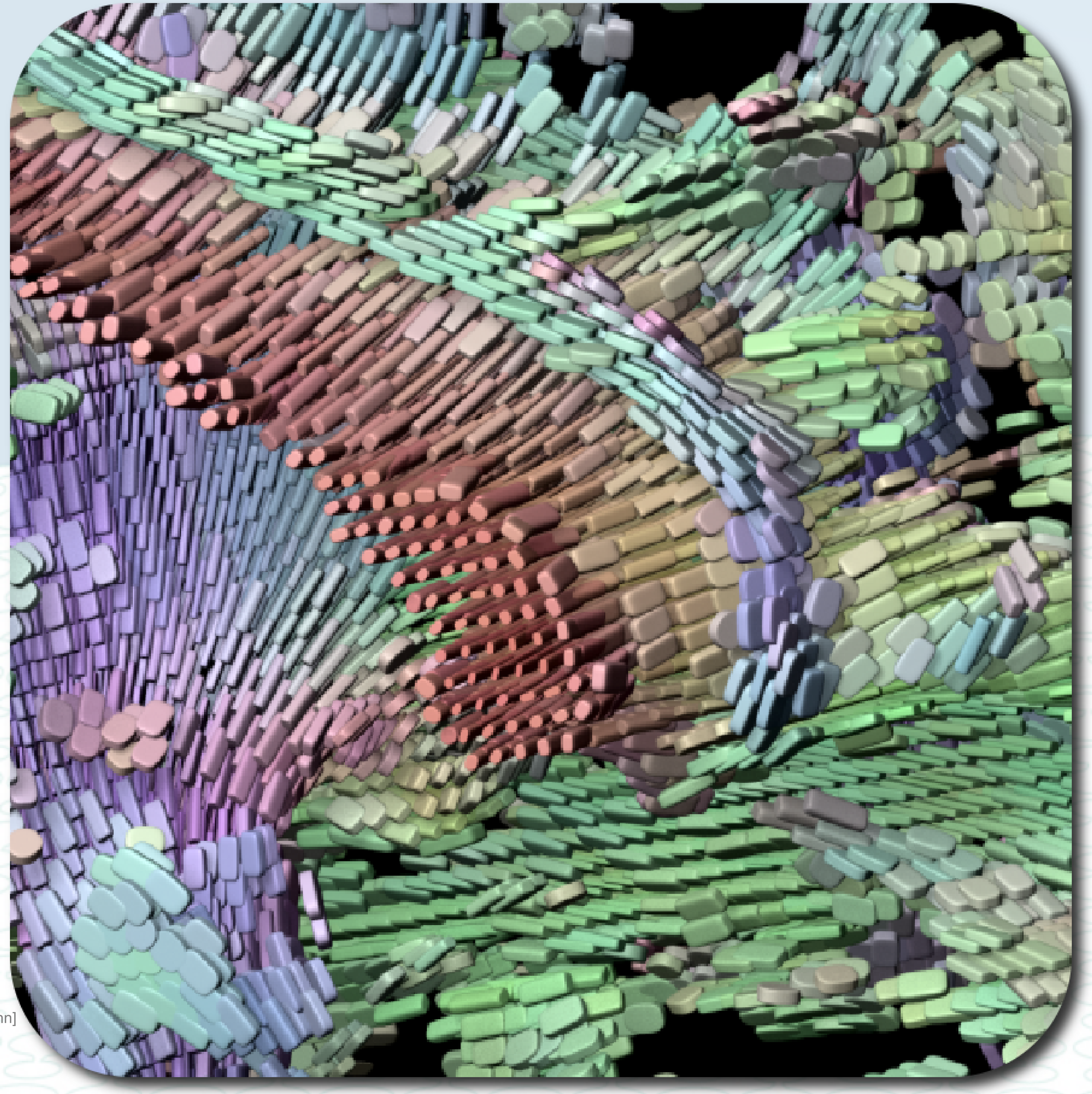

[Kindlmann]

Tensor Field Visualization

M2S

J. Tierny

Previously

$$f : \mathcal{D} \to \mathbb{R}$$

Function

Definition Domain

Arrival Domain

Attributes

Previously

$$f : \mathcal{D} \rightarrow \mathcal{T}(\mathcal{D})$$

Function

Definition Domain

Arrival Domain

Attributes

Previously

Flow information

$$f : \mathcal{D} \rightarrow \mathcal{T}(\mathcal{D})$$

Function

Arrival Domain

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Definition Domain

Previously

Flow information

Scalar field derivatives

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Function

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Previously

"Transformation" information

Scalar field derivatives

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Function

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[**Attributes**]()

Previously

“Transformation” information

Vector field derivatives

$$f : \mathcal{D} \rightarrow \mathcal{T}(\mathcal{D})$$

Function

Arrival Domain

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Definition Domain

Previously

“Transformation” information

Vector field derivatives

$$f : \mathcal{D} \rightarrow \mathbb{M}_{d \times d}$$

Function

Definition Domain

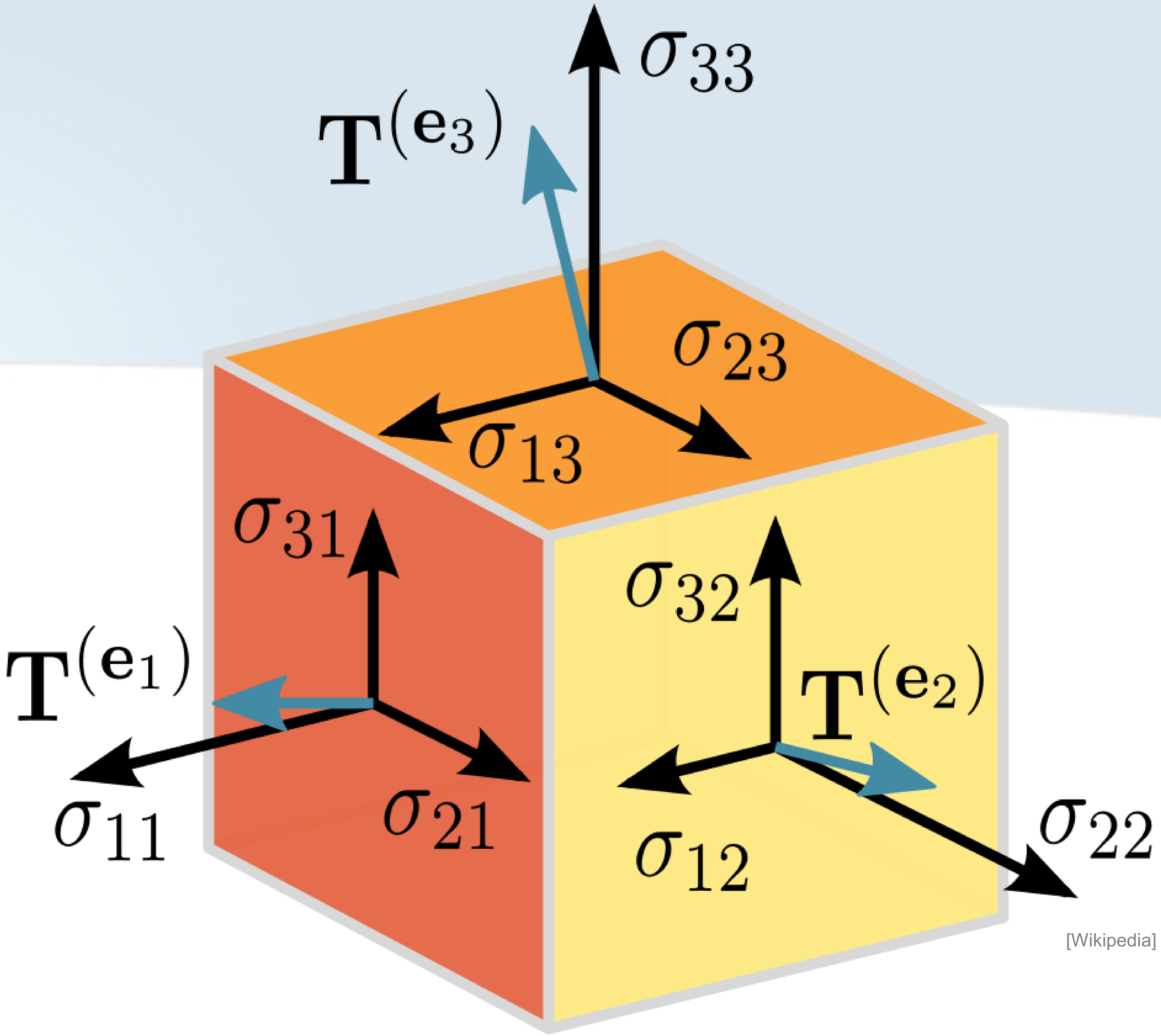
Arrival Domain

Attributes

Notion of tensor

Notion of tensor

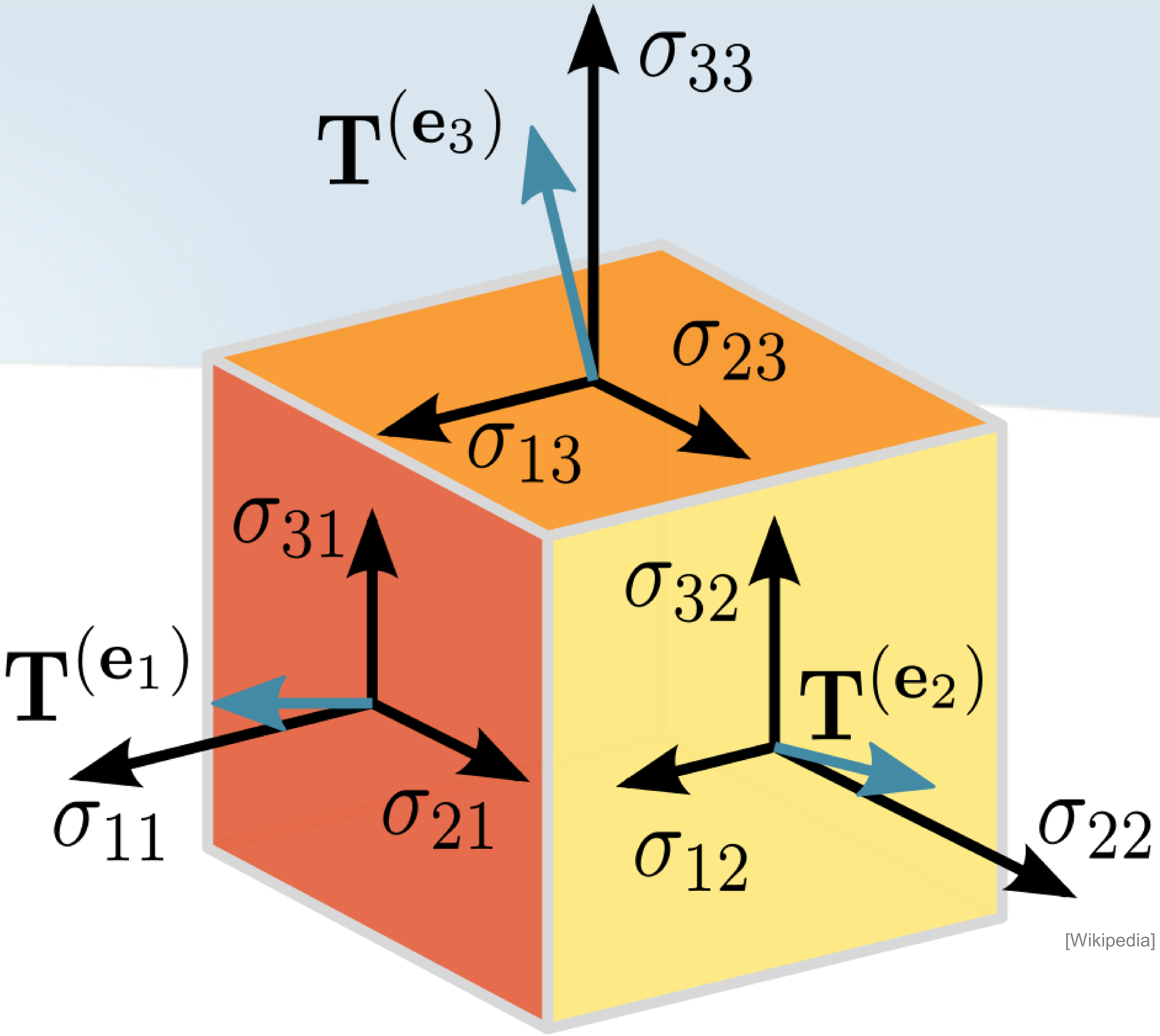
- Generalization of scalars and vectors



[Wikipedia]

Notion of tensor

- Generalization of scalars and vectors
- Describes linear relations between



[Wikipedia]

Notion of tensor

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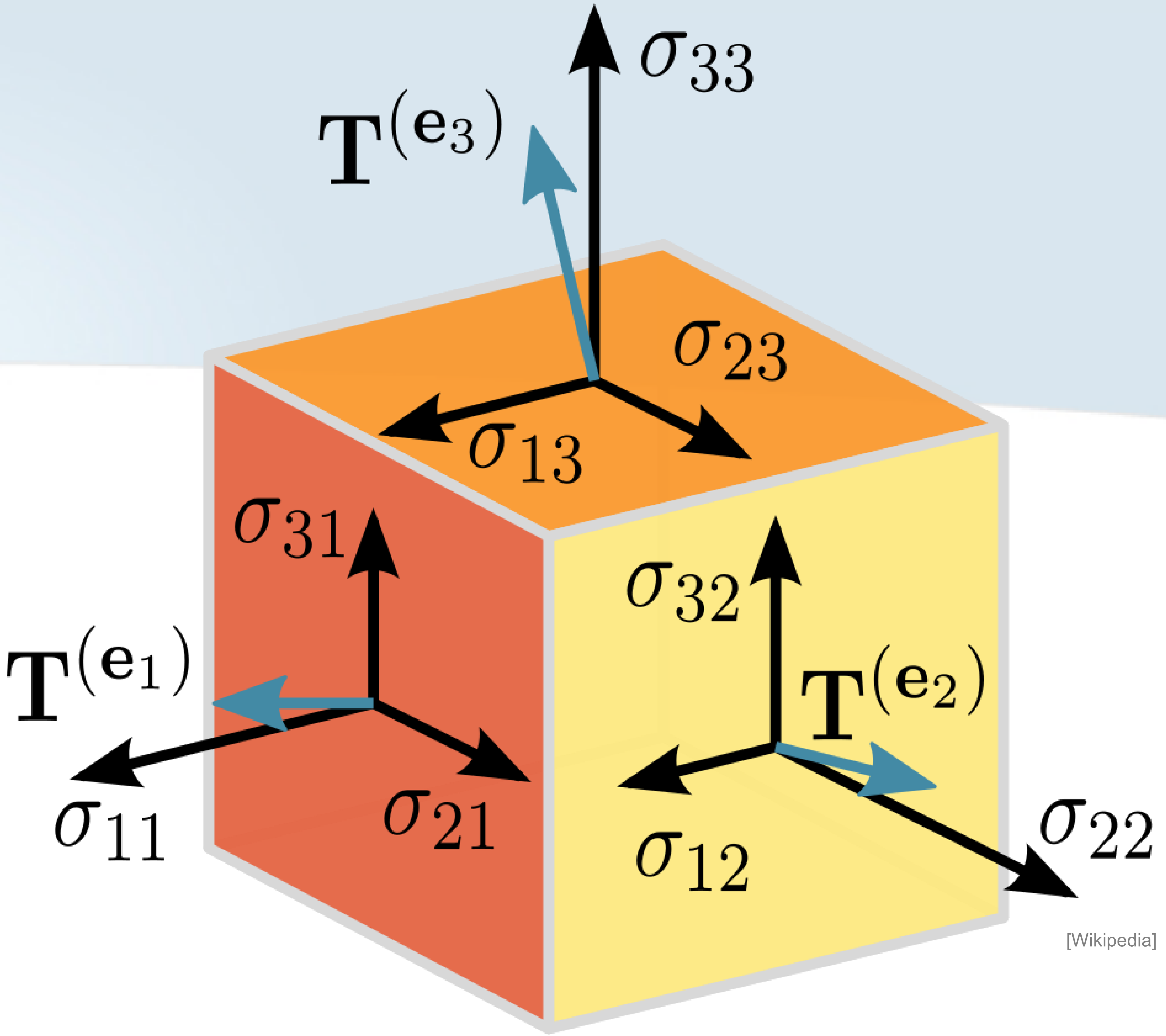
  - Vectors
  - Or other tensors



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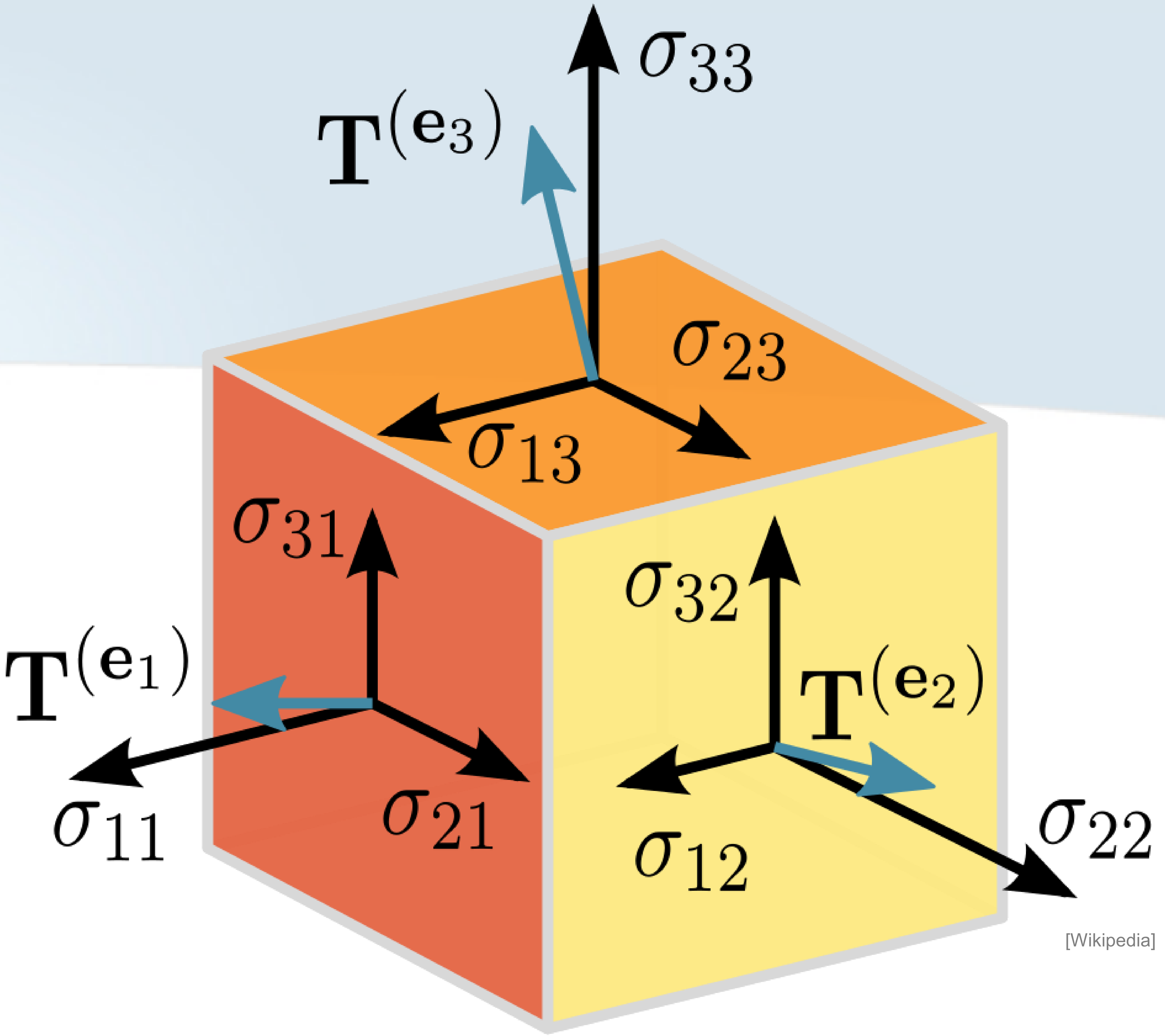
- Multi-dimensional array of numerical values



[Wikipedia]

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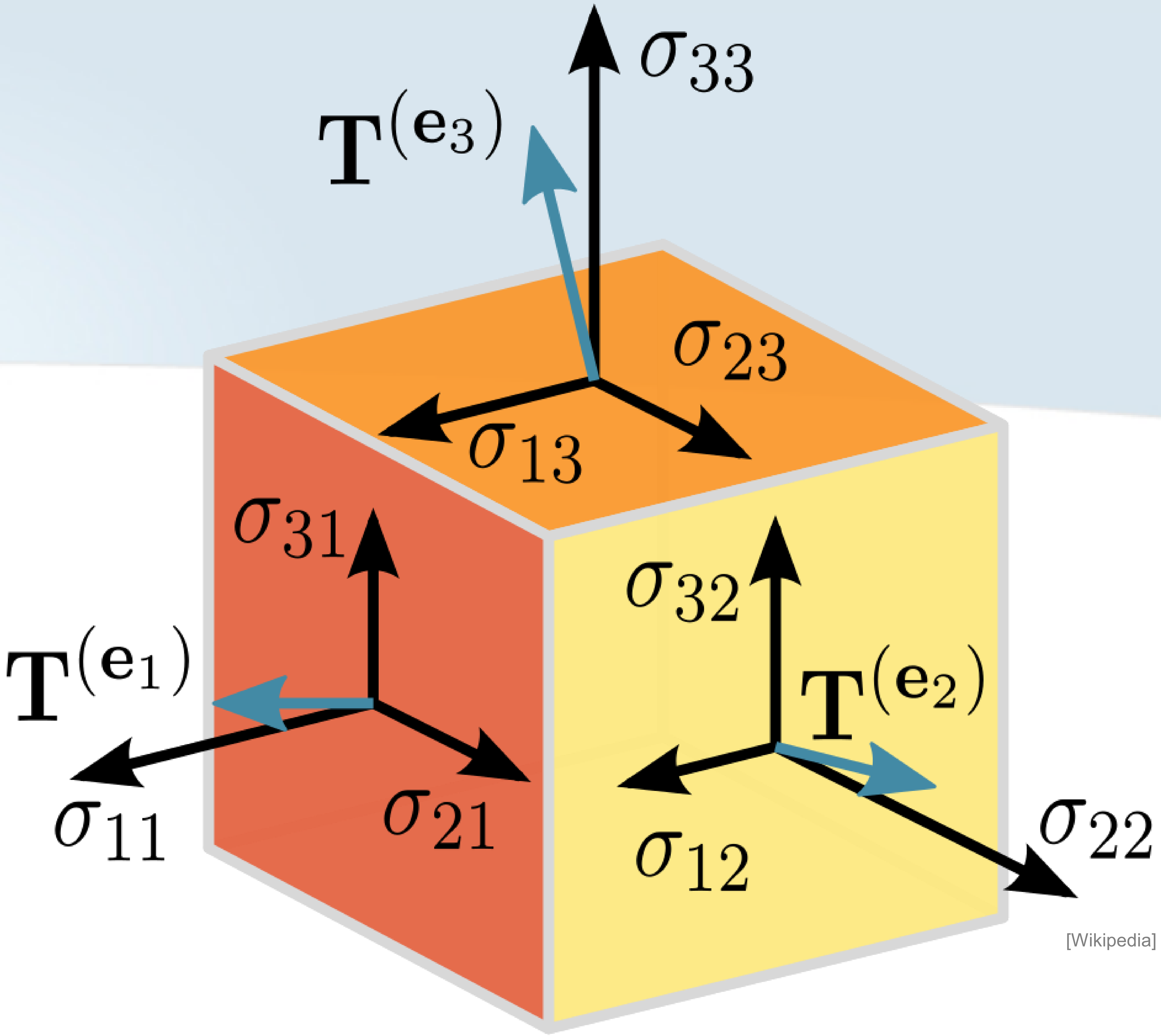
- Multi-dimensional array of numerical values
  - Order of a tensor



[Wikipedia]

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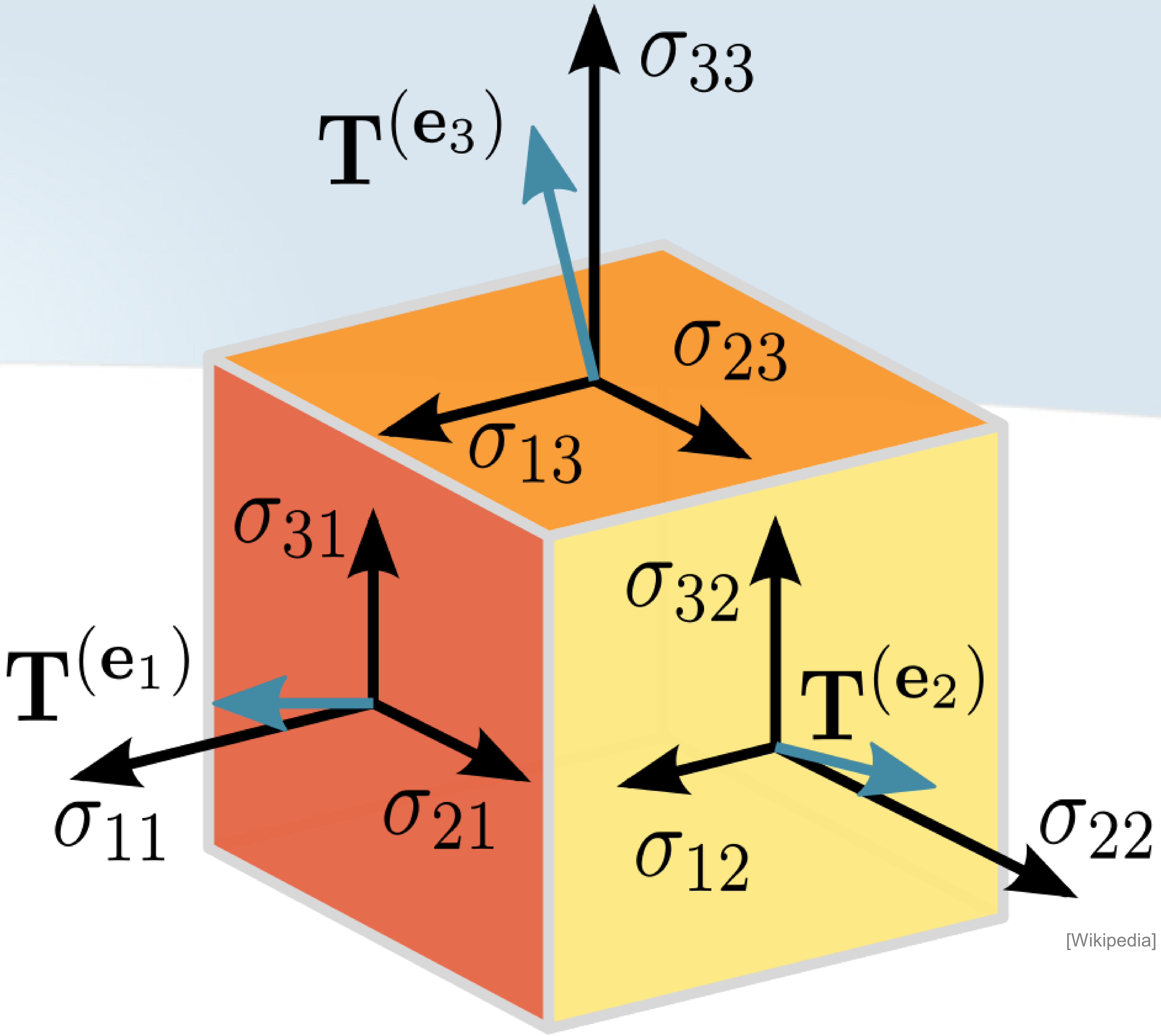
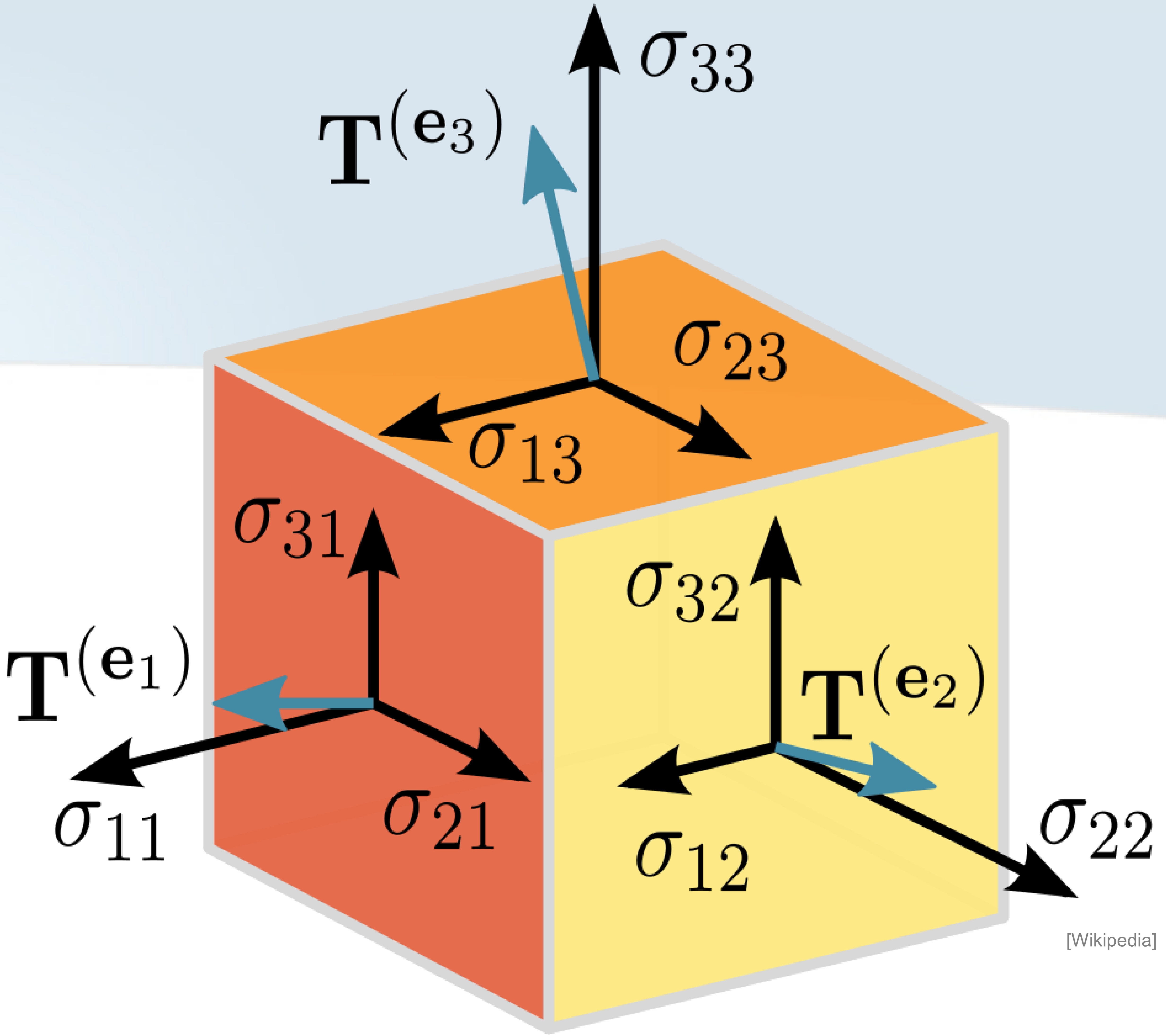


[Wikipedia]

- Multi-dimensional array of numerical values
  - Order of a tensor
    - Number of dimensions

Notion of tensor

- Order of a tensor

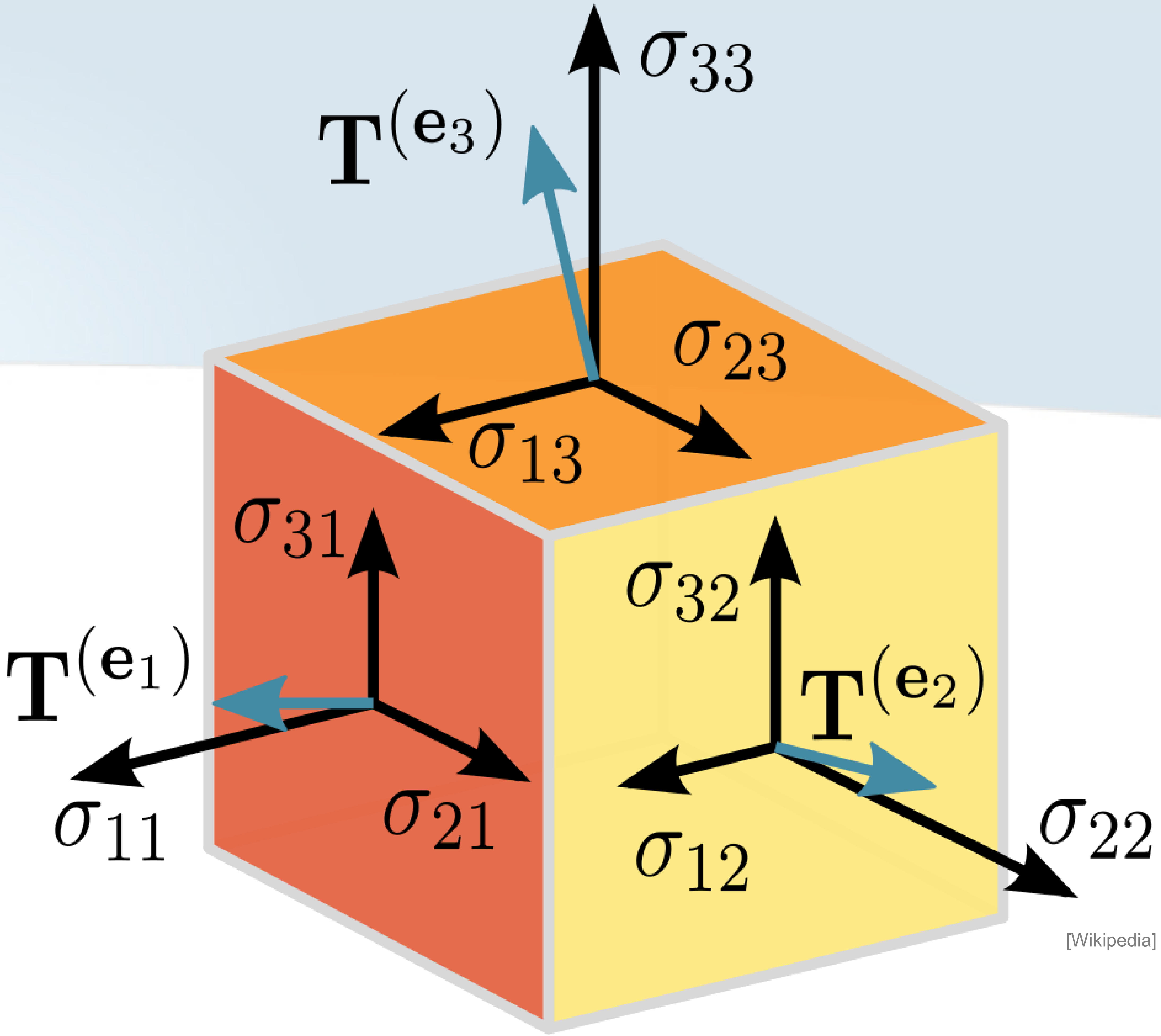


[Wikipedia]

[Chen]

Notion of tensor

- Order of a tensor
  - Scalar value: 0th order tensor



[Wikipedia]

[Chen]

Notion of tensor

- Order of a tensor
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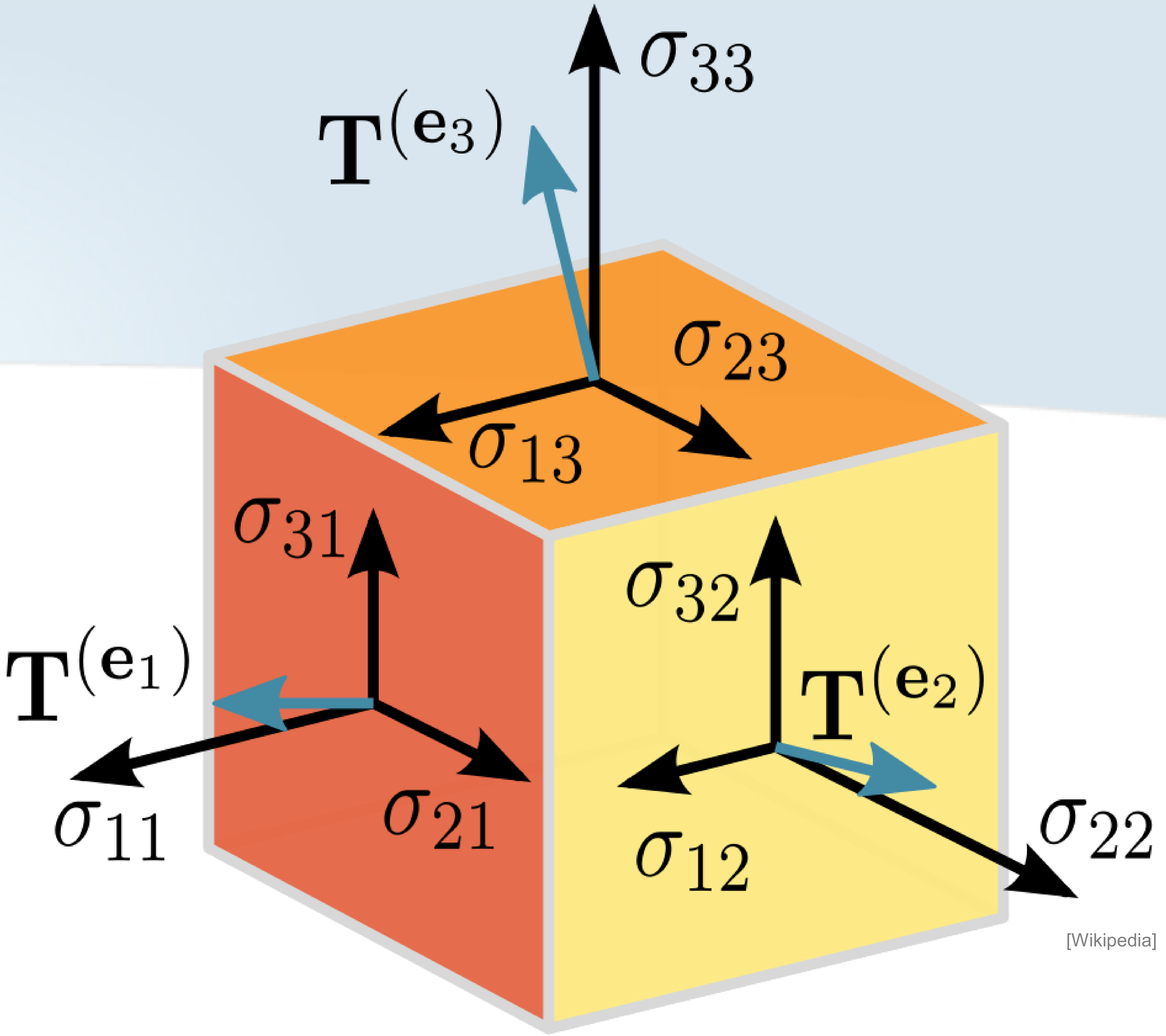

[Wikipedia]

[Chen]

Notion of tensor

- Order of a tensor
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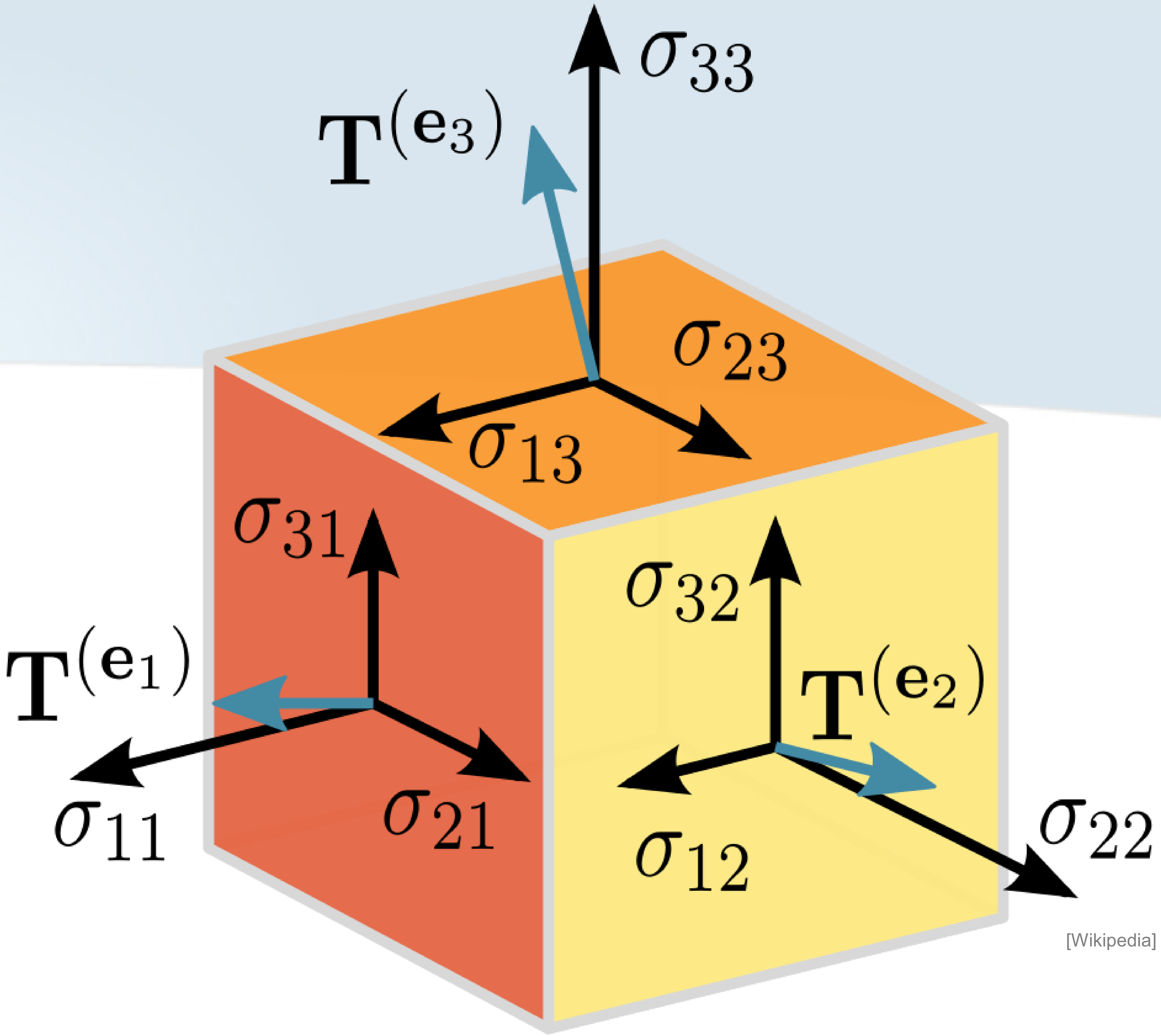
  - (dxd)-matrix: 2nd order tensor



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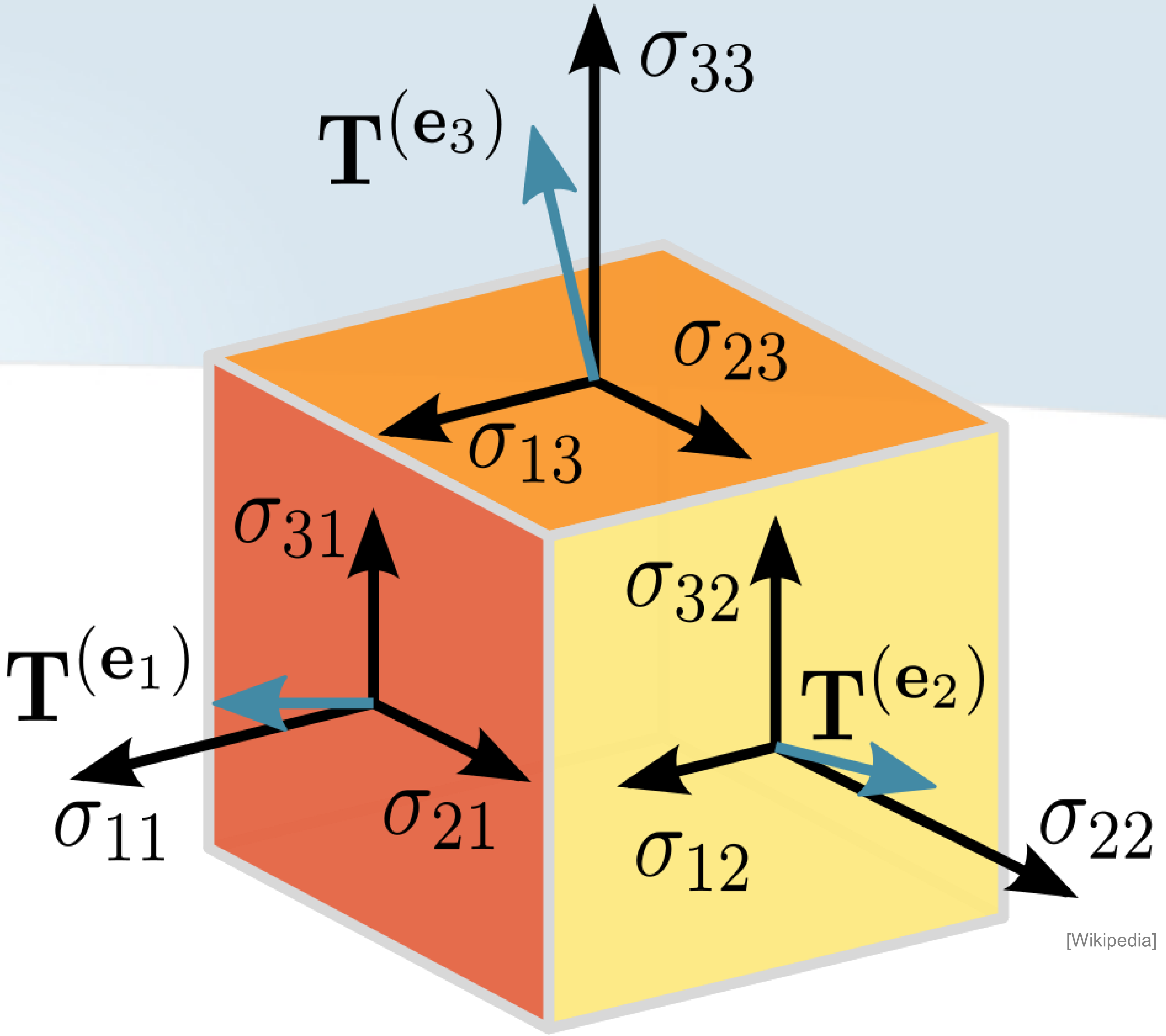

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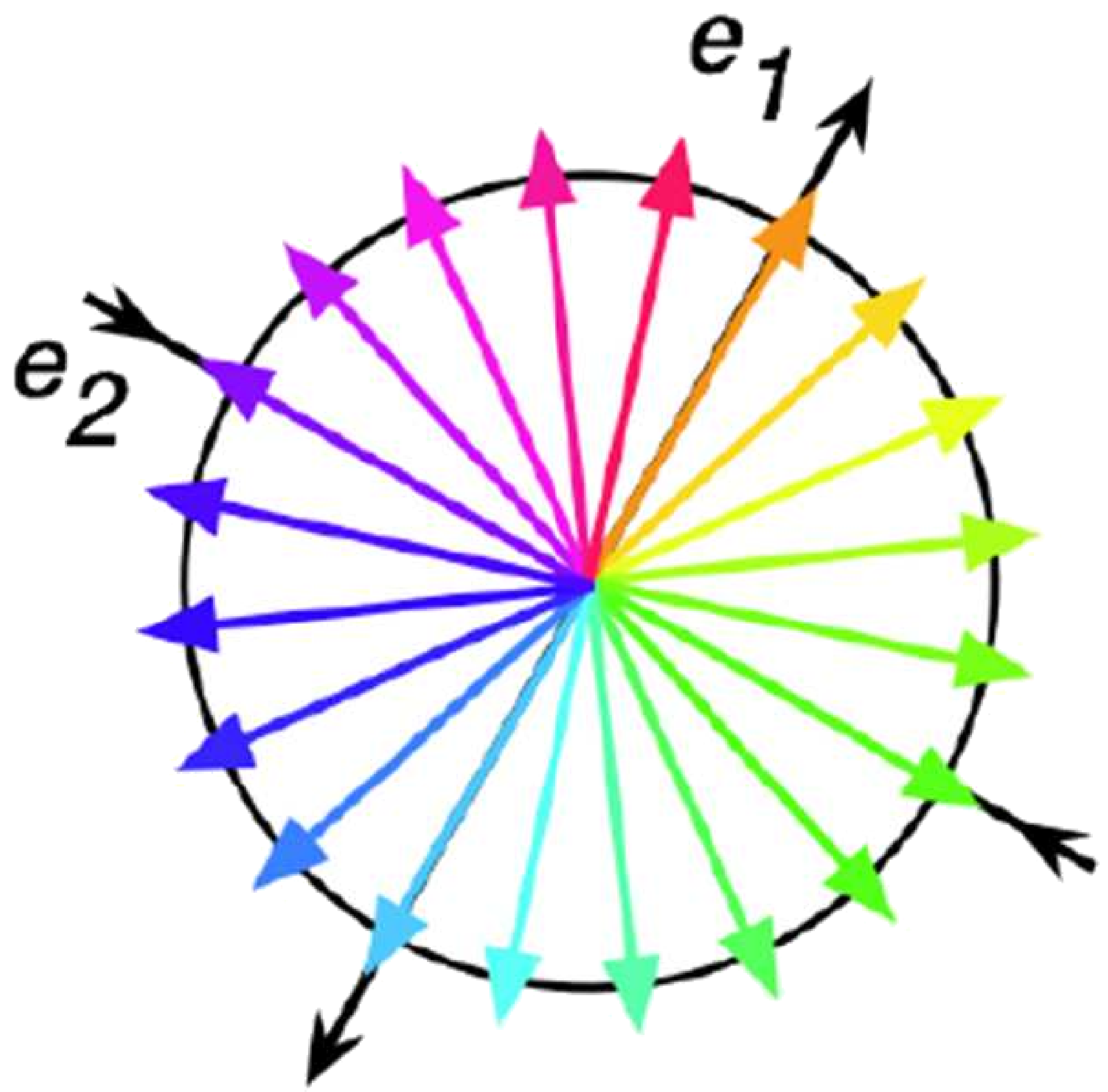

[Chen]

- Here, **mostly 2nd order tensors**
  - For instance
    - $t : \mathbb{V} \rightarrow \mathbb{V}$

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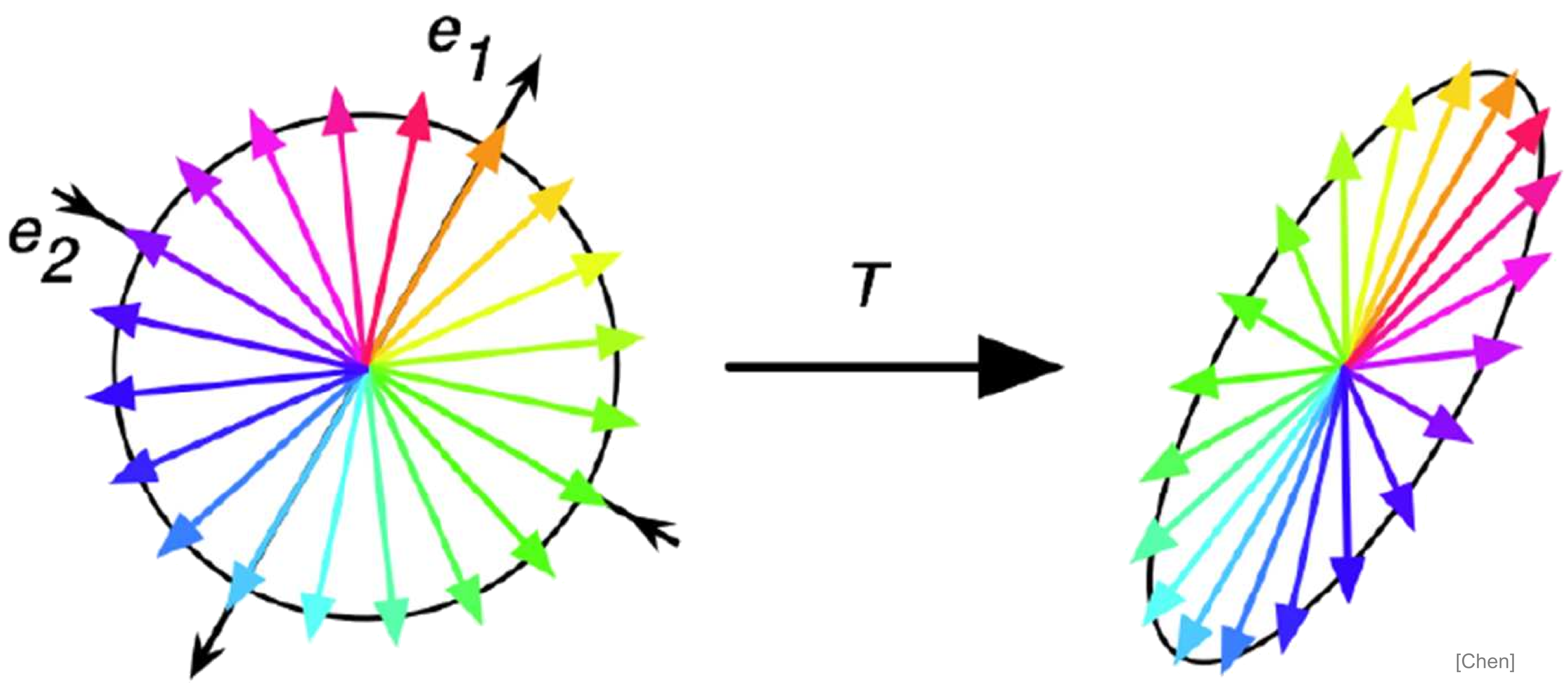
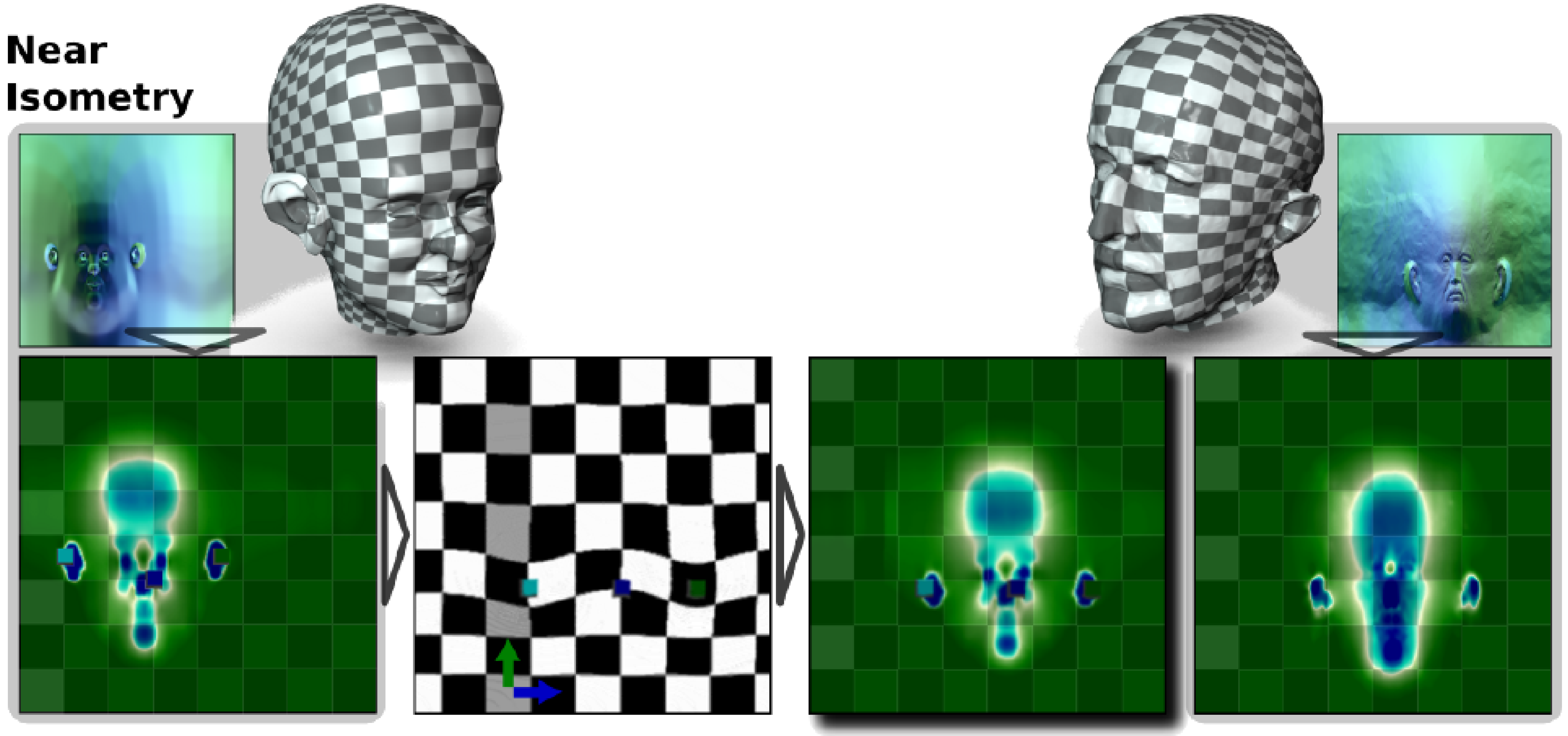


[Chen]

Why?

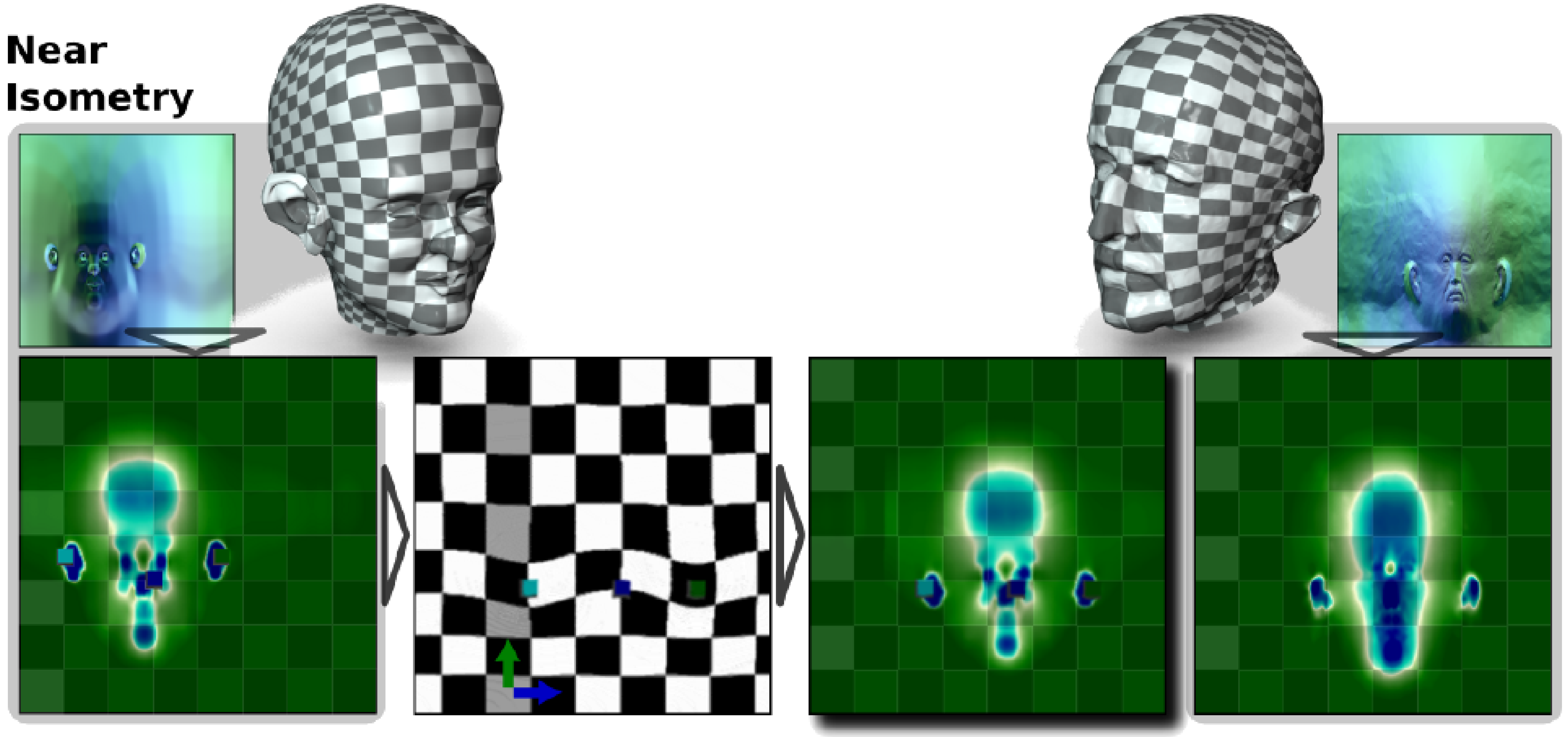
Why?

- Trivial motivation
  - Derivatives of vector fields



Why?

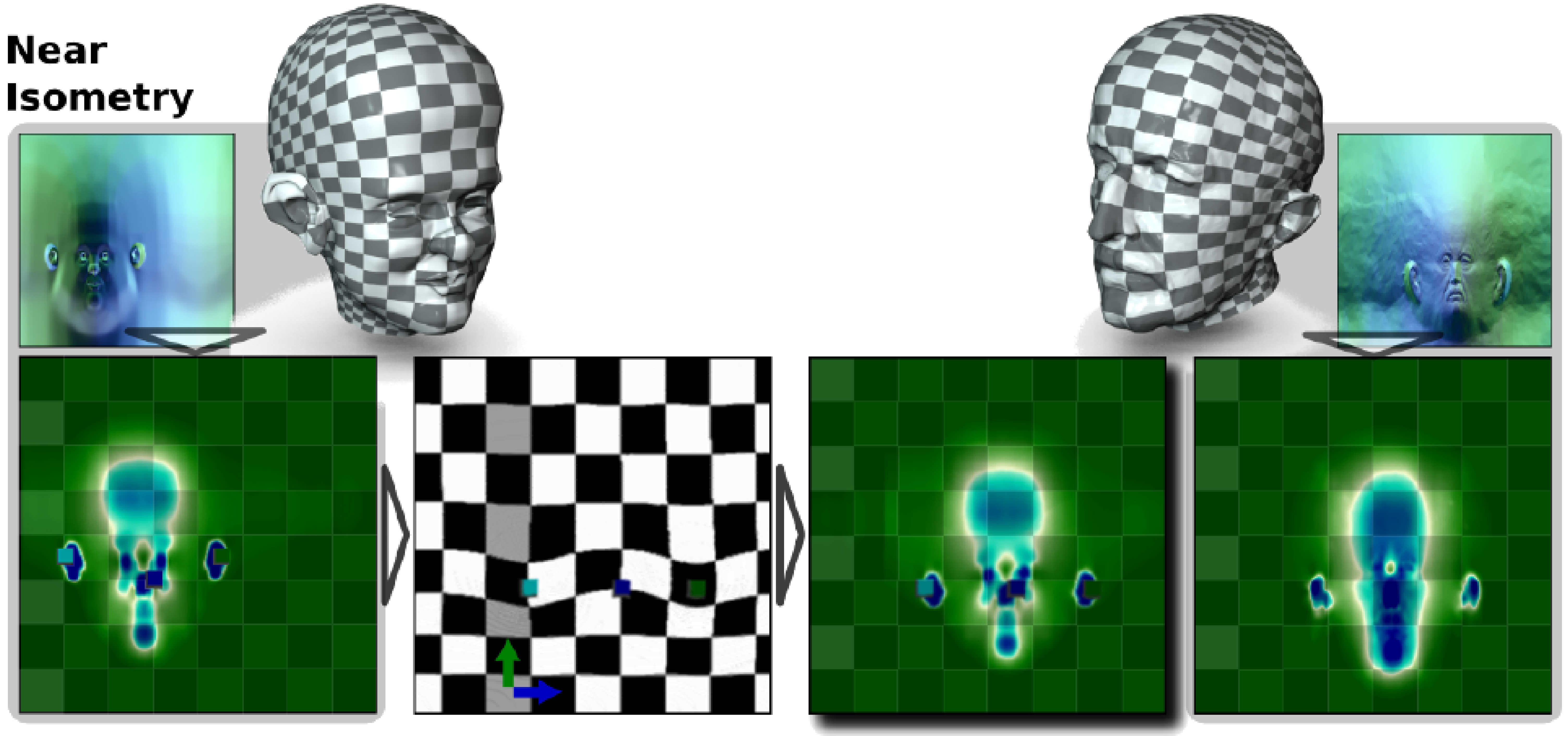
- Trivial motivation
  - Derivatives of vector fields
    - $f : \mathbb{R}^2 \to \mathbb{R}^2$



Why?

- Trivial motivation
  - Derivatives of vector fields
    - $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$
    - Jacobian

$$J = \begin{bmatrix} \frac{\partial f_x}{\partial x} & \frac{\partial f_x}{\partial y} \\ \frac{\partial f_y}{\partial x} & \frac{\partial f_y}{\partial y} \end{bmatrix}$$



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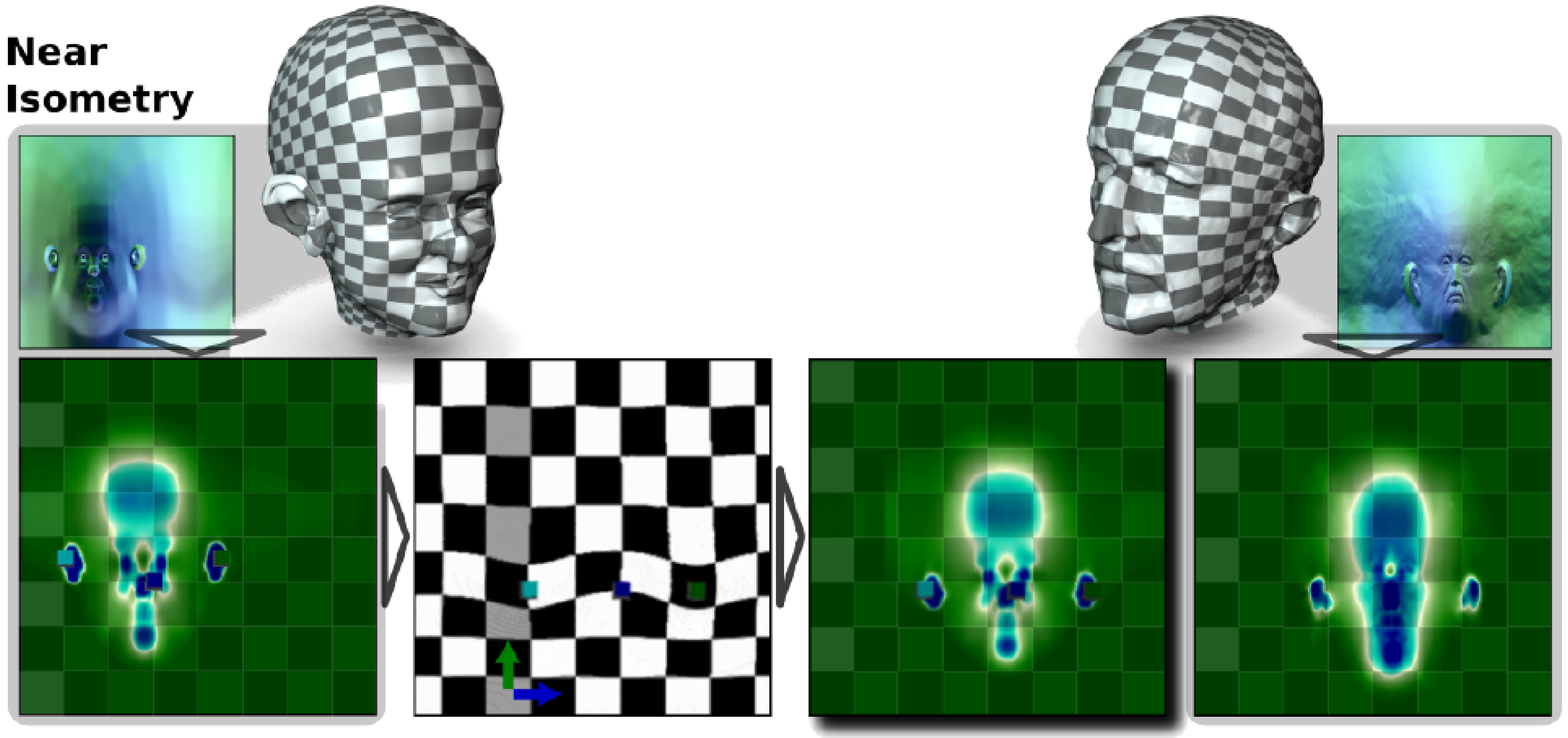
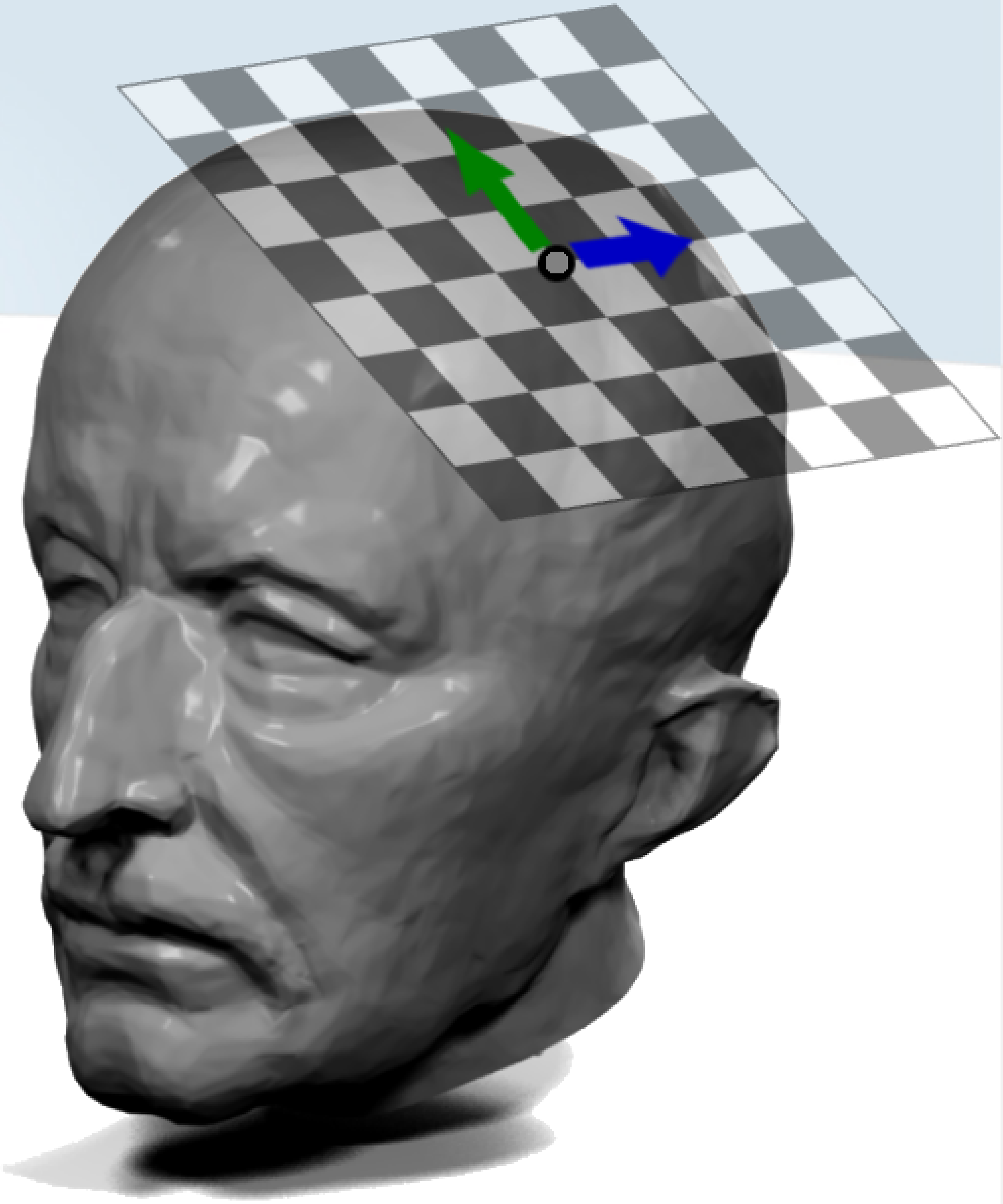


- Example
  - Derivatives of planar transformations

Why?

- Metric tensors on manifolds



Why?

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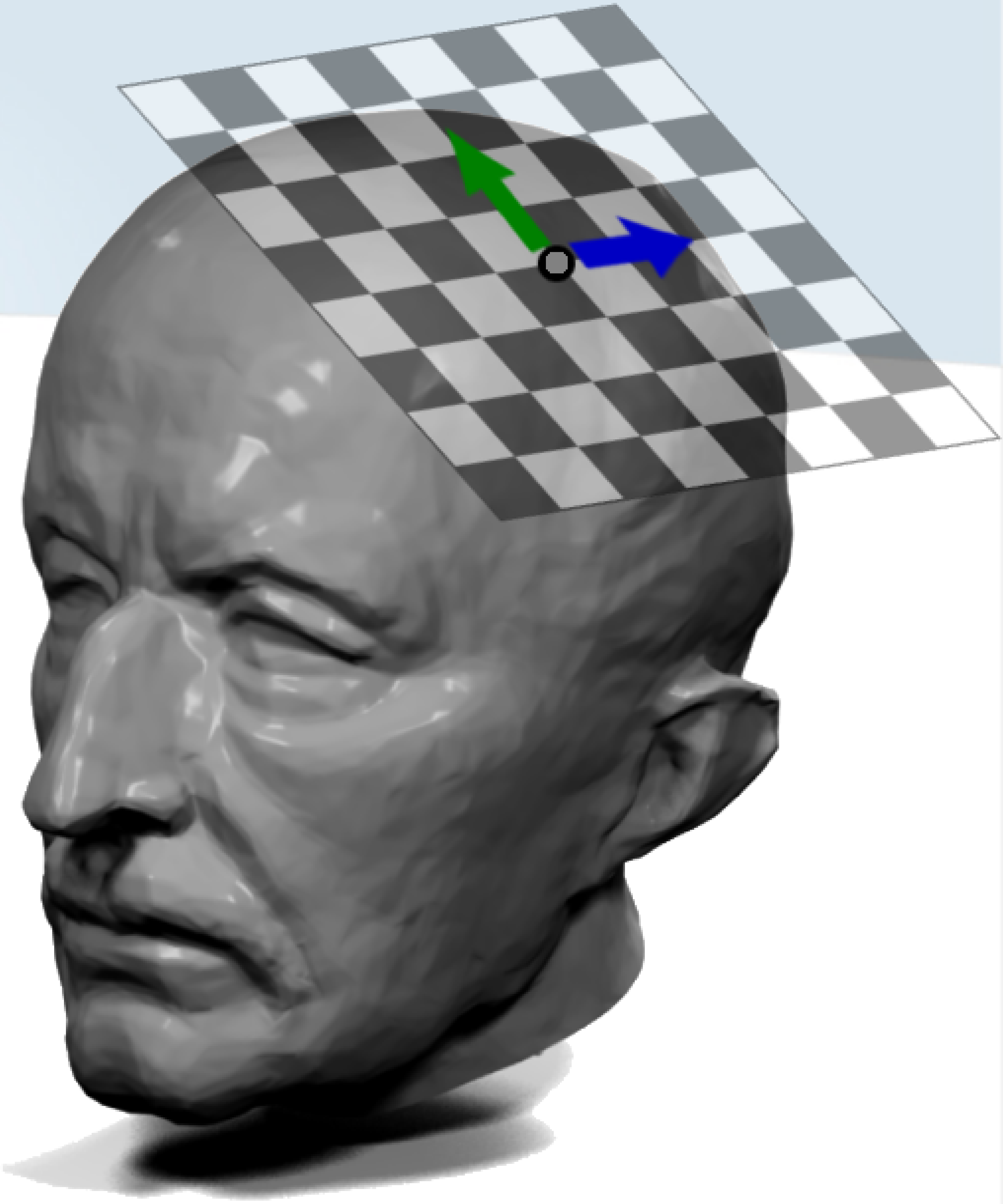

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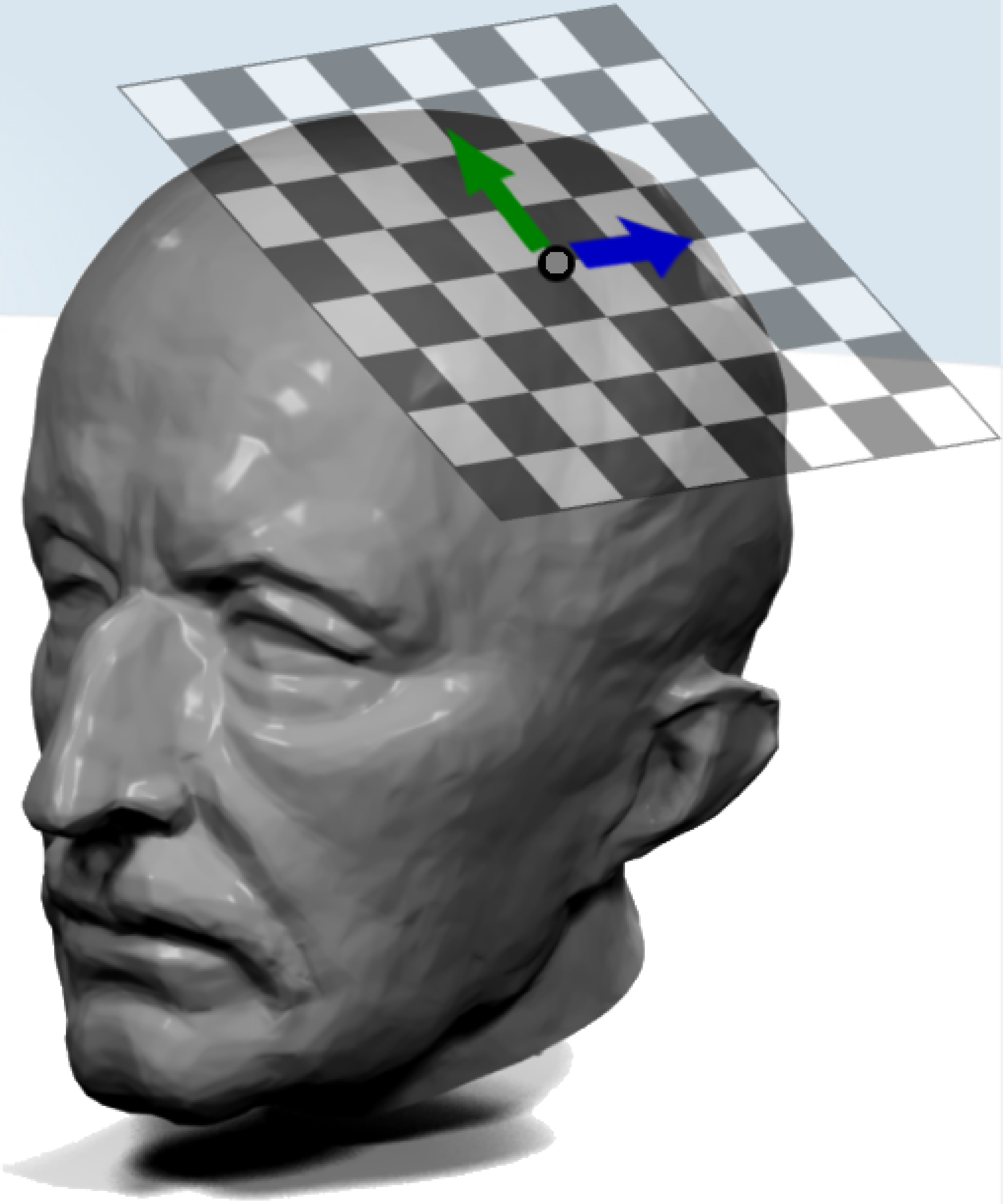
  - (u, v)



Why?

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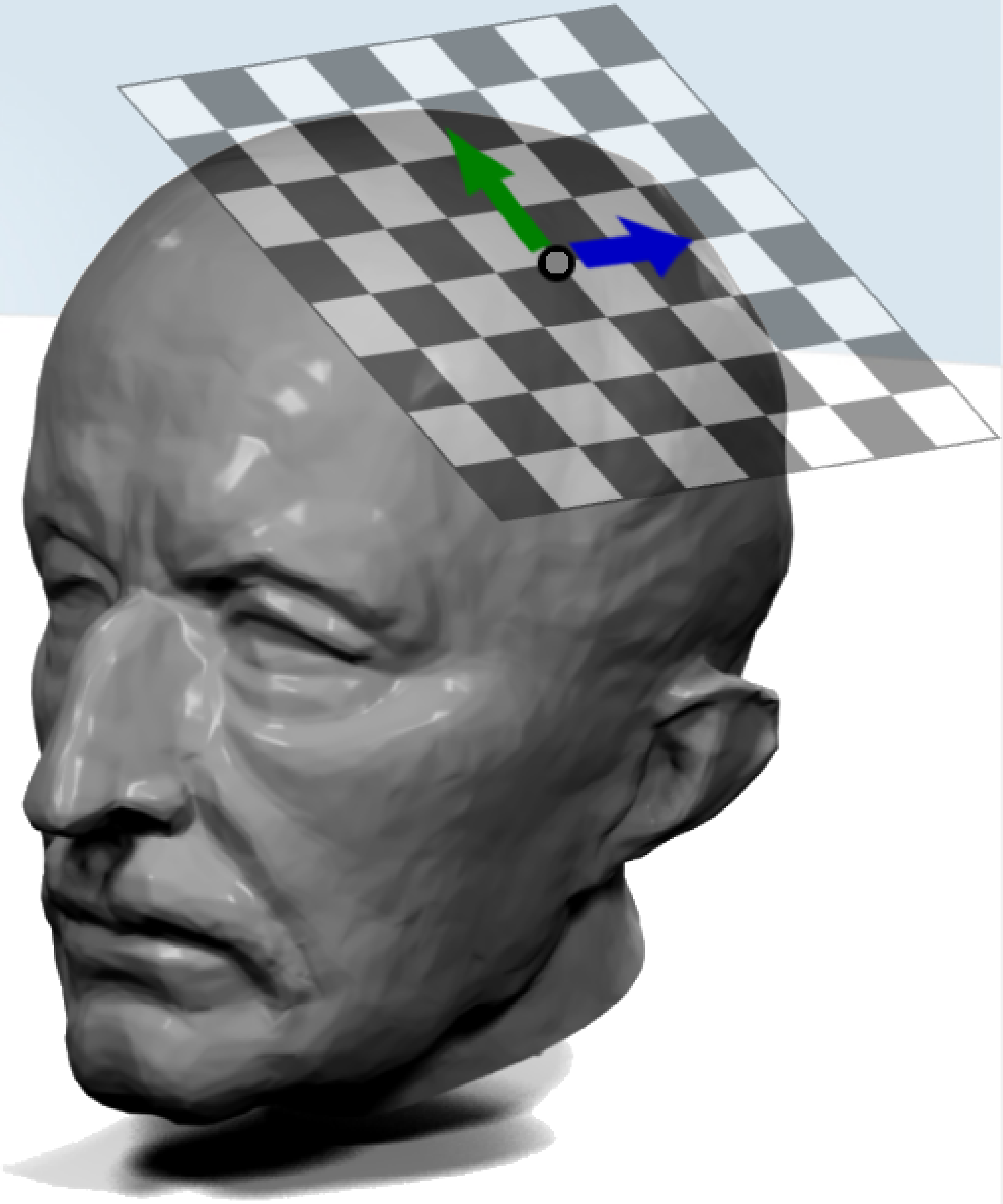

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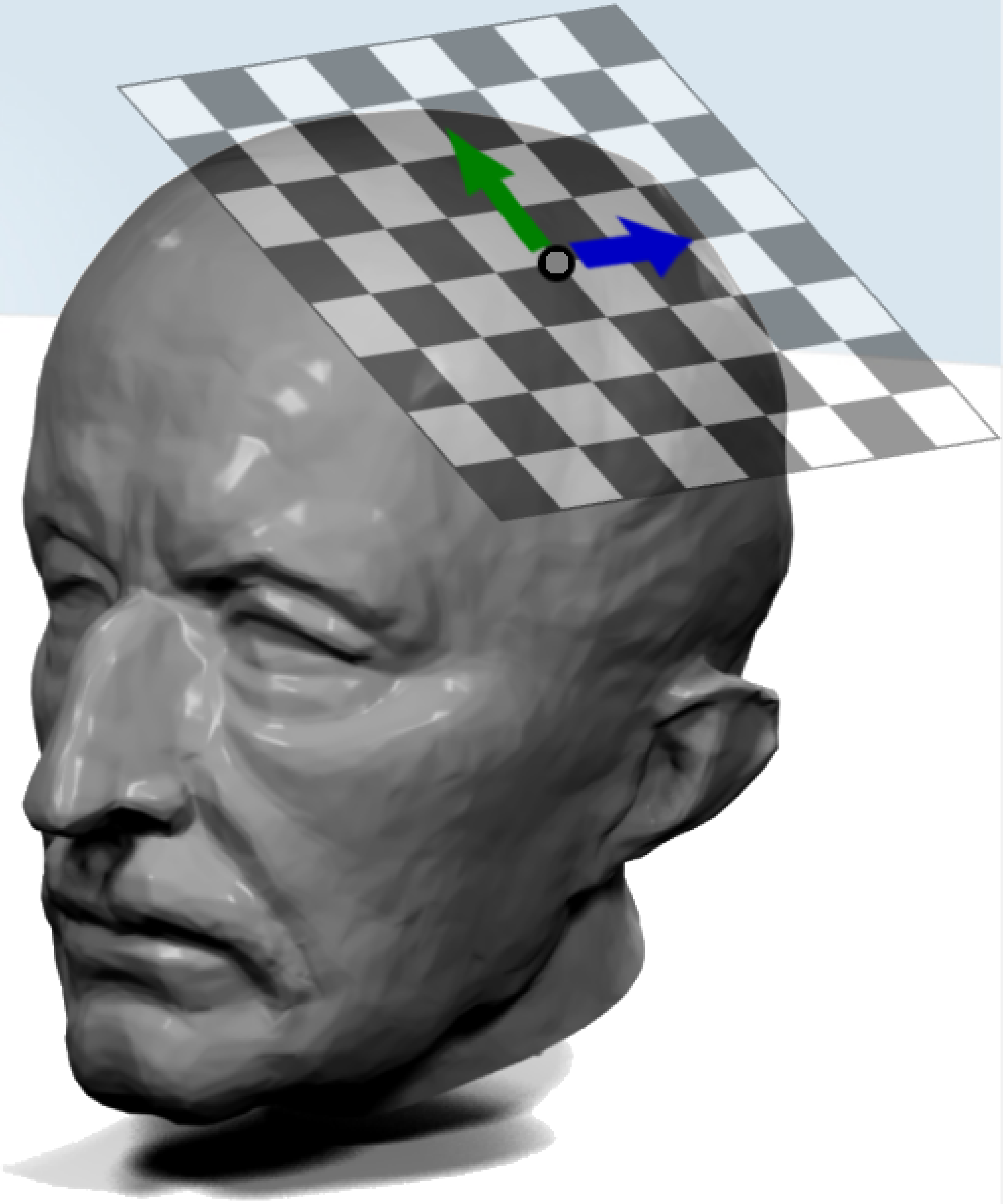
$$\frac{\partial e}{\partial u} = [\frac{\partial x}{\partial u}, \frac{\partial y}{\partial u}, \frac{\partial z}{\partial u}] \quad \frac{\partial e}{\partial v} = [\frac{\partial x}{\partial v}, \frac{\partial y}{\partial v}, \frac{\partial z}{\partial v}]$$



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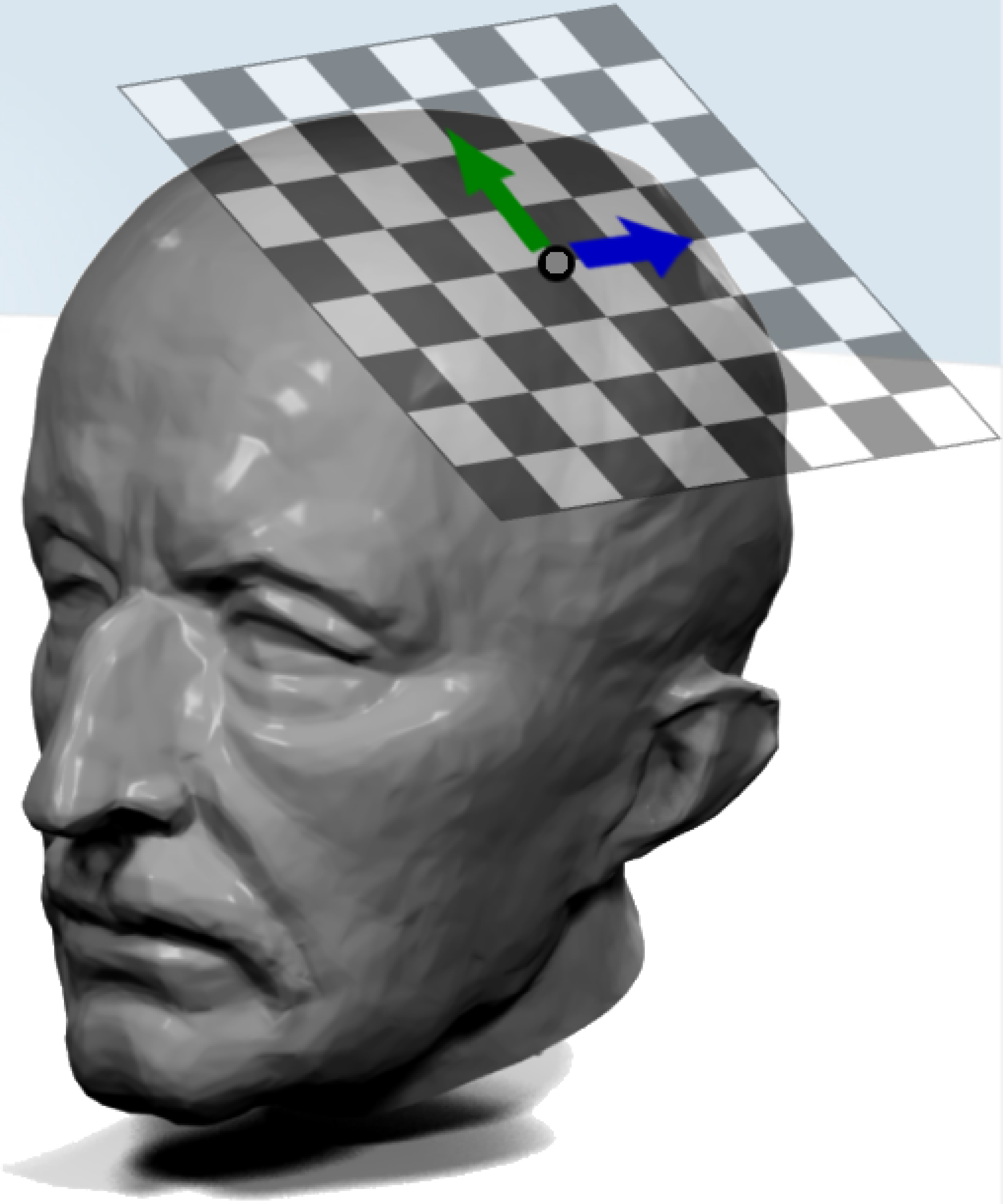
  - First fundamental form

$$G = \begin{bmatrix} \frac{\partial e}{\partial u} \cdot \frac{\partial e}{\partial u} & \frac{\partial e}{\partial u} \cdot \frac{\partial e}{\partial v} \\ \frac{\partial e}{\partial v} \cdot \frac{\partial e}{\partial u} & \frac{\partial e}{\partial v} \cdot \frac{\partial e}{\partial v} \end{bmatrix}$$



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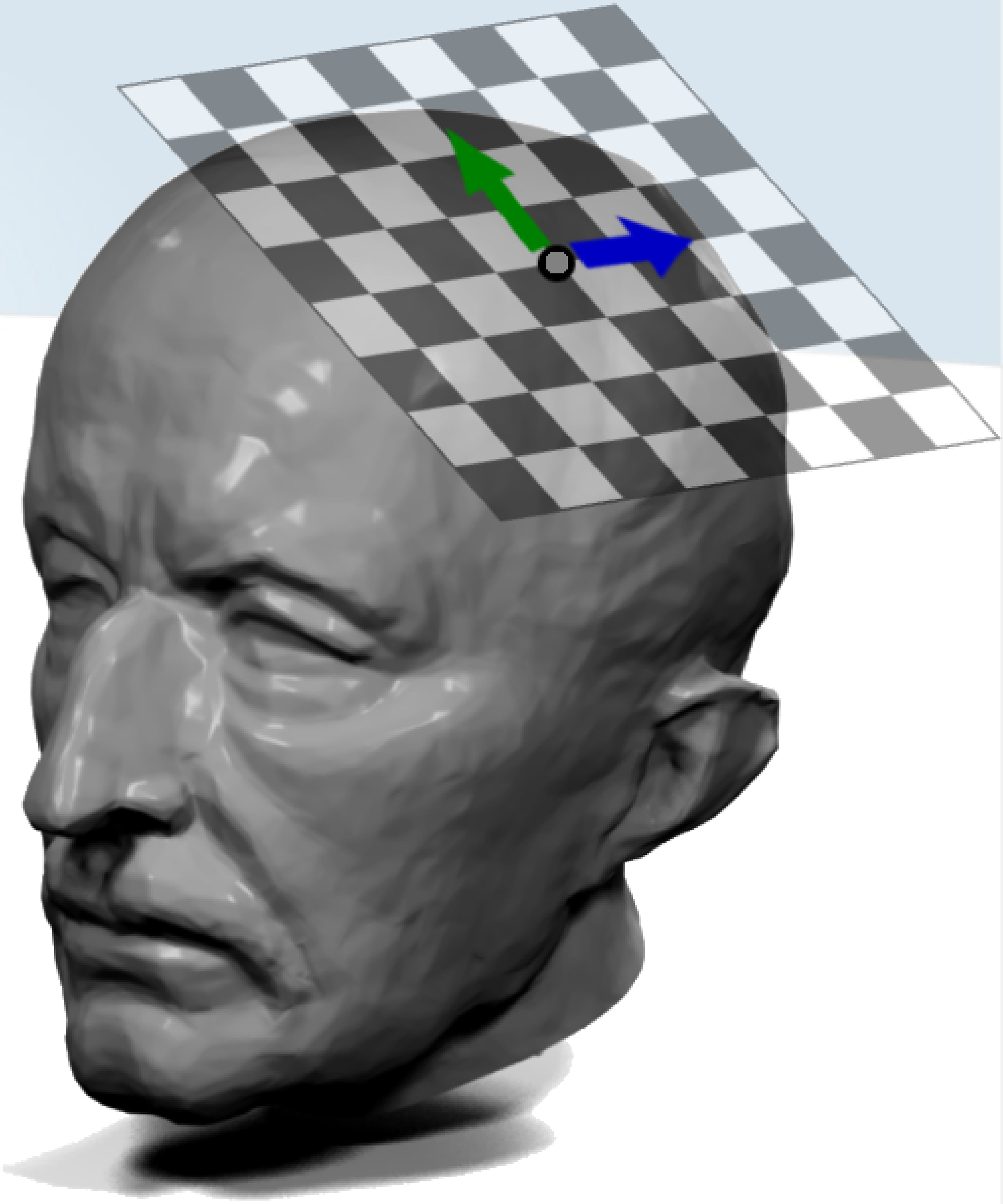
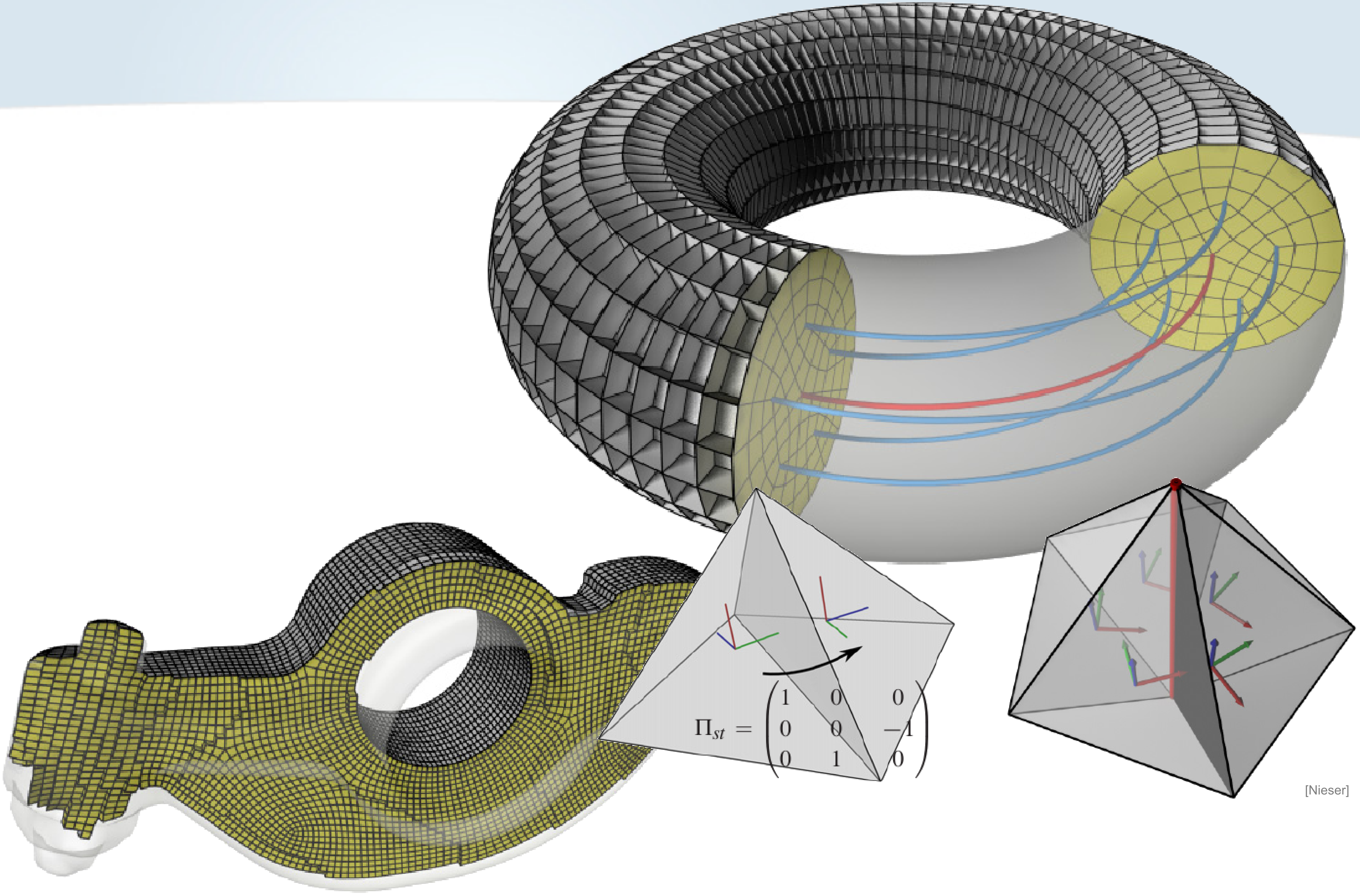
  - Curvature, length, area, angles according to the embedding space



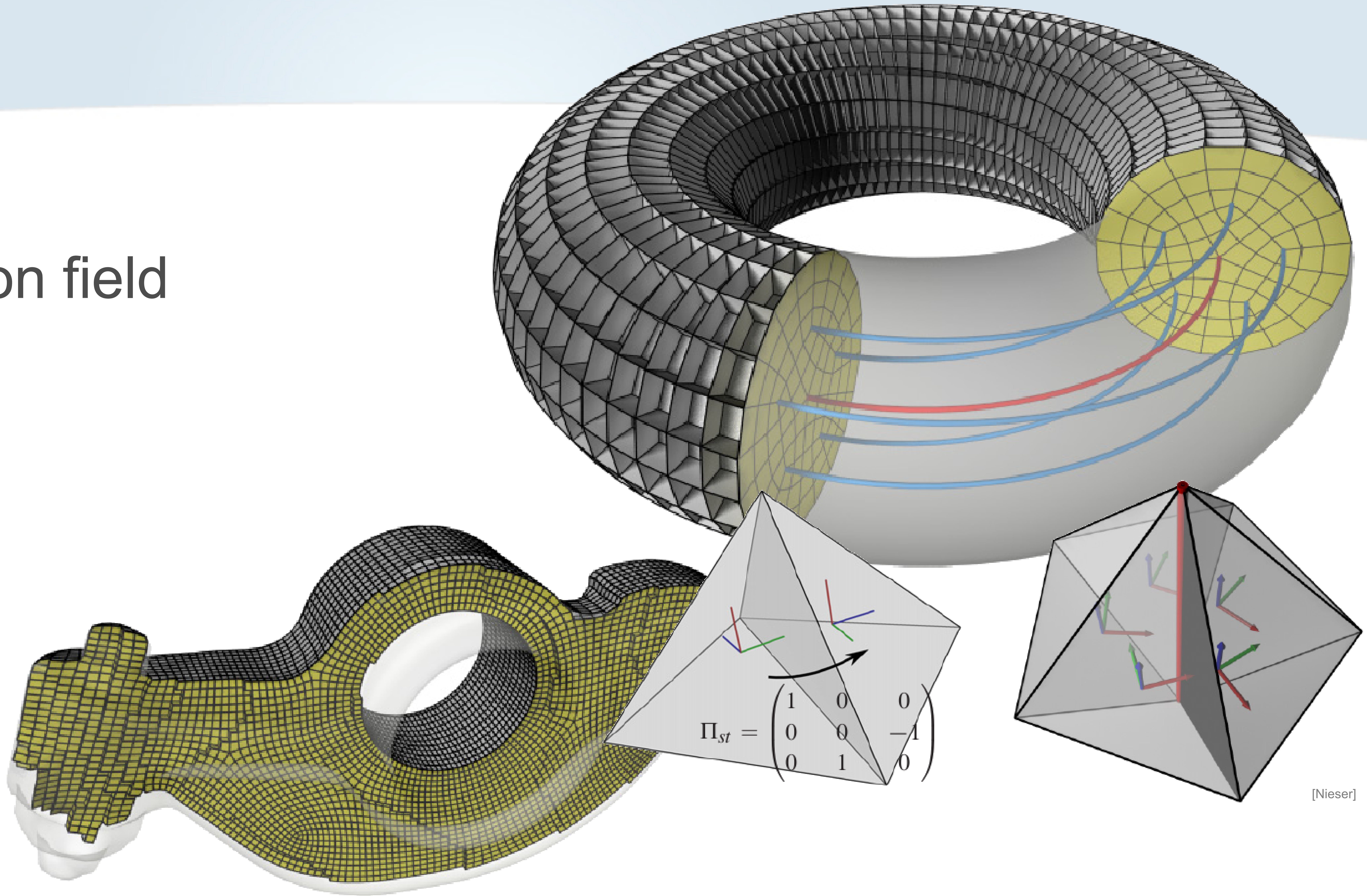
Why?

