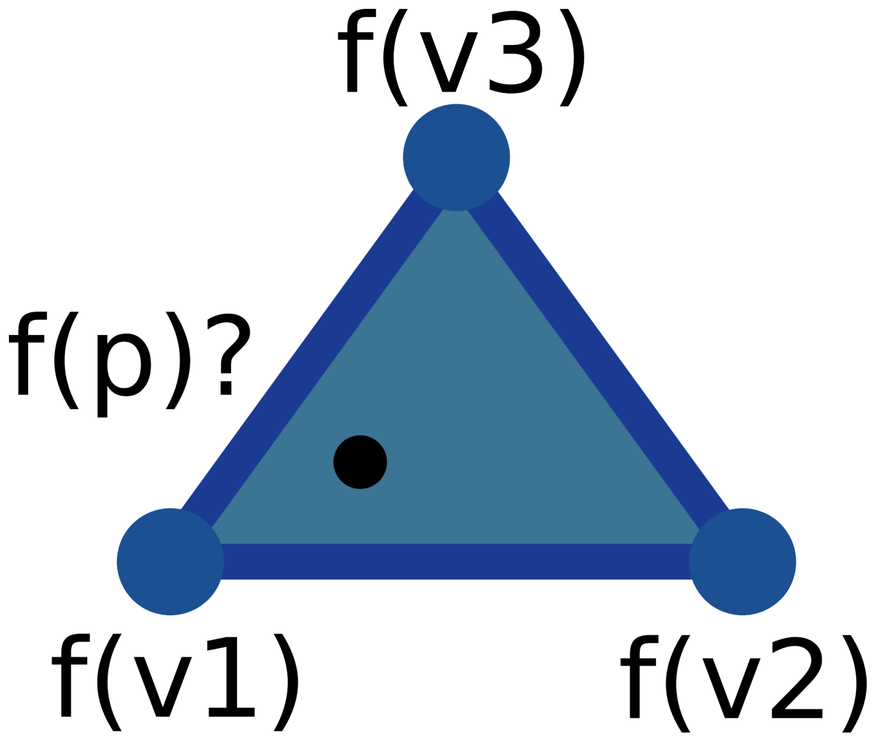
- Similar reasoning for d-simplex



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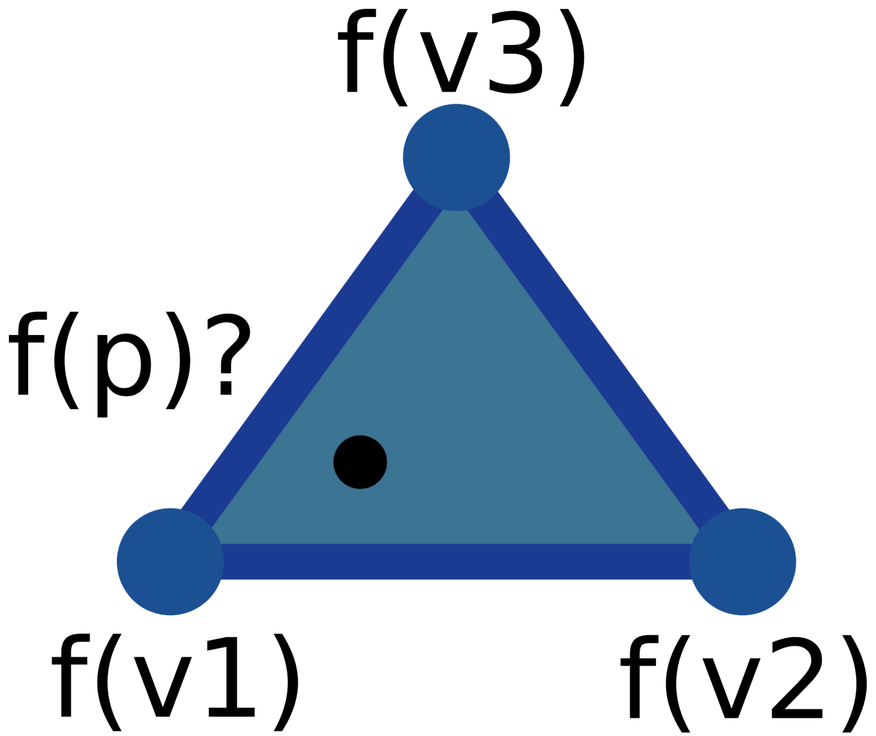

- Automatically interpolates on the edges
- Similar reasoning for d-simplex
- Can be used to determine if a point lies within a simplex

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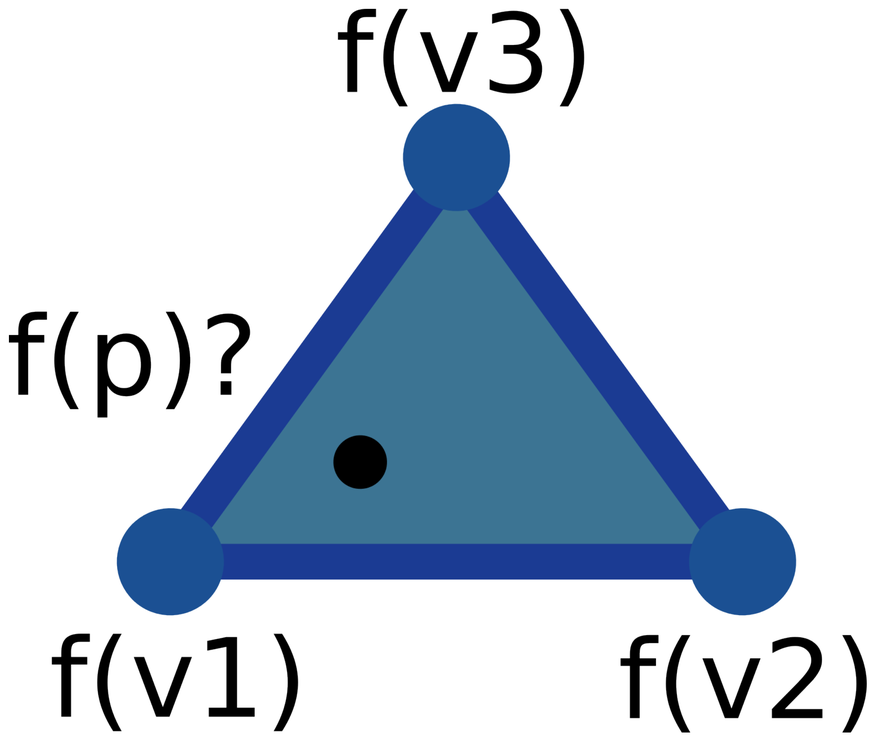