- Mesh generation



Why?

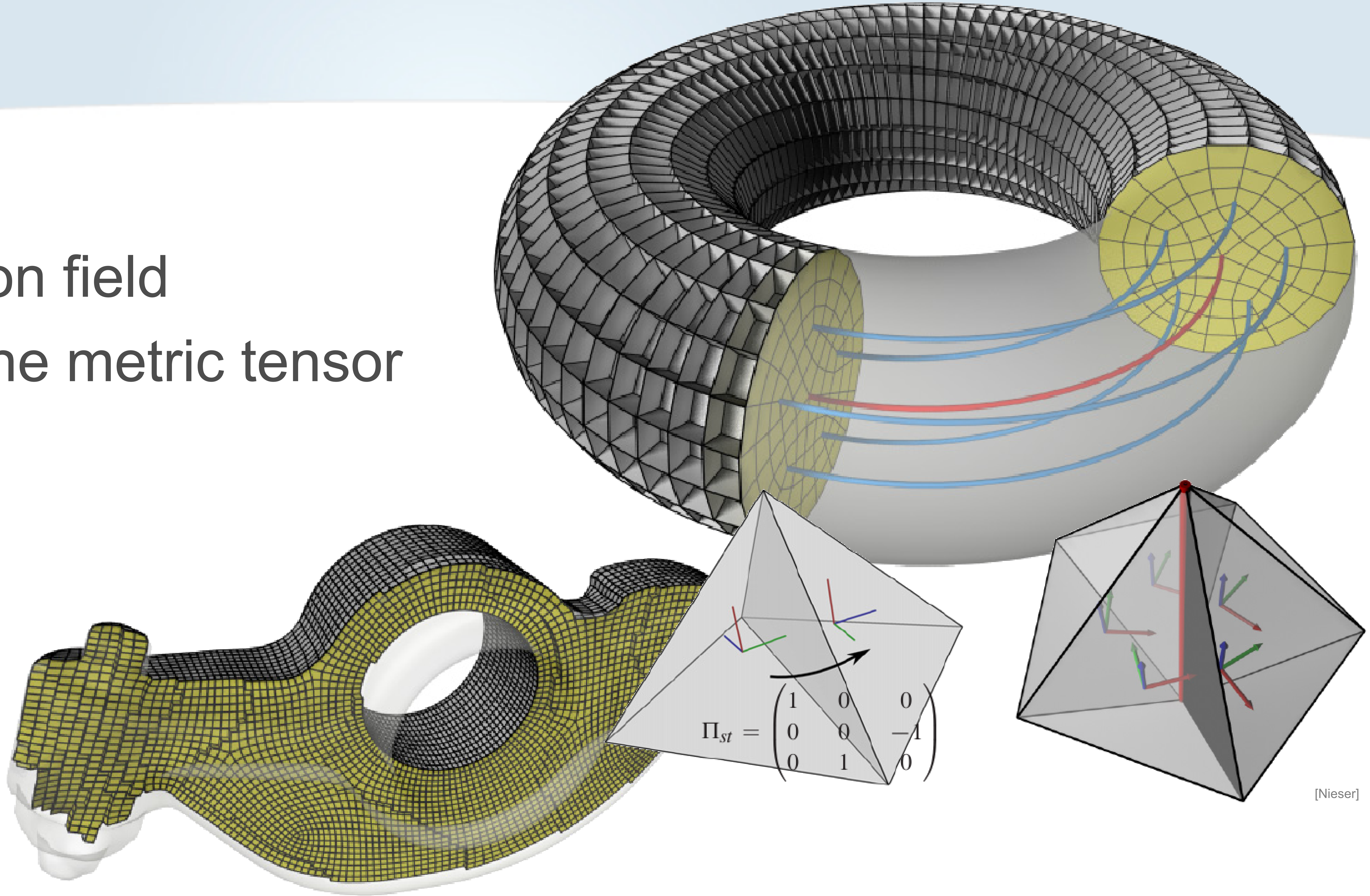
- Mesh generation
  - Notion of direction field



[Nieser]

Why?

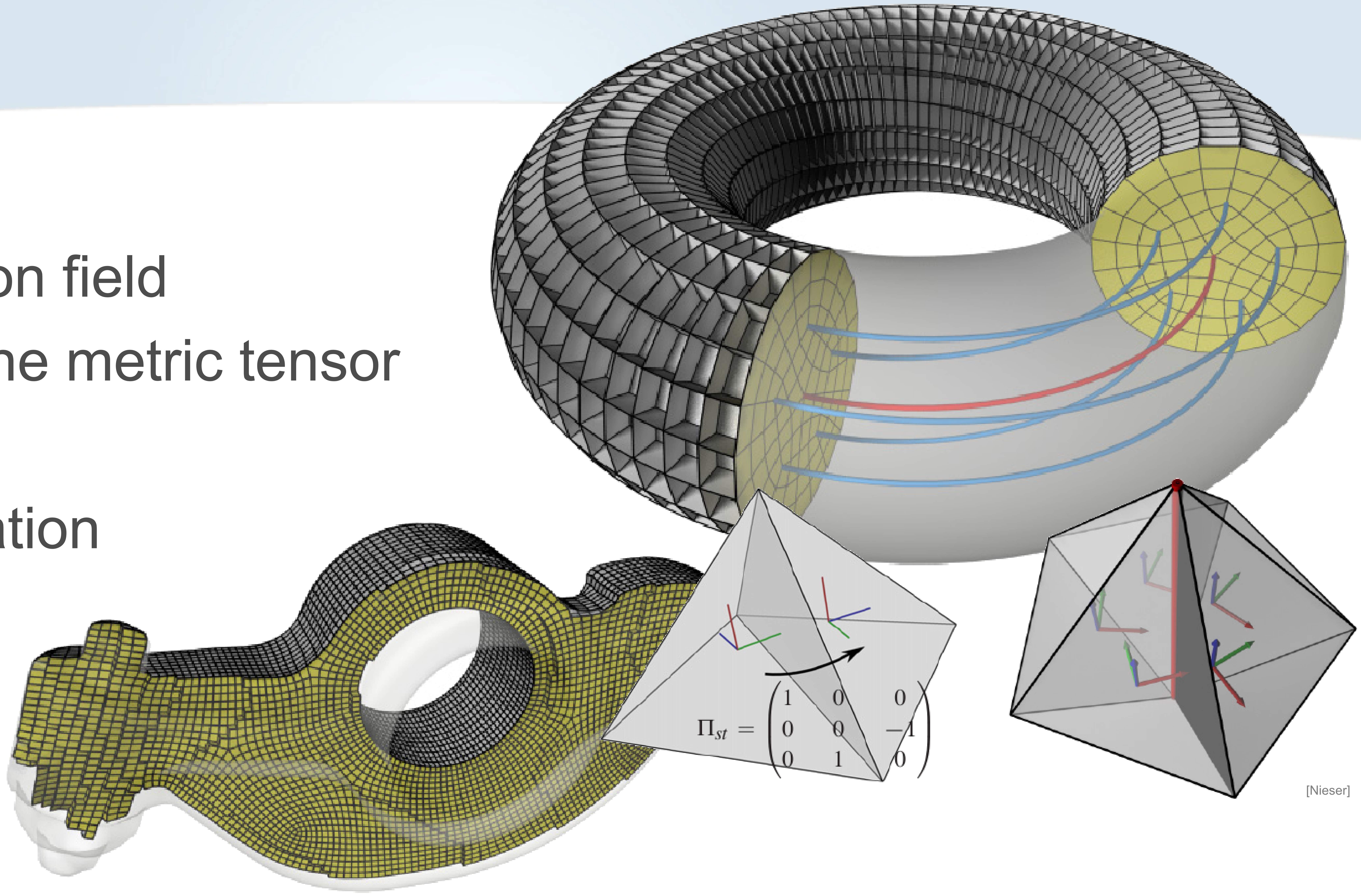
- Mesh generation
  - Notion of direction field
  - Constraints on the metric tensor



[Nieser]

Why?

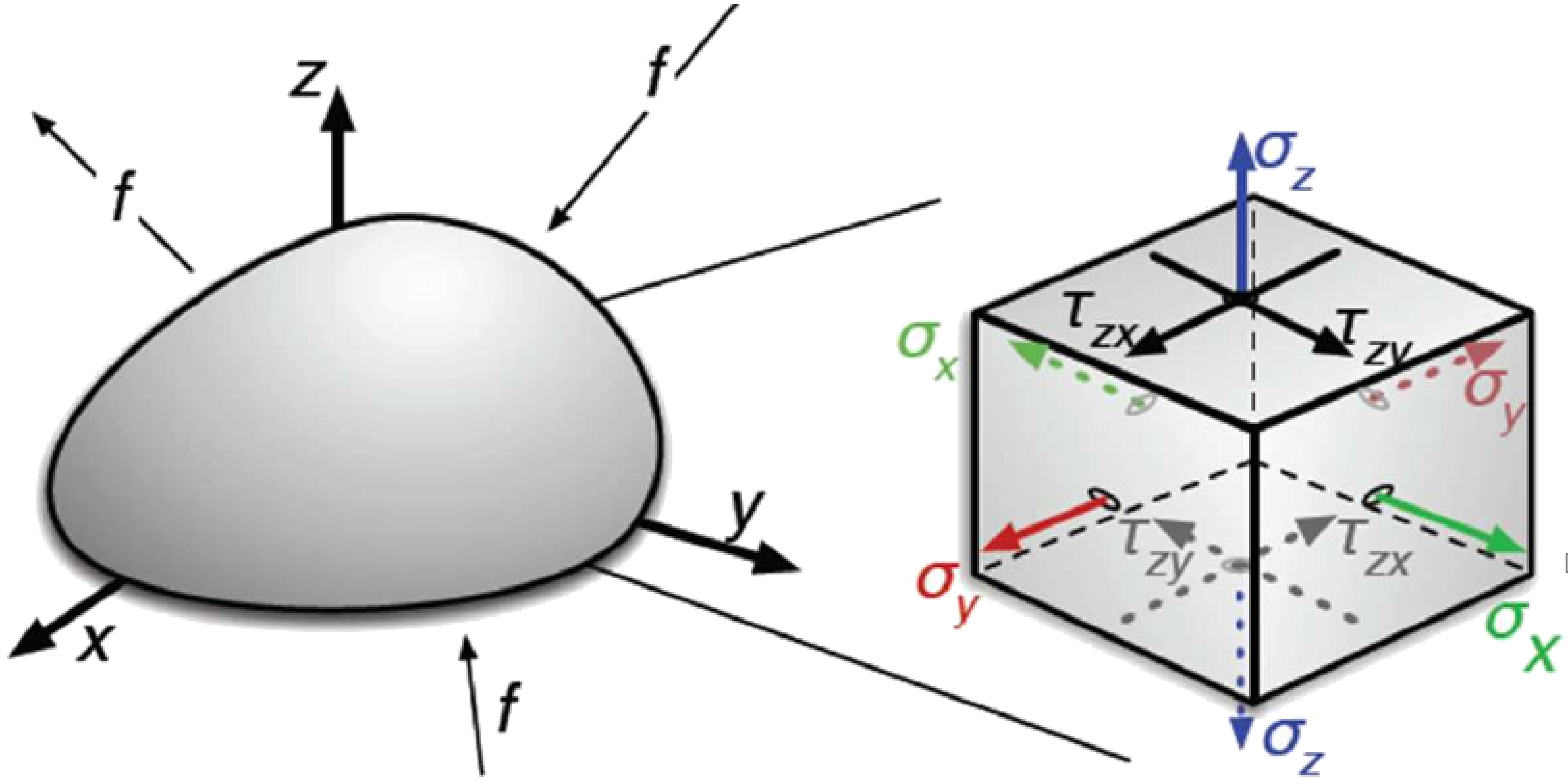
- Mesh generation
  - Notion of direction field
  - Constraints on the metric tensor
    - Smoothness
    - Angle preservation
    - Etc.



[Nieser]

Why?

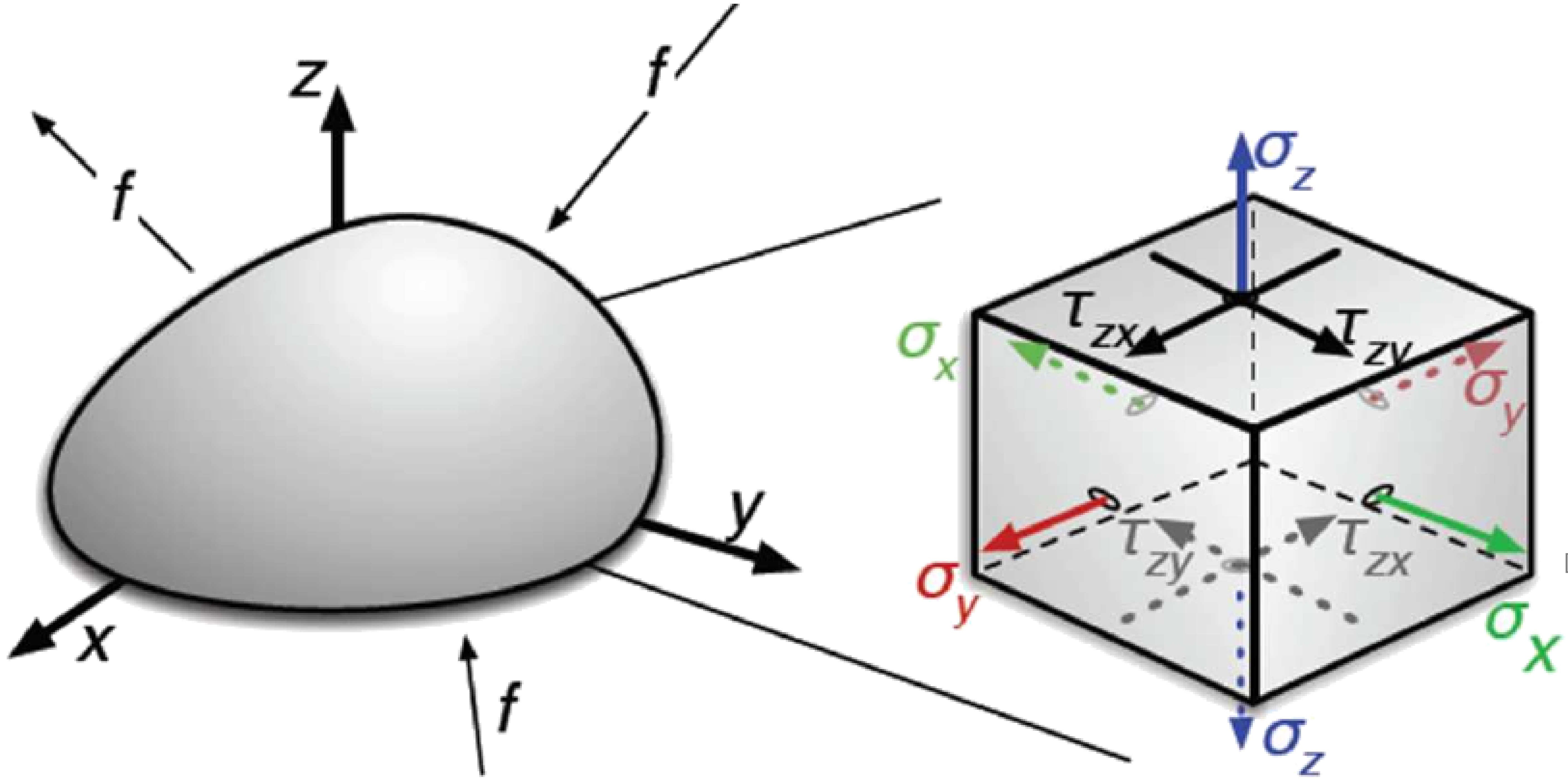
- Mechanics



[Chen]

Why?

- Mechanics
  - Stress force of a body in response to external forces



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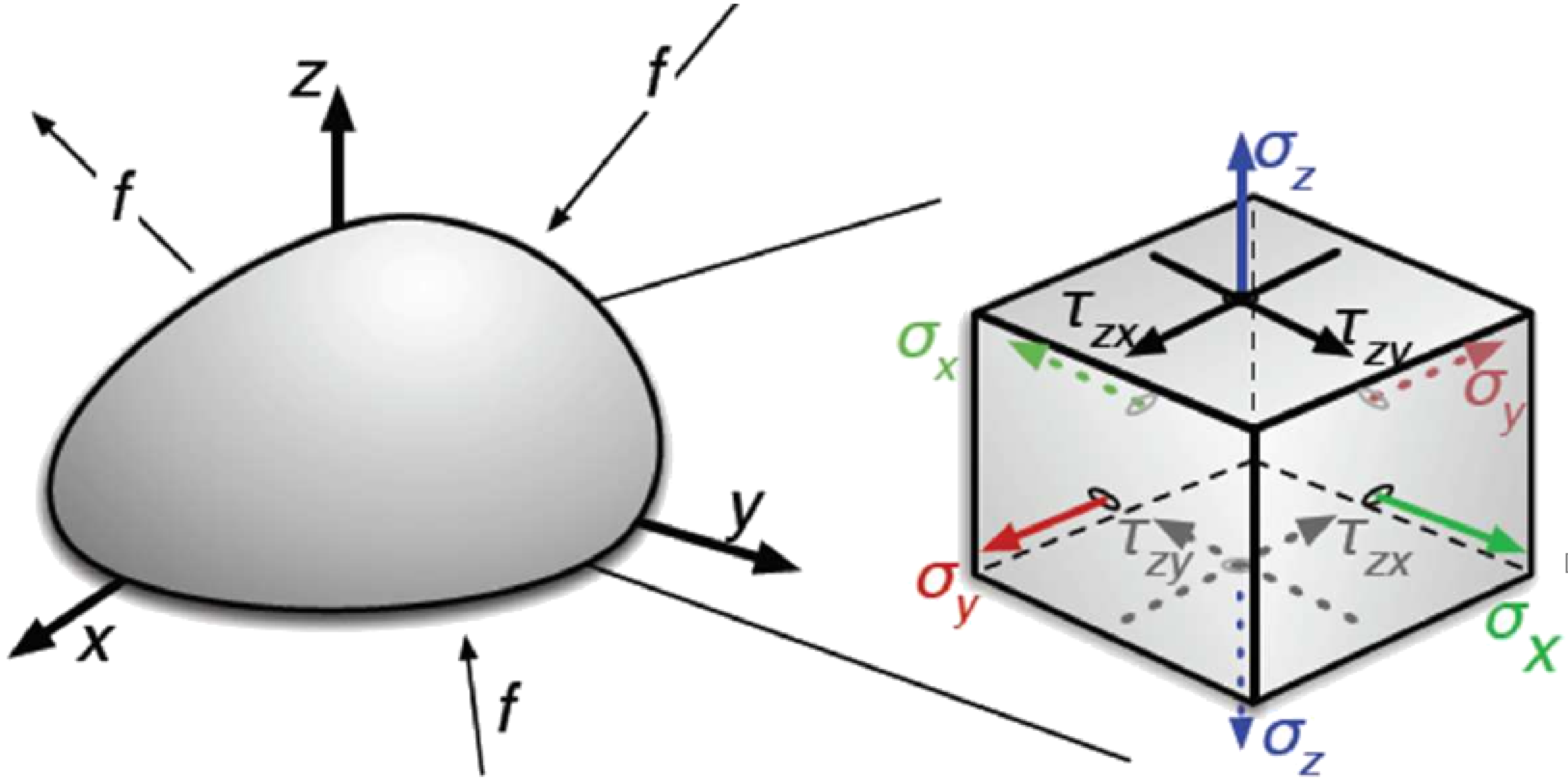

[Chen]

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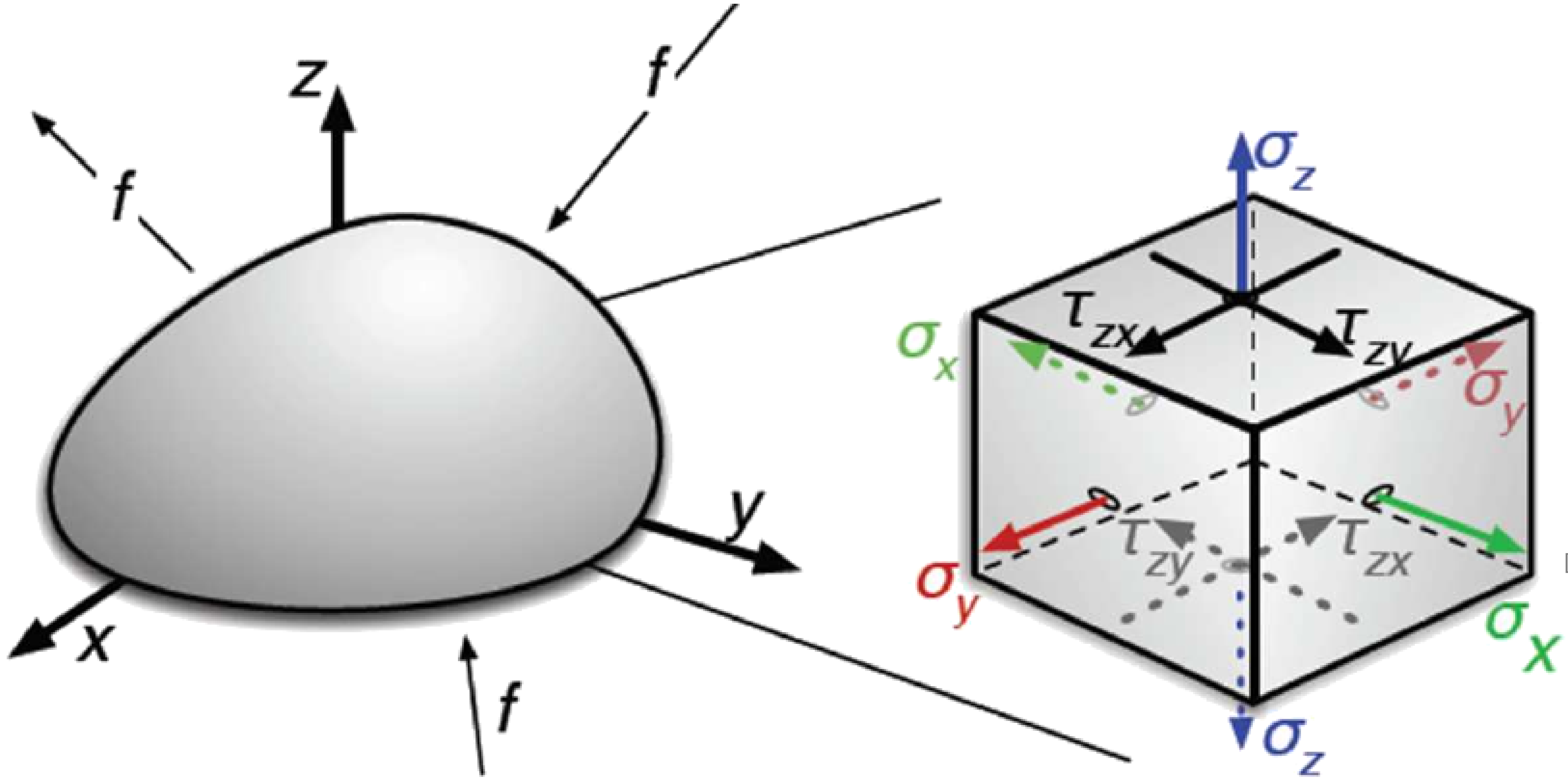
  - Resulting force



[Chen]

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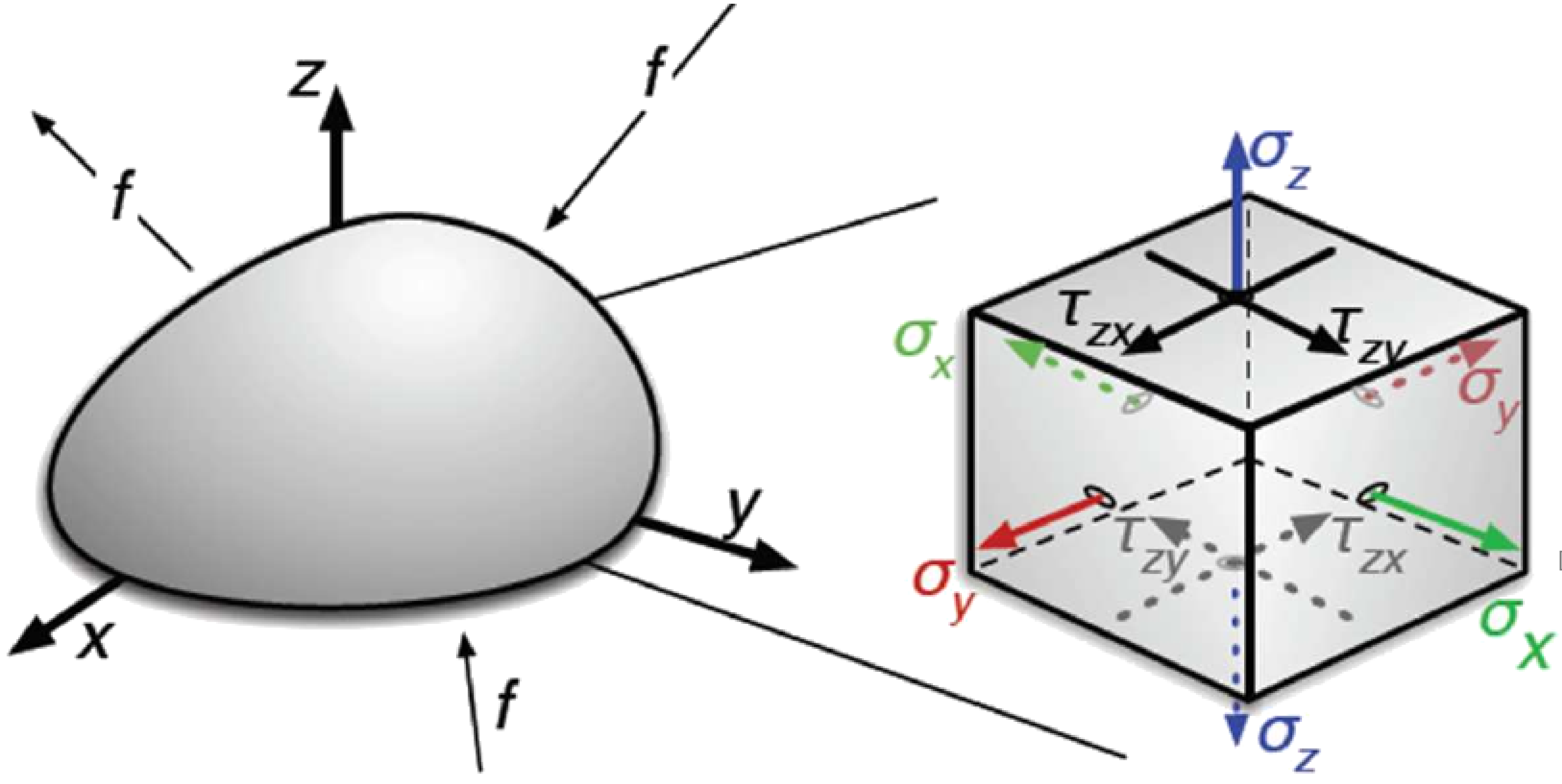
    - Given an input force applied on the material



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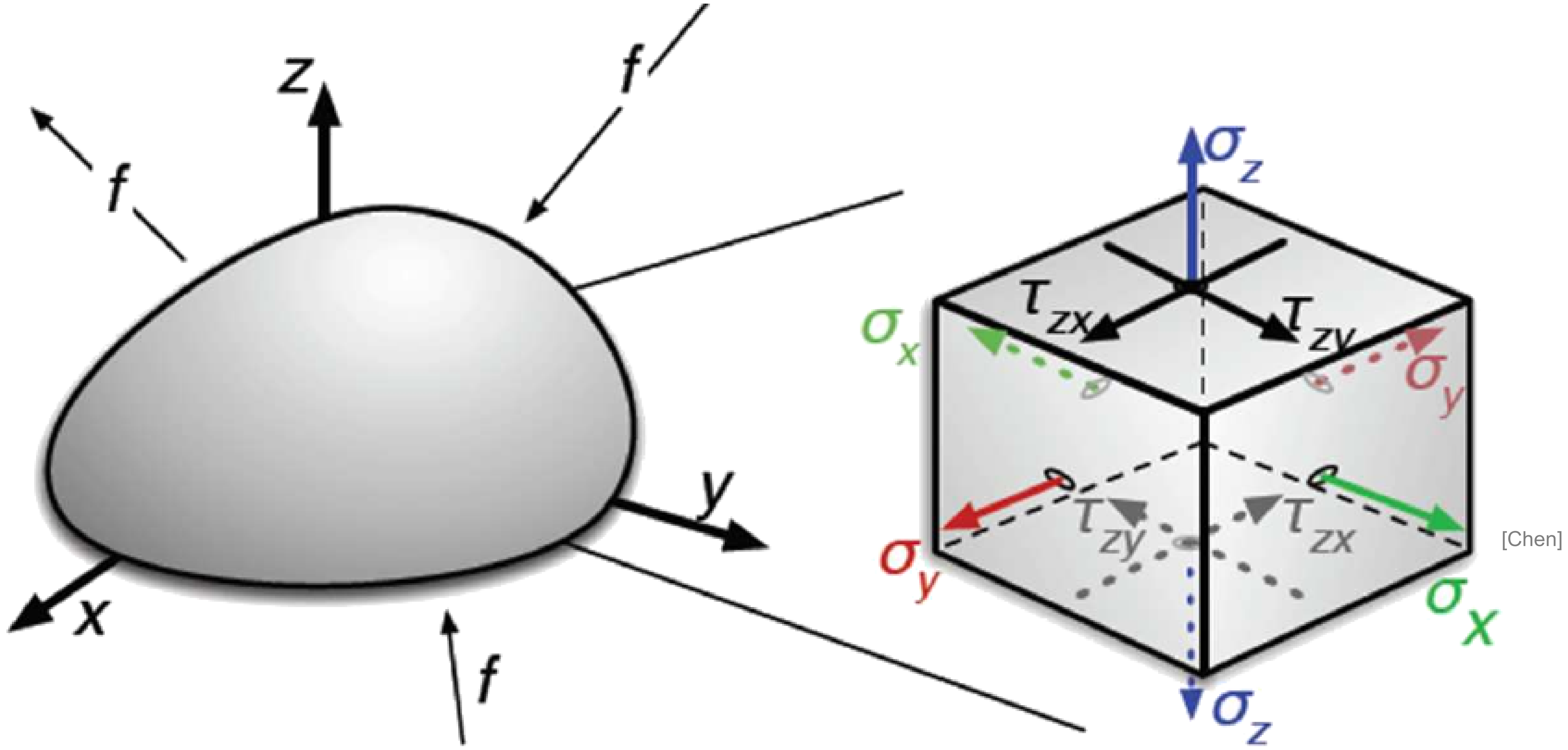

  - Resulting force
    - Given an input force applied on the material
  - Design, medicine, geology, astrophysics, etc.

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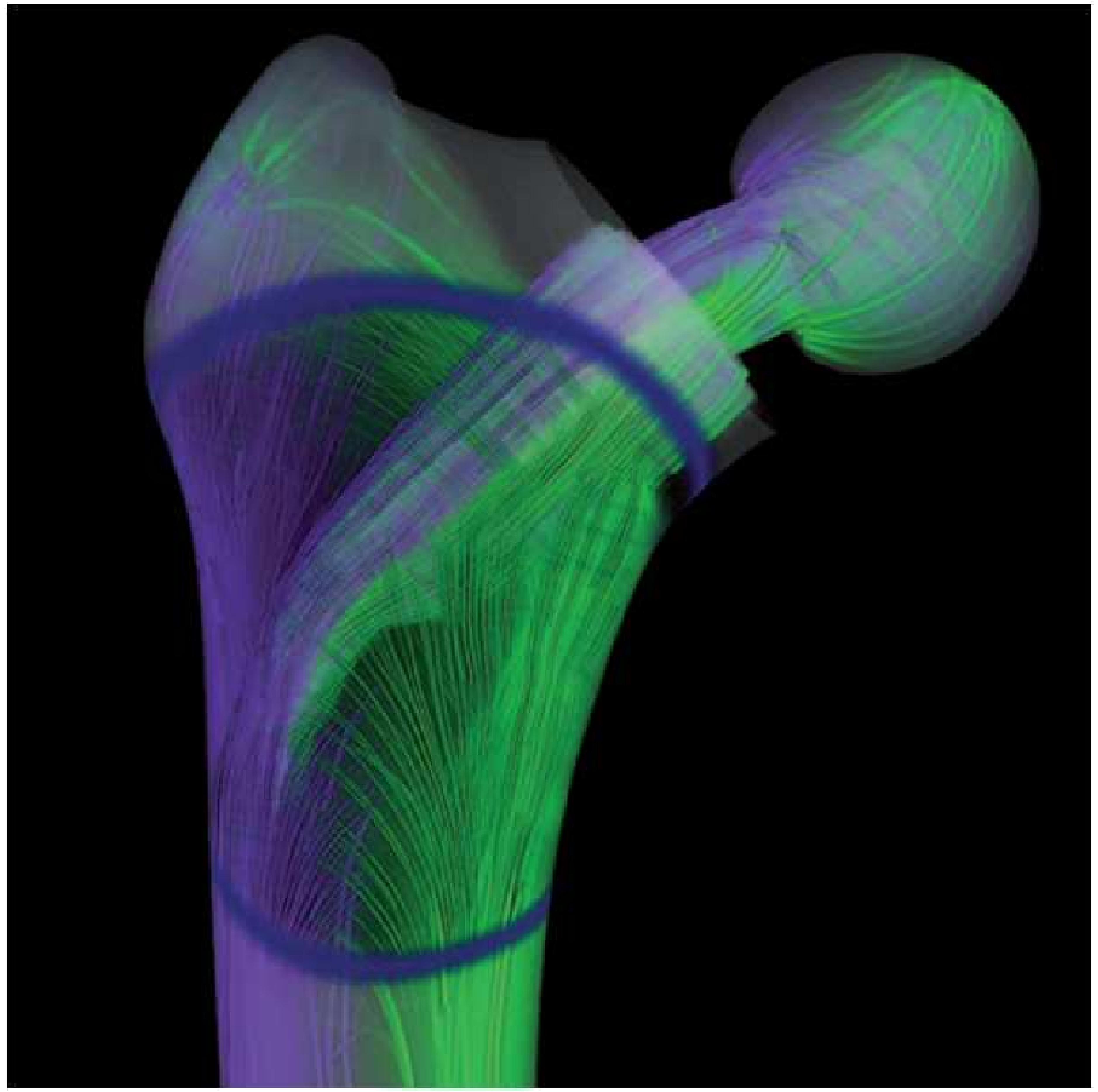
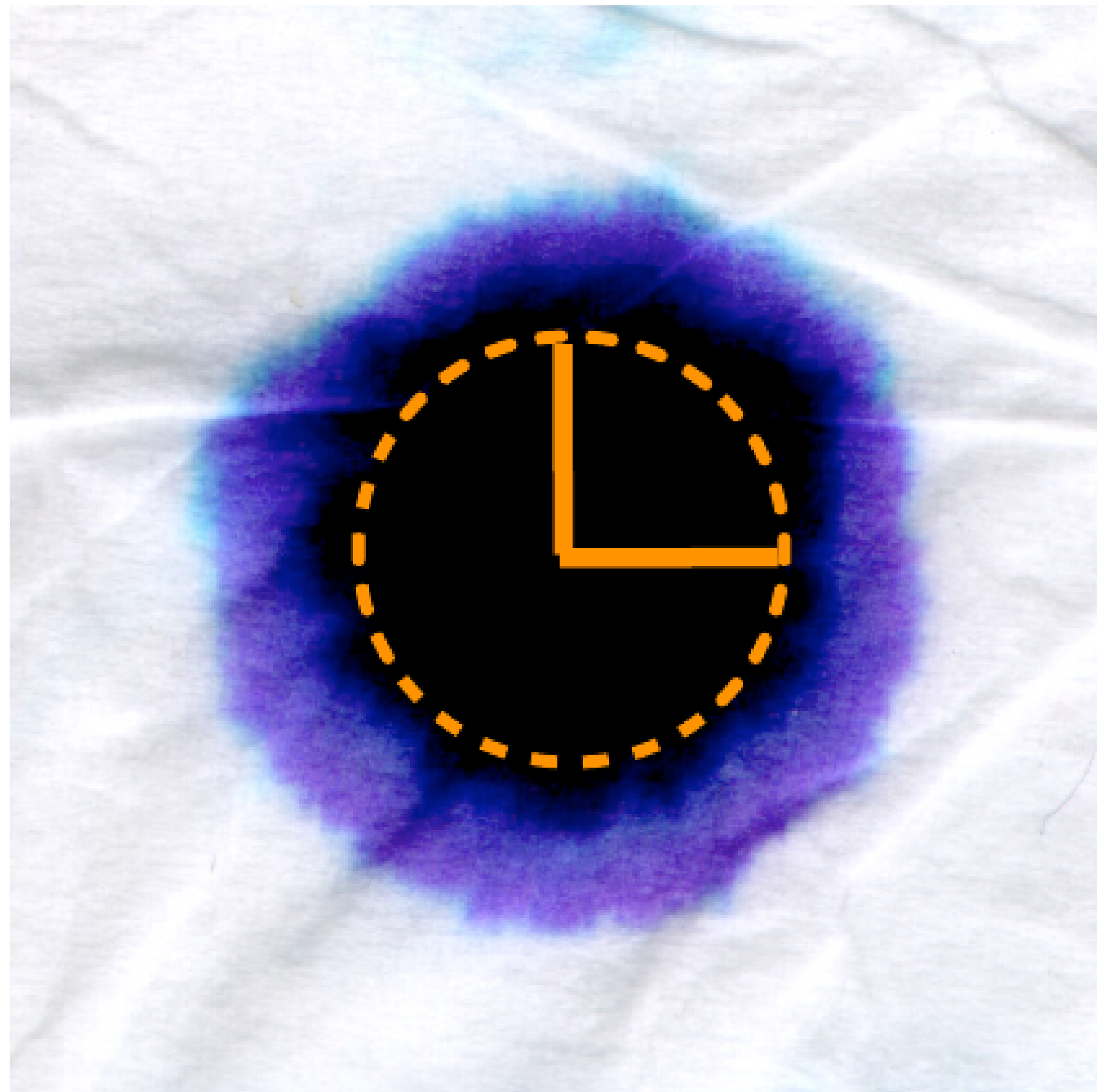


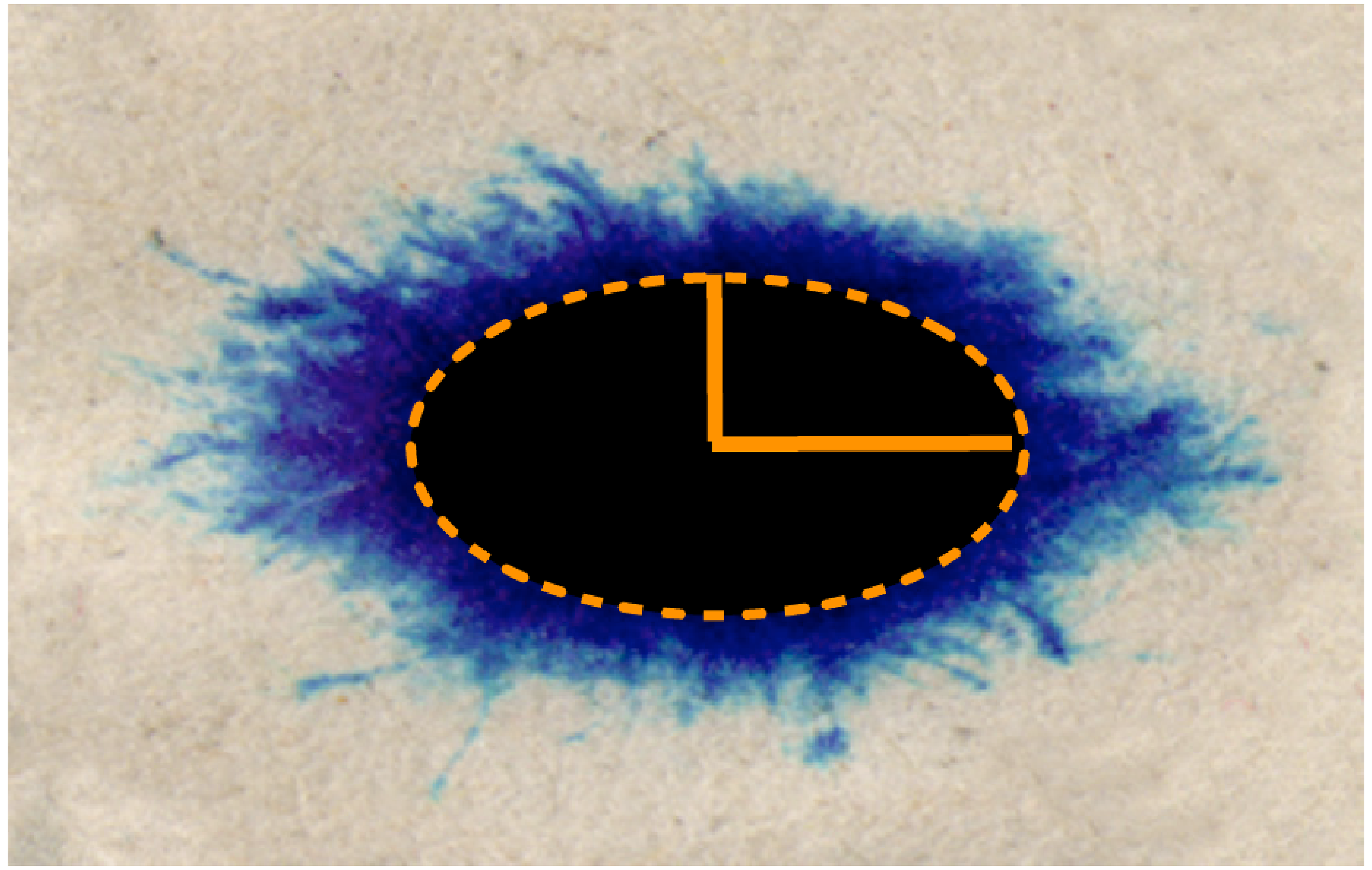
[Dick et al 09]

Why?

- Diffusion processes



Kleenex

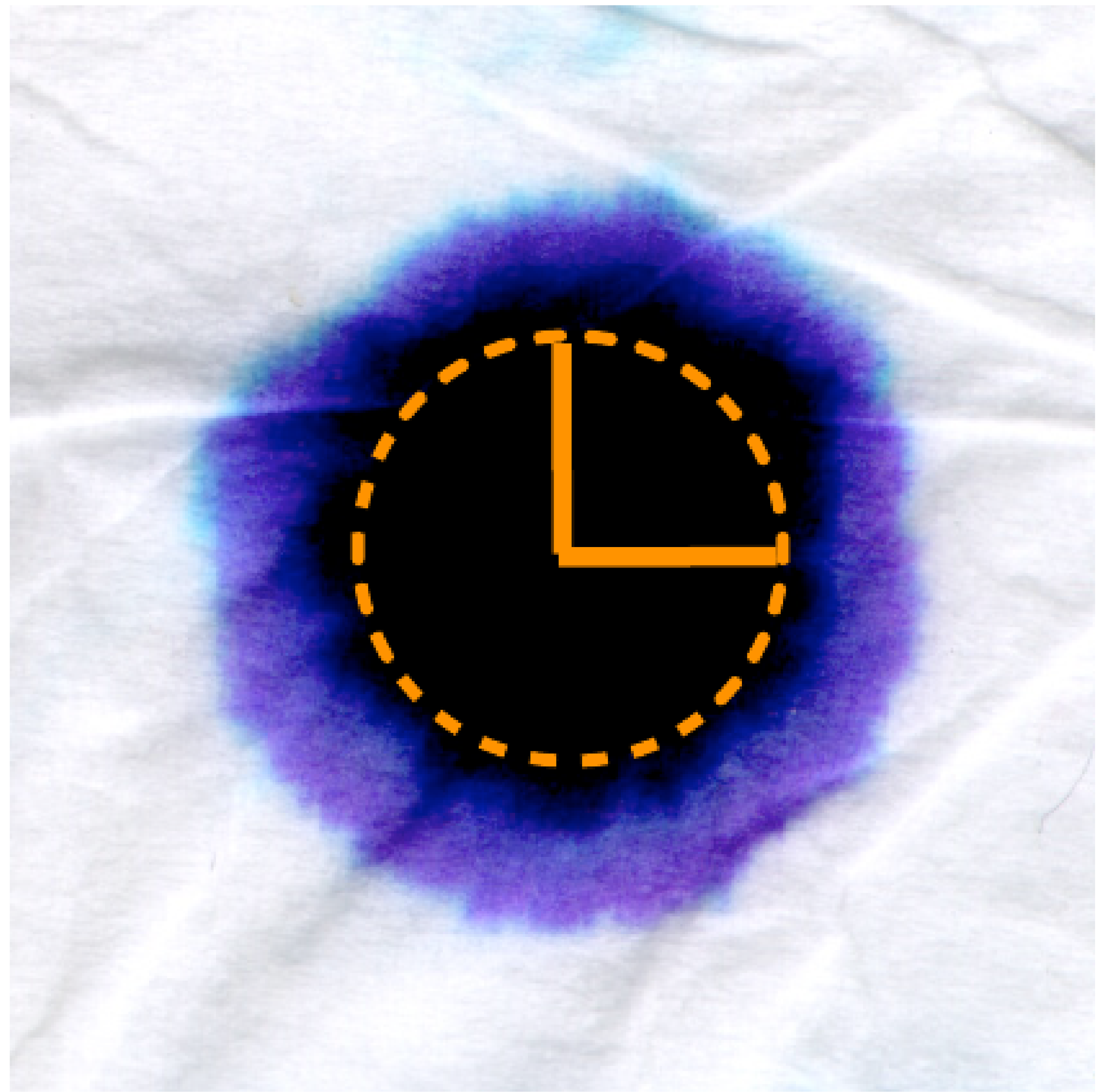


[Kindlmann]

Newspaper

Why?

- Diffusion processes
  - Diffusion properties of a material



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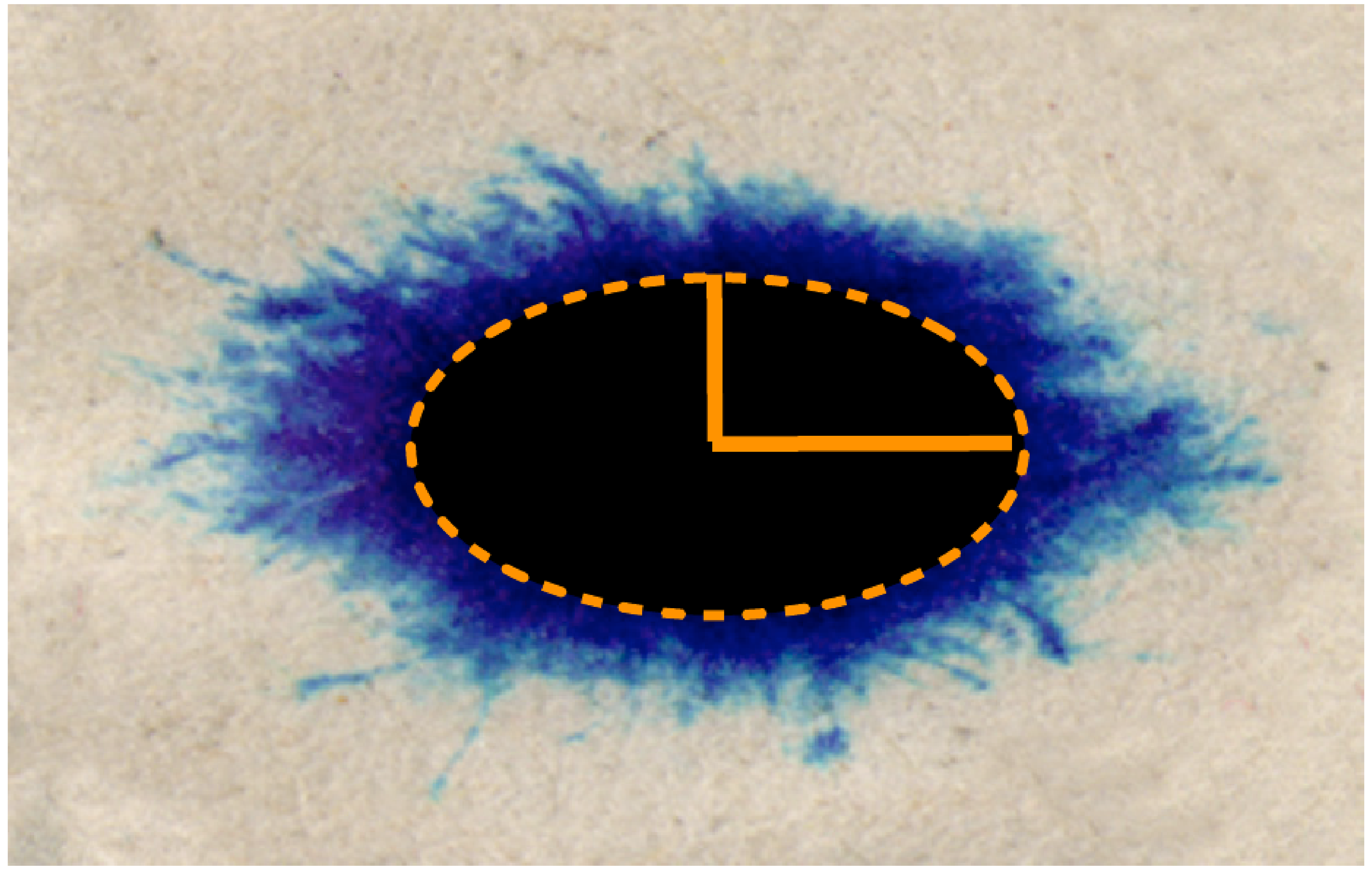
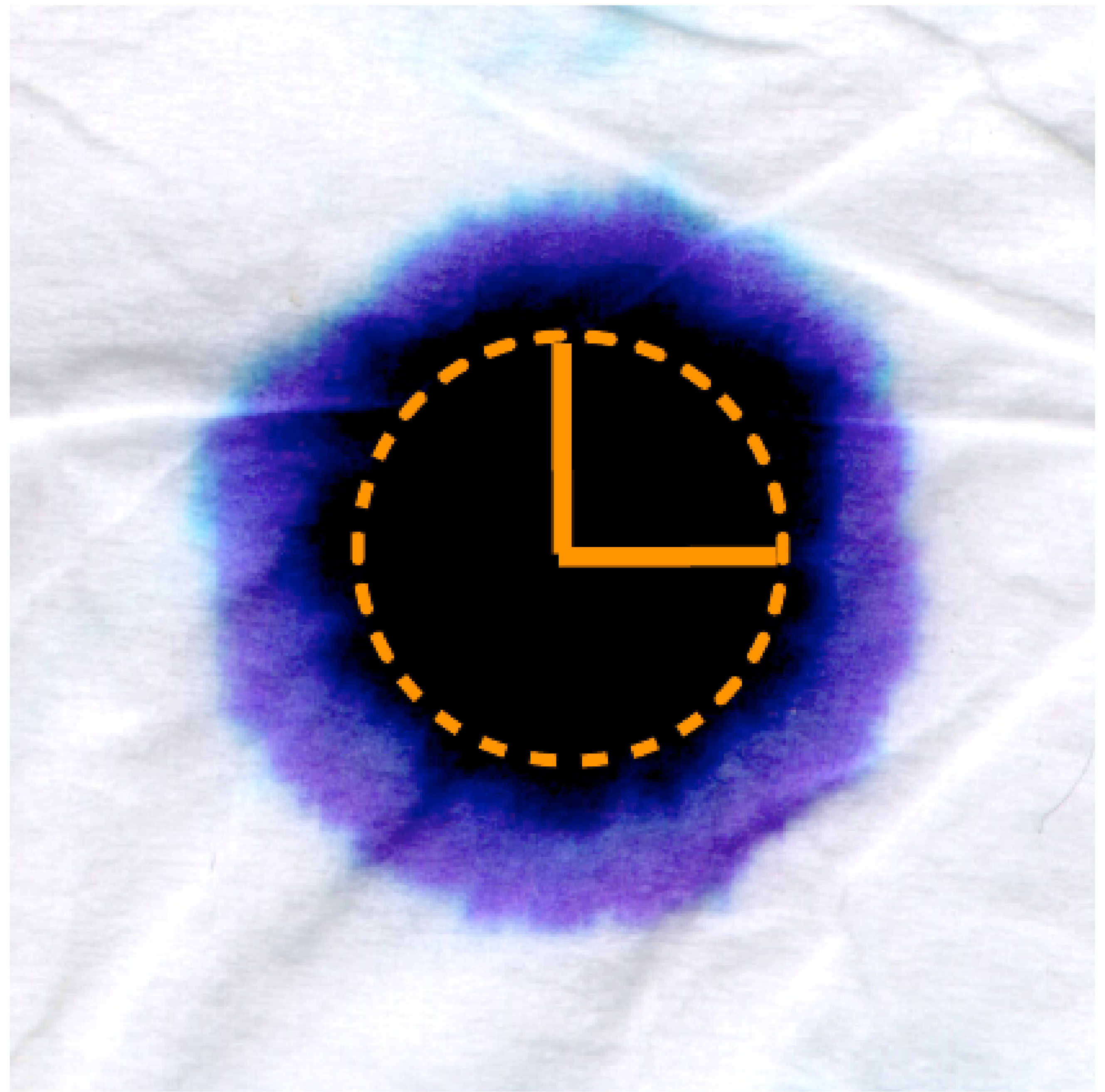


[Kindlmann]

Newspaper

Why?

- Diffusion processes
  - Diffusion properties of a material
    - Diffusion can be faster in a given direction



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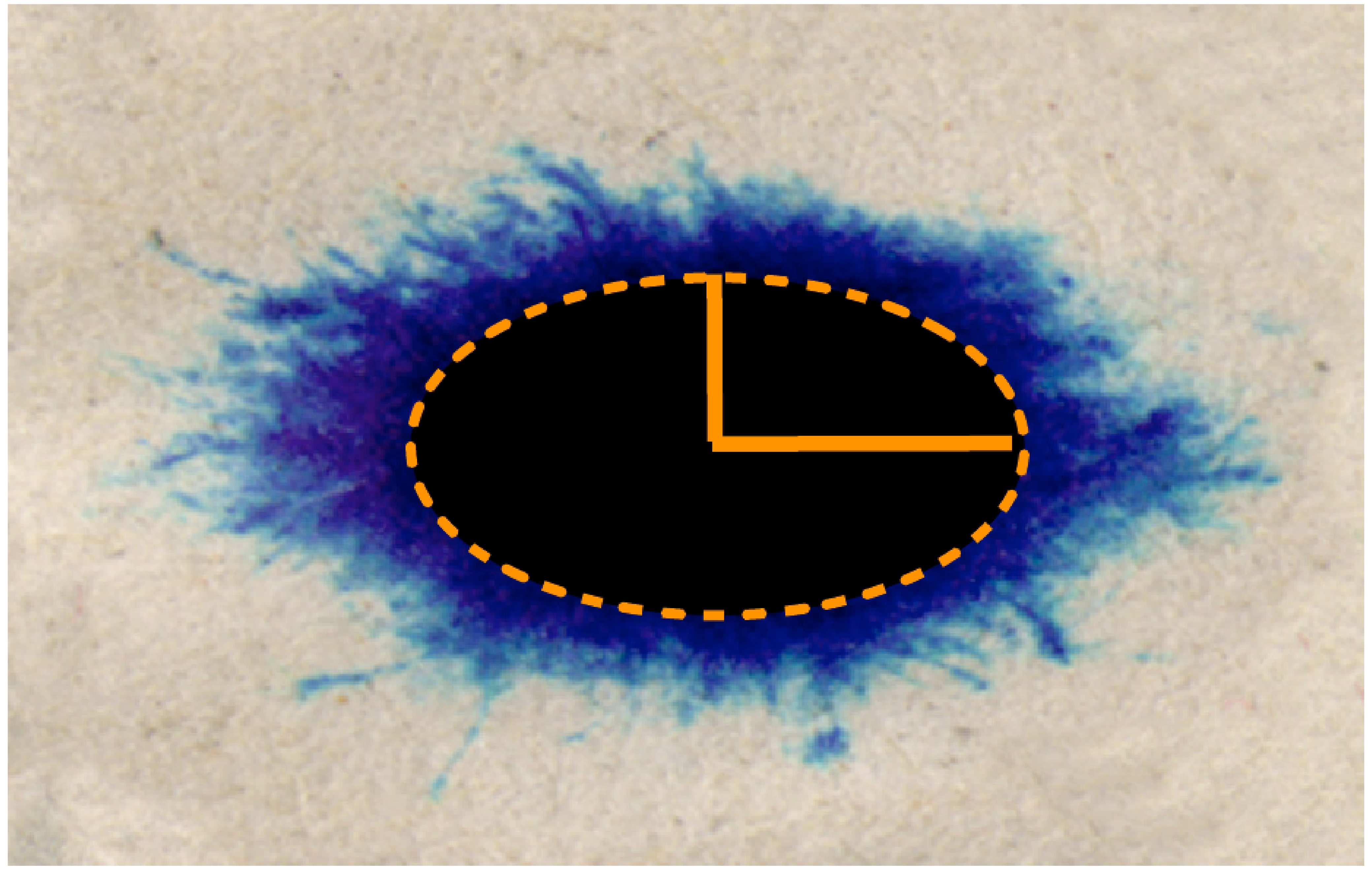
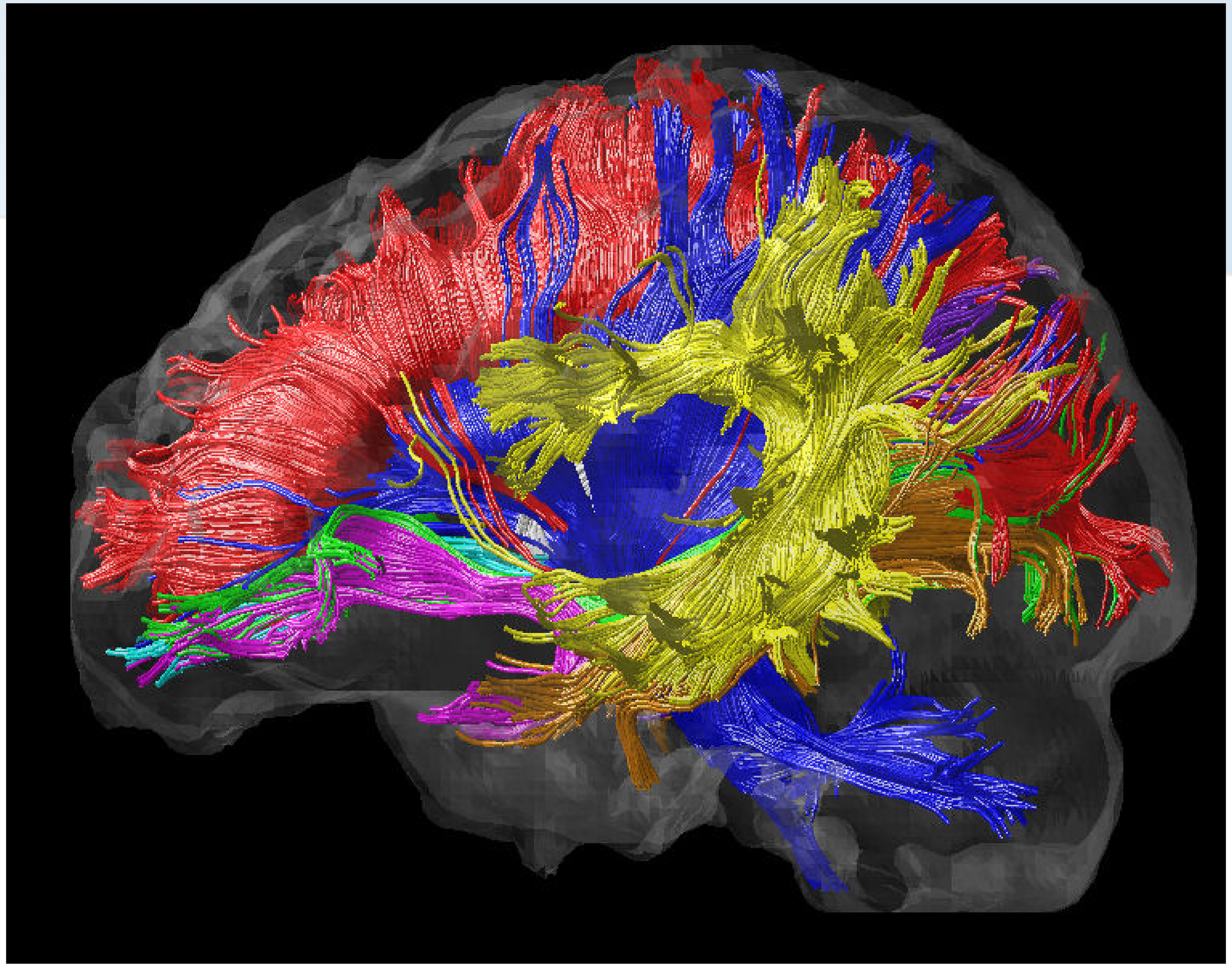


[Kindlmann]

Newspaper

Why?

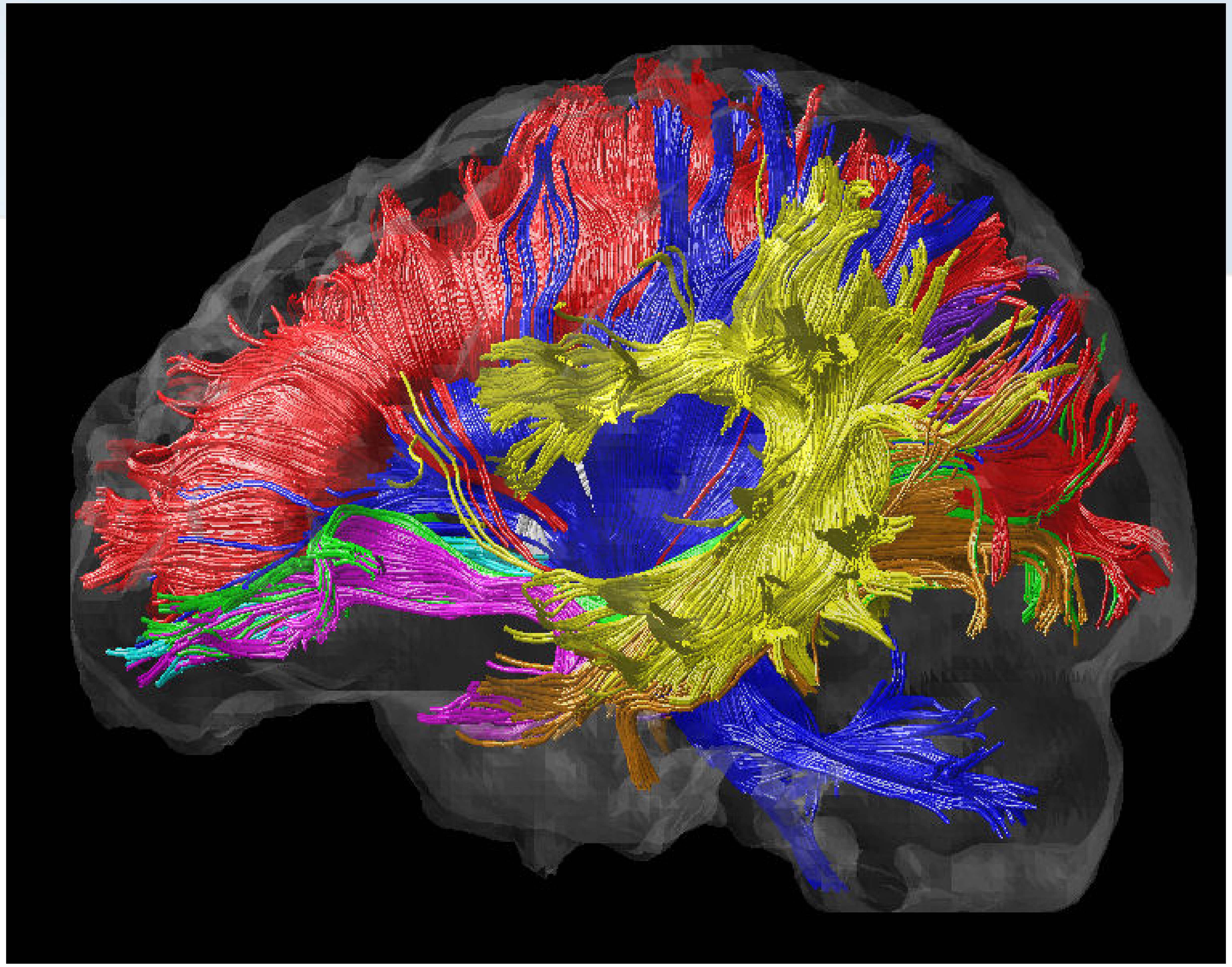
- Diffusion tensor imaging



[http://www.neuroimaging.tau.ac.il]

Why?

- Diffusion tensor imaging
  - Medical applications



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- Diffusion tensor imaging
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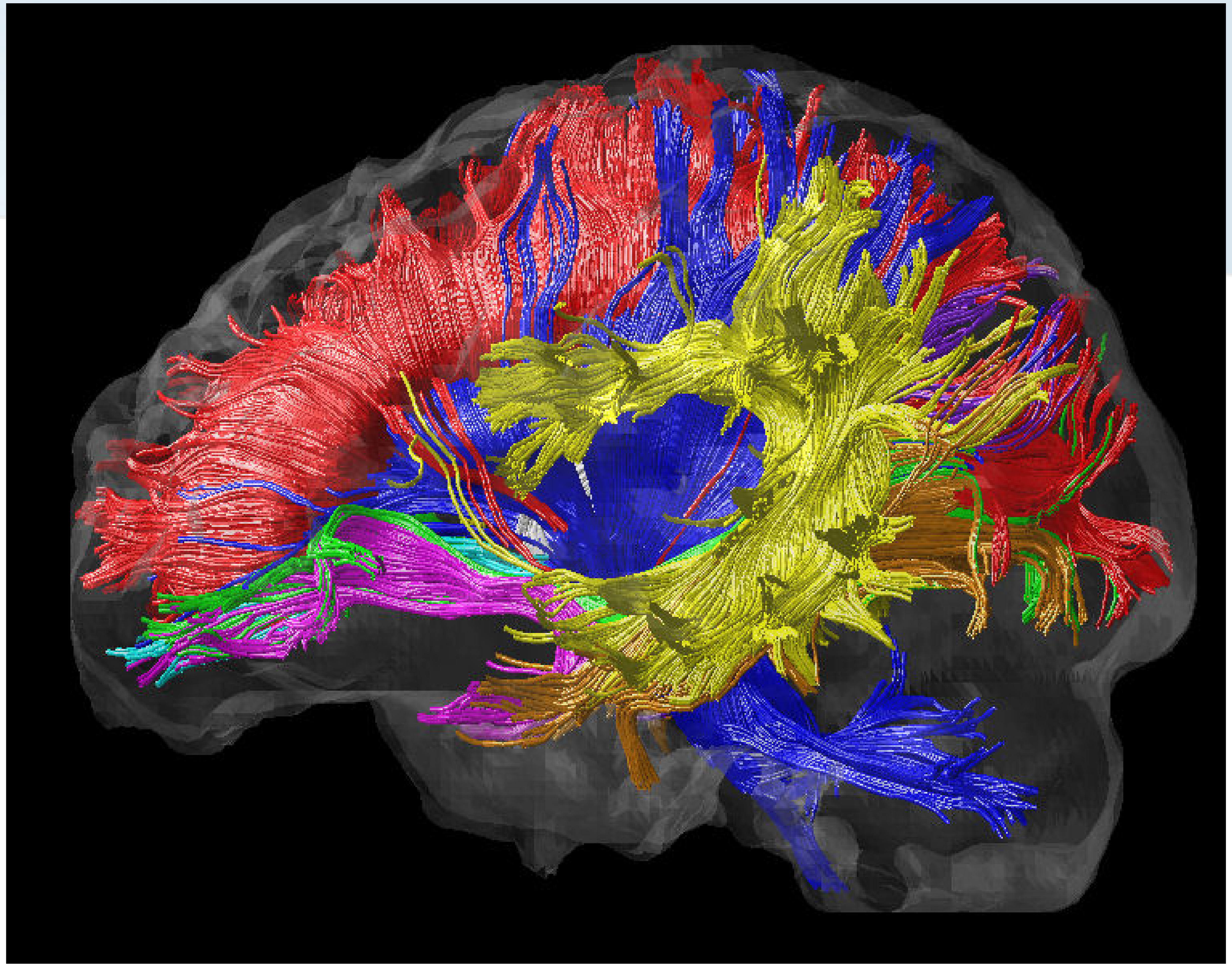
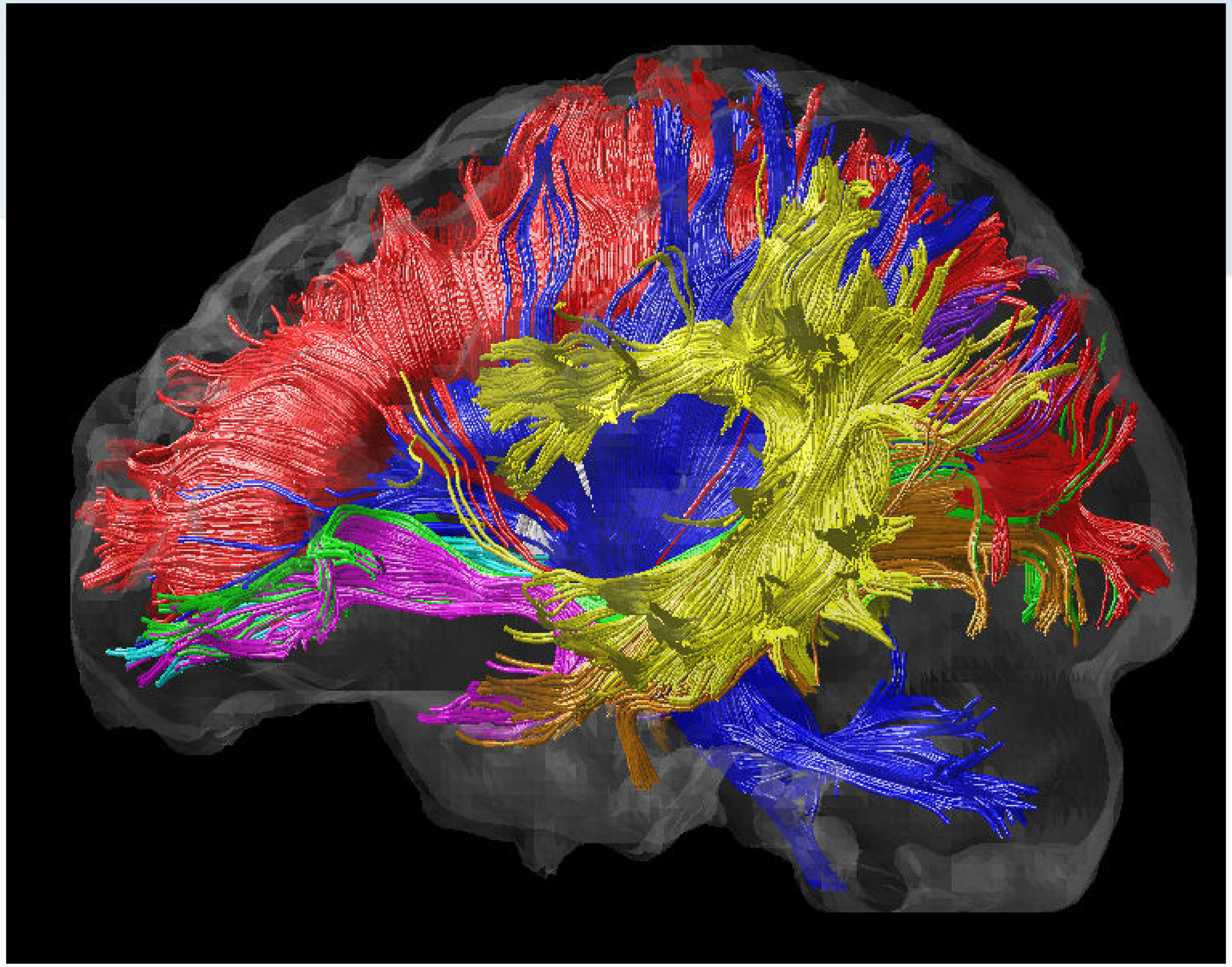
  - Neuro-imaging
    - Anisotropic diffusion of water molecules in tissues



[http://www.neuroimaging.tau.ac.il]

Why?

- Diffusion tensor imaging
  - Medical applications
  - Neuro-imaging
    - Anisotropic diffusion of water molecules in tissues
  - Diffusion tensor
    - Strength of the diffusion for a given direction
    - Anisotropy of the diffusion



[http://www.neuroimaging.tau.ac.il]

In practice

- Given a domain $\mathcal{D}$

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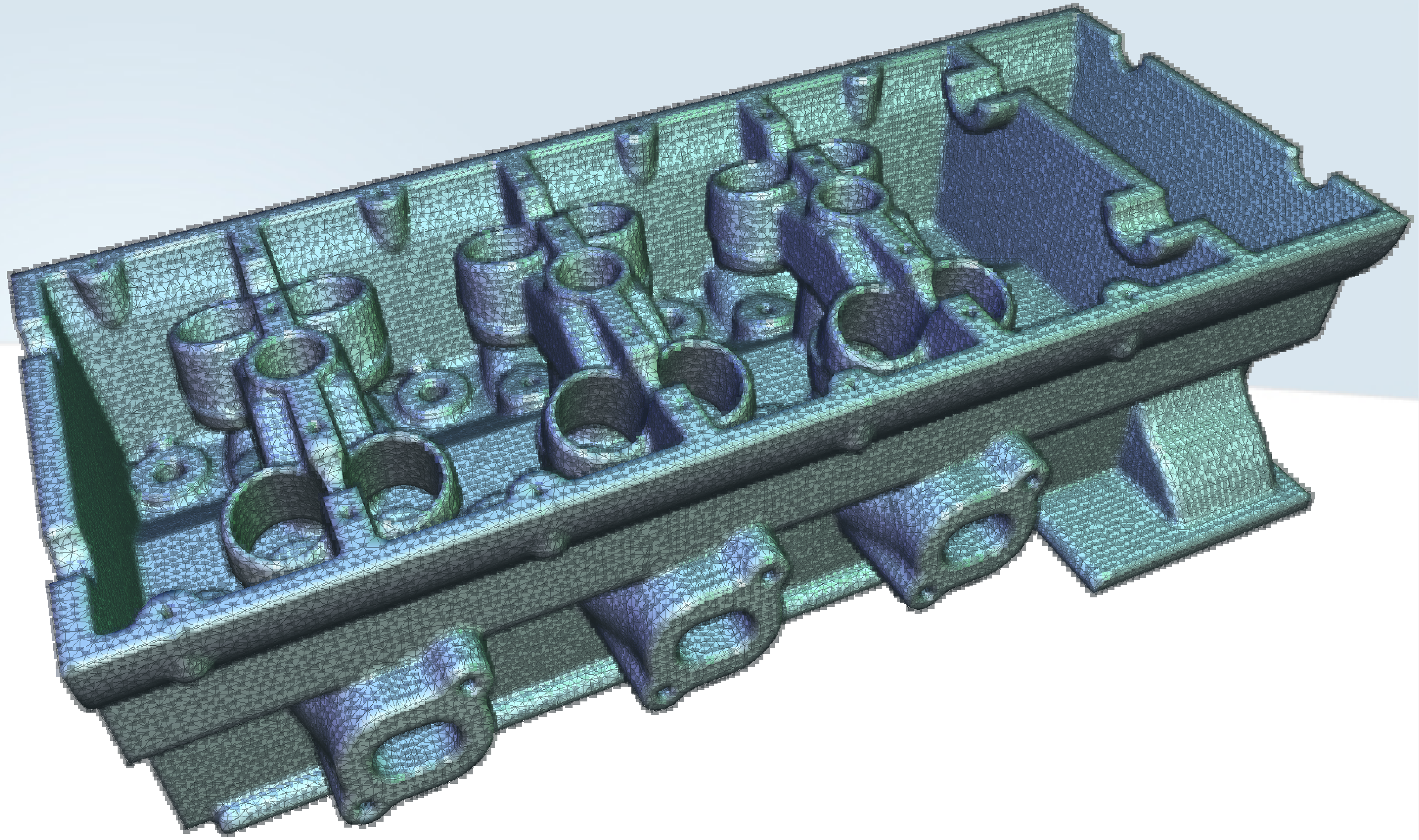

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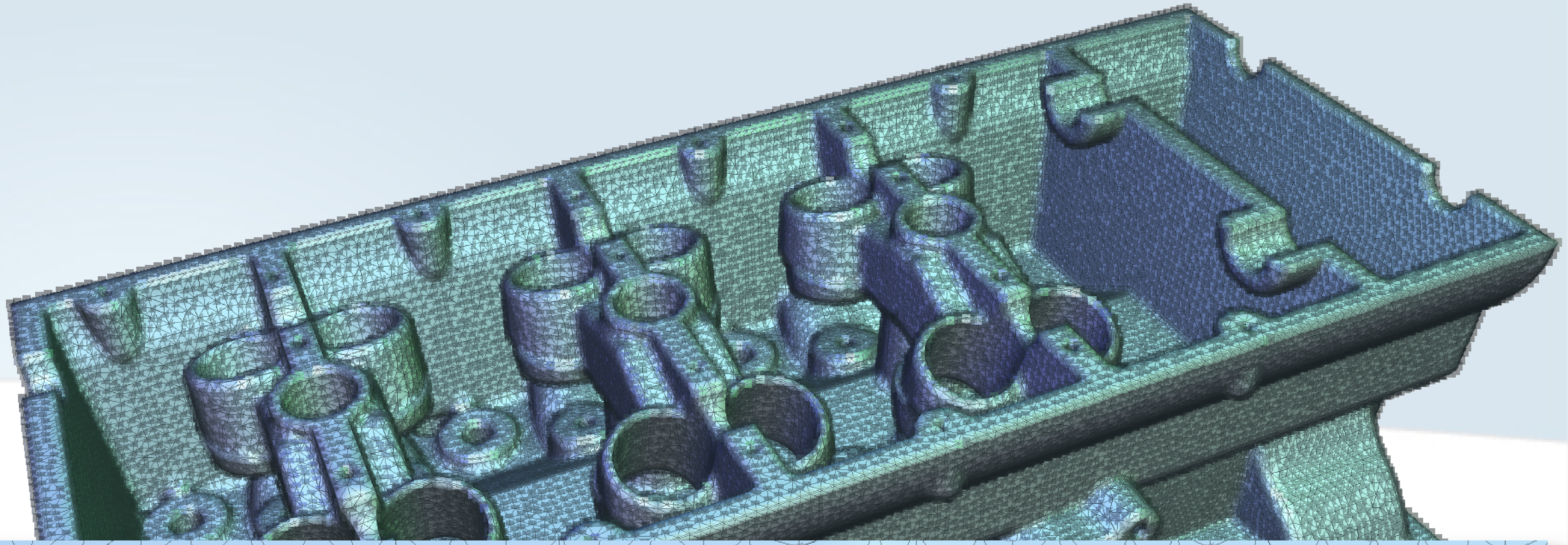
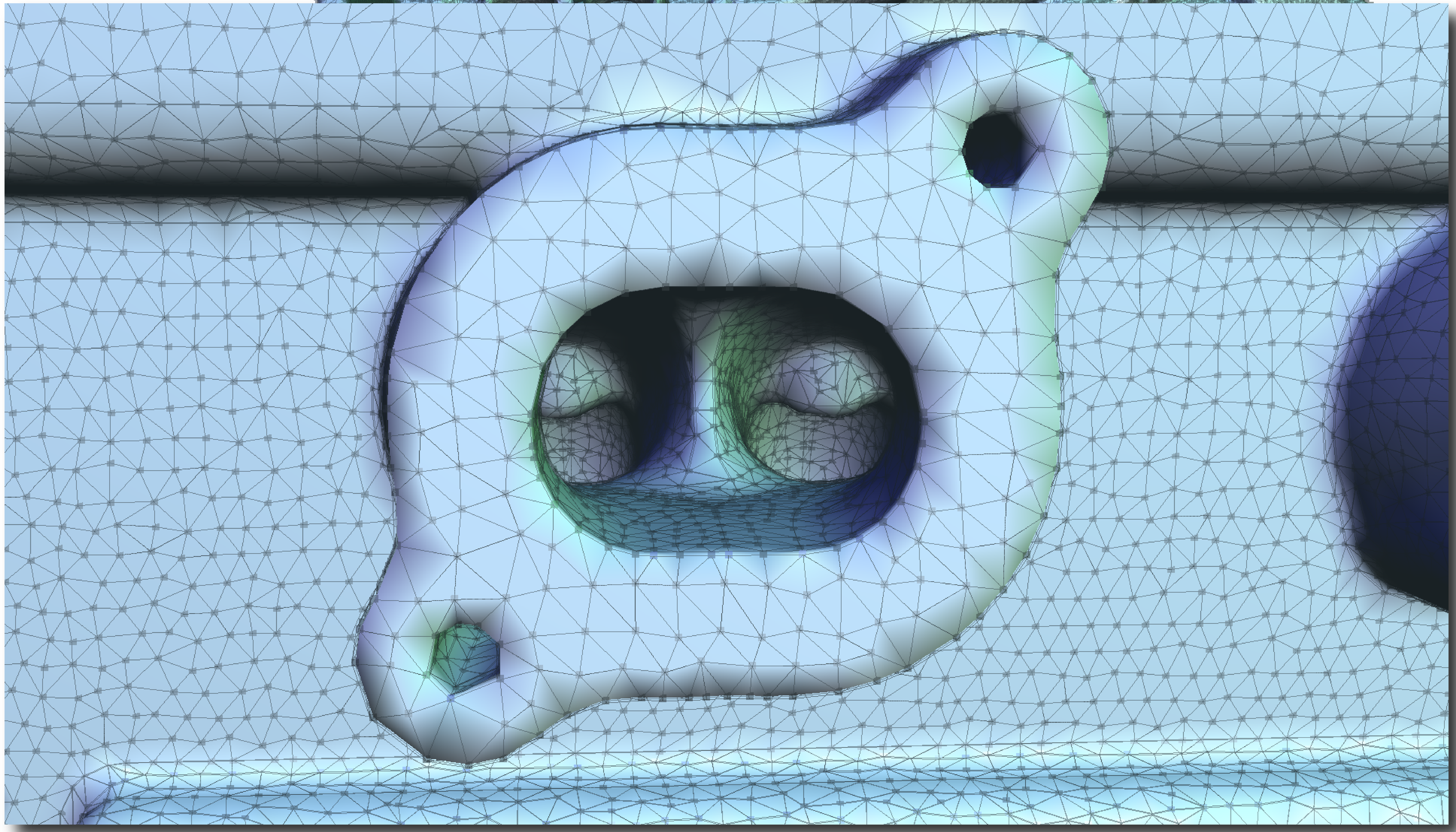
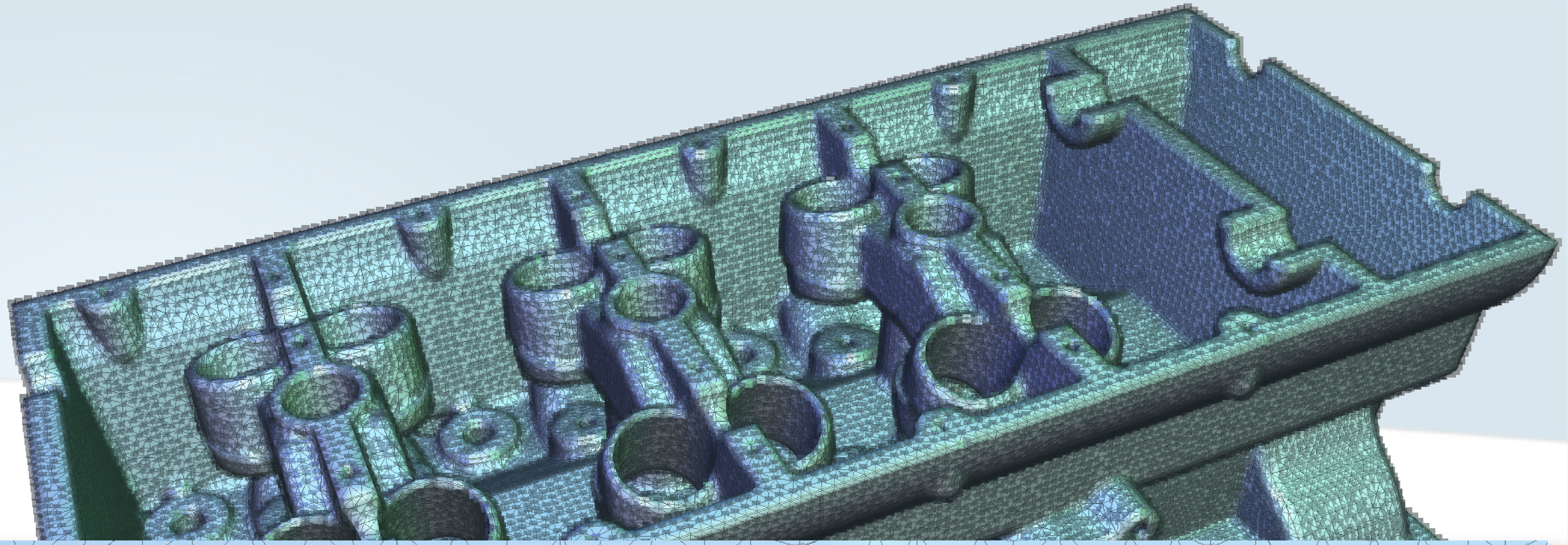
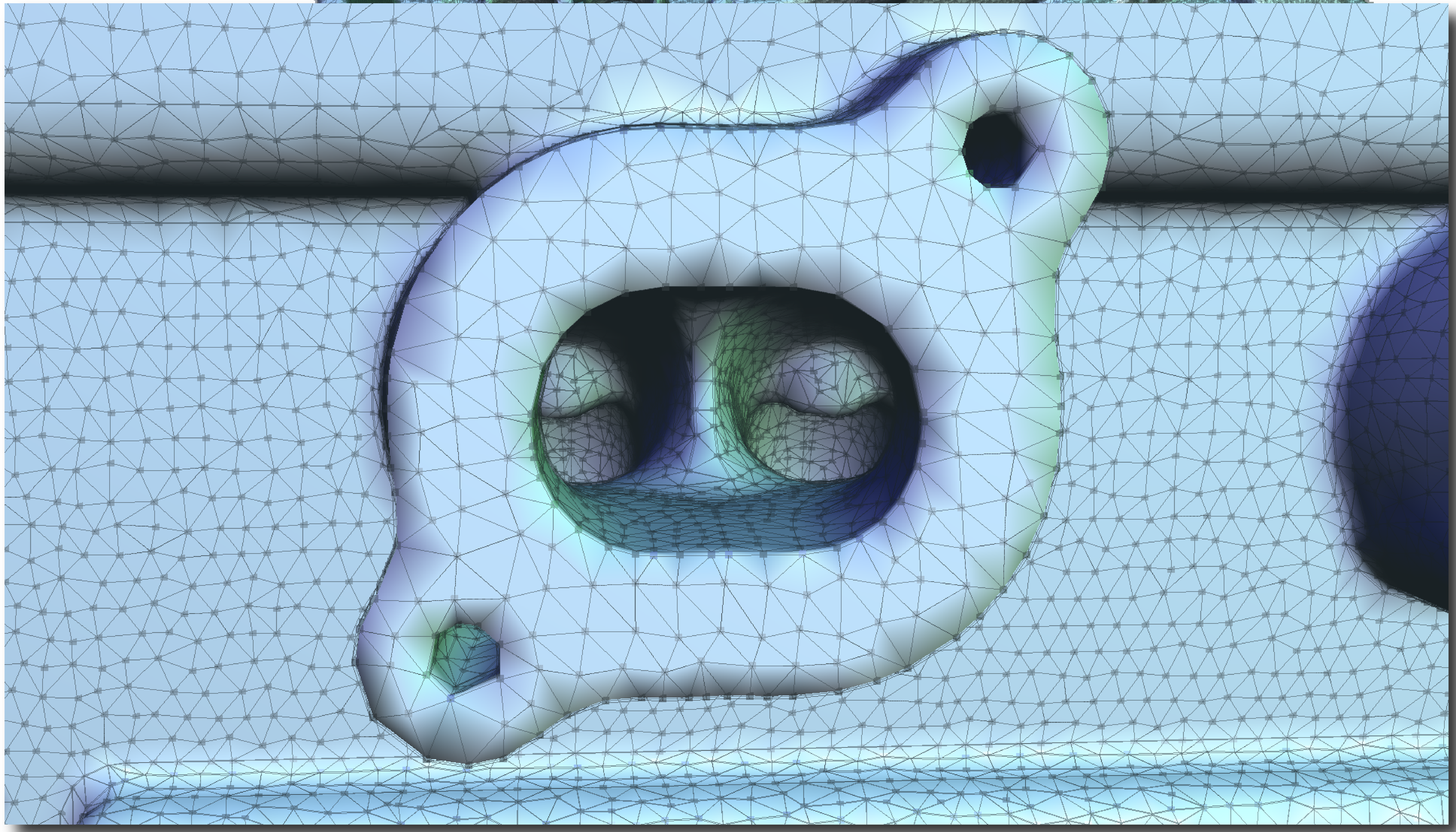




In practice

- Given a domain $\mathcal{D}$
- For each vertex v





In practice

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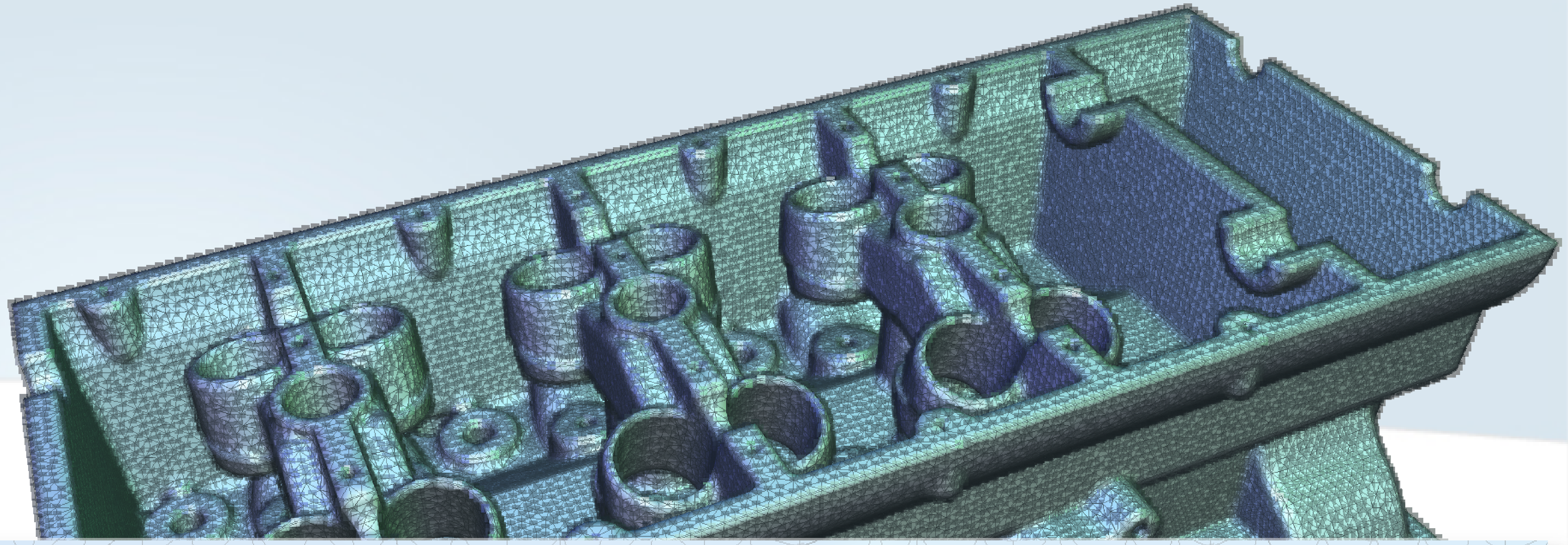