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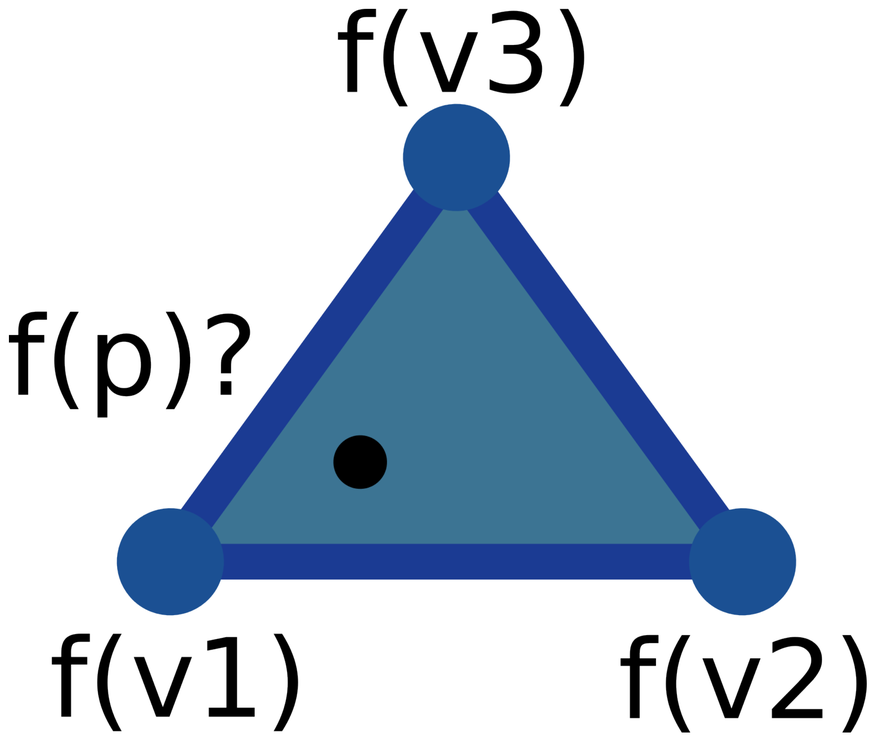
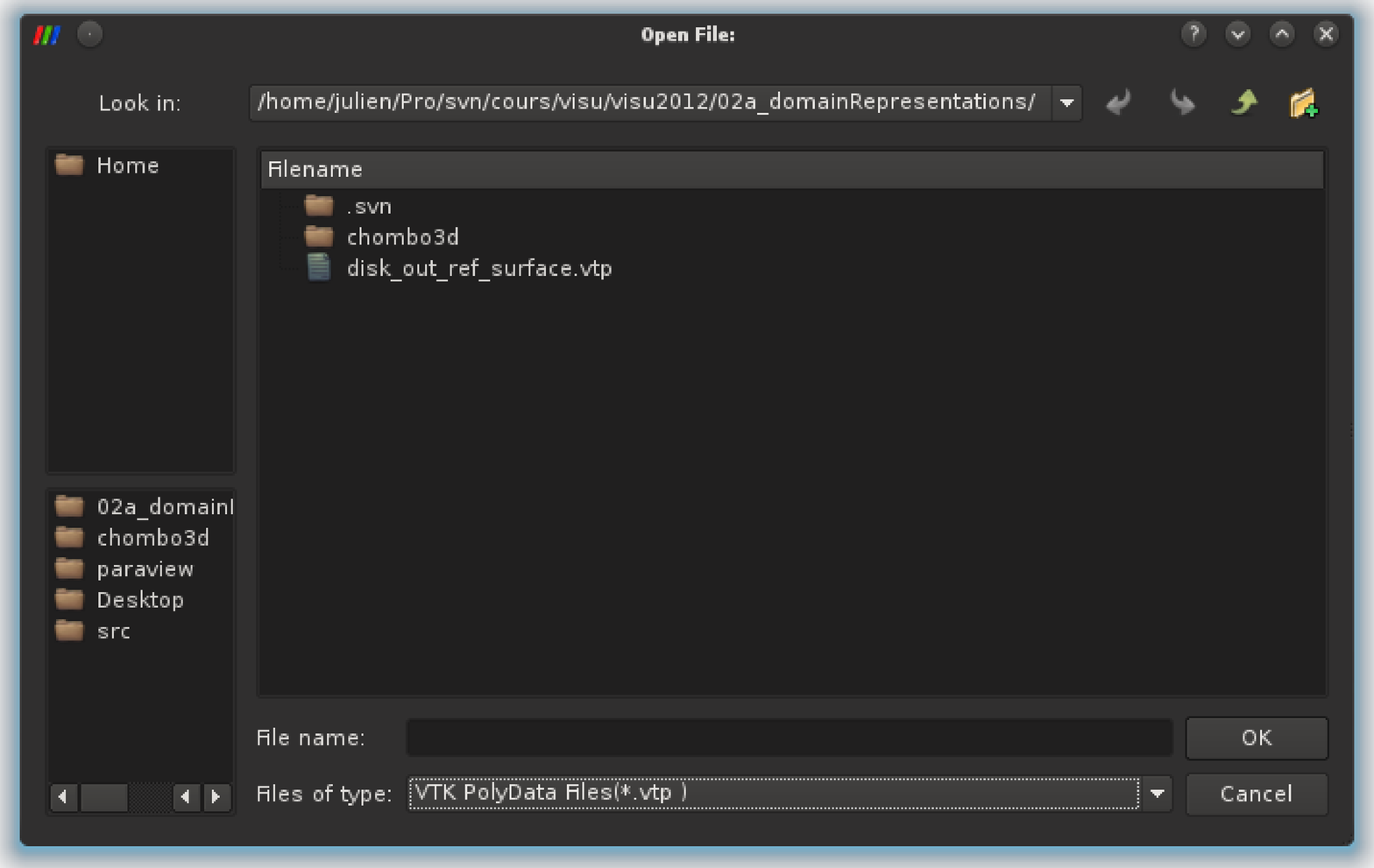


- Automatically interpolates on the edges
- Similar reasoning for d-simplex
- Can be used to determine if a point lies within a simplex $(\alpha_1, \alpha_2, \alpha_3) \in [0,1] \times [0,1] \times [0,1]$
- No critical point in the interior!