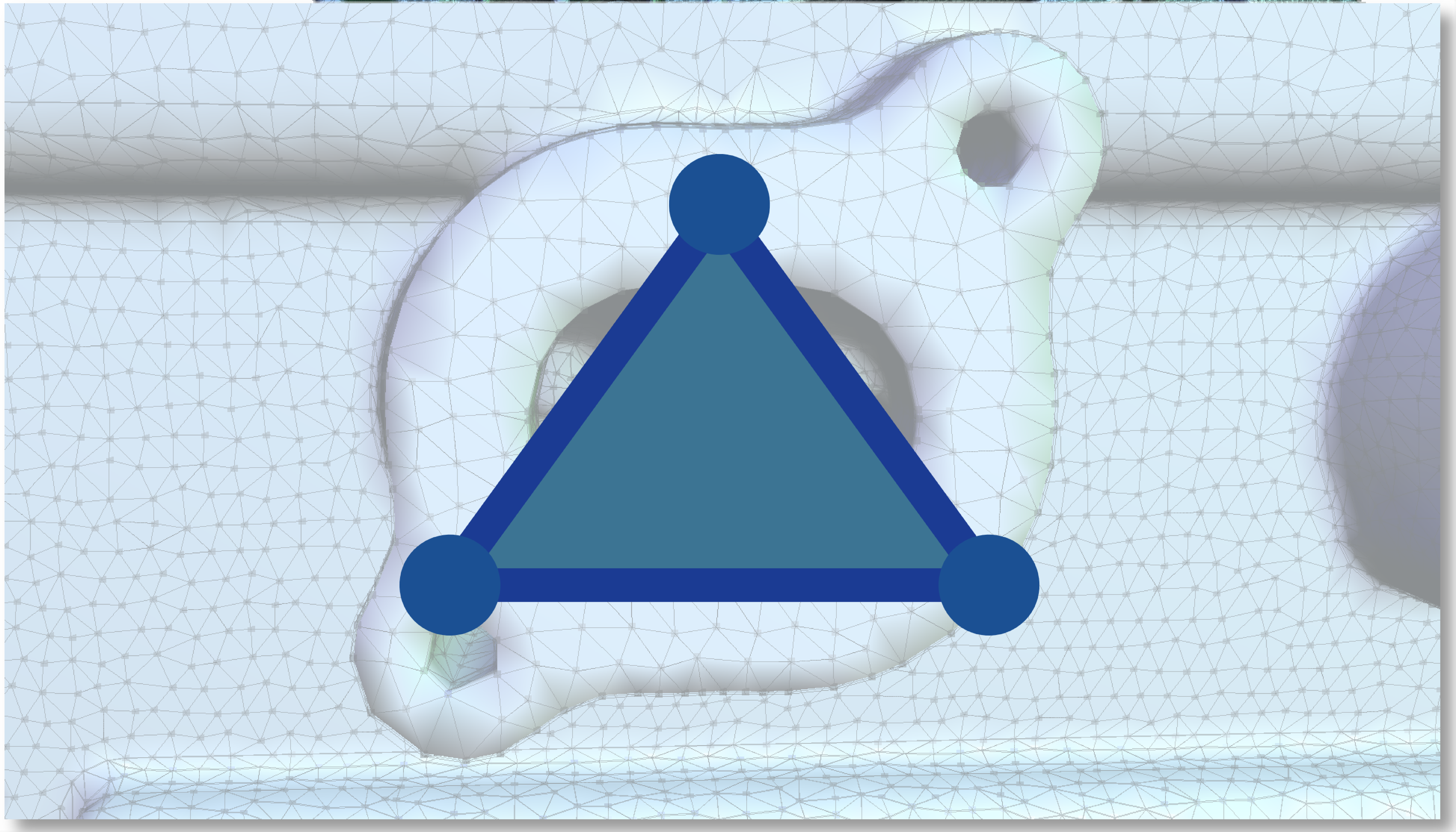

In practice

- Given a domain $\mathcal{D}$
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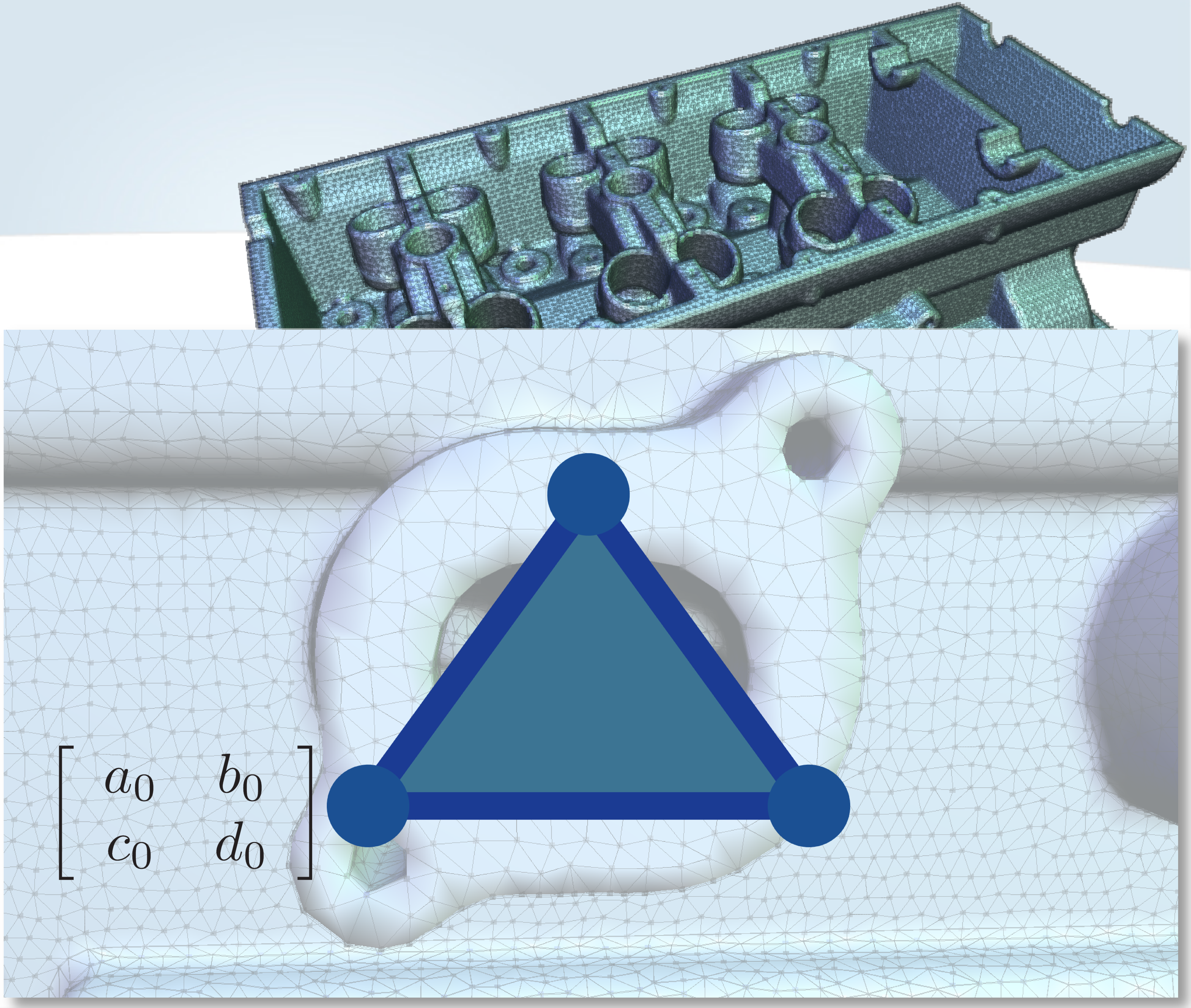
- One matrix $f(v)$



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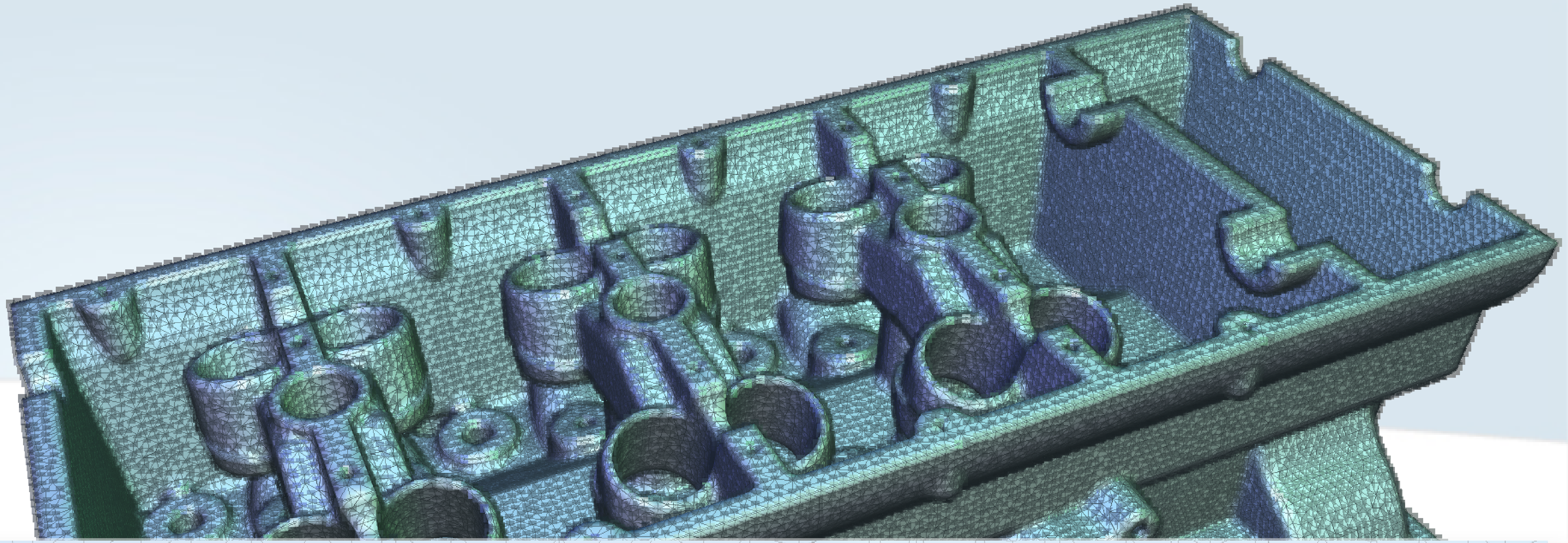

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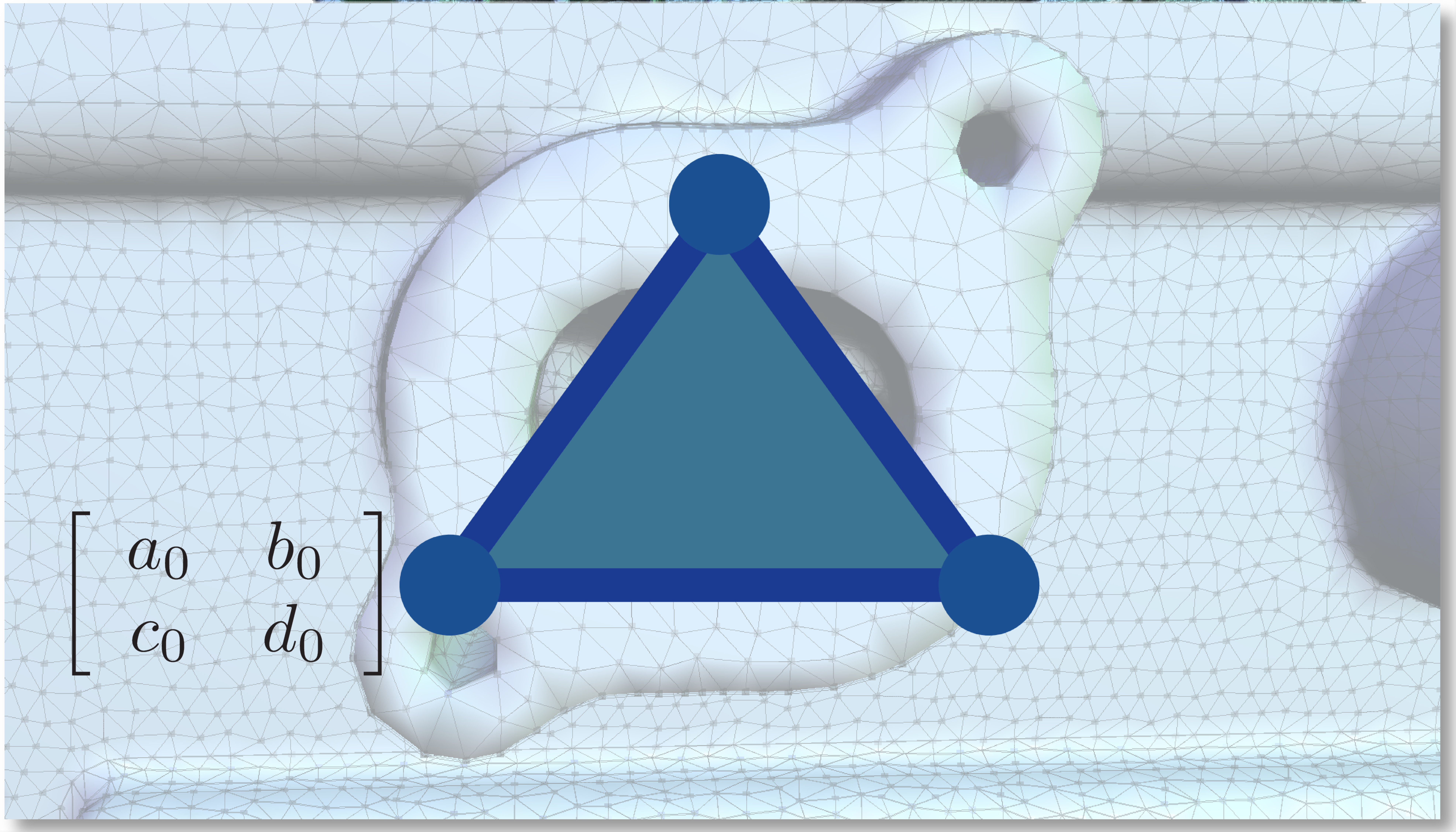
  - (dxd)-matrix
  - d: dimension of $\mathcal{D}$



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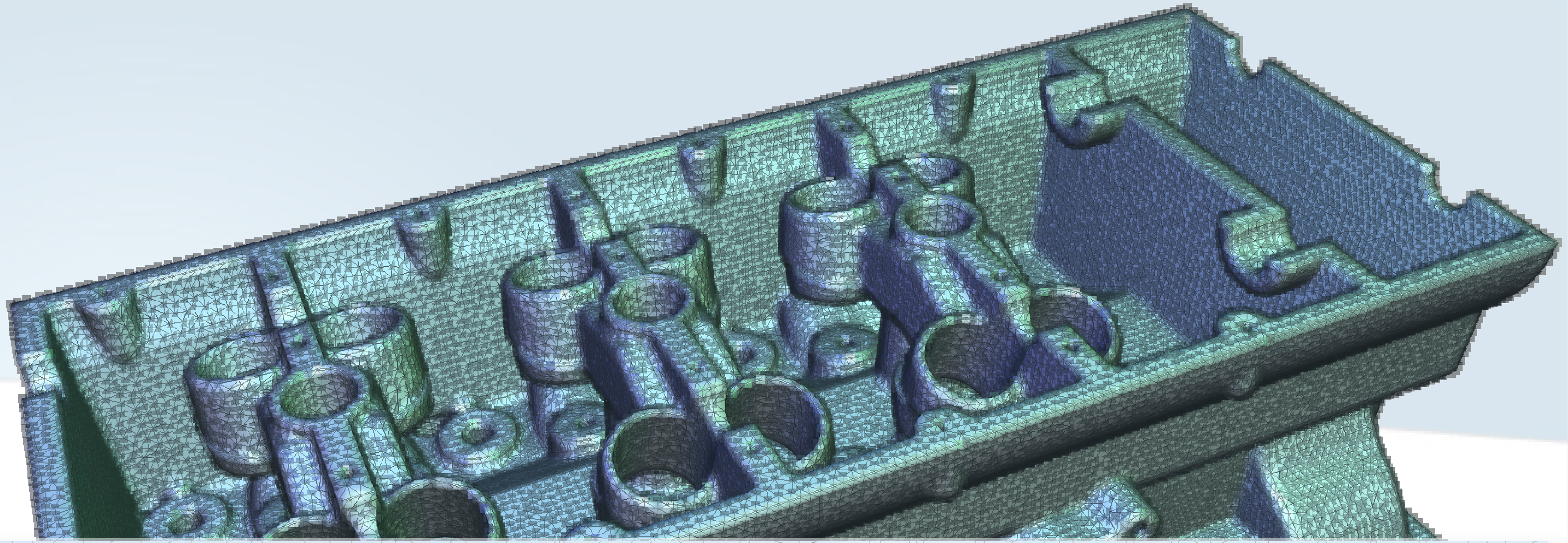
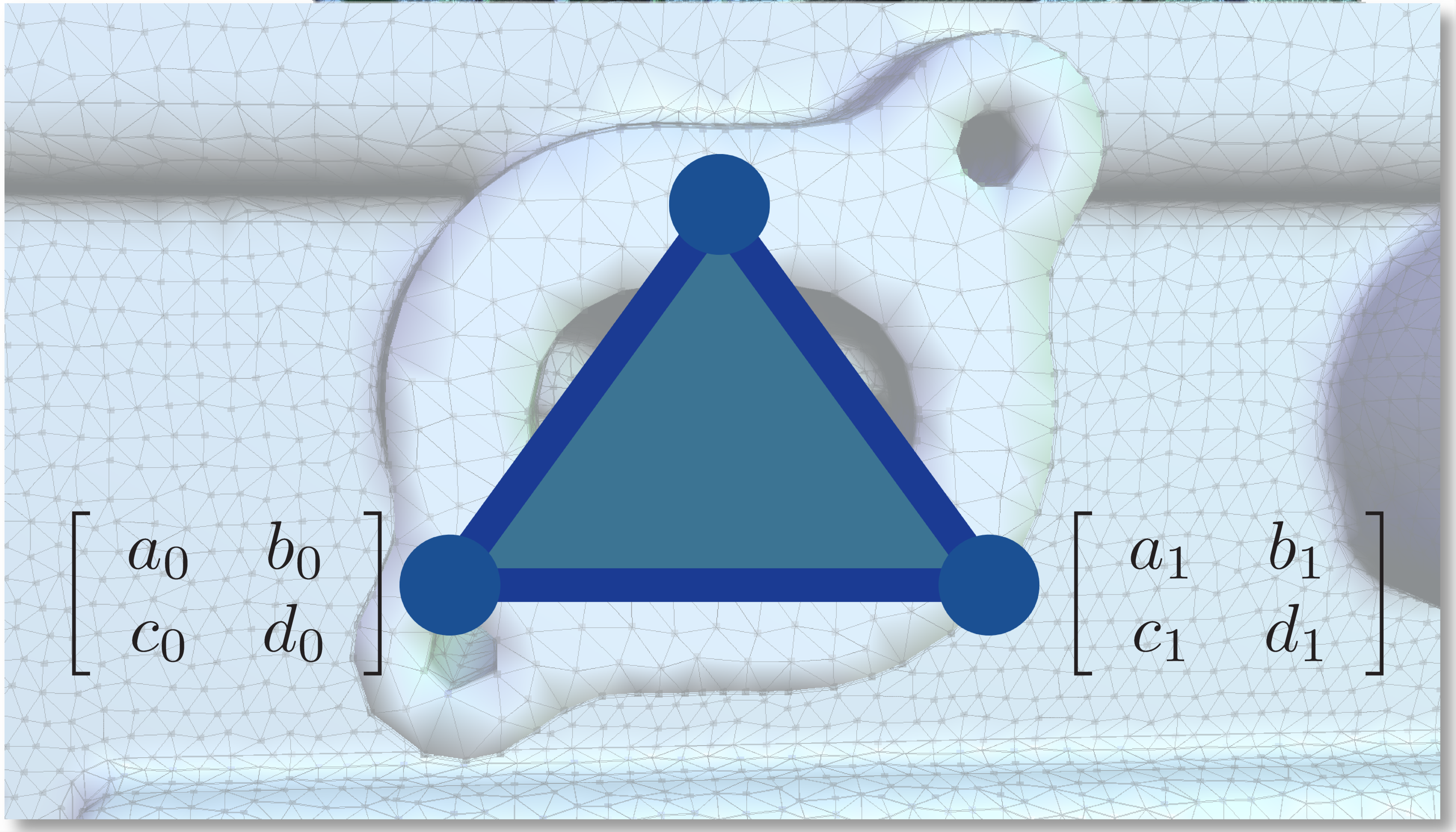
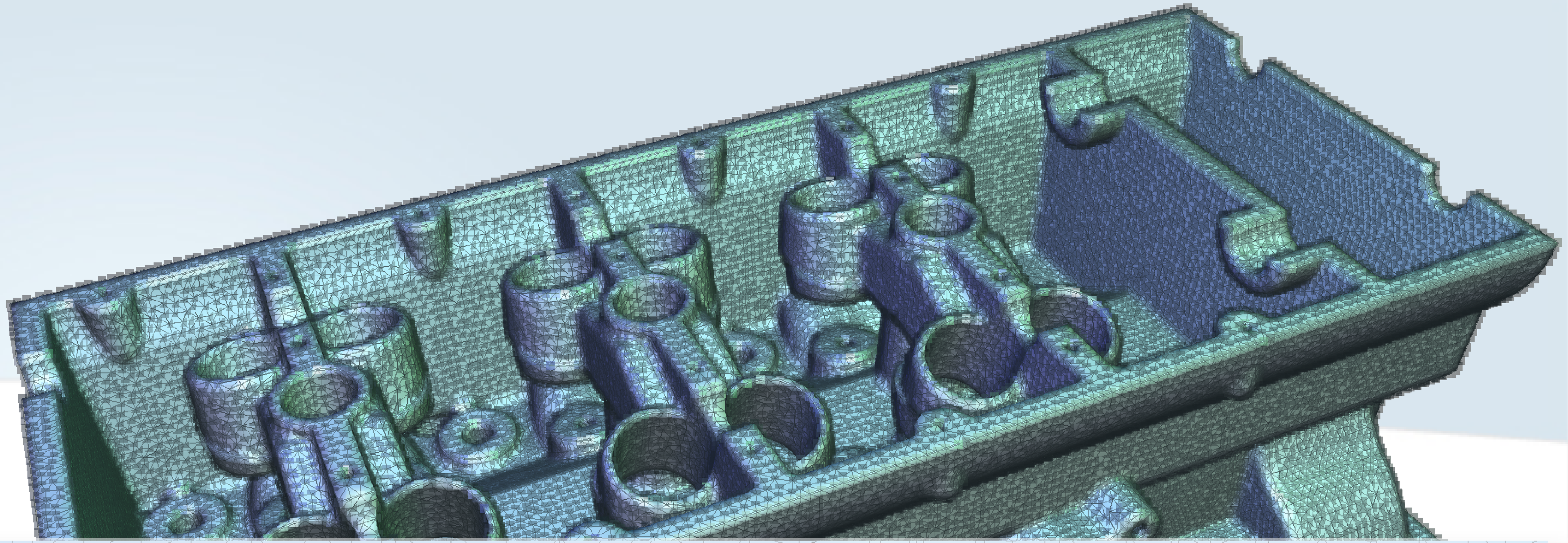




$$\begin{bmatrix} a_0 & b_0 \\ c_0 & d_0 \end{bmatrix} \qquad \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$$

In practice



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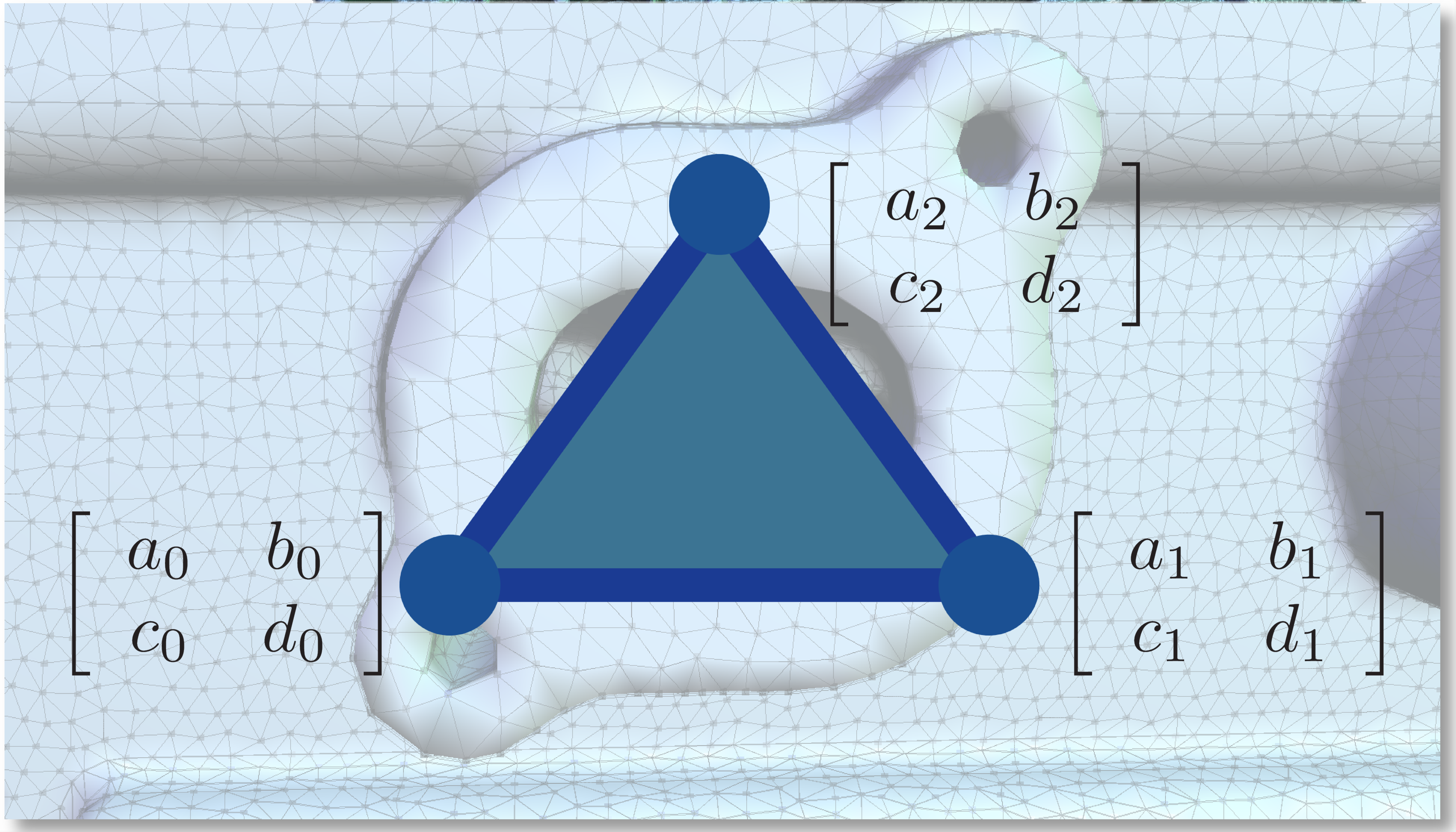

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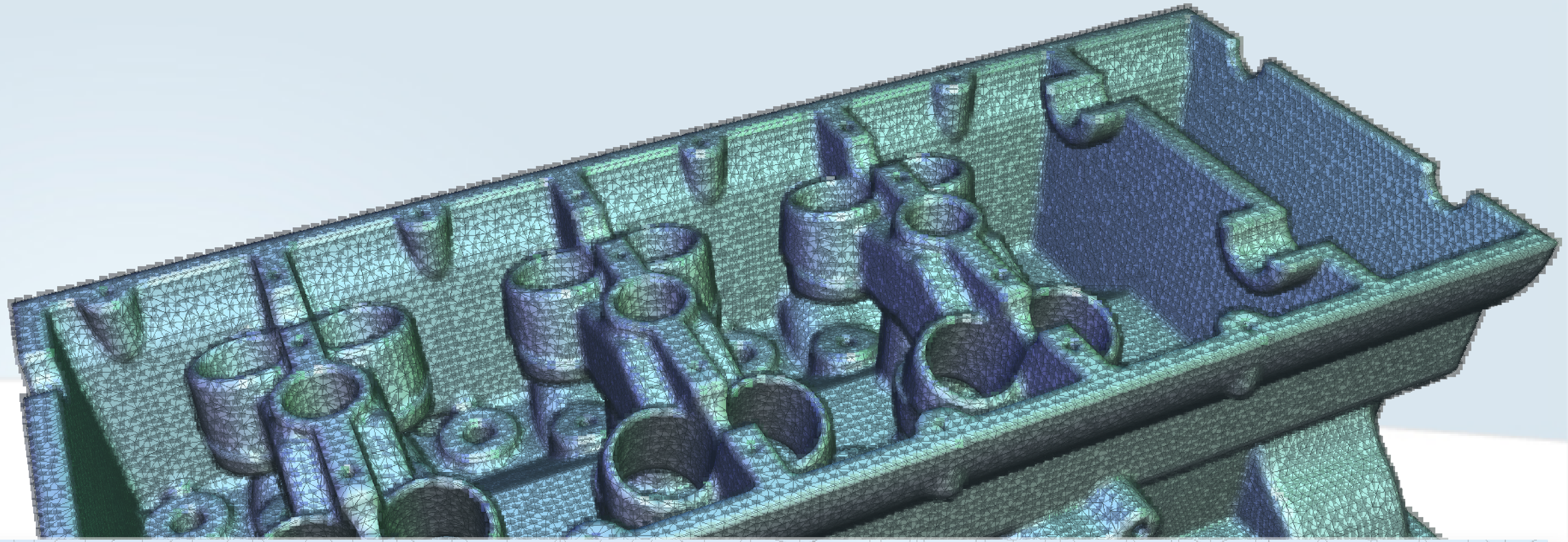

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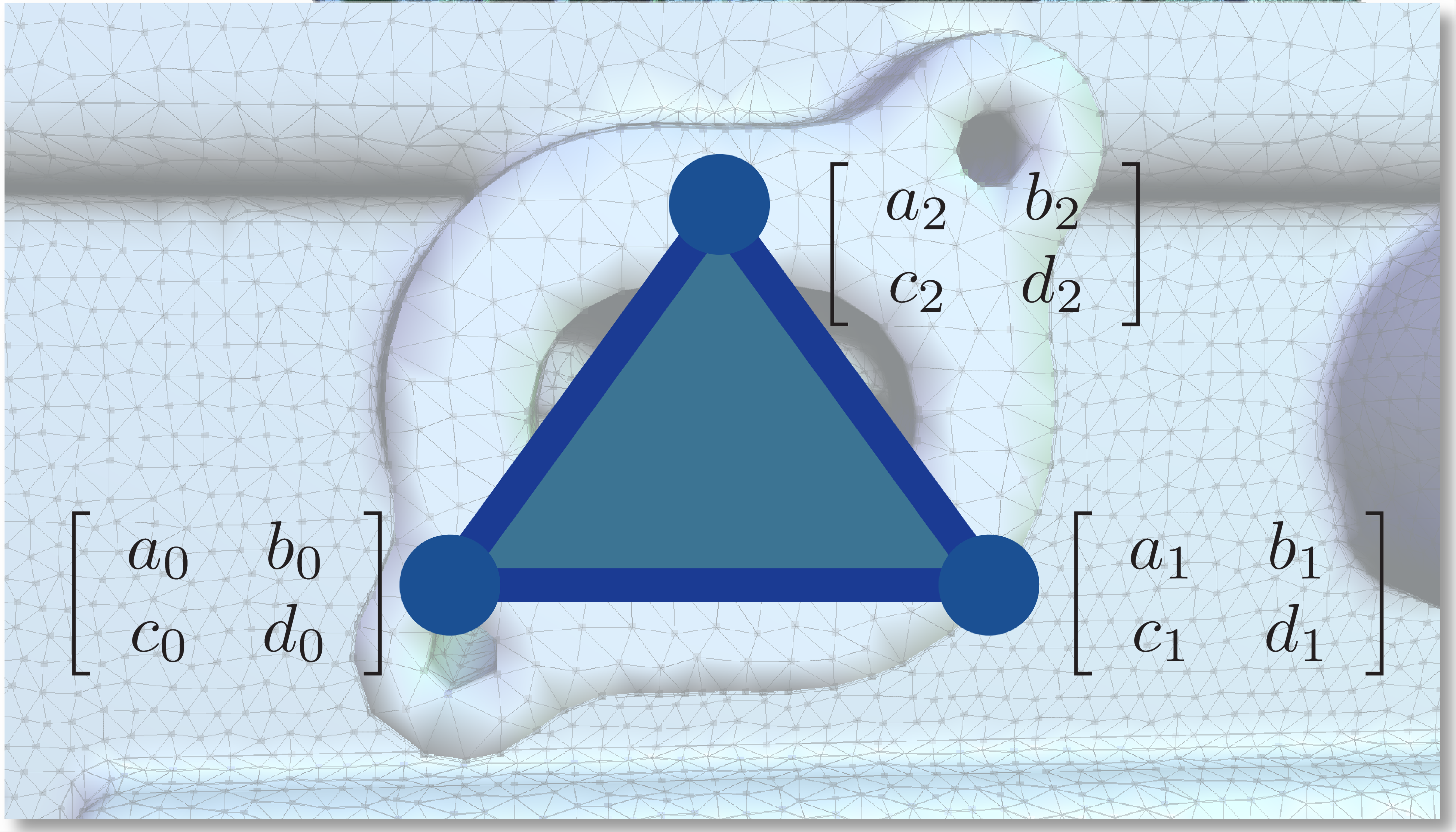

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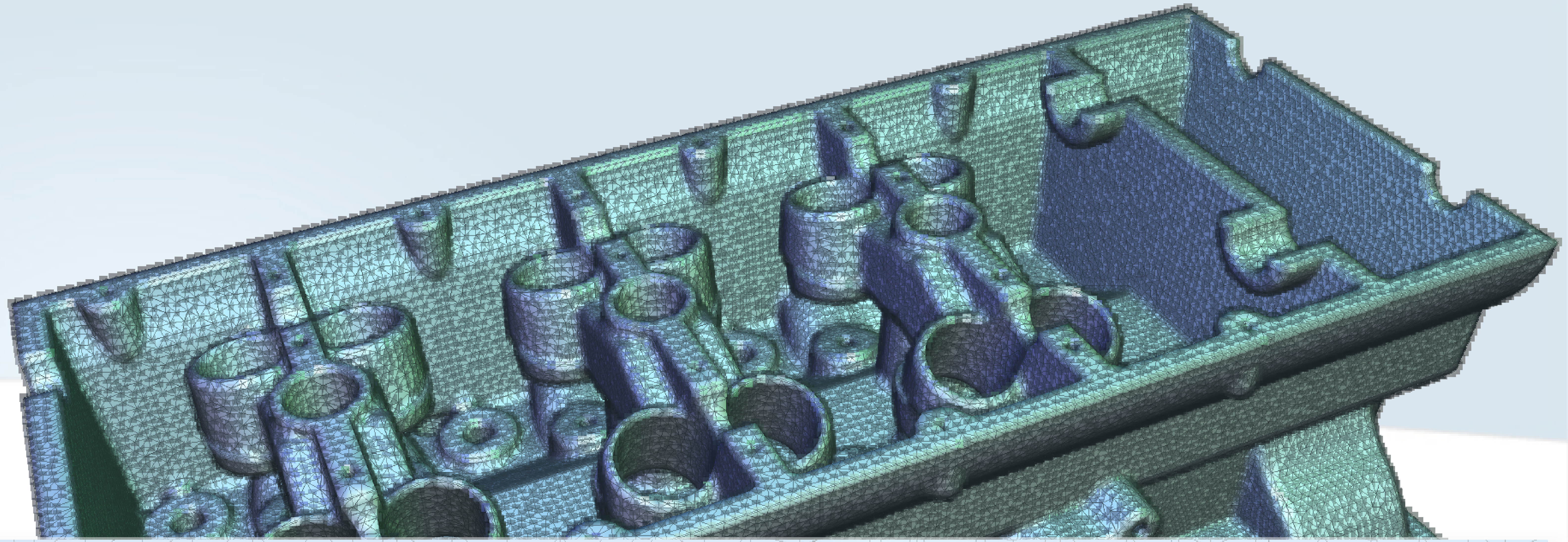

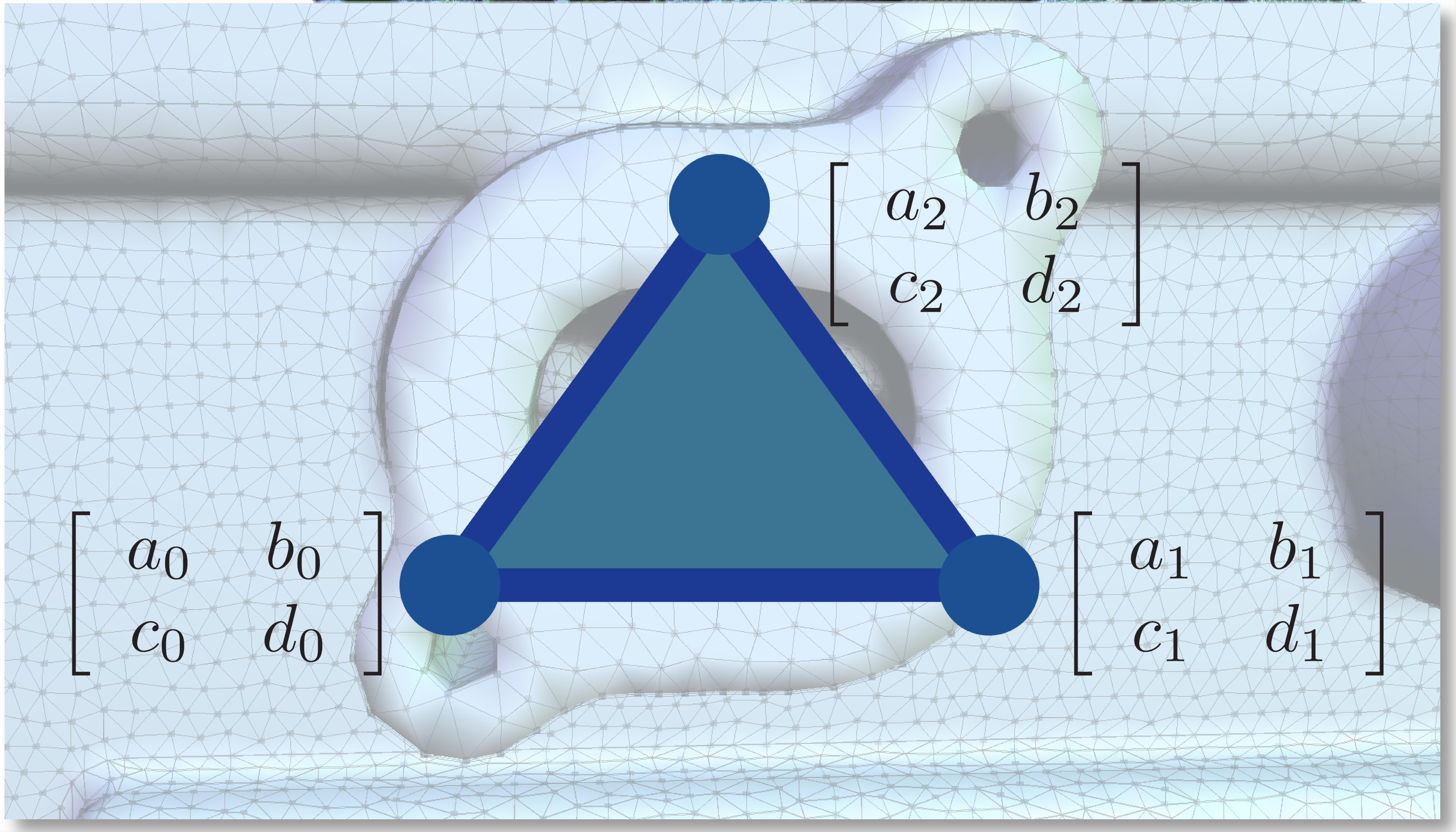

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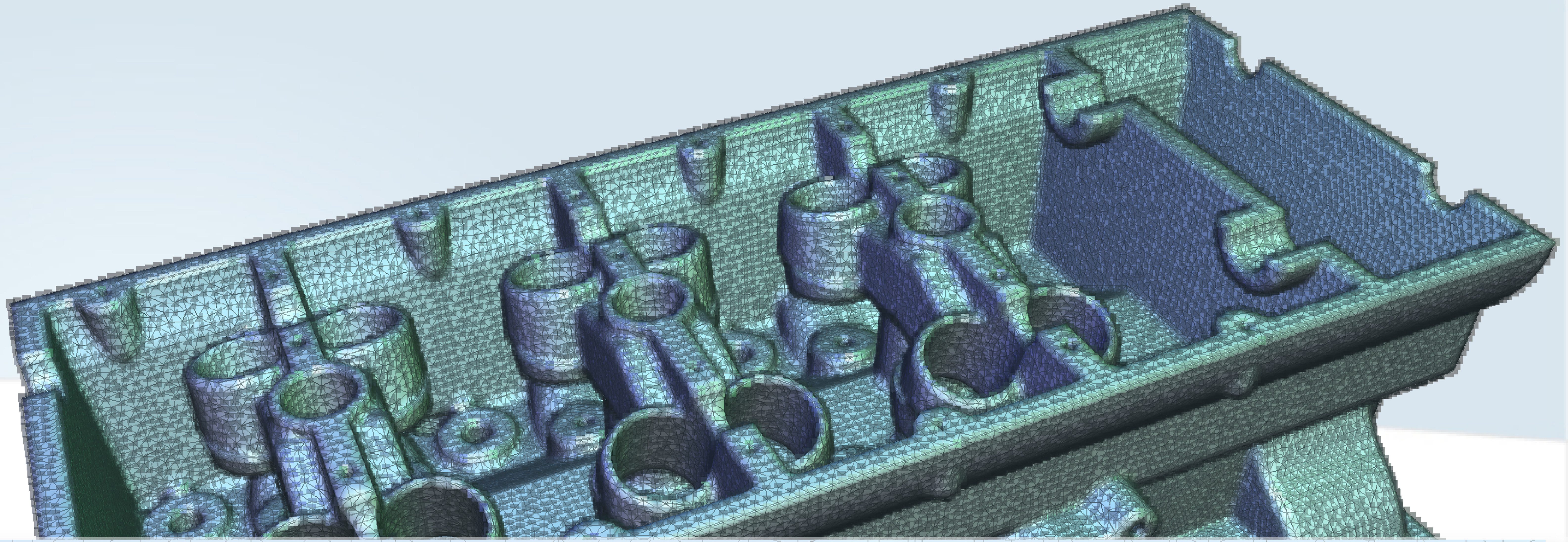

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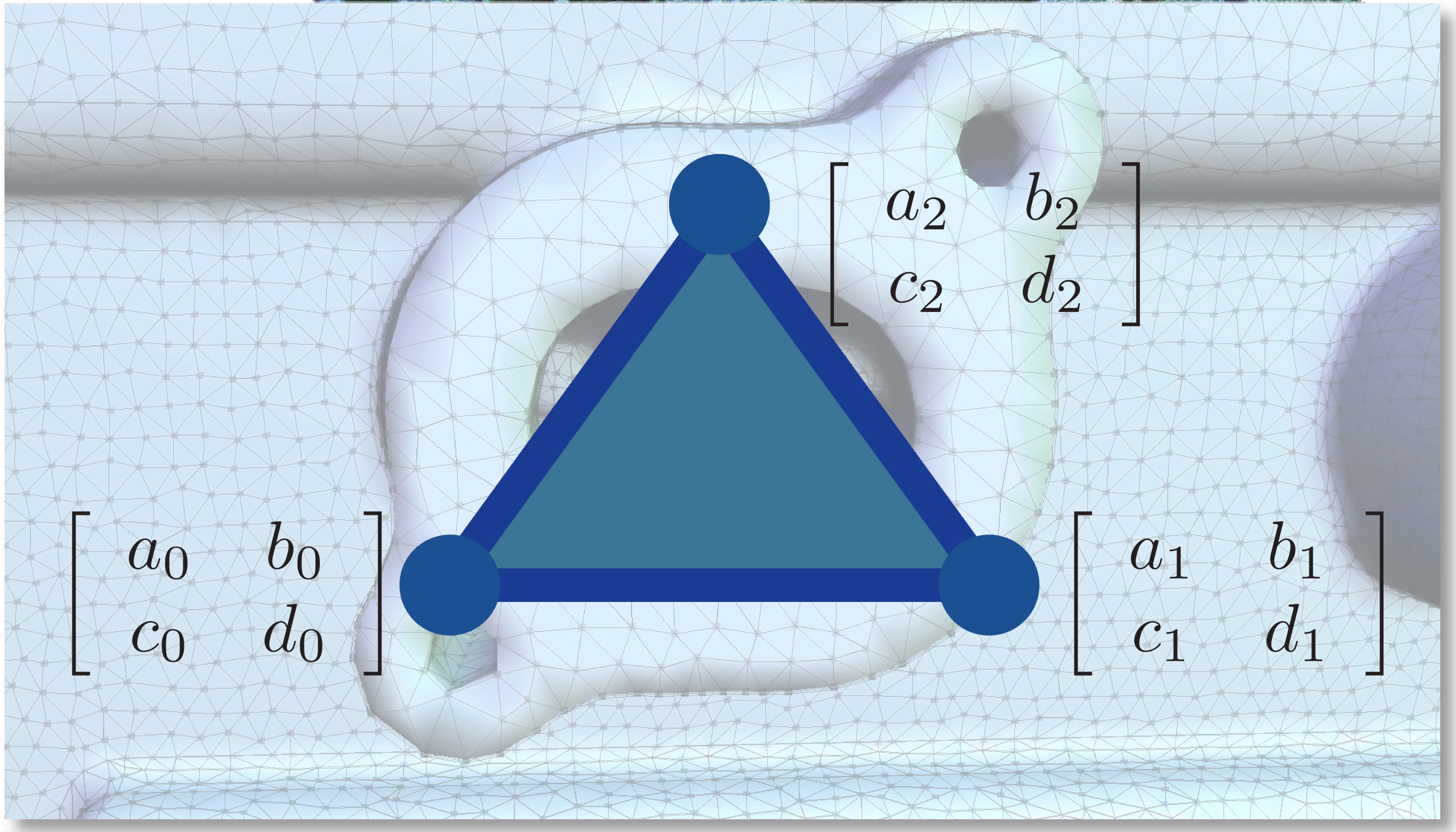

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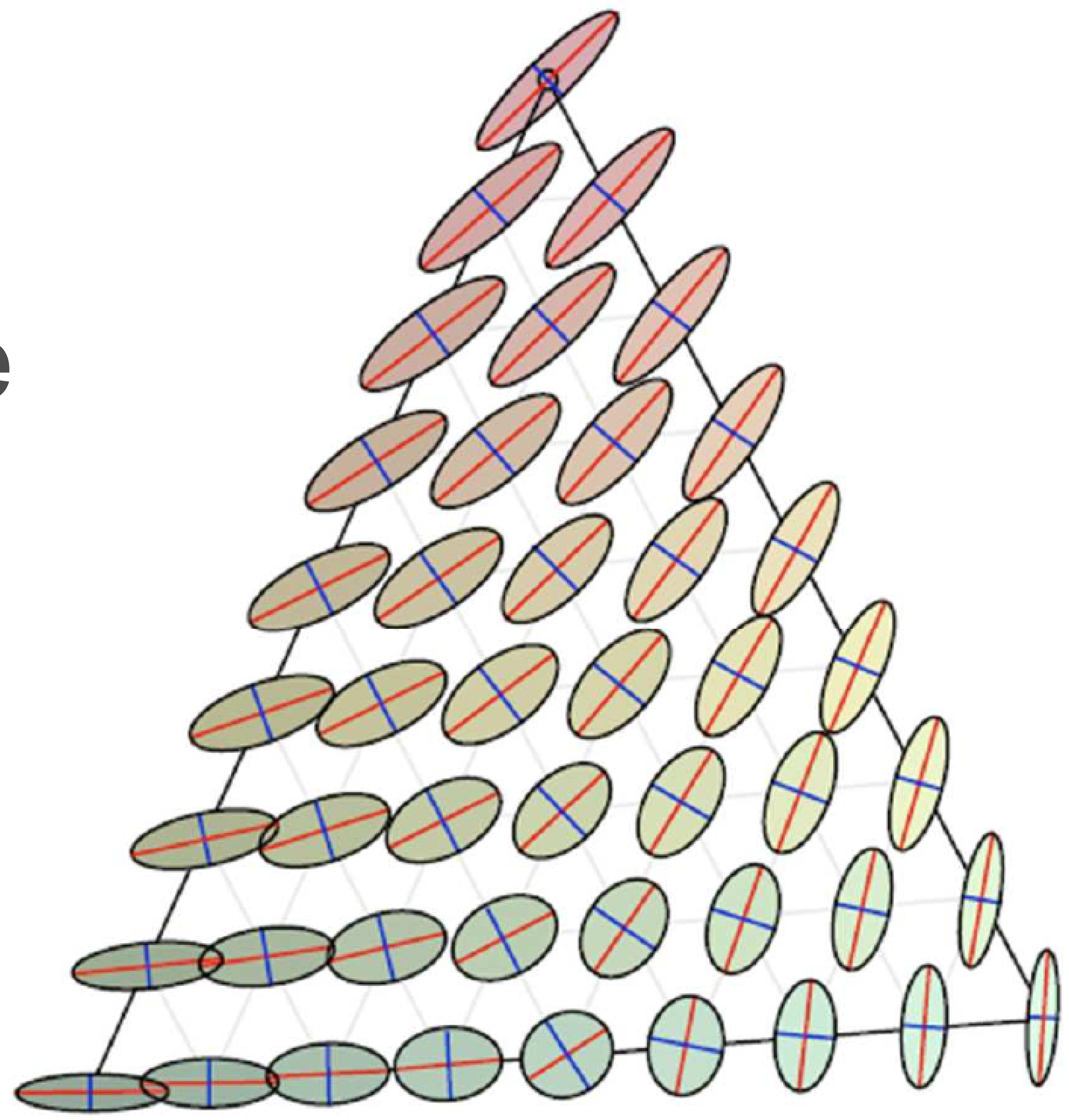

[Hotz]

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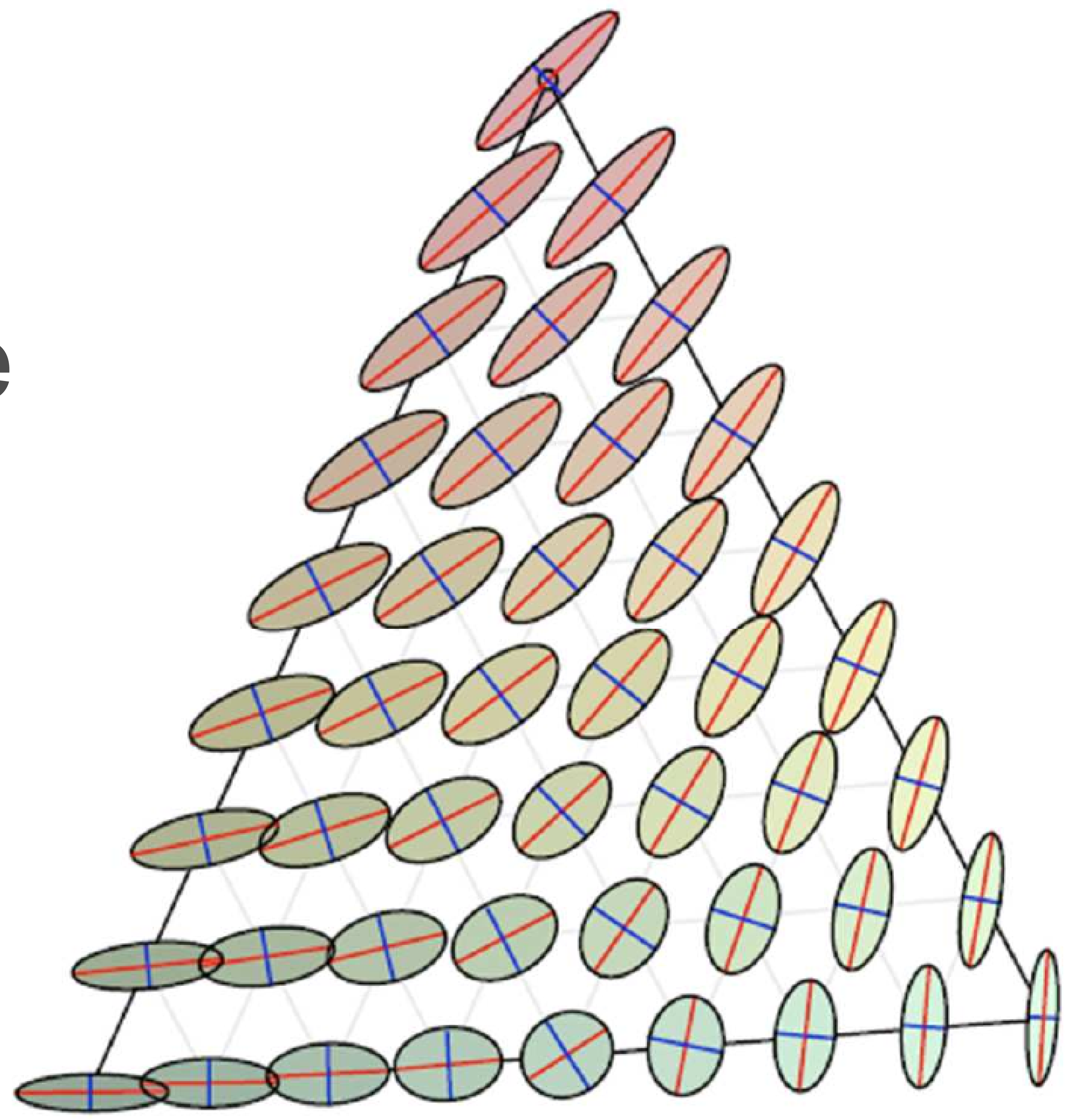

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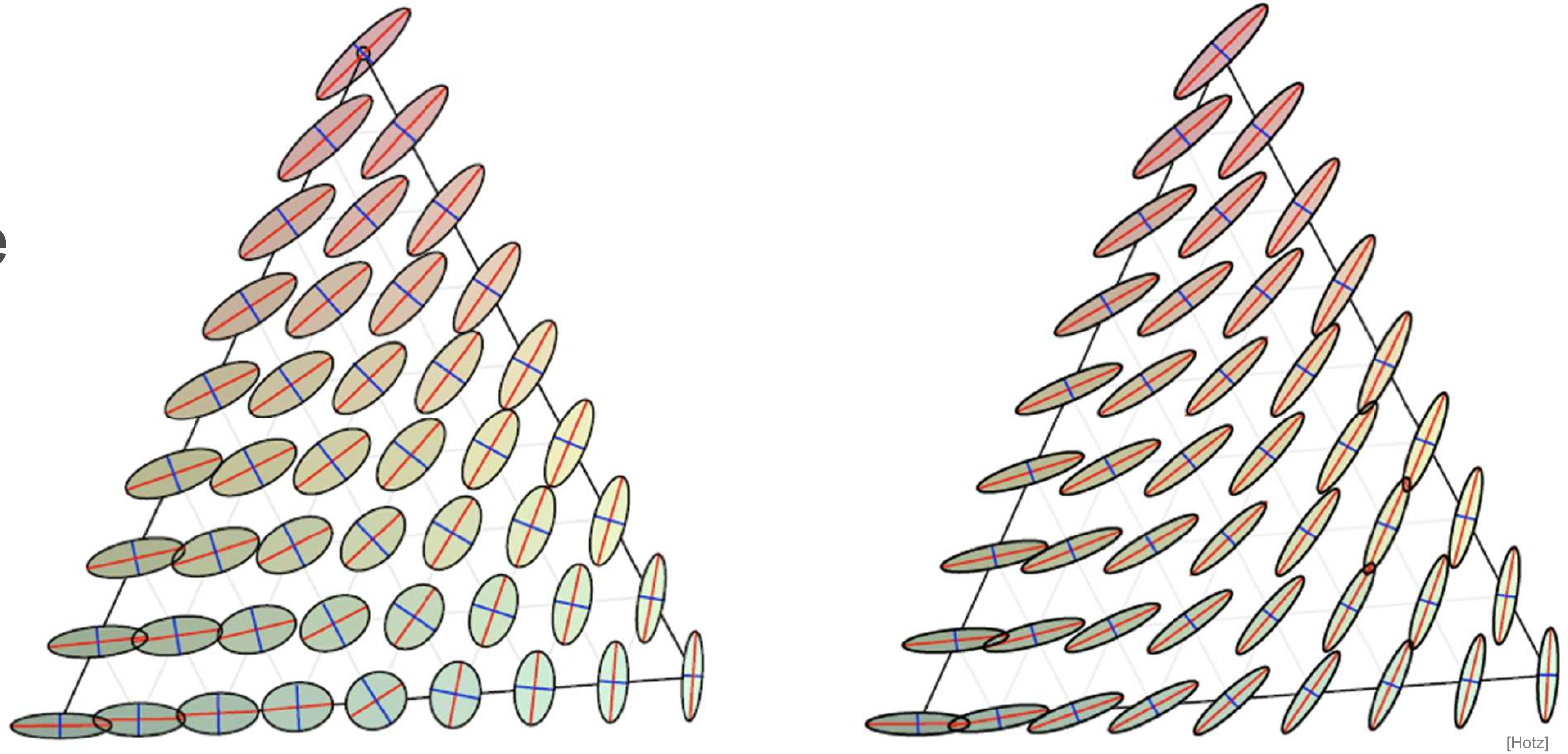

[Hotz]

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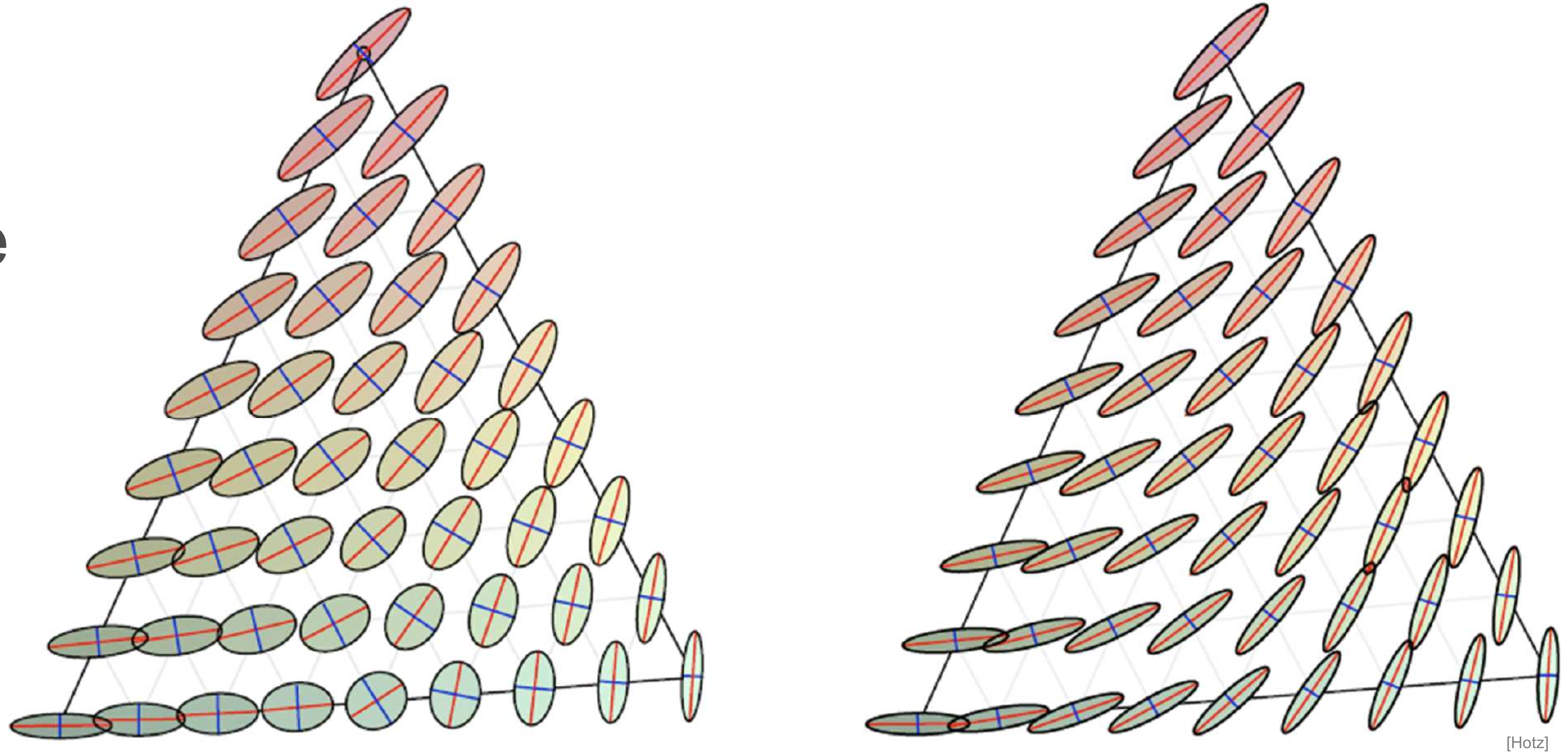
  - Similar to vector fields
    - Magnitude/angle



[Hotz]

Summary

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Glyph packing

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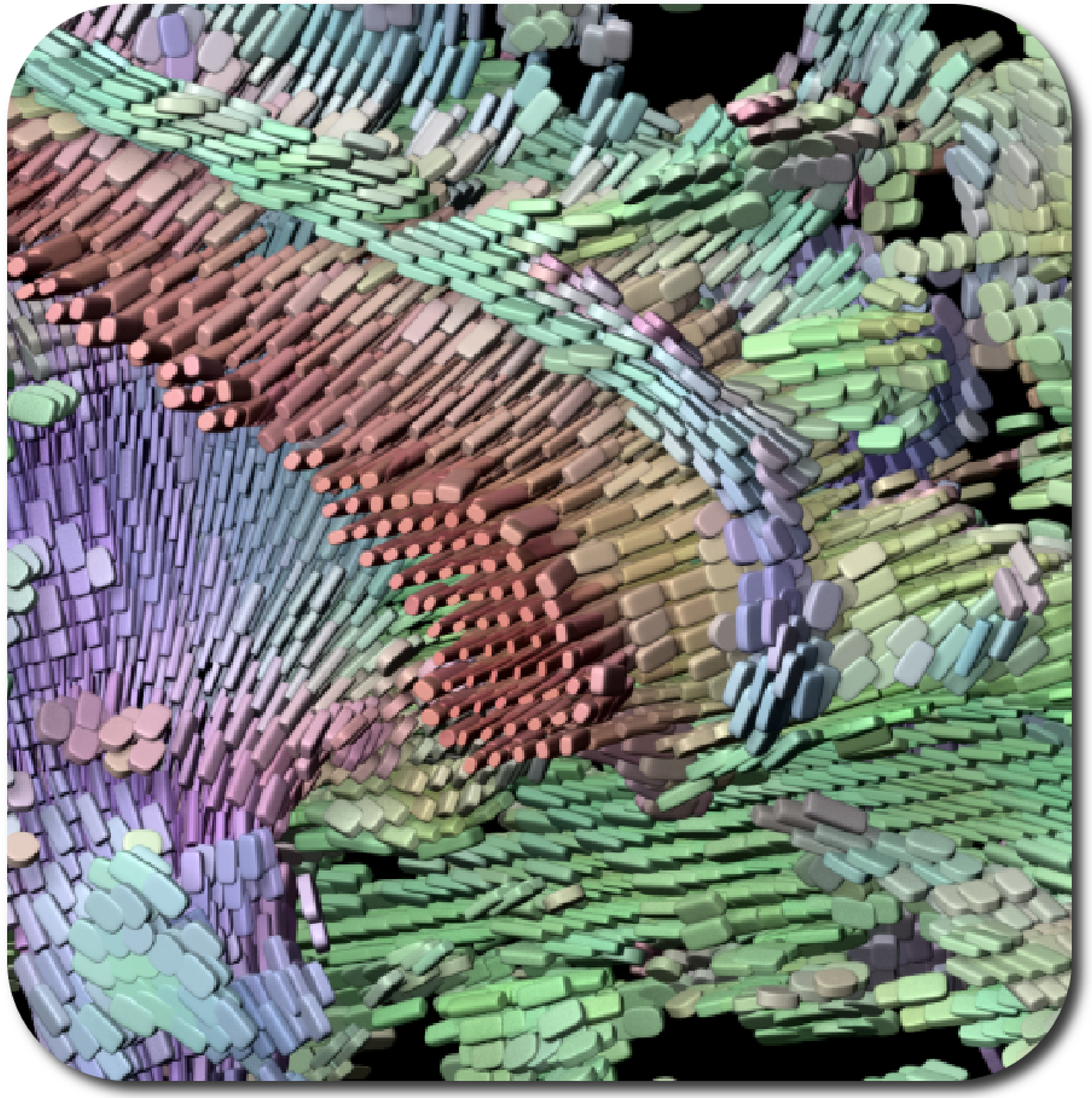

[Kindlmann]

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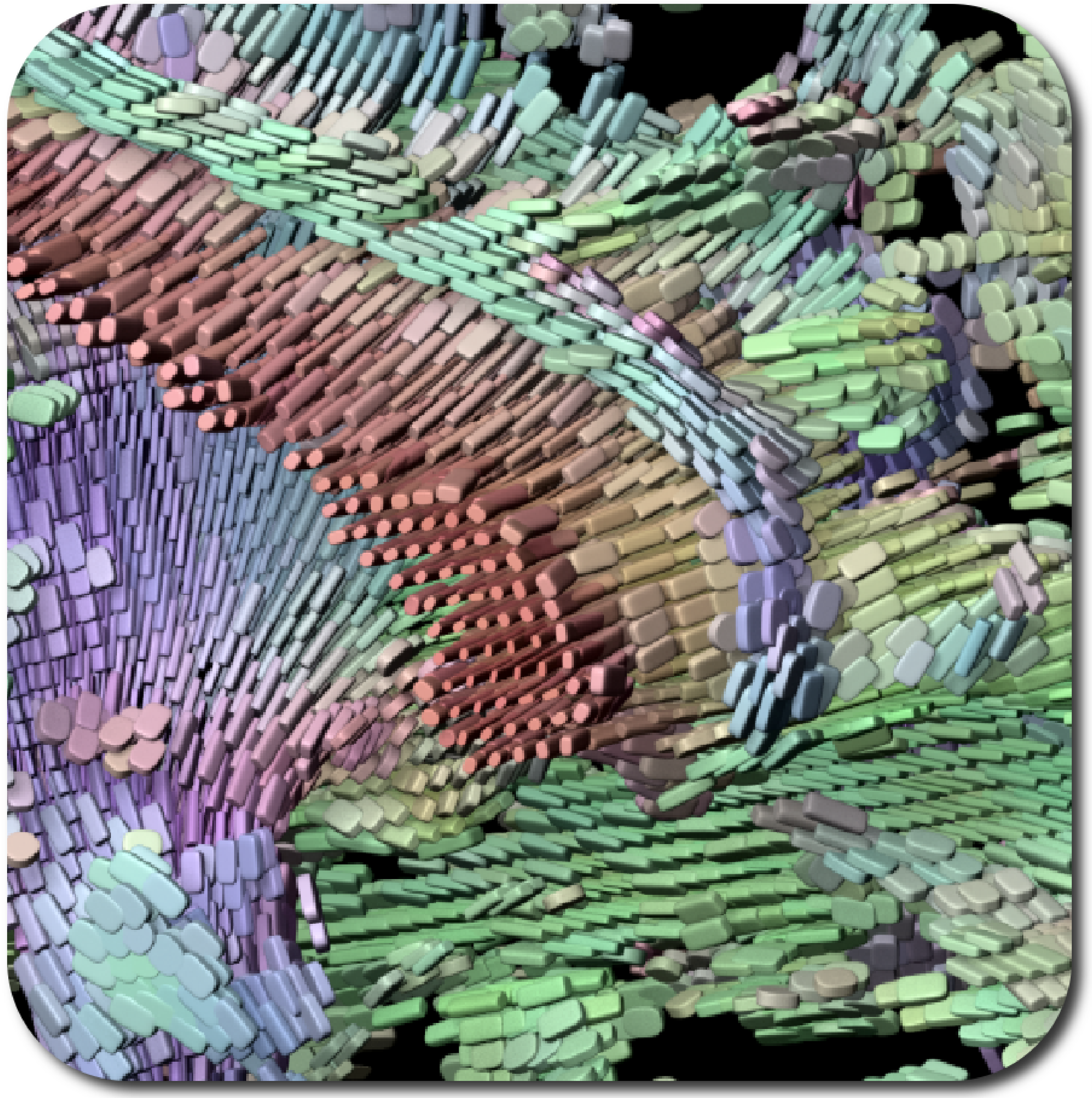

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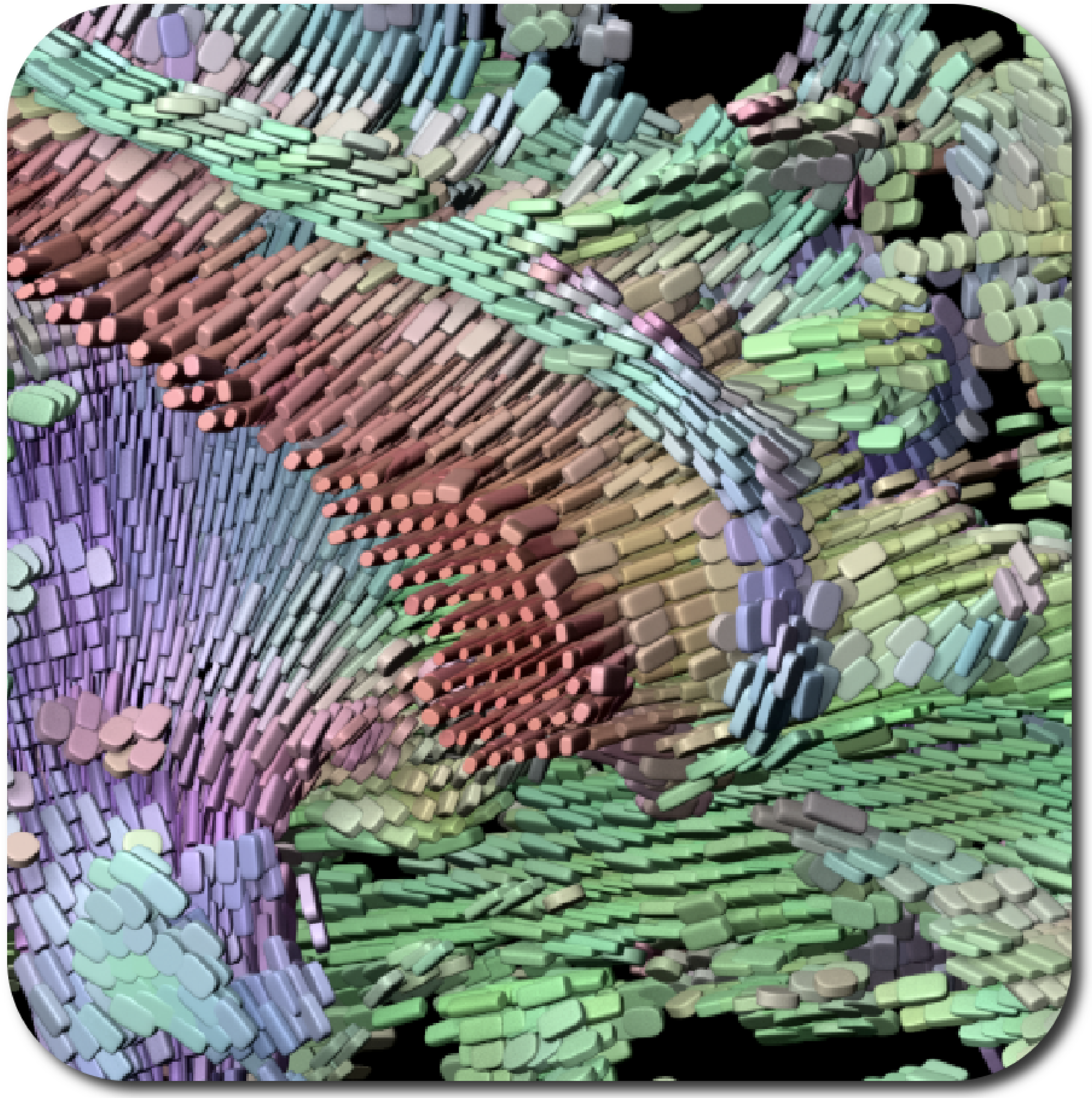

[Kindlmann]

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SVD or PCA

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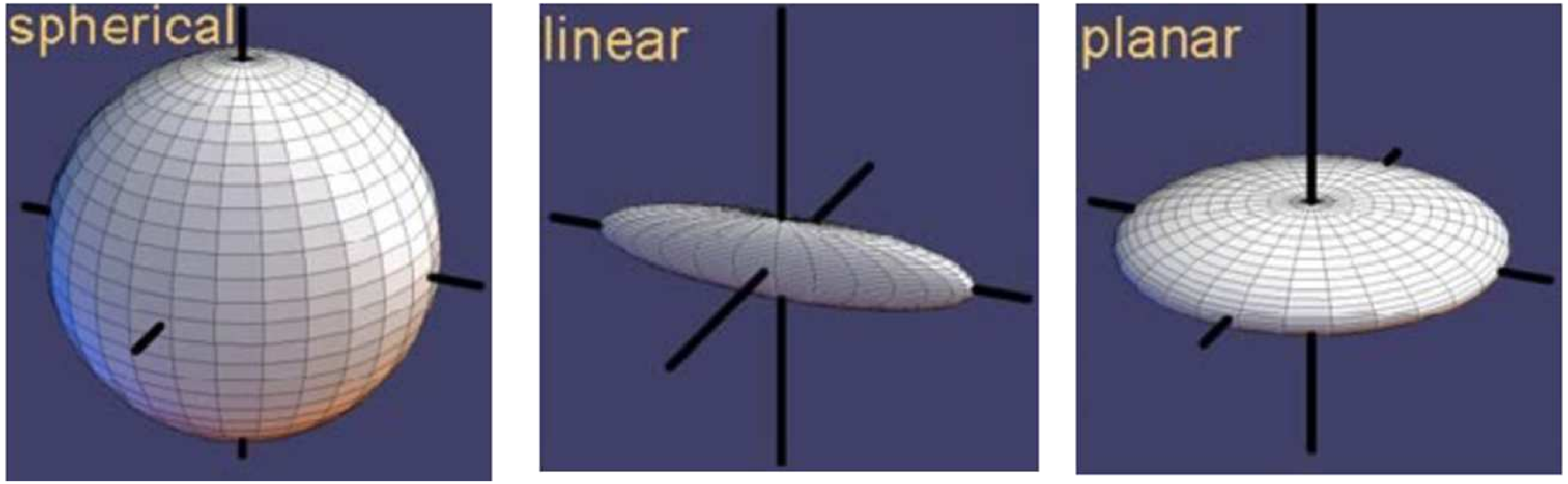
Tensor glyphs

- Ellipsoids



[Kindlmann]

Tensor glyphs

- Ellipsoids
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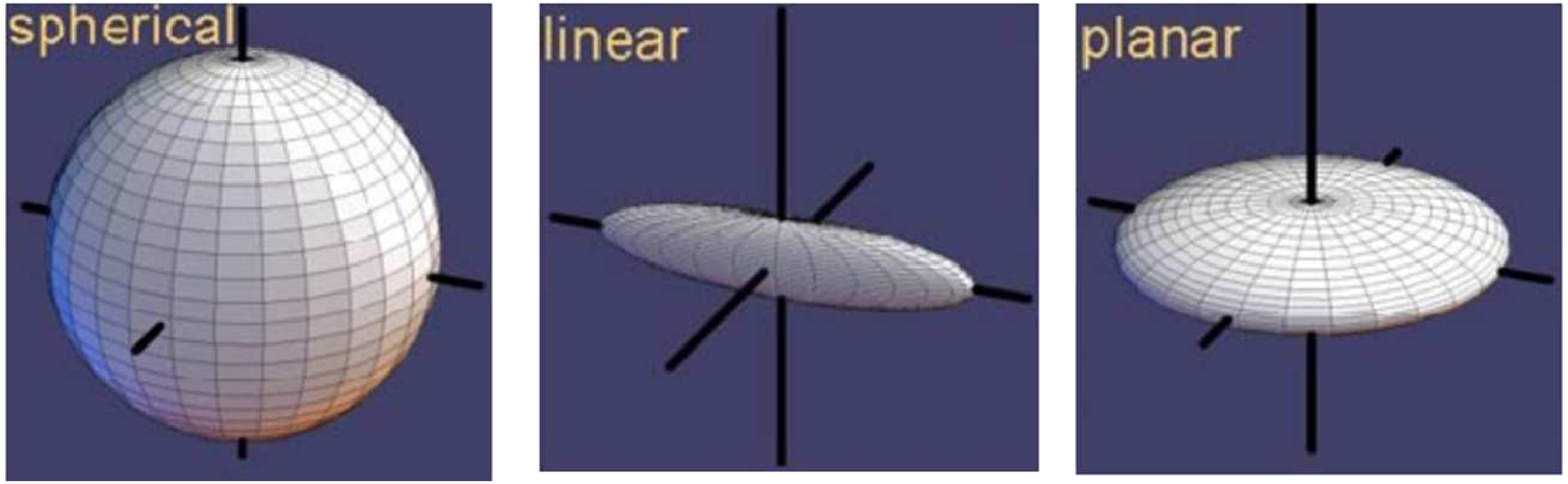

[Kindlmann]

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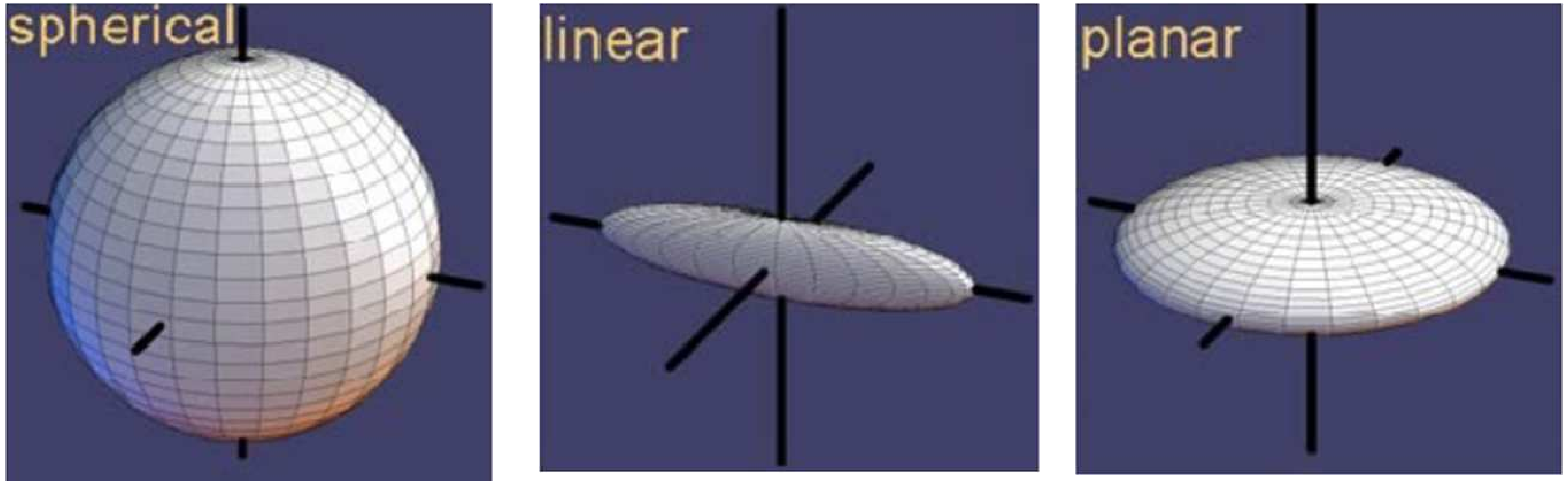

[Kindlmann]

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- Ellipsoids
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  - Axis length
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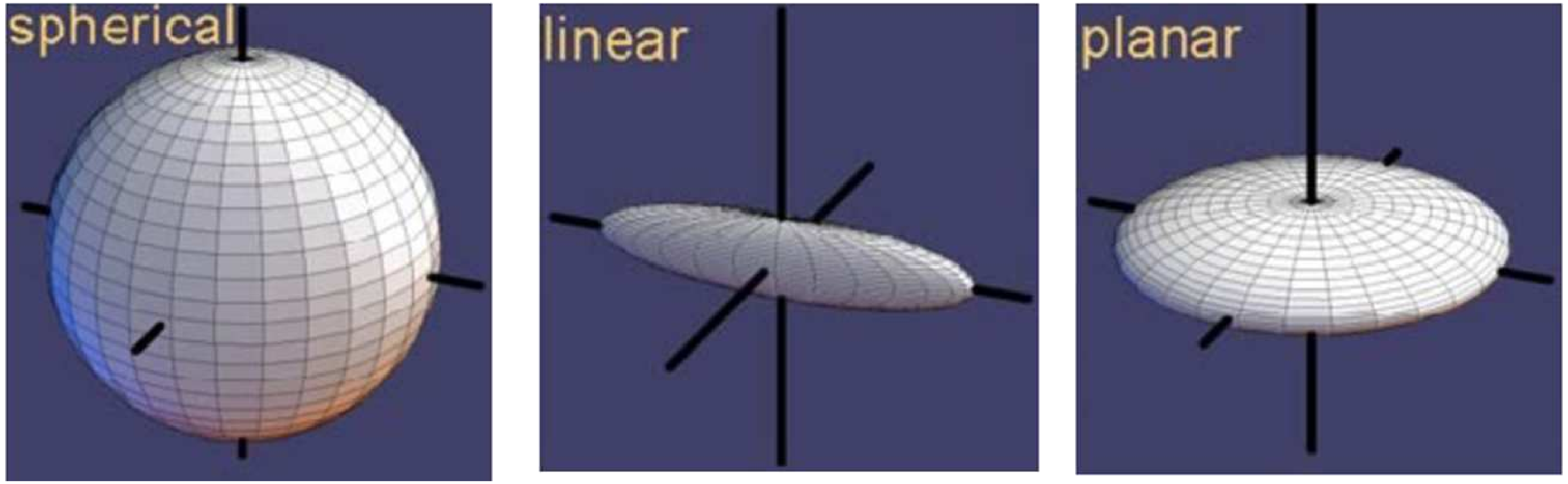

[Kindlmann]

Tensor glyphs

- Ellipsoids
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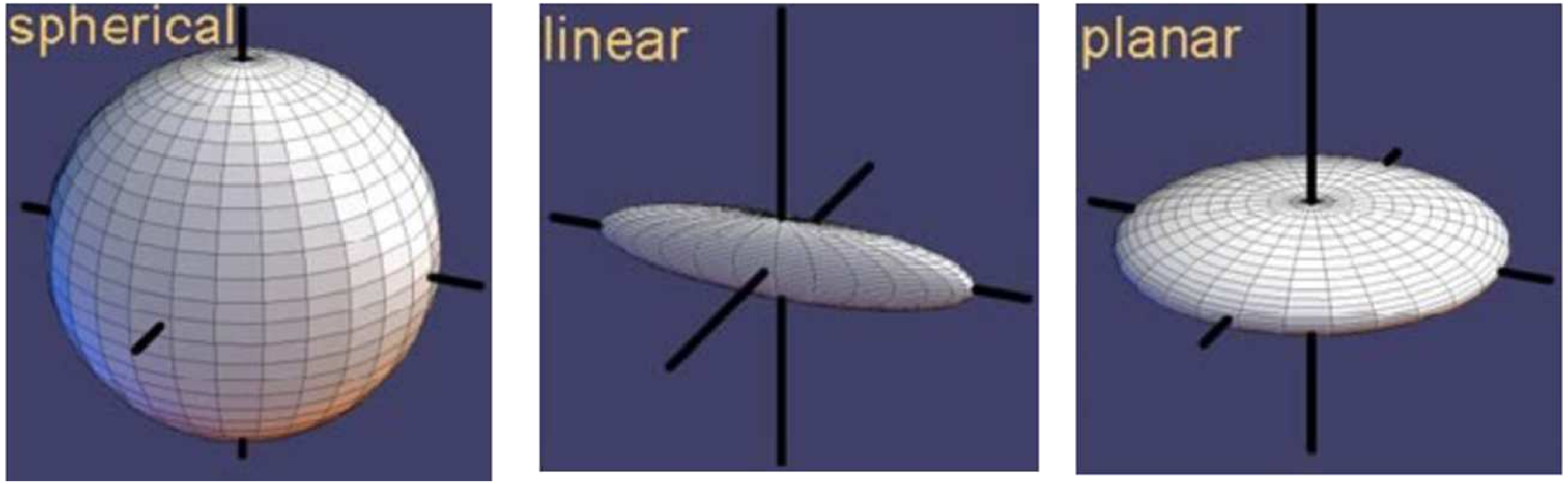
  - Anisotropy information



[Kindlmann]

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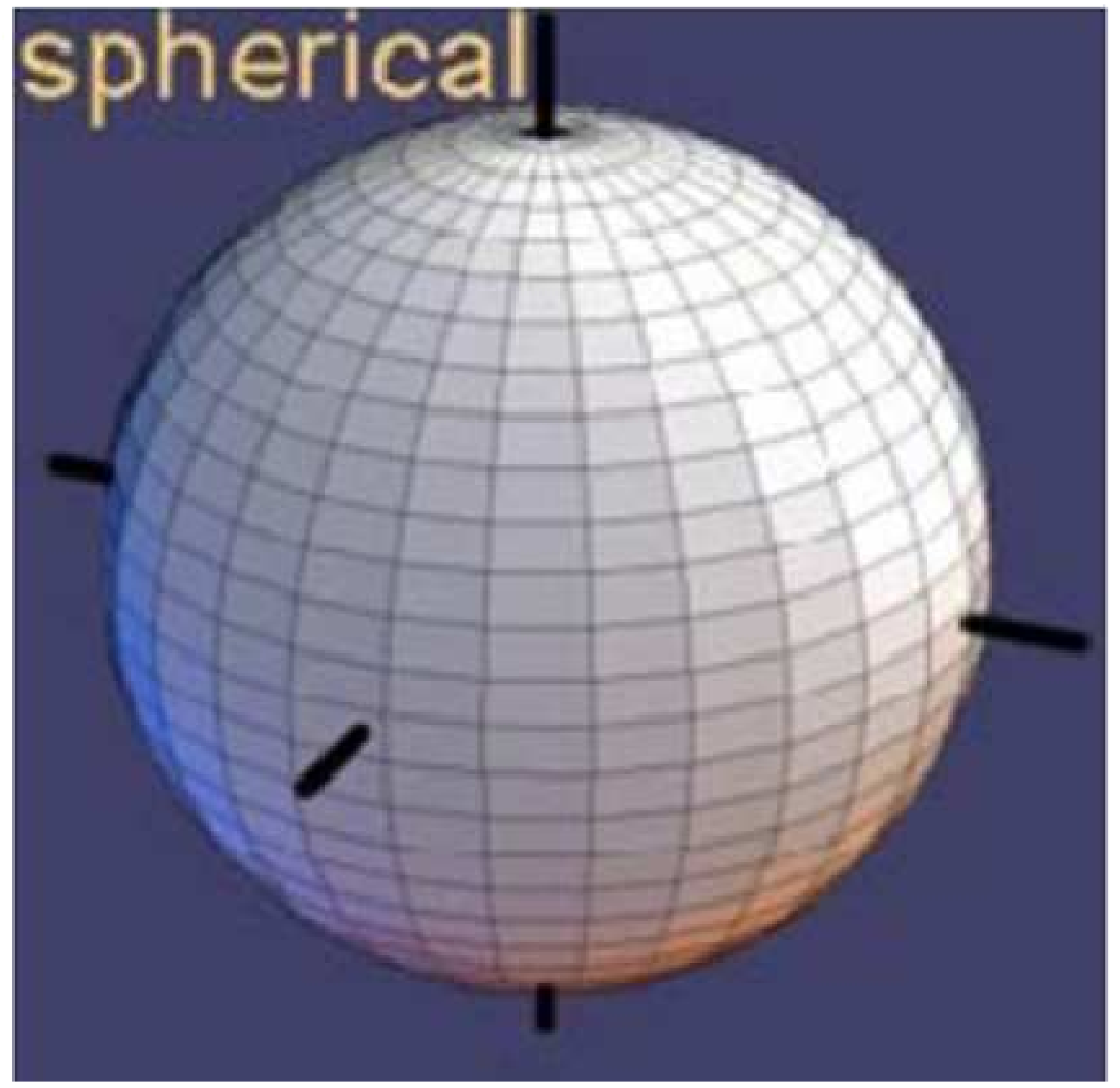
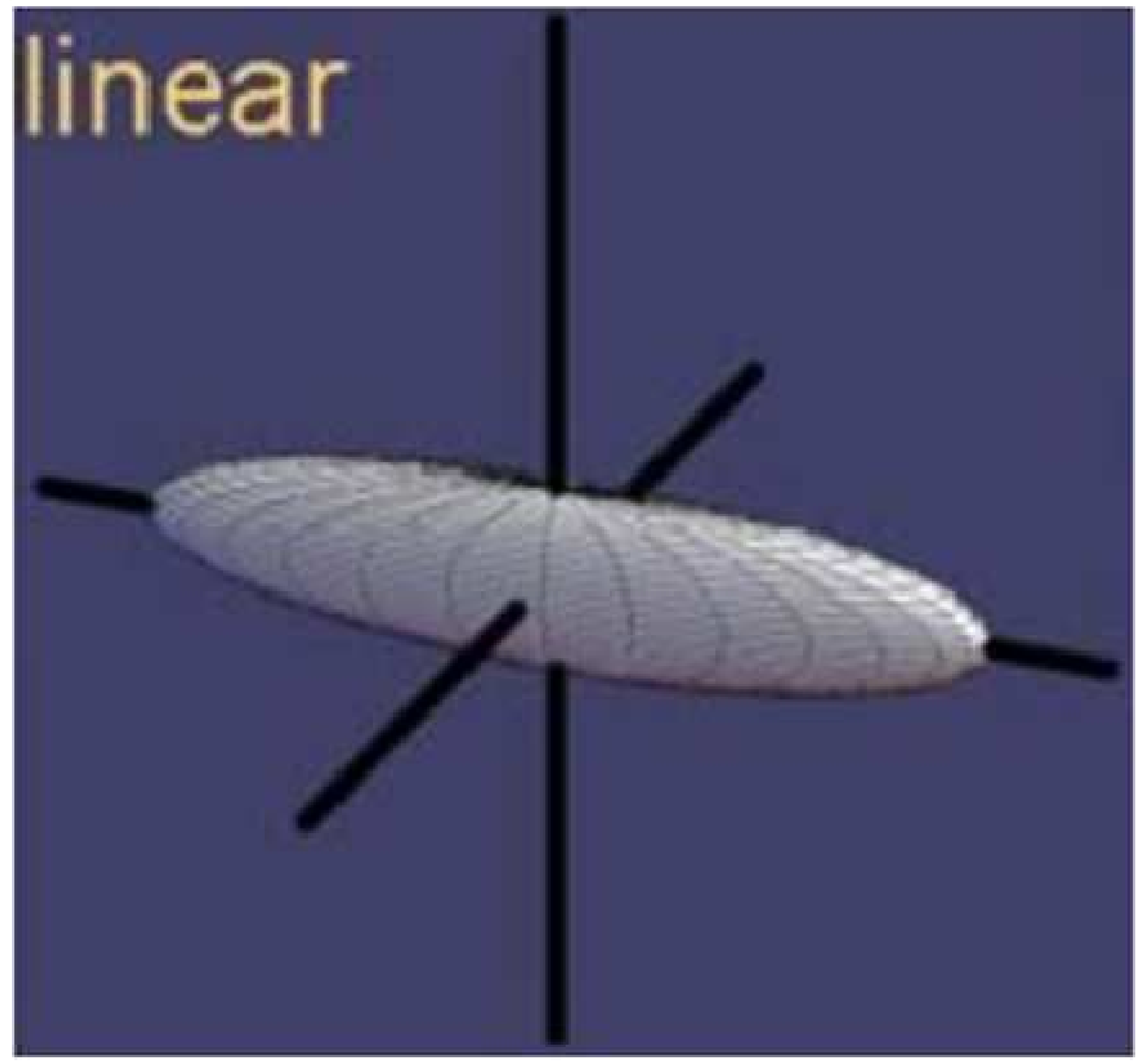
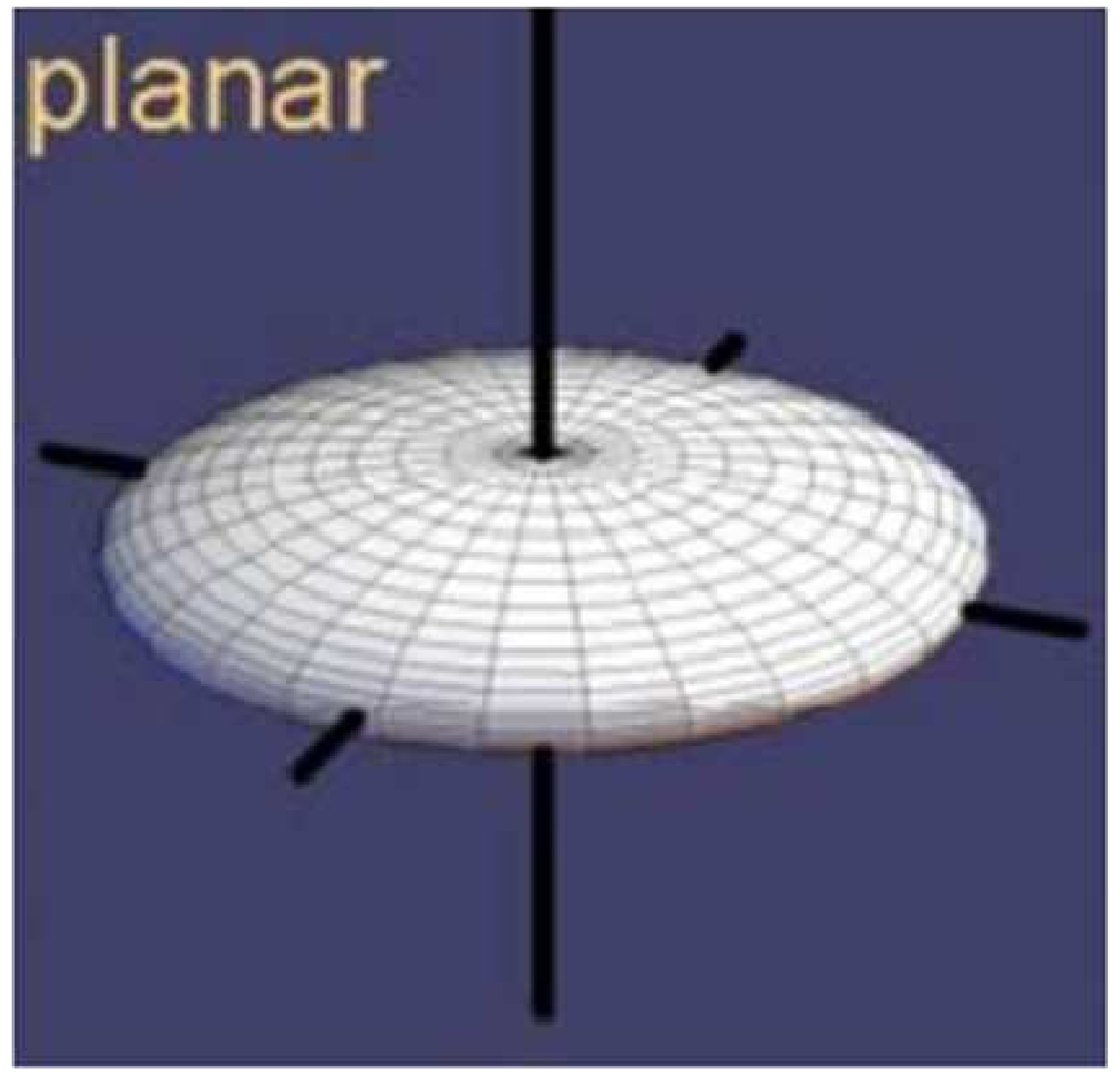
Isotropy



Axial anisotropy



Planar anisotropy



[Kindlmann]

Tensor glyphs

- Ellipsoids in 3D

Tensor glyphs

[Kindlmann]

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Tensor glyphs

[Kindlmann]

- Ellipsoids in 3D
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Tensor glyphs

[Kindlmann]

- Ellipsoids in 3D
  - Information loss due to screen projection
  - Continuous shading

Tensor glyphs

[Kindlmann]

- Ellipsoids in 3D
  - Information loss due to screen projection
  - Continuous shading
    - Hard to evaluate variations across axes

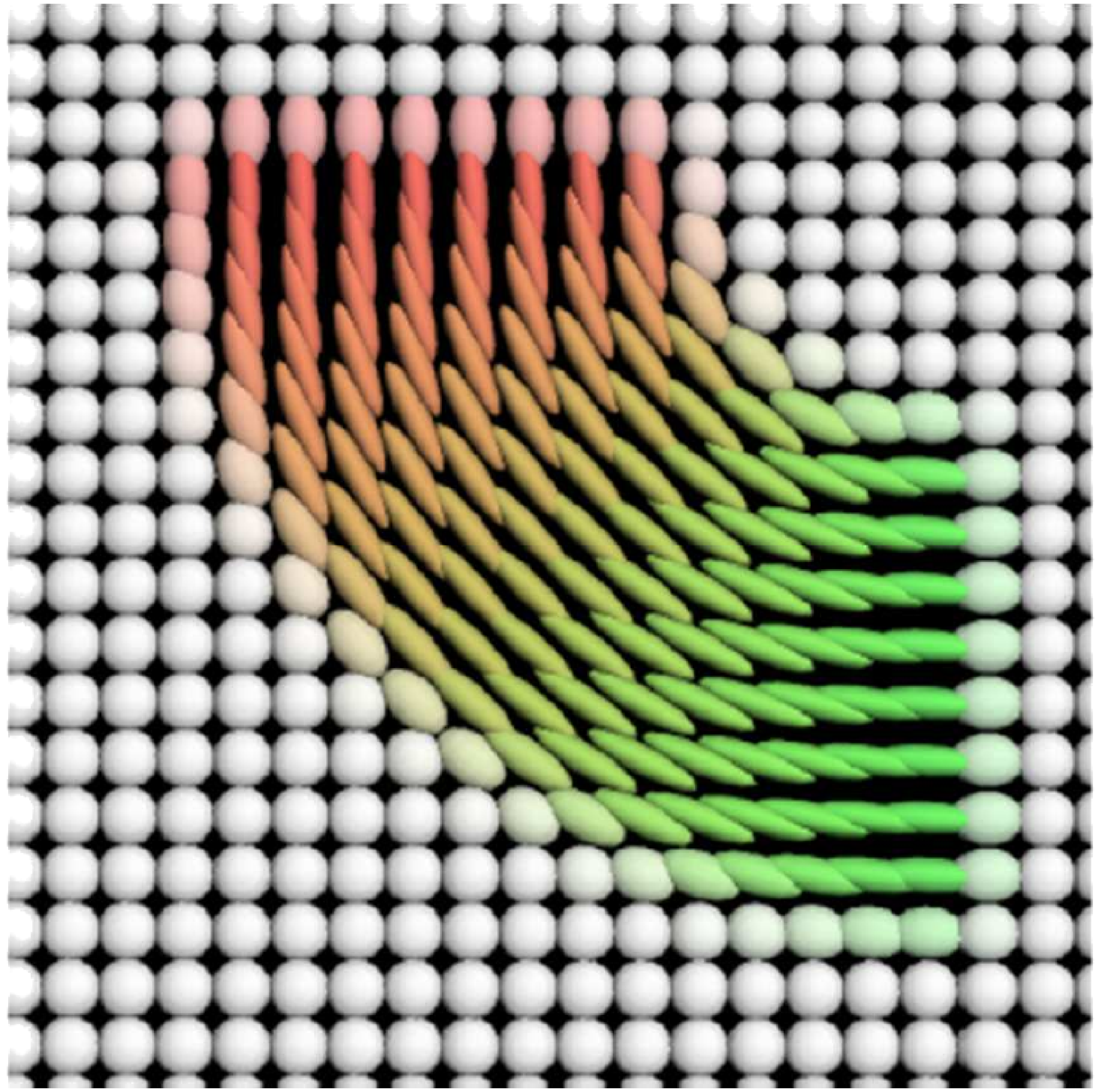
Tensor glyphs

[Kindlmann]

- “Superquadrics”
  - Parallelepiped with smooth edges
  - Exaggerate shading variations across axis

Glyph packing

- How to distribute the glyphs



[Kindlmann]

Glyph packing

- How to distribute the glyphs
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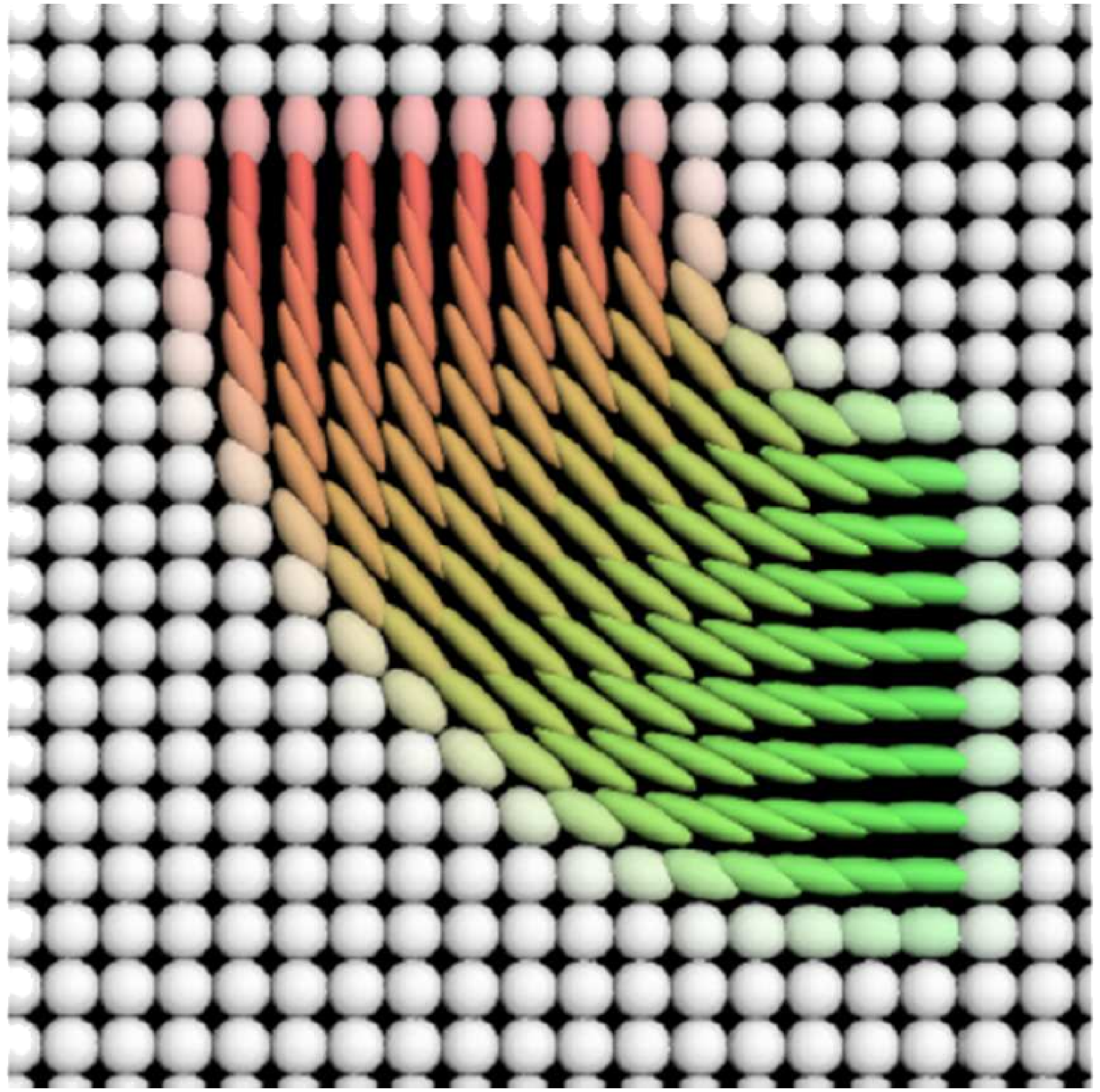

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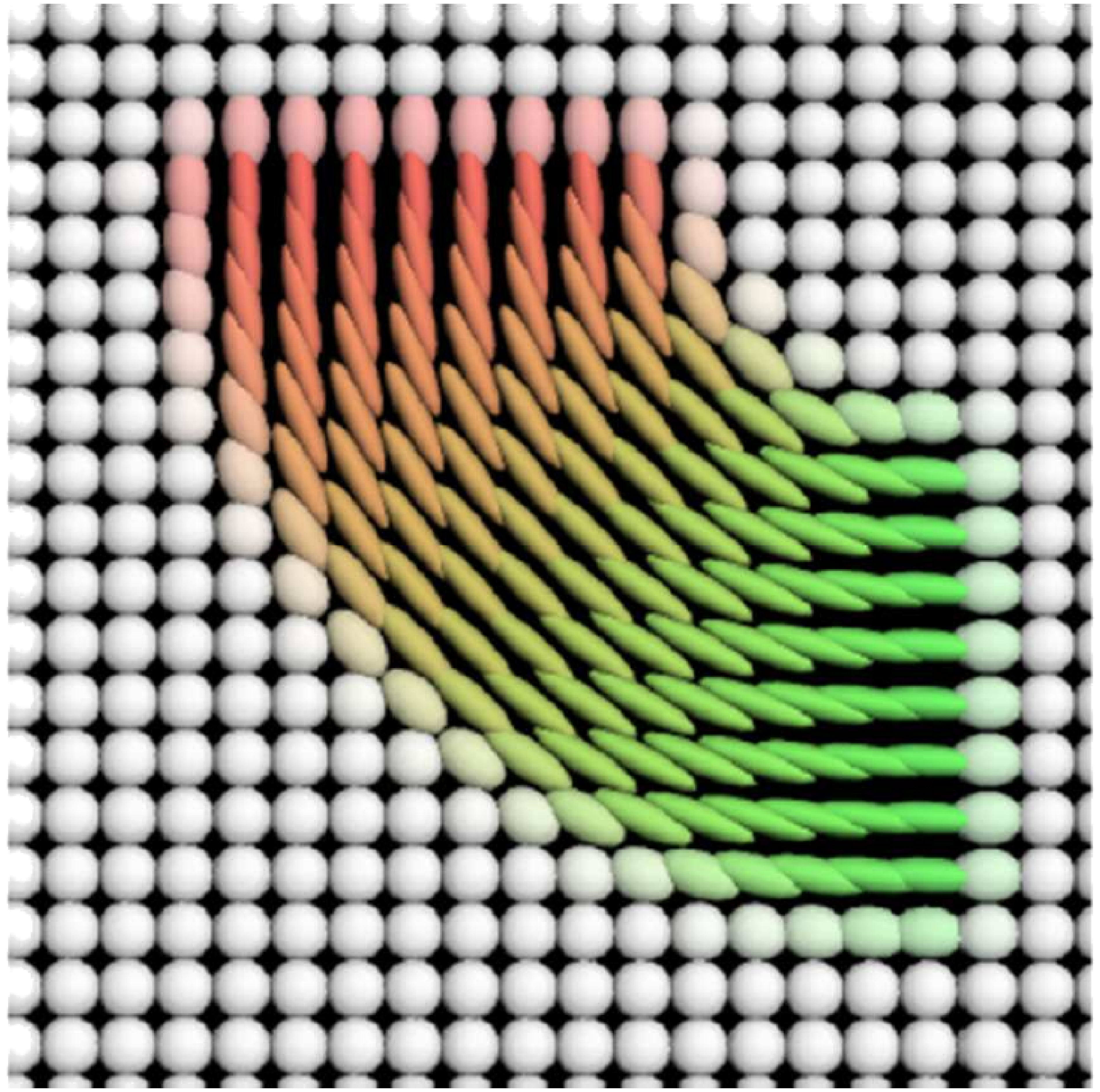

[Kindlmann]

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- How to distribute the glyphs
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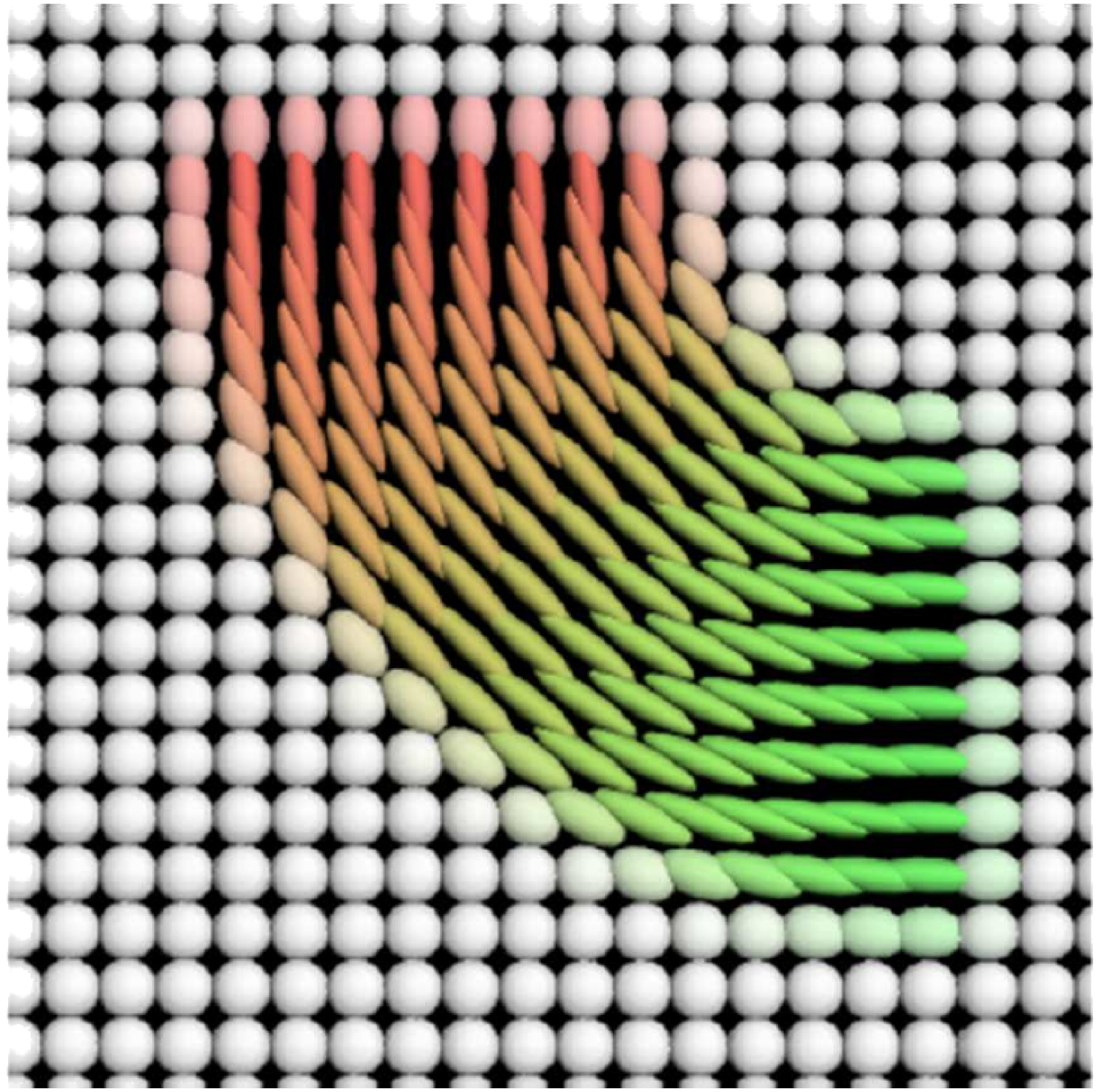

[Kindlmann]

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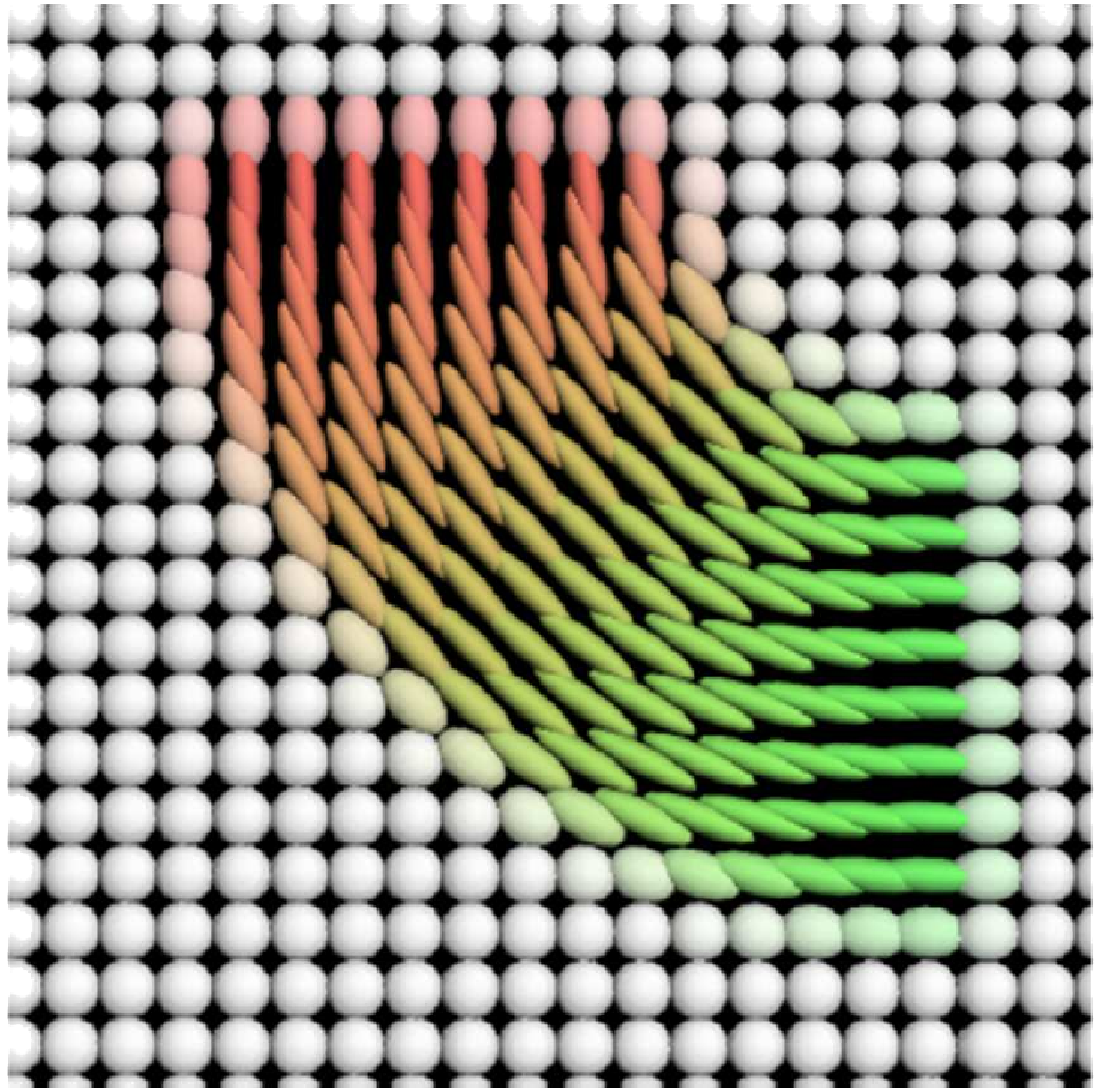

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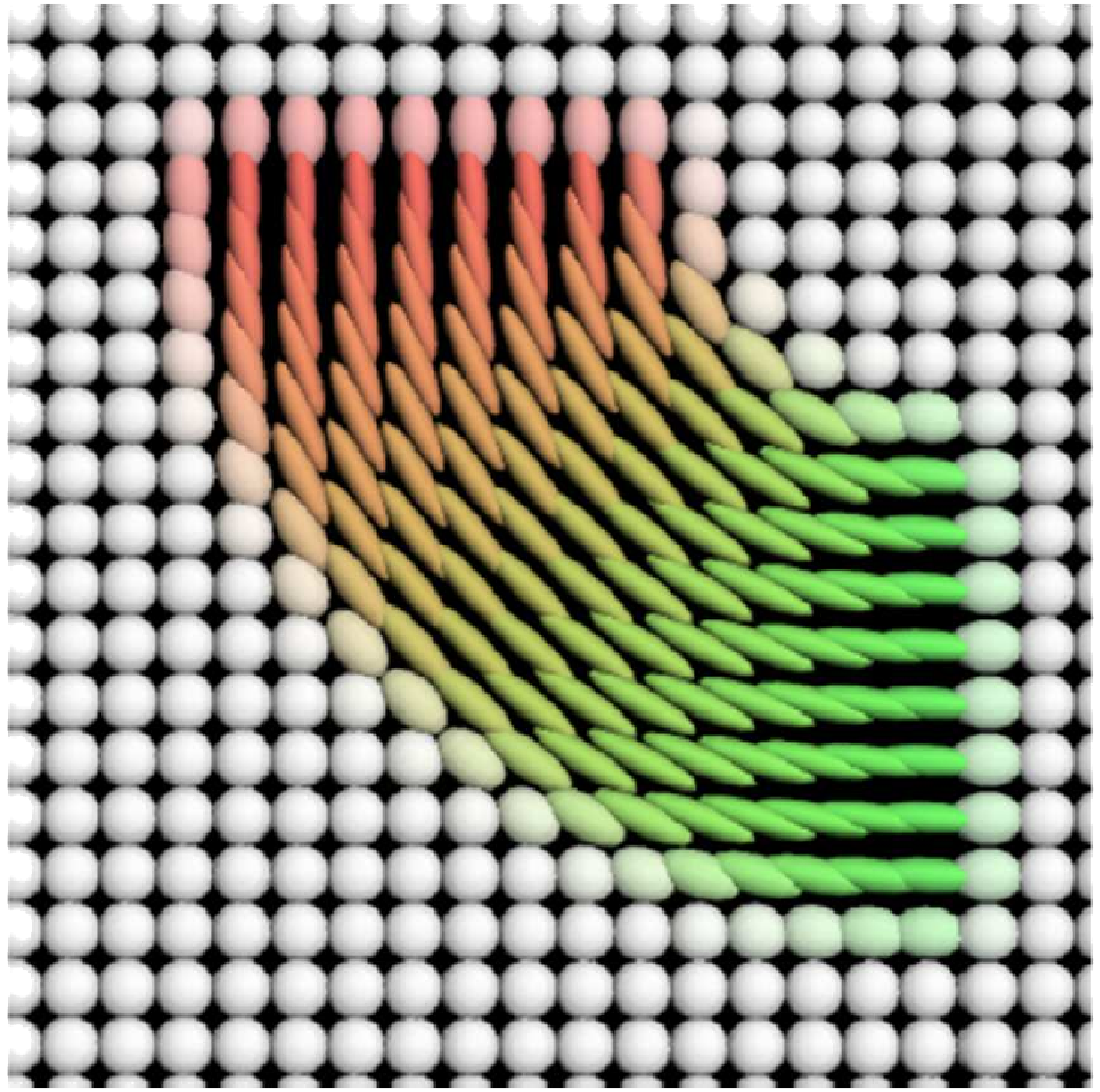

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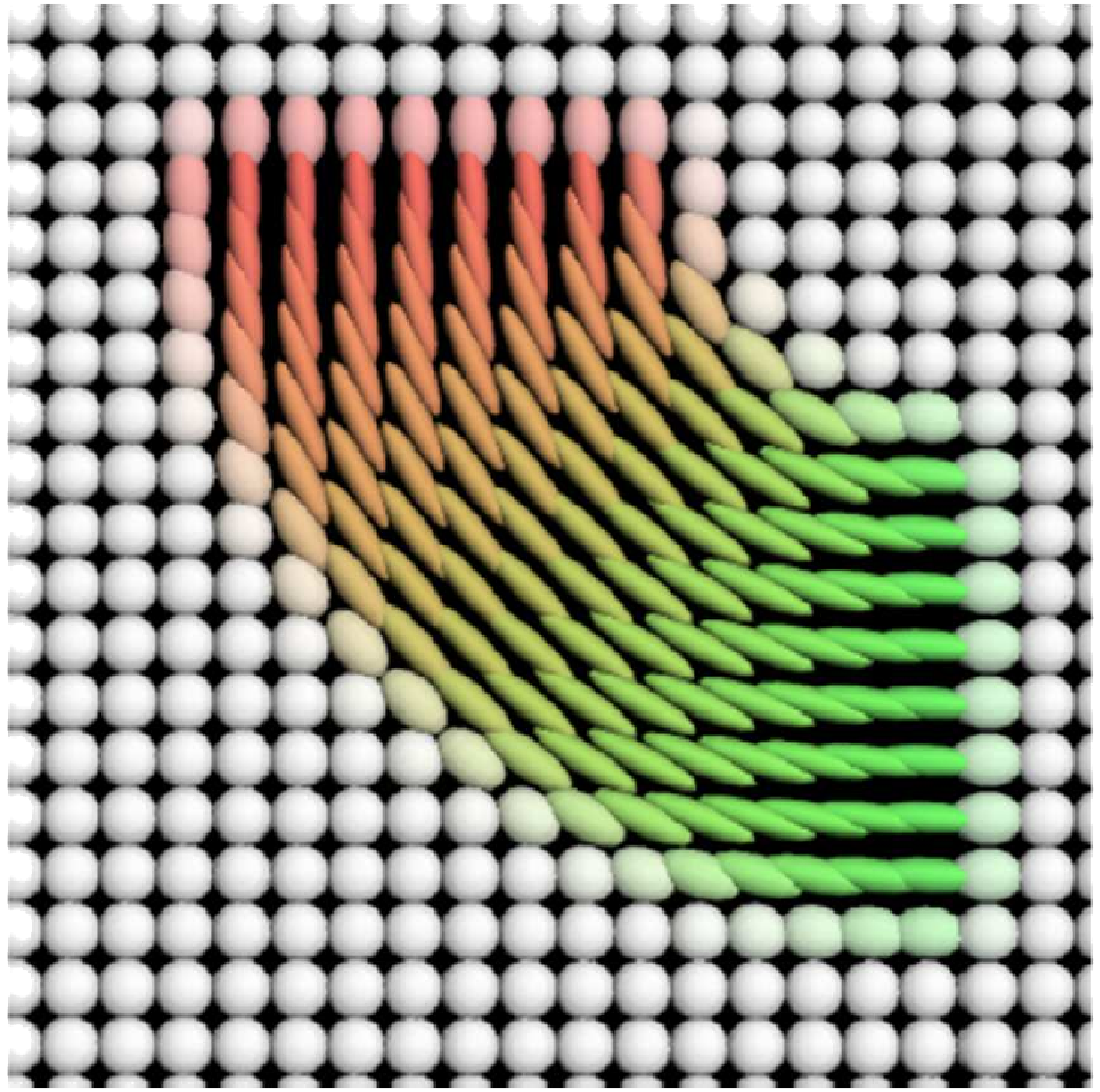

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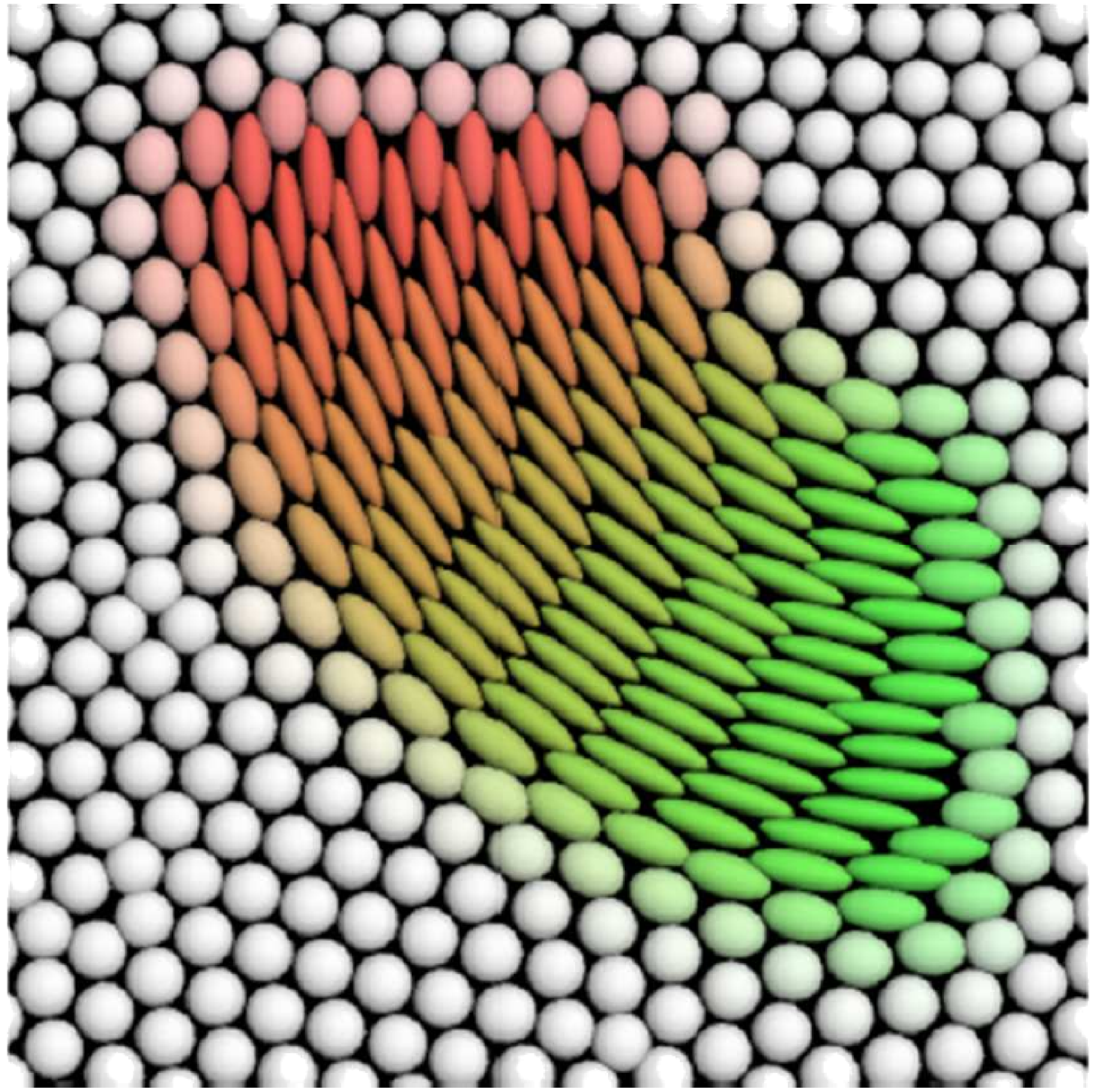

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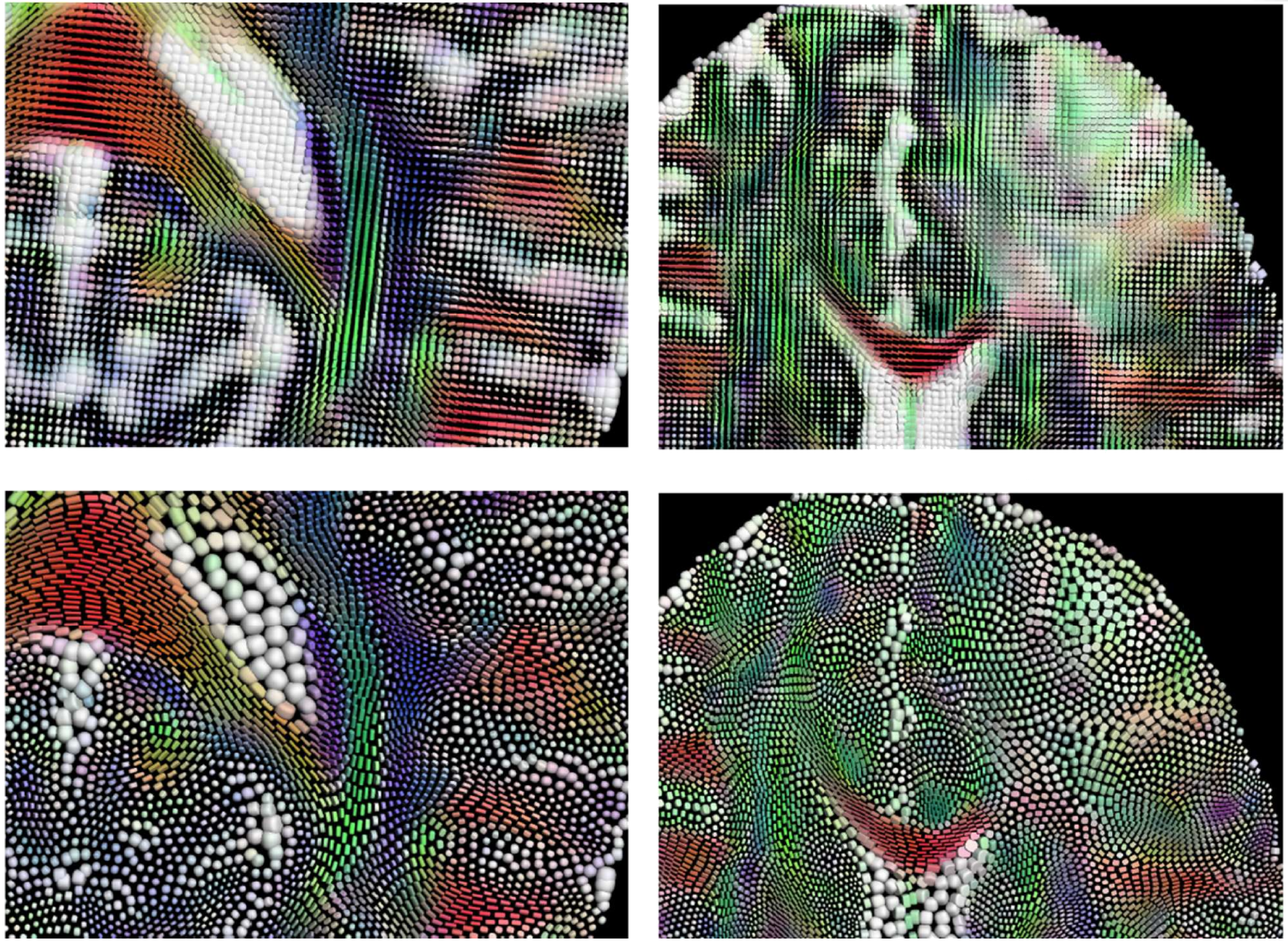


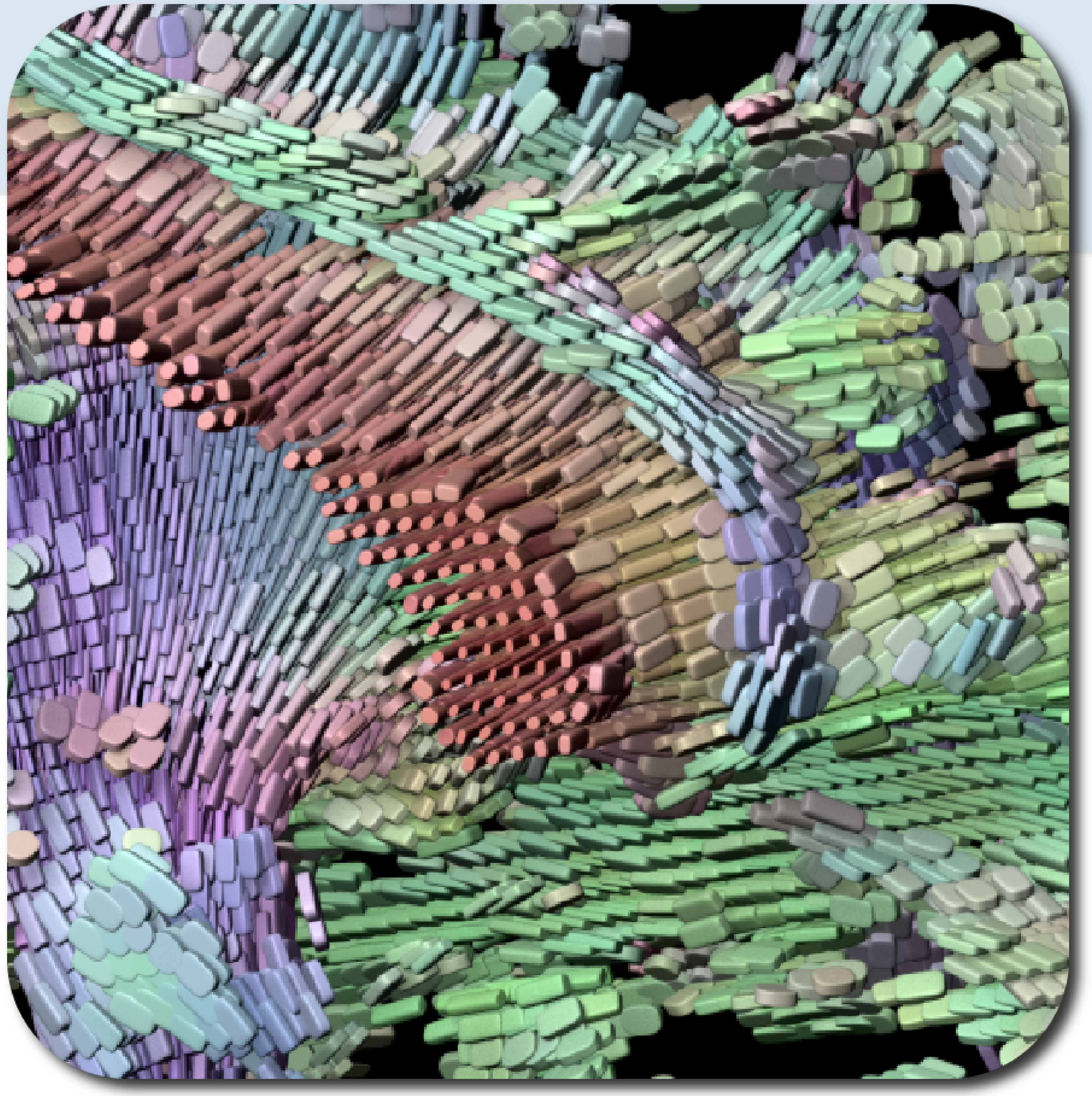
[Kindlmann]

Putting it all together

[Kindlmann]

Tensor glyphs

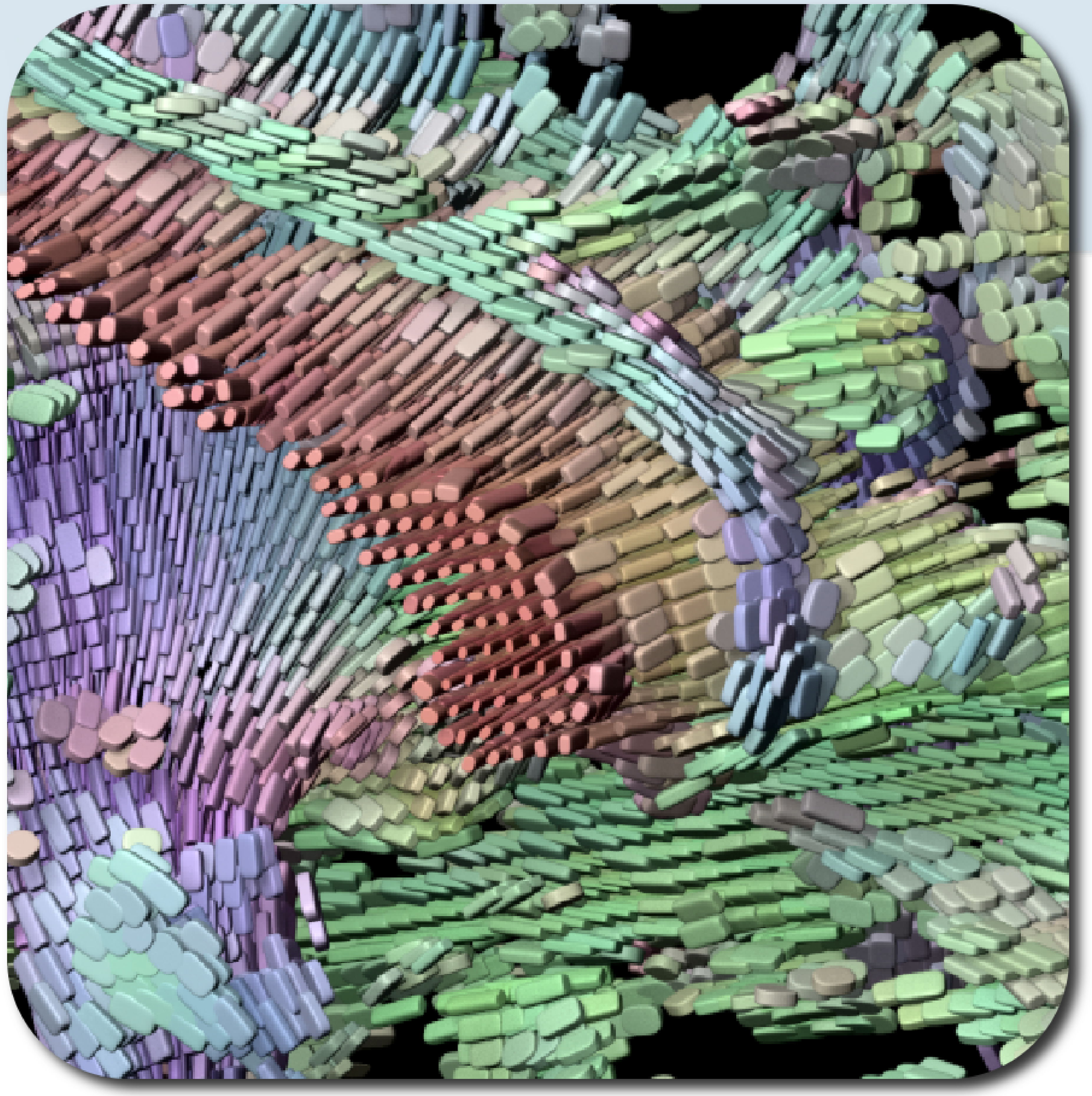
- Provide important insights



[Kindlmann]

Tensor glyphs

- Provide important insights
  - Direction information



[Kindlmann]

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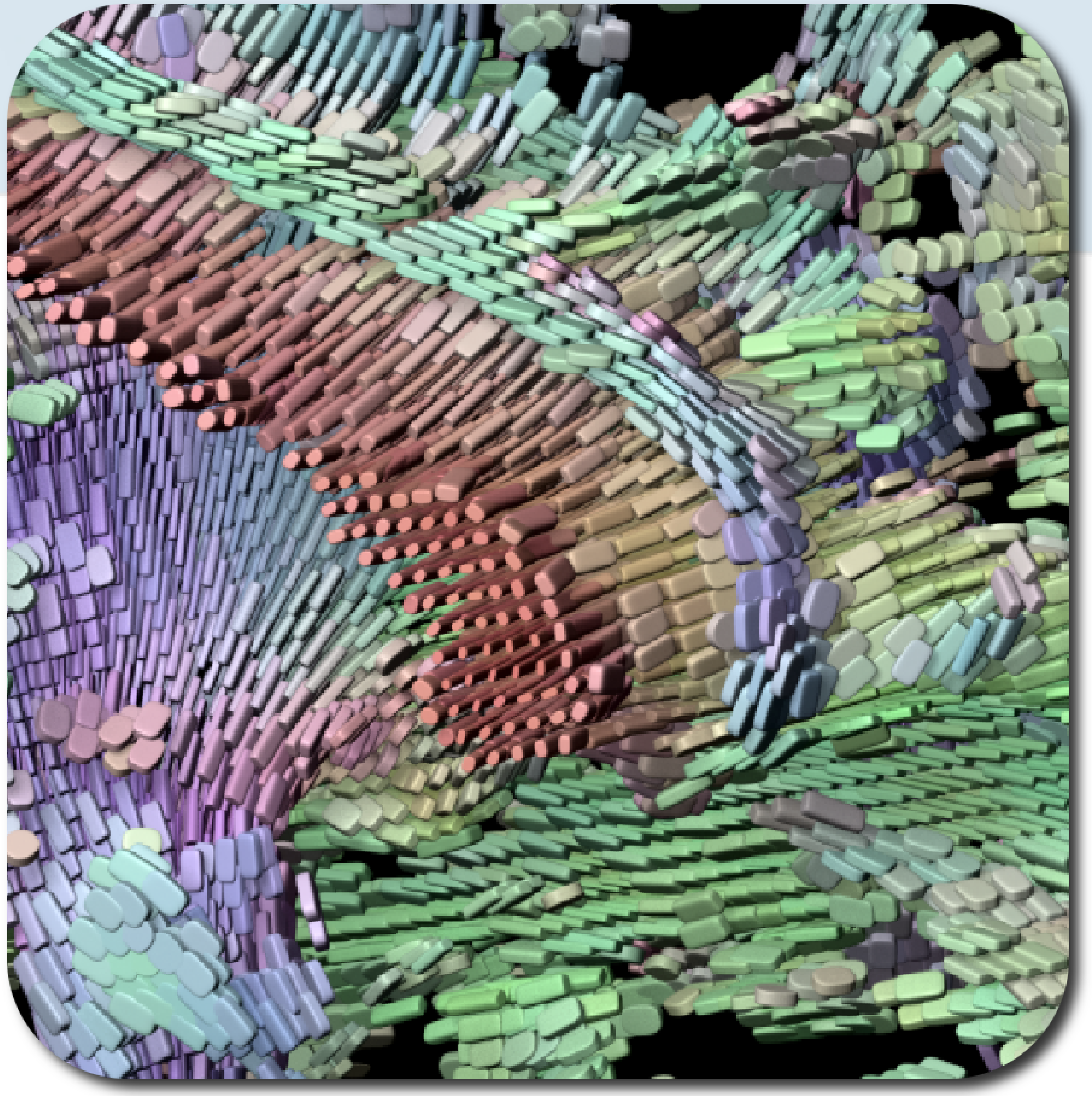

[Kindlmann]

Tensor glyphs

- Provide important insights
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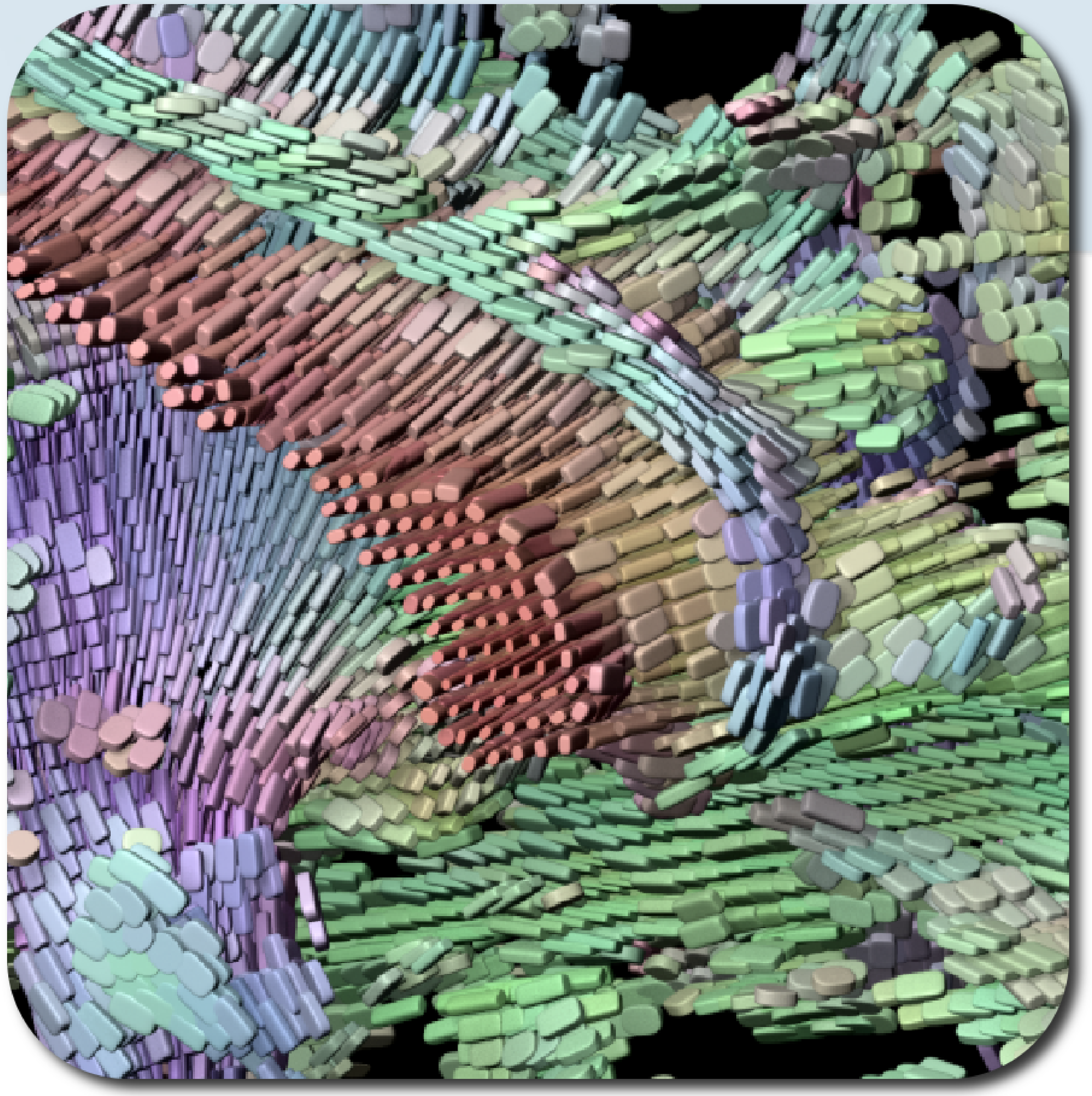
- Still
  - Occlusion issues

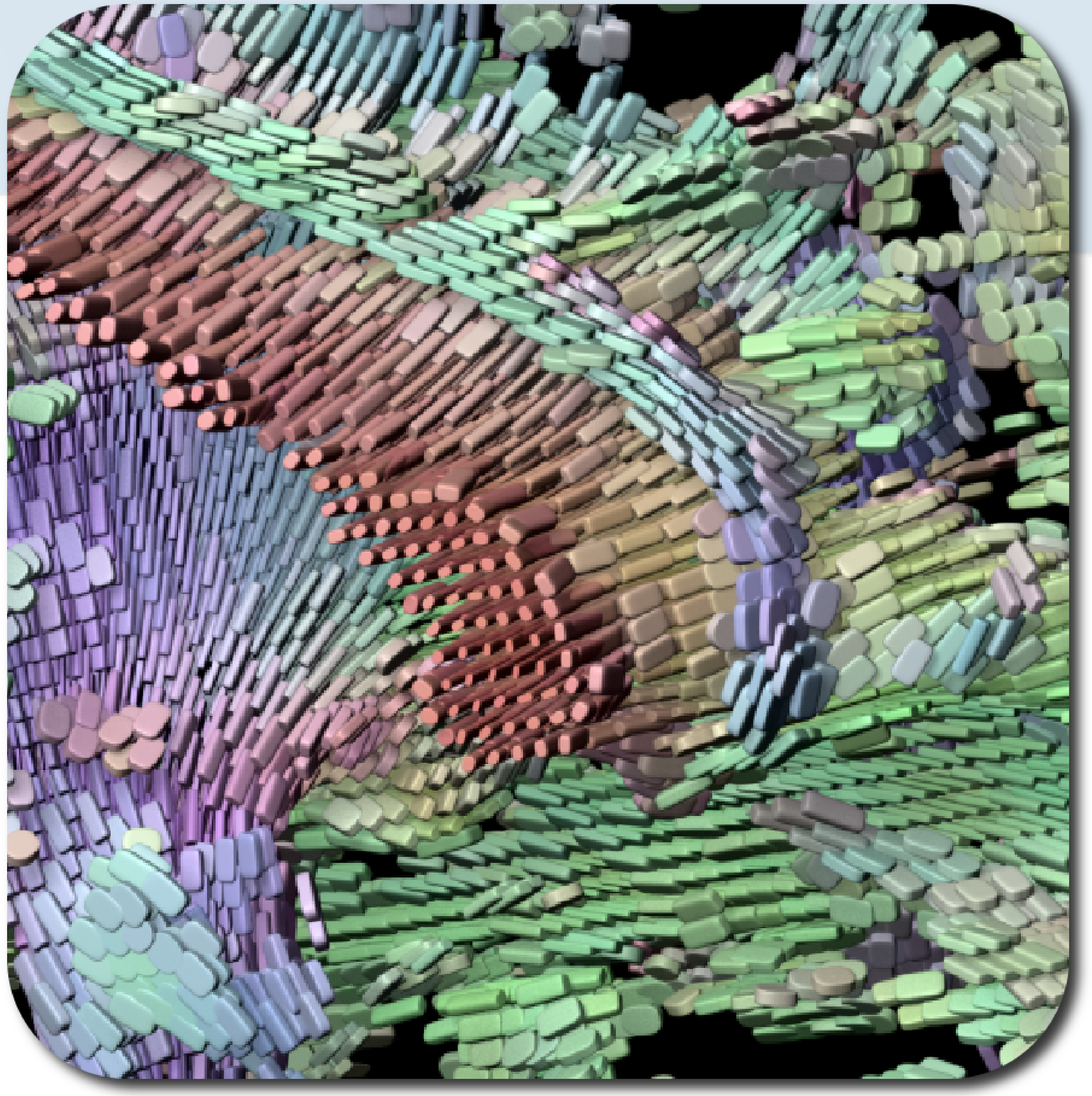


[Kindlmann]

Tensor glyphs

- Provide important insights
  - Direction information
  - Anisotropy information

- Still
  - Occlusion issues
  - No global information

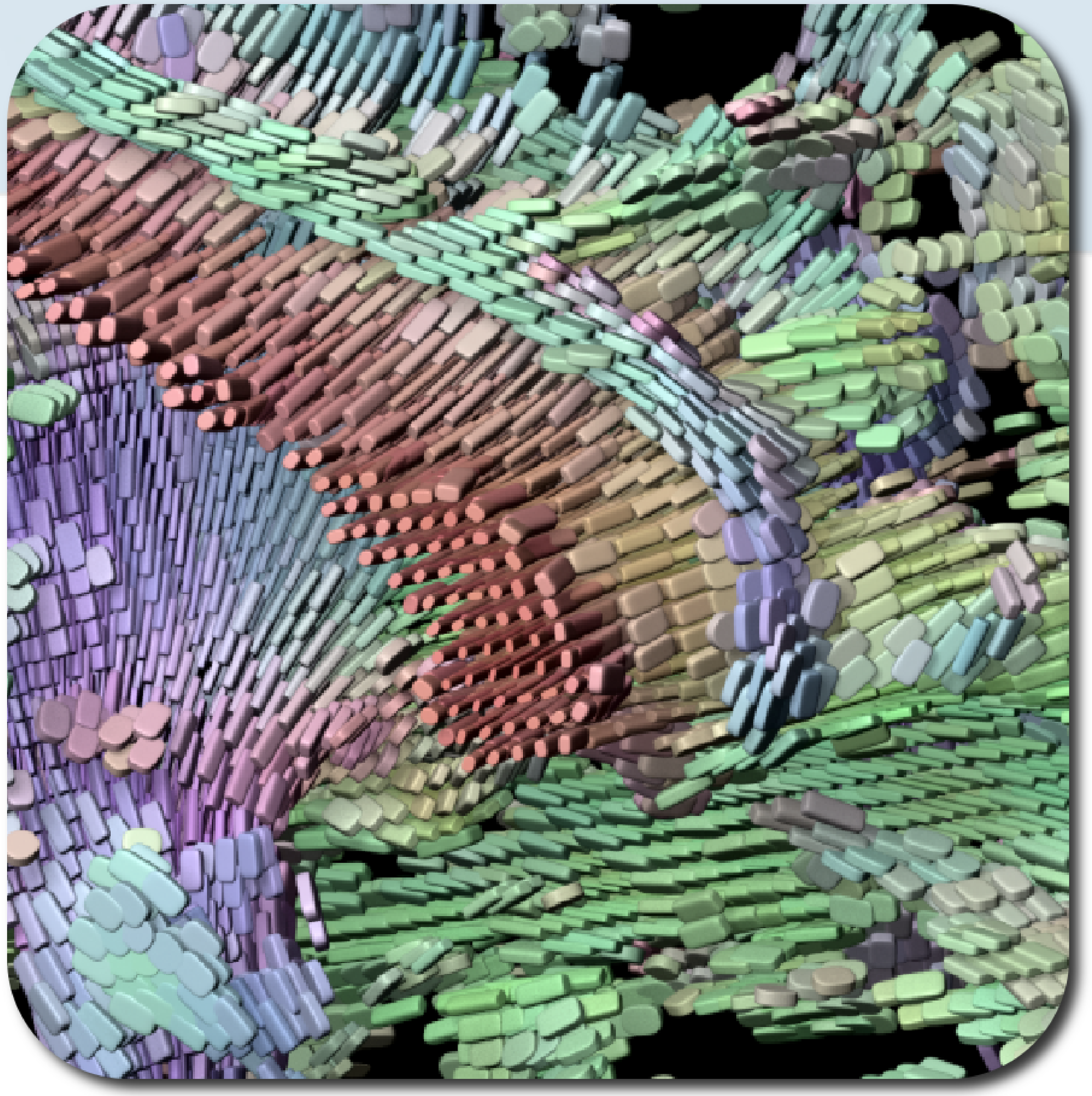


[Kindlmann]

Tensor glyphs

- Provide important insights
  - Direction information
  - Anisotropy information

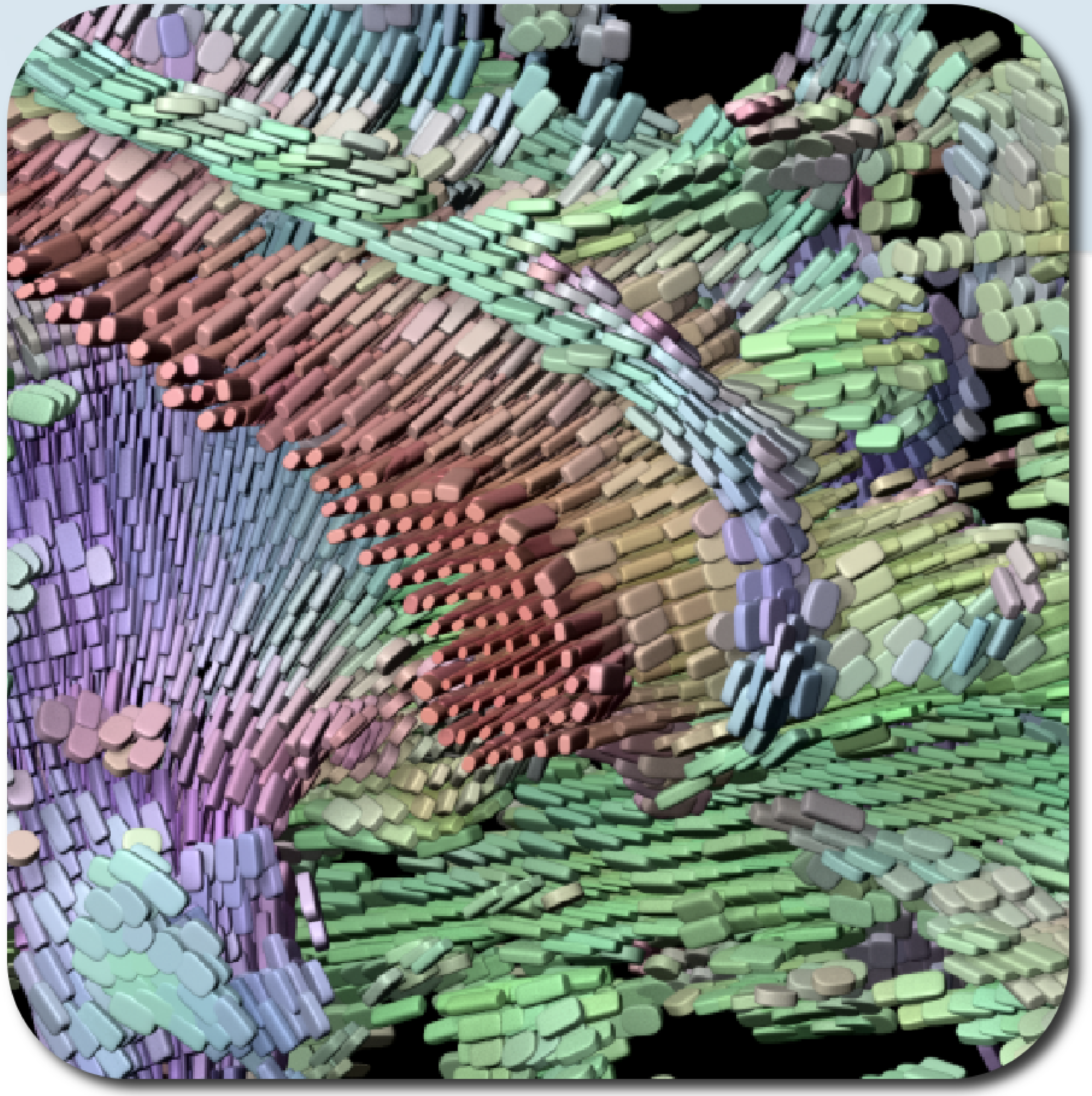
- Still
  - Occlusion issues
  - No global information
  - Dependent on the number of glyphs
    - Trade-off



[Kindlmann]

Beyond glyphs

- Generating more advanced visualizations



[Kindlmann]

Beyond glyphs

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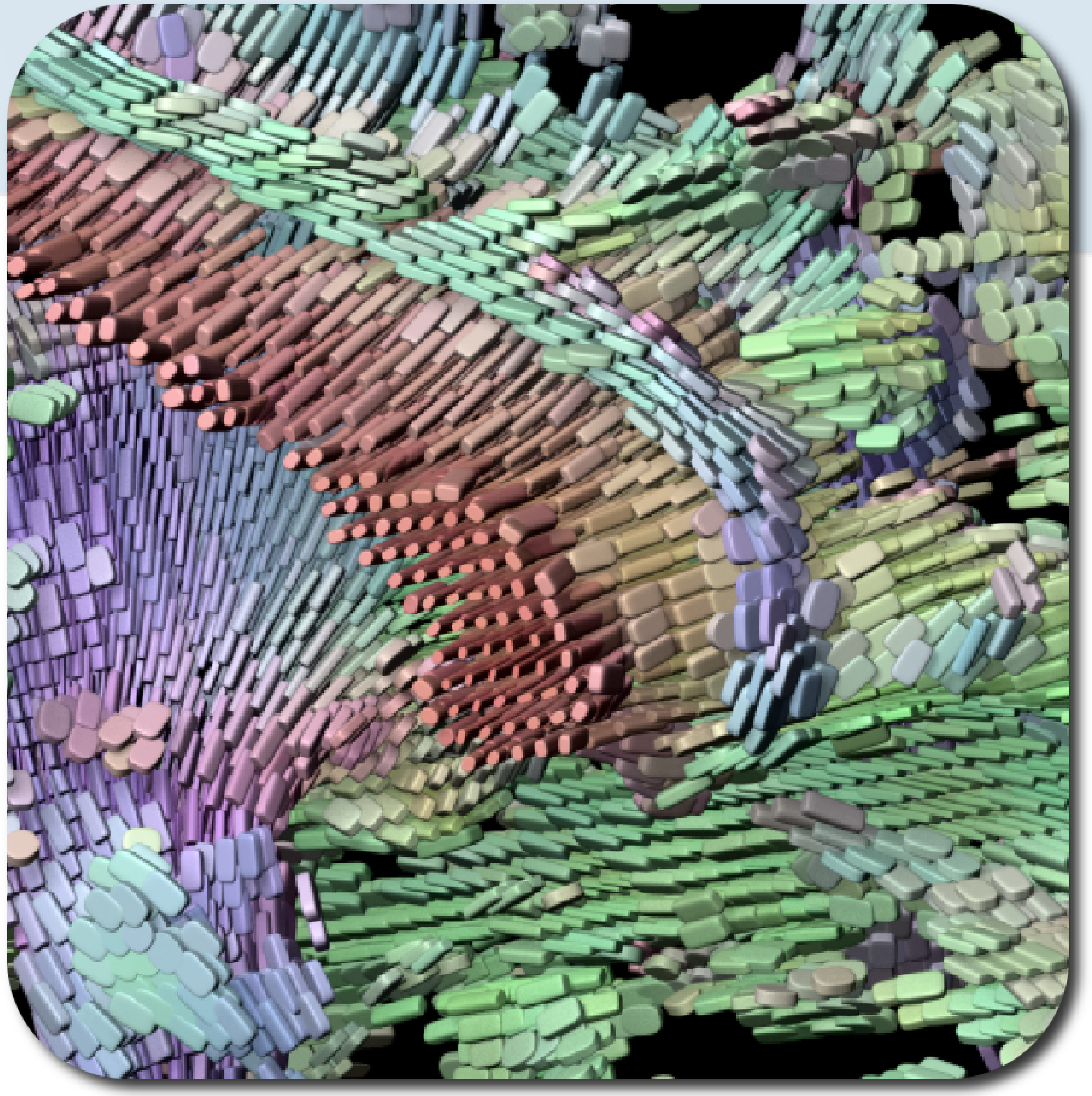
  - Global visualization



[Kindlmann]

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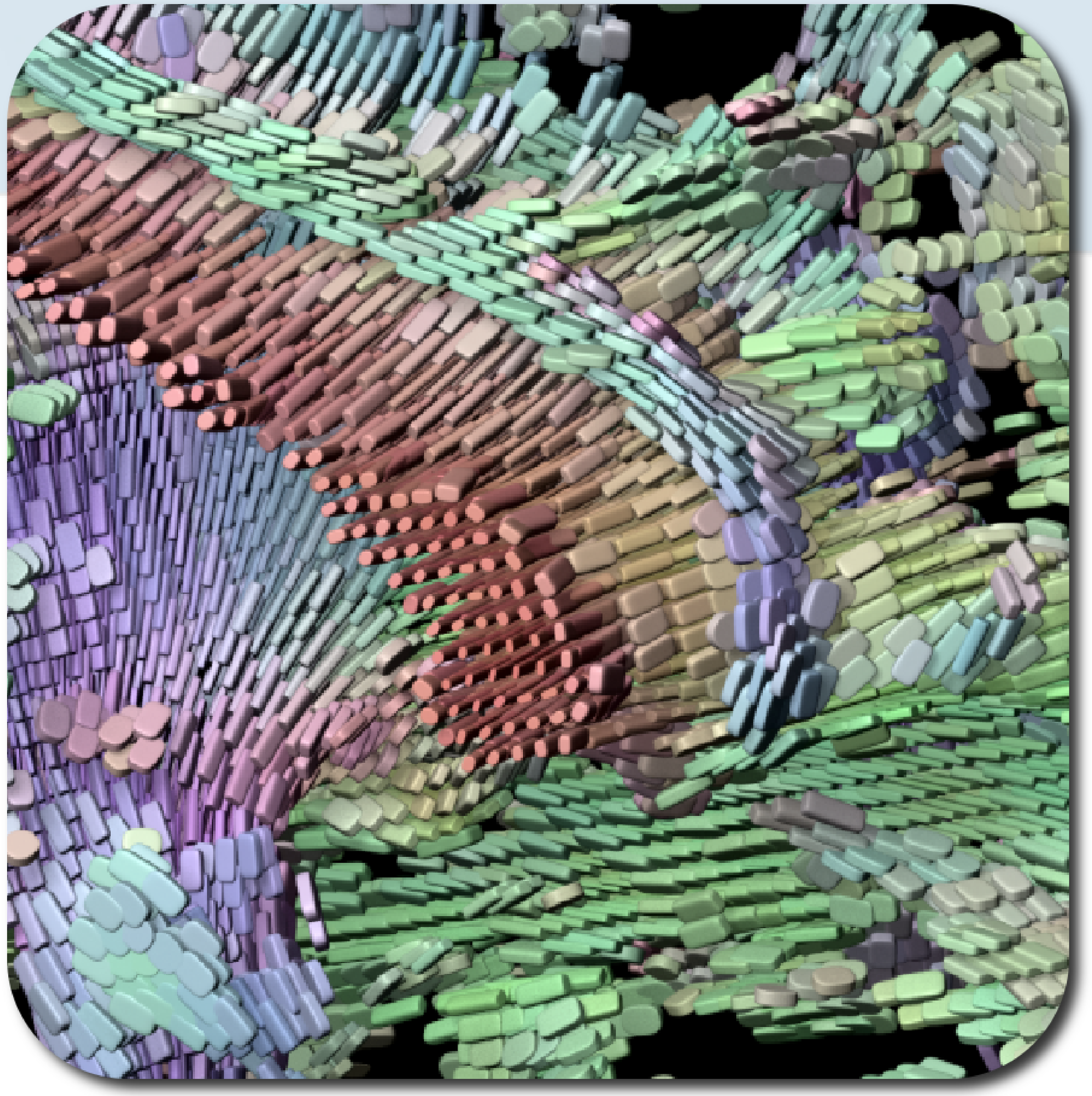

[Kindlmann]

Beyond glyphs

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  - Global visualization
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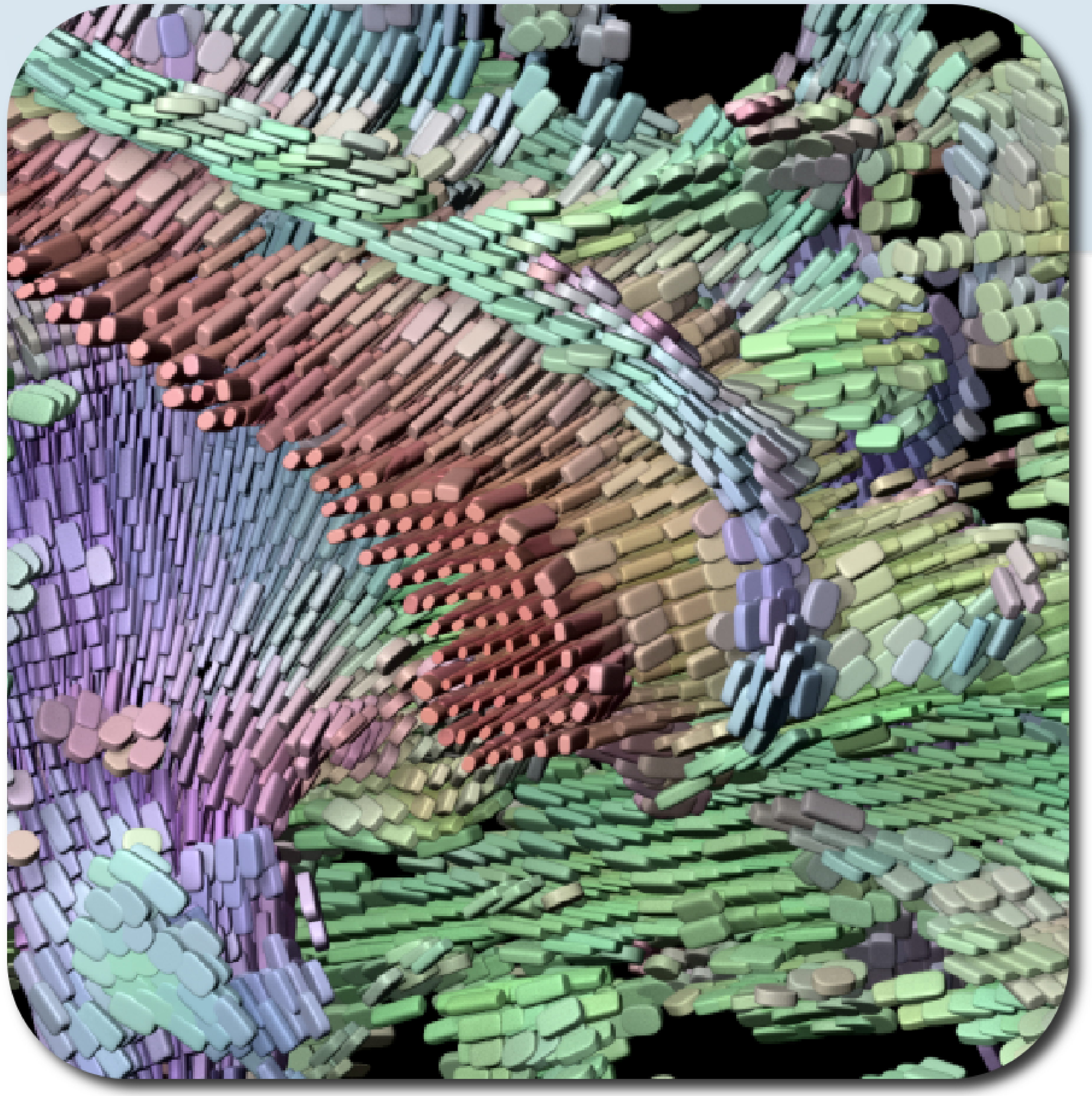
- Getting inspiration from...



[Kindlmann]

Beyond glyphs

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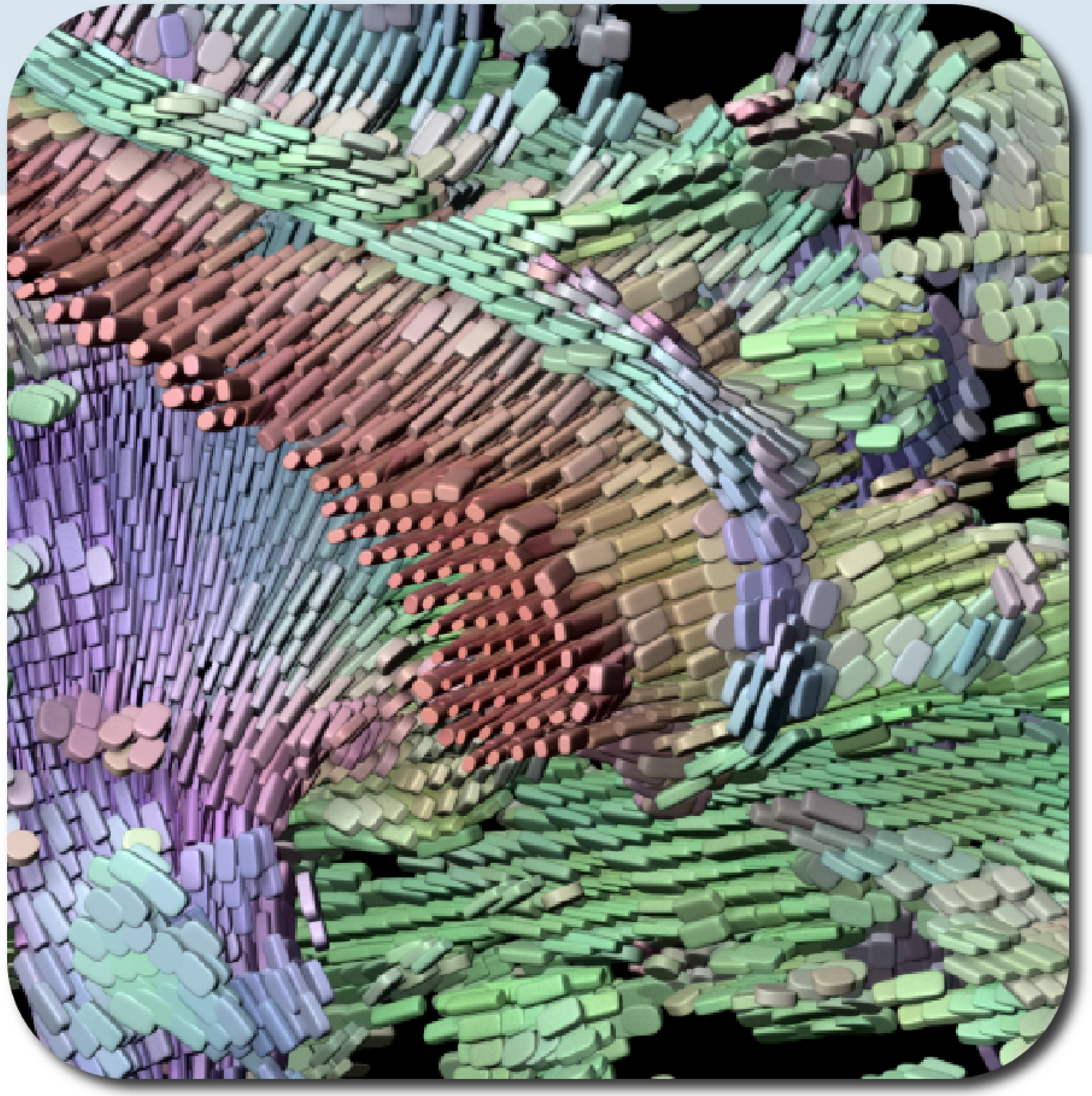
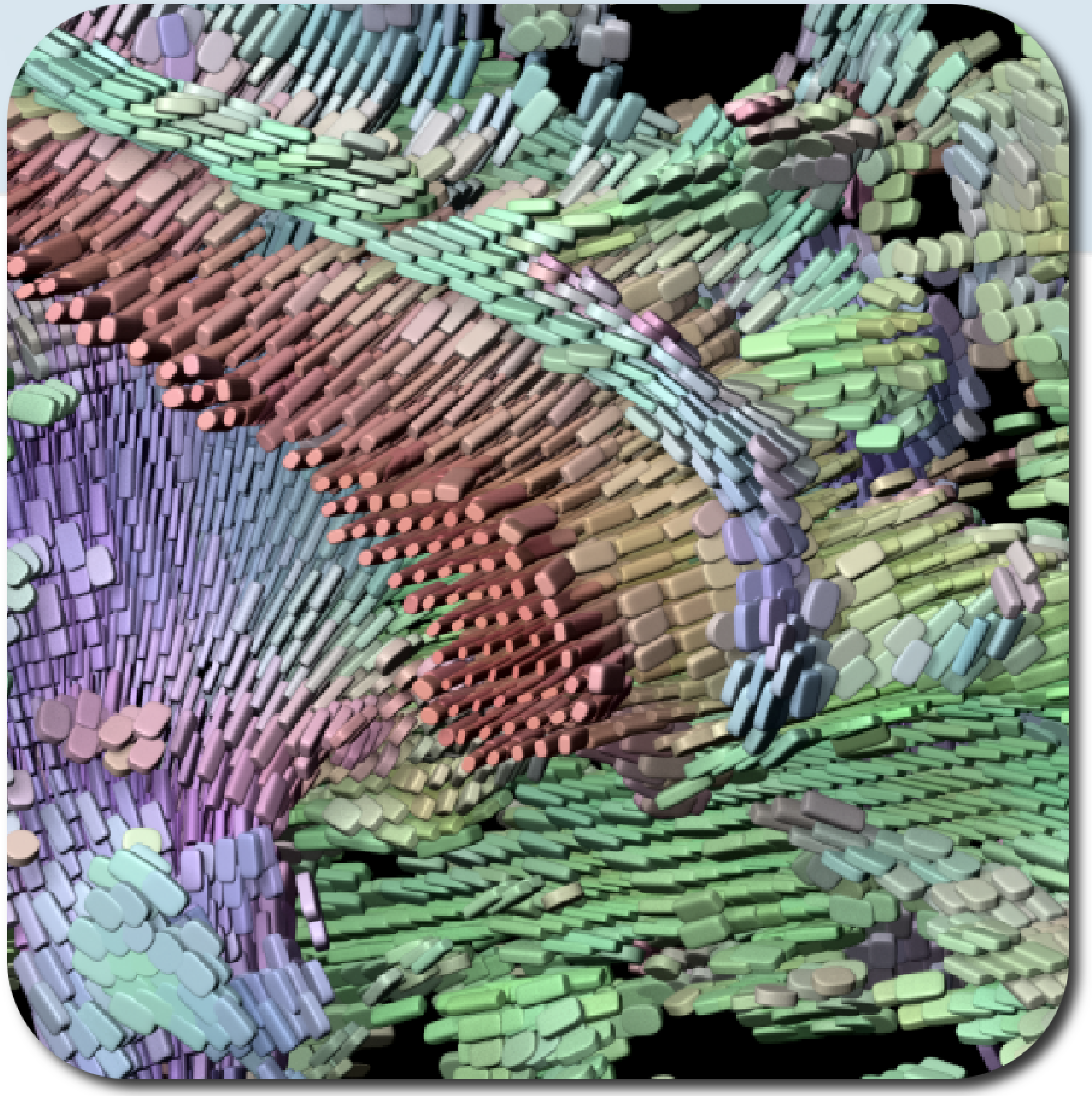


[Kindlmann]

Beyond glyphs

- Generating more advanced visualizations
  - Global visualization
  - Structural information

- Getting inspiration from...
  - Existing techniques
    - Derived scalar fields



[Kindlmann]

Beyond glyphs

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- Getting inspiration from...
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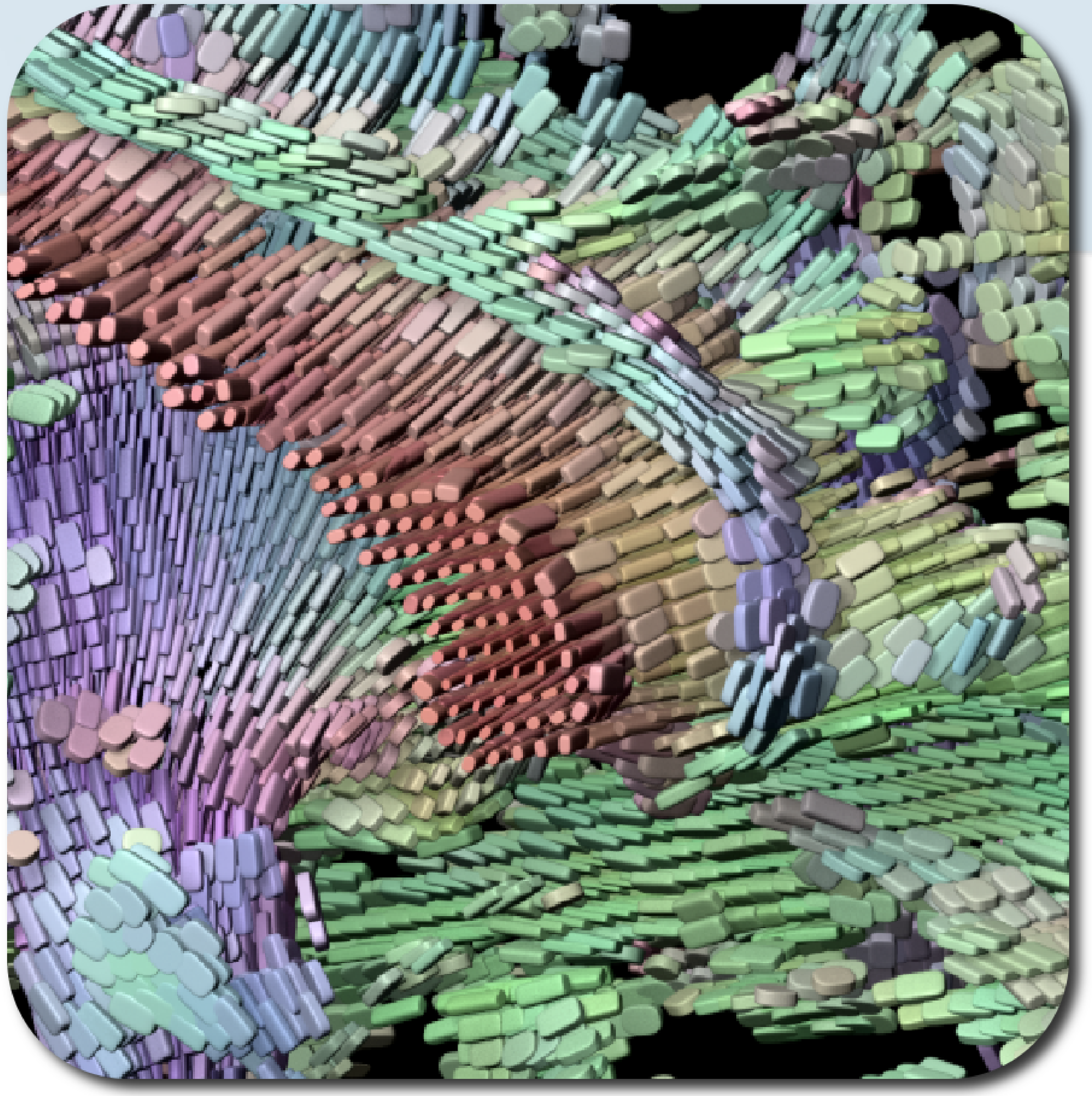
    - Derived scalar fields
    - Derived vector fields



[Kindlmann]

Derived scalar fields

- What scalar values can we visualize?

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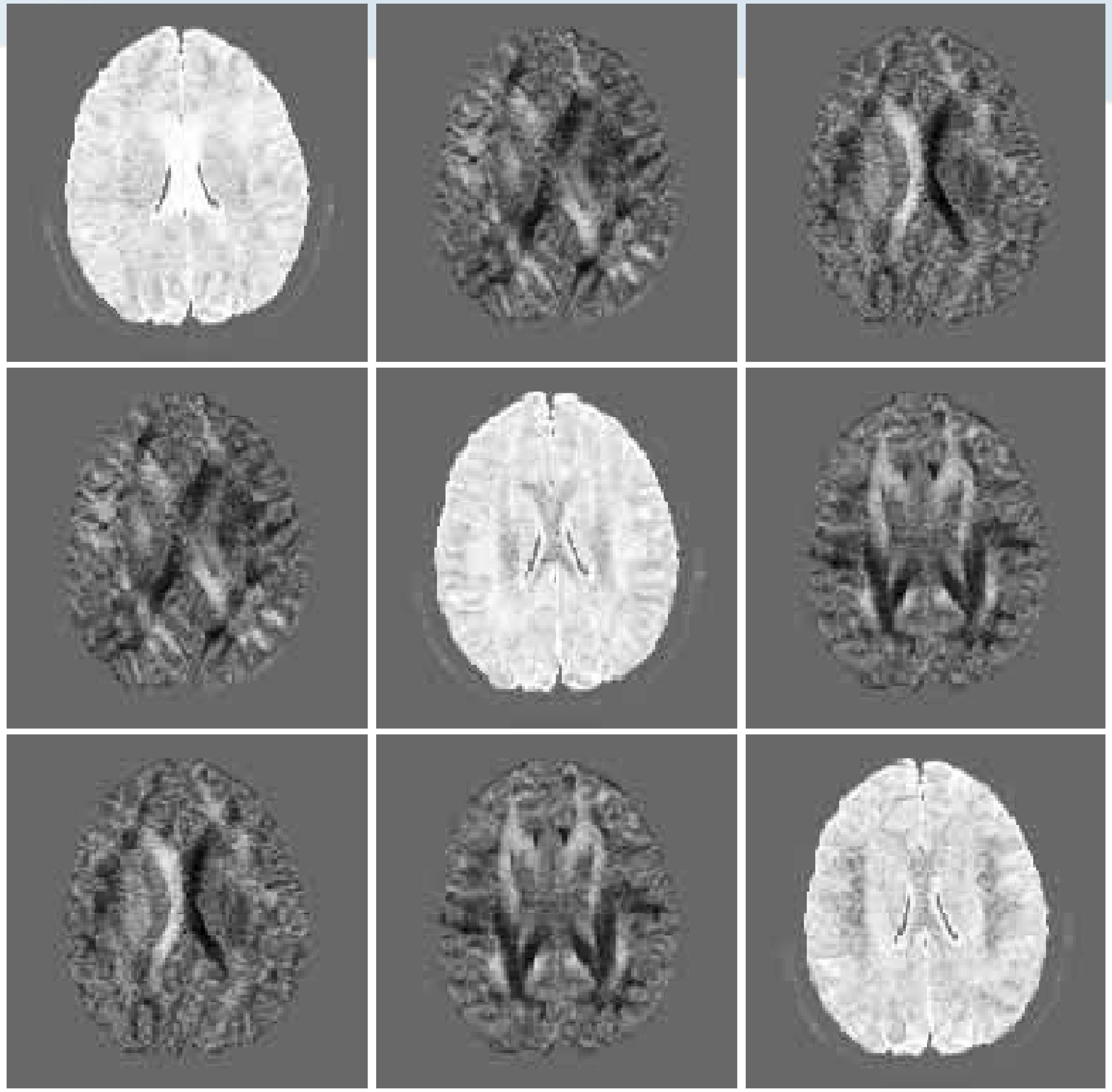

[Moller]

Derived scalar fields

- What scalar values can we visualize?
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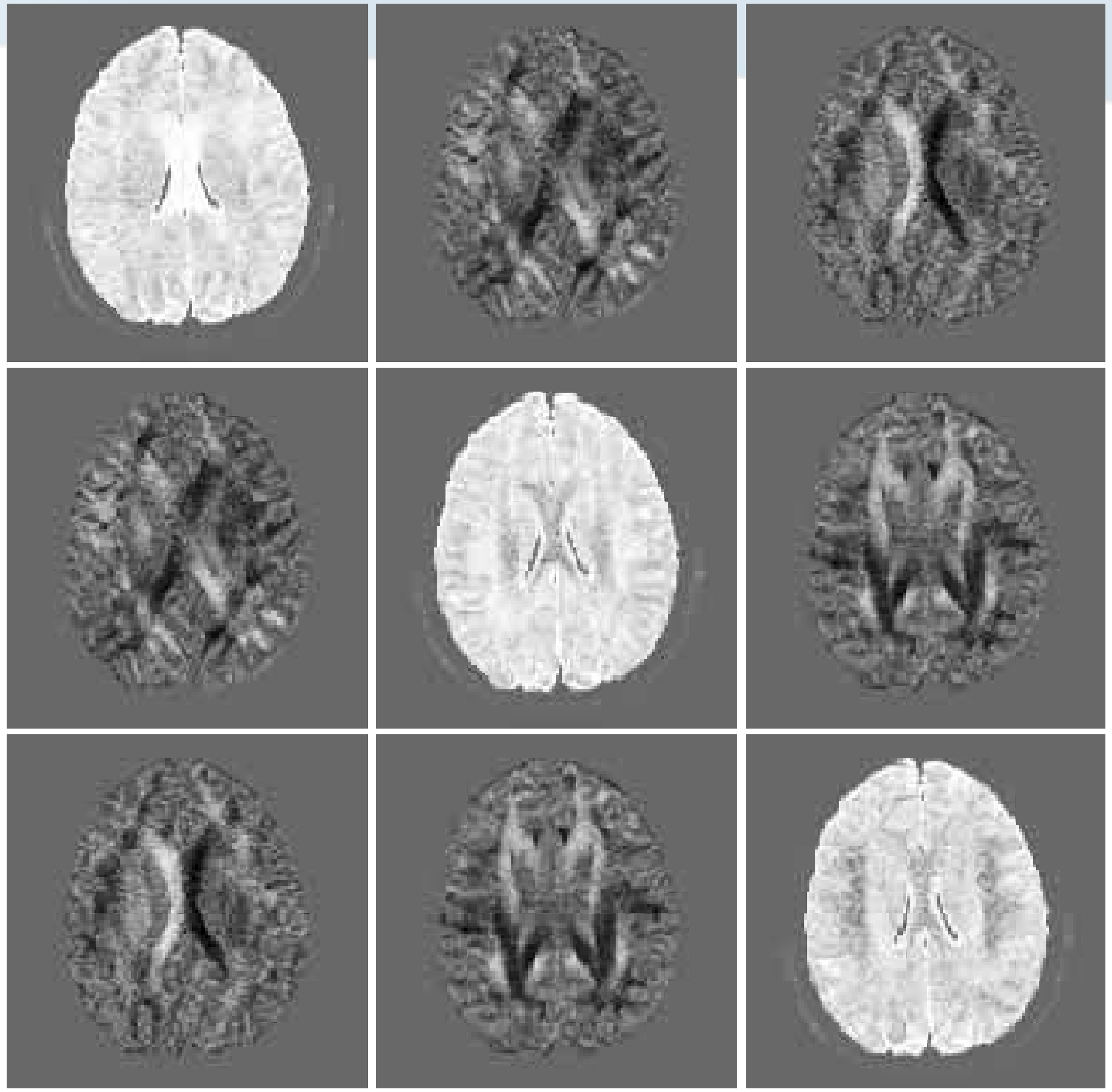
    - Actual components: (dxd) scalar fields
    - Eigen values: d scalar fields



[Moller]

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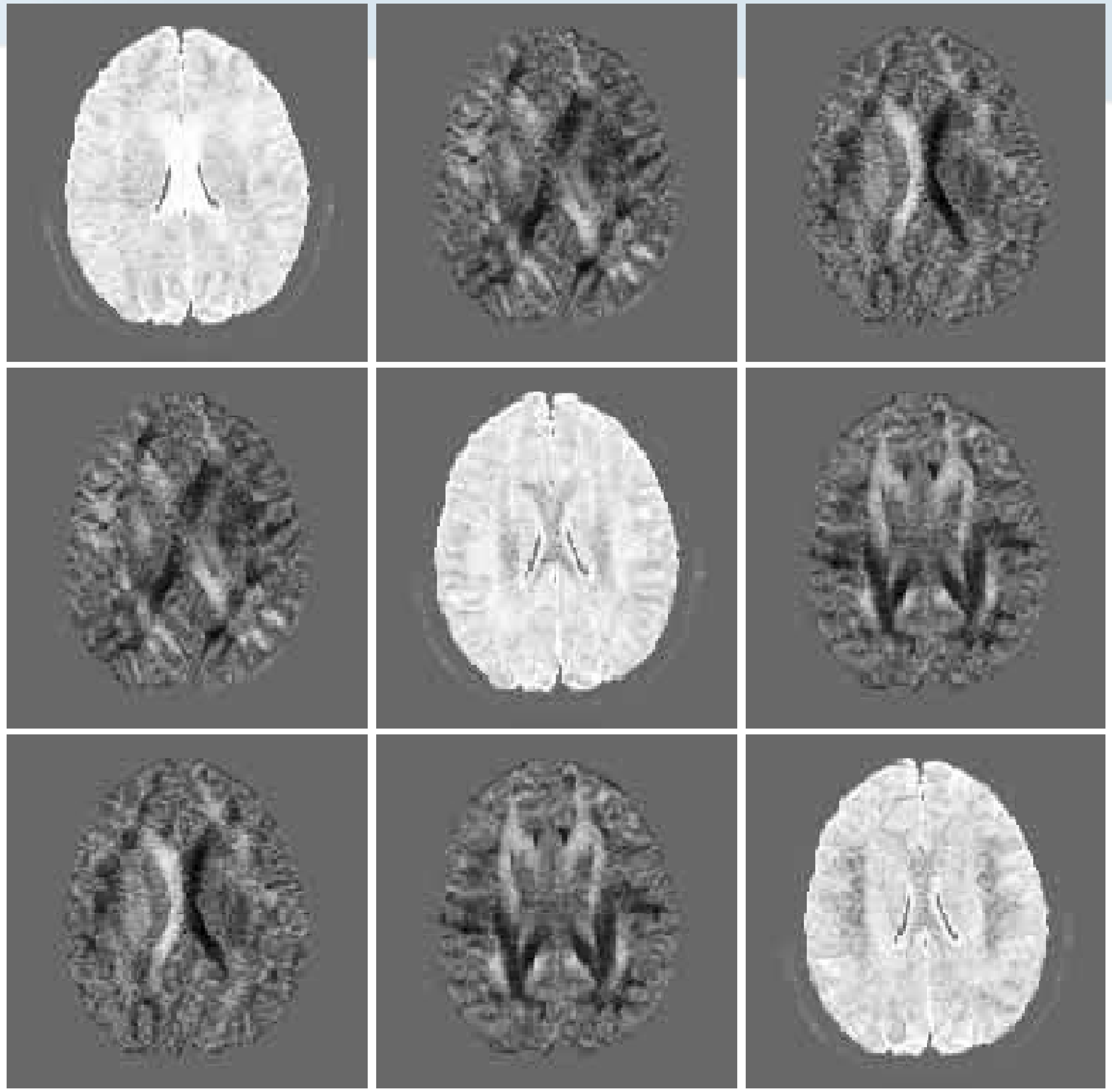
    - Eigen values: d scalar fields
  - Global information
    - Determinant



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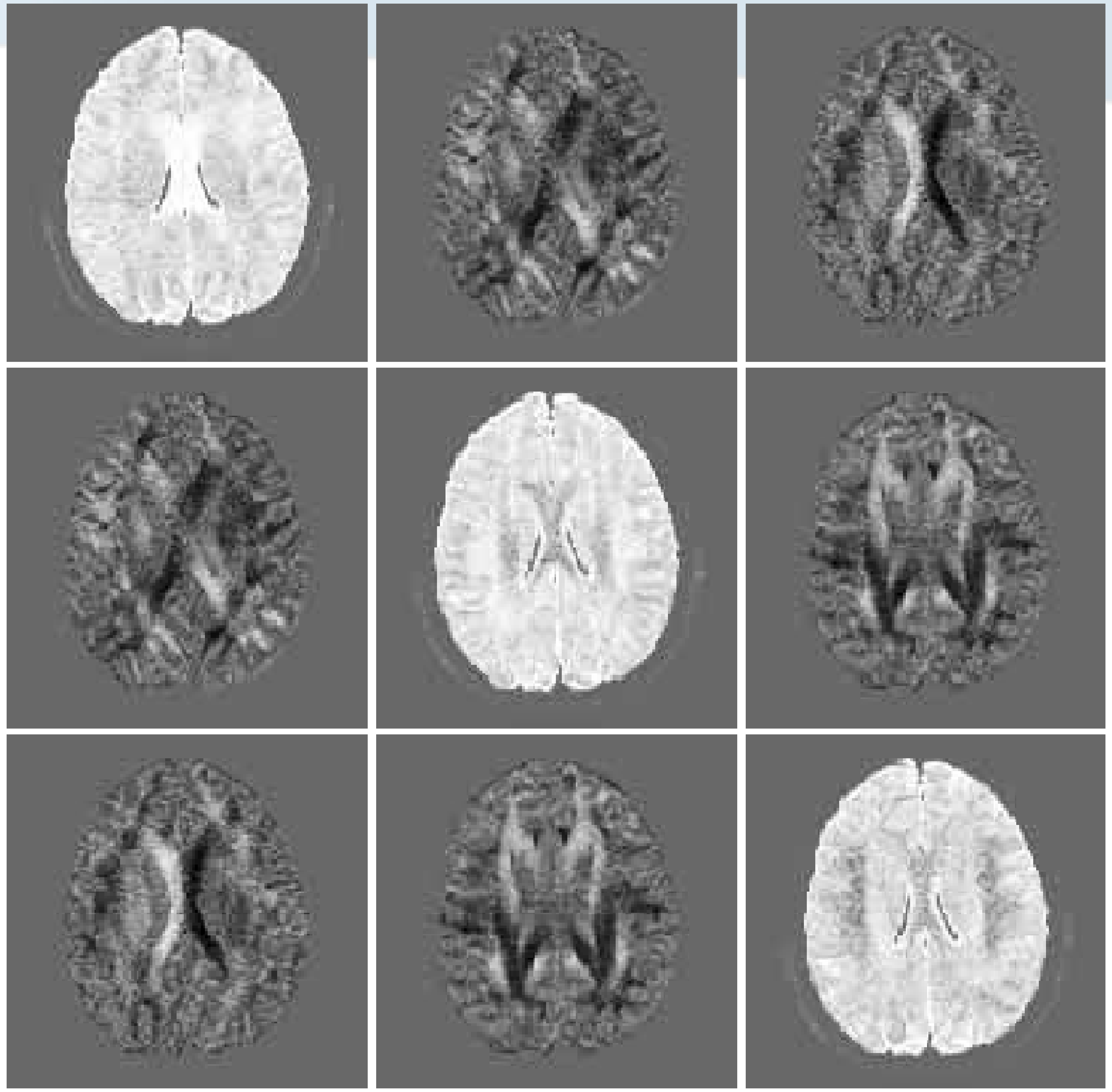
  - Global information
    - Determinant
    - Magnitude

$$||f(v)|| = \sqrt{\frac{1}{2}\sum f(v)_{ij}^2}$$



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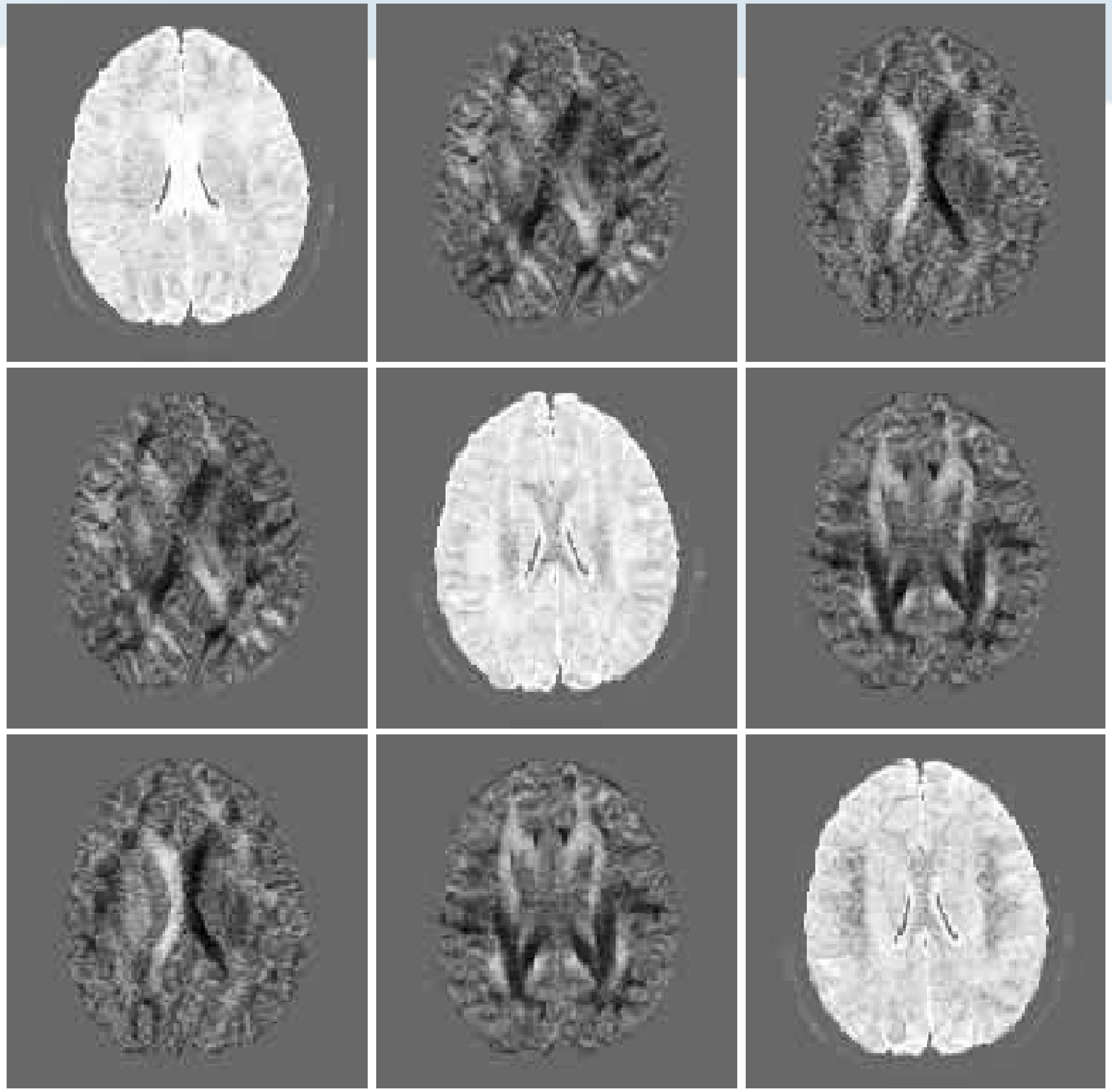
  - Component-wise
    - Actual components: (dxd) scalar fields
    - Eigen values: d scalar fields
  - Global information
    - Determinant
    - Magnitude
    - Trace

$$||f(v)|| = \sqrt{\frac{1}{2}\sum f(v)_{ij}^2}$$

$$tr(f(v)) = \sum f(v)_{ii}$$



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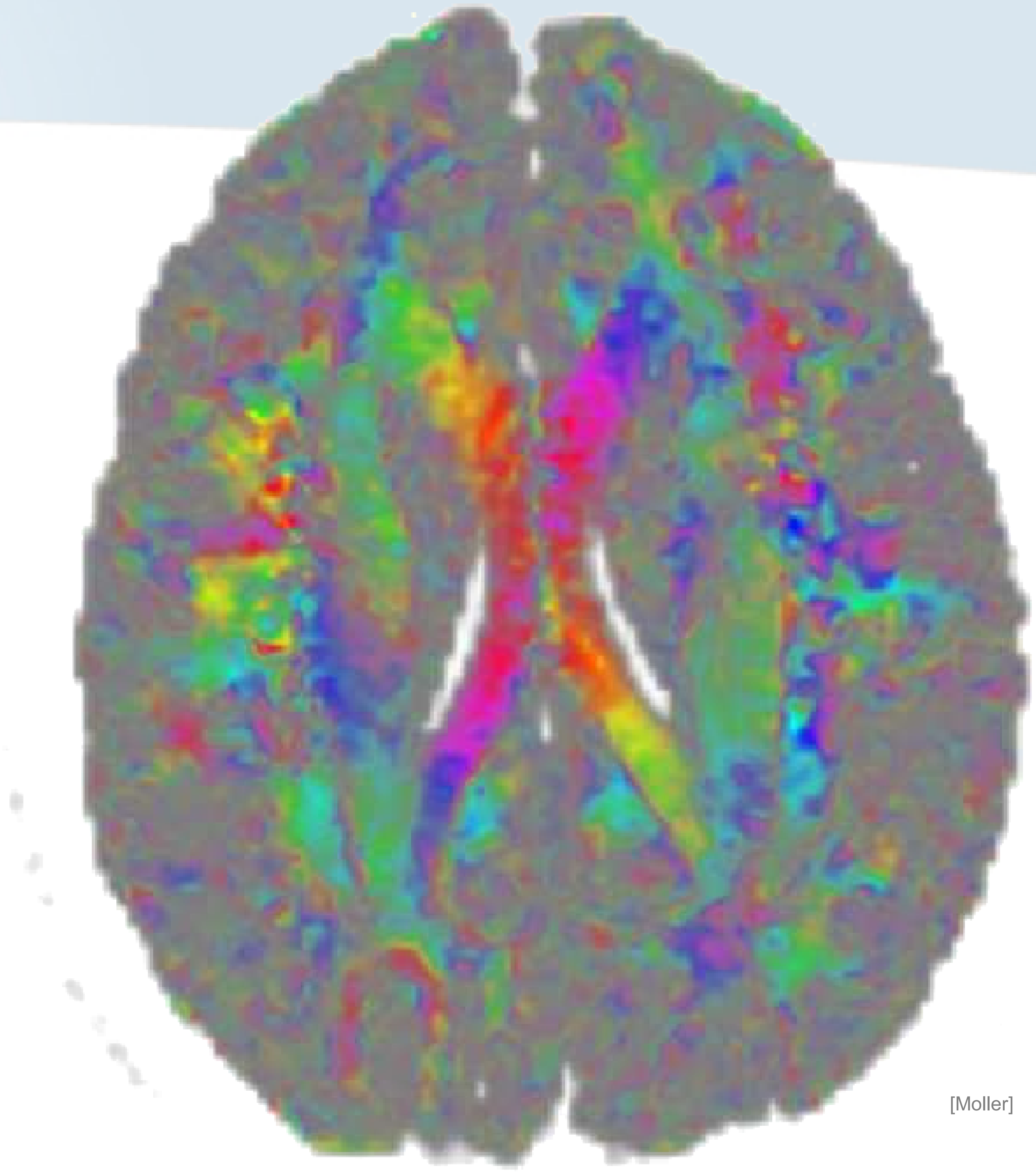

[Moller]

Derived vector fields

- What interesting vector fields could we consider?