In practice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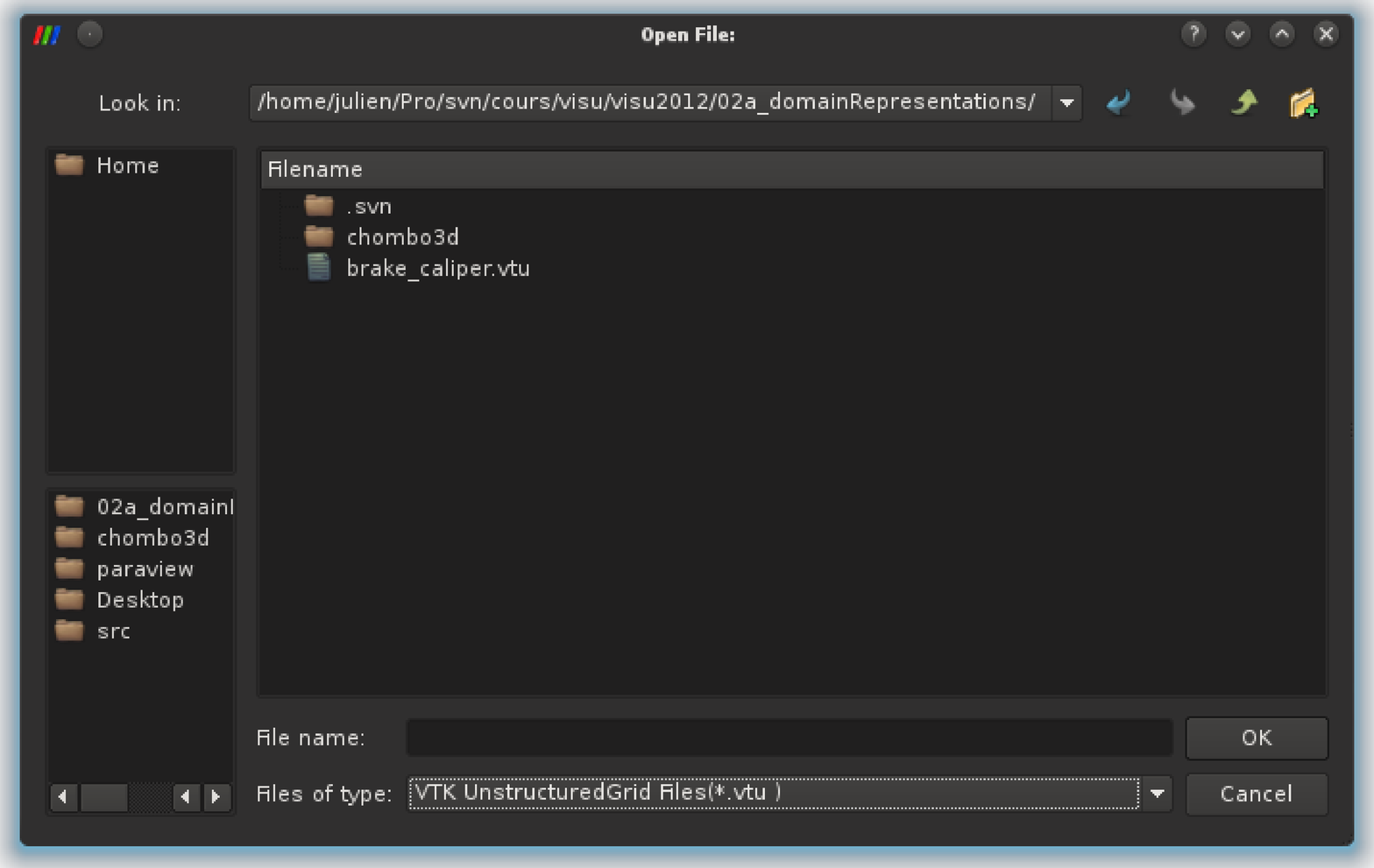
In practice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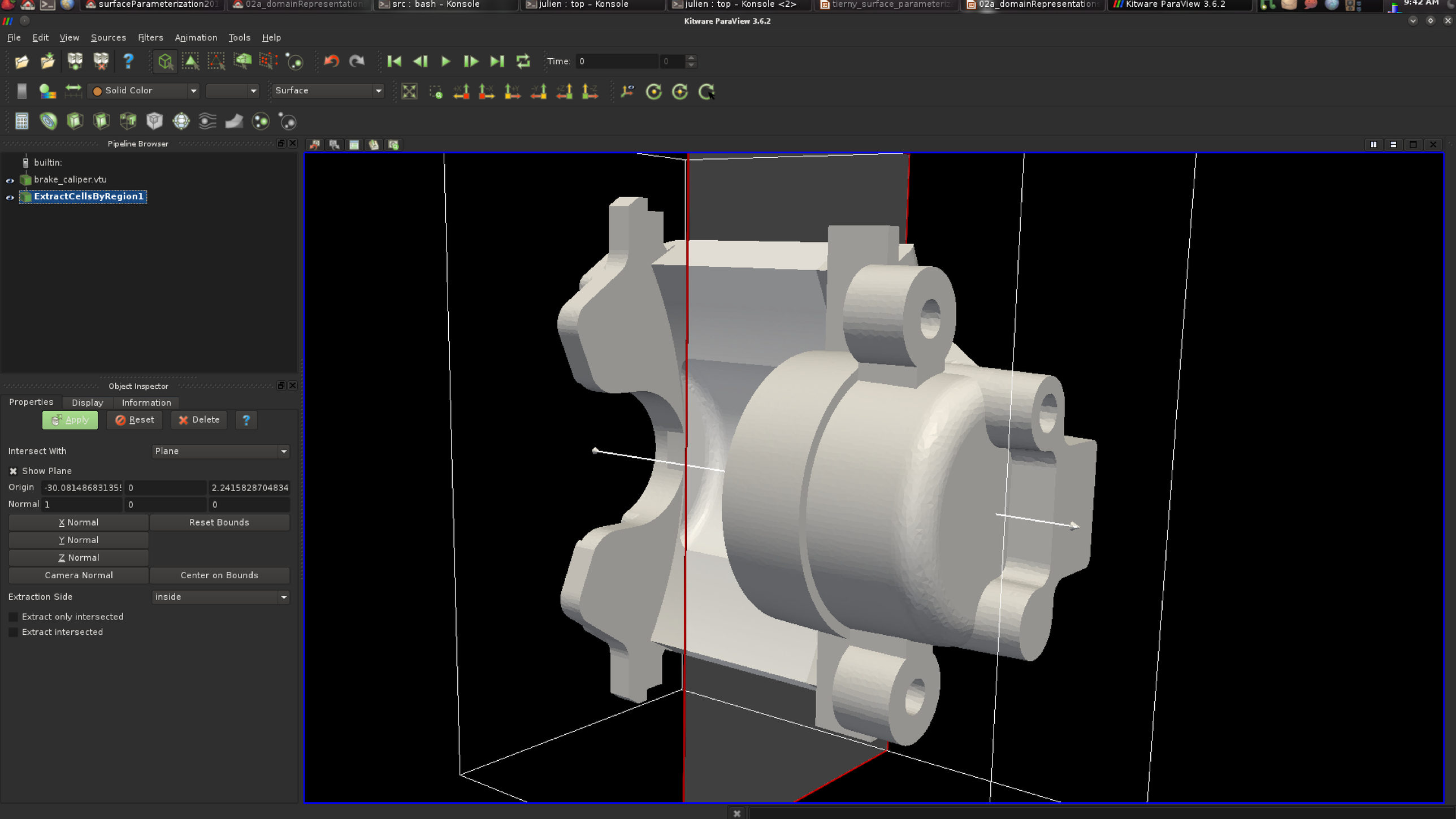
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Extract only intersected
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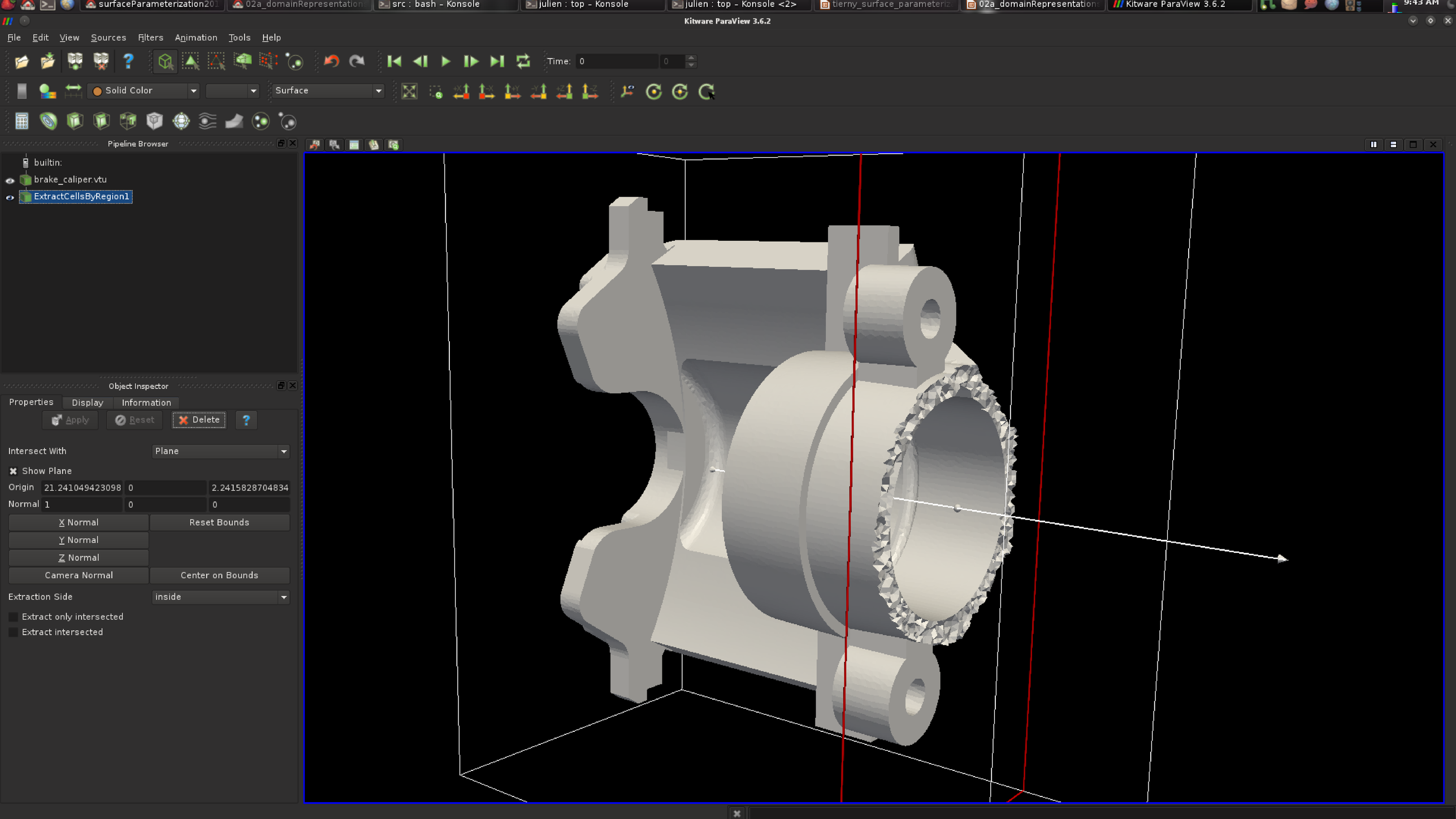
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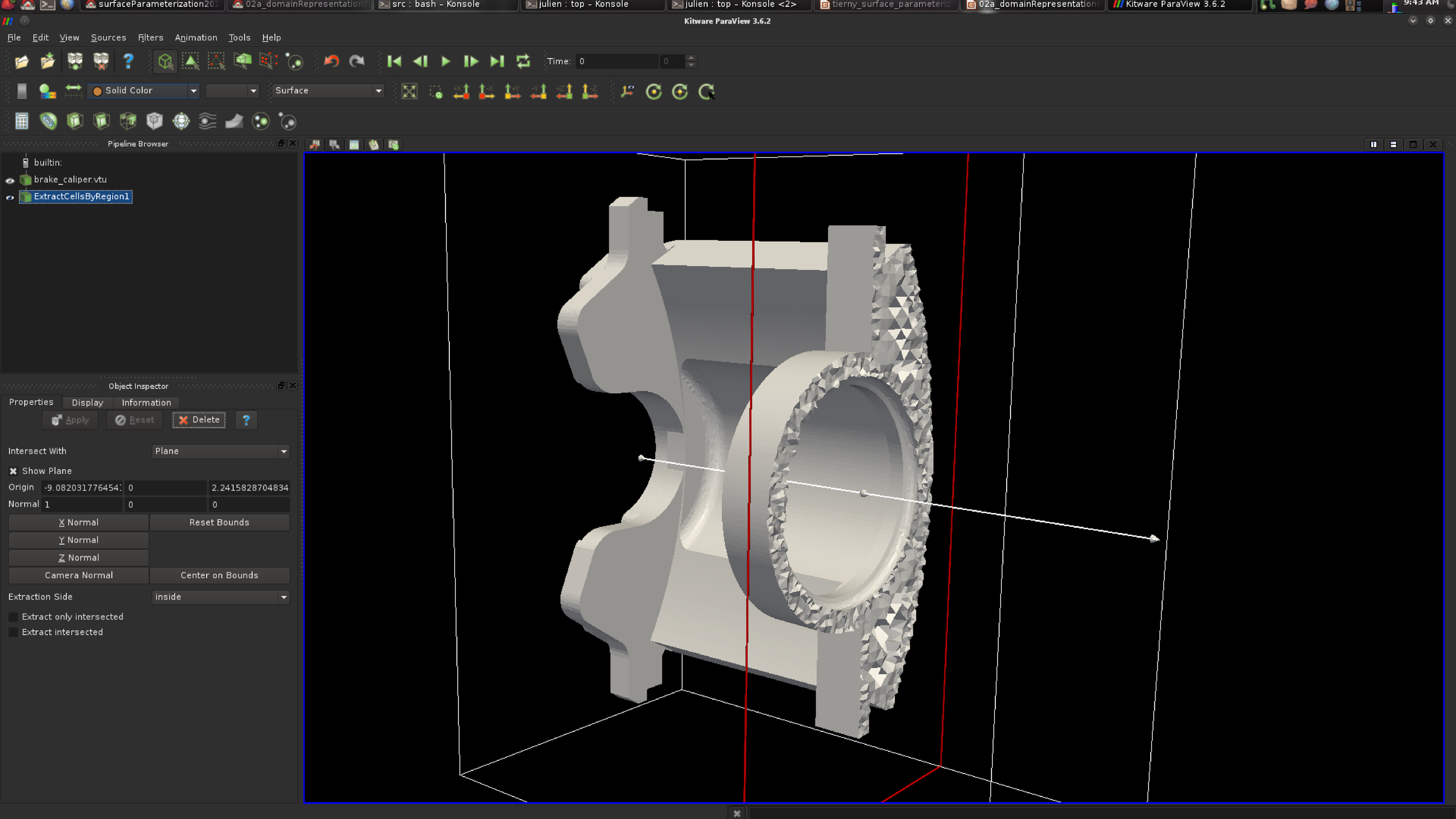
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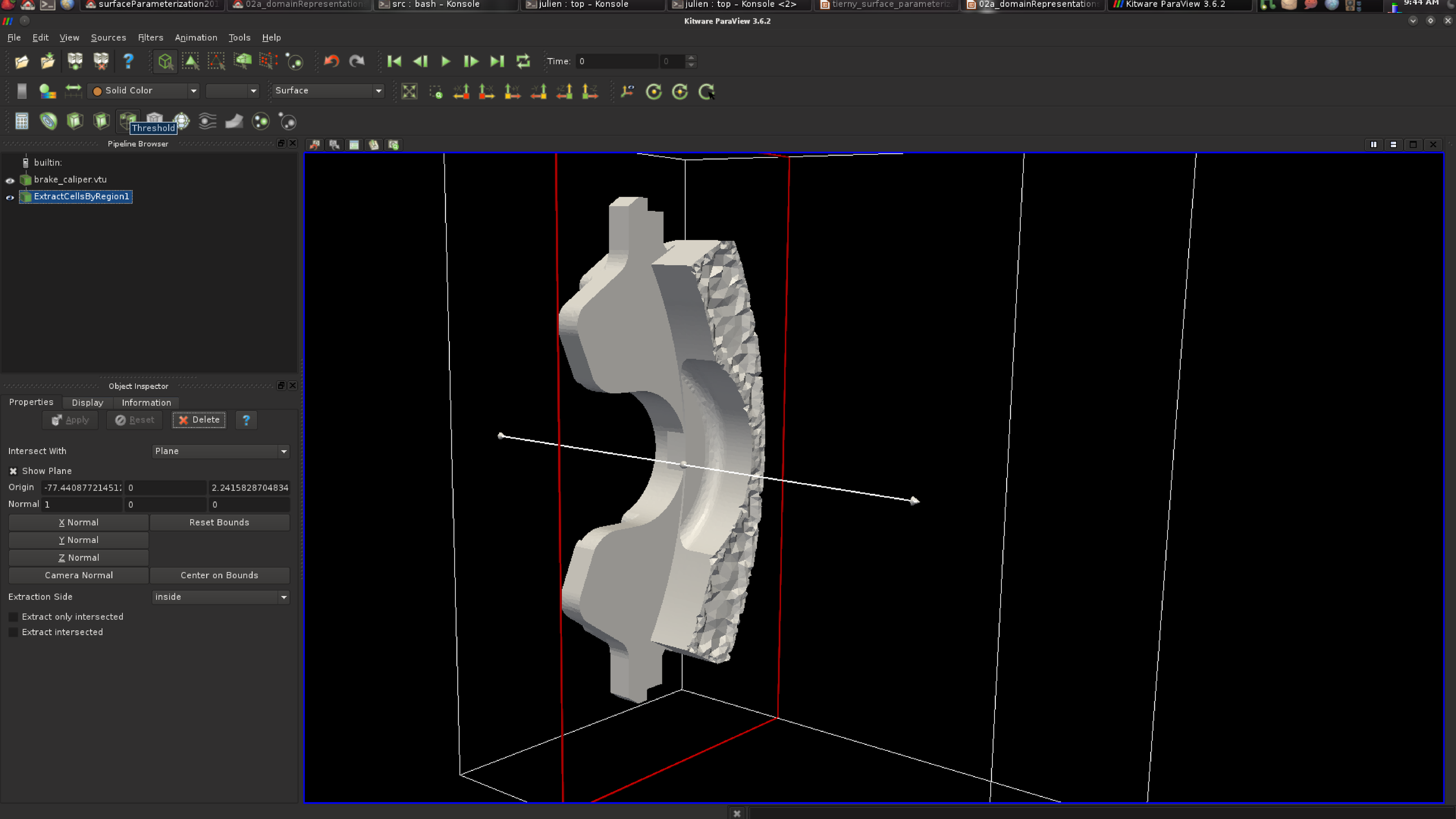
Solid Color
Surface
Pipeline Browser
builtin:
brake_caliper.vtu
ExtractCellsByRegion1
Object Inspector
Properties Display Information
Apply Reset Delete
Intersect With Plane
Show Plane
Origin -9.082031776454 0 2.2415828704834
Normal 1 0 0
X Normal
Reset Bounds
Y Normal
Z Normal
Camera Normal
Center on Bounds
Extraction Side inside
Extract only intersected
Extract intersected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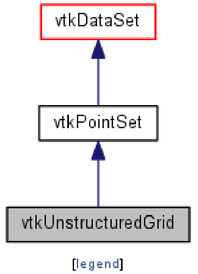
VTK