Derived vector fields

- What interesting vector fields could we consider?
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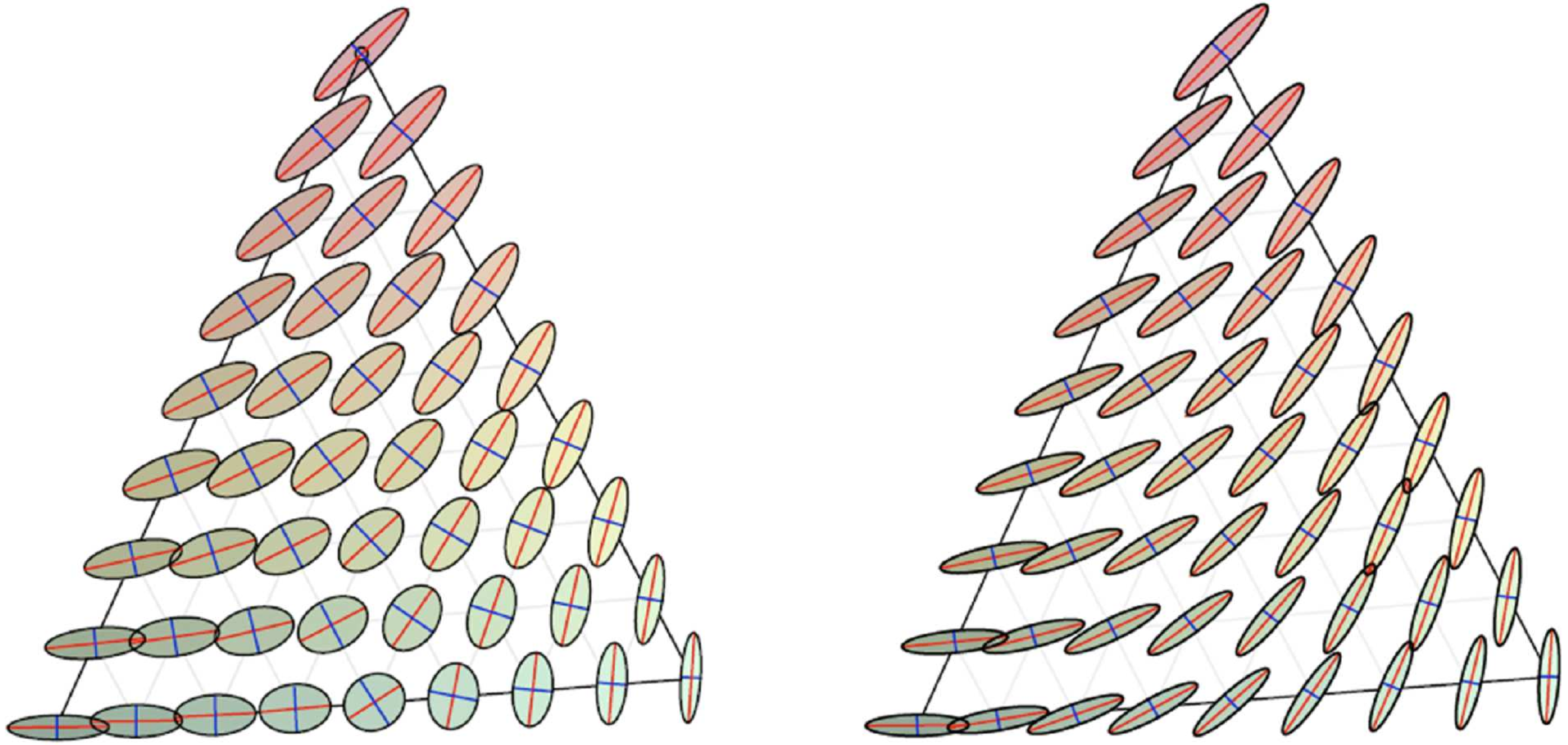
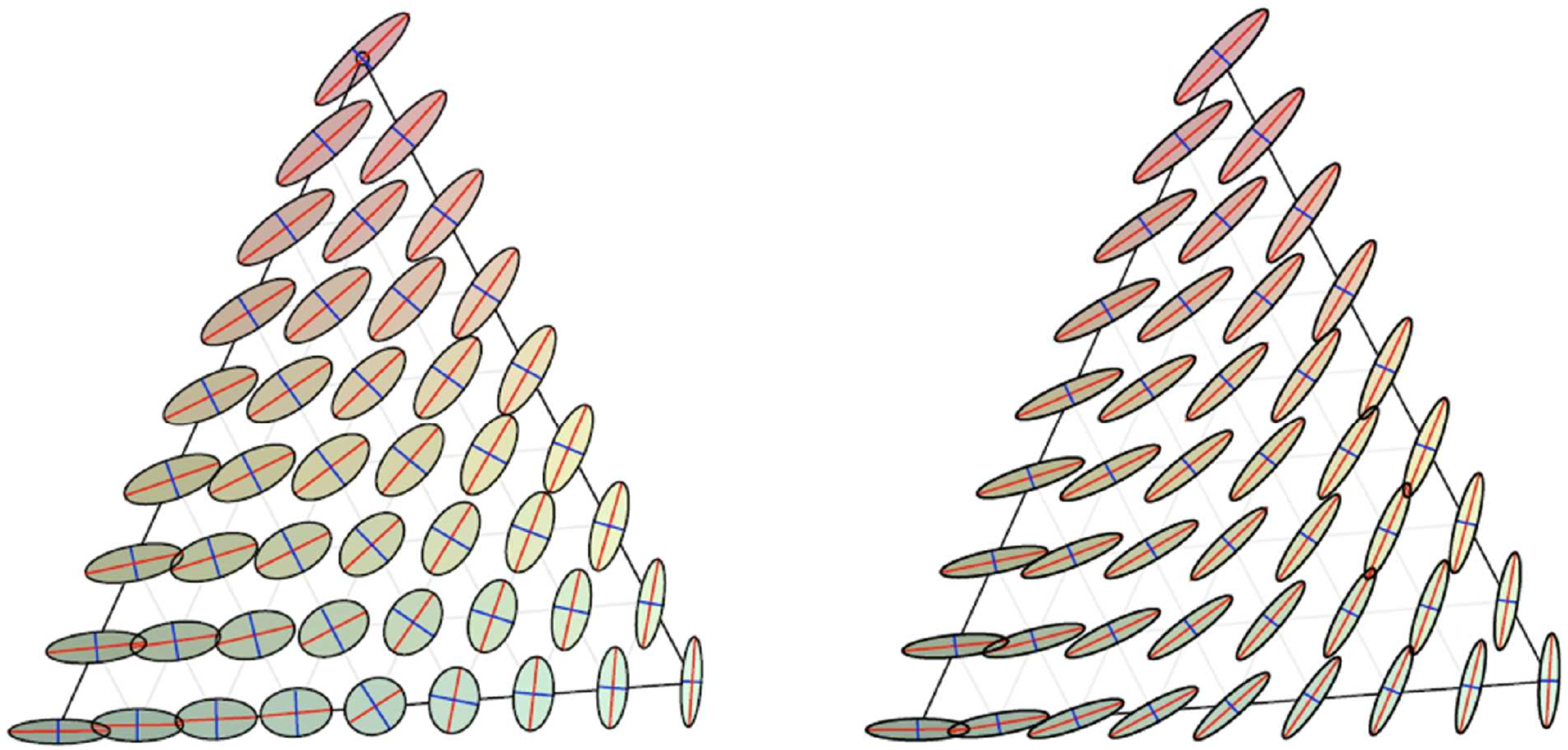


[Hotz]

Derived vector fields

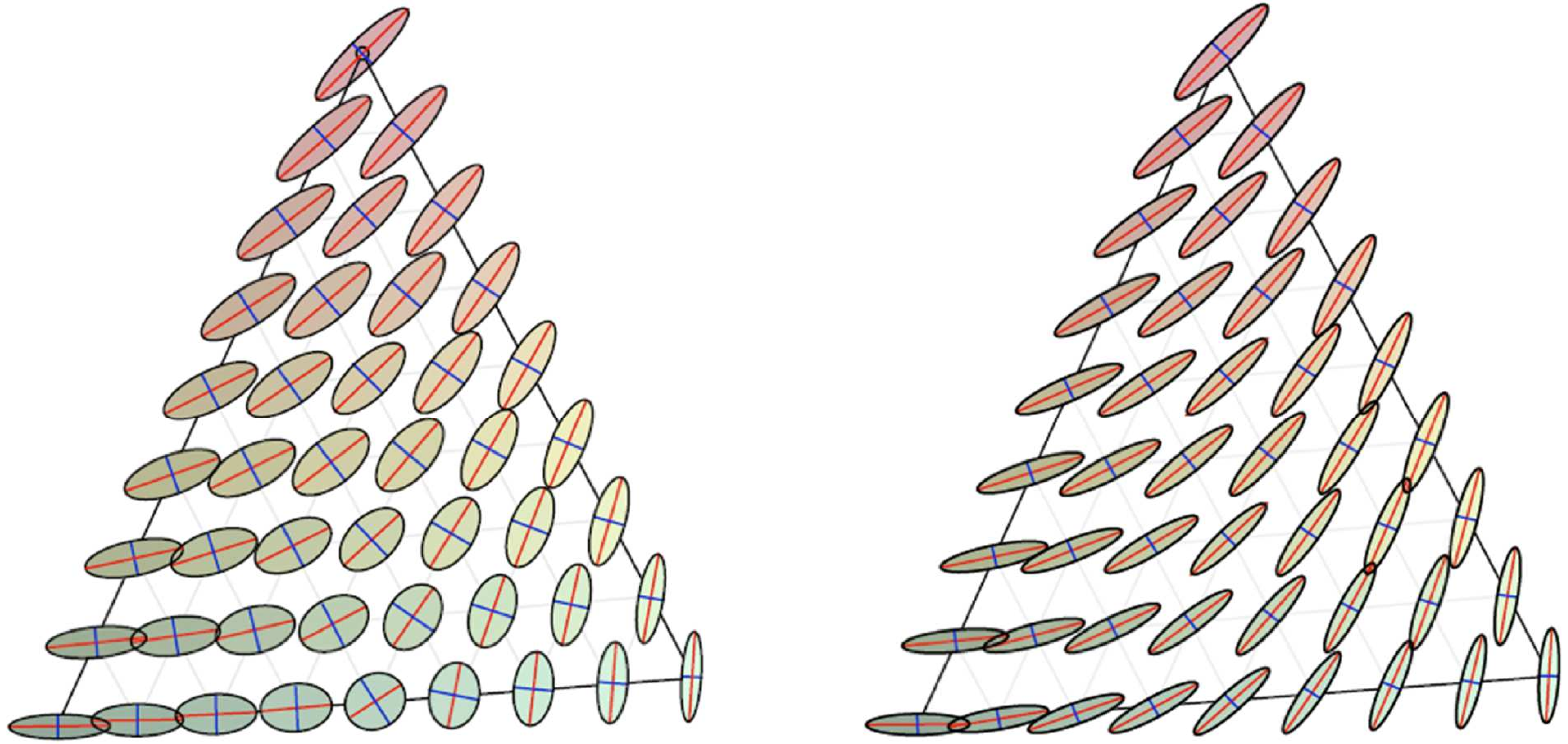
- What interesting vector fields could we consider?
  - Eigenvectors
    - Not a real vector field



[Hotz]

Derived vector fields

- What interesting vector fields could we consider?
  - Eigenvectors
    - Not a real vector field
      - – No magnitude
      - – No orientation



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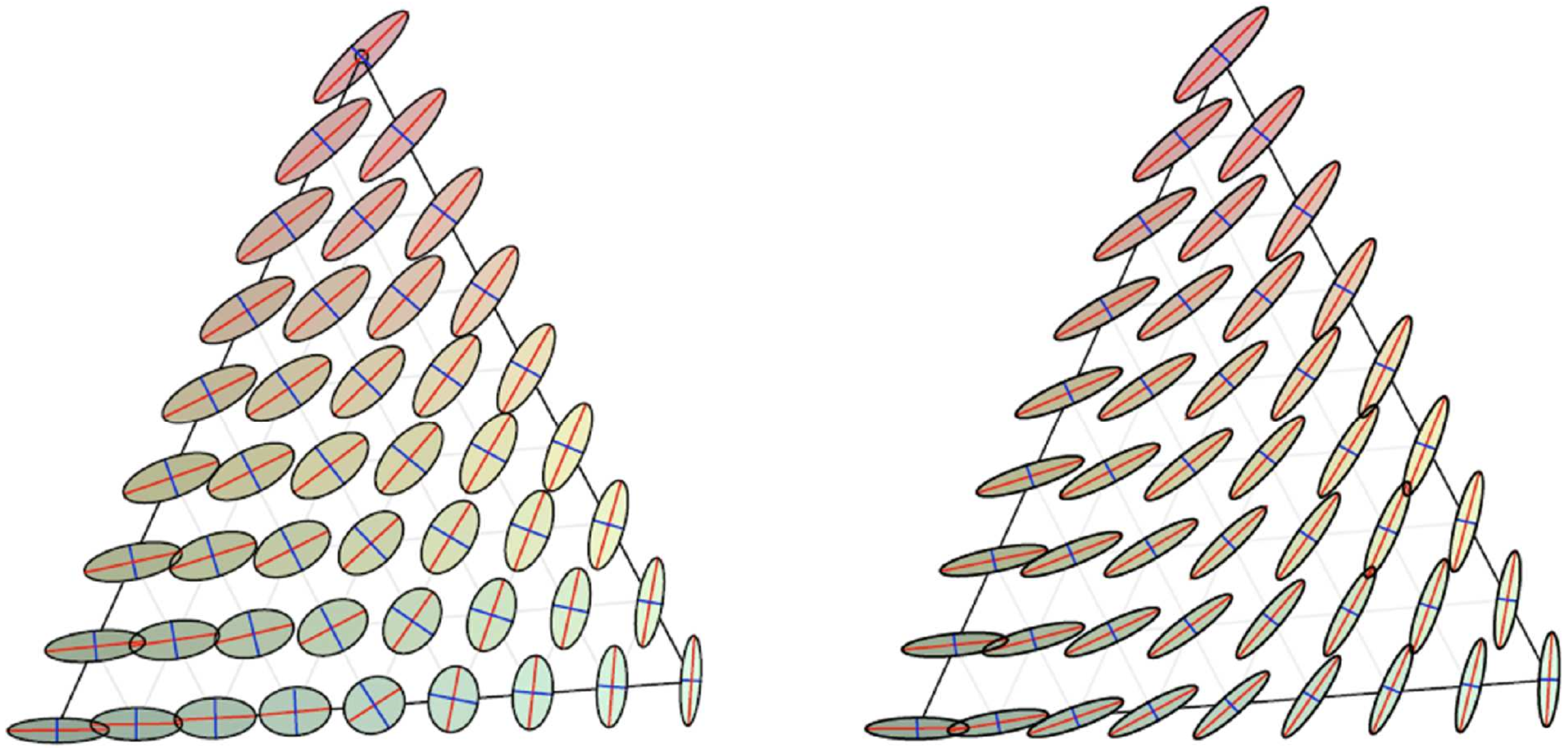

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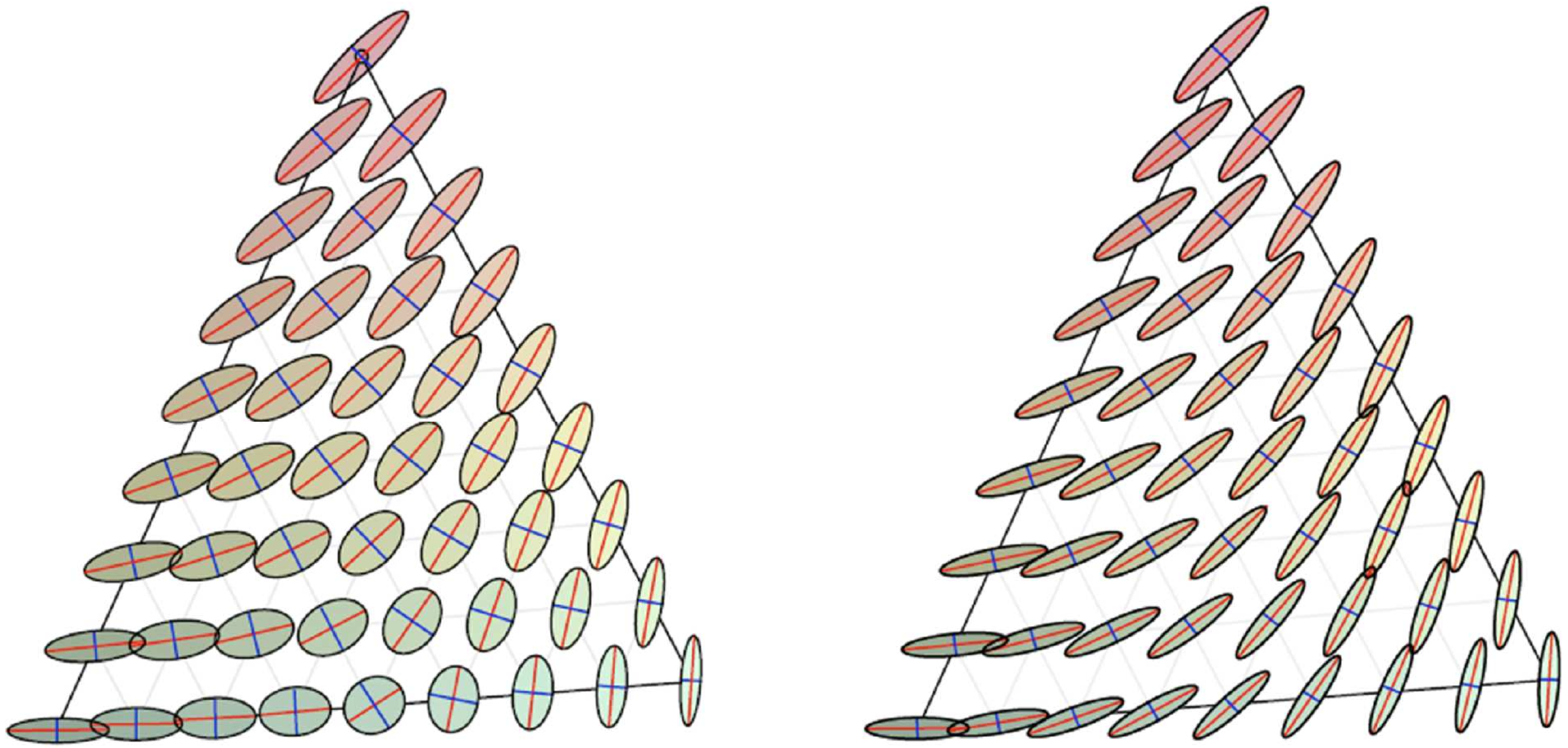

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Derived vector fields

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  - Eigenvectors
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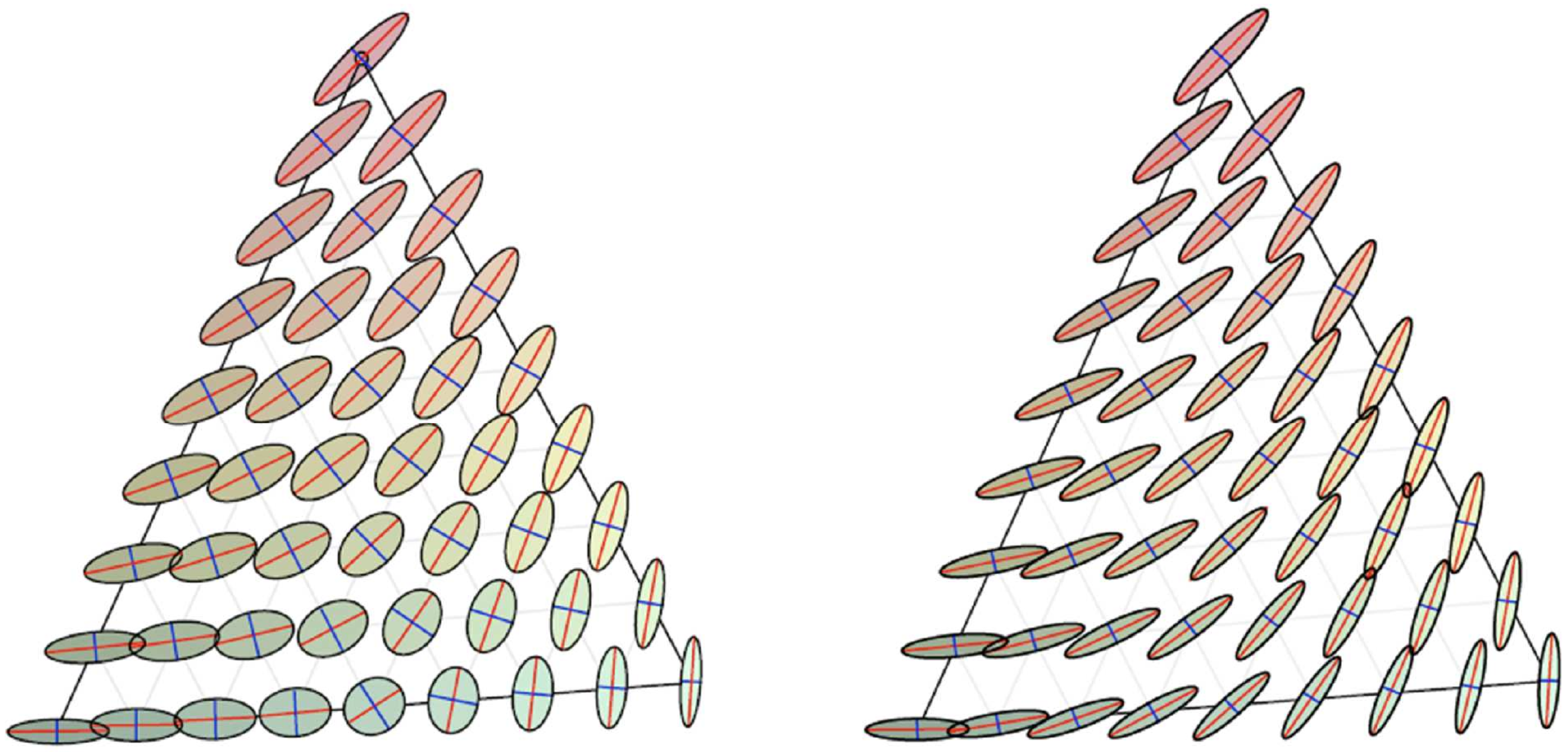
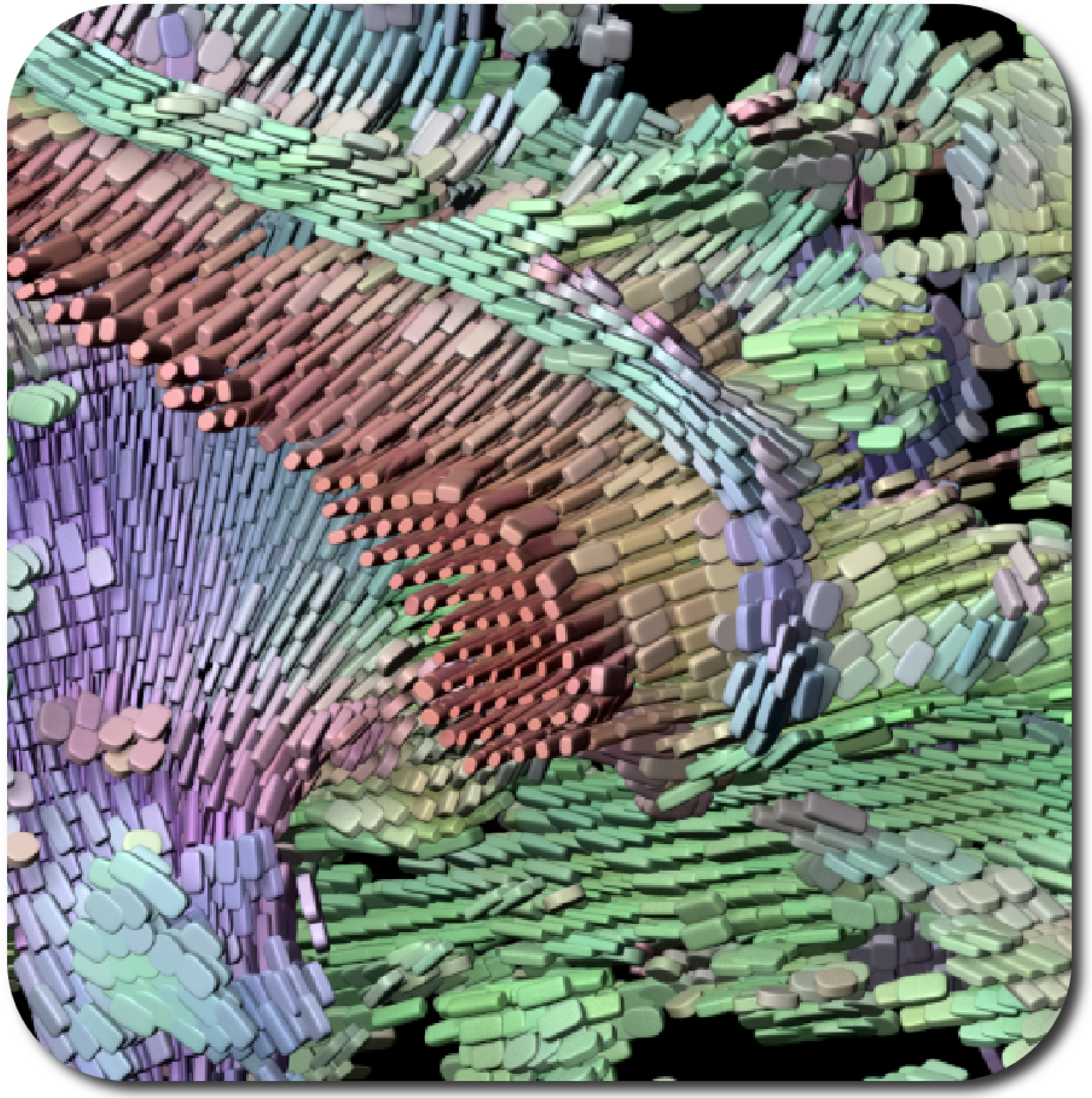
      - No magnitude
      - No orientation
  - Magnitude
    - Eigenvalues
  - Ambiguous entity
    - Notion of direction field



[Hotz]

Direction fields

- Set of orthogonal “pseudo” vectors



[Kindlmann]

Direction fields

- Set of orthogonal “pseudo” vectors
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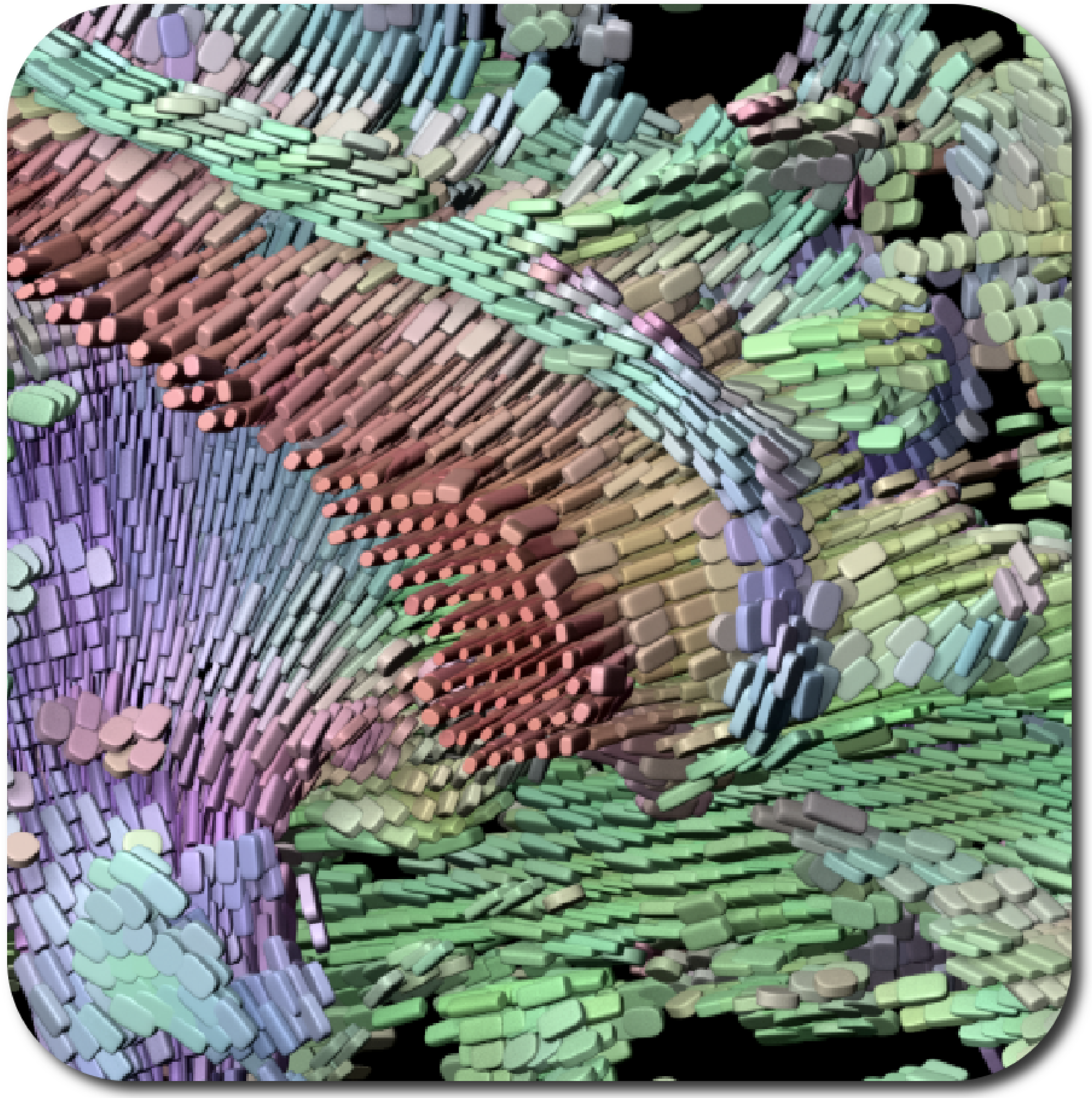

[Kindlmann]

Direction fields

- Set of orthogonal “pseudo” vectors
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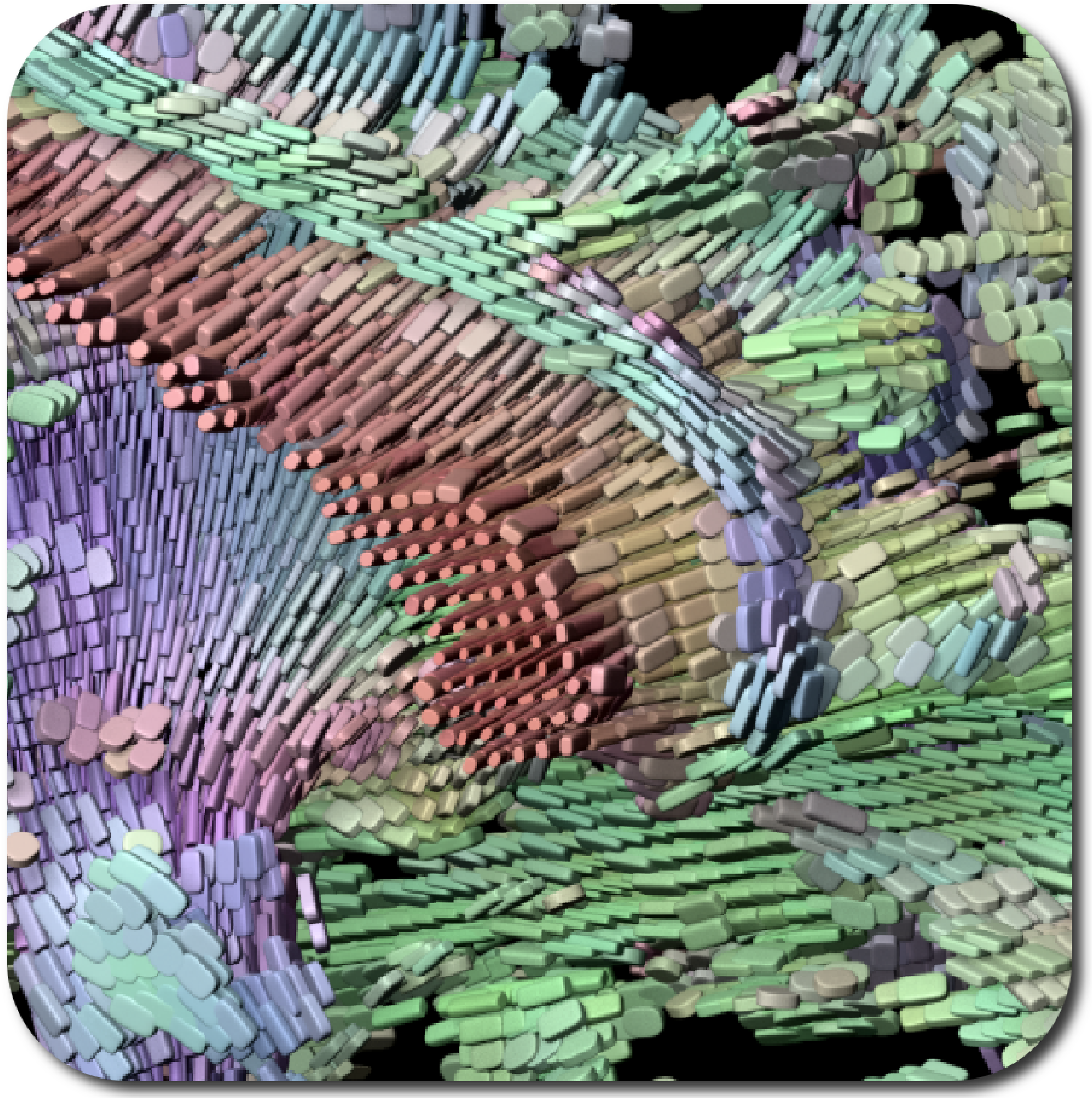
    - Streamline computation
    - Streamline seeding
    - LIC



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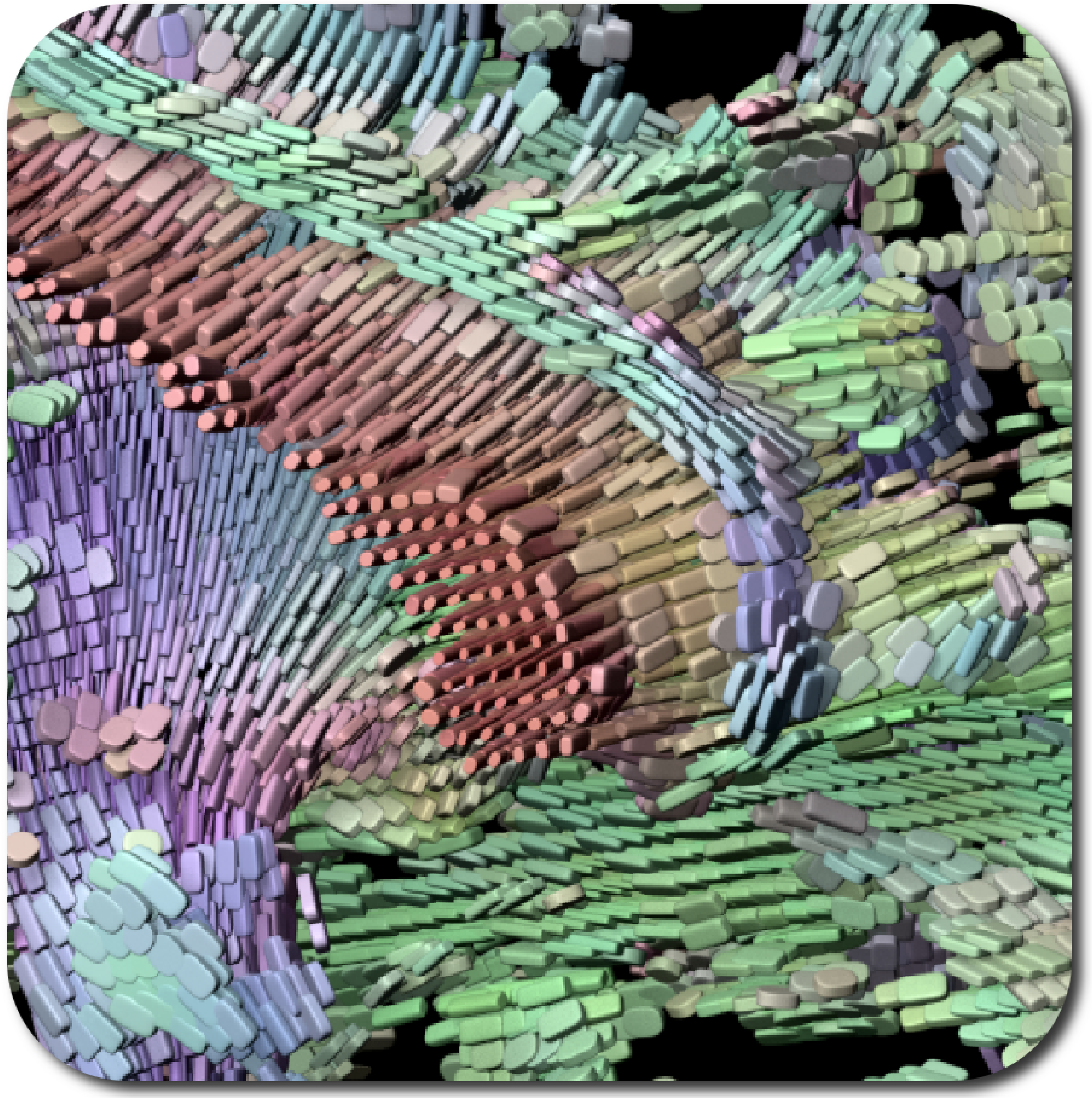
  - How to combine the directions?



[Kindlmann]

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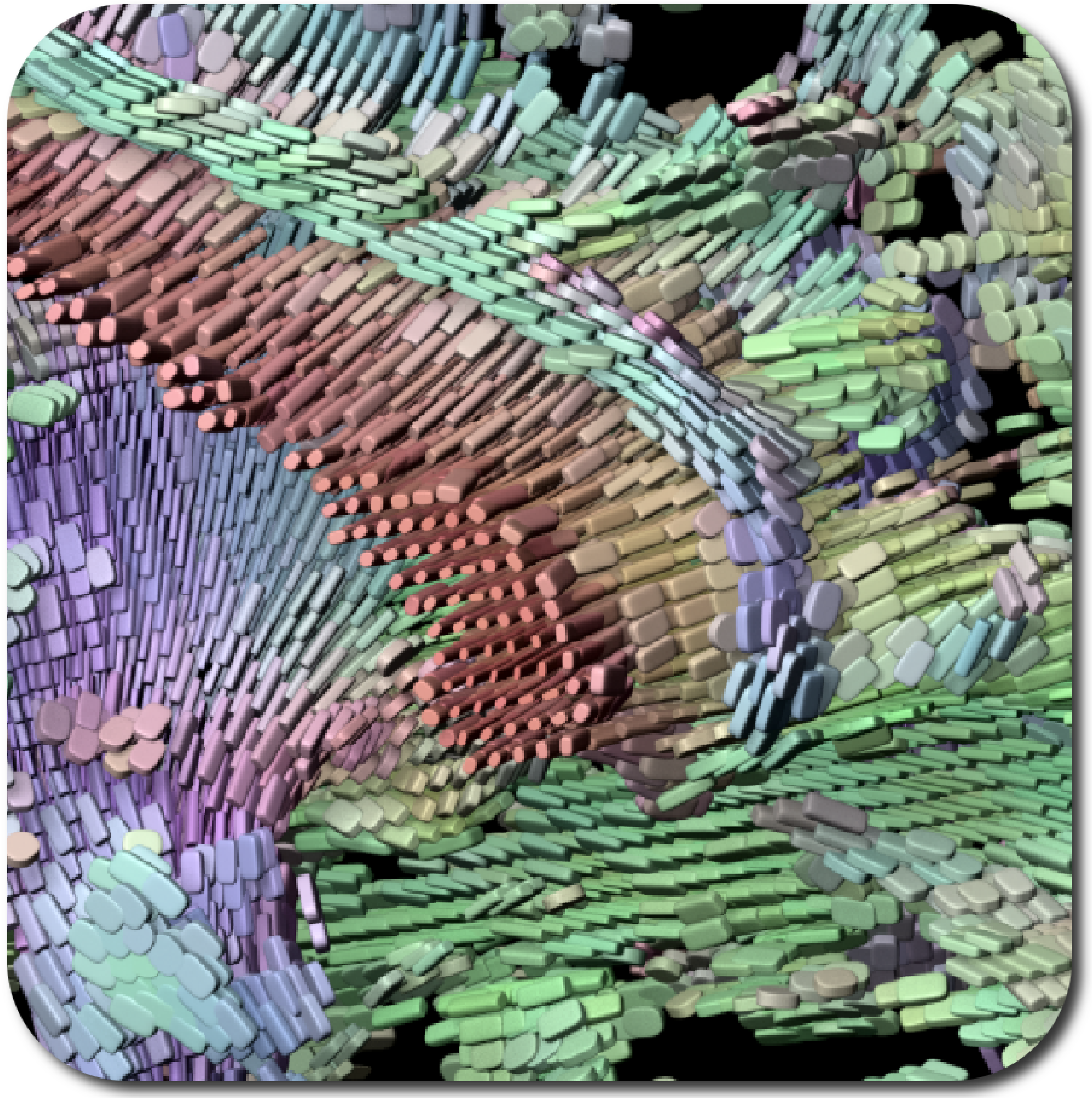

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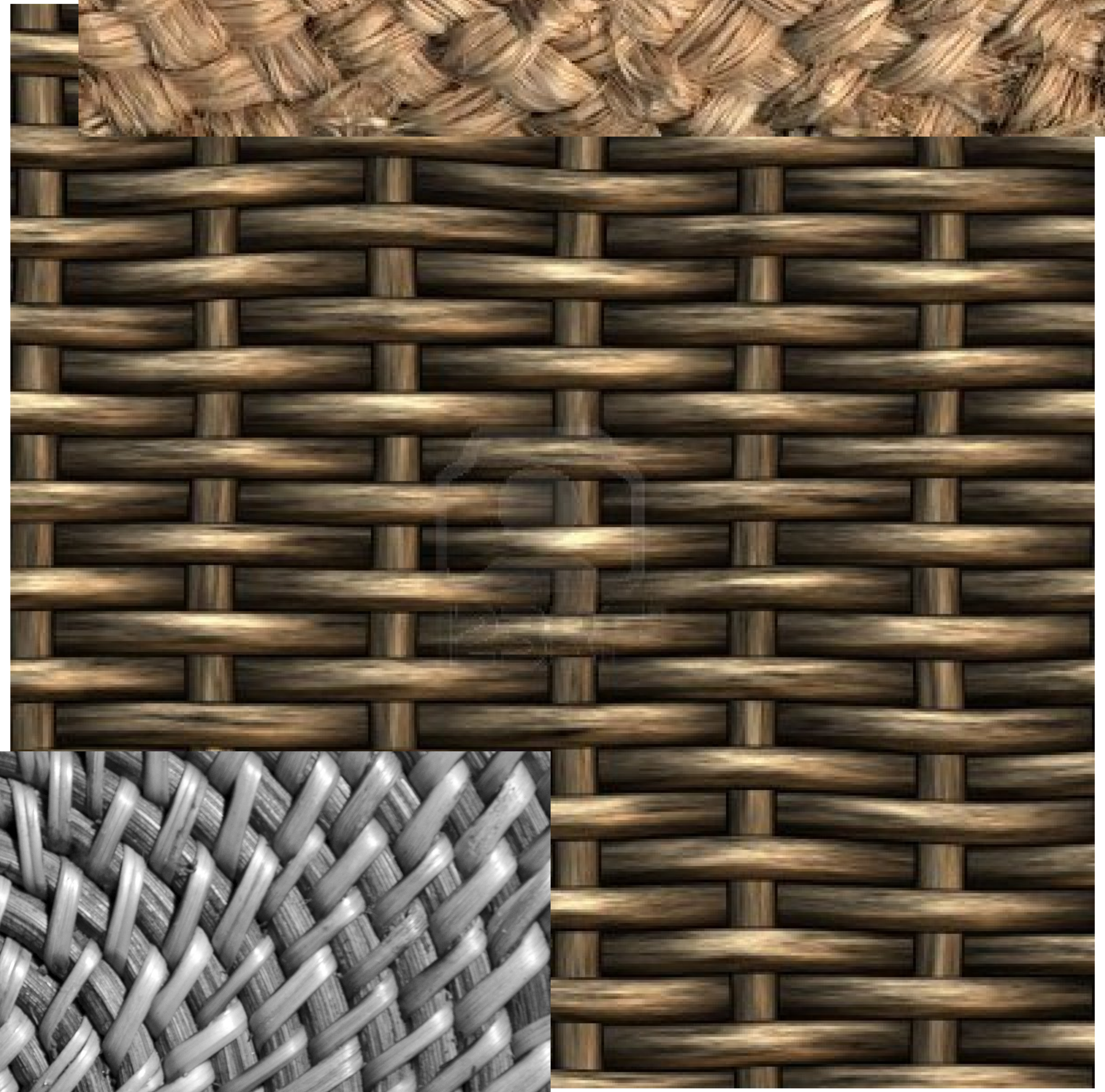
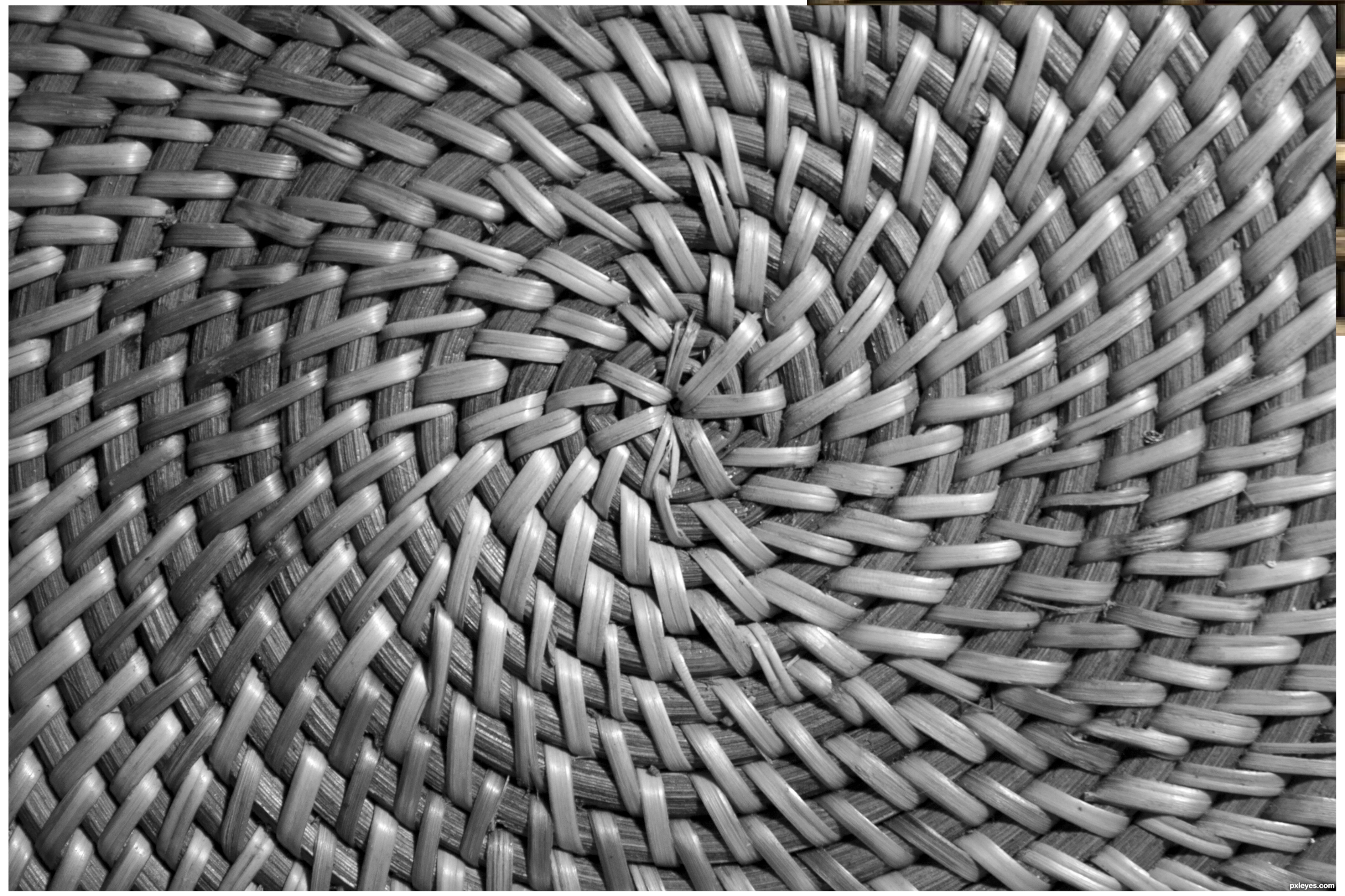


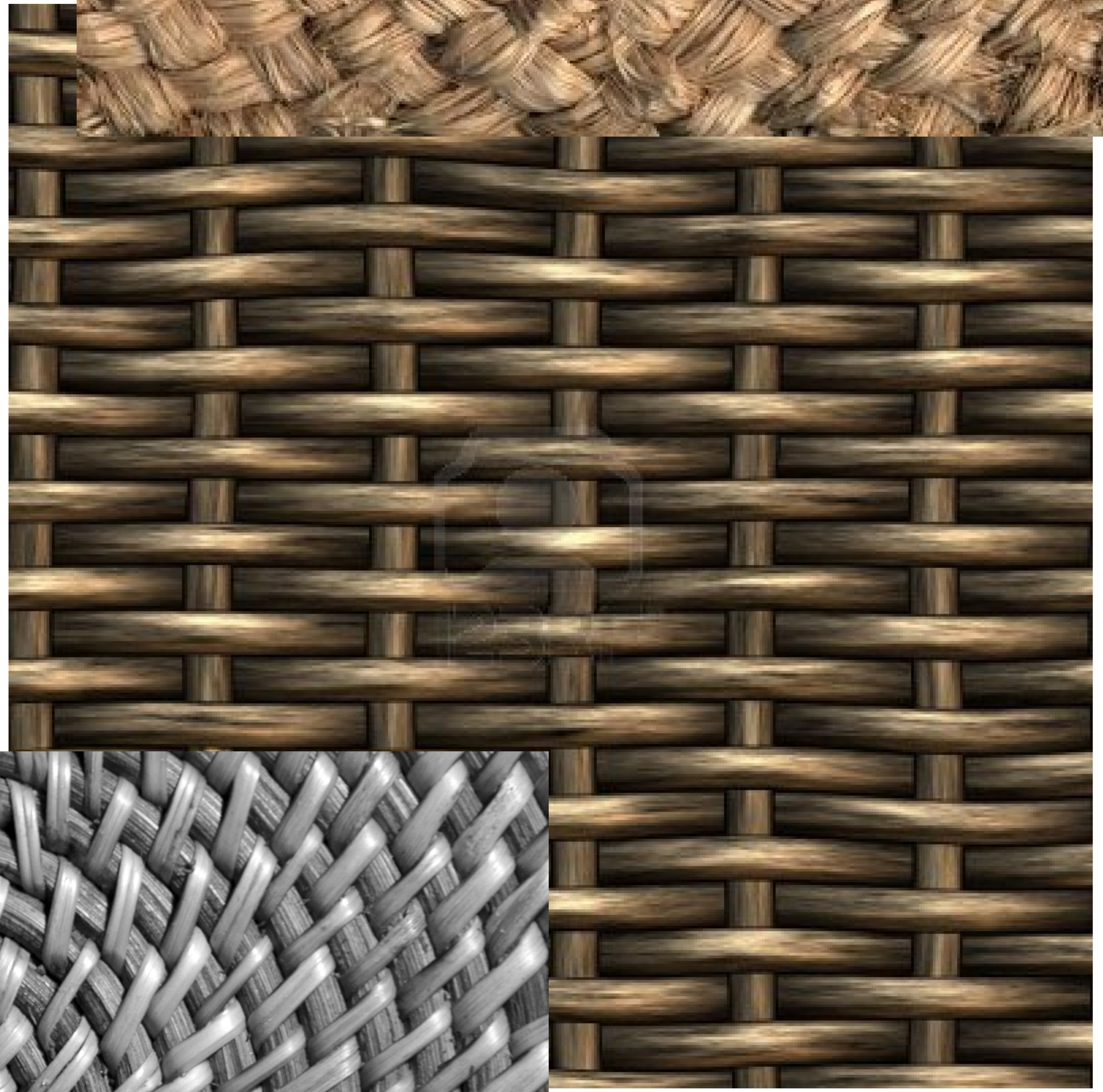
[http://www.123rf.com]



[http://www.pxleyes.com]

Direction fields

- Set of orthogonal “pseudo” vectors
  - Pulling all the vector field visualization techniques
    - Streamline computation
    - Streamline seeding
    - LIC
  - How to combine the directions?
    - Get inspiration from … craft
    - Overlay the directions



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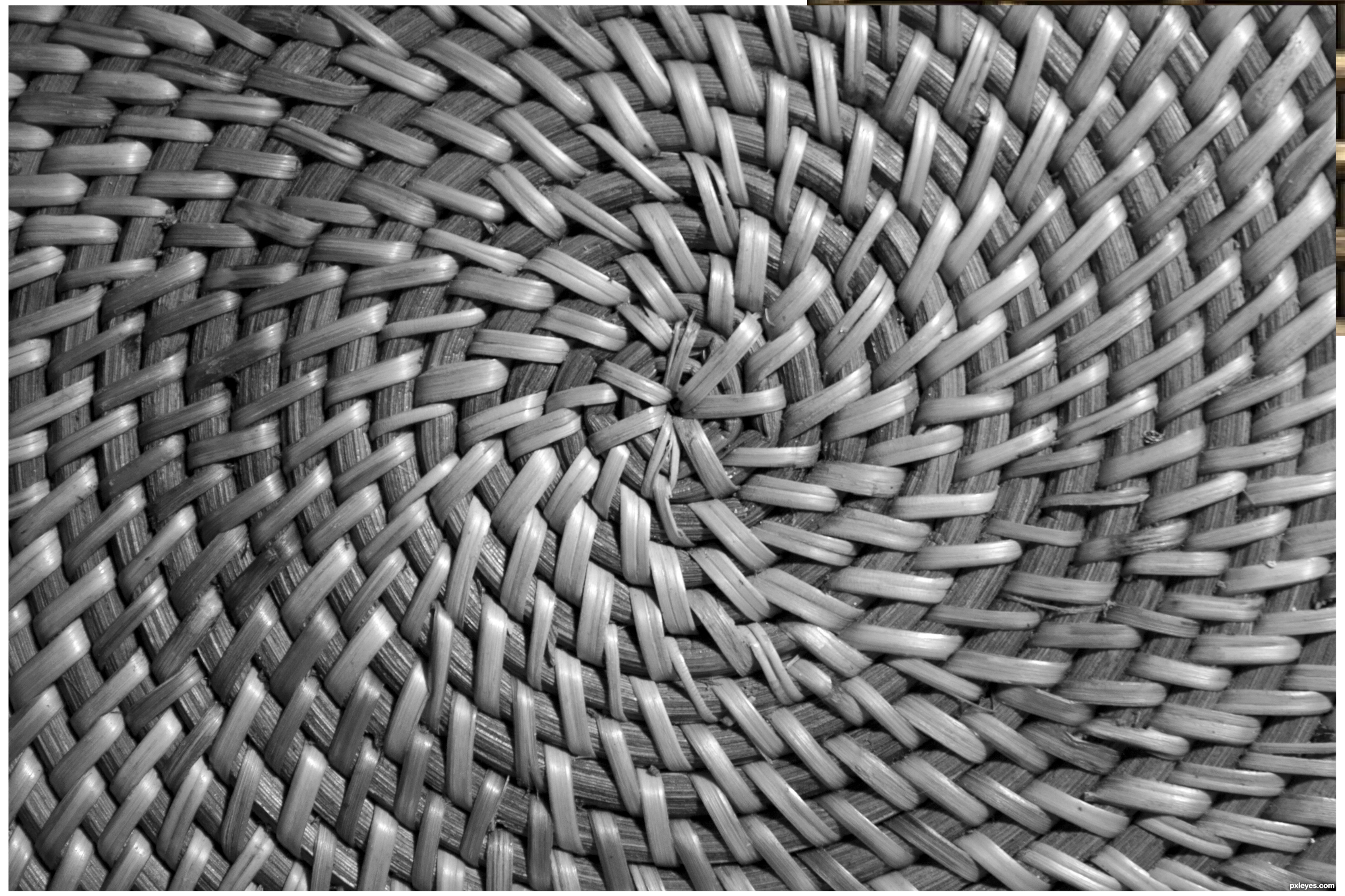
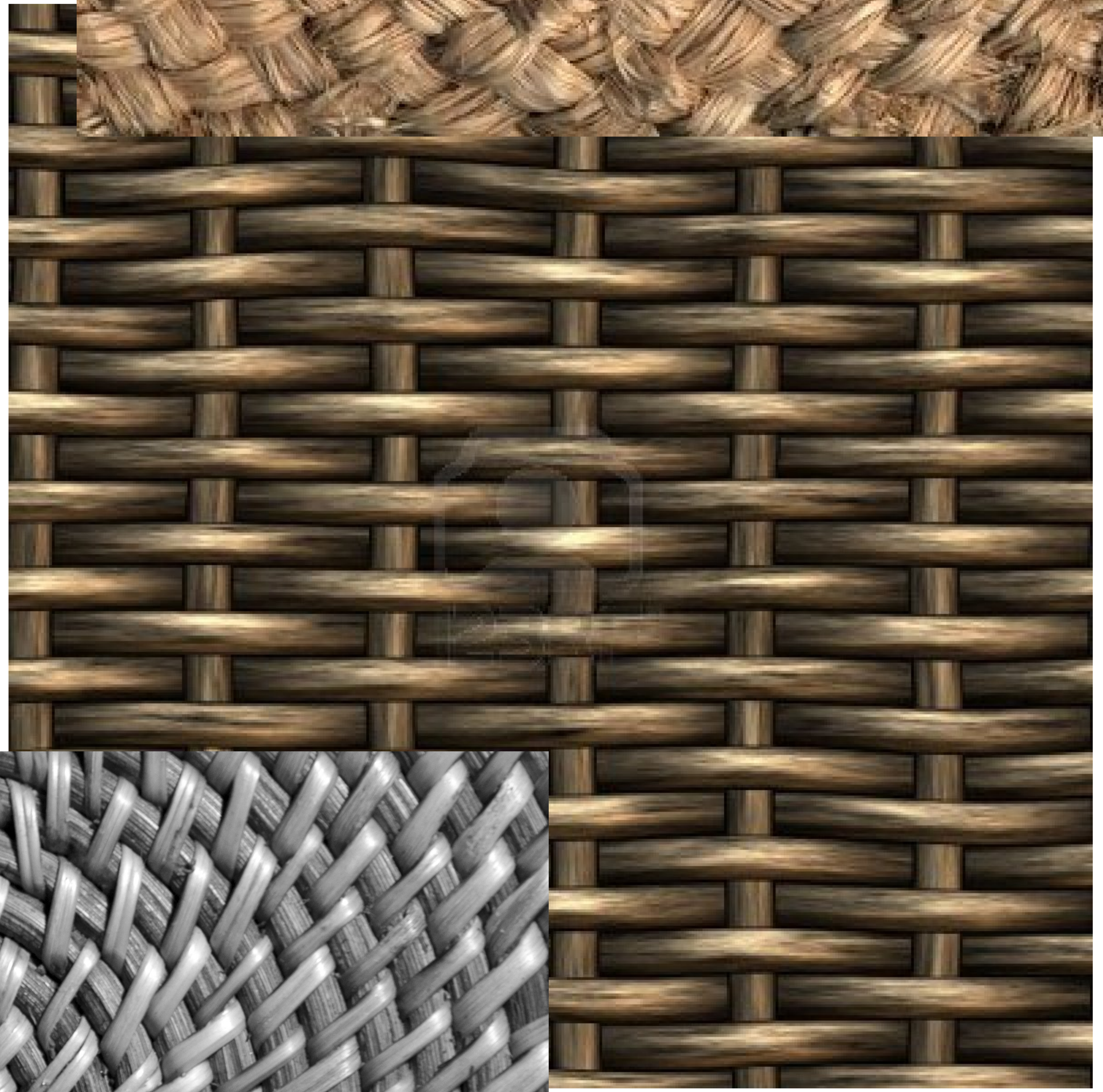


[http://www.pxleyes.com]

Hyper-streamlines

- For each of the d directions



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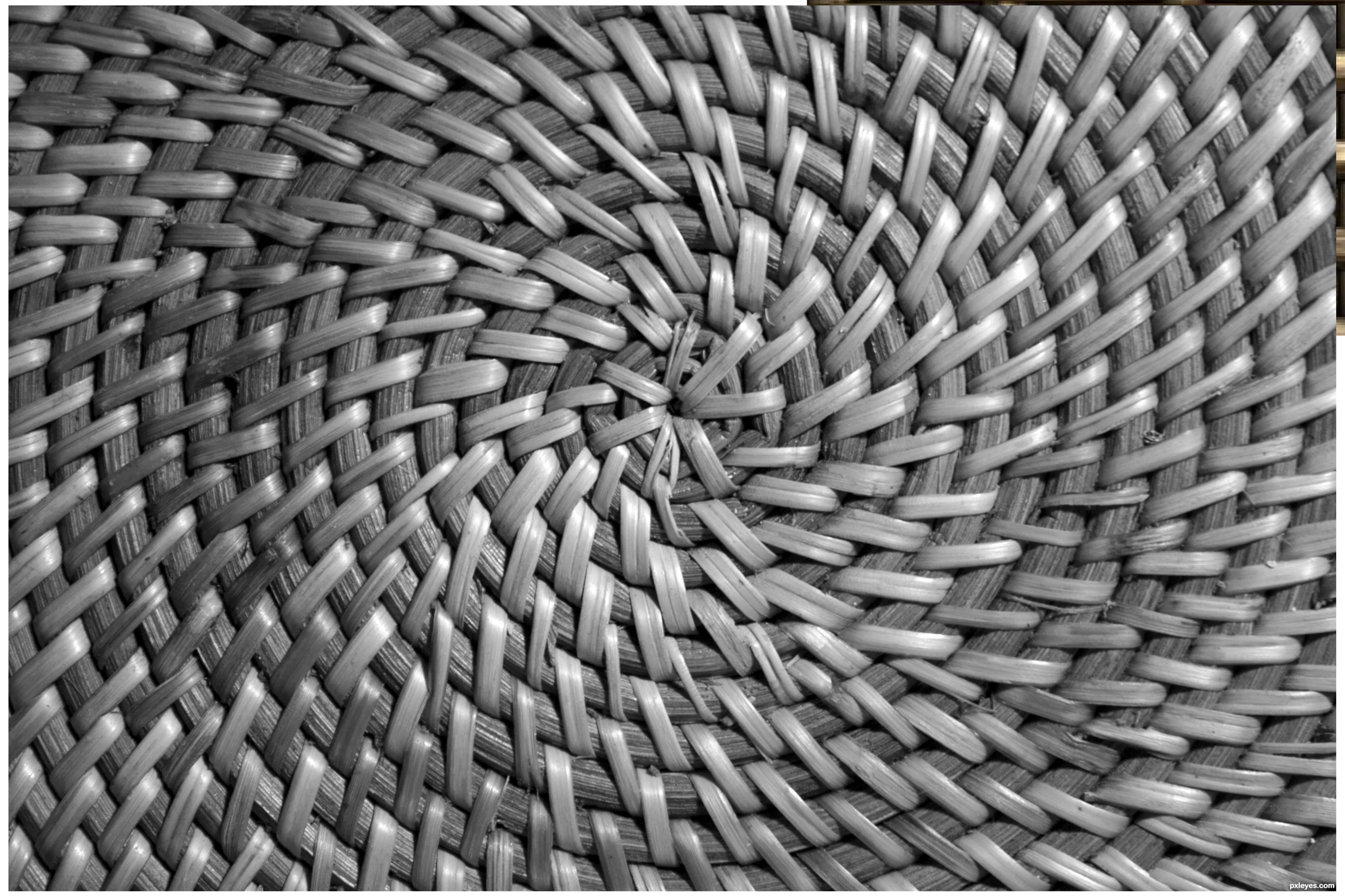

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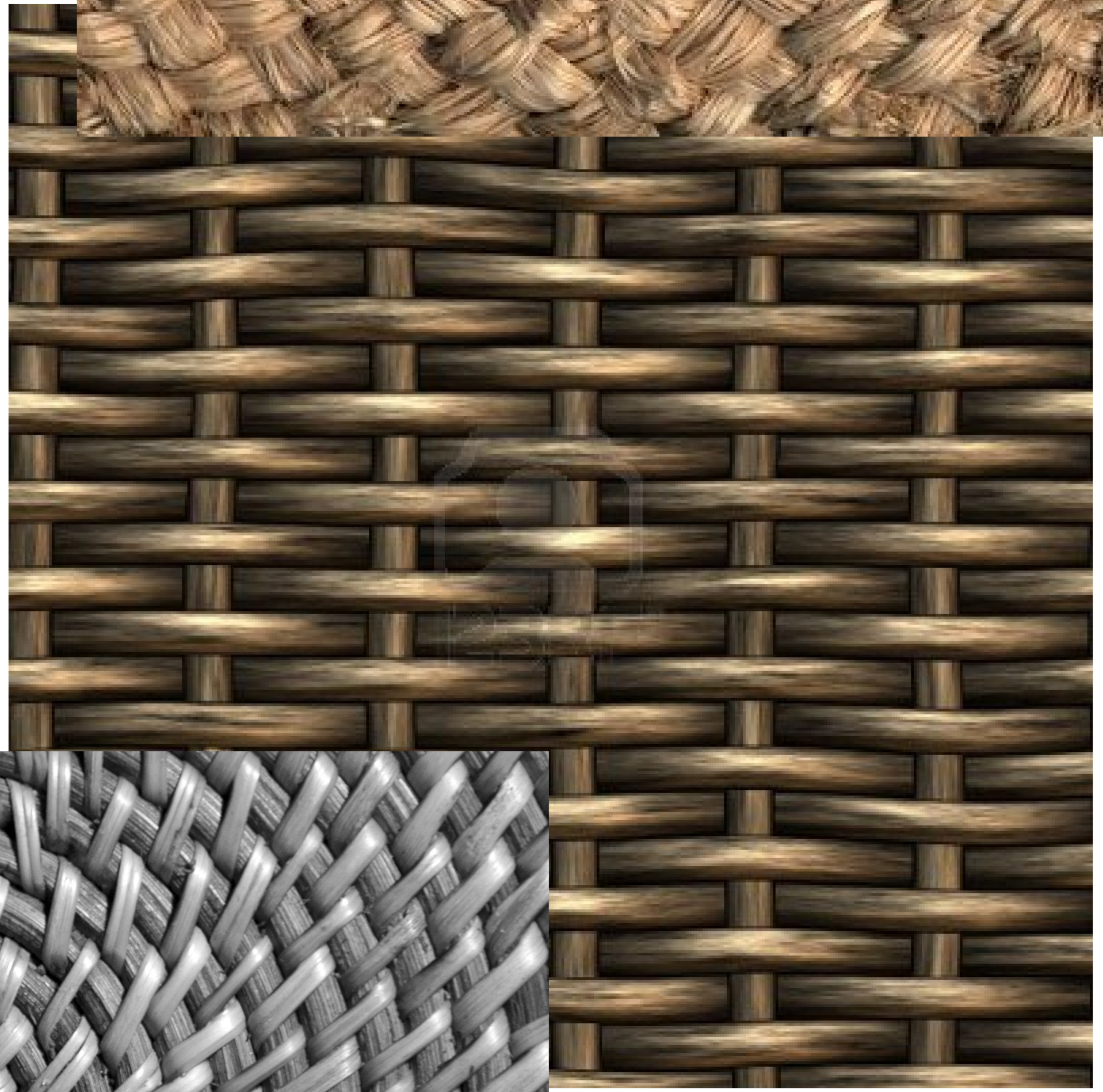
Hyper-streamlines

- For each of the d directions
  - Streamline integration



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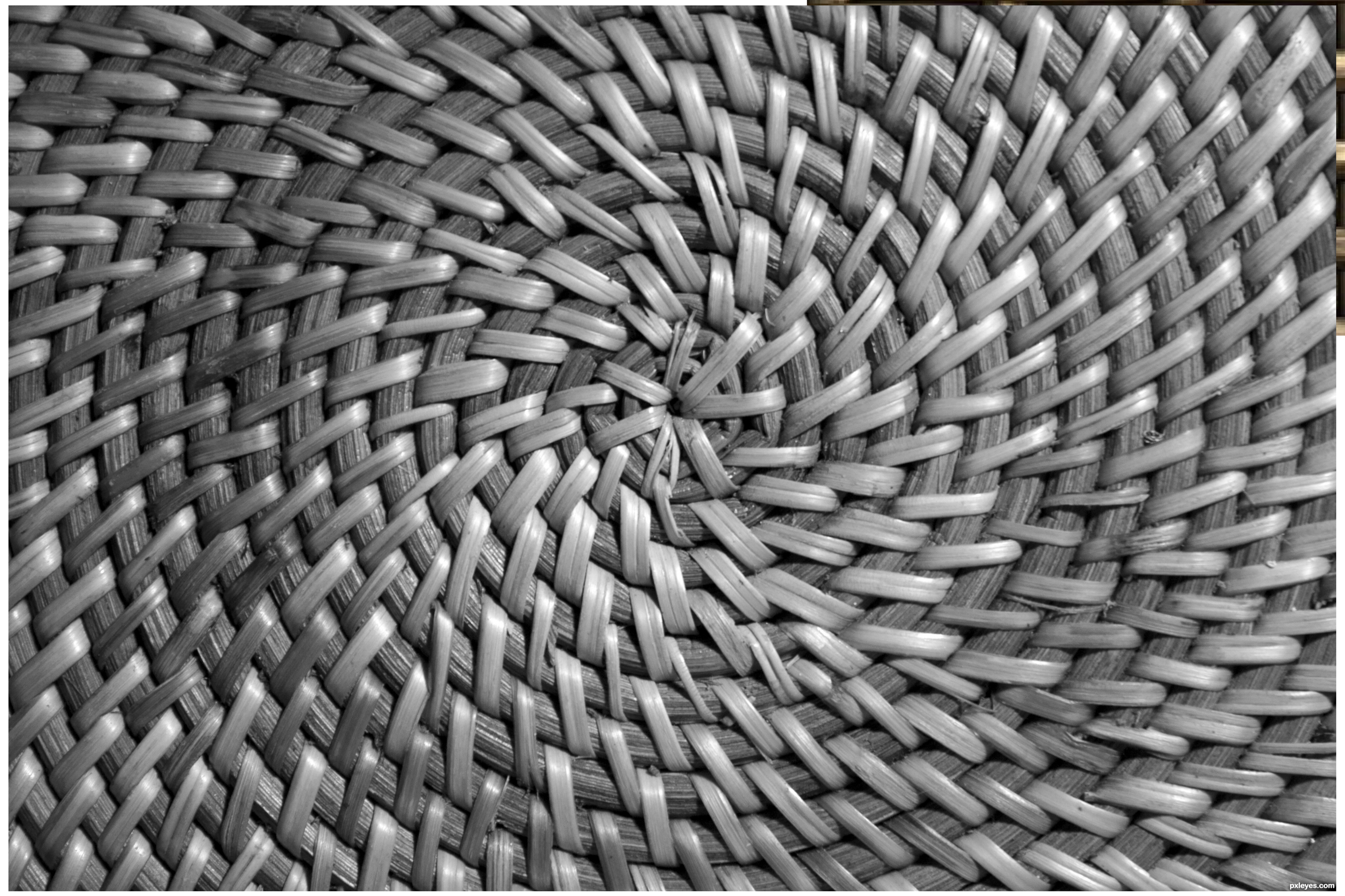

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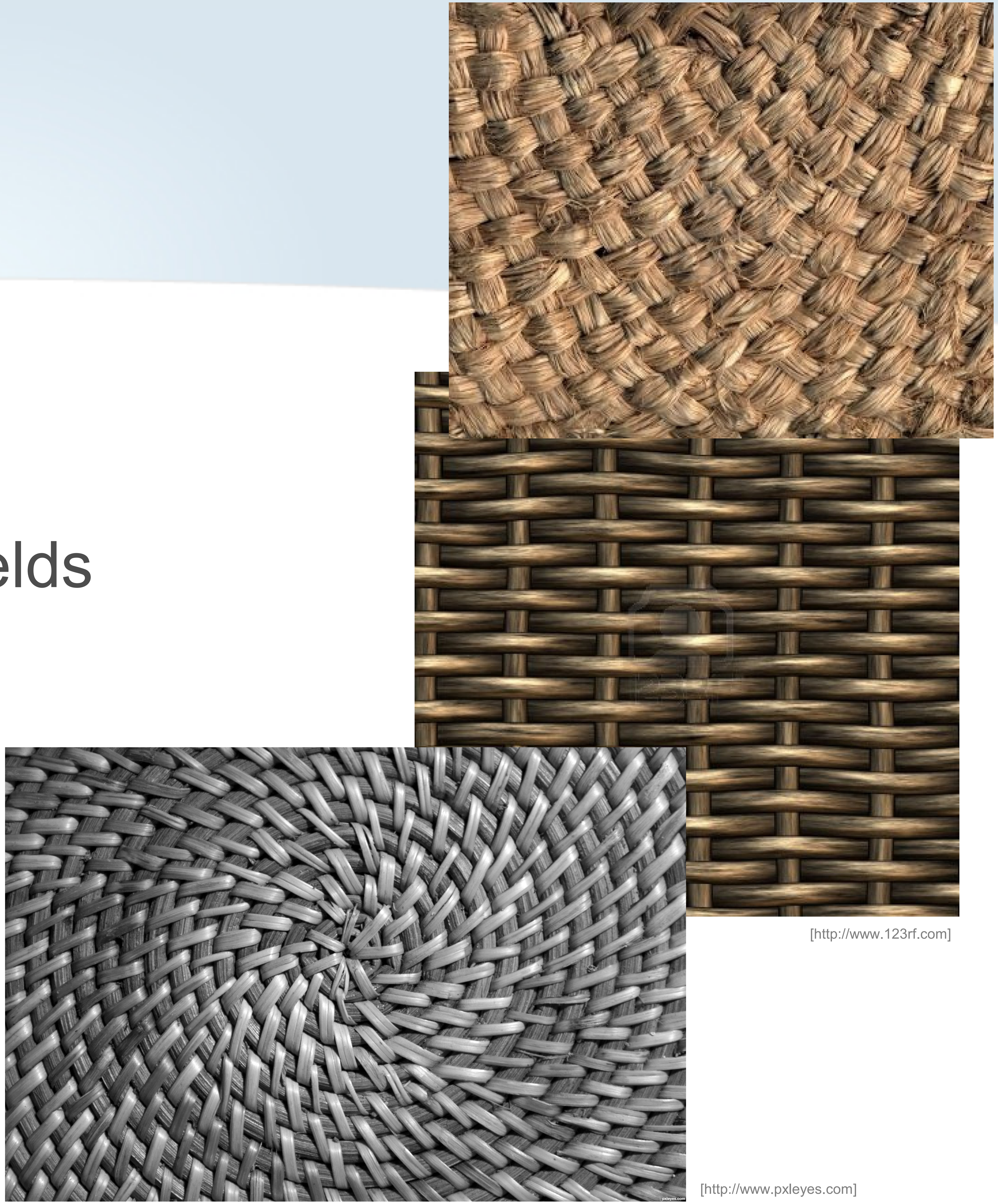
Hyper-streamlines

- For each of the d directions
  - Streamline integration
    - We know how to do it for vector fields

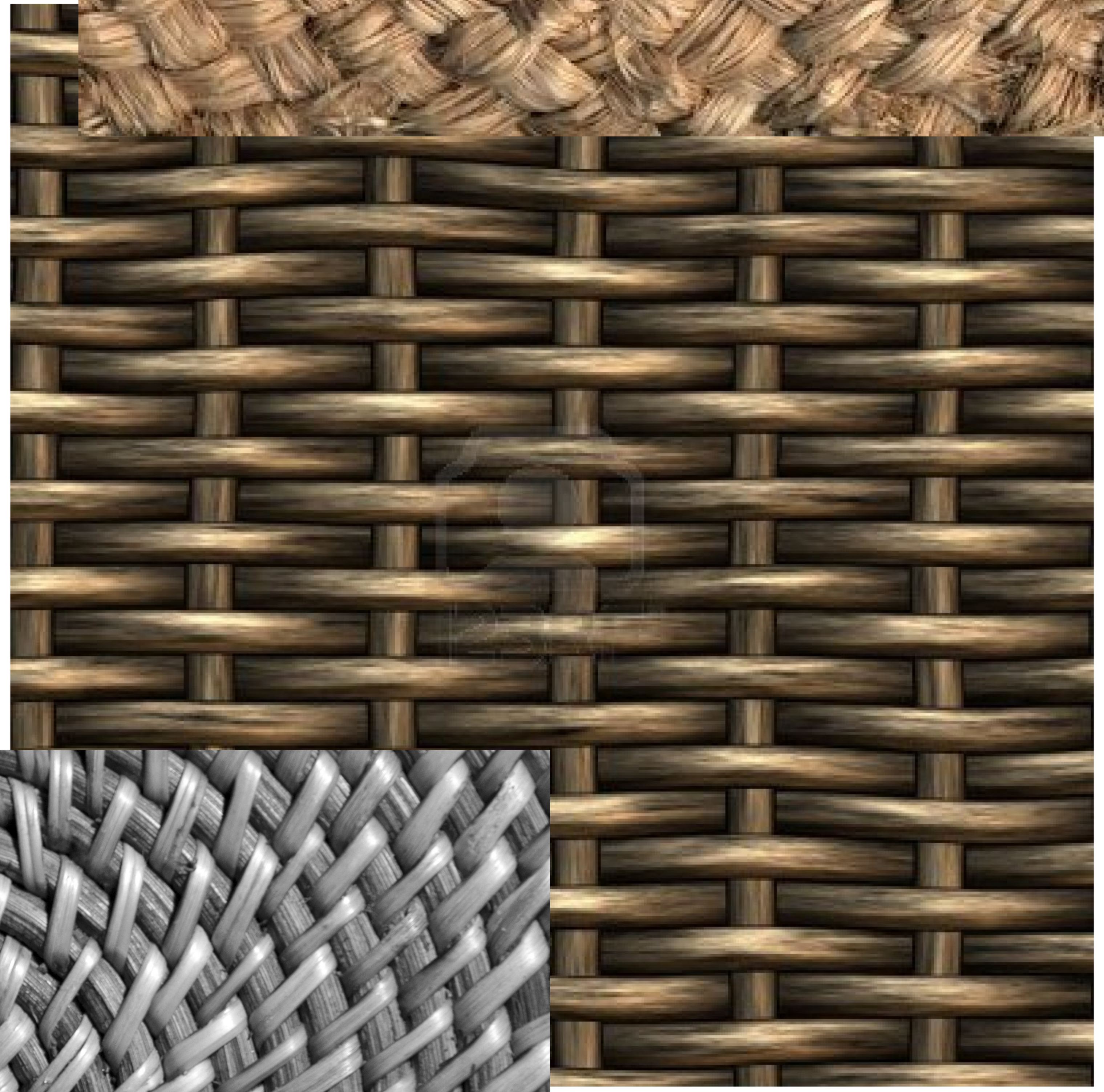


[http://www.123rf.com]

[http://www.pxleyes.com]

Hyper-streamlines

- For each of the d directions
  - Streamline integration
    - We know how to do it for vector fields
  - Problem



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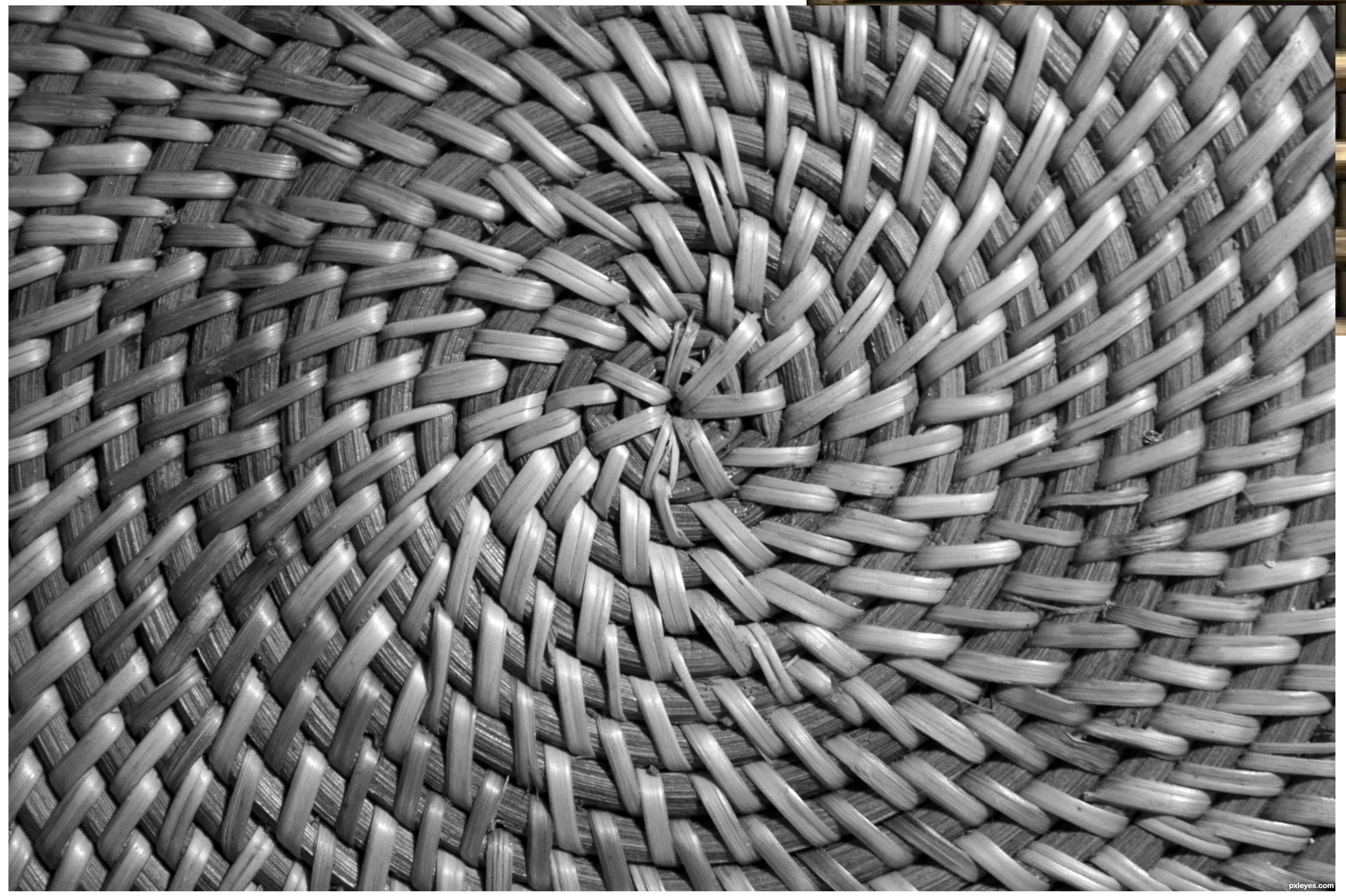
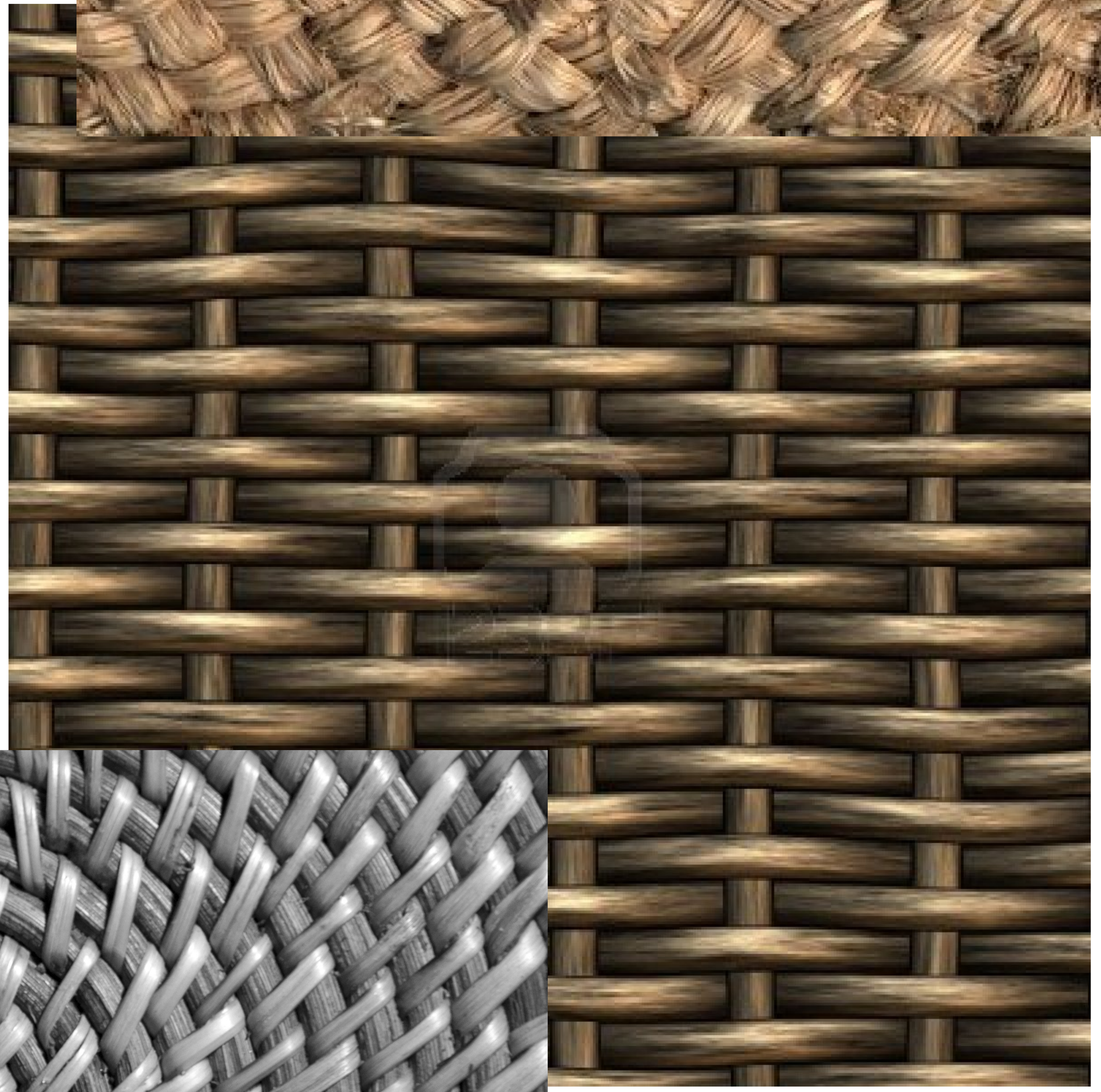


[http://www.pxleyes.com]