vtkUnstructuredGrid Class Reference

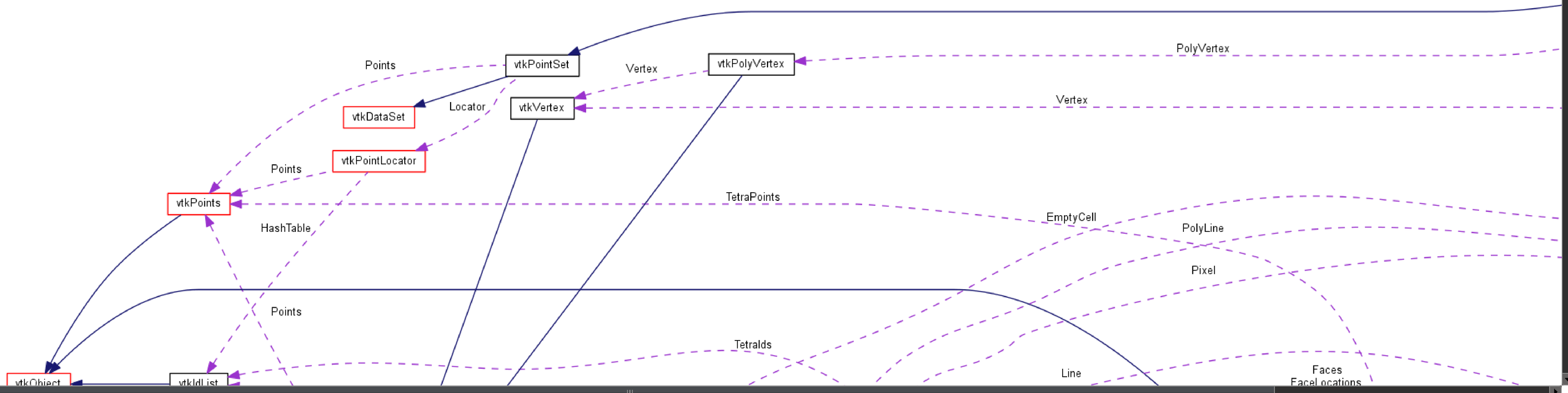
dataset represents arbitrary combinations of all possible cell types More...

#include <vtkUnstructuredGrid.h>

Inheritance diagram for vtkUnstructuredGrid:



Collaboration diagram for vtkUnstructuredGrid:



Public Member Functions

virtual **int** **IsA** (const char *type)
vtkUnstructuredGrid * **NewInstance** () const
void **PrintSelf** (ostream &os, **vtkIndent** indent)
vtkIdType **InsertNextCell** (**int** type, **vtkIdType** npts, **vtkIdType** *ptIds)
vtkIdType **InsertNextCell** (**int** type, **vtkIdList** *ptIds)
vtkIdType **InsertNextCell** (**int** type, **vtkIdType** npts, **vtkIdType** *ptIds, **vtkIdType** nfaces, **vtkIdType** *faces)
int **GetCellType** (**vtkIdType** cellId)
vtkUnsignedCharArray * **GetCellTypesArray** ()
vtkIdTypeArray * **GetCellLocationsArray** ()
void **Squeeze** ()
void **Initialize** ()
int **GetMaxCellSize** ()
void **BuildLinks** ()
vtkCellLinks * **GetCellLinks** ()
virtual void **GetCellPoints** (**vtkIdType** cellId, **vtkIdType** &npts, **vtkIdType** *&pts)
void **GetFaceStream** (**vtkIdType** cellId, **vtkIdList** *ptIds)
void **GetFaceStream** (**vtkIdType** cellId, **vtkIdType** &nfaces, **vtkIdType** *&ptIds)
vtkCellArray * **GetCells** ()
void **ReplaceCell** (**vtkIdType** cellId, **int** npts, **vtkIdType** *pts)
vtkIdType **InsertNextLinkedCell** (**int** type, **int** npts, **vtkIdType** *pts)
void **RemoveReferenceToCell** (**vtkIdType** ptId, **vtkIdType** cellId)
void **AddReferenceToCell** (**vtkIdType** ptId, **vtkIdType** cellId)
void **ResizeCellList** (**vtkIdType** ptId, **int** size)
virtual **int** **GetGhostLevel** ()
unsigned long **GetActualMemorySize** ()
void **GetIdsOfCellsOfType** (**int** type, **vtkIdTypeArray** *array)
int **IsHomogeneous** ()
void **RemoveGhostCells** (**int** level)
vtkIdType * **GetFaces** (**vtkIdType** cellId)
int **InitializeFacesRepresentation** (**vtkIdType** numPrevCells)

int **GetDataObjectType** ()
virtual void **Allocate** (**vtkIdType** numCells=1000, **int** extSize=1000)

void **Reset** ()
virtual void **CopyStructure** (**vtkDataSet** *ds)
vtkIdType **GetNumberOfCells** ()
virtual **vtkCell** * **GetCell** (**vtkIdType** cellId)
virtual void **GetCell** (**vtkIdType** cellId, **vtkGenericCell** *cell)
virtual void **GetCellBounds** (**vtkIdType** cellId, **double** bounds[6])
virtual void **GetCellPoints** (**vtkIdType** cellId, **vtkIdList** *ptIds)
void **GetPointCells** (**vtkIdType** ptId, **vtkIdList** *cellIds)

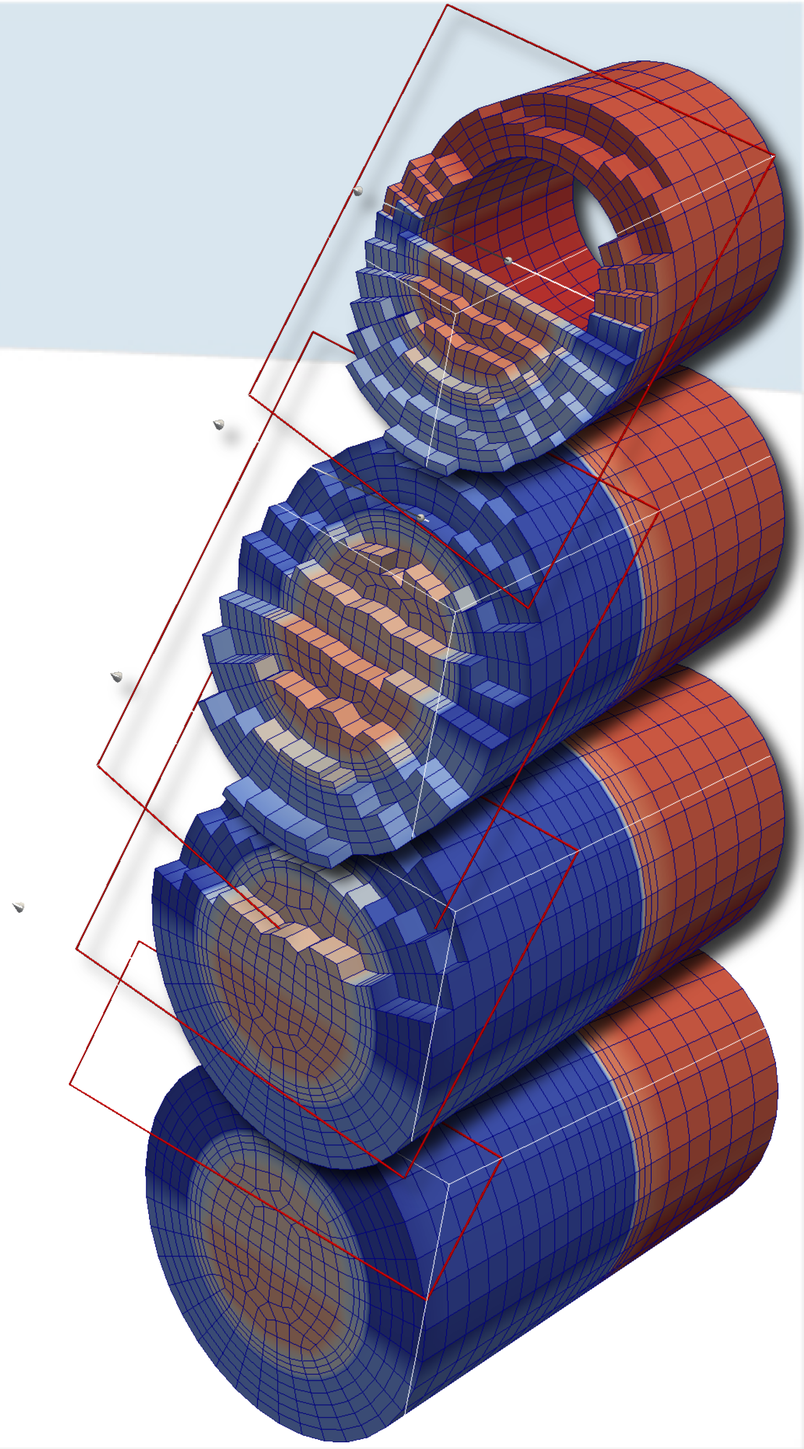
void **SetCells** (**int** type, **vtkCellArray** *cells)

Are we done?

- Nearly

Are we done?

- Nearly
  - What about these objects?



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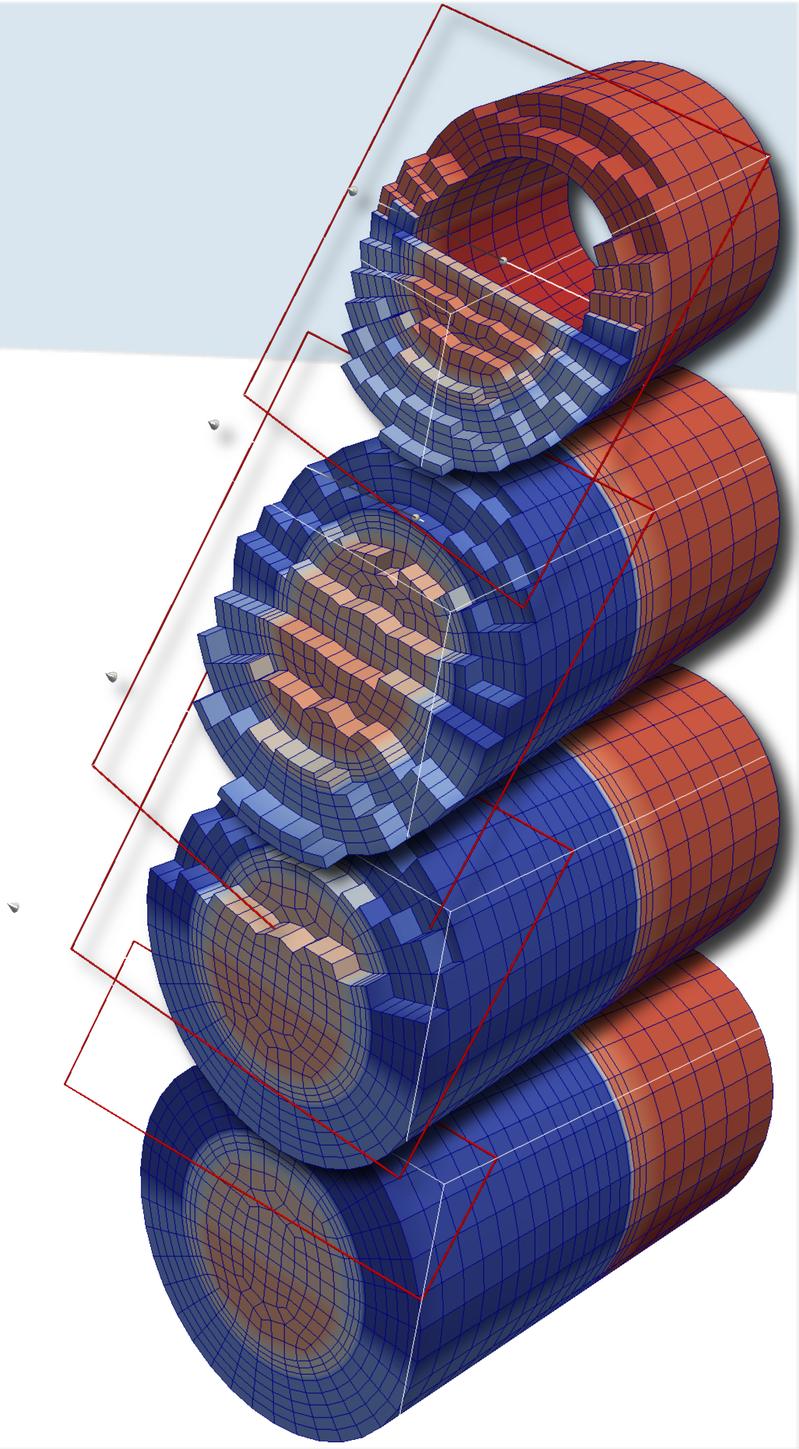
- Other manifold representations