Hyper-streamlines

- For each of the d directions
  - Streamline integration
    - We know how to do it for vector fields
  - Problem
    - No orientation



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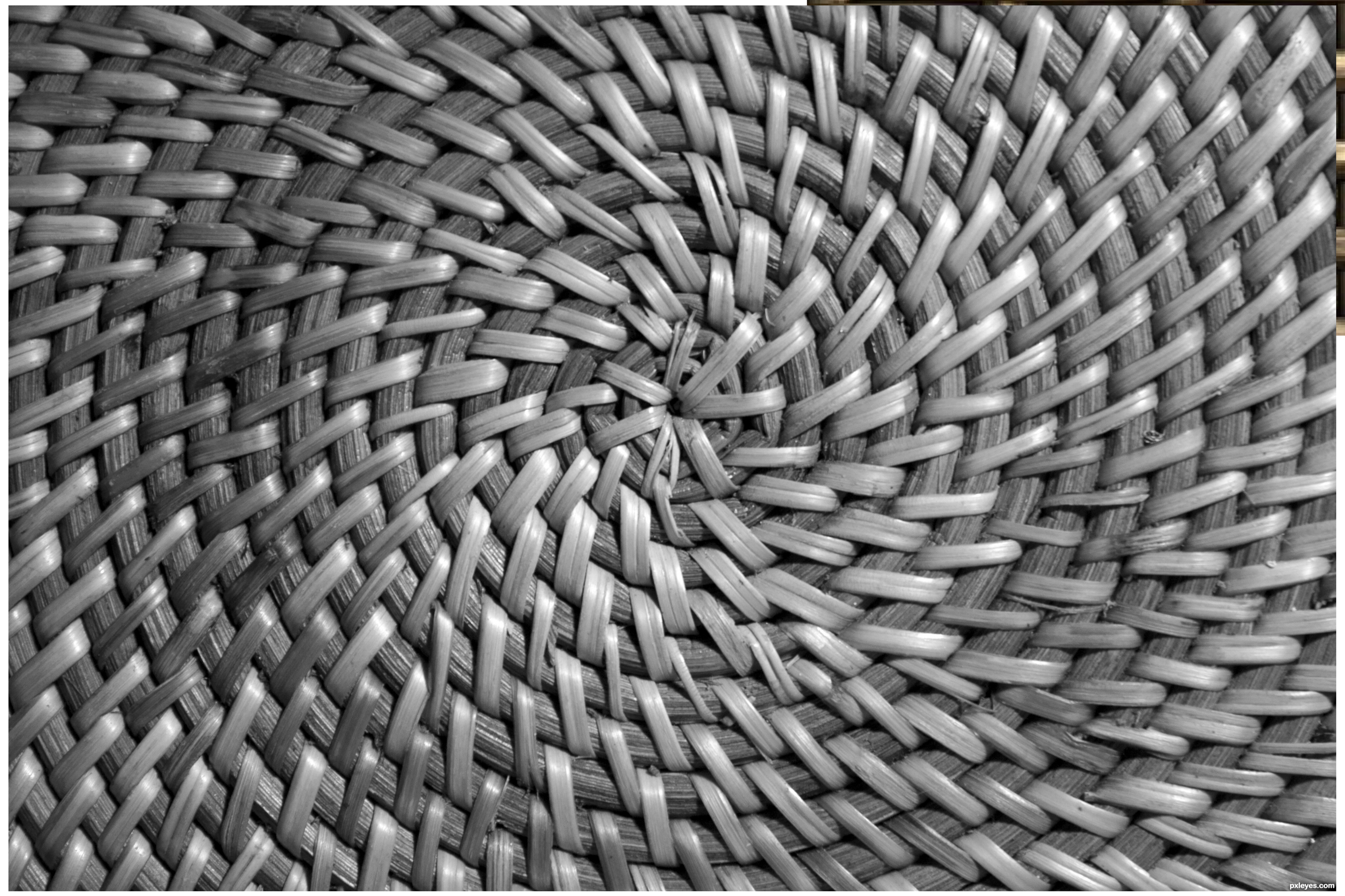

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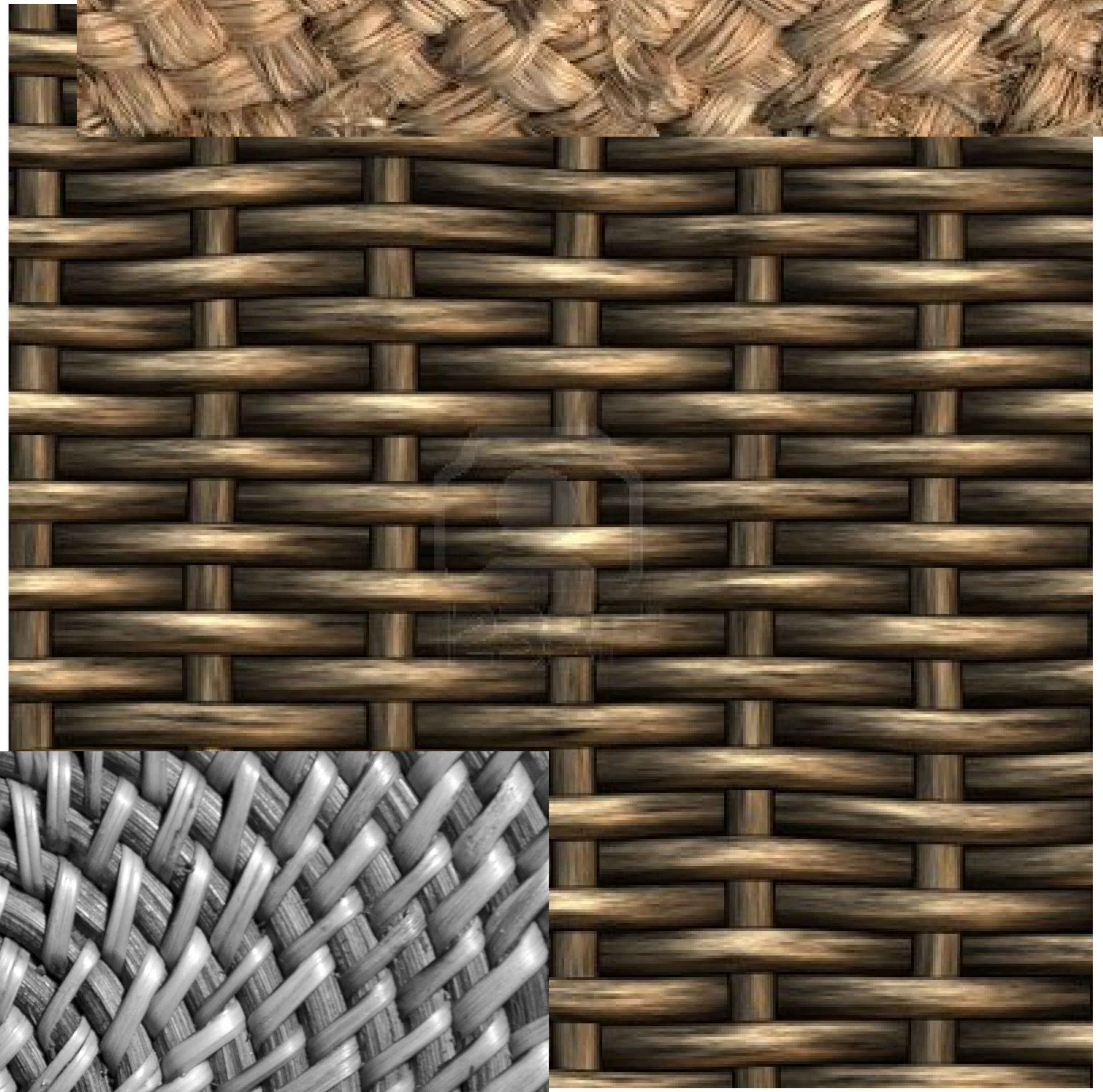

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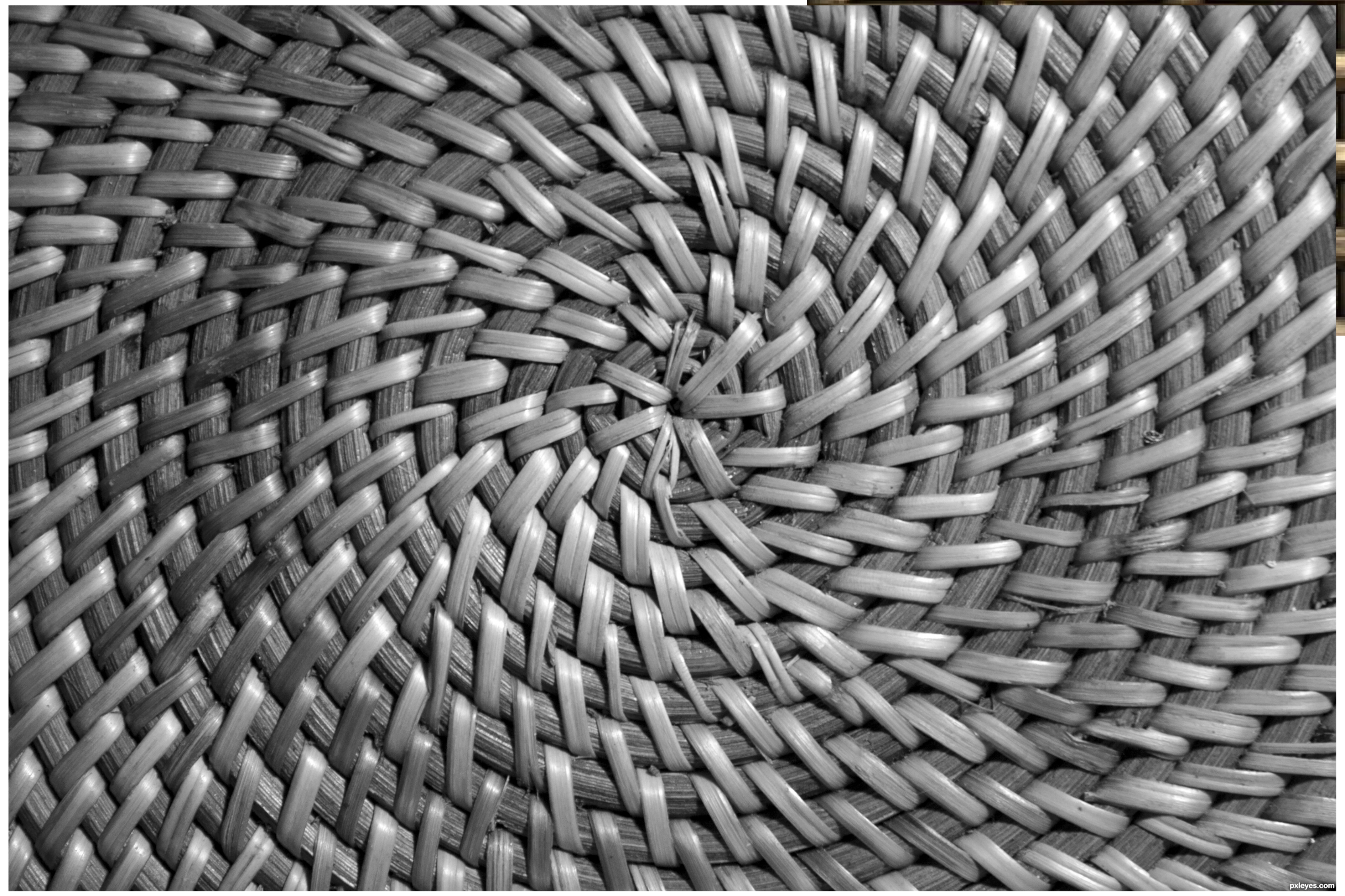
- Problem
  - No orientation
  - No clear matches across cells



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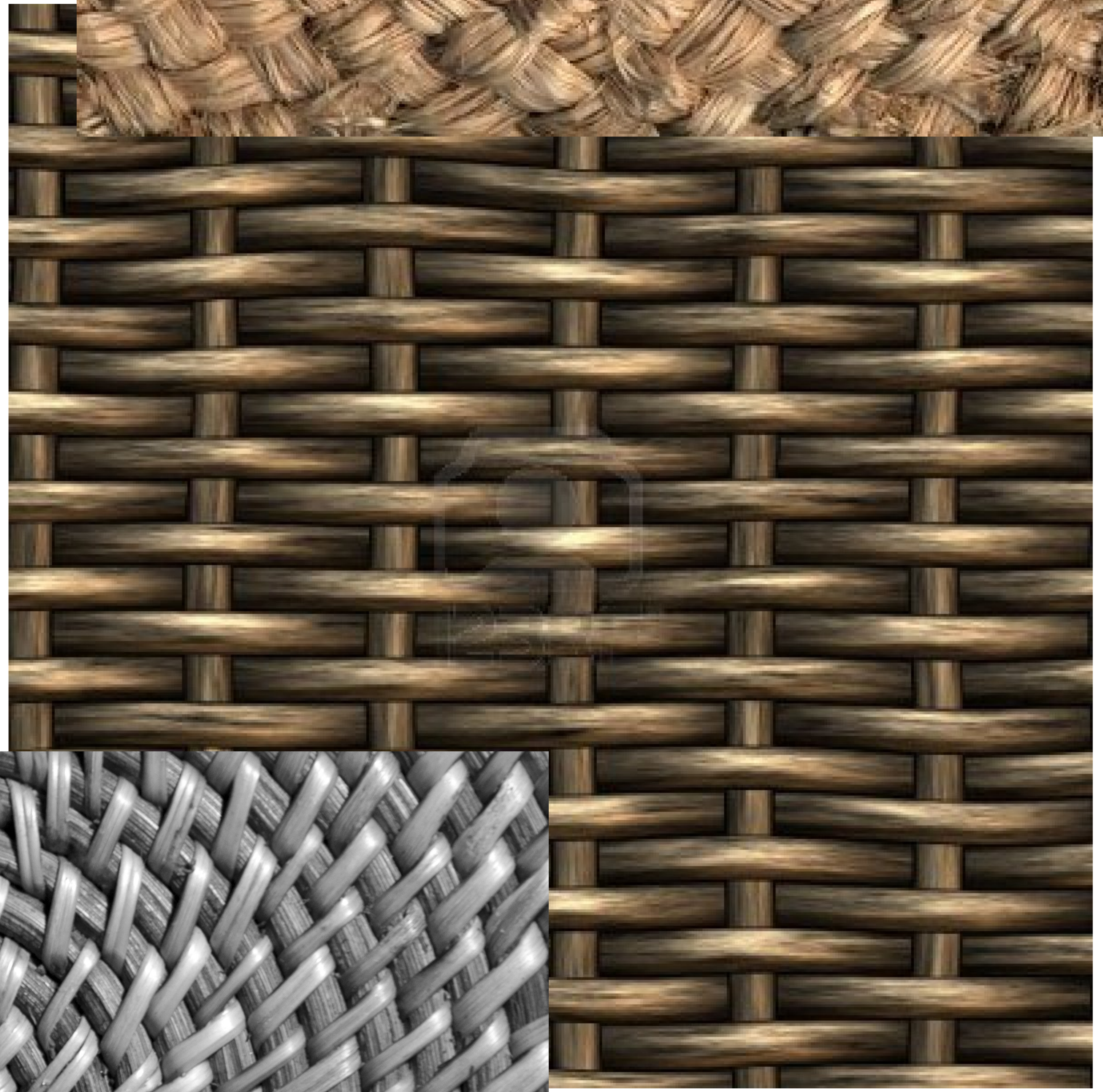
Hyper-streamlines

- Matches across cell



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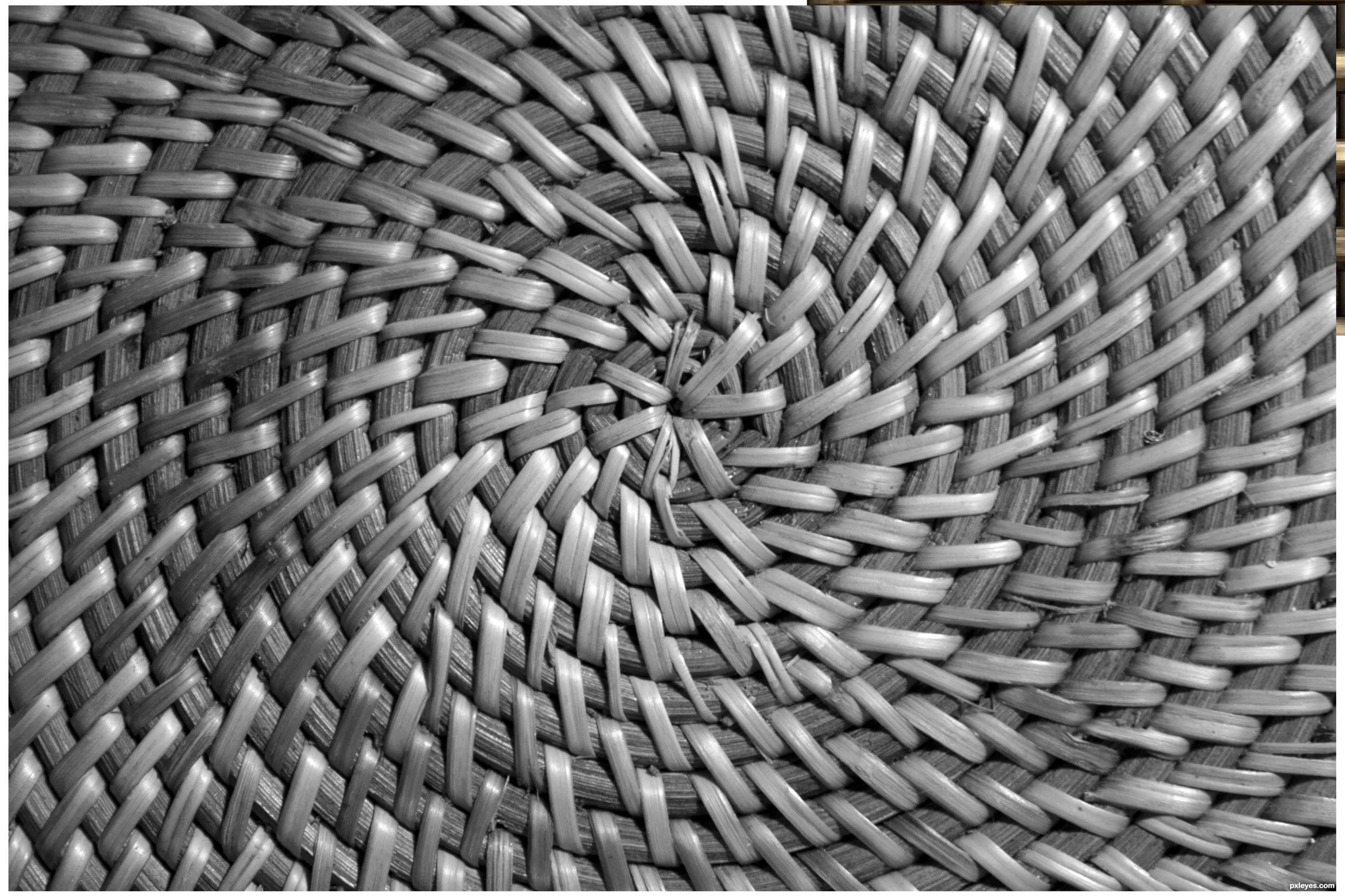
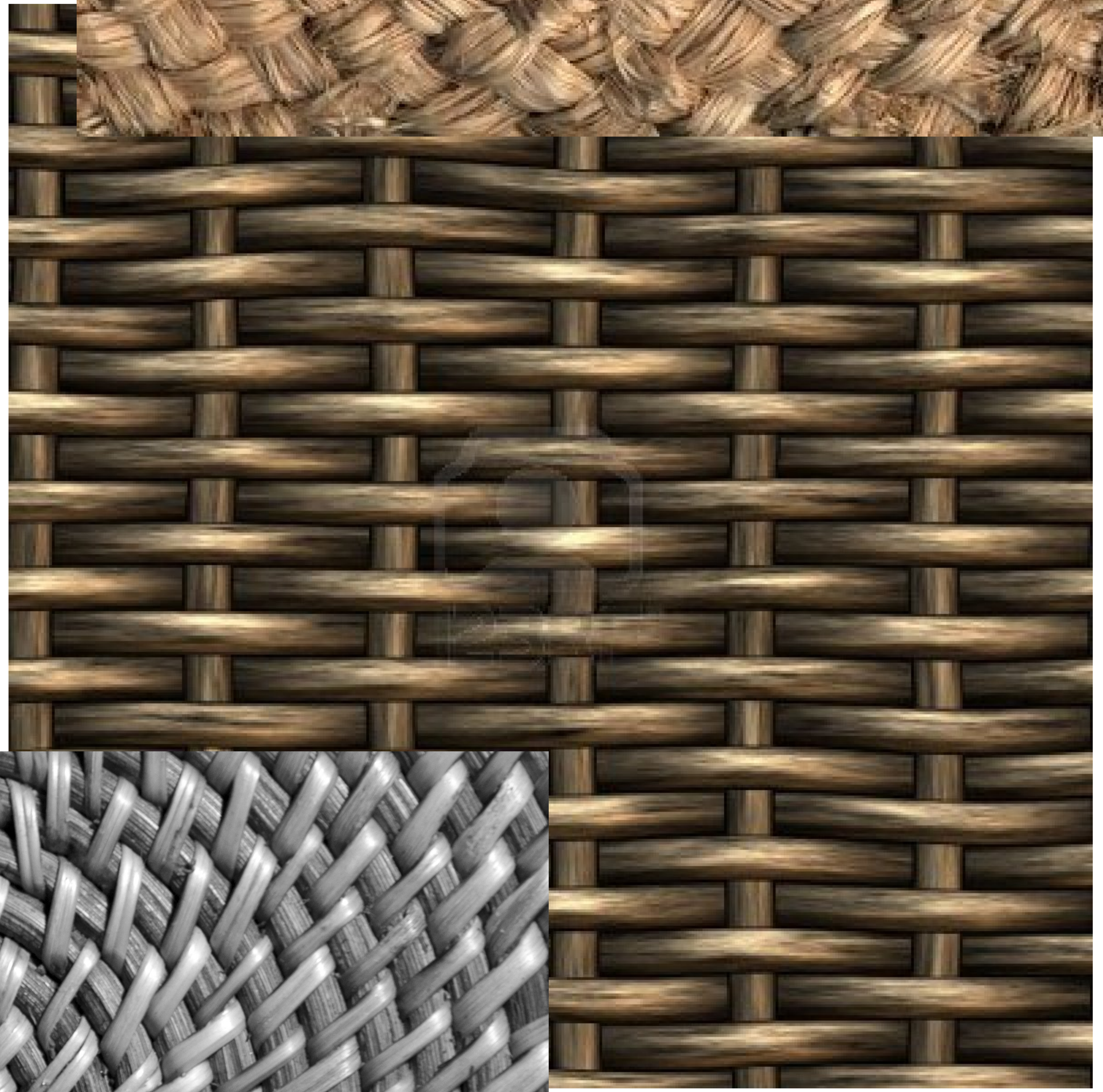


[http://www.pxleyes.com]

Hyper-streamlines

- Matches across cell
    - A consistent choice is possible if



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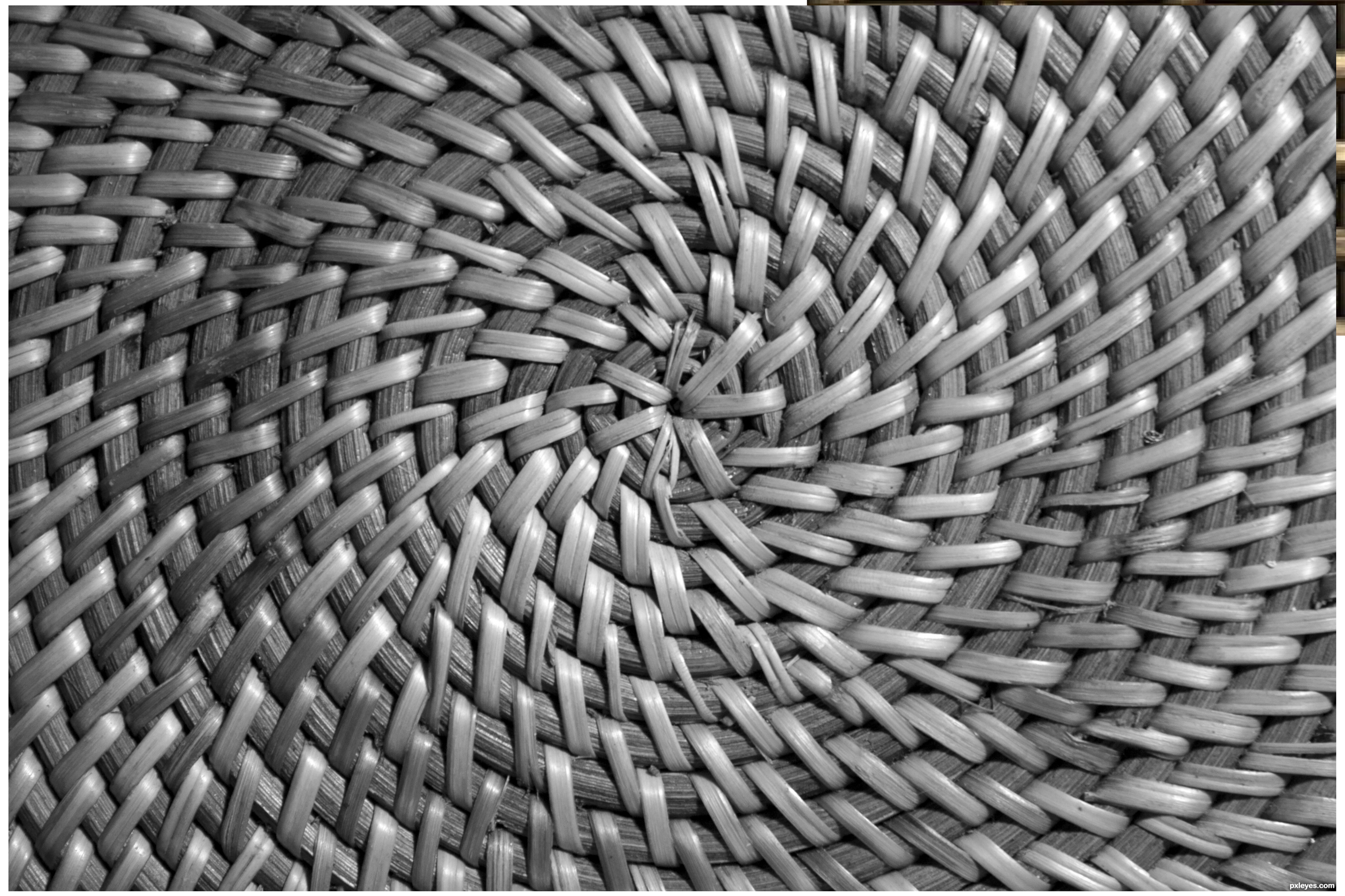

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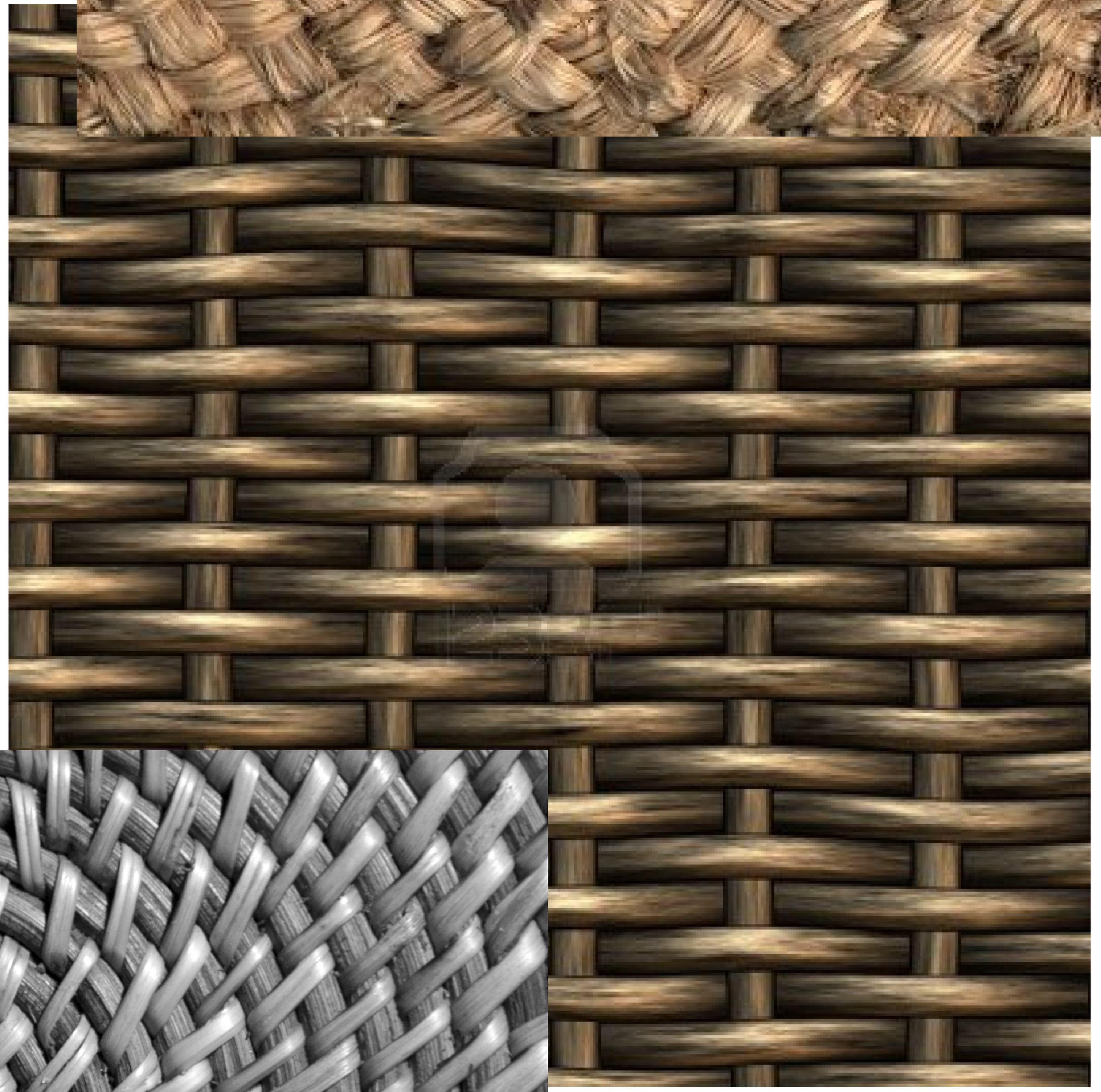
Hyper-streamlines

- Matches across cell
  - A consistent choice is possible if
    - The eigenvalues are different



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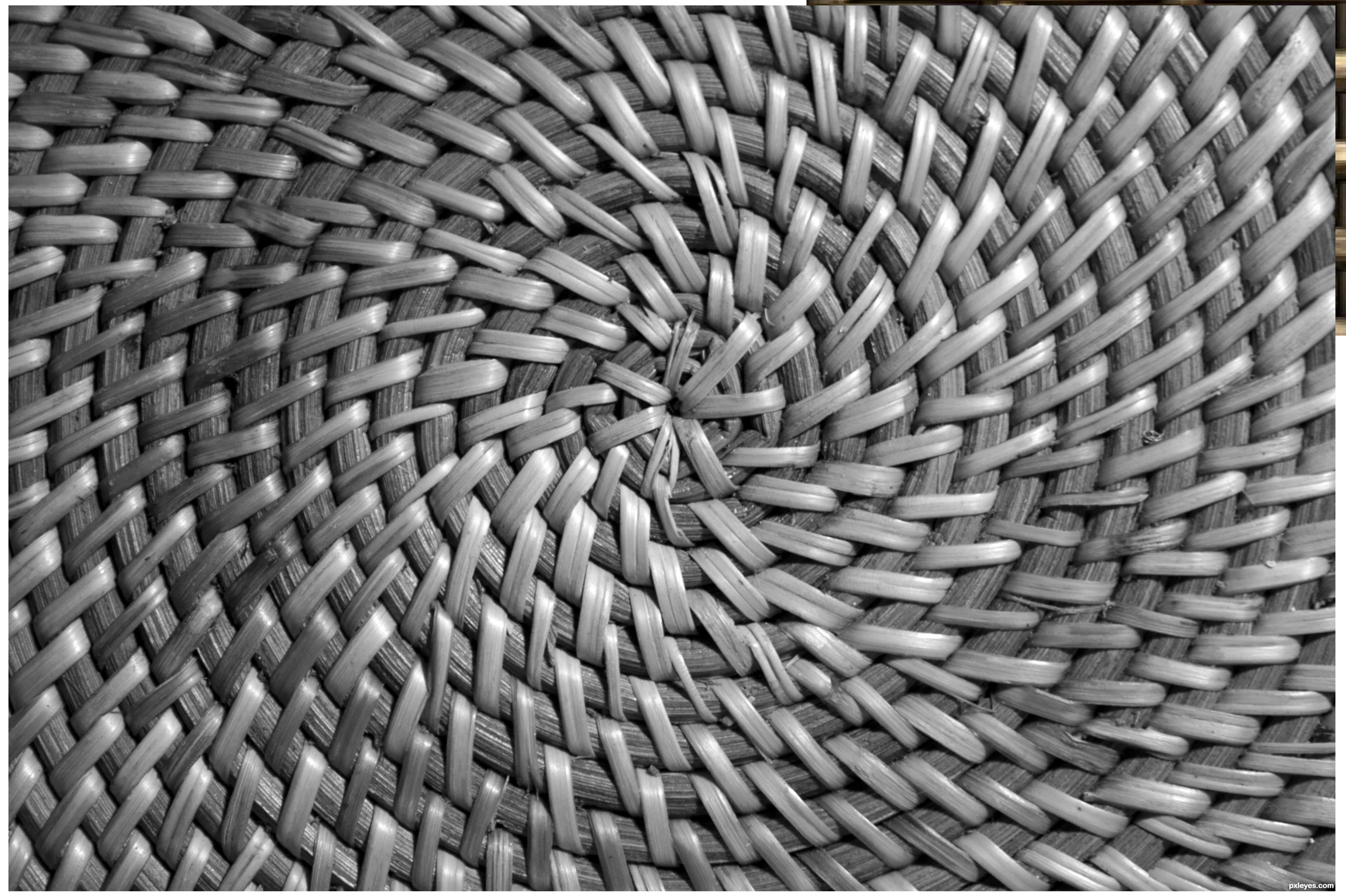


[http://www.pxleyes.com]

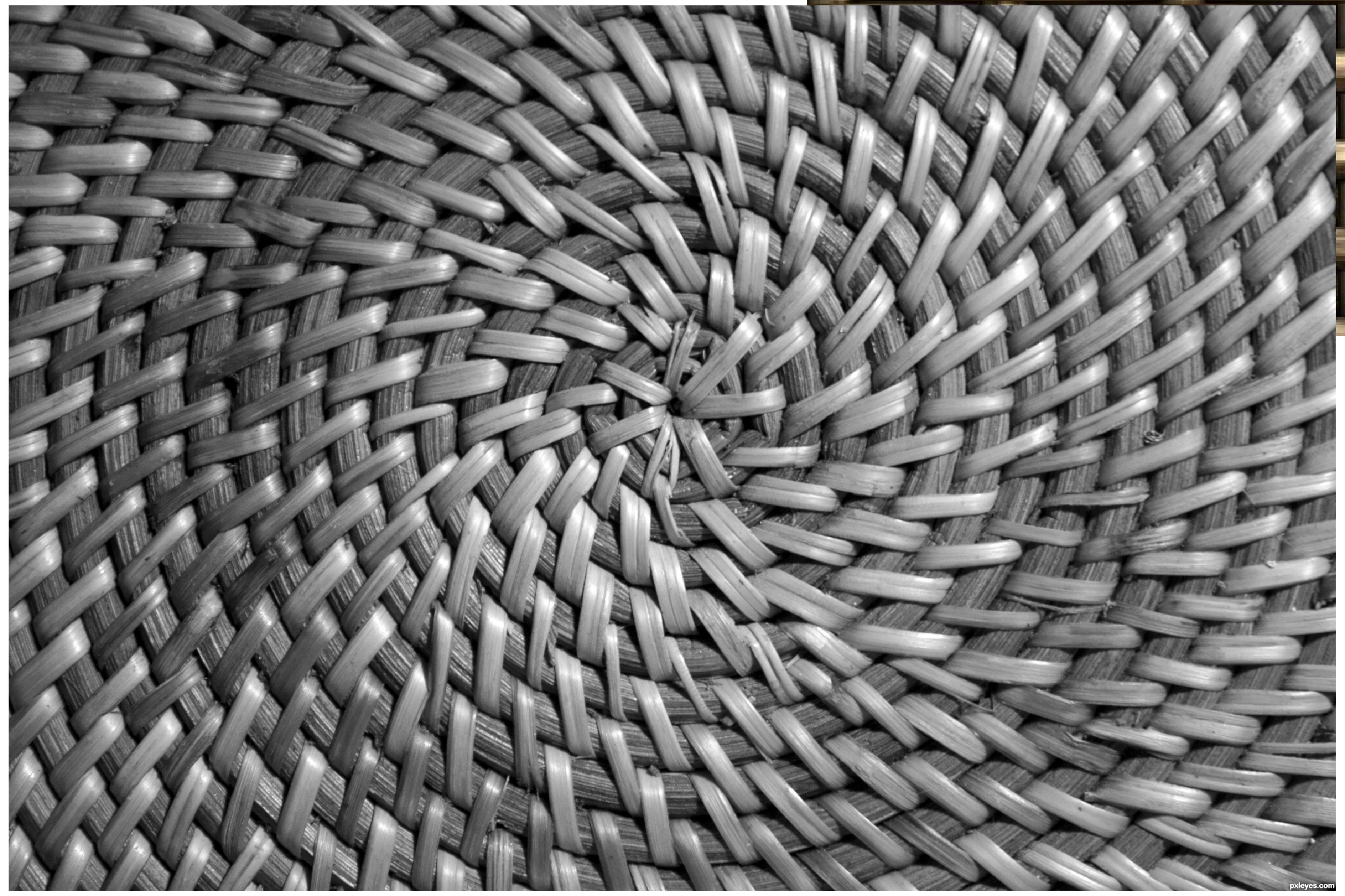
Hyper-streamlines

- Matches across cell

  - A consistent choice is possible if
    - The eigenvalues are different

  - Hyper-streamlines for

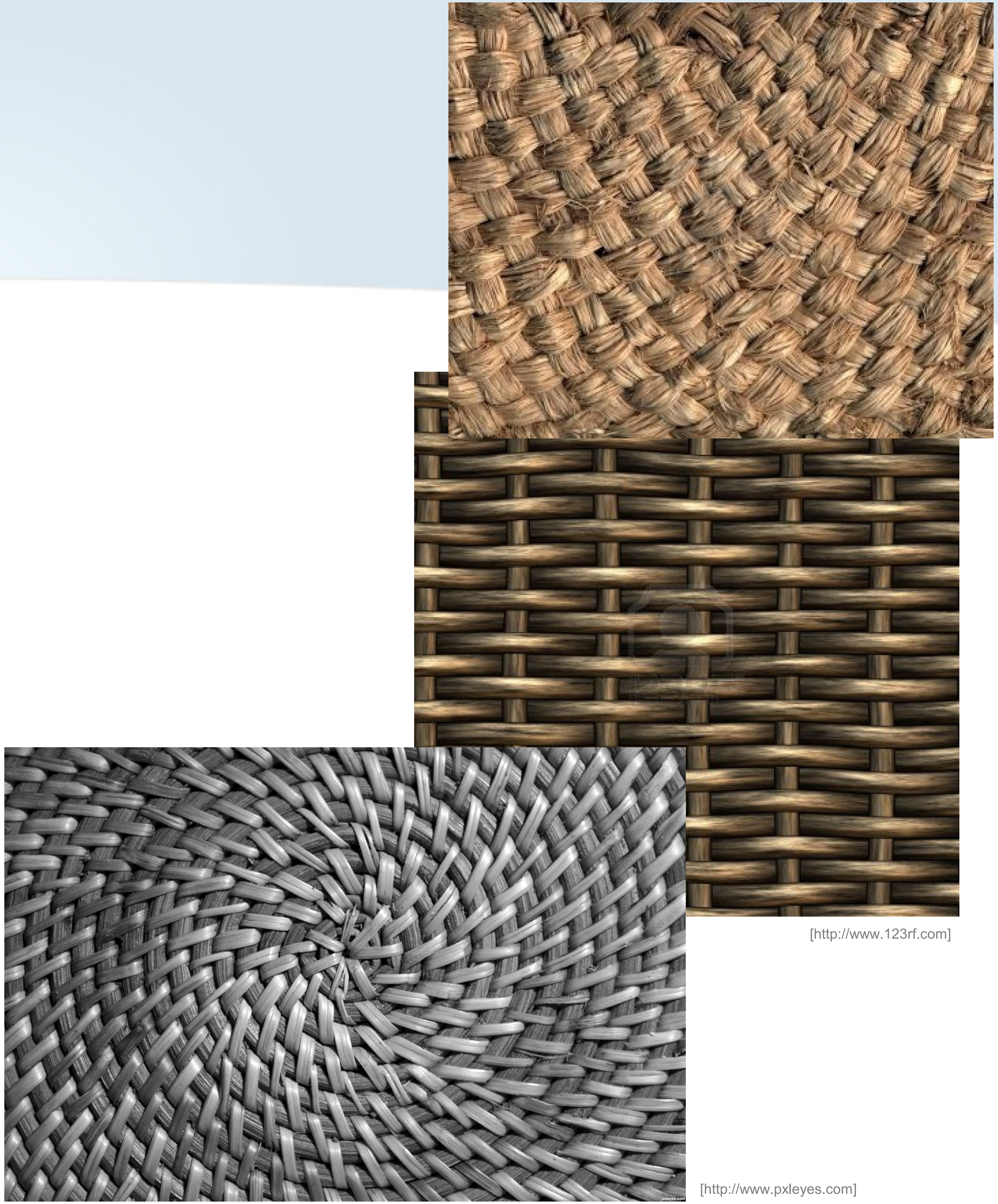
[http://www.123rf.com]



[http://www.pxleyes.com]

Hyper-streamlines

- Matches across cell
  - A consistent choice is possible if
    - The eigenvalues are different
  - Hyper-streamlines for
    - Major eigenvalue/vector



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Hyper-streamlines

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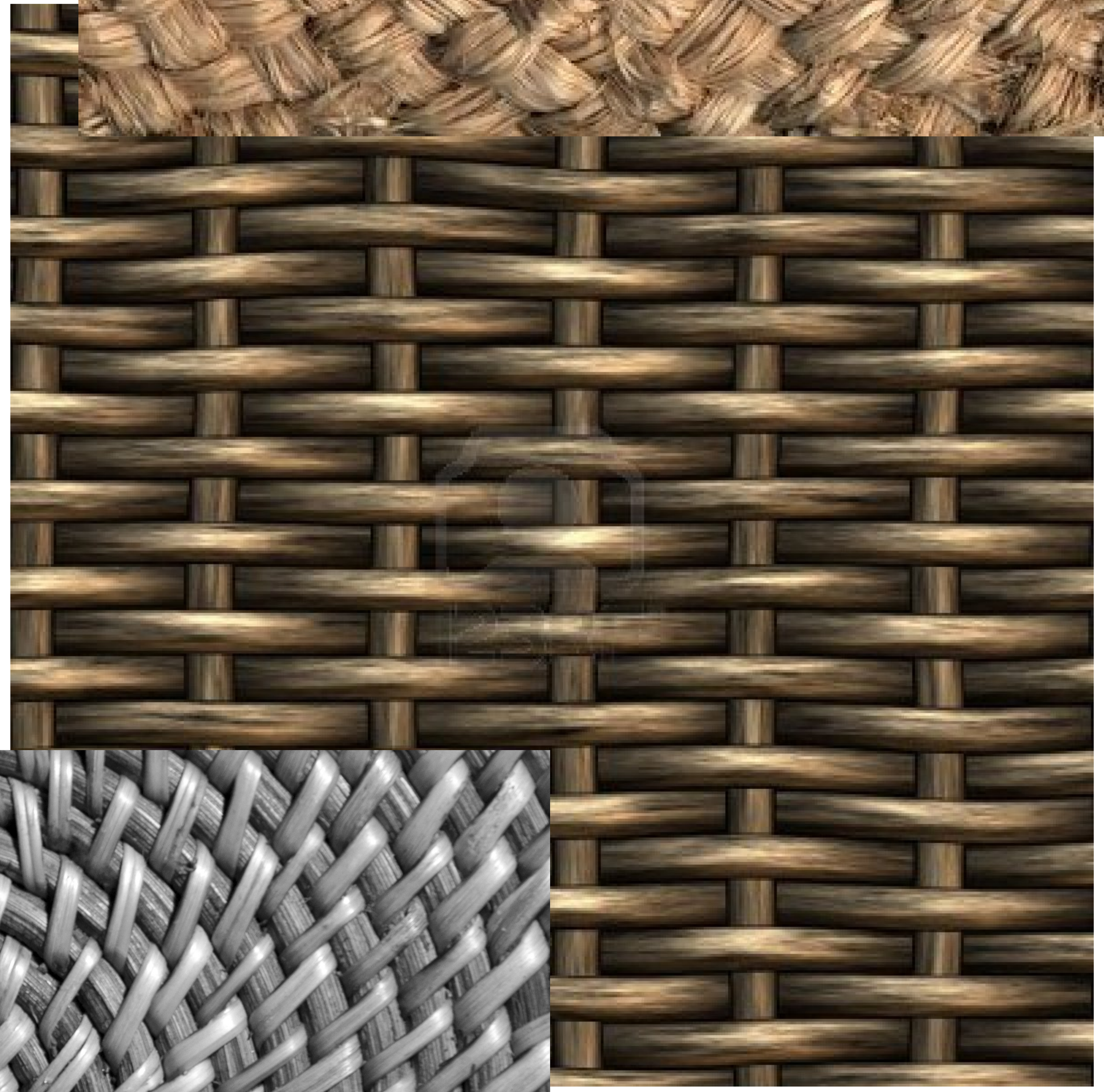
    - The eigenvalues are different
  - Hyper-streamlines for
    - Major eigenvalue/vector
    - Minor eigenvalue/vector



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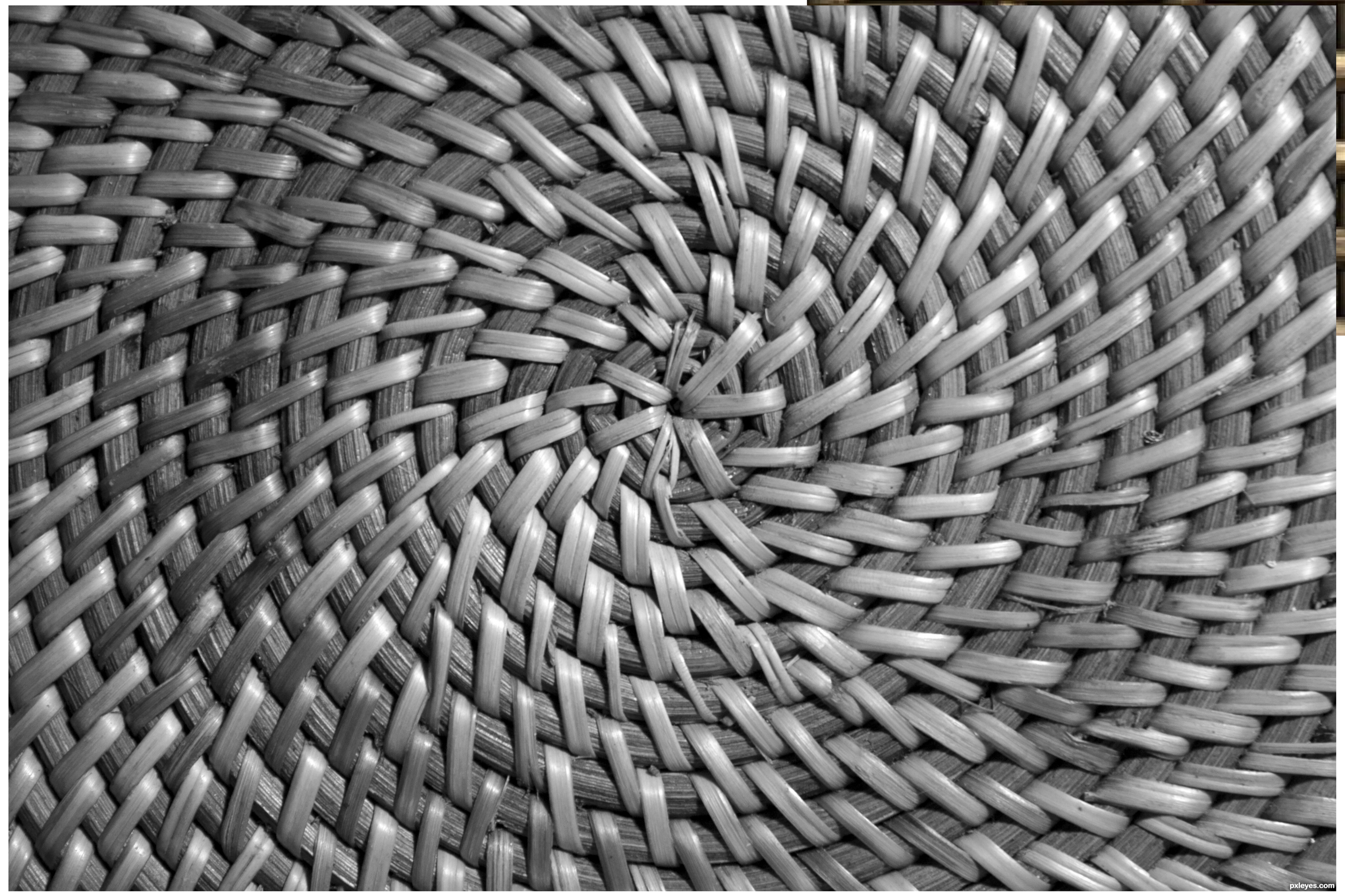

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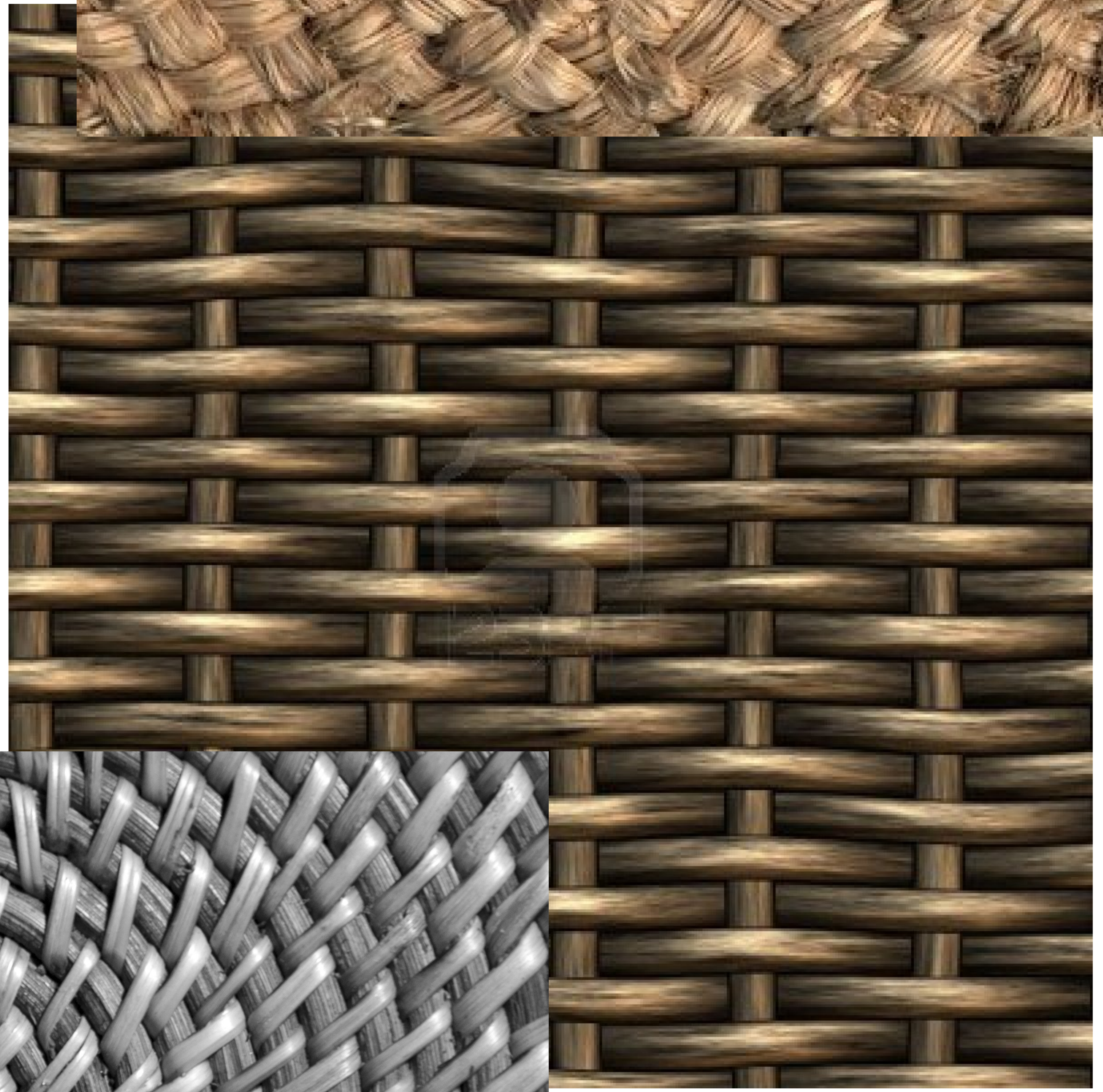
Hyper-streamlines

- Orientation



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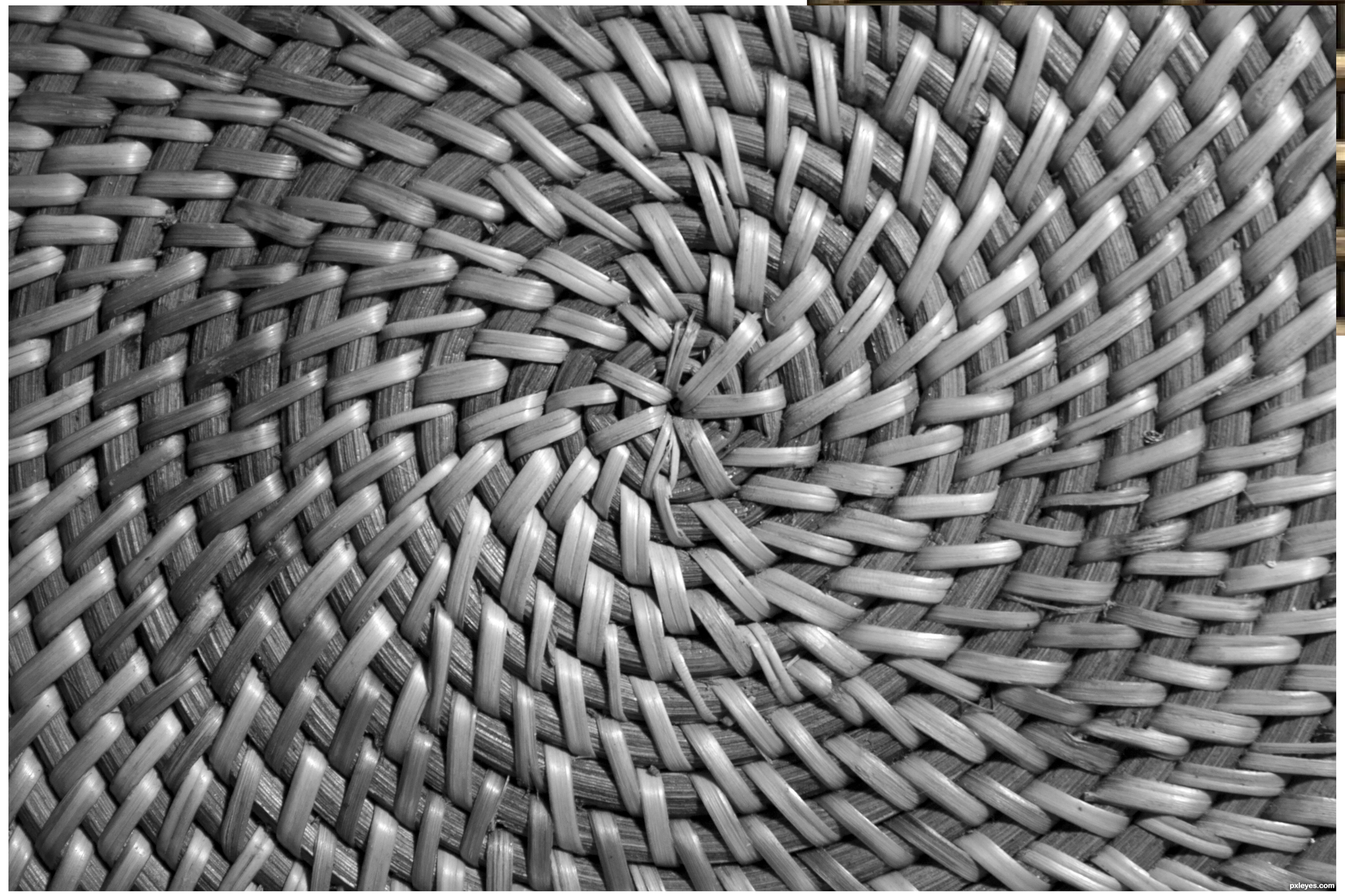

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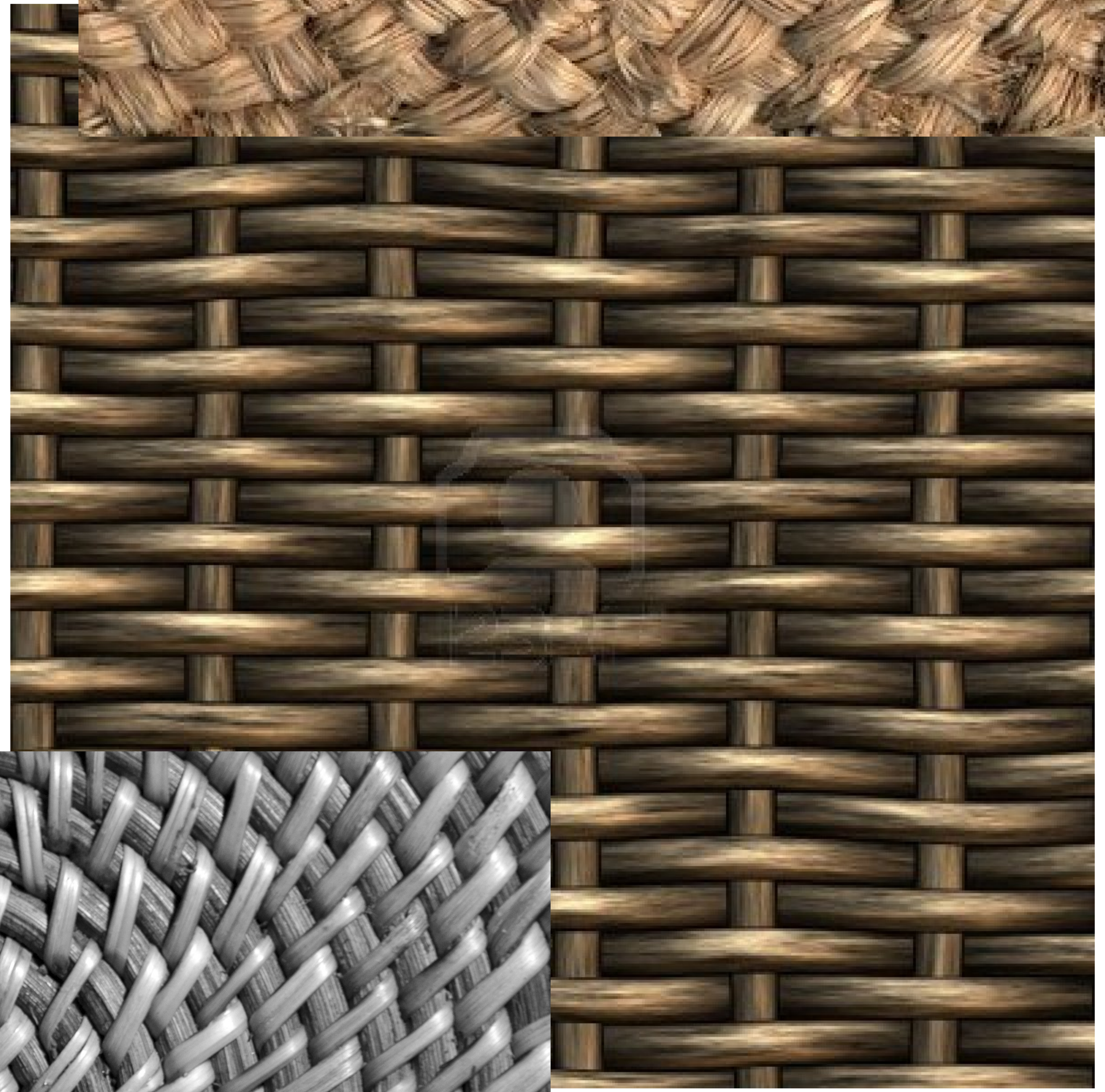
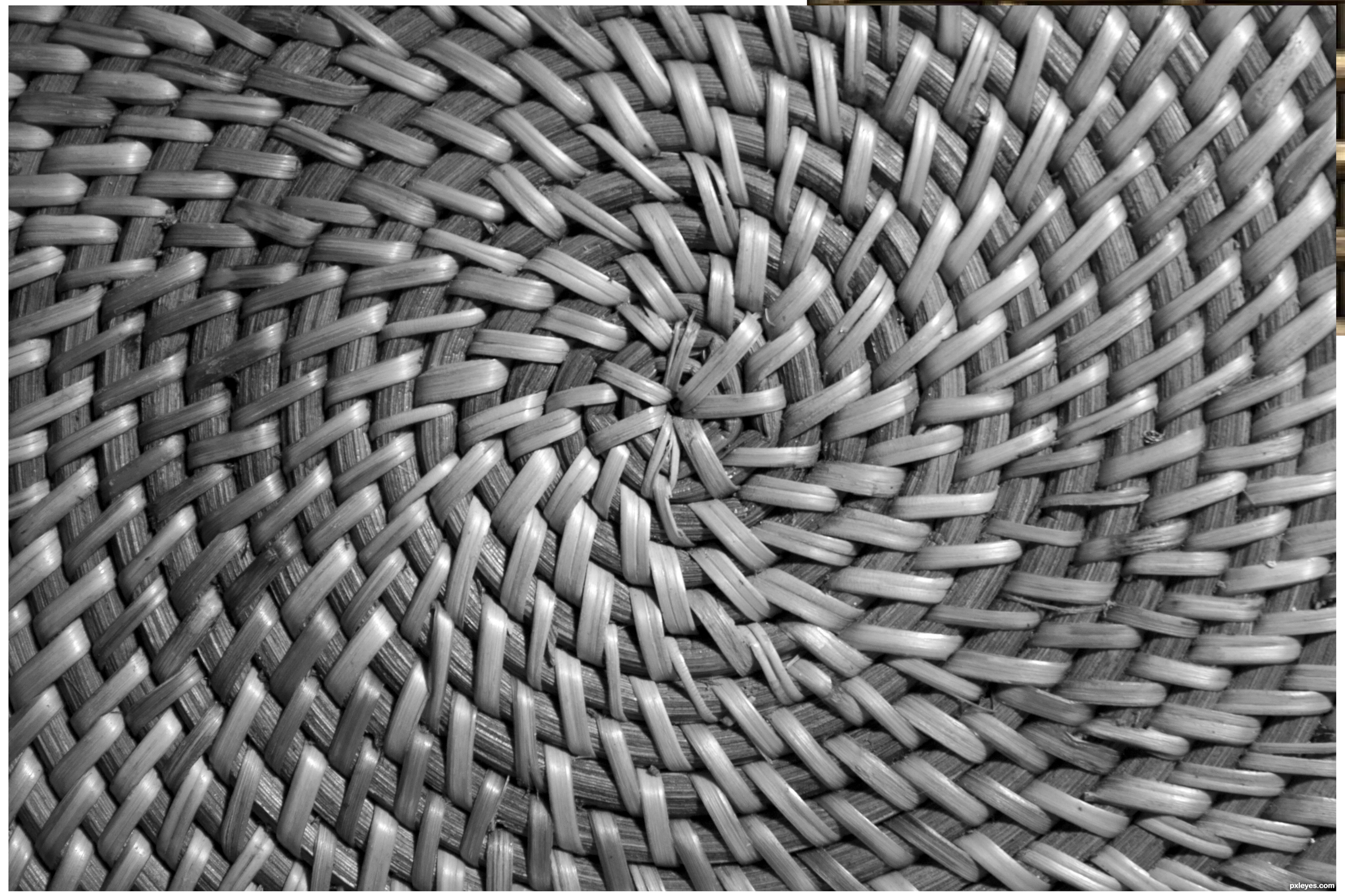
Hyper-streamlines

- Orientation
  - Eigenvector computation



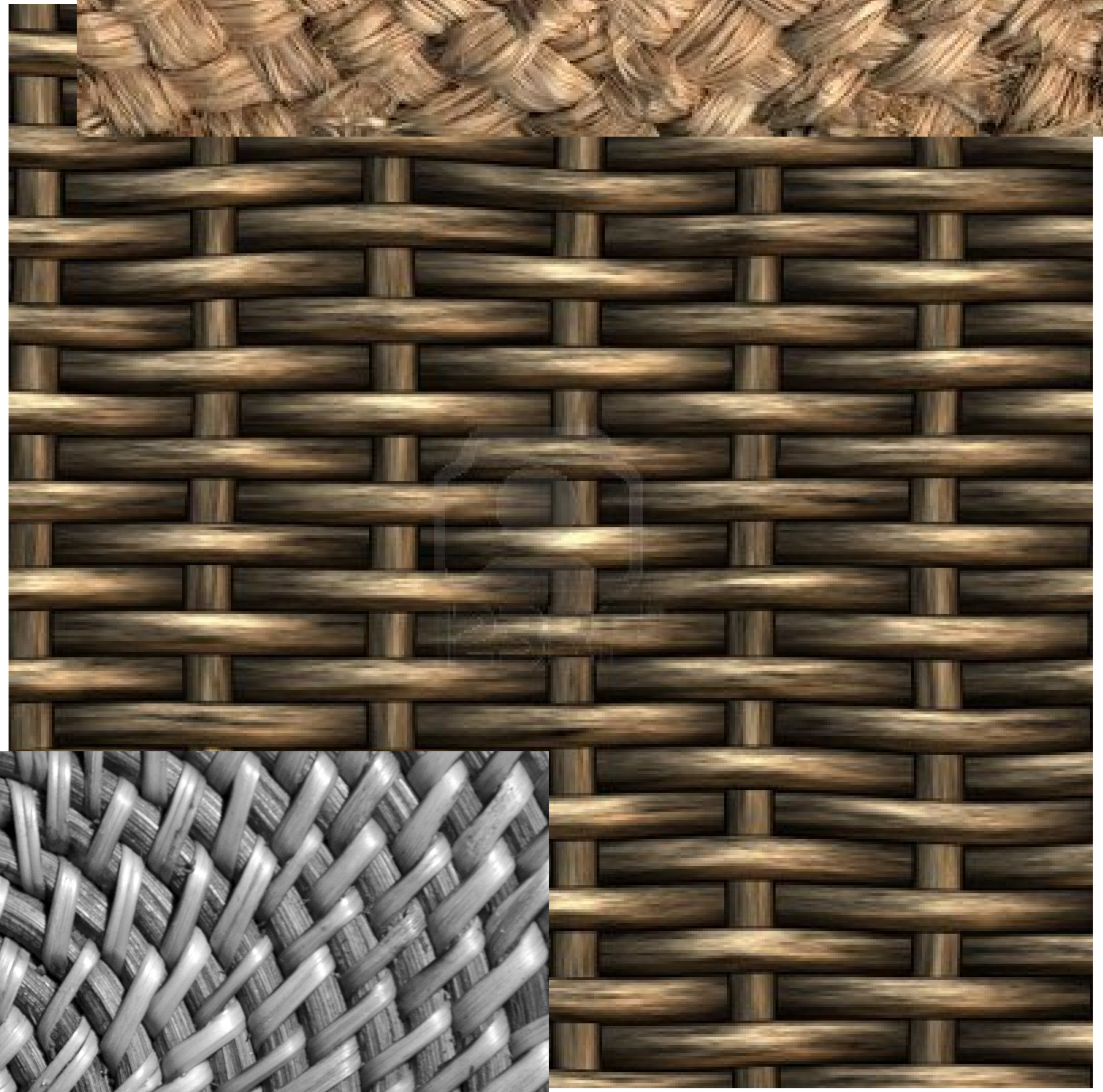
[http://www.123rf.com]



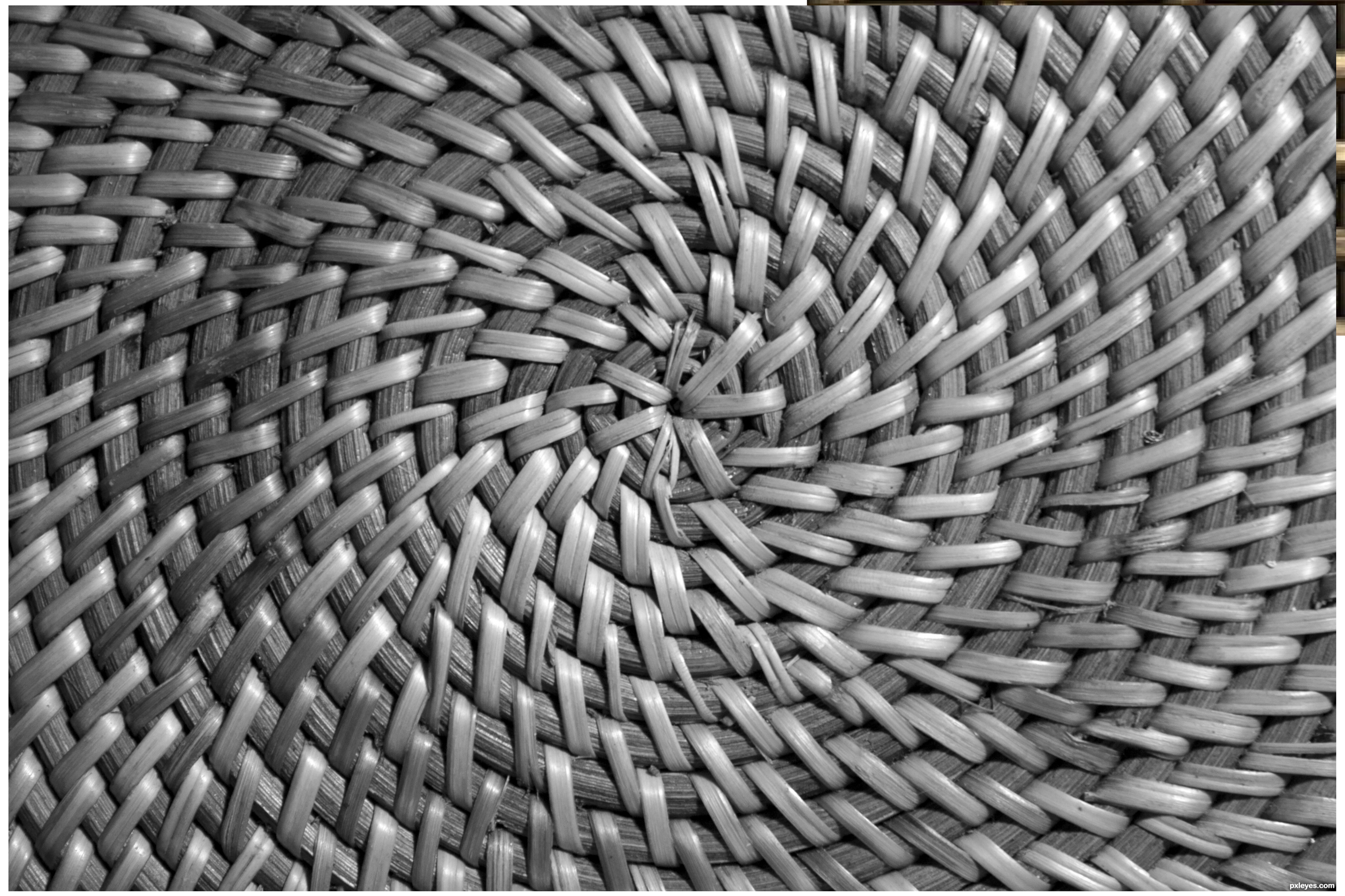
[http://www.pxleyes.com]

Hyper-streamlines

- Orientation
  - Eigenvector computation
    - Independent at each point



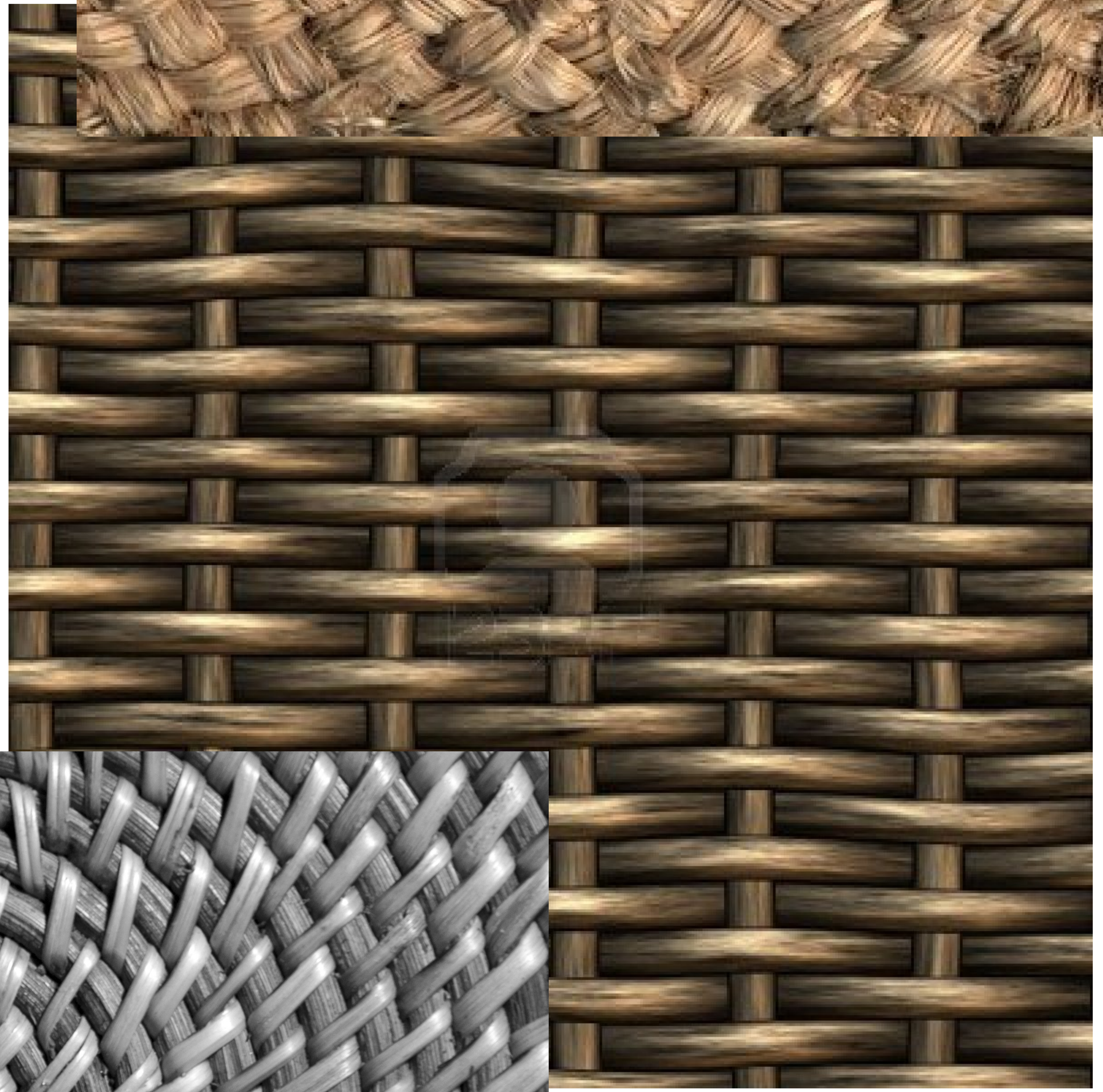
[http://www.123rf.com]



[http://www.pxleyes.com]

Hyper-streamlines

- Orientation
  - Eigenvector computation
    - Independent at each point
  - Possible flips in orientation



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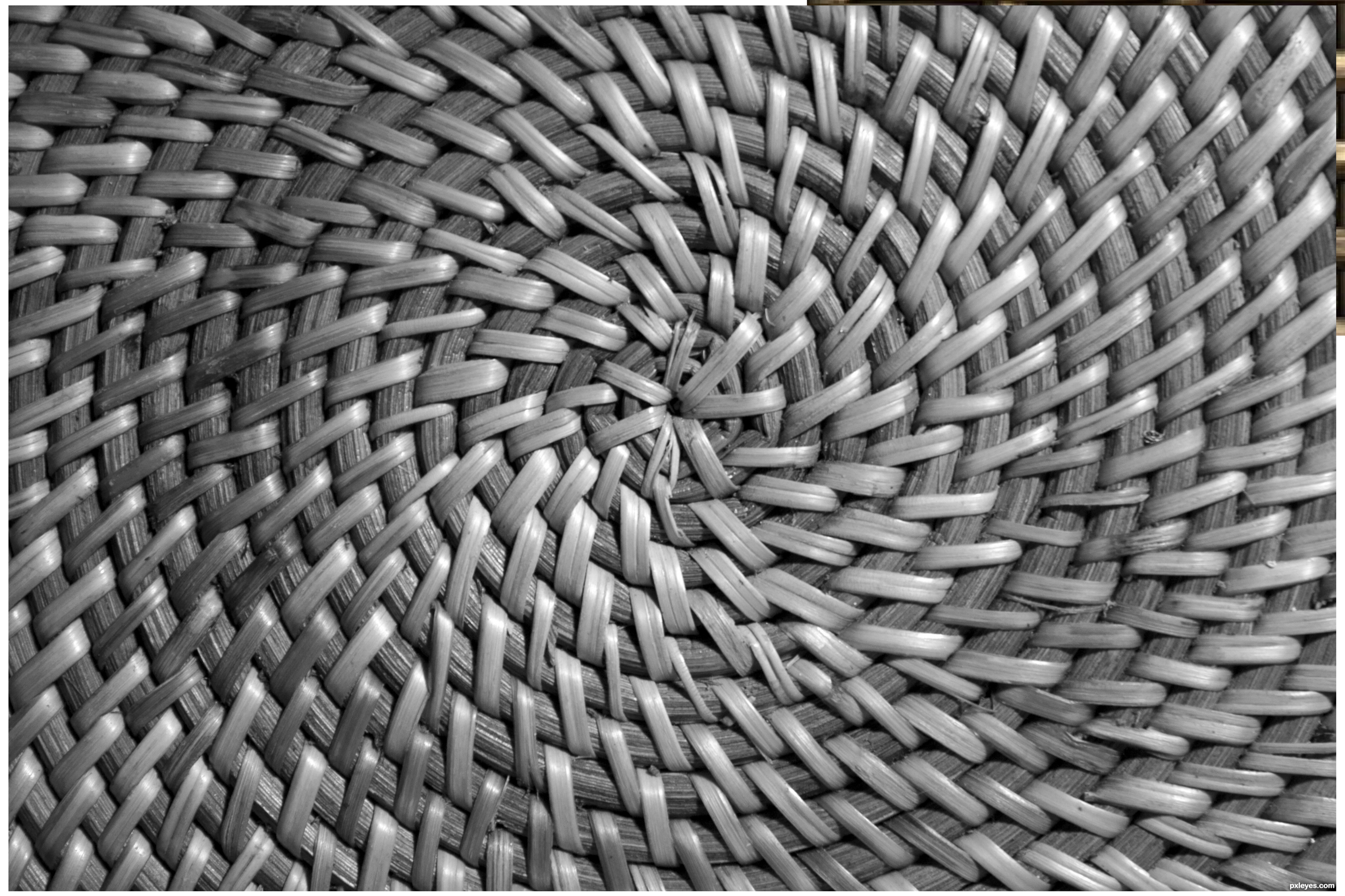
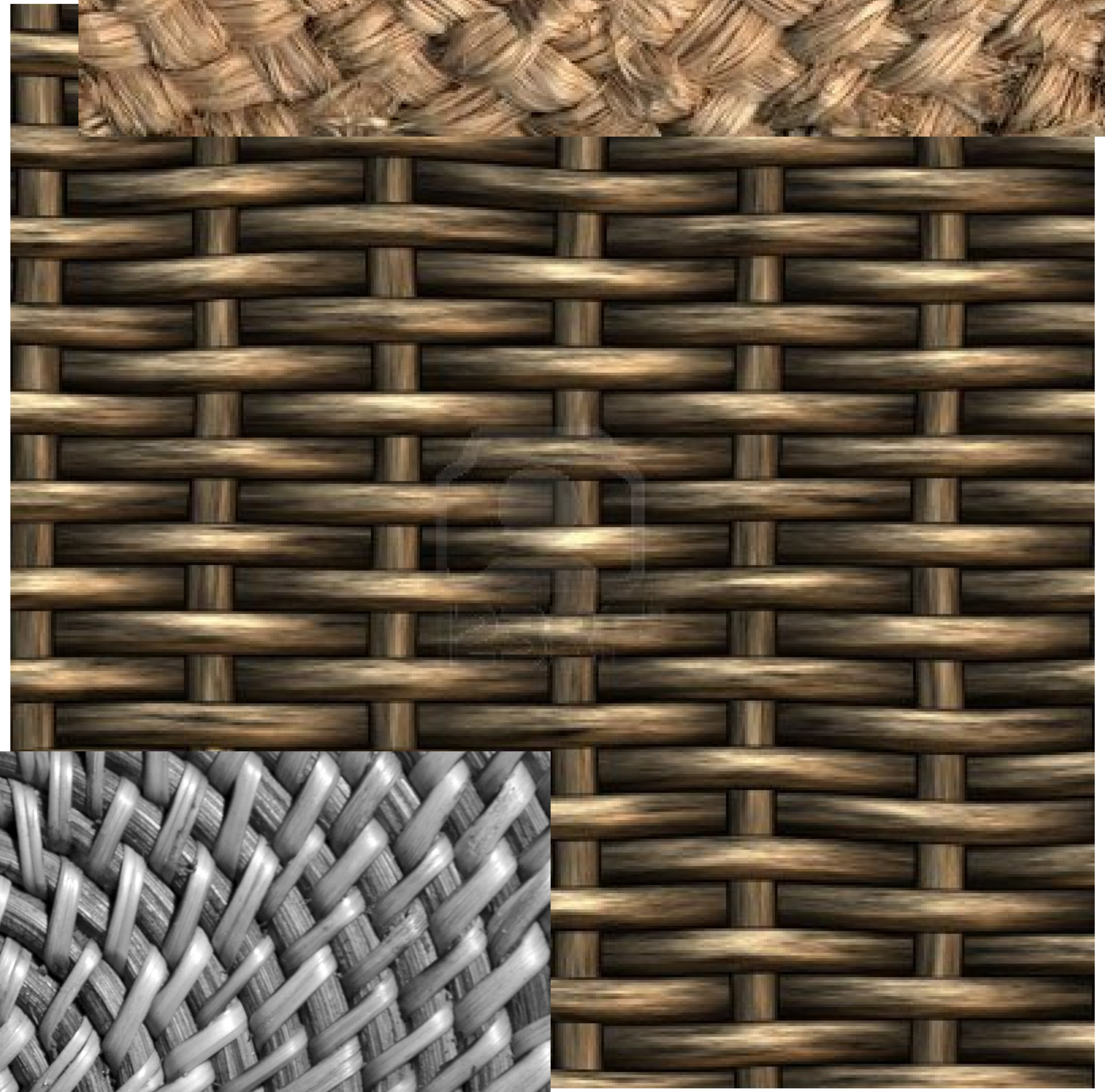


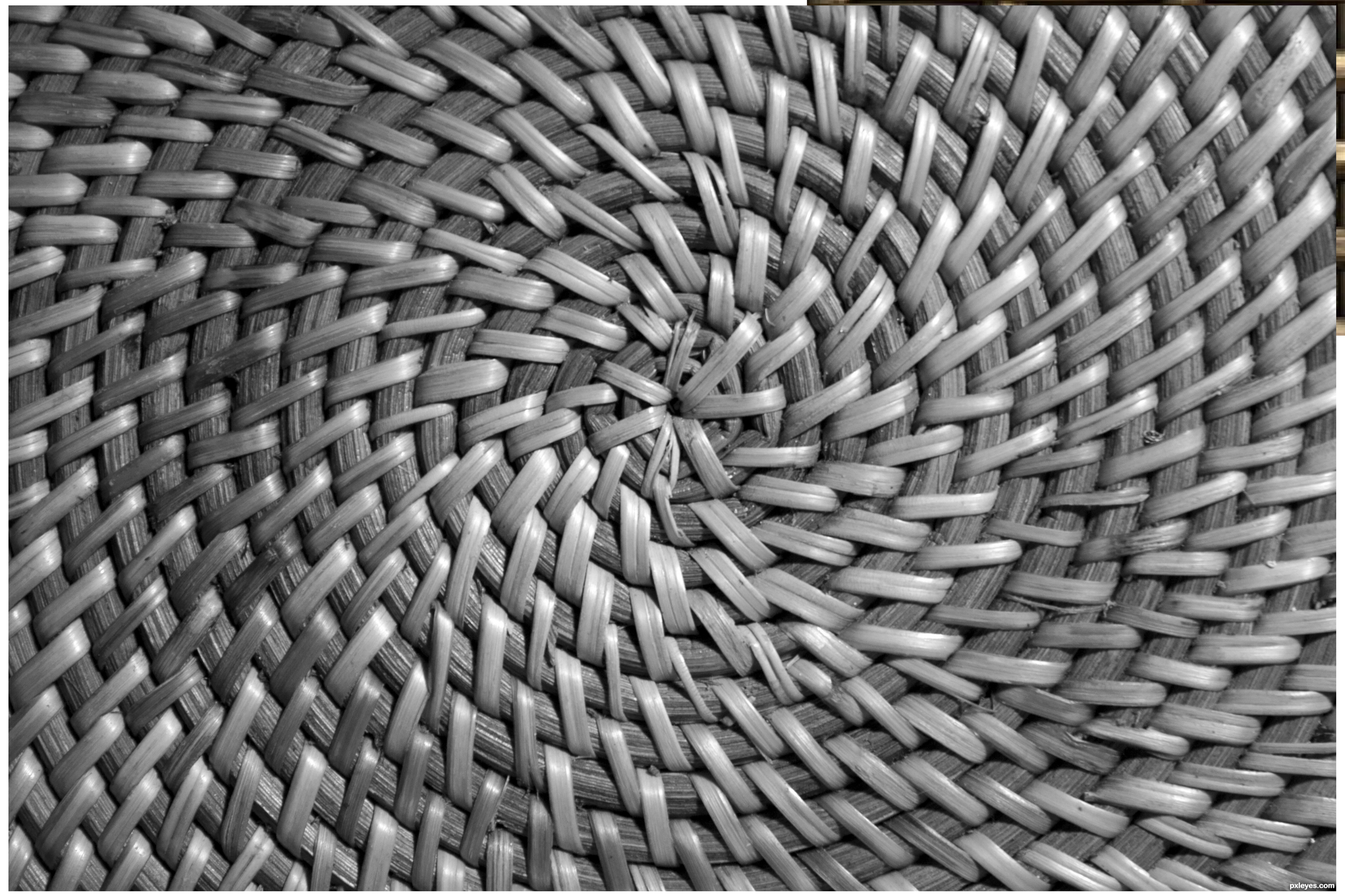
[http://www.pxleyes.com]

Hyper-streamlines

- Orientation
  - Eigenvector computation
    - Independent at each point
  - Possible flips in orientation
    - Dot product



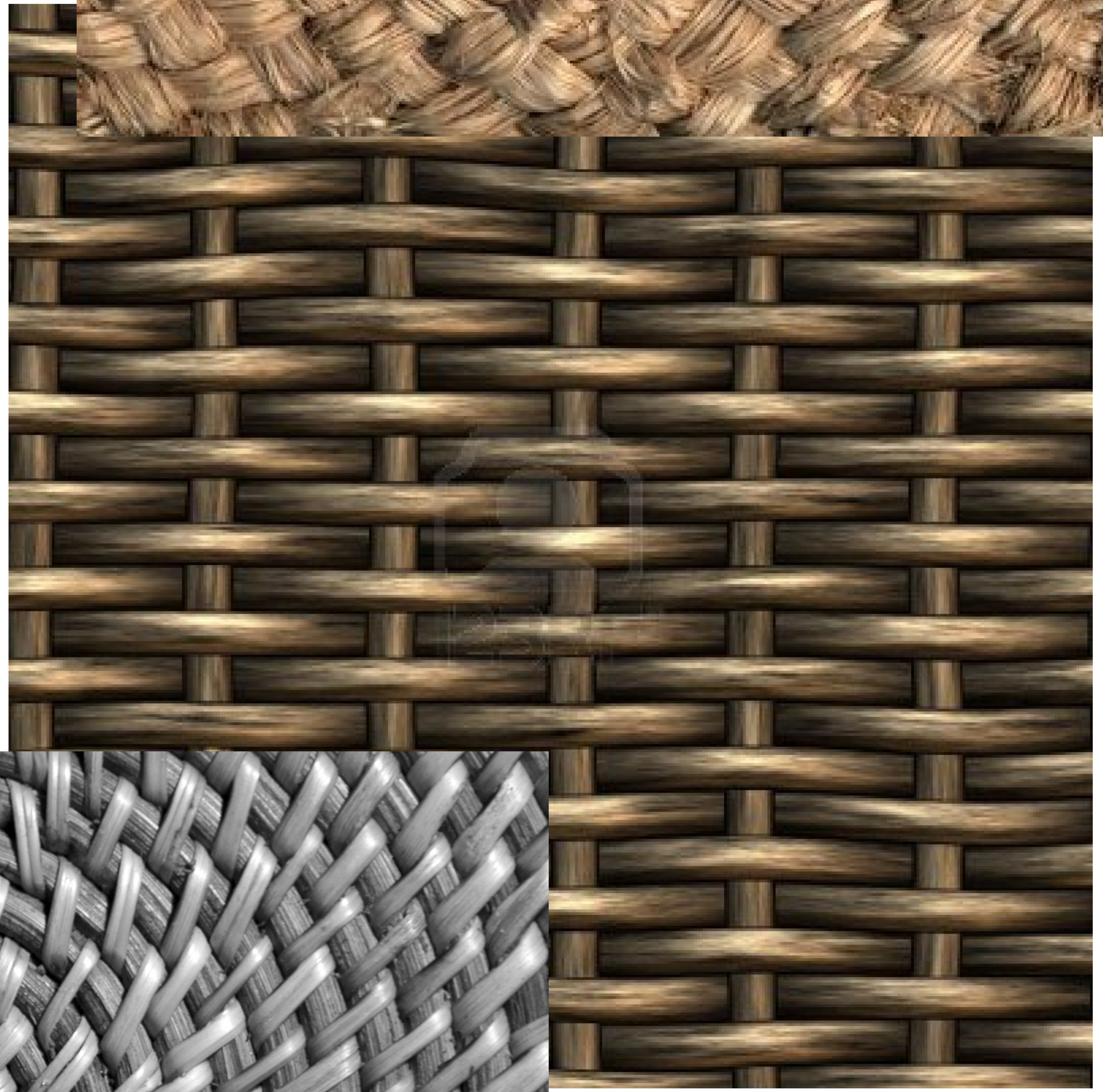
[http://www.123rf.com]



[http://www.pxleyes.com]

Hyper-streamlines

- Orientation
  - Eigenvector computation
    - Independent at each point
  - Possible flips in orientation
    - Dot product
      - Current advancing direction
      - Eigenvector



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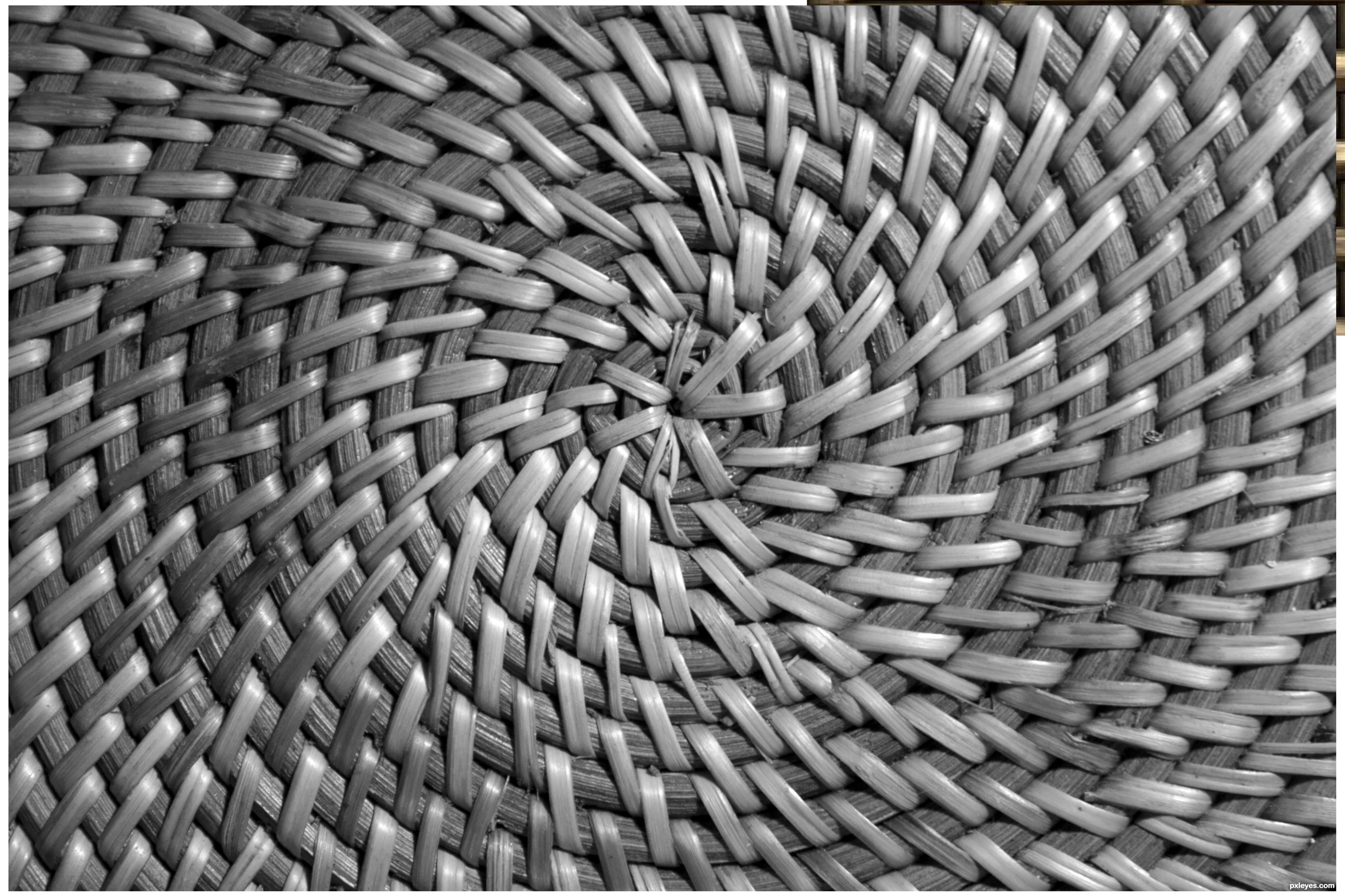
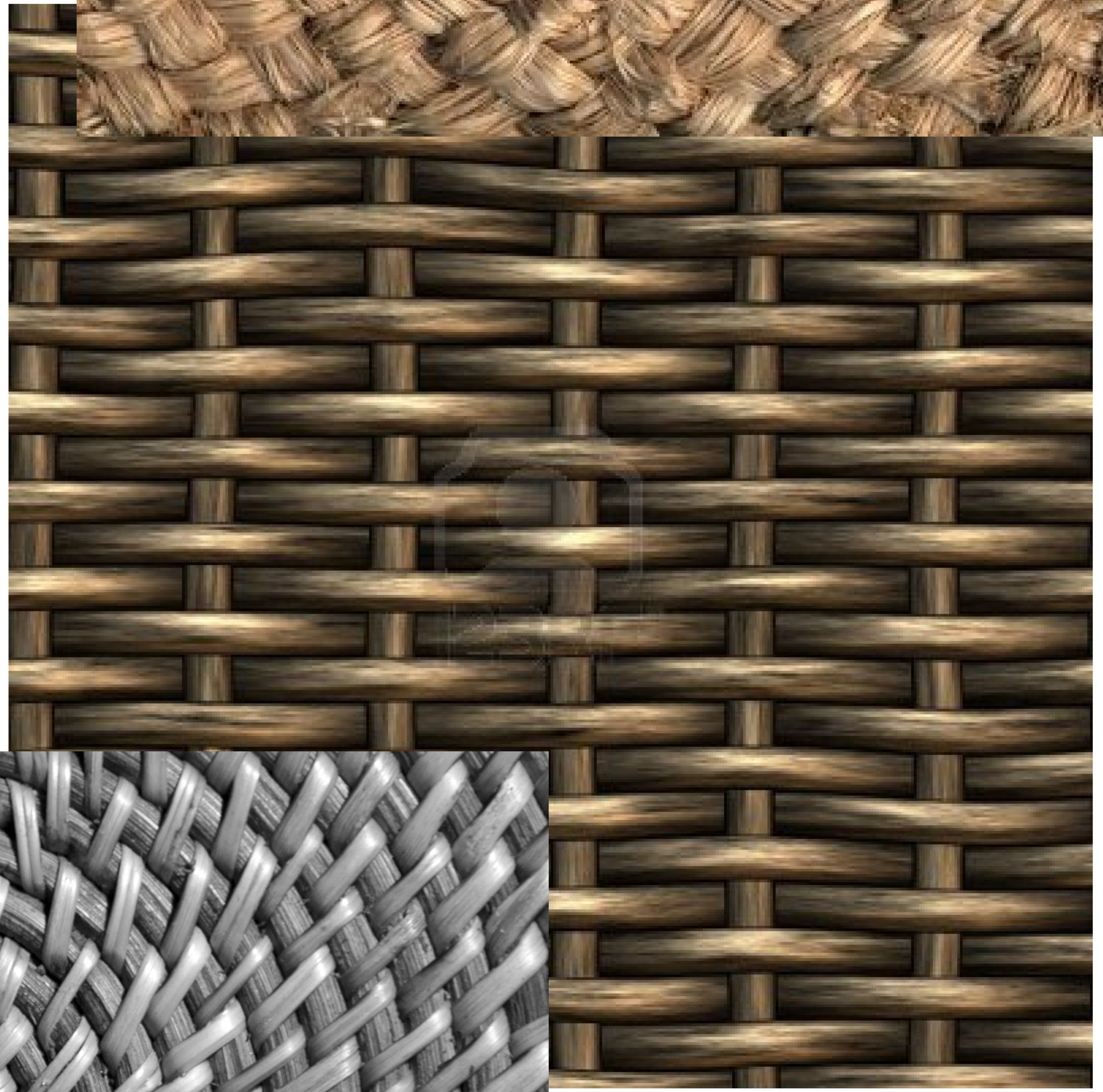


[http://www.pxleyes.com]

Hyper-streamlines

- Orientation
  - Eigenvector computation
    - Independent at each point
  - Possible flips in orientation
    - Dot product
      - Current advancing direction
      - Eigenvector
    - Flip if negative



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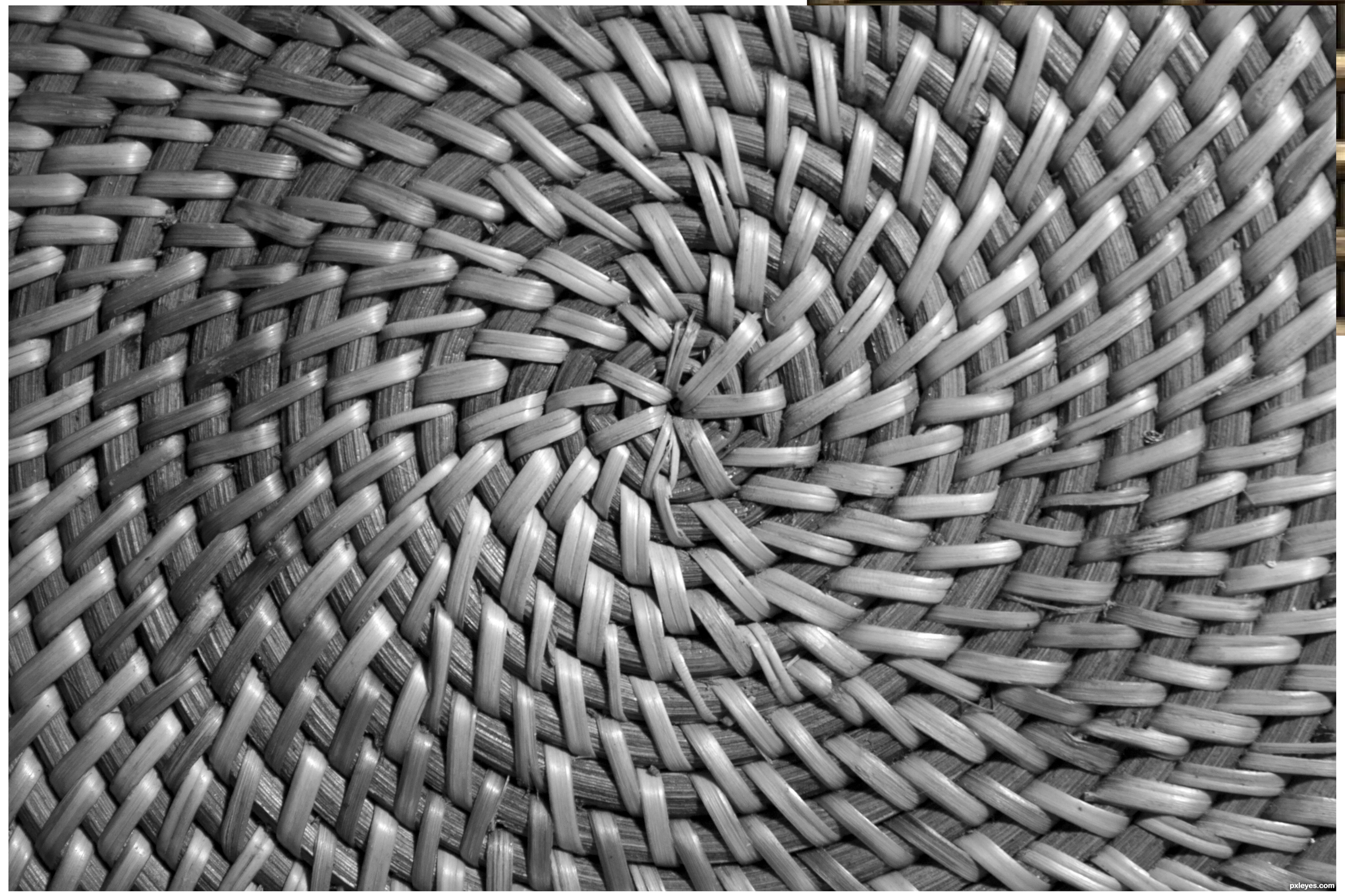
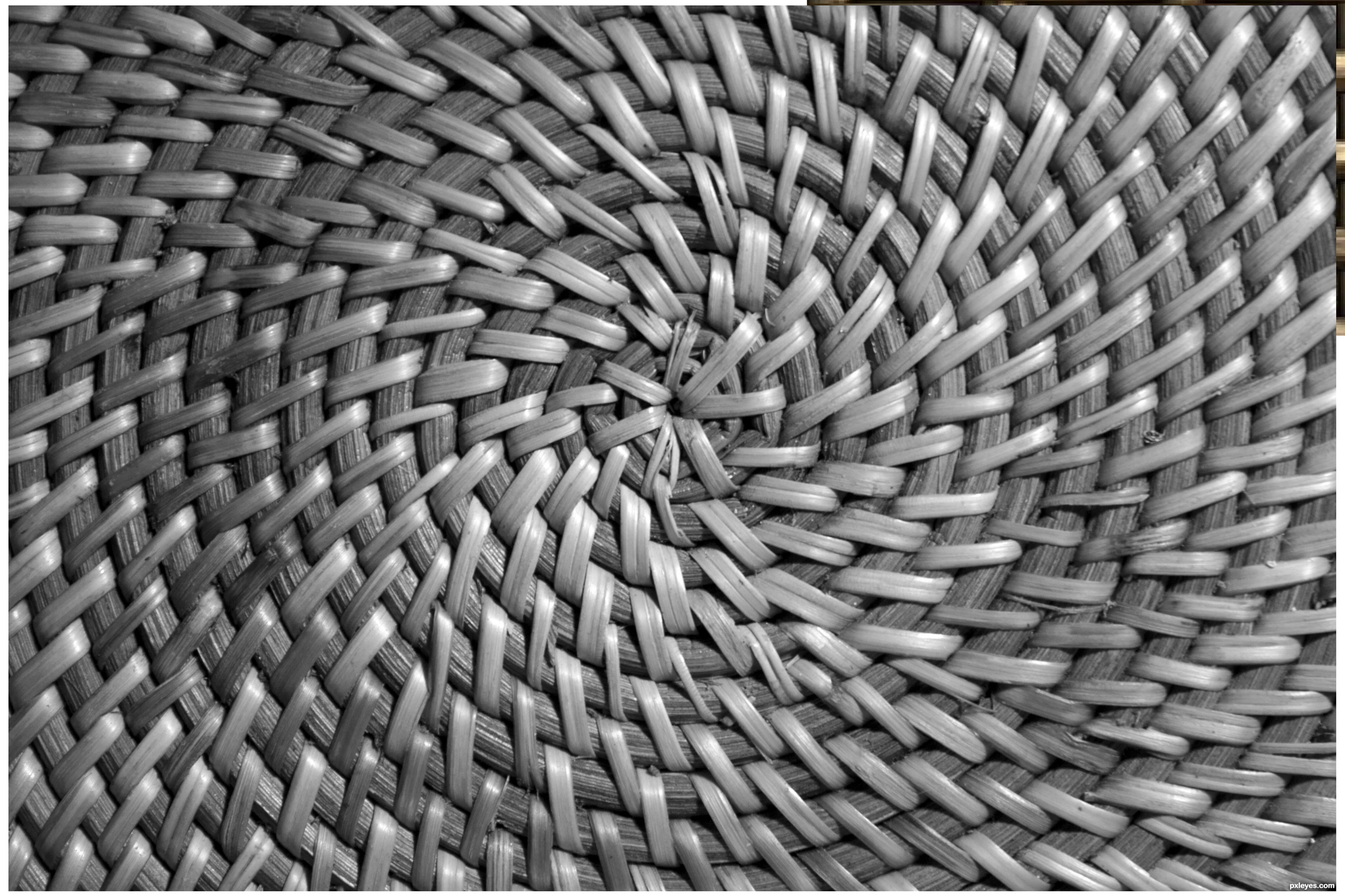


[http://www.pxleyes.com]

Hyper-streamlines

- Other than that...