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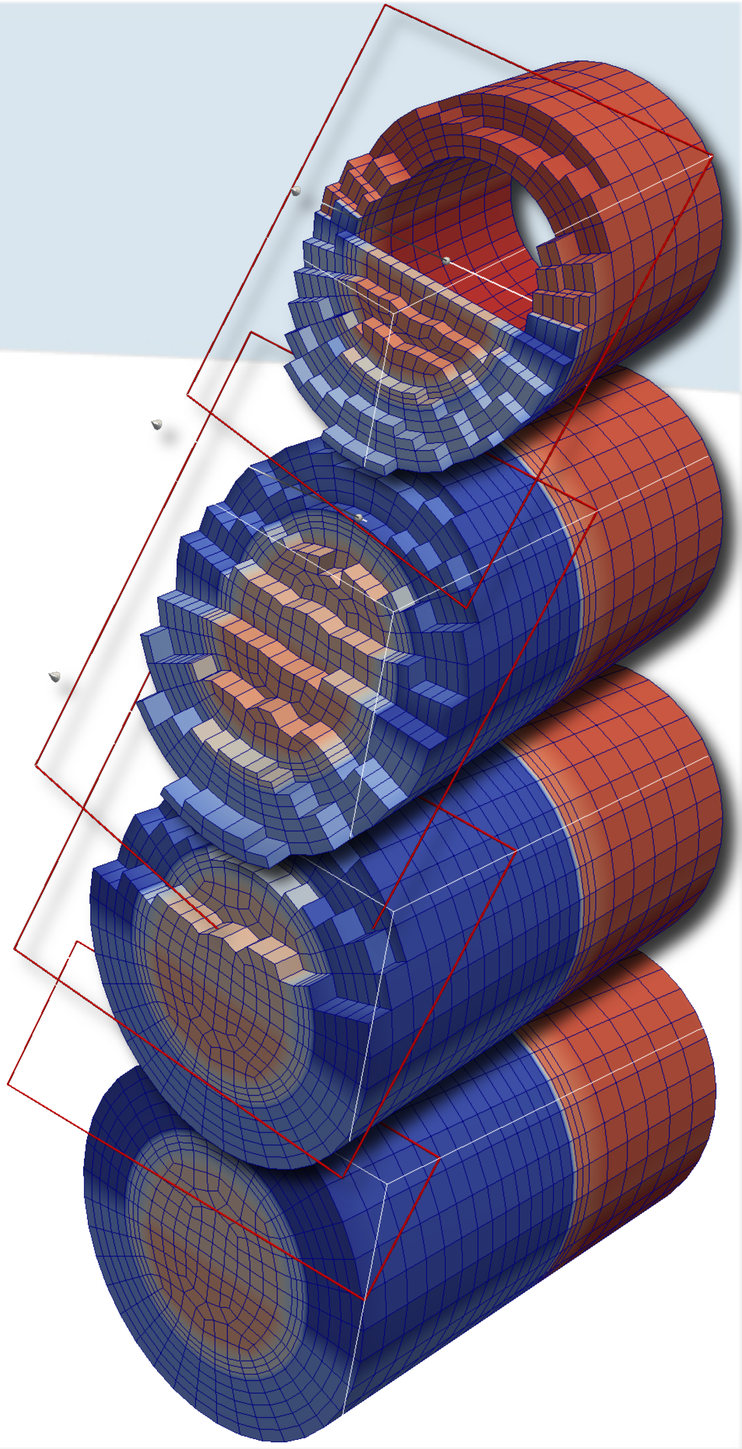
- Other manifold representations
  - Closure-finite Weak-topology (CW) complex



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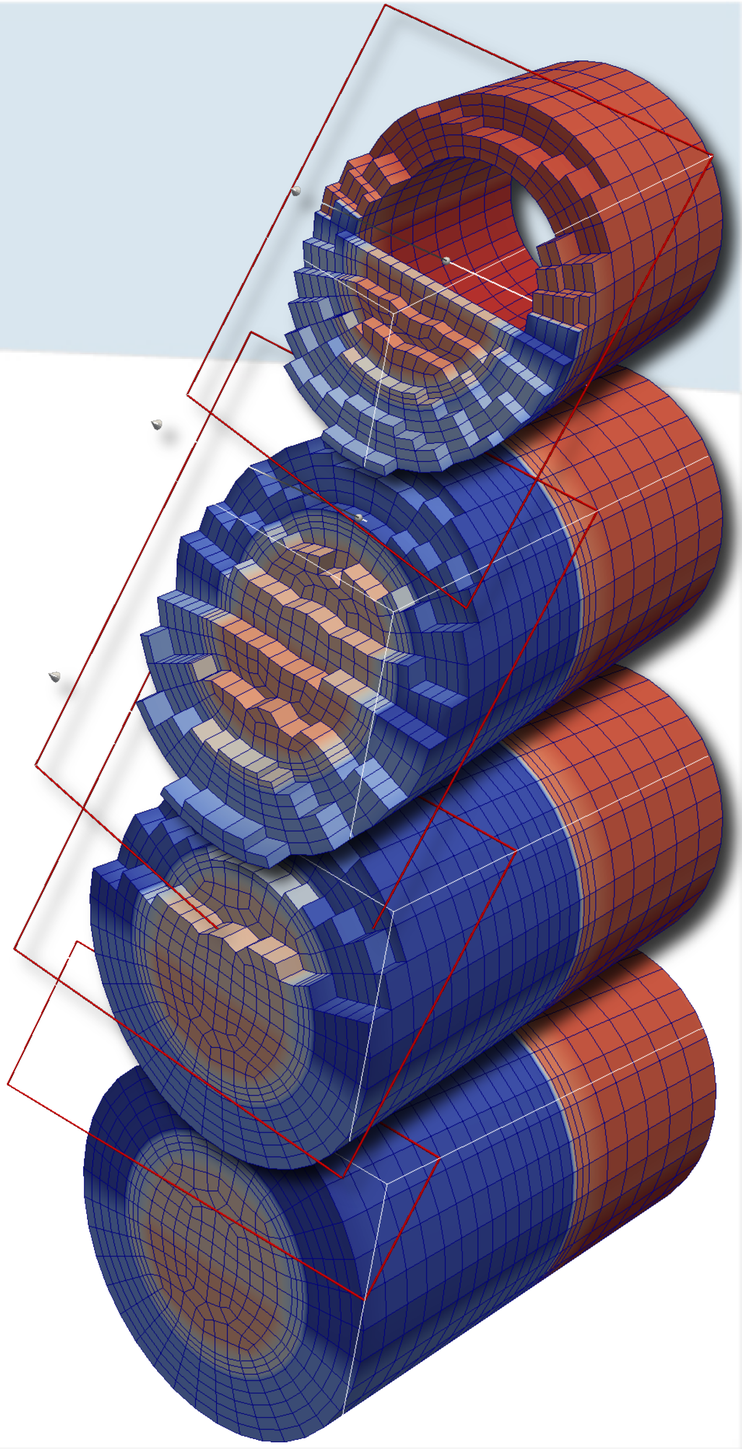
    - Arbitrary number of samples per cell