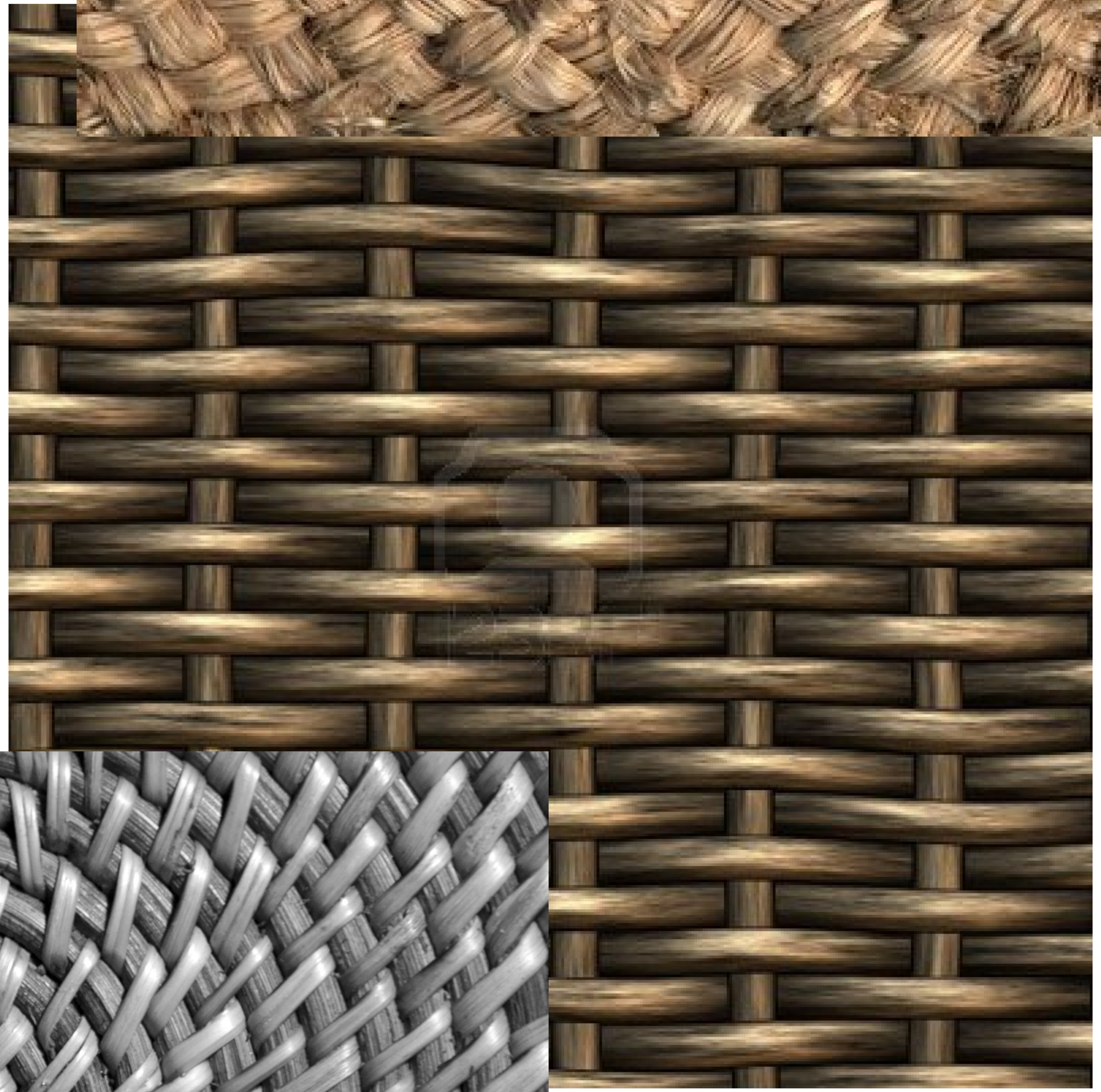
[http://www.123rf.com]



[http://www.pxleyes.com]

Hyper-streamlines

- Other than that...
  - Same integration as for vector fields



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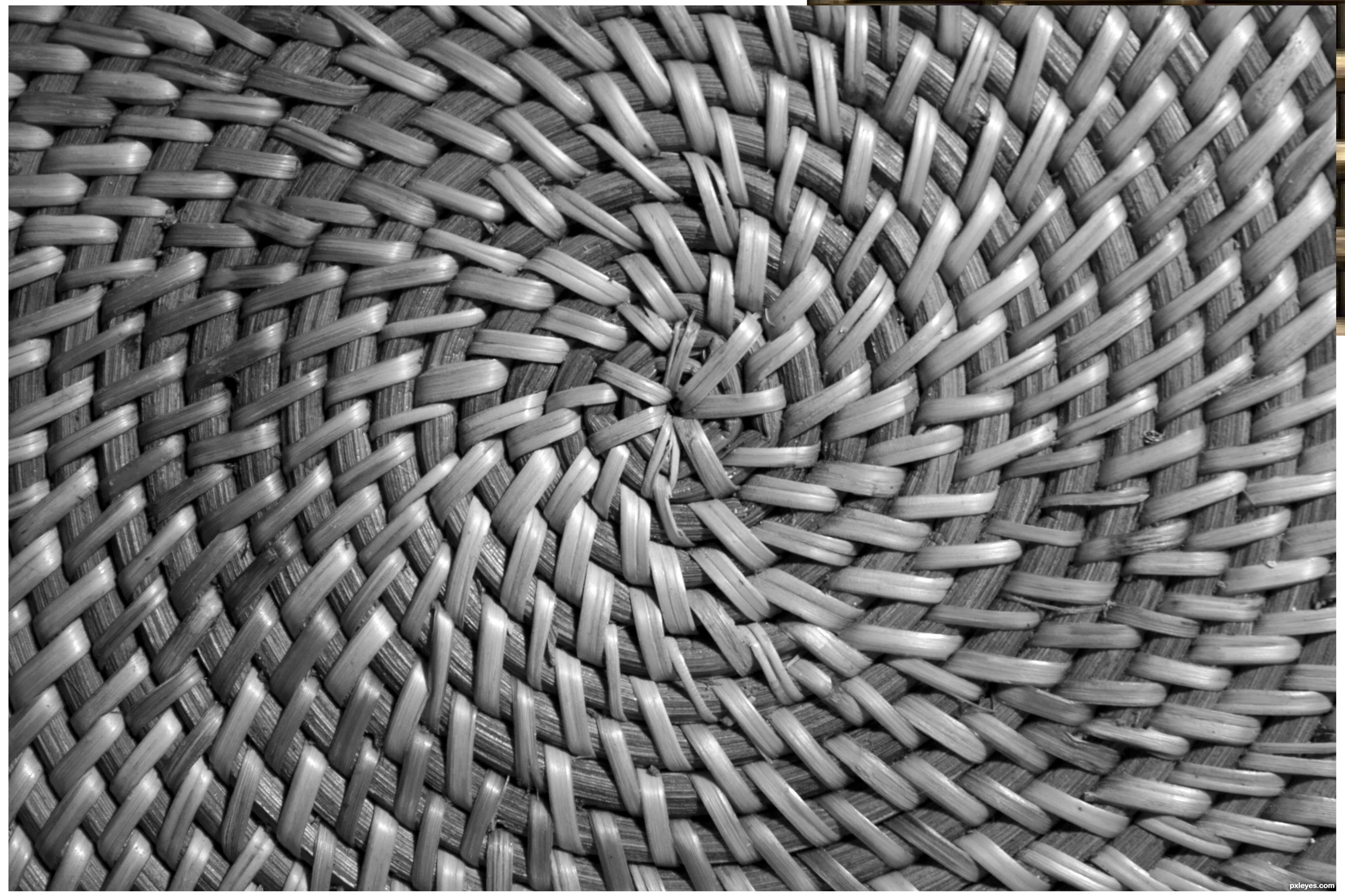

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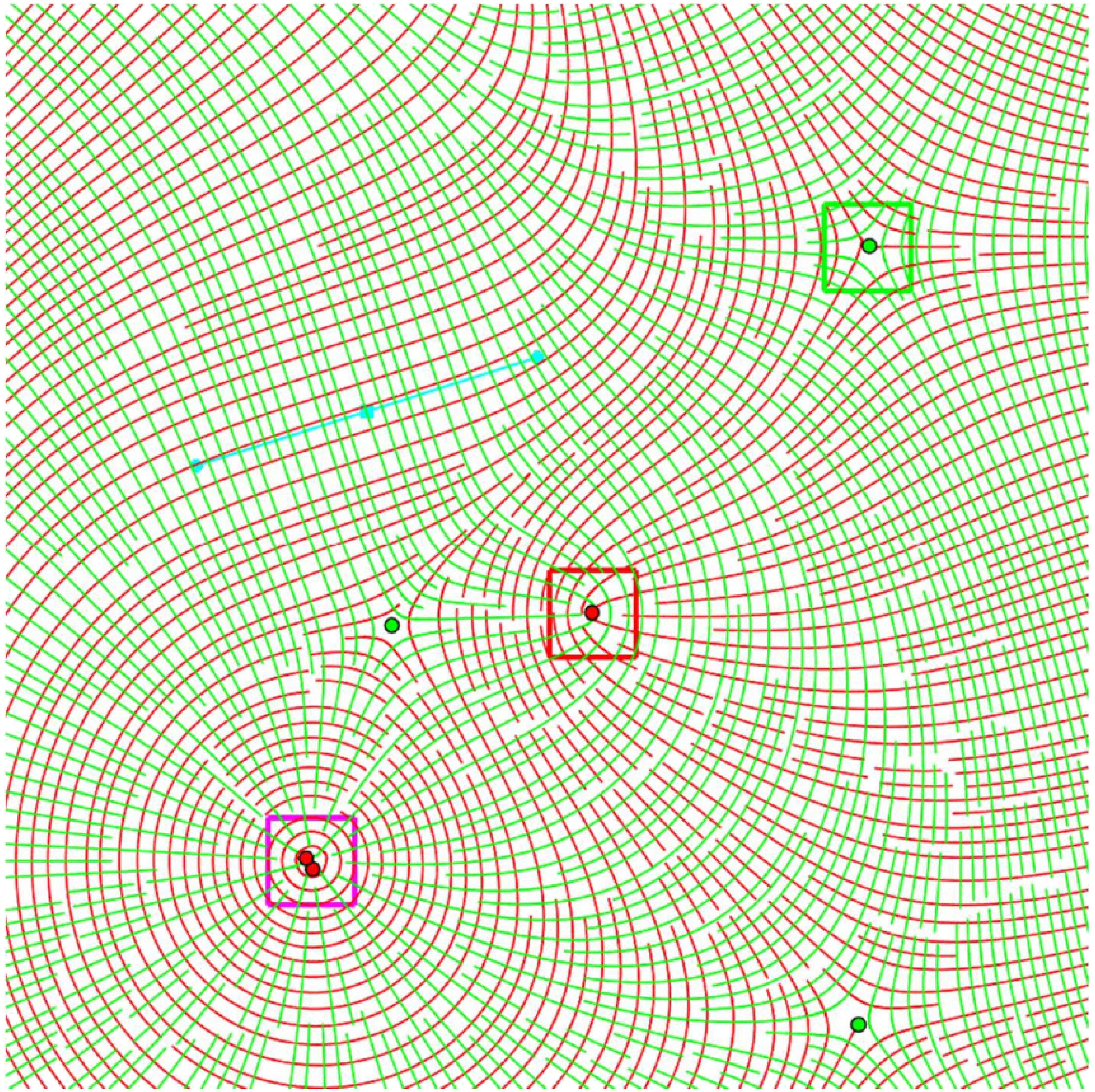
Hyper-streamlines

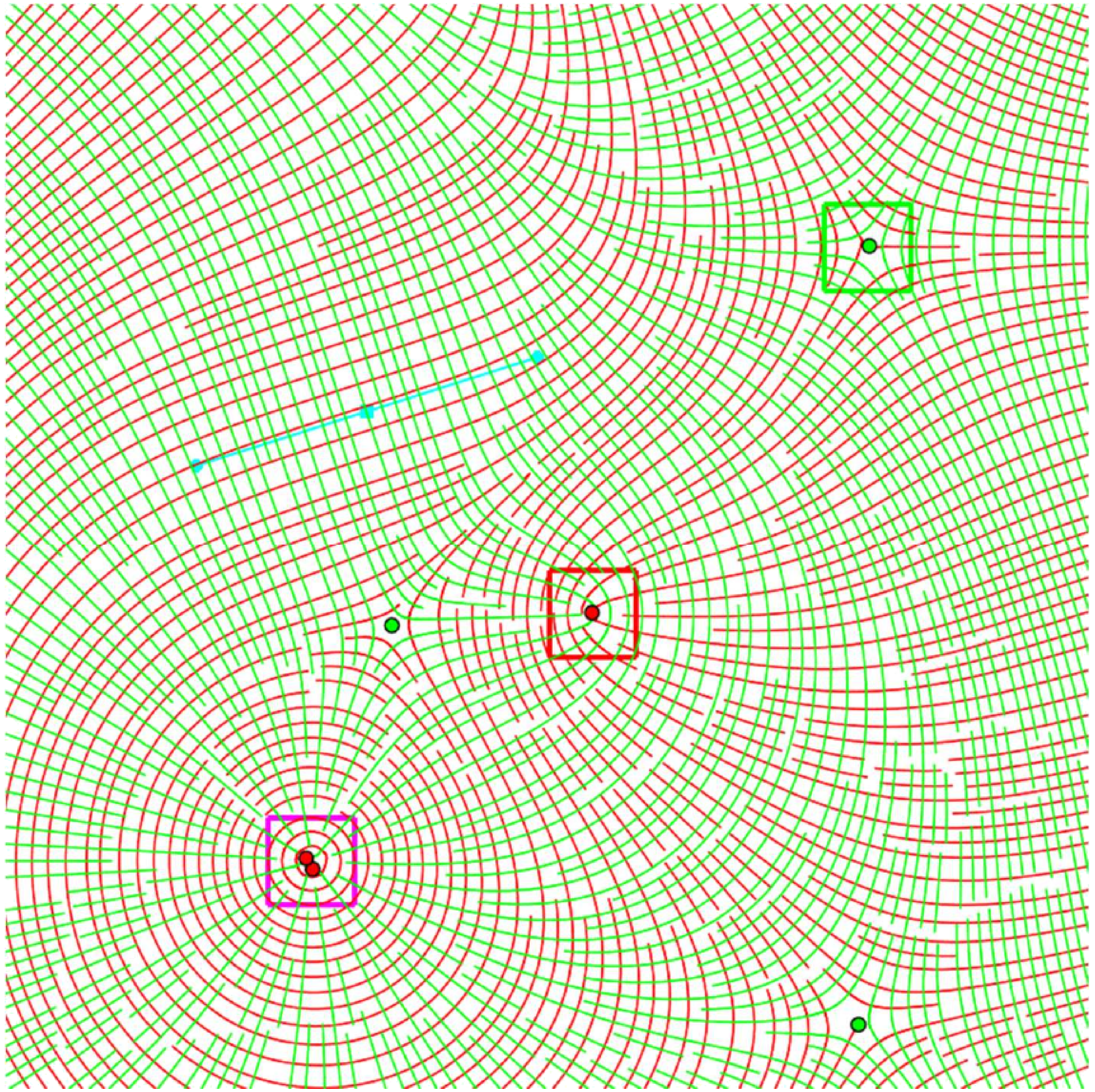
- Other than that...
  - Same integration as for vector fields



[Chen]

Hyper-streamlines

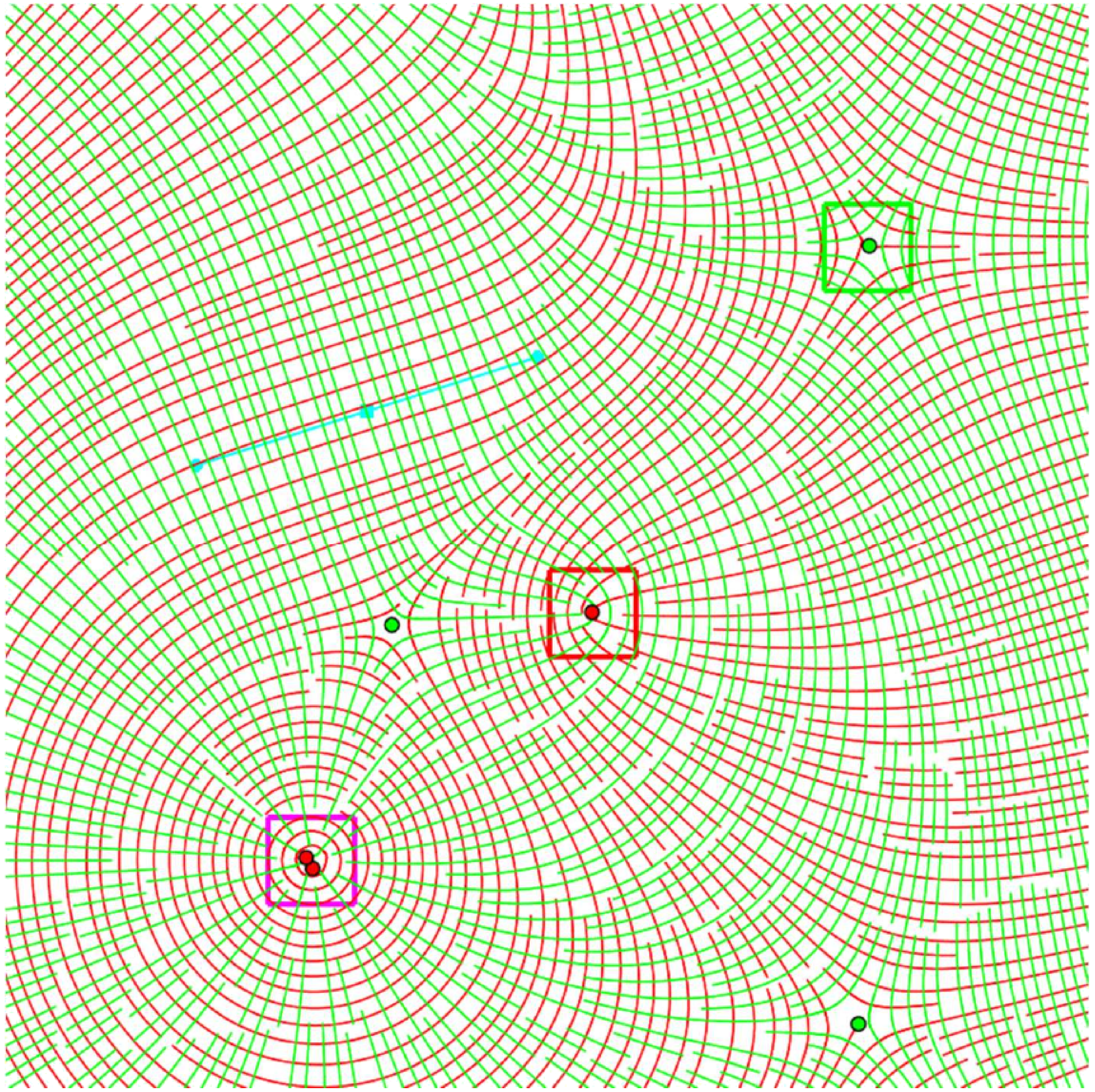
- Other than that...
  - Same integration as for vector fields
- Recap



[Chen]

Hyper-streamlines

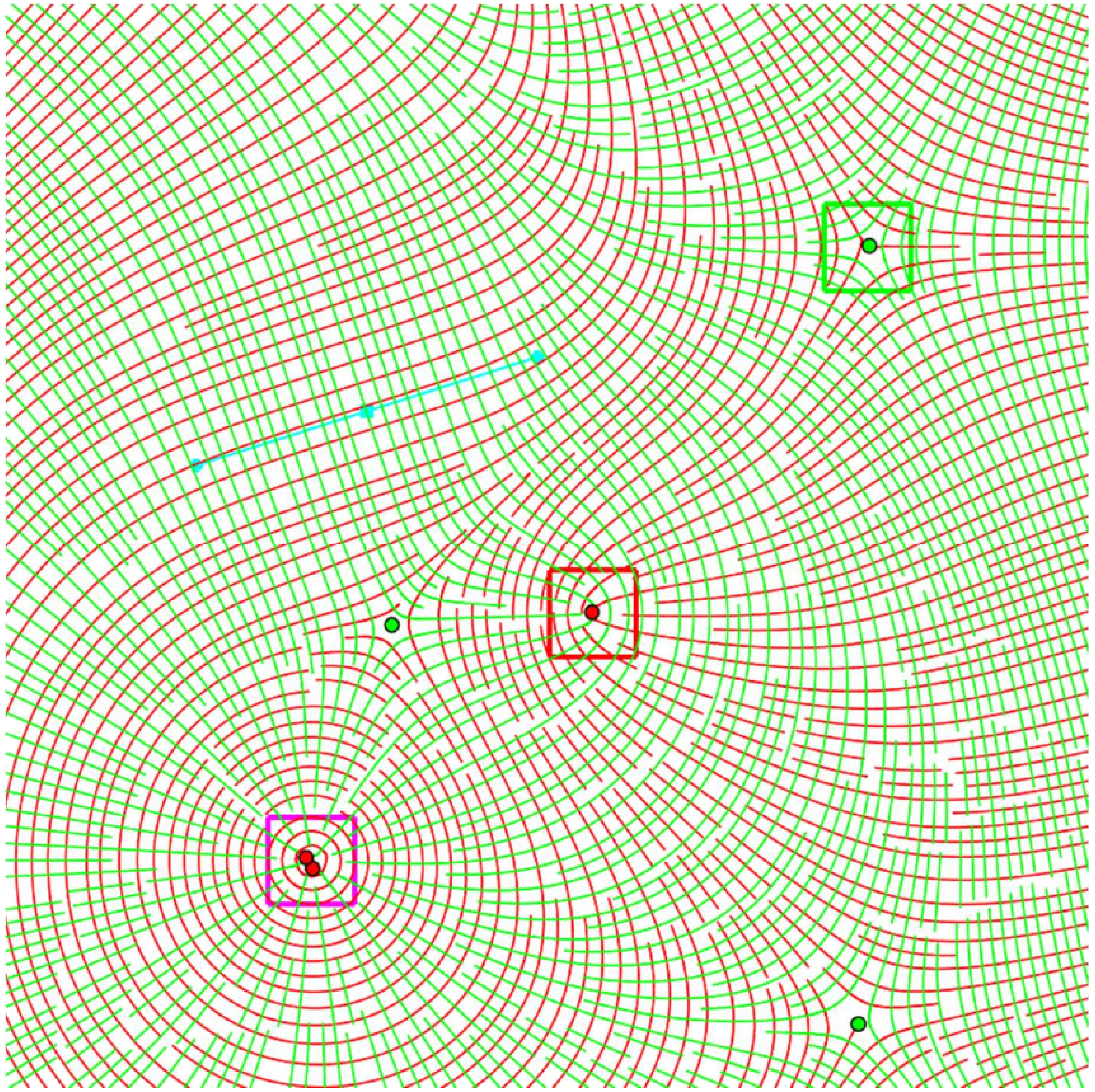
- Other than that...
  - Same integration as for vector fields
- Recap
  - Independent computation for minor/major



[Chen]

Hyper-streamlines

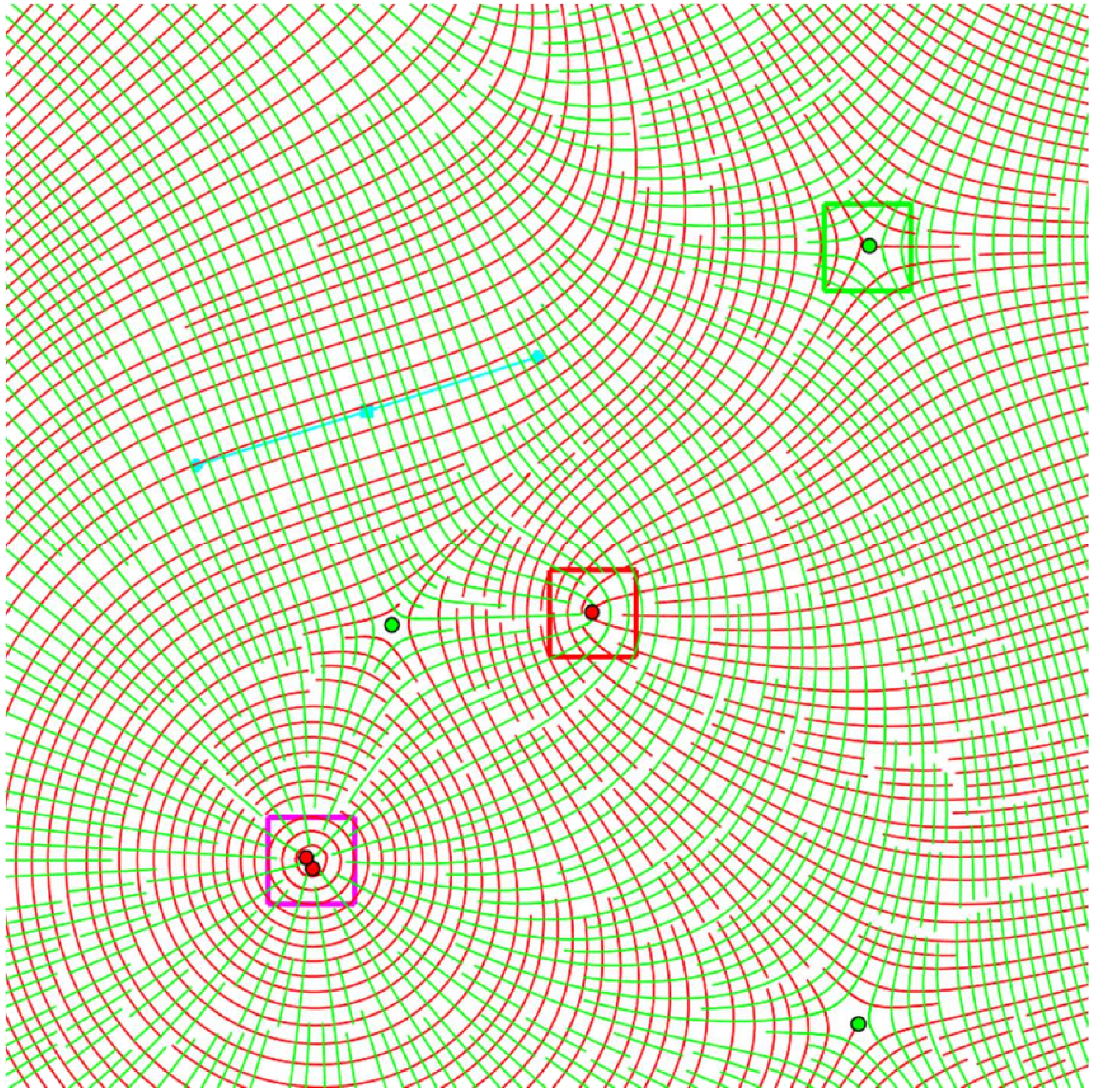
- Other than that...
  - Same integration as for vector fields
- Recap
  - Independent computation for minor/major
  - Seeding
    - Distance criterion



[Chen]

Hyper-streamlines

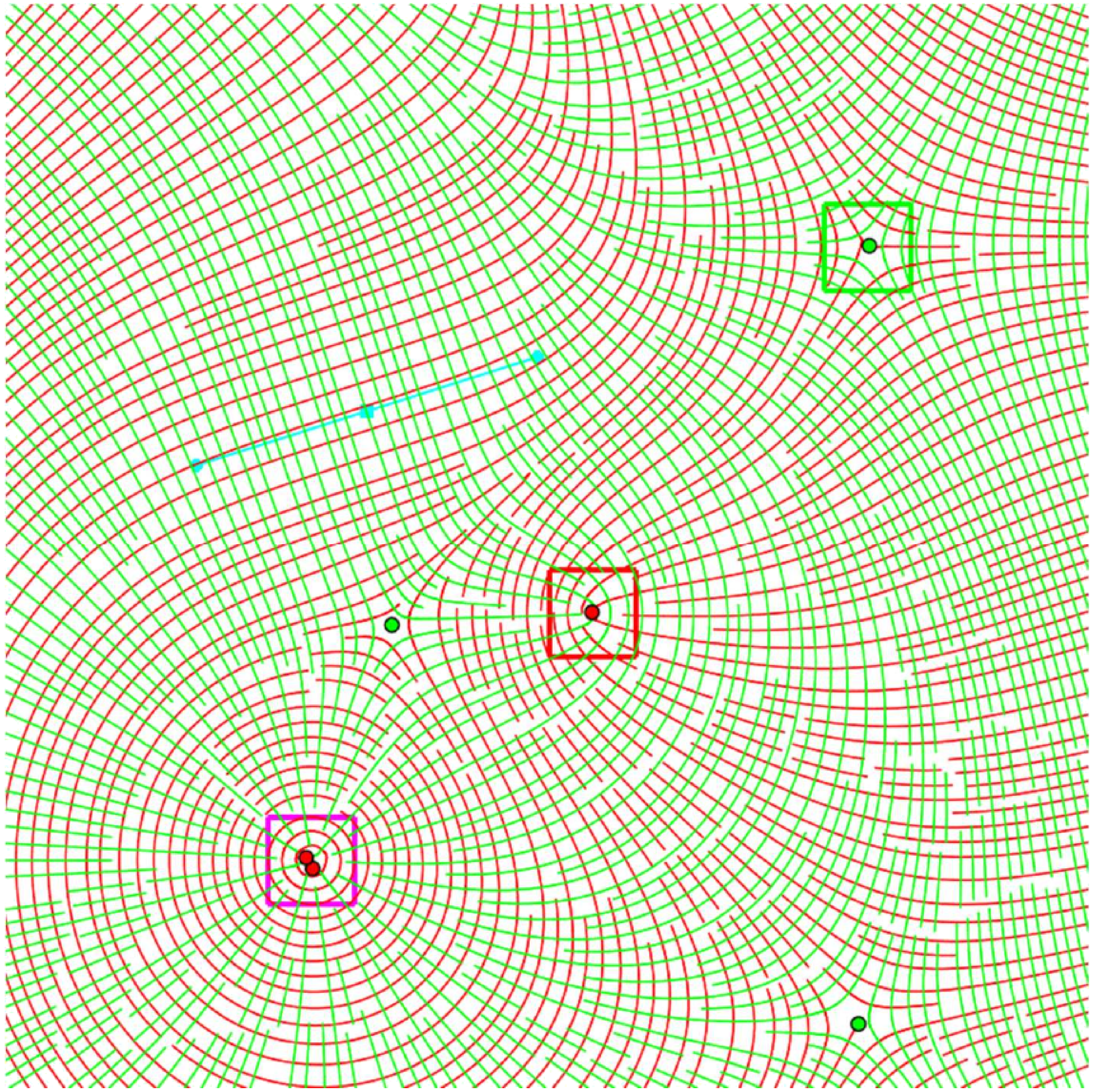
- Other than that...
  - Same integration as for vector fields
- Recap
  - Independent computation for minor/major
  - Seeding
    - Distance criterion
  - Integration
    - Euler
    - Runge-Kutta



[Chen]

Beyond hyper-streamlines

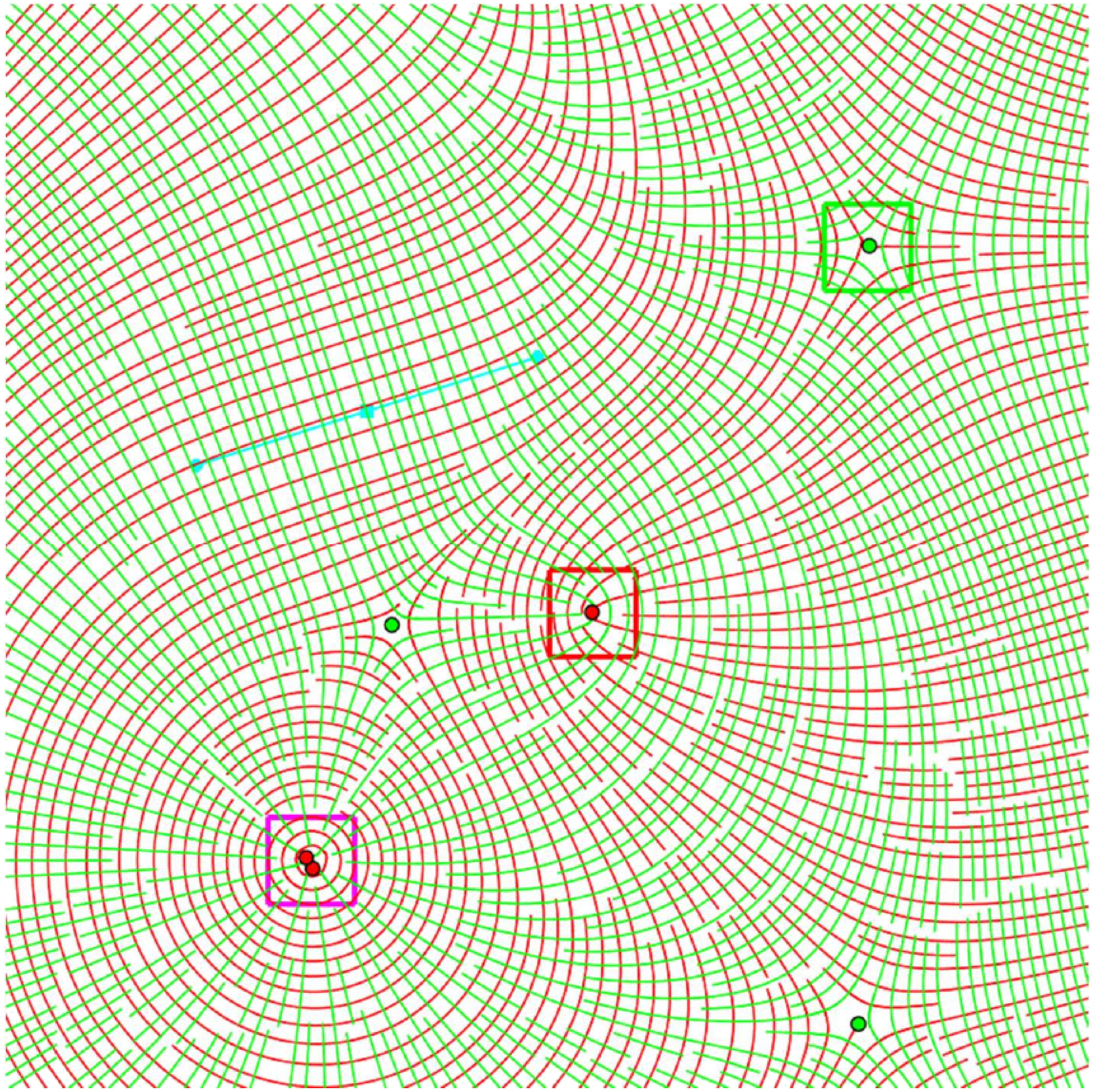
- Now that we know how to extract hyper-streamlines



[Chen]

Beyond hyper-streamlines

- Now that we know how to extract hyper-streamlines
  - More global visualization

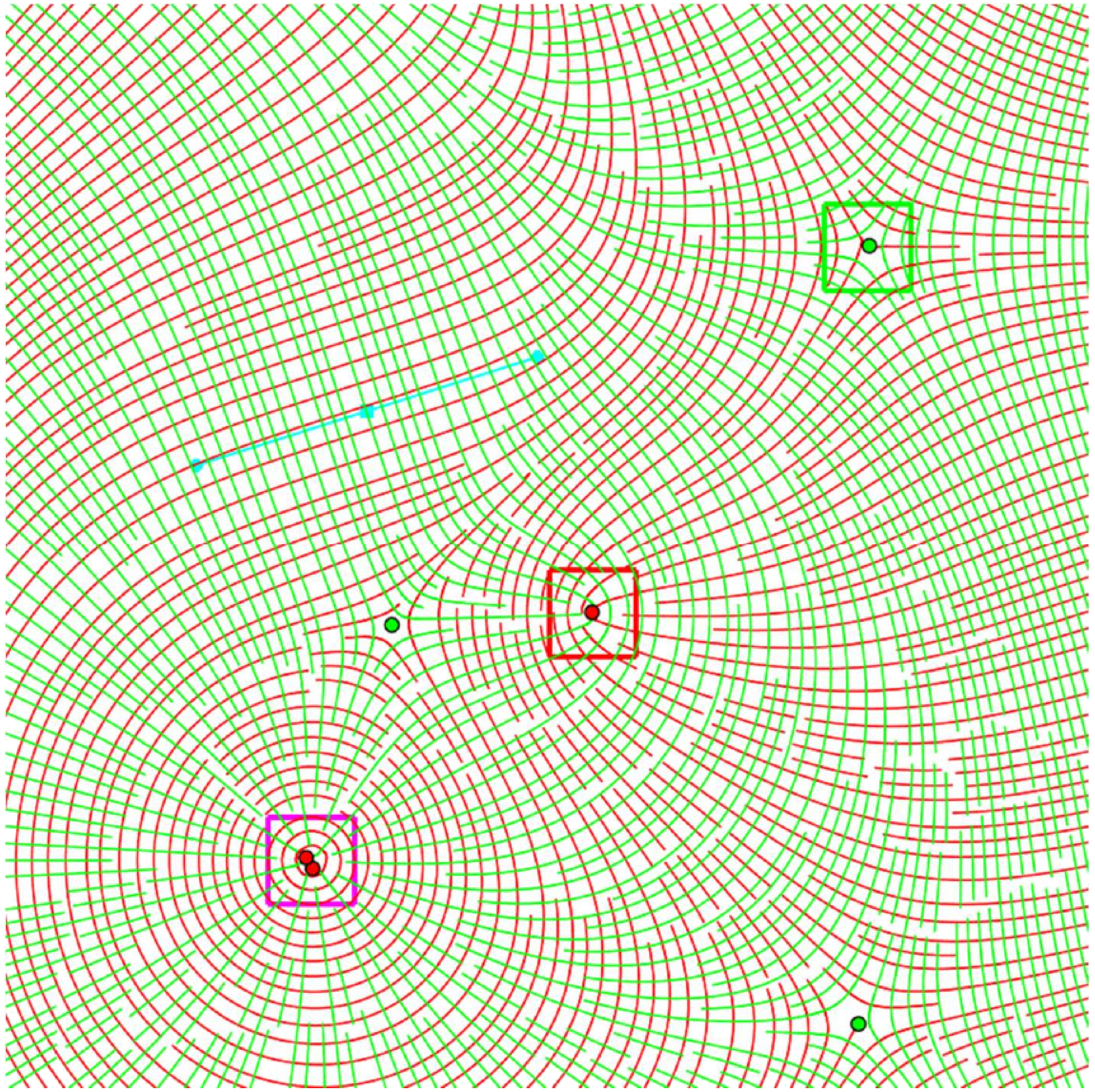


[Chen]

Beyond hyper-streamlines

- Now that we know how to extract hyper-streamlines
  - More global visualization

- Getting inspiration from vector fields

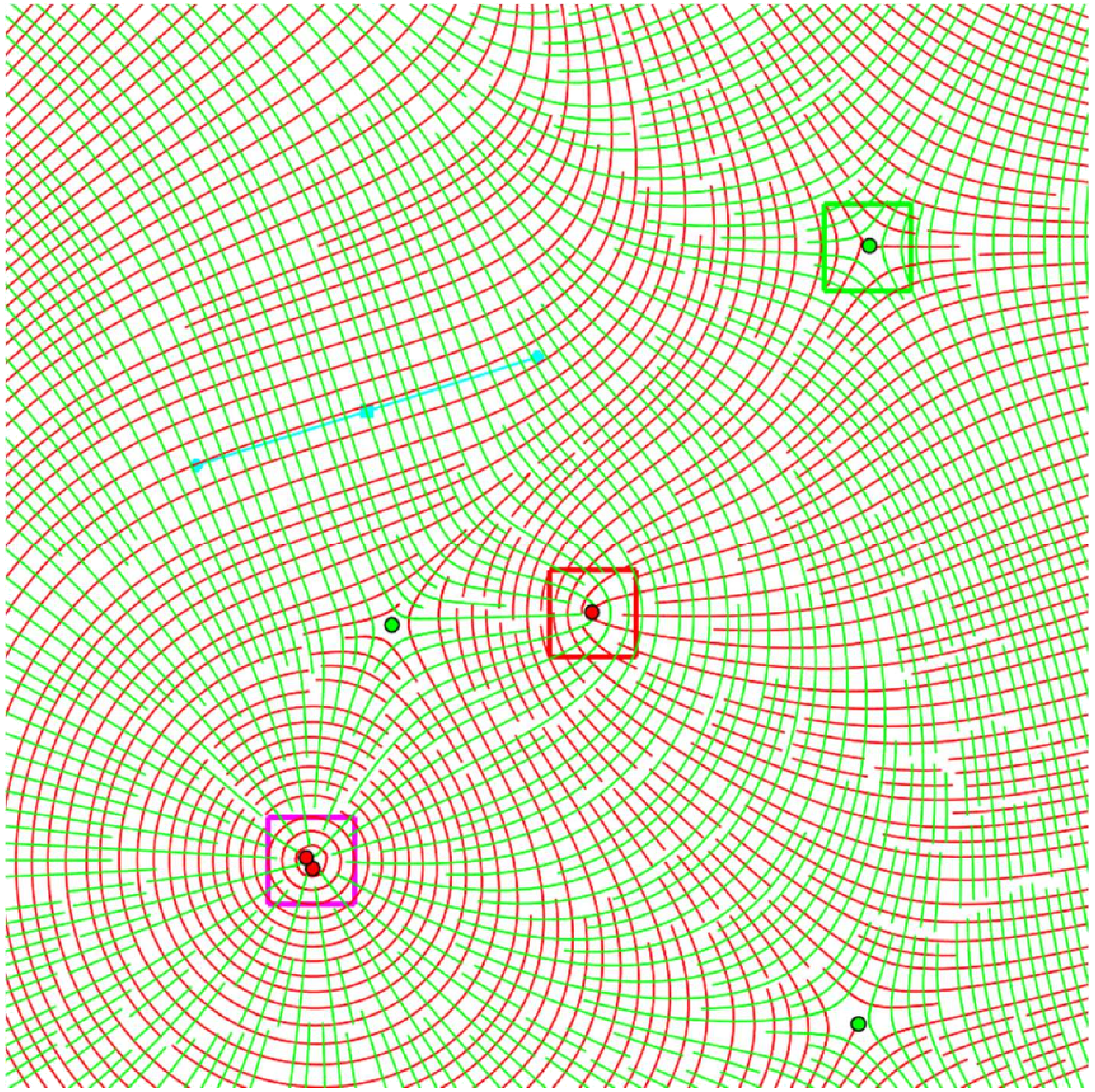


[Chen]

Beyond hyper-streamlines

- Now that we know how to extract hyper-streamlines
  - More global visualization

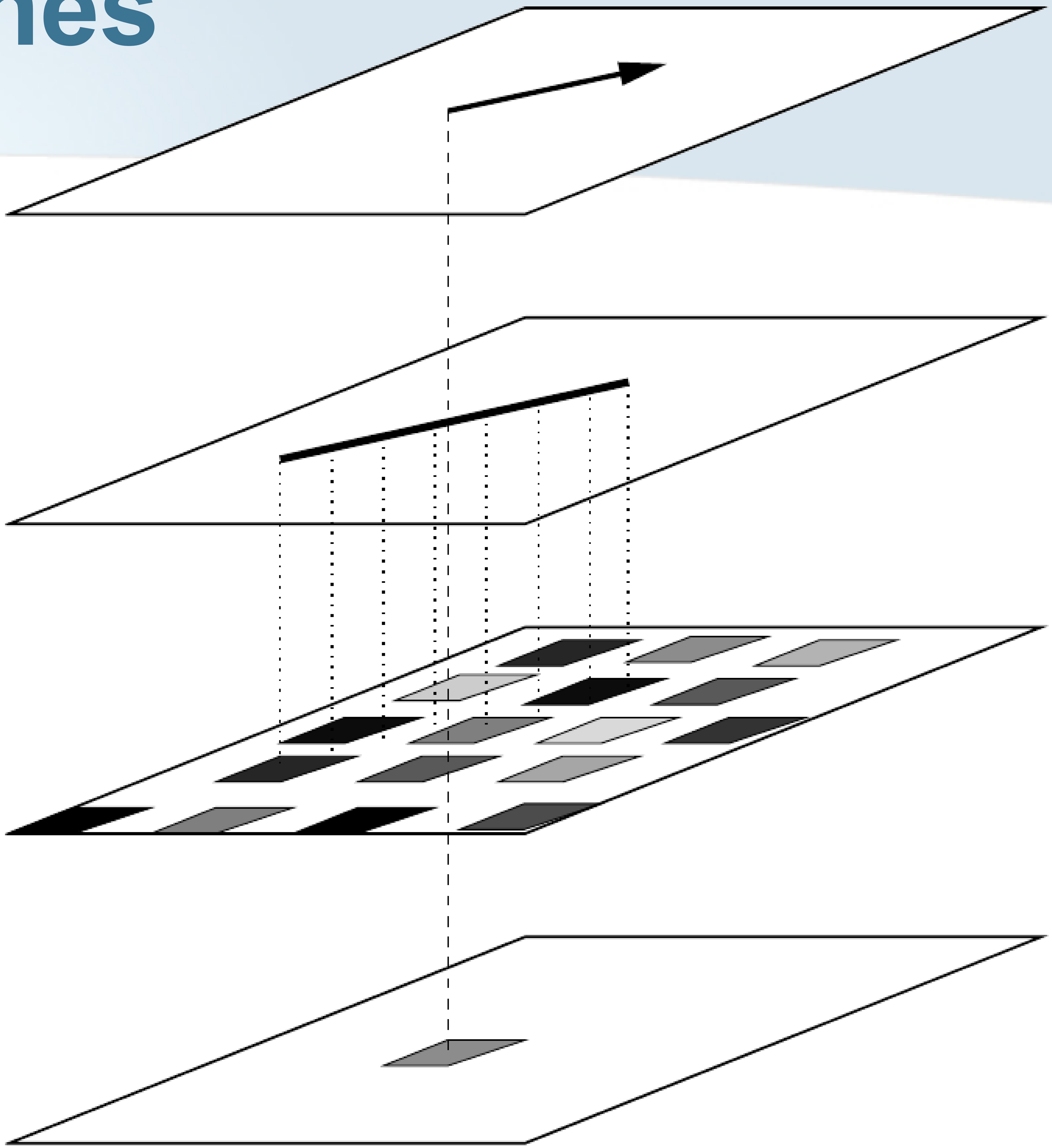
- Getting inspiration from vector fields
  - Line Integral Convolution



[Chen]

Beyond hyper-streamlines

- Now that we know how to extract hyper-streamlines
  - More global visualization
- Getting inspiration from vector fields
  - Line Integral Convolution

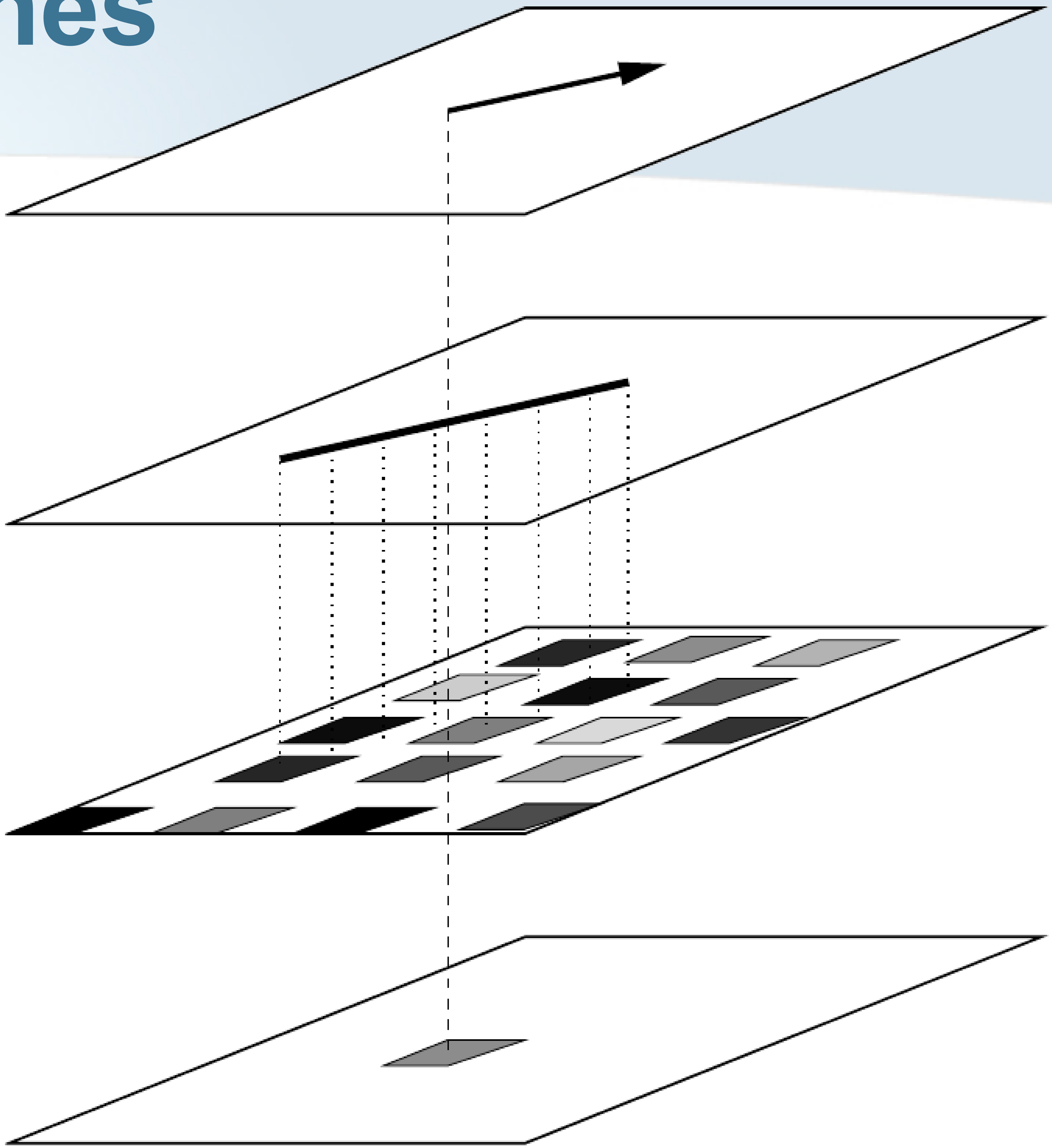


[Helgeland]

Beyond hyper-streamlines

- Now that we know how to extract hyper-streamlines
  - More global visualization

- Getting inspiration from vector fields
  - Line Integral Convolution
    - d-dimensional convolution kernel

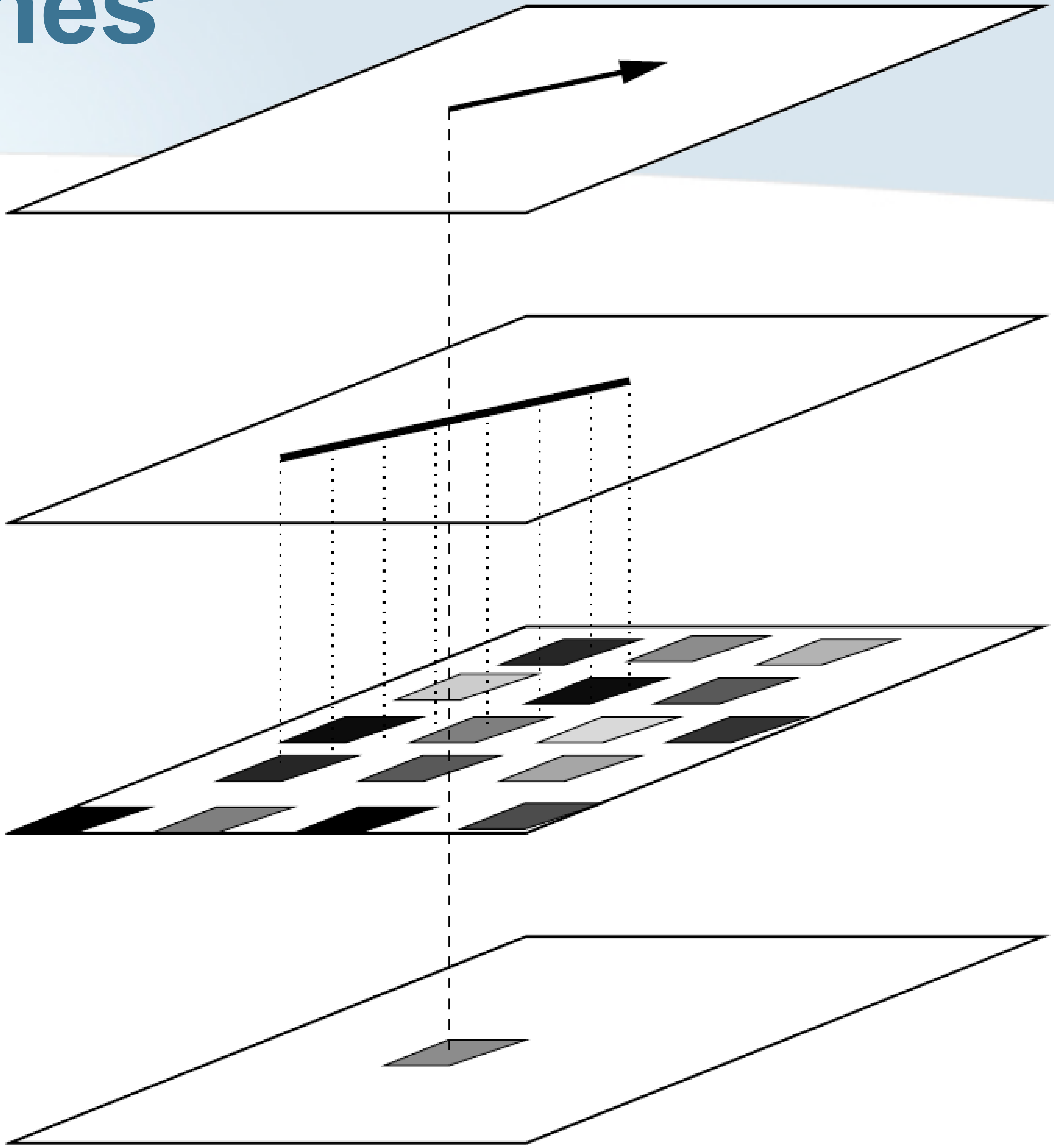


[Helgeland]

Beyond hyper-streamlines

- Now that we know how to extract hyper-streamlines
  - More global visualization

- Getting inspiration from vector fields
  - Line Integral Convolution
    - d-dimensional convolution kernel
    - Independent computations

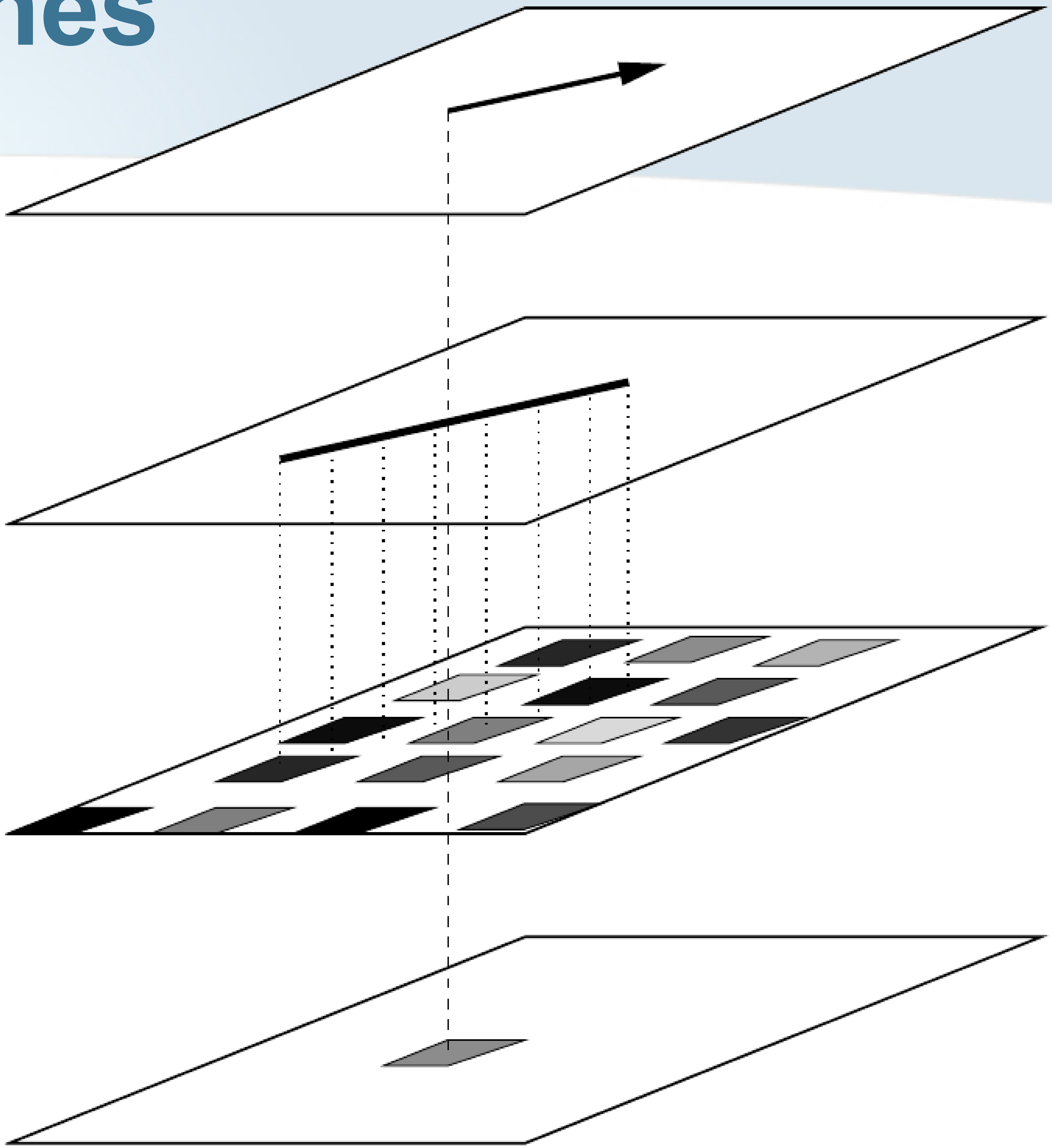


[Helgeland]

Beyond hyper-streamlines

- Now that we know how to extract hyper-streamlines
  - More global visualization

- Getting inspiration from vector fields
  - Line Integral Convolution
    - d-dimensional convolution kernel
    - Independent computations
      – Random blend minor/major



[Helgeland]

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    - d-dimensional convolution kernel
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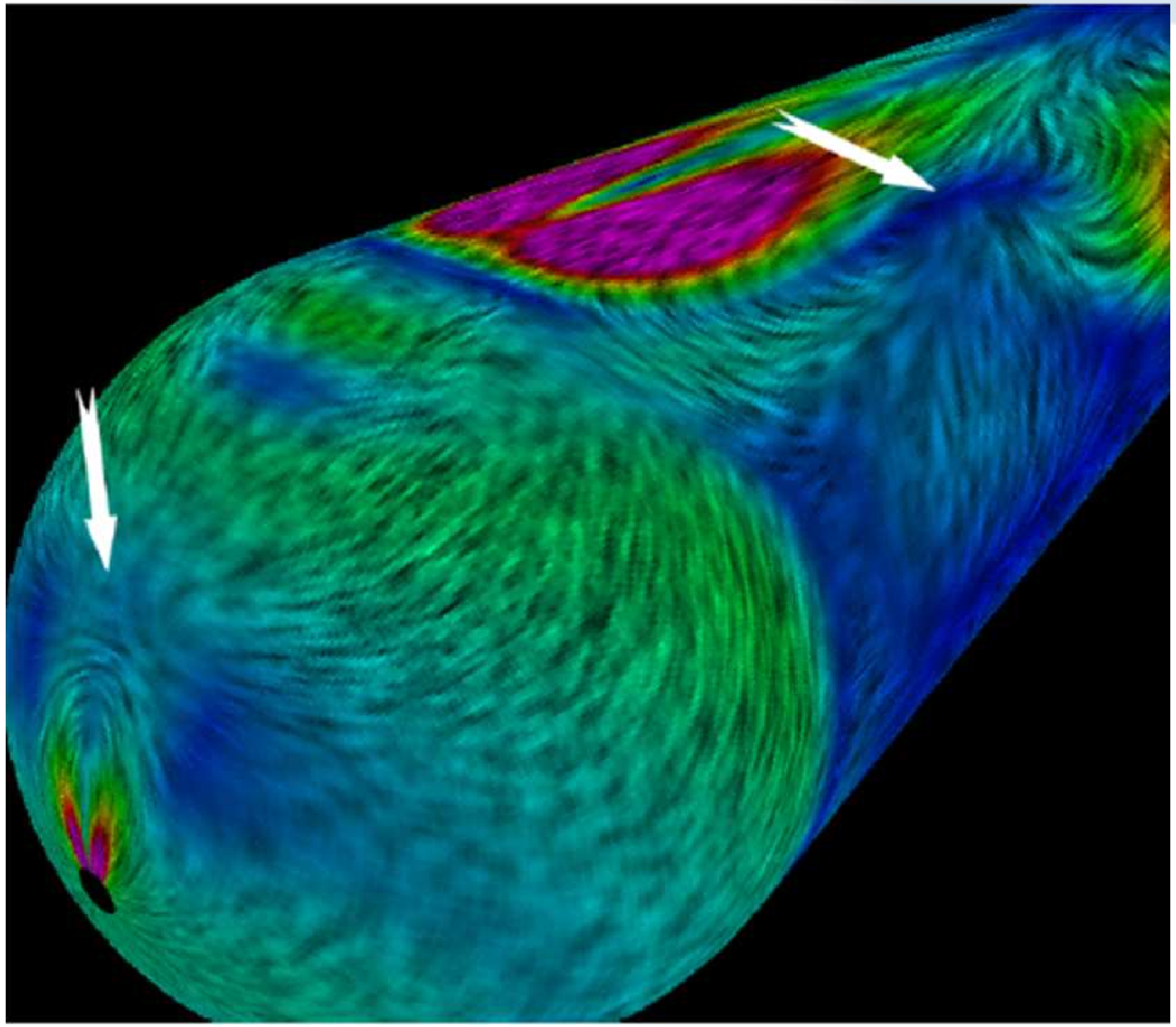

[Chen]

Beyond hyper-streamlines

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    - d-dimensional convolution kernel
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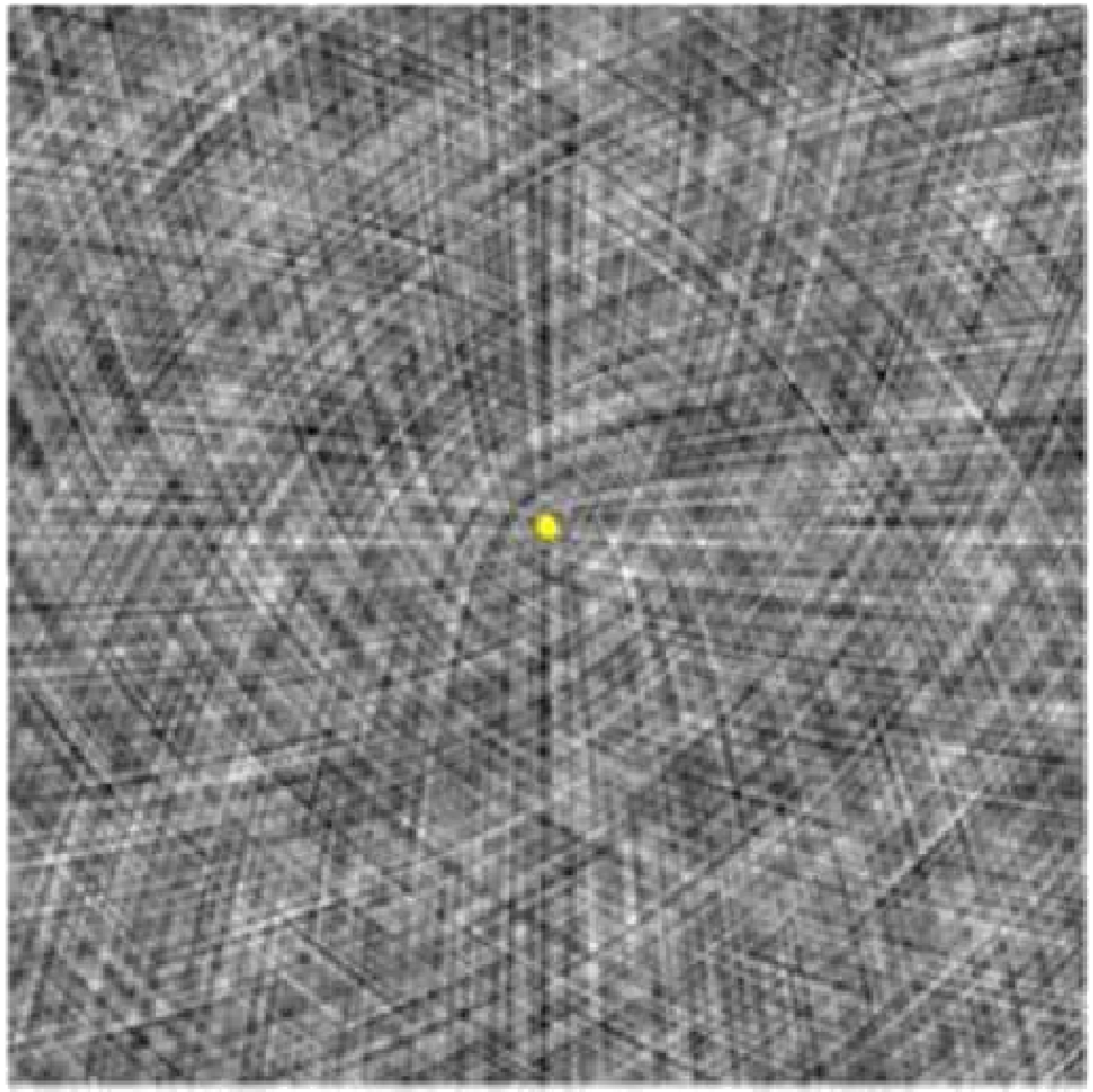
      – Random blend minor/major



[Zhang]

Beyond hyper-streamlines

- Now that we know how to extract hyper-streamlines
  - More global visualization

- Getting inspiration from vector fields
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    - d-dimensional convolution kernel
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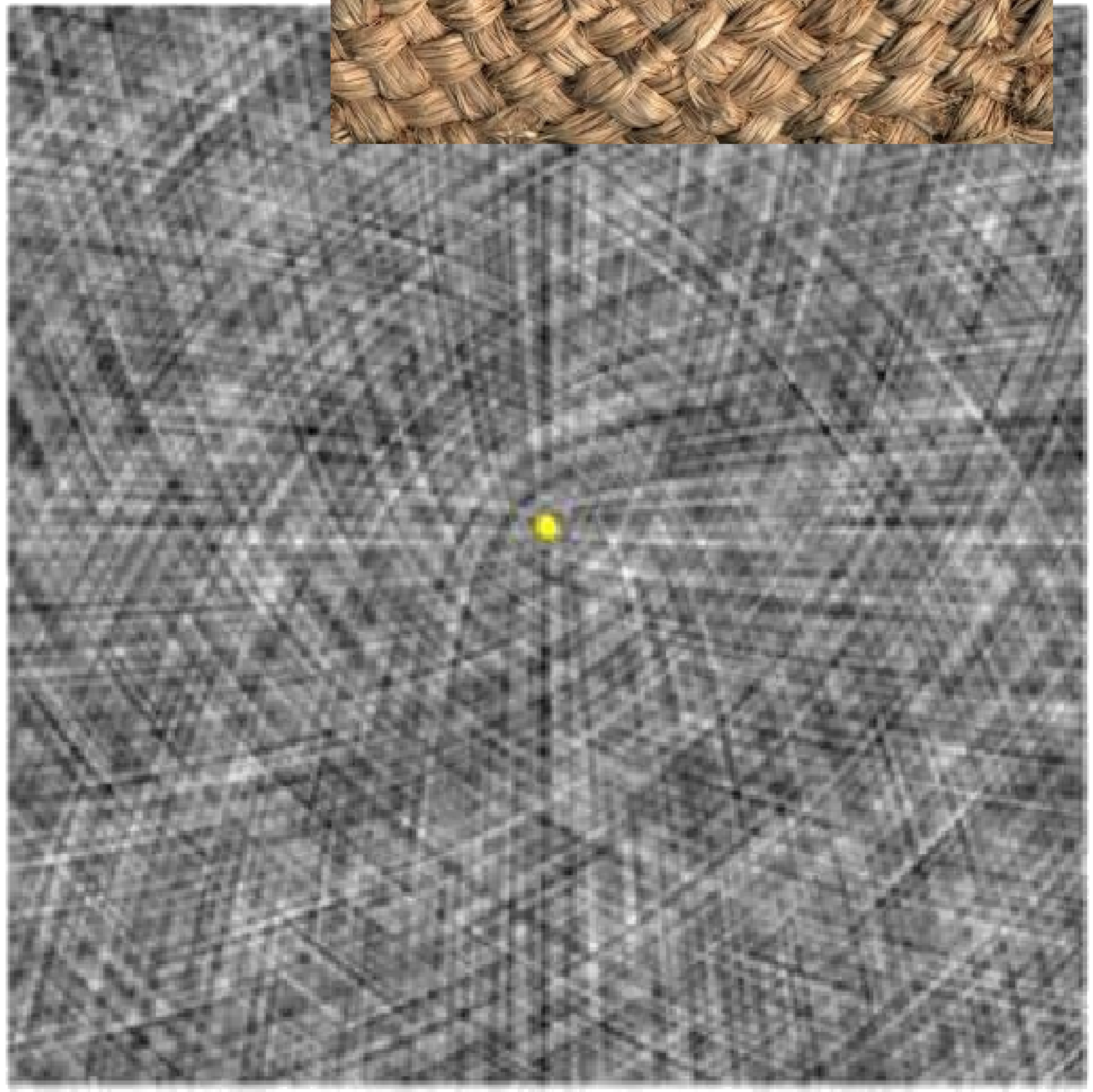
      – Random blend minor/major



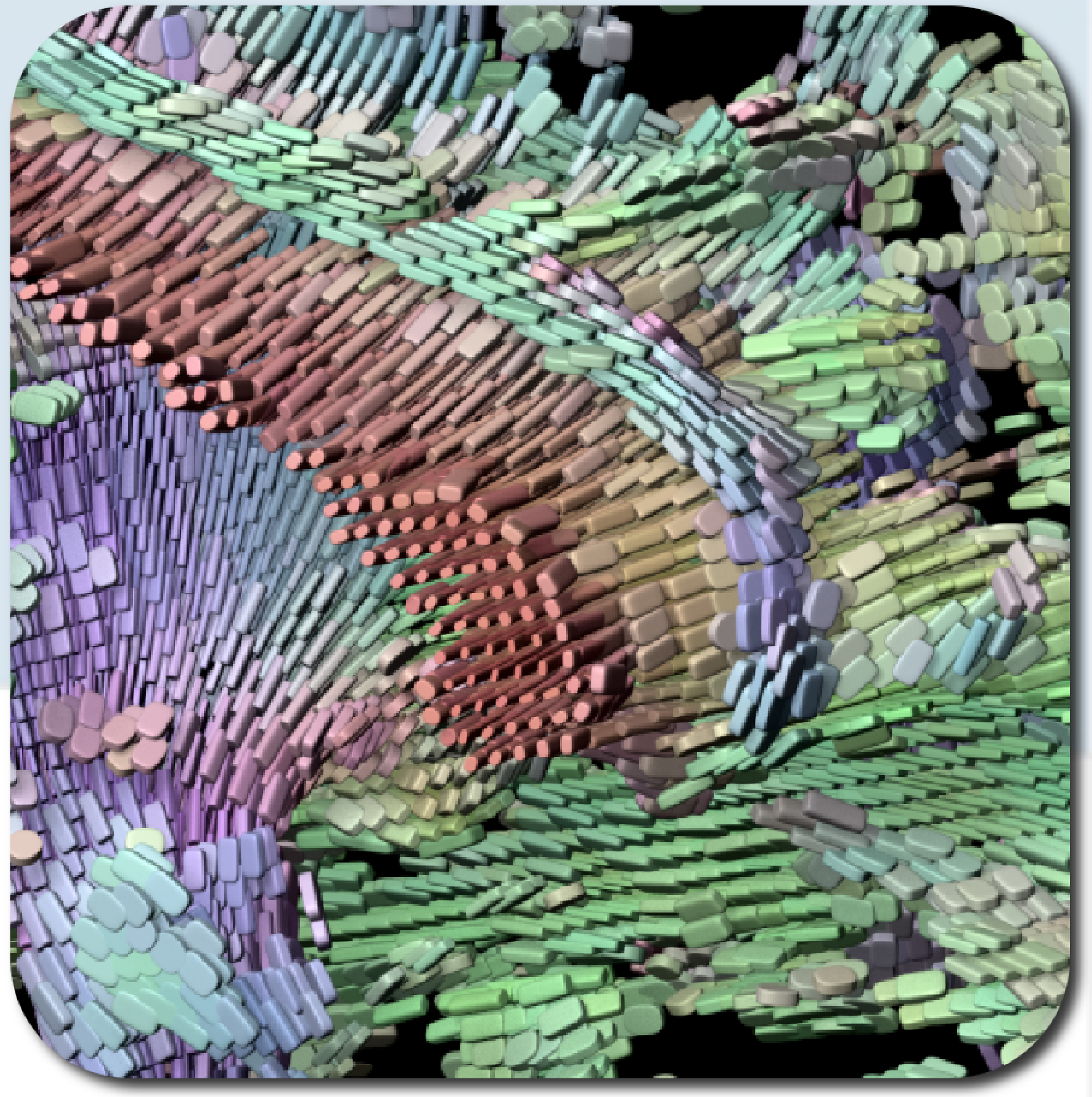
[Zhang]

So far

- We can generate

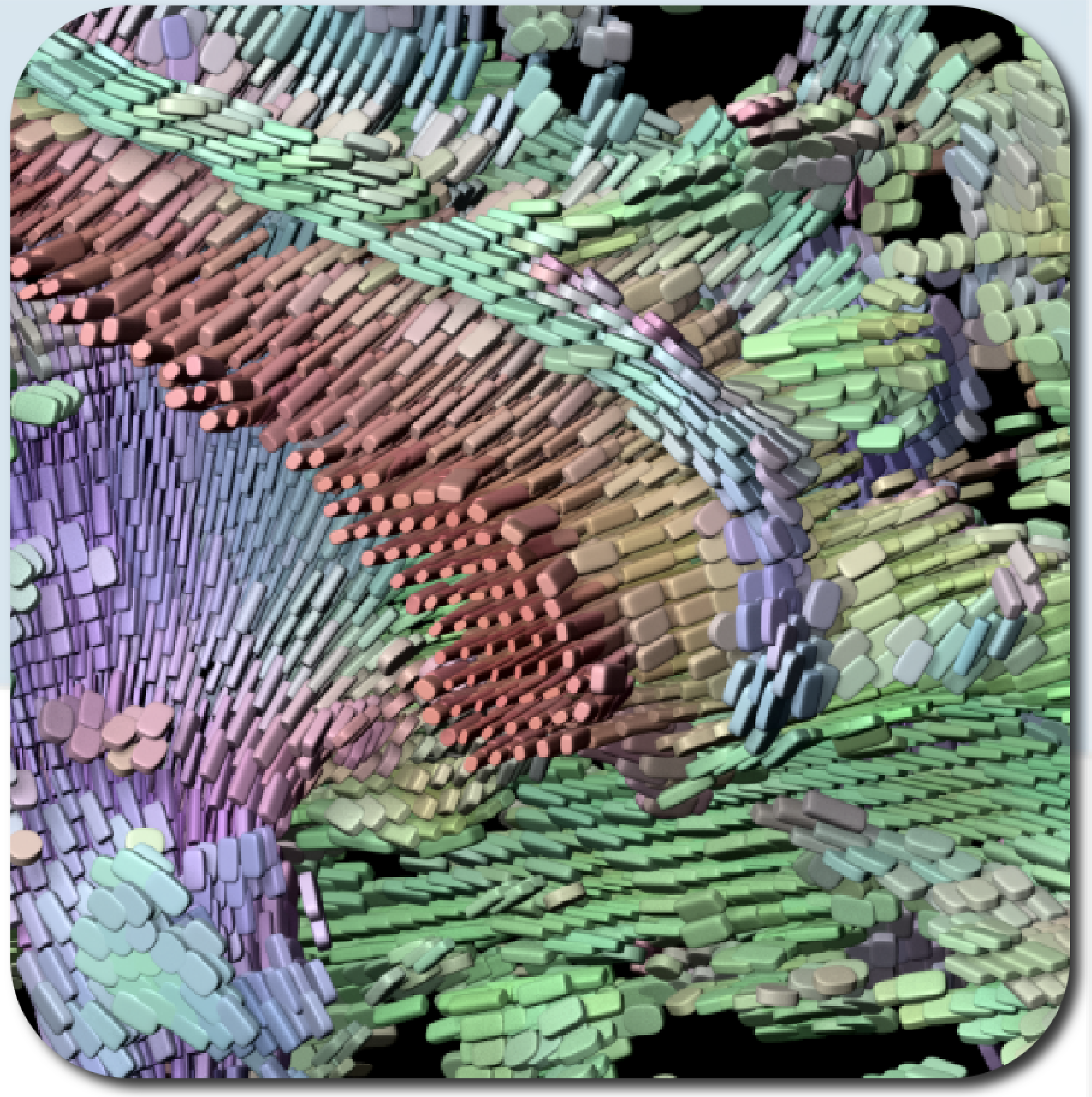
So far

- We can generate
  - Direct visualization
    - Glyph packing



[Kindlmann]

So far

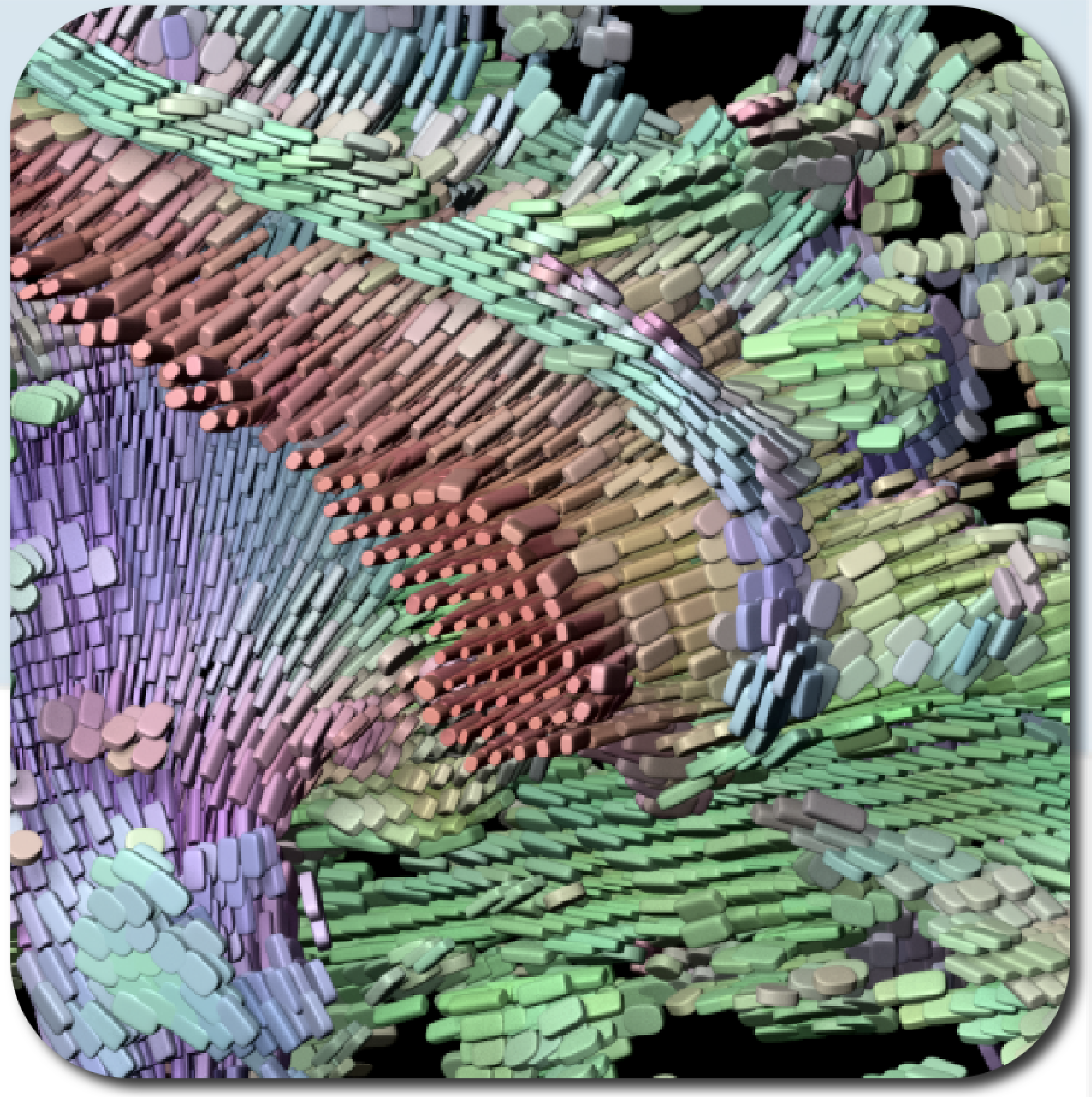


[Kindlmann]

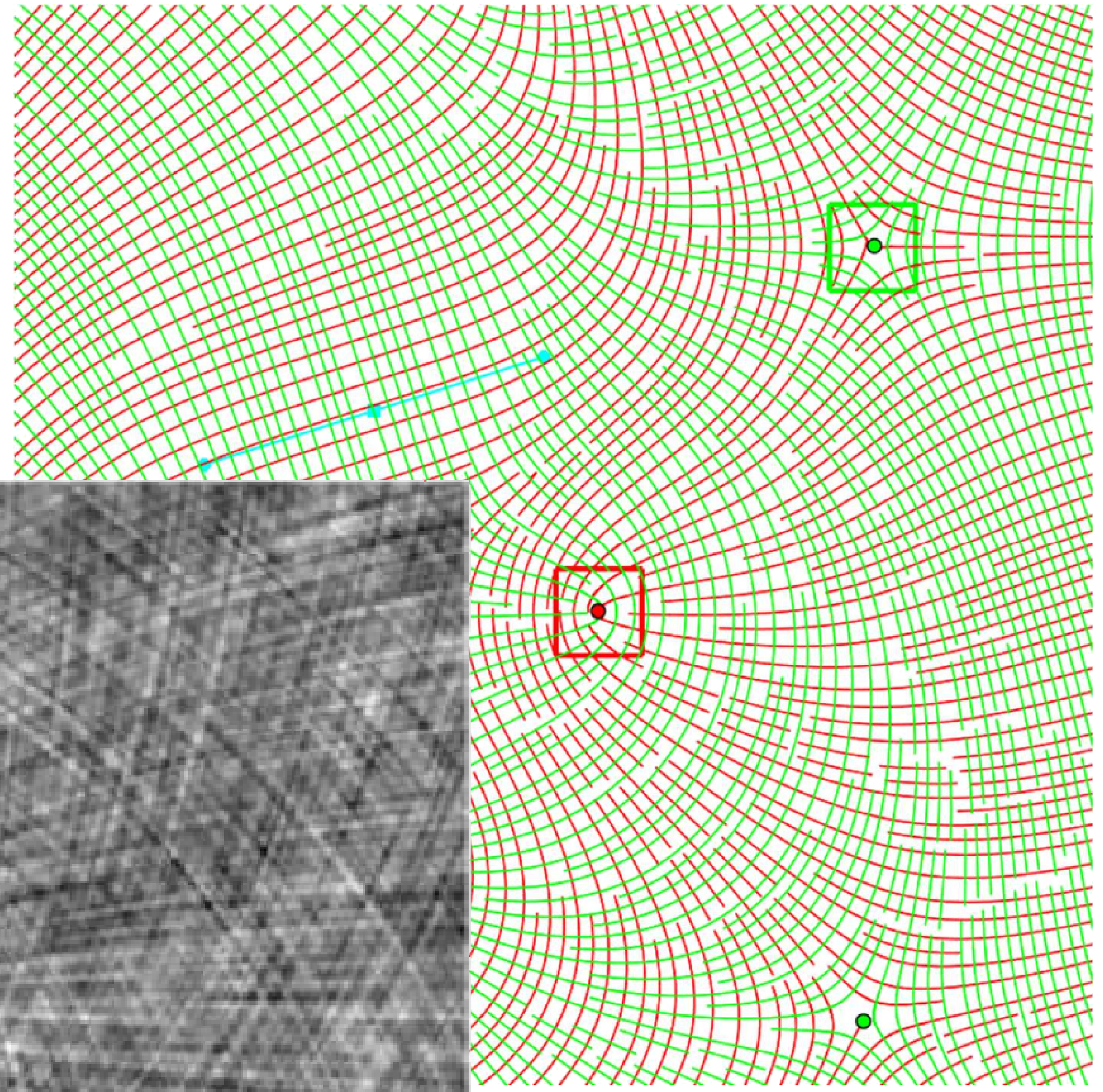
- We can generate
  - Direct visualization
    - Glyph packing
  - Geometrical measures
    - Eigenvalues, traces, etc.