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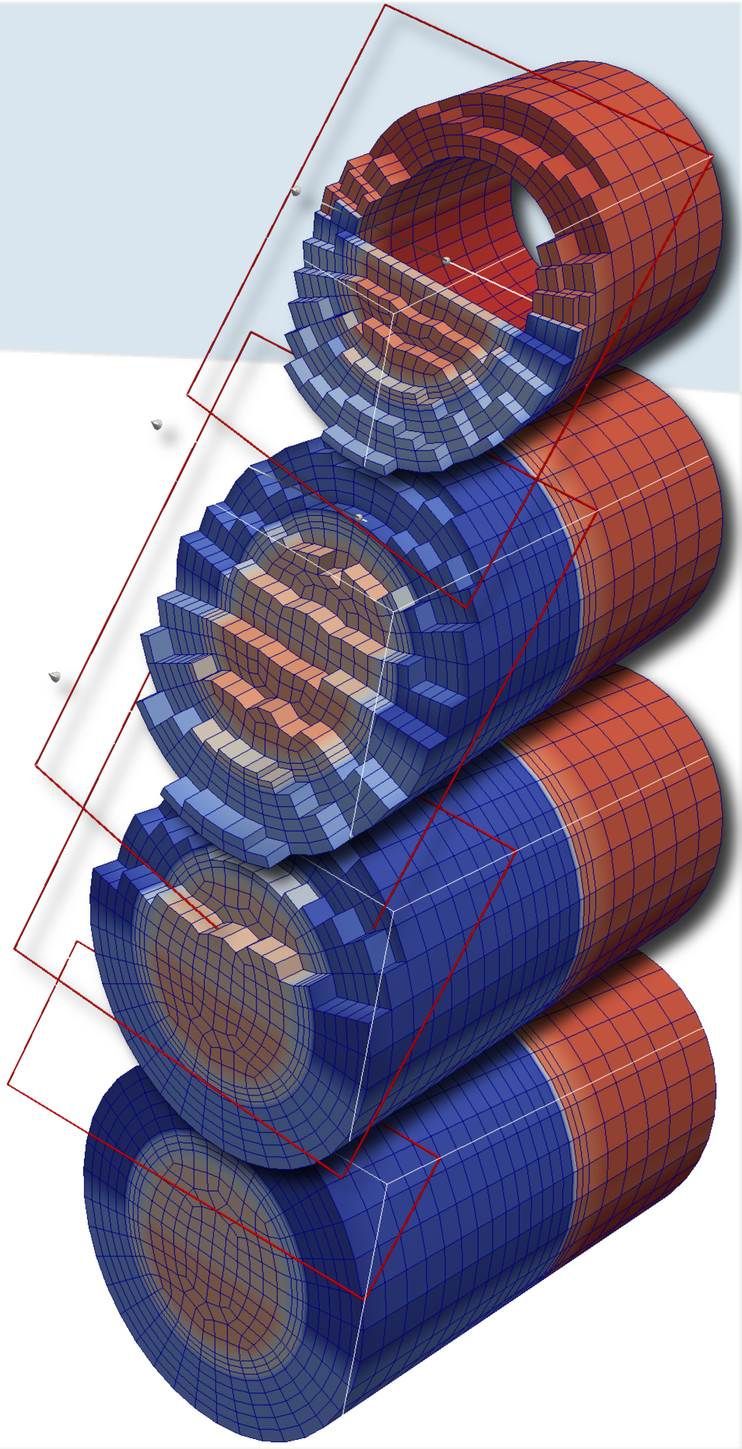
    - Similar implementation



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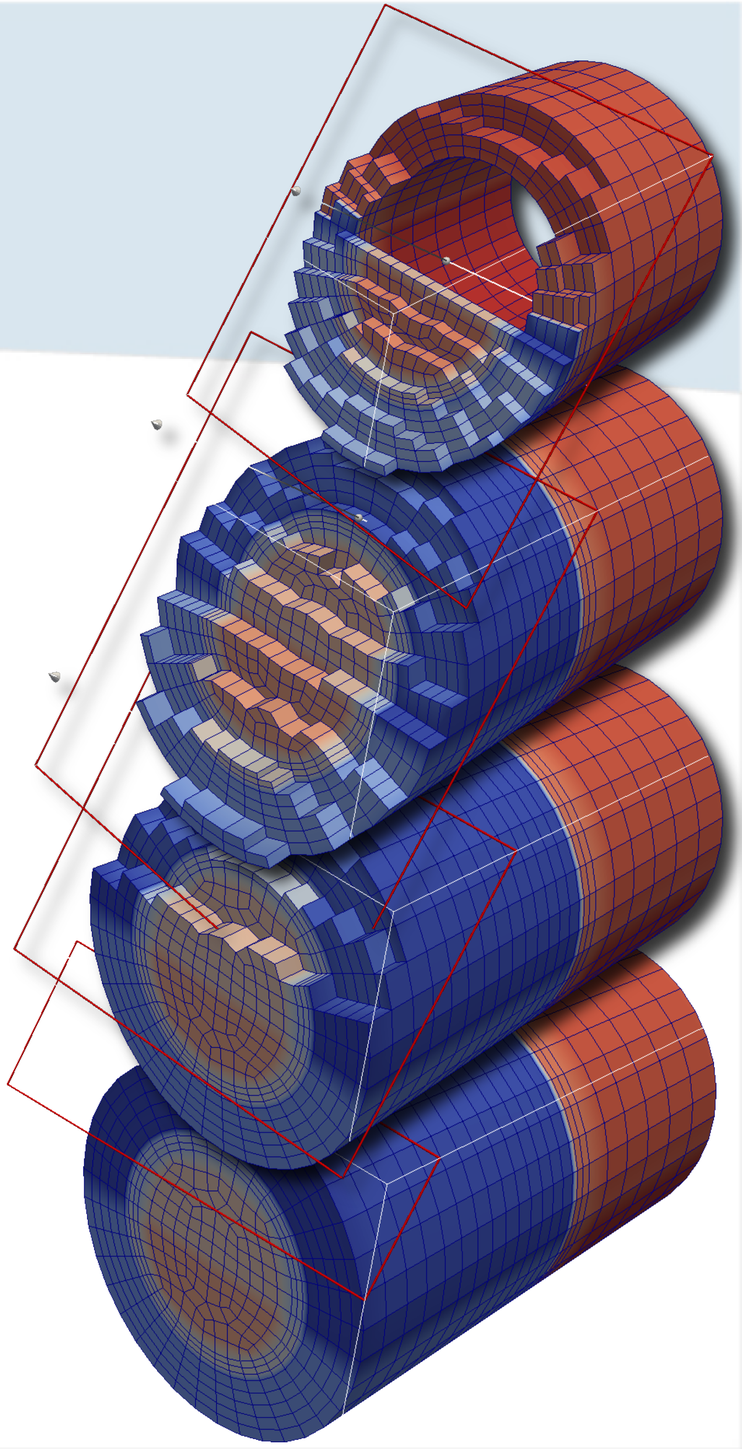

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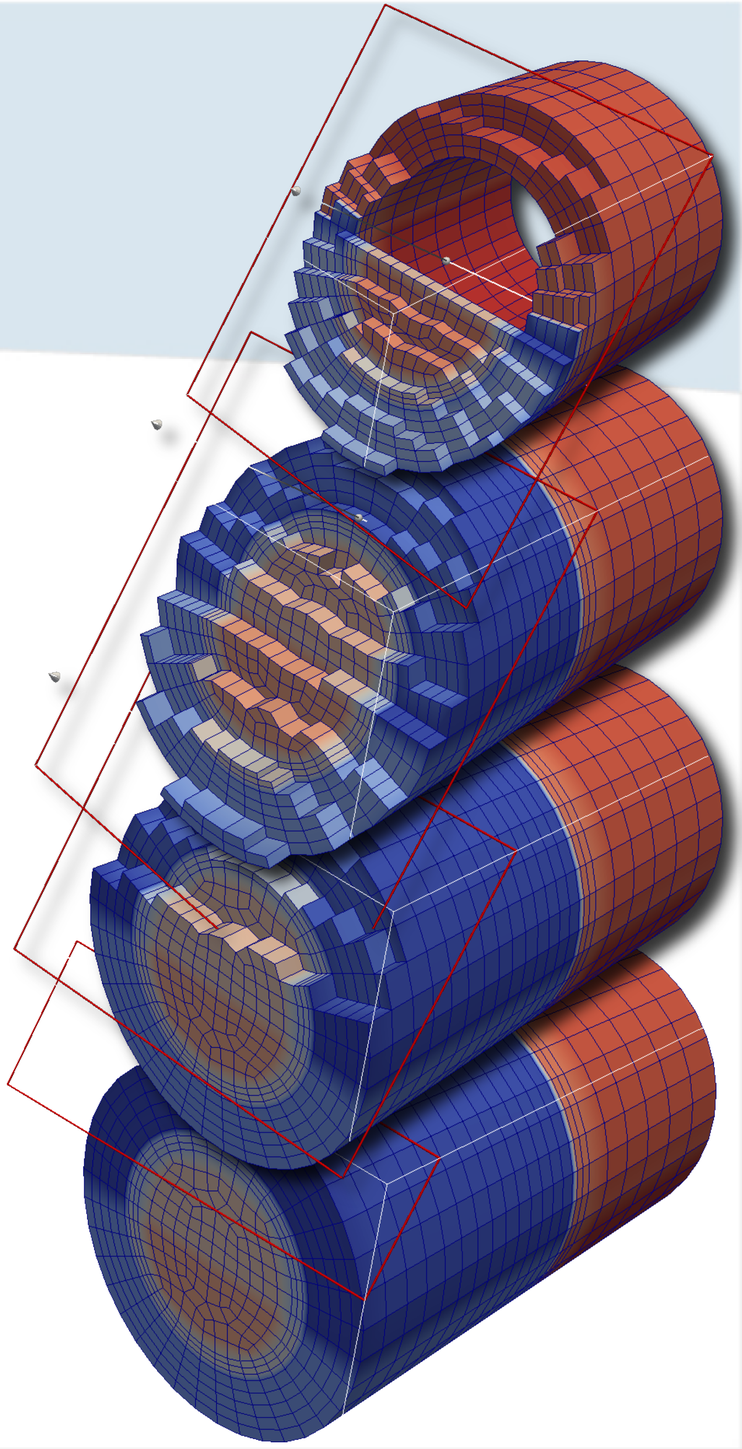
    - Special cases with 2^d samples per cell



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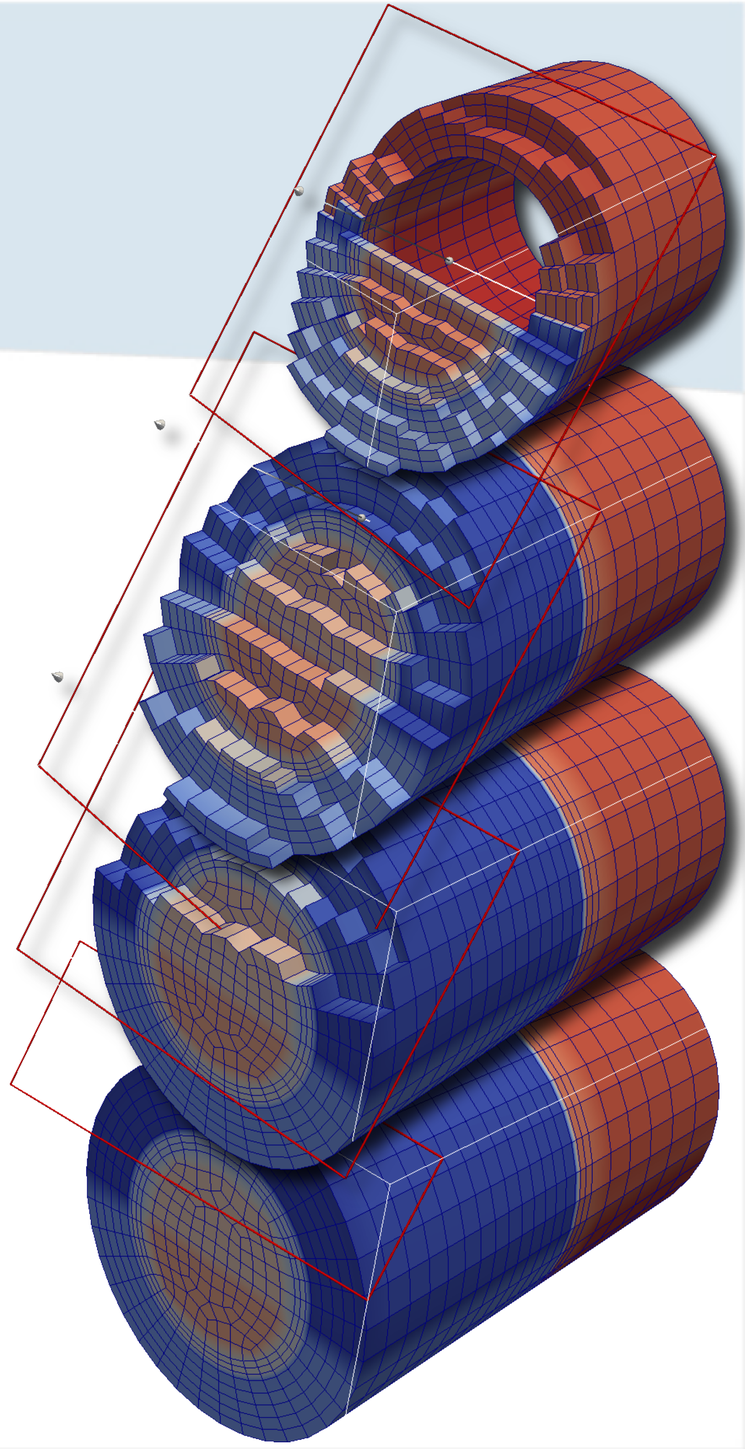
    - Similar implementation (vtkUnstructuredGrid)
    - Special cases with 2^d samples per cell
      - Quad mesh
      - Hexahedral mesh



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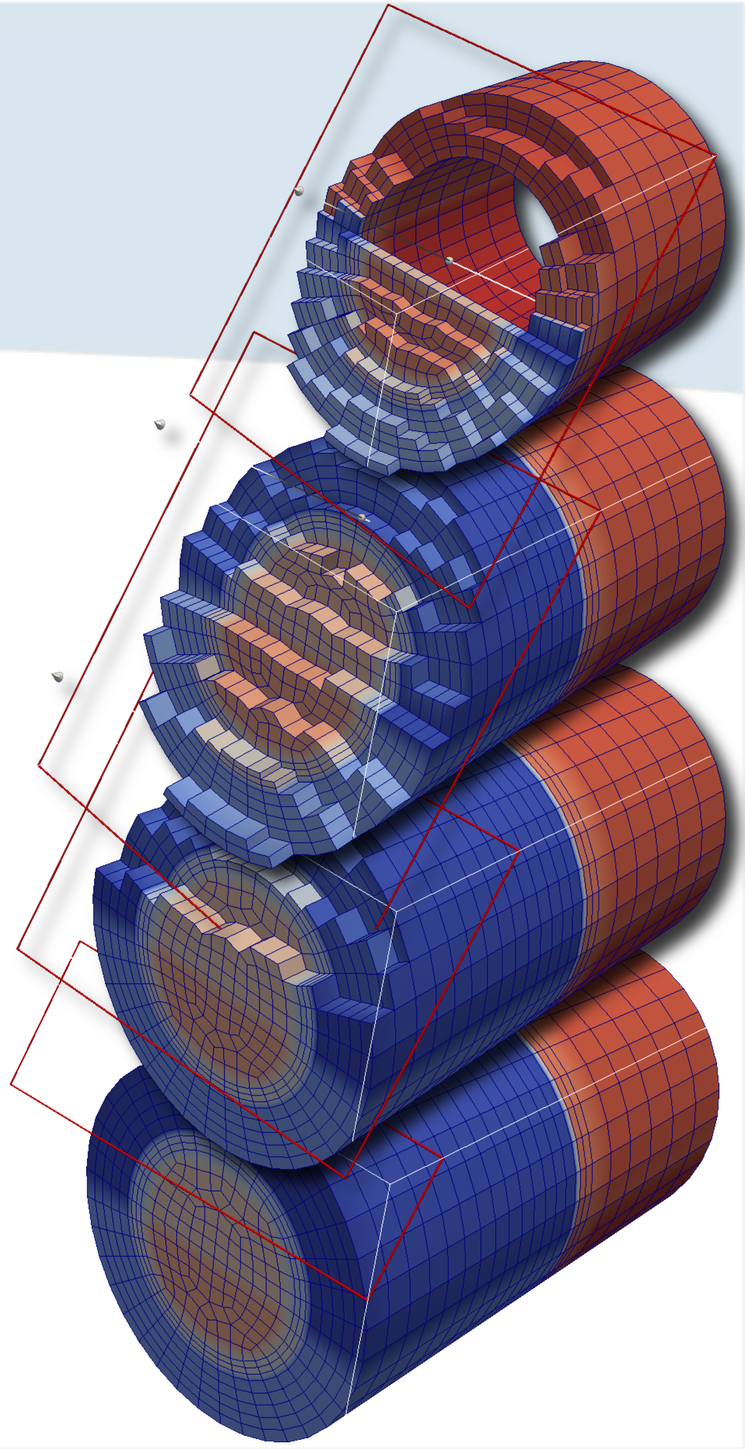
      - Quad mesh
      - Hexahedral mesh … intriguing properties



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- Now you know
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  - How to implement representations for euclidean spaces
  - How to implement representations for manifold spaces
  - How to interpolate numerical values all over these domains
  - Example of available implementations