So far

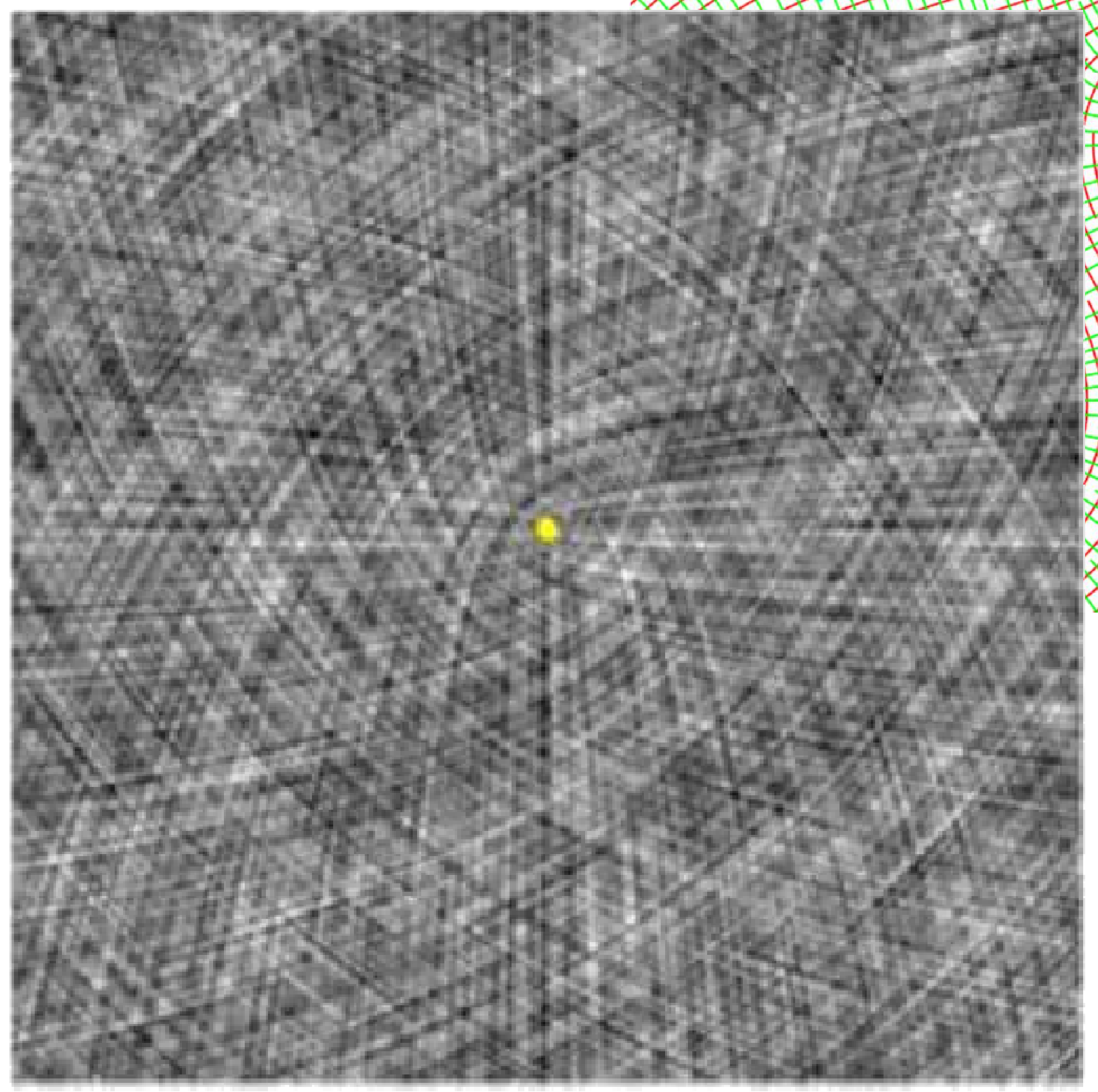
- We can generate
  - Direct visualization
    - Glyph packing
  - Geometrical measures
    - Eigenvalues, traces, etc.
  - Geometrical structures
    - Hyper-streamlines, LIC



[Kindlmann]



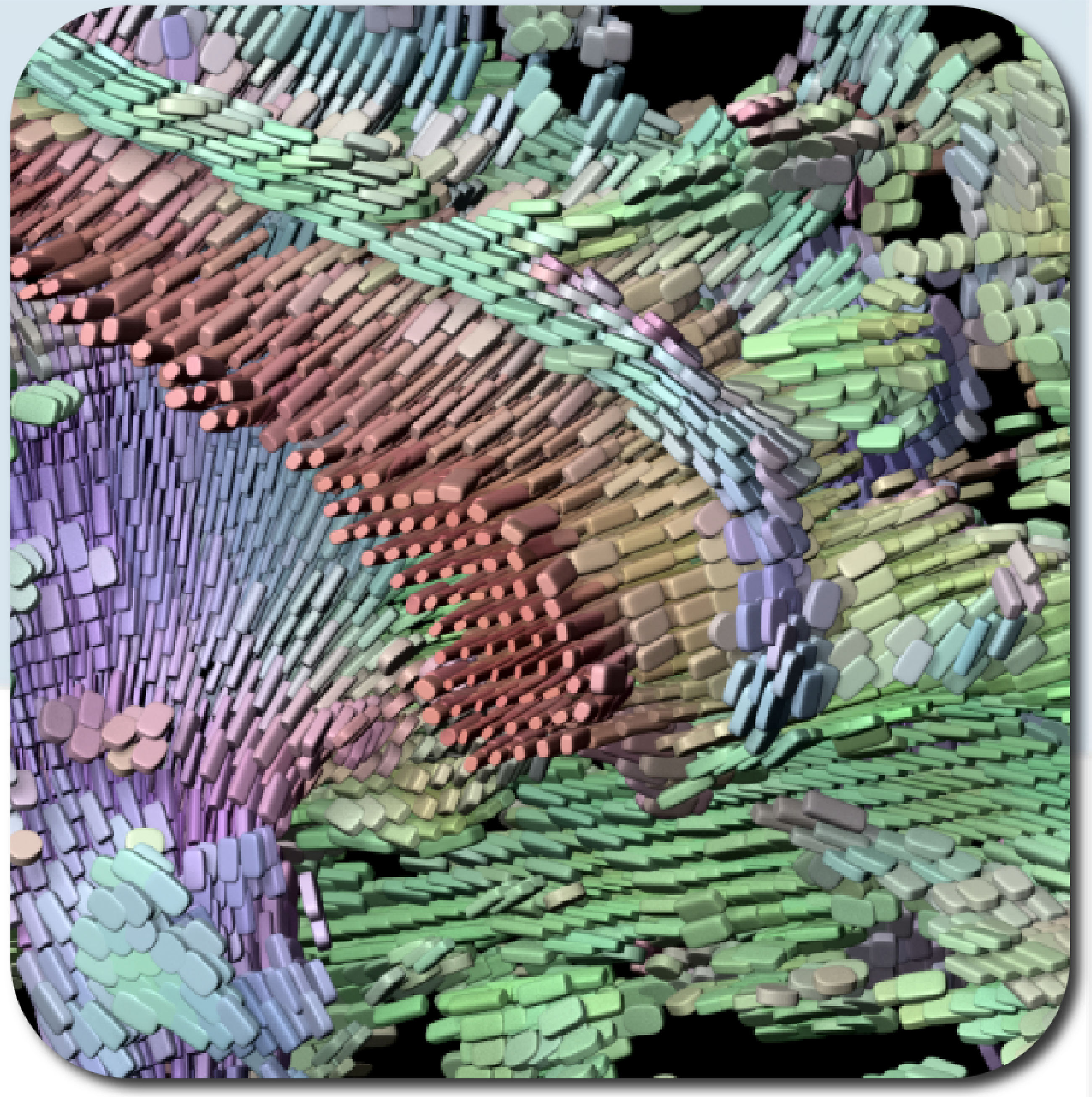
[Chen]



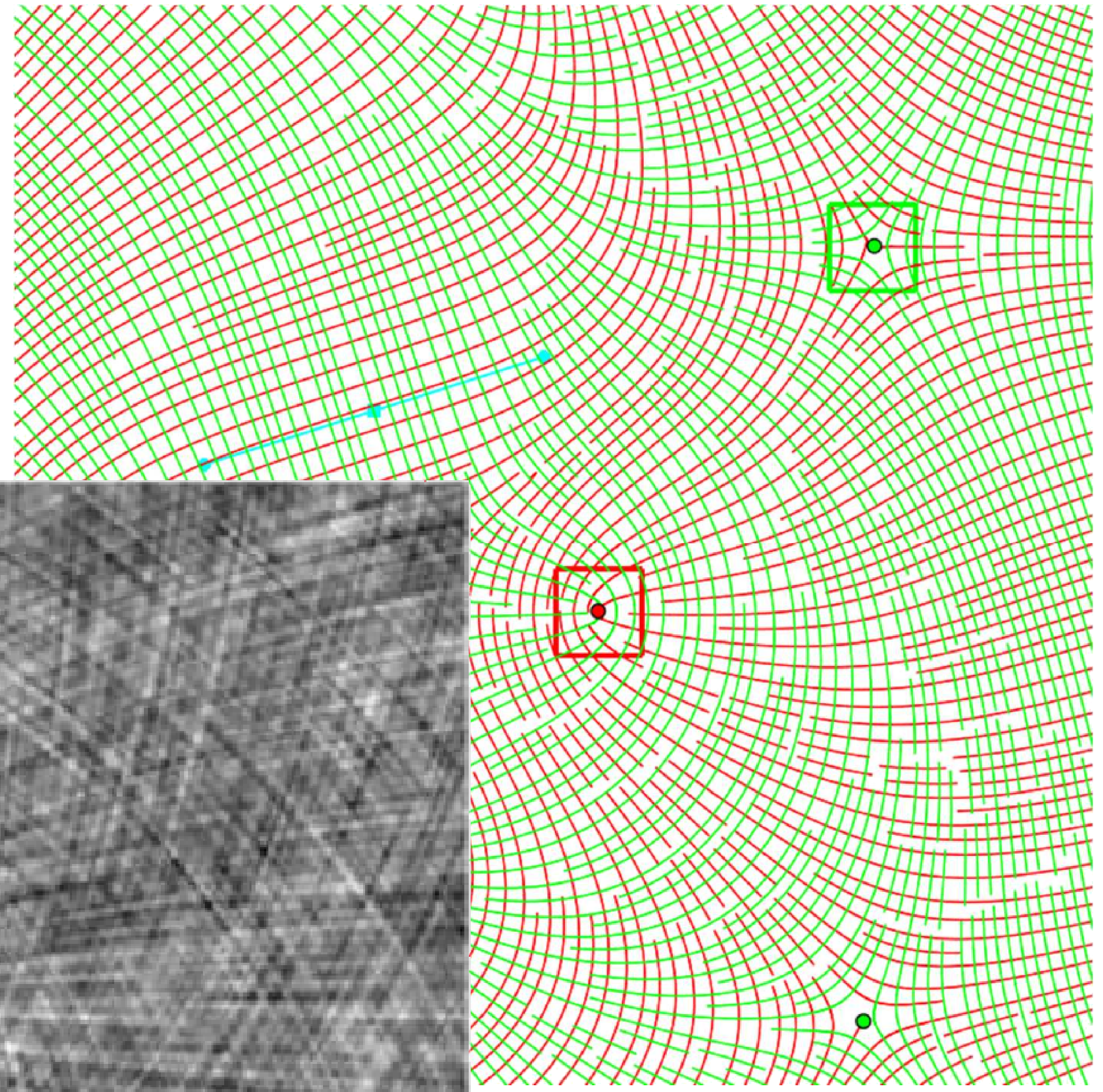
[Zhang]

So far

- We can generate
  - Direct visualization
    - Glyph packing
  - Geometrical measures
    - Eigenvalues, traces, etc.
  - Geometrical structures
    - Hyper-streamlines, LIC
- How to put it all together?



[Kindlmann]



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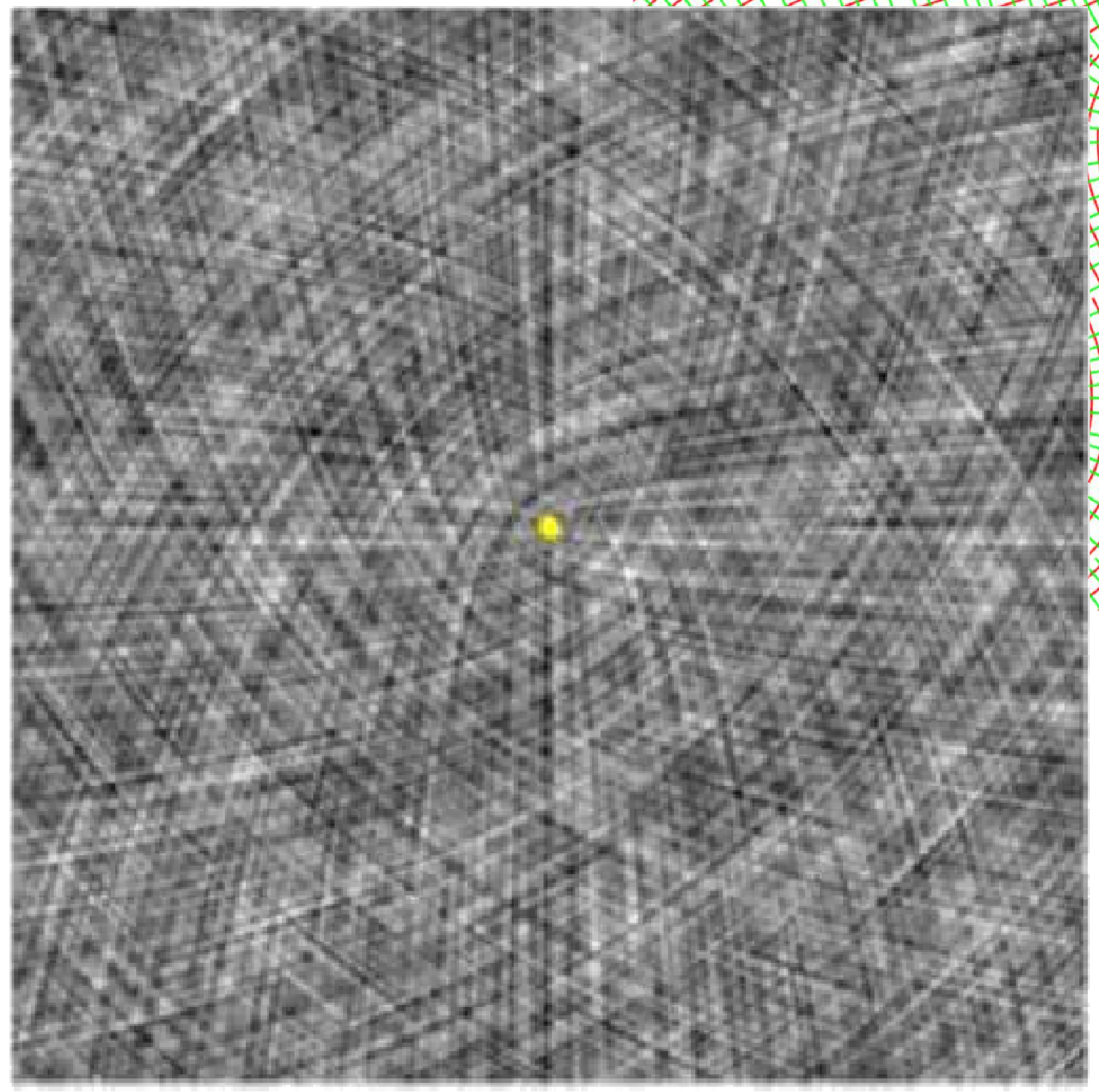
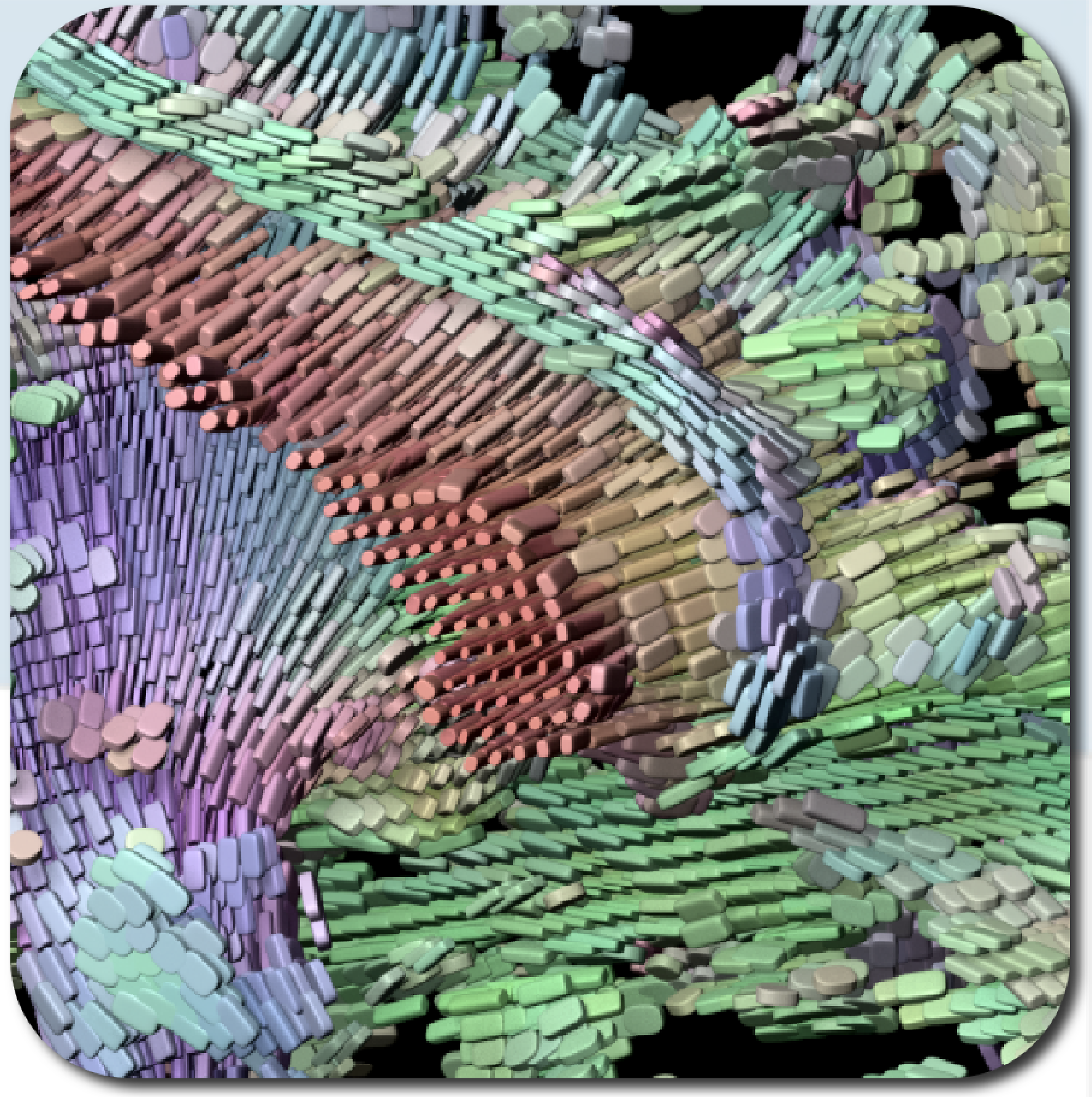


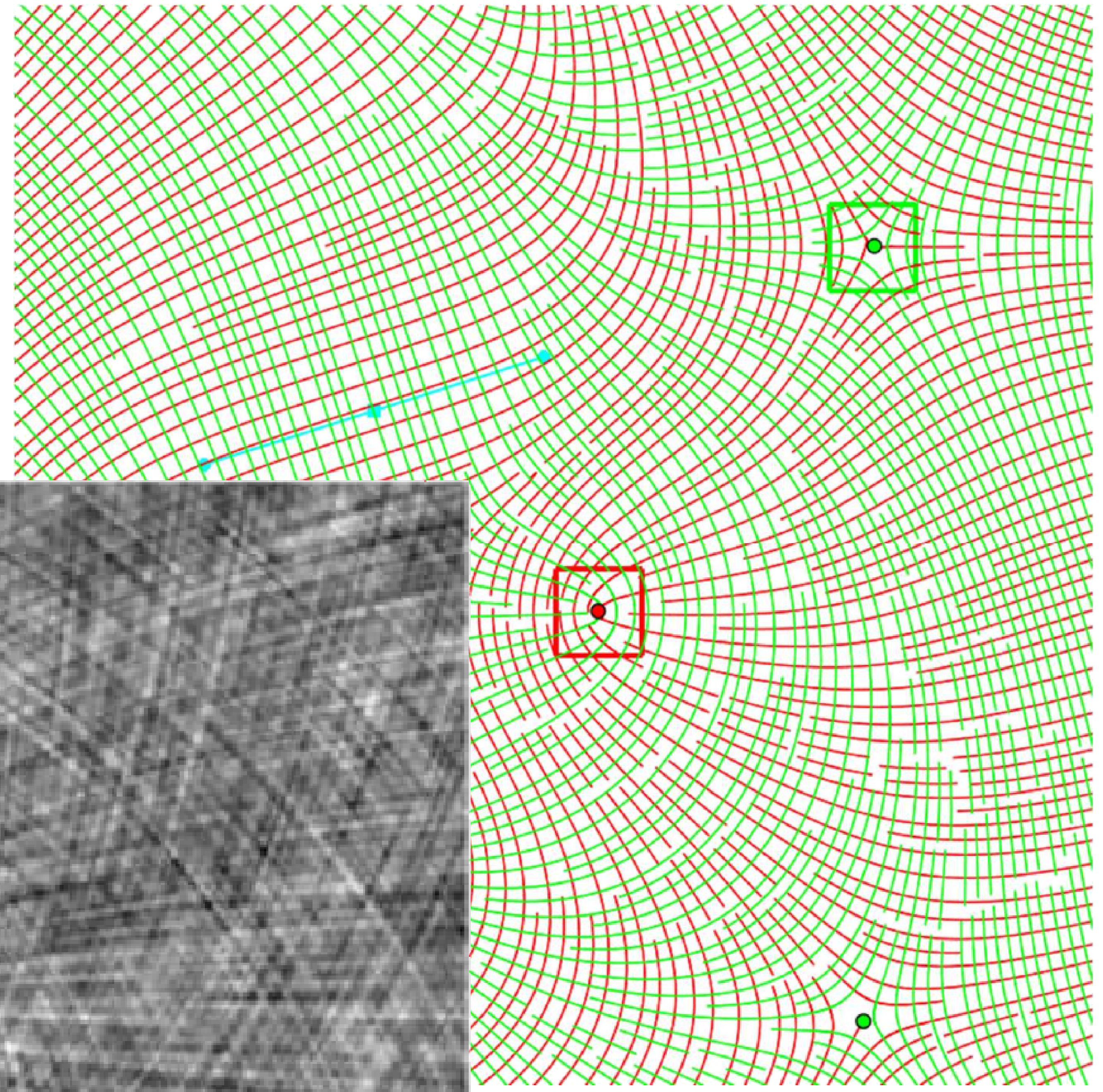
[Zhang]

So far

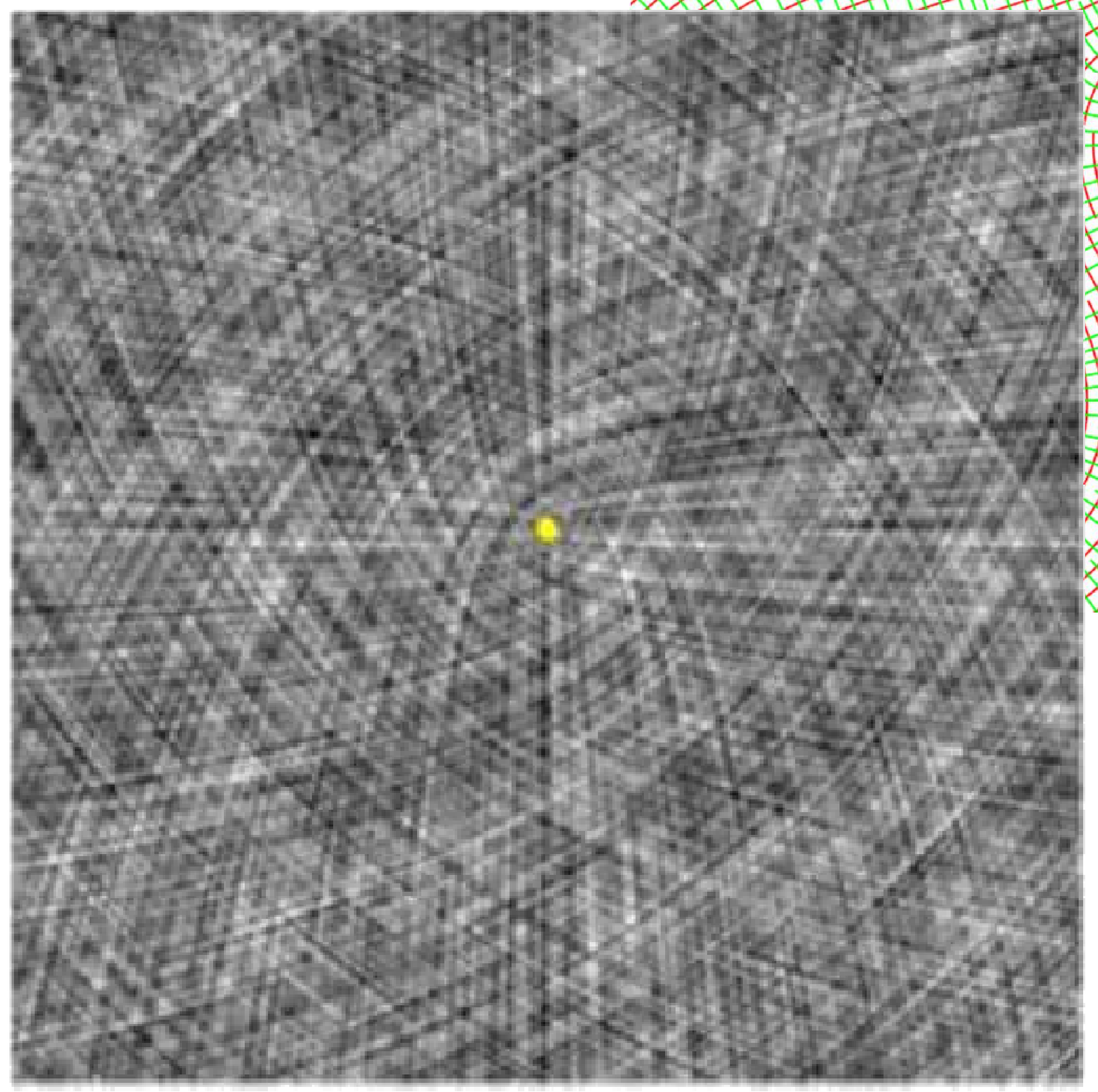
- We can generate
  - Direct visualization
    - Glyph packing
  - Geometrical measures
    - Eigenvalues, traces, etc.
  - Geometrical structures
    - Hyper-streamlines, LIC
- How to put it all together?
  - **Tensor field topology**



[Kindlmann]

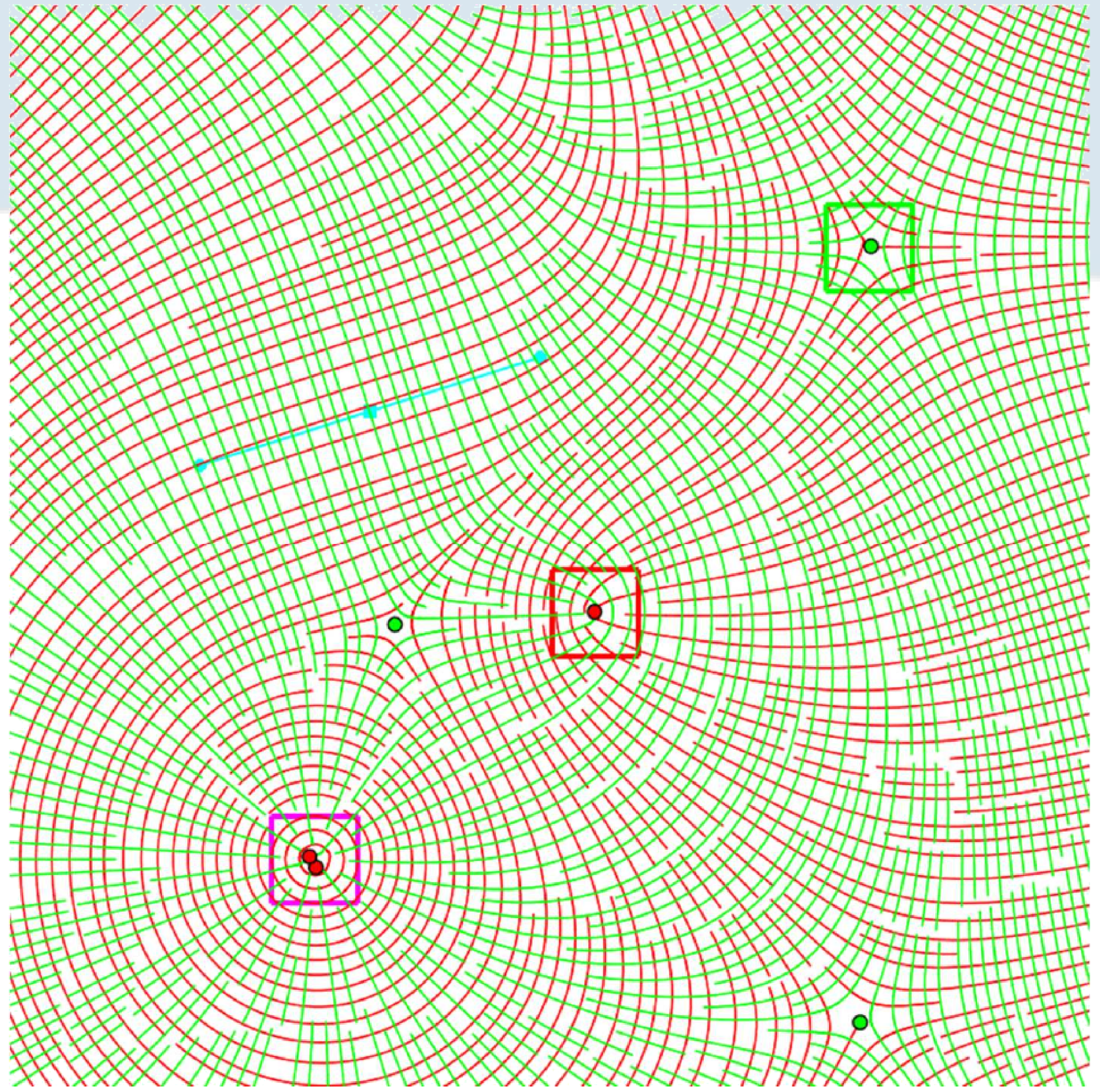


[Chen]



[Zhang]

Tensor field topology for dummies



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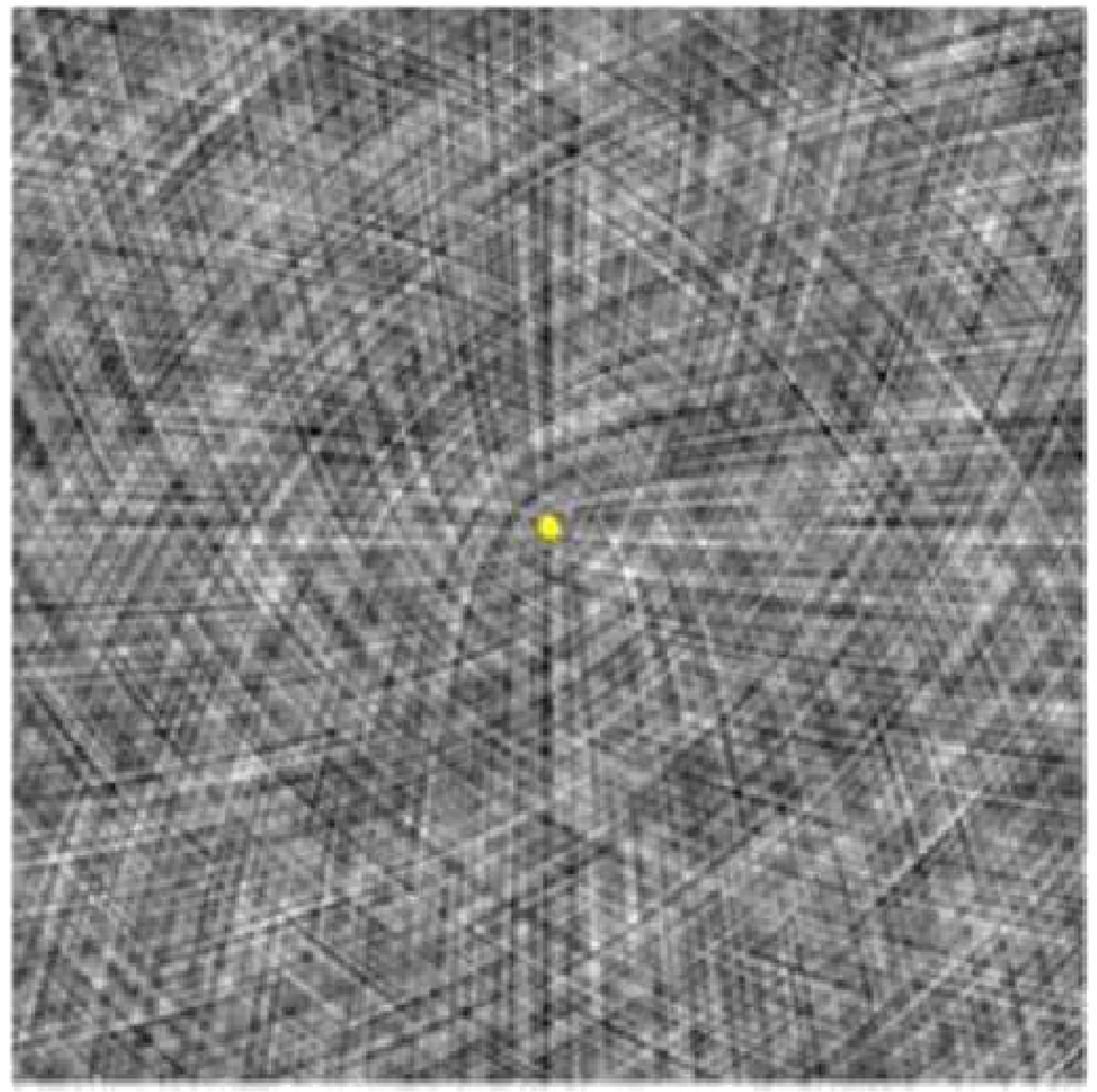
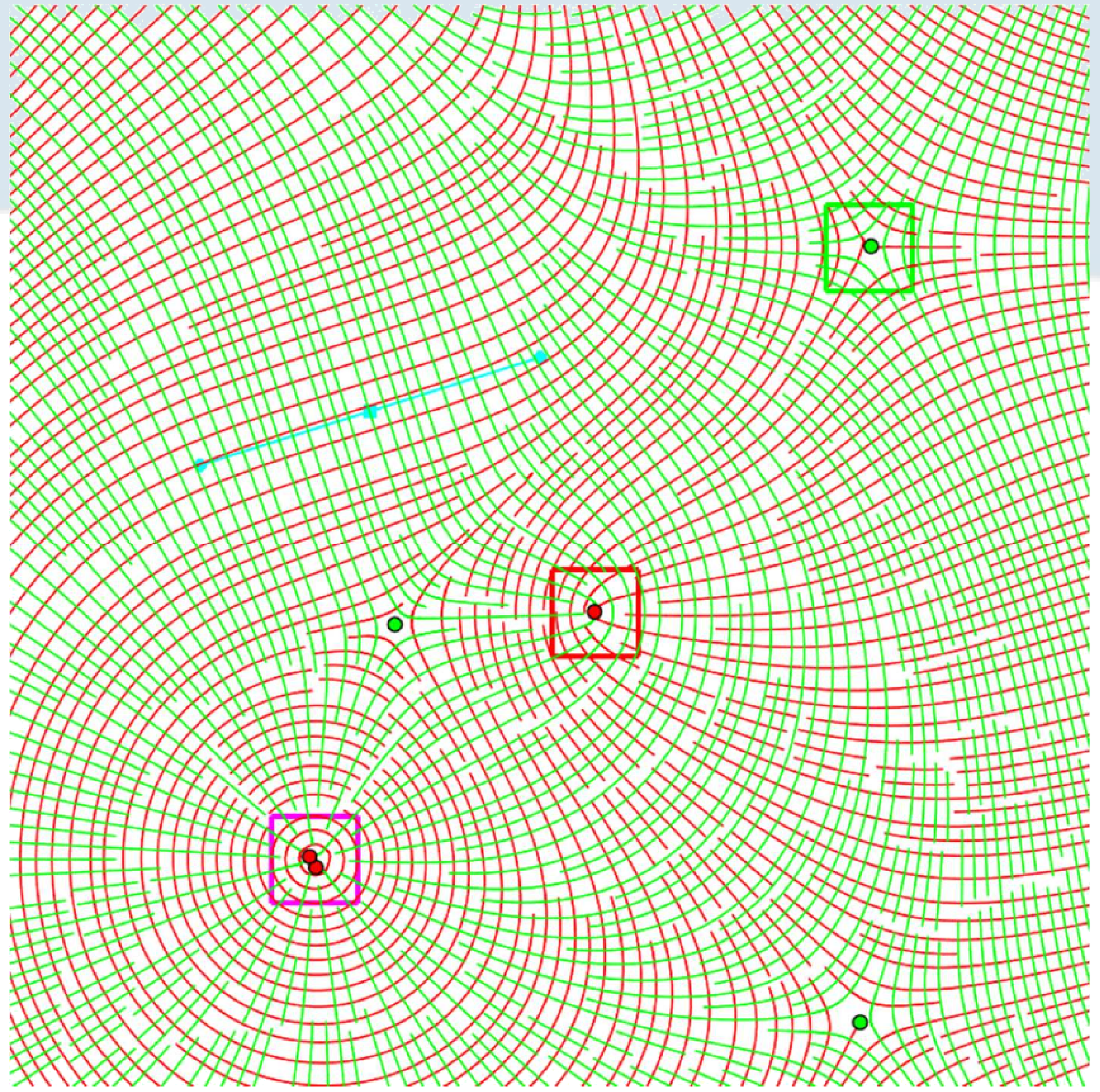


[Zhang]

Tensor field topology for dummies

- Analogy to vector fields



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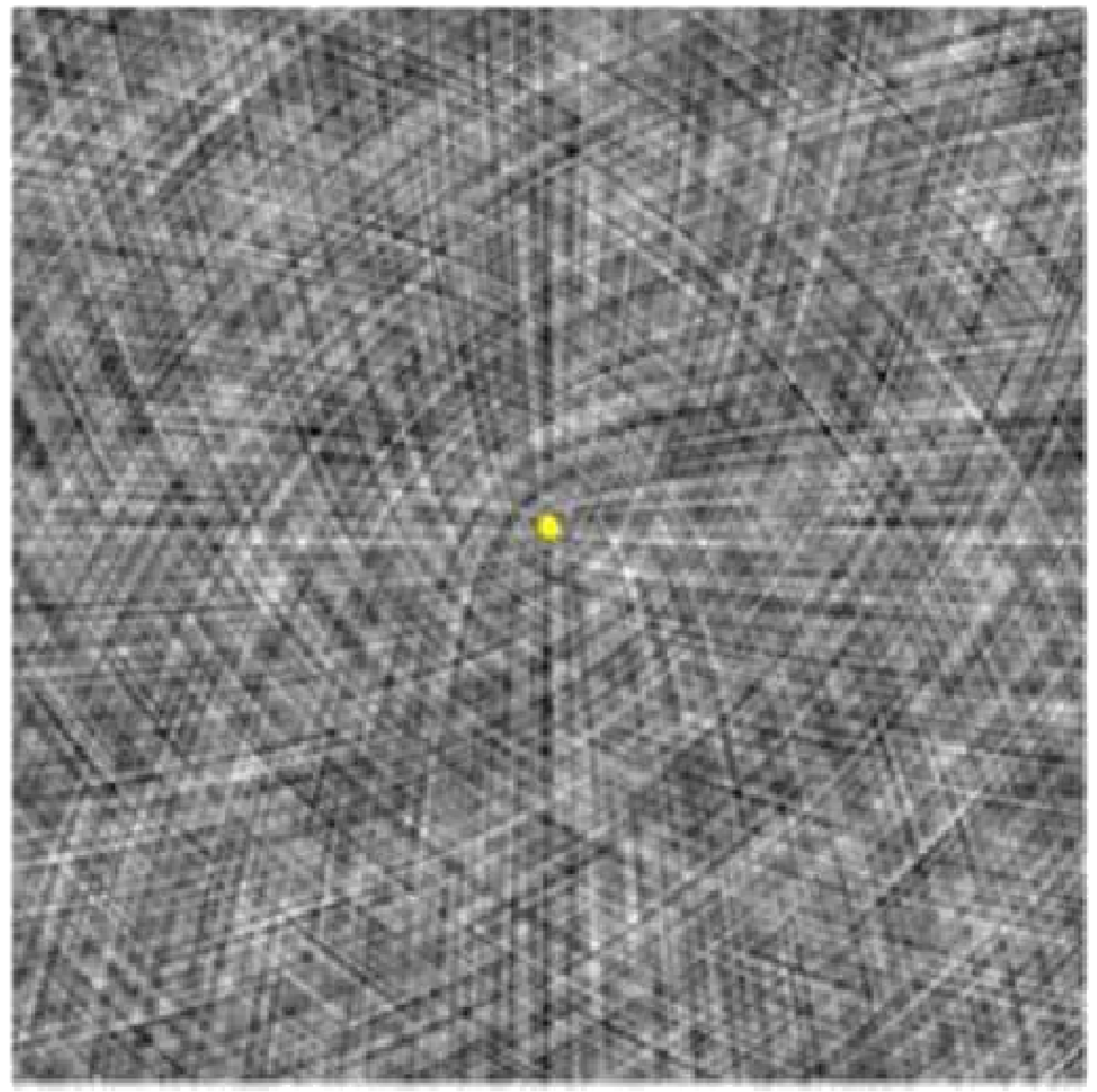
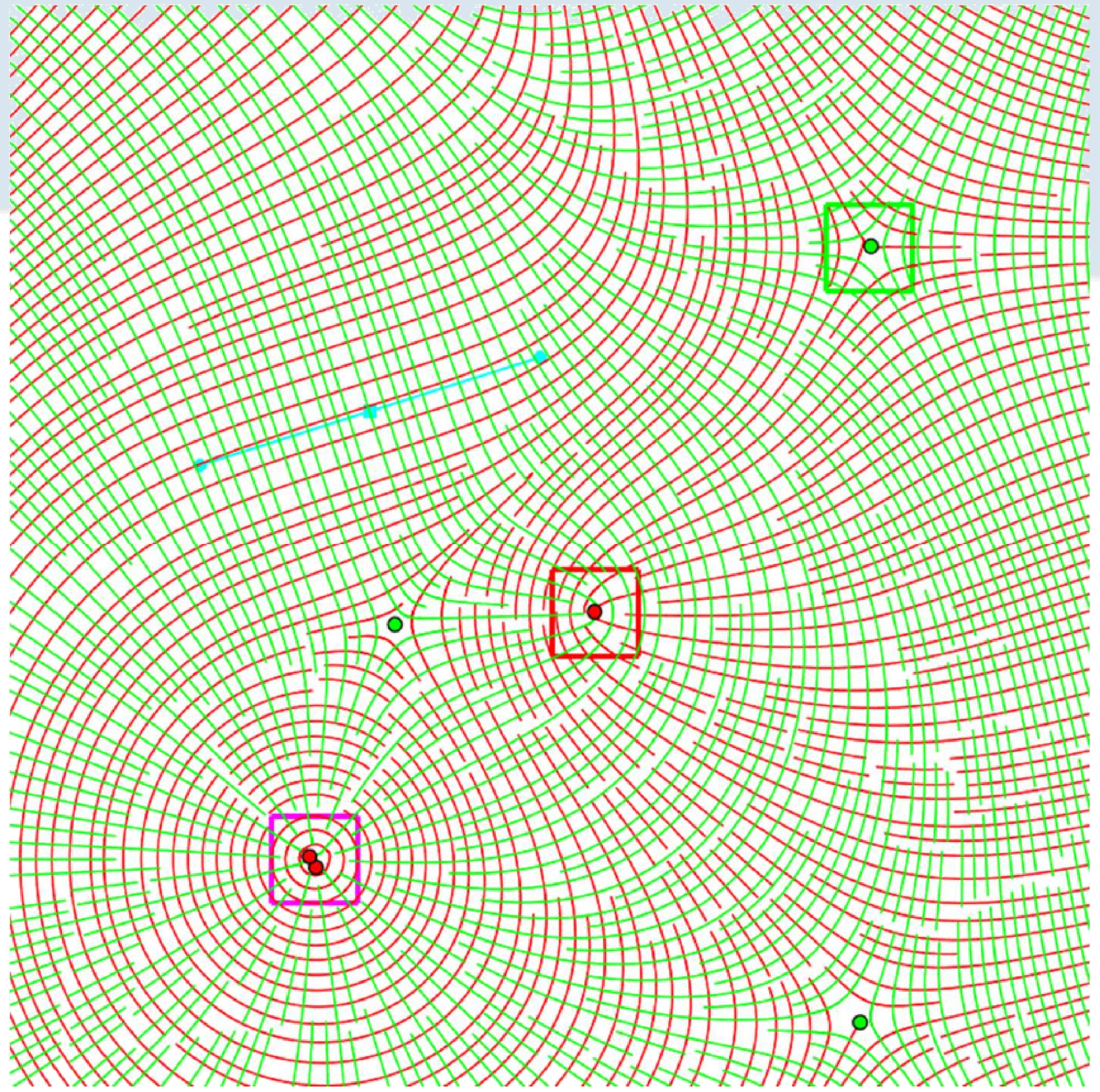


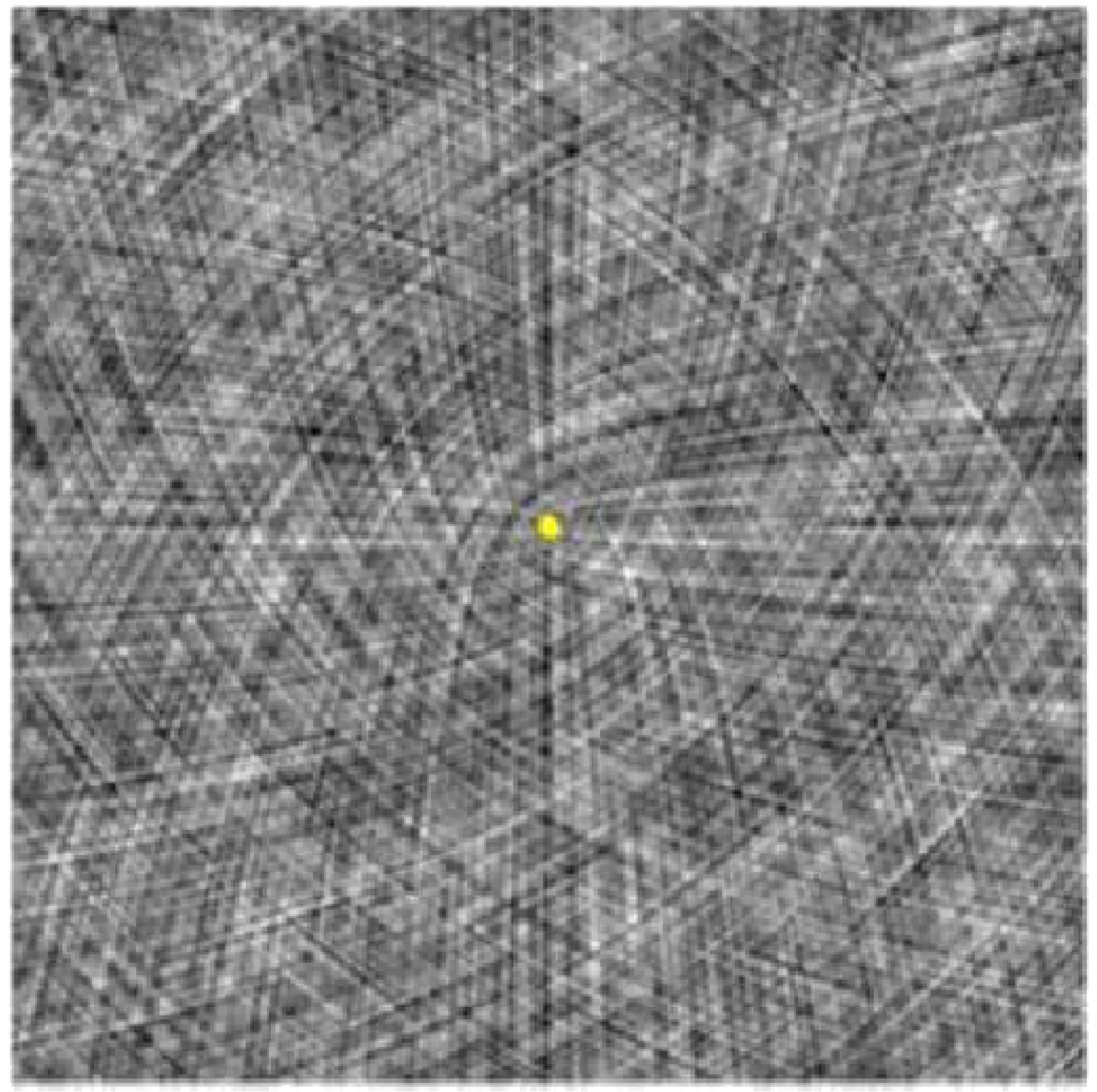
[Zhang]

Tensor field topology for dummies

- Analogy to vector fields
  - When do hyper-streamlines end?



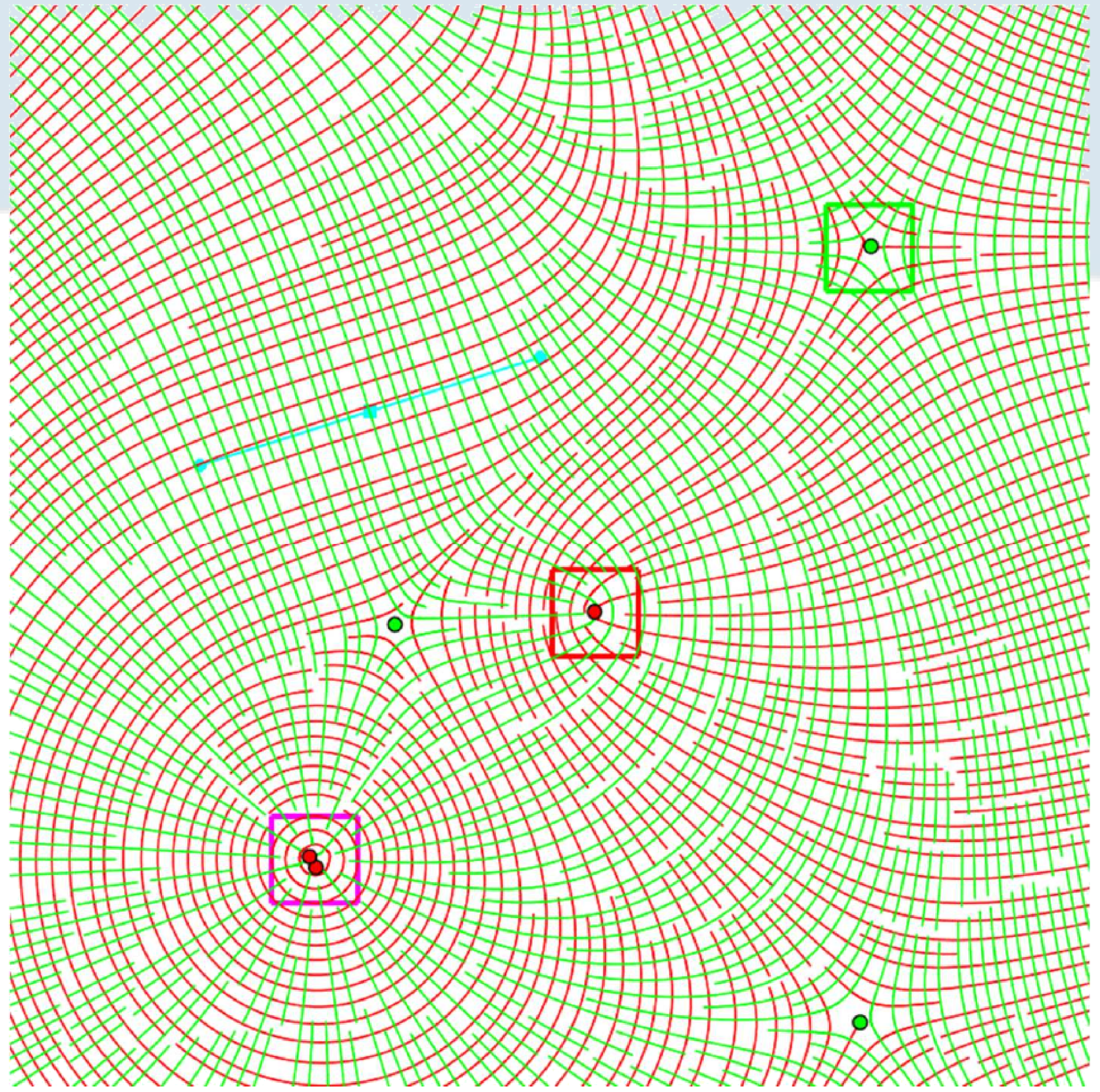
[Chen]



[Zhang]

Tensor field topology for dummies

- Analogy to vector fields
  - When do hyper-streamlines end?
  - When something critical happens



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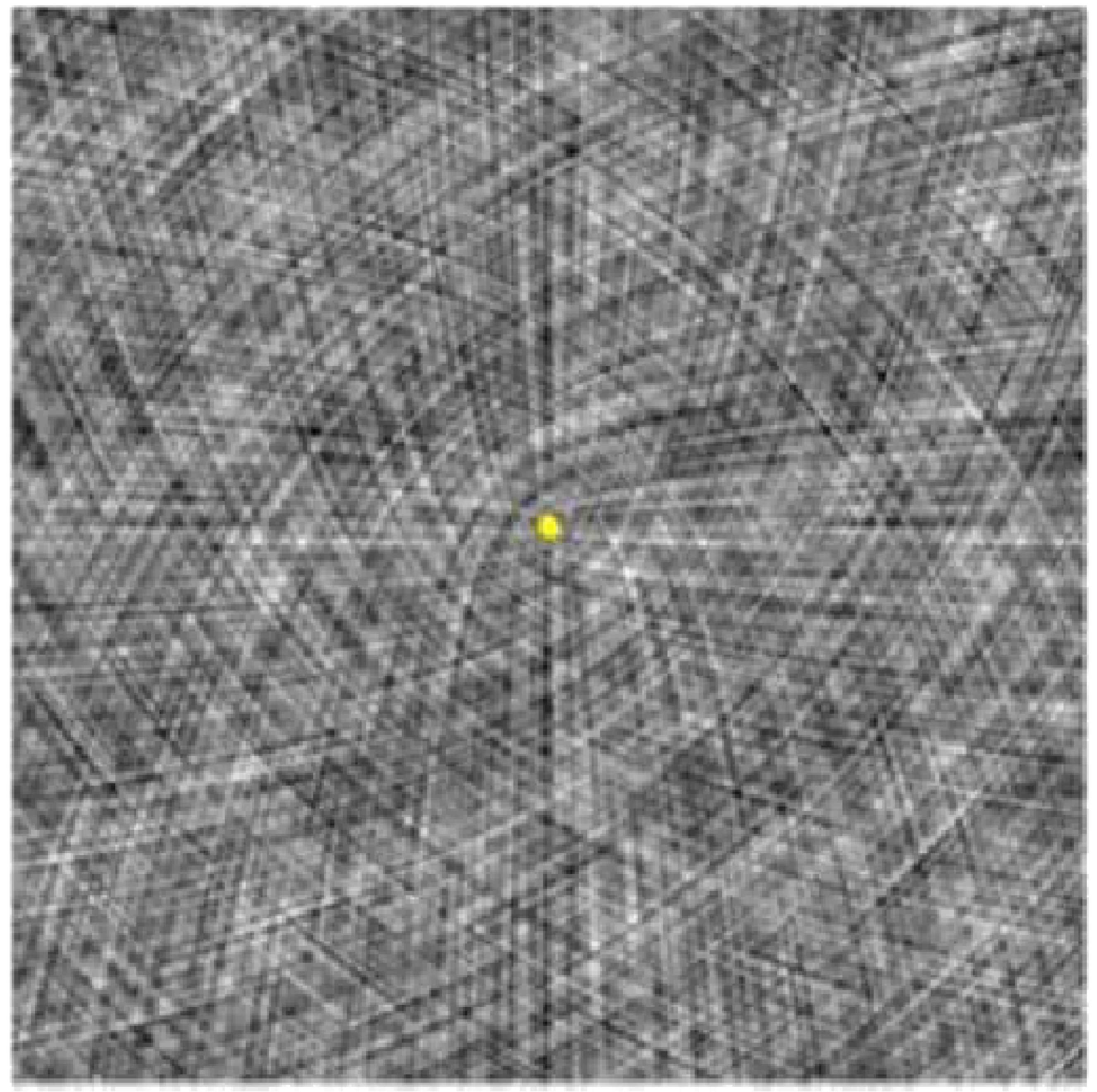
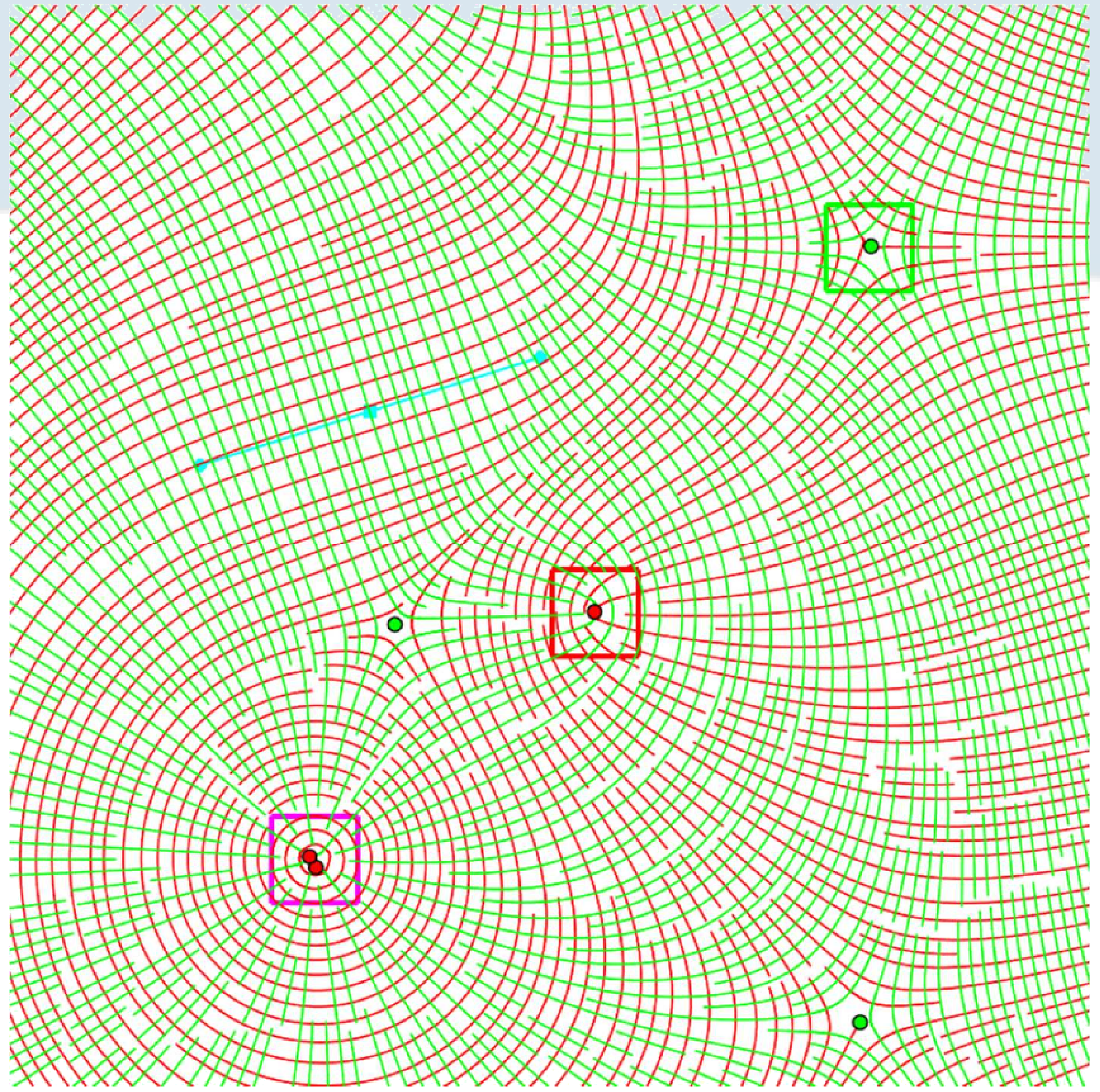


[Zhang]

Tensor field topology for dummies

- Analogy to vector fields
  - When do hyper-streamlines end?
  - When something critical happens
    - For vector fields



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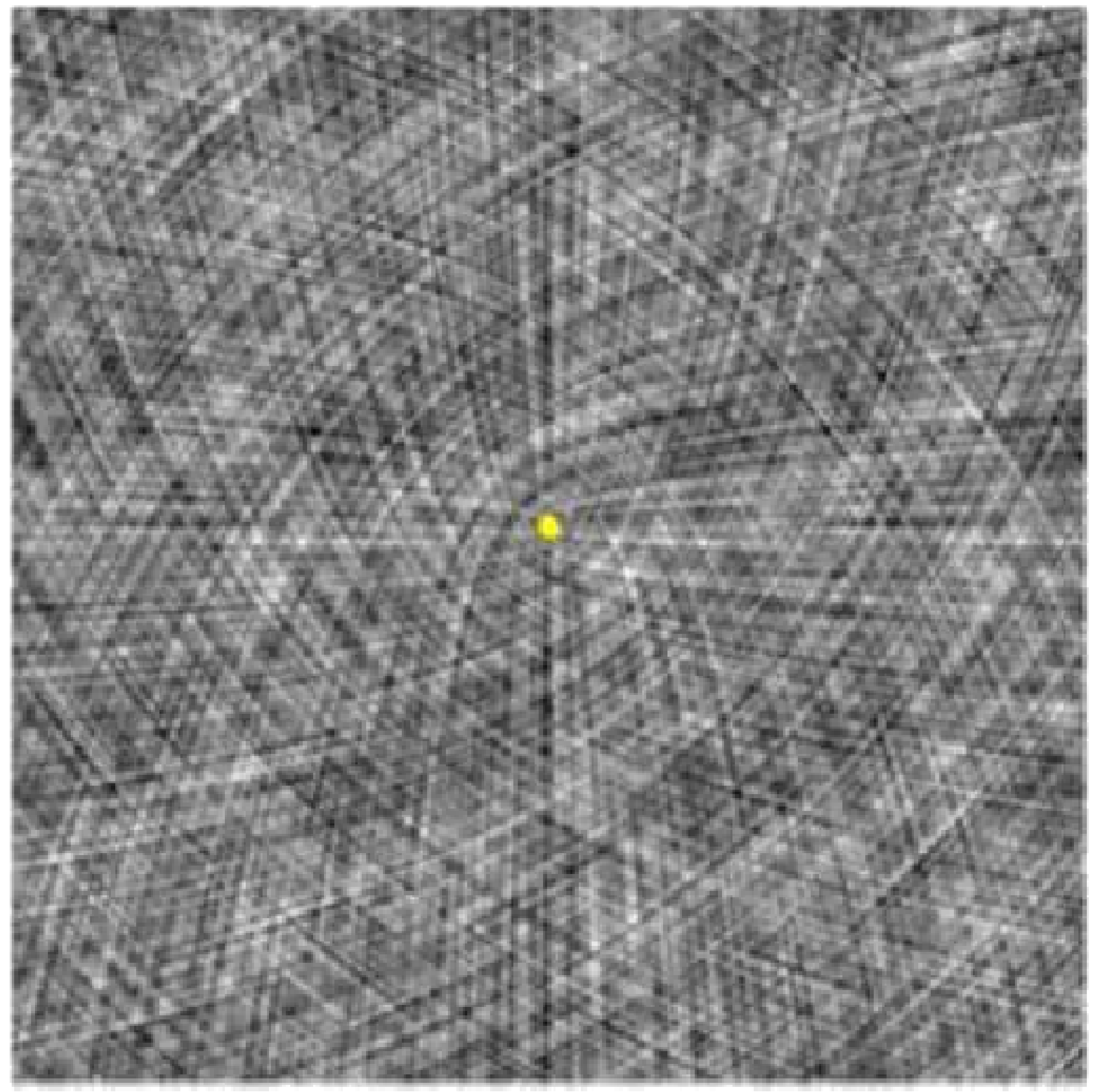
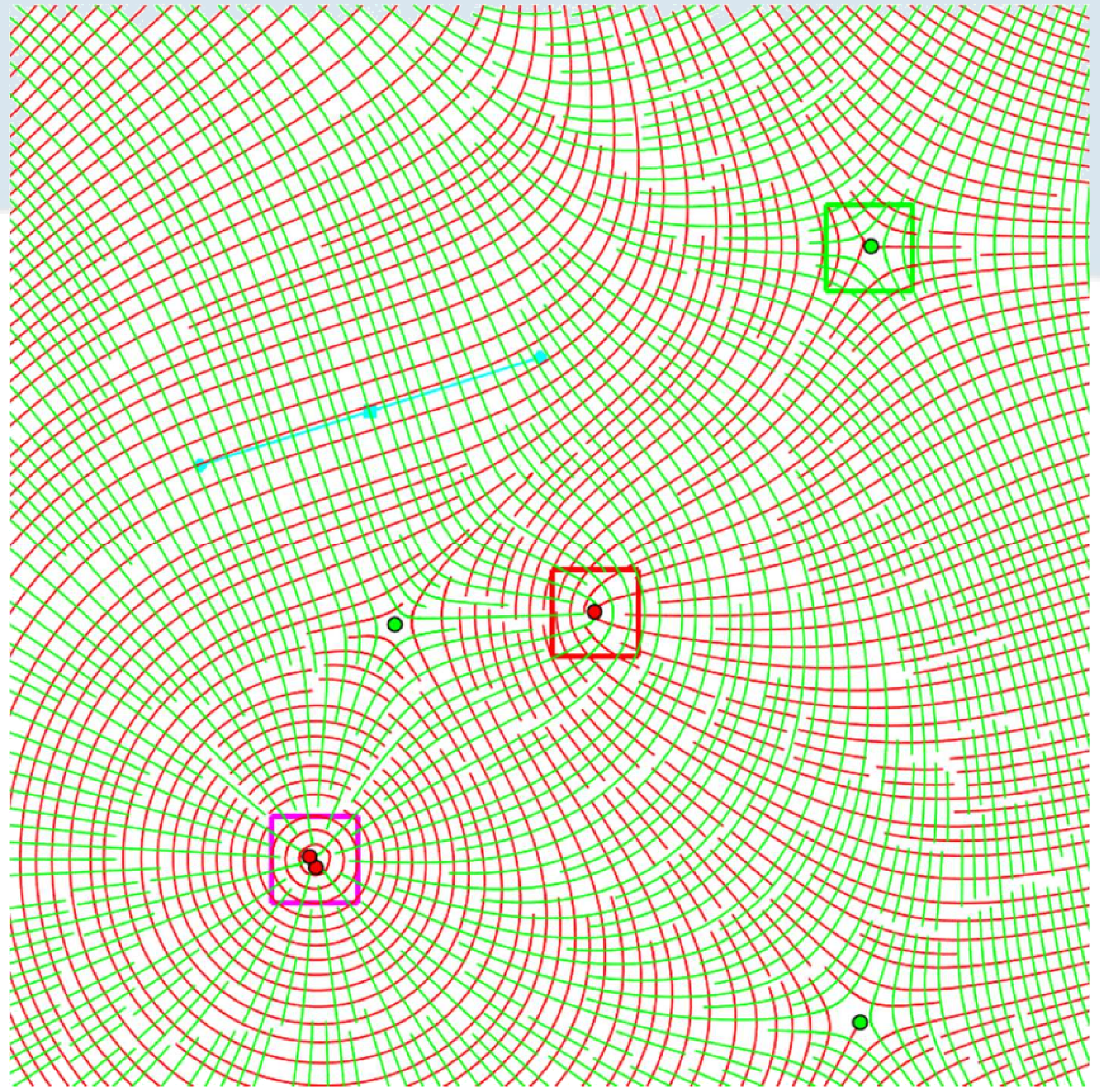


[Zhang]

Tensor field topology for dummies

- Analogy to vector fields
  - When do hyper-streamlines end?
  - When something critical happens
    - For vector fields
      - When the magnitude vanishes



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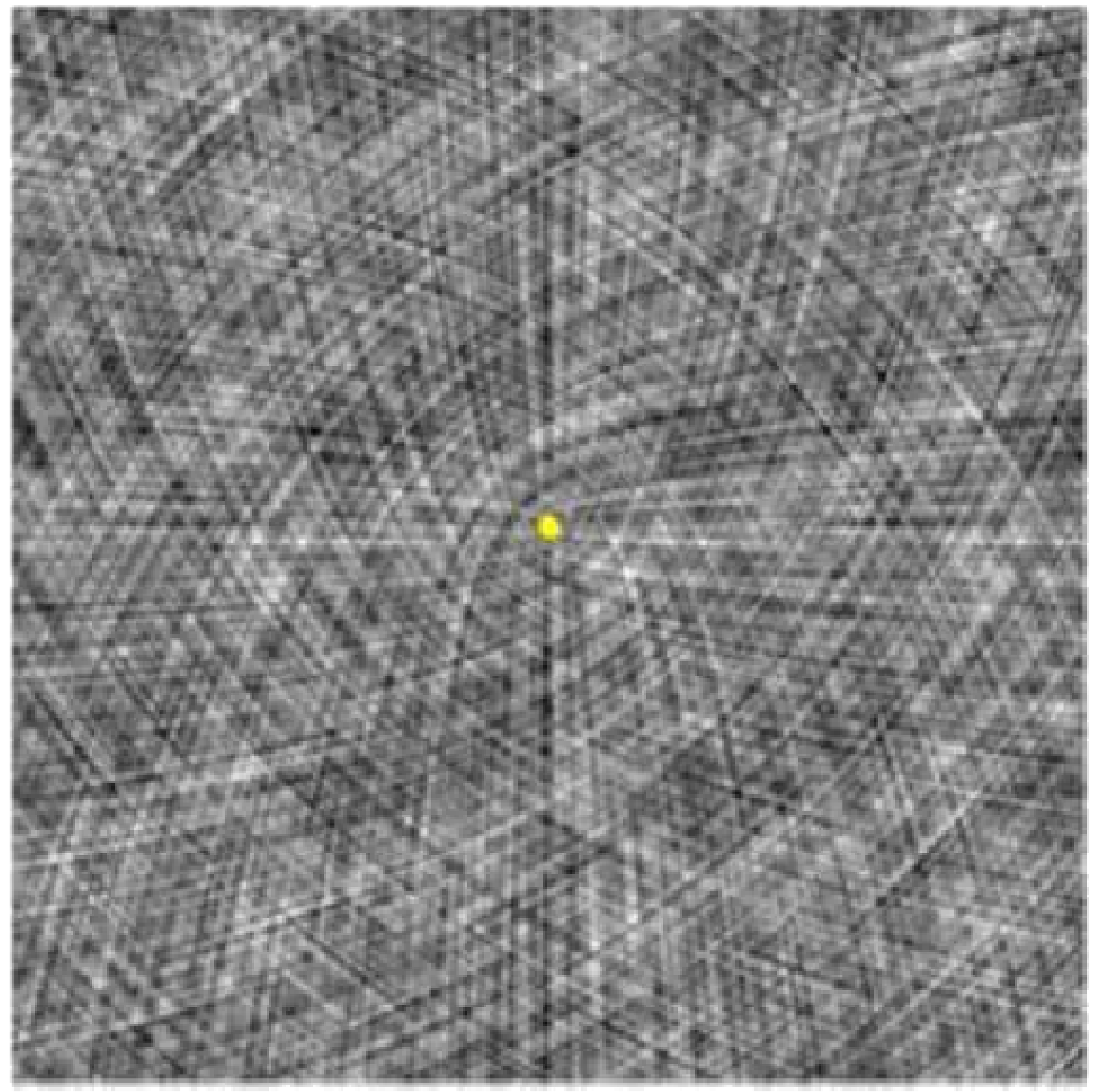
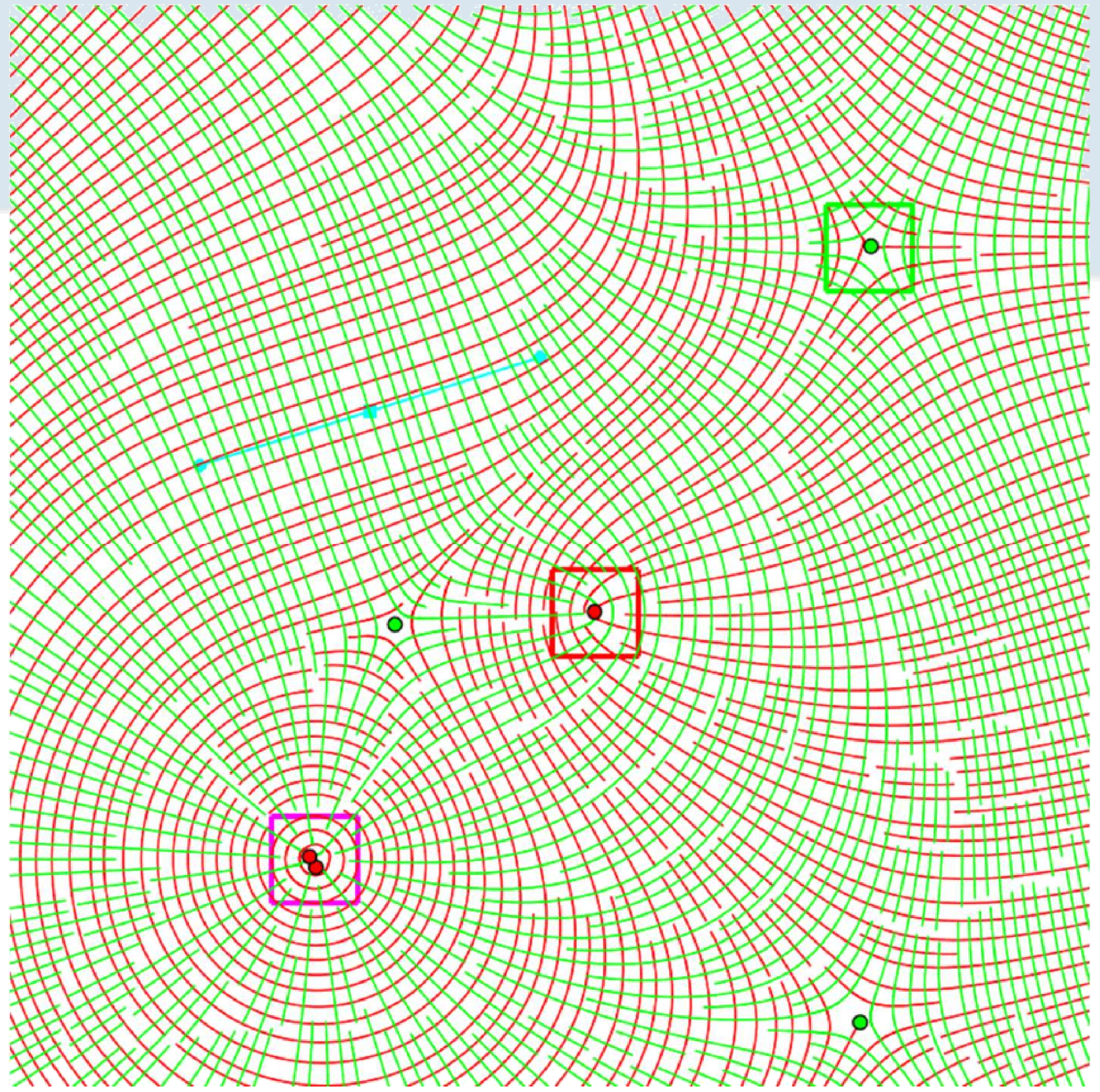


[Zhang]

Tensor field topology for dummies

- Analogy to vector fields
  - When do hyper-streamlines end?
  - When something critical happens
    - For vector fields
      - When the magnitude vanishes
    - For tensor fields?



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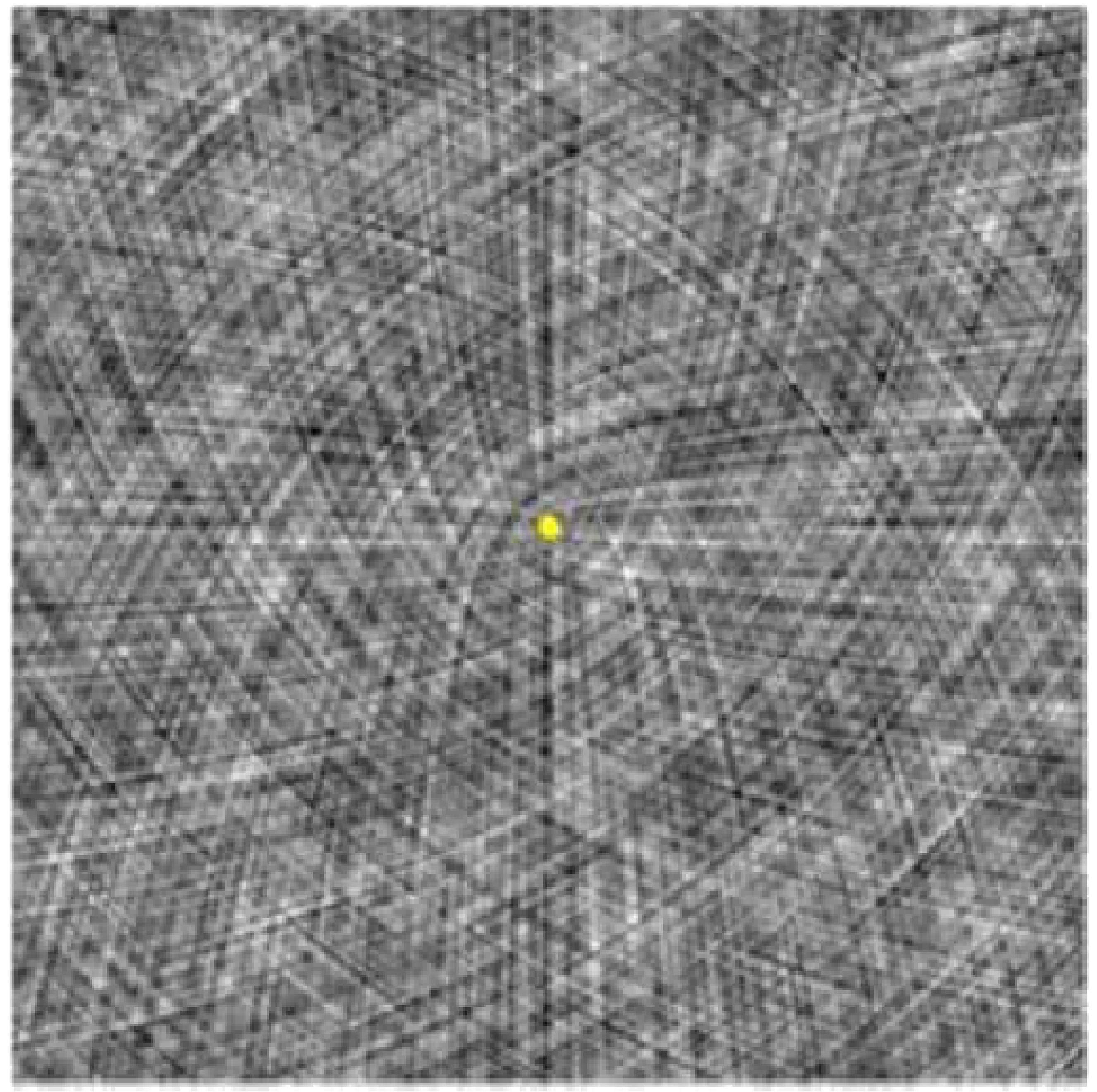
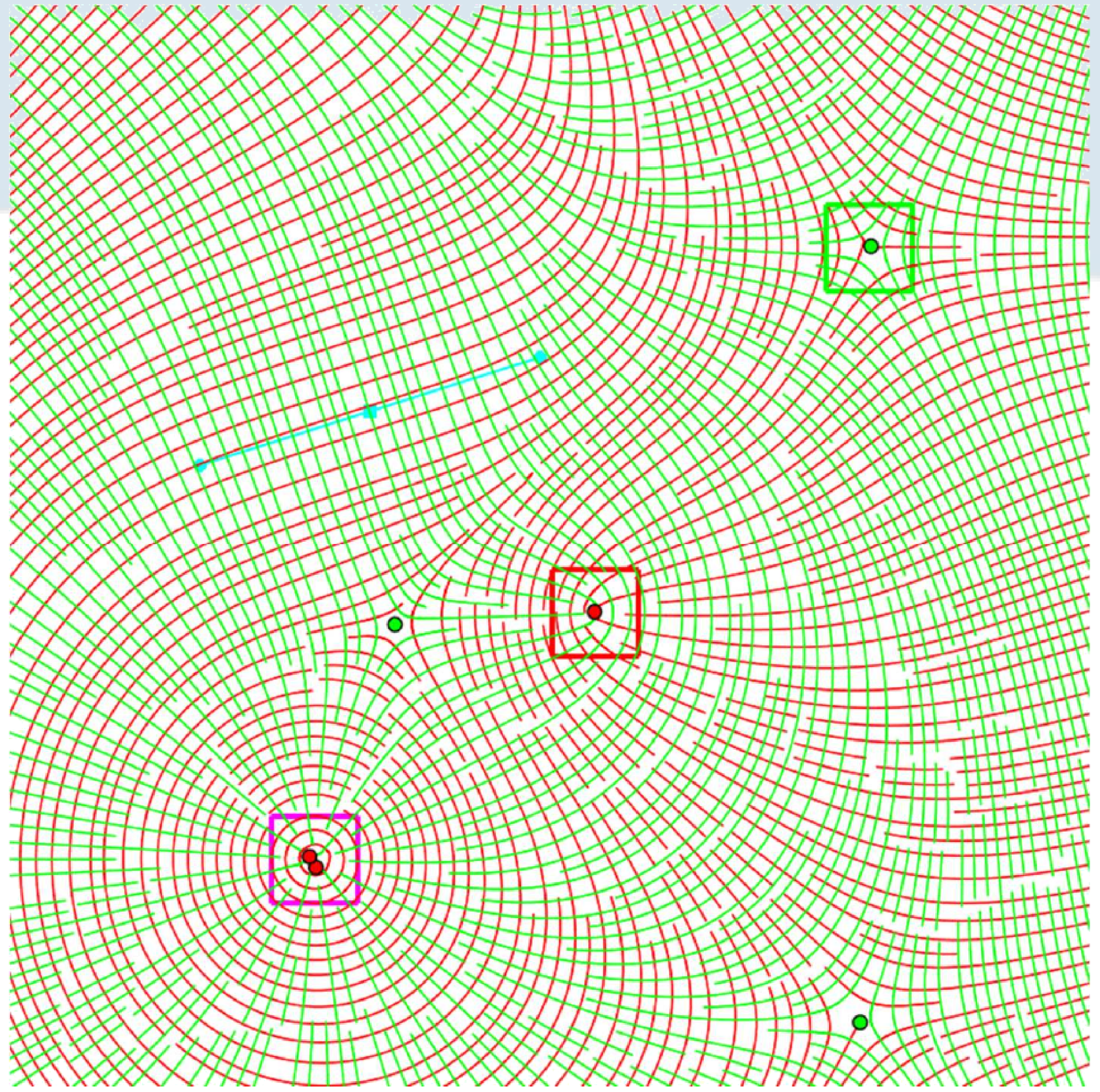


[Zhang]

Tensor field topology for dummies

- Analogy to vector fields
  - When do hyper-streamlines end?
  - When something critical happens
    - For vector fields
      – When the magnitude vanishes
    - For tensor fields?
      – When it's impossible to choose a direction



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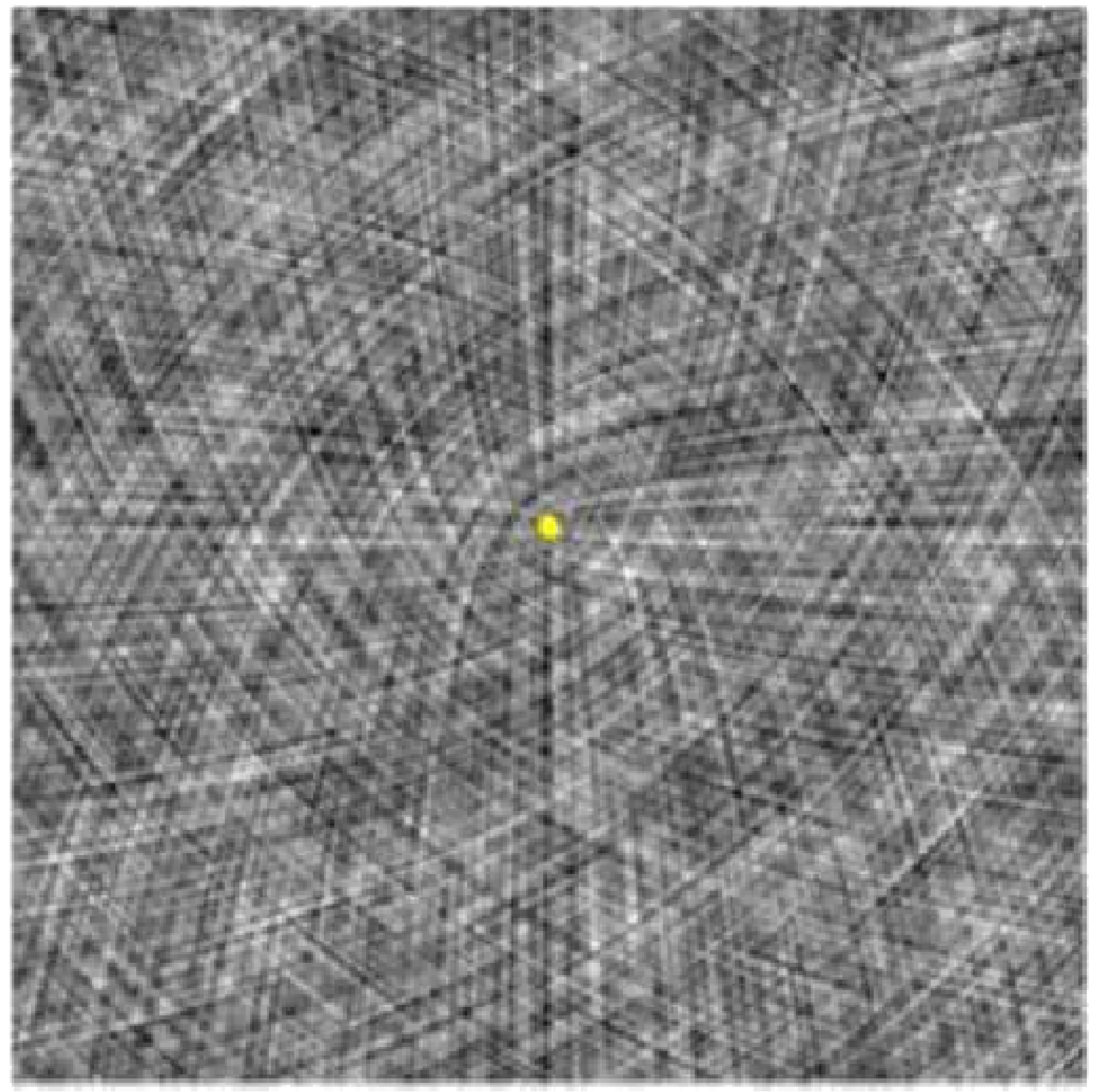
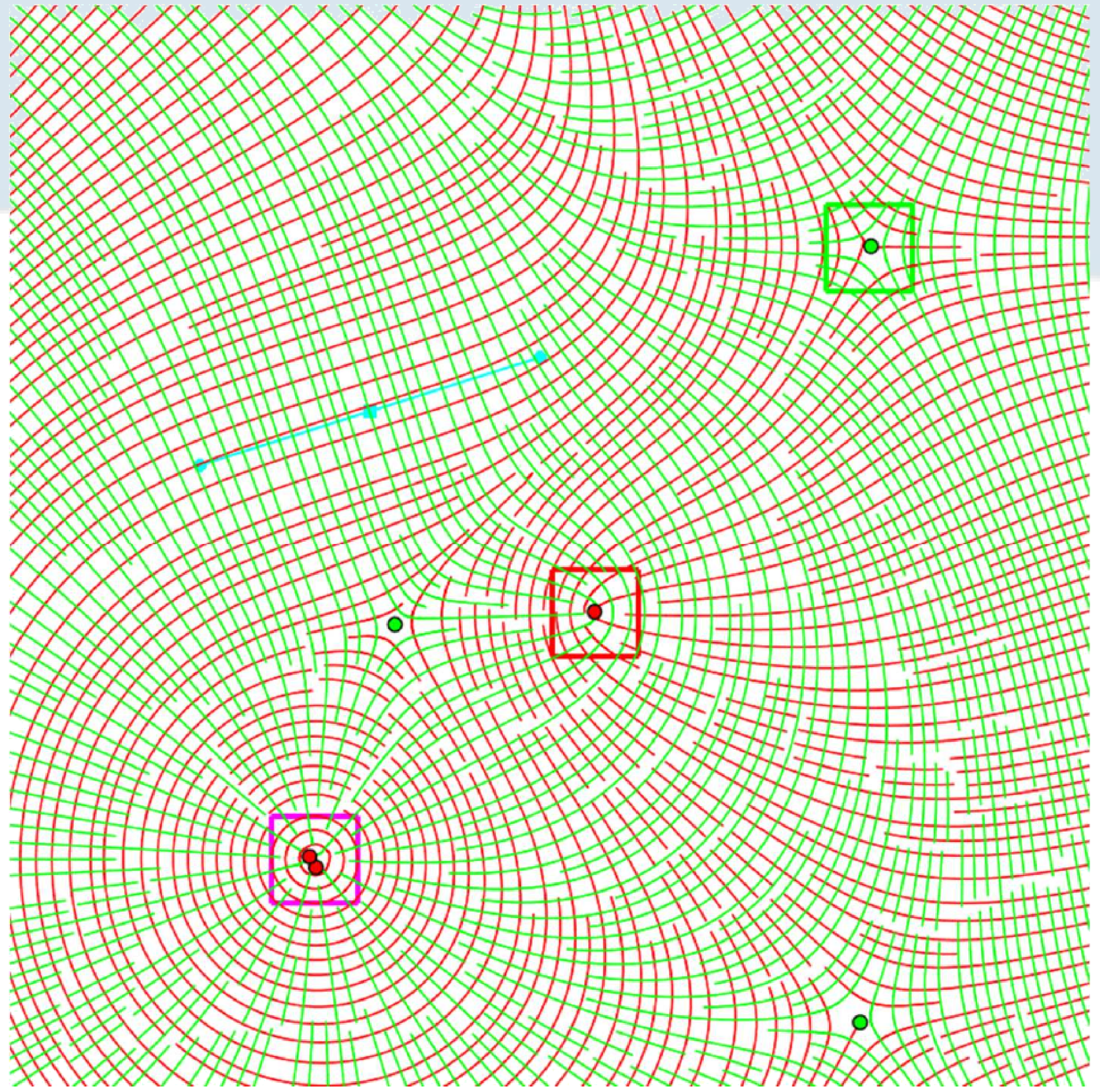
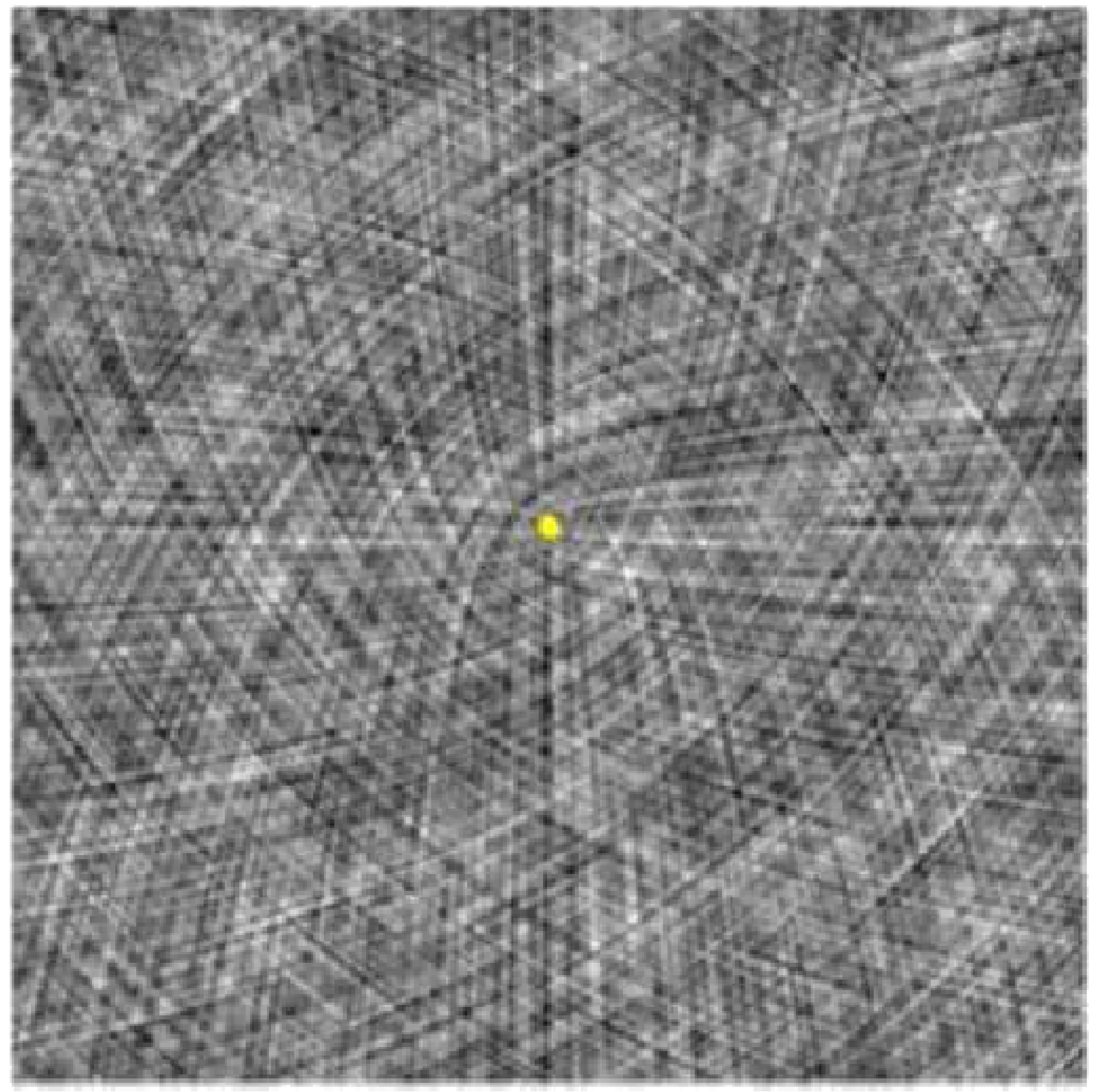


[Zhang]

Tensor field topology for dummies

- Analogy to vector fields
  - When do hyper-streamlines end?
  - When something critical happens
    - For vector fields
      – When the magnitude vanishes
    - For tensor fields?
      – When it's impossible to choose a direction
      – When it's impossible to order eigenvalues


[Chen]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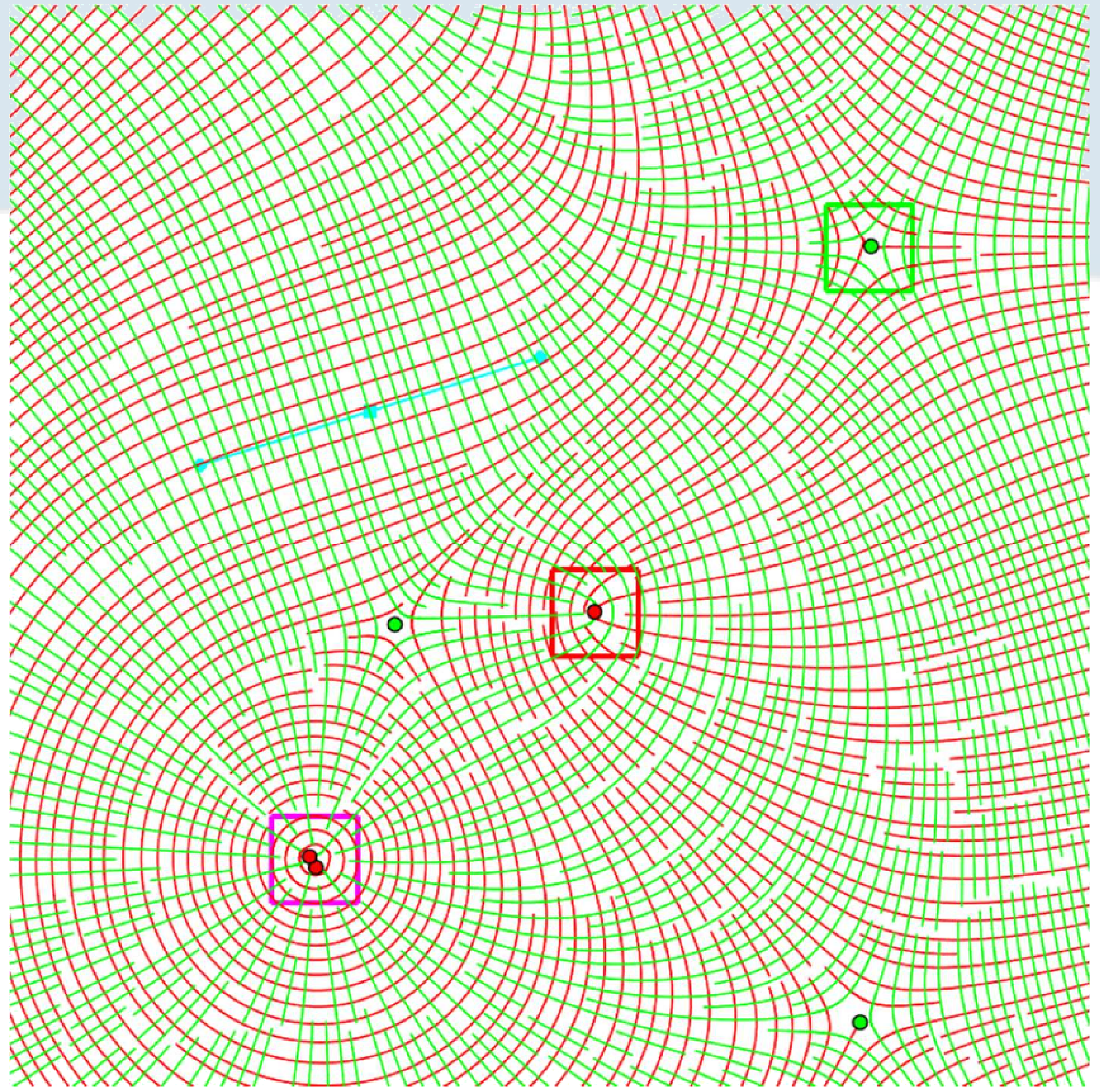

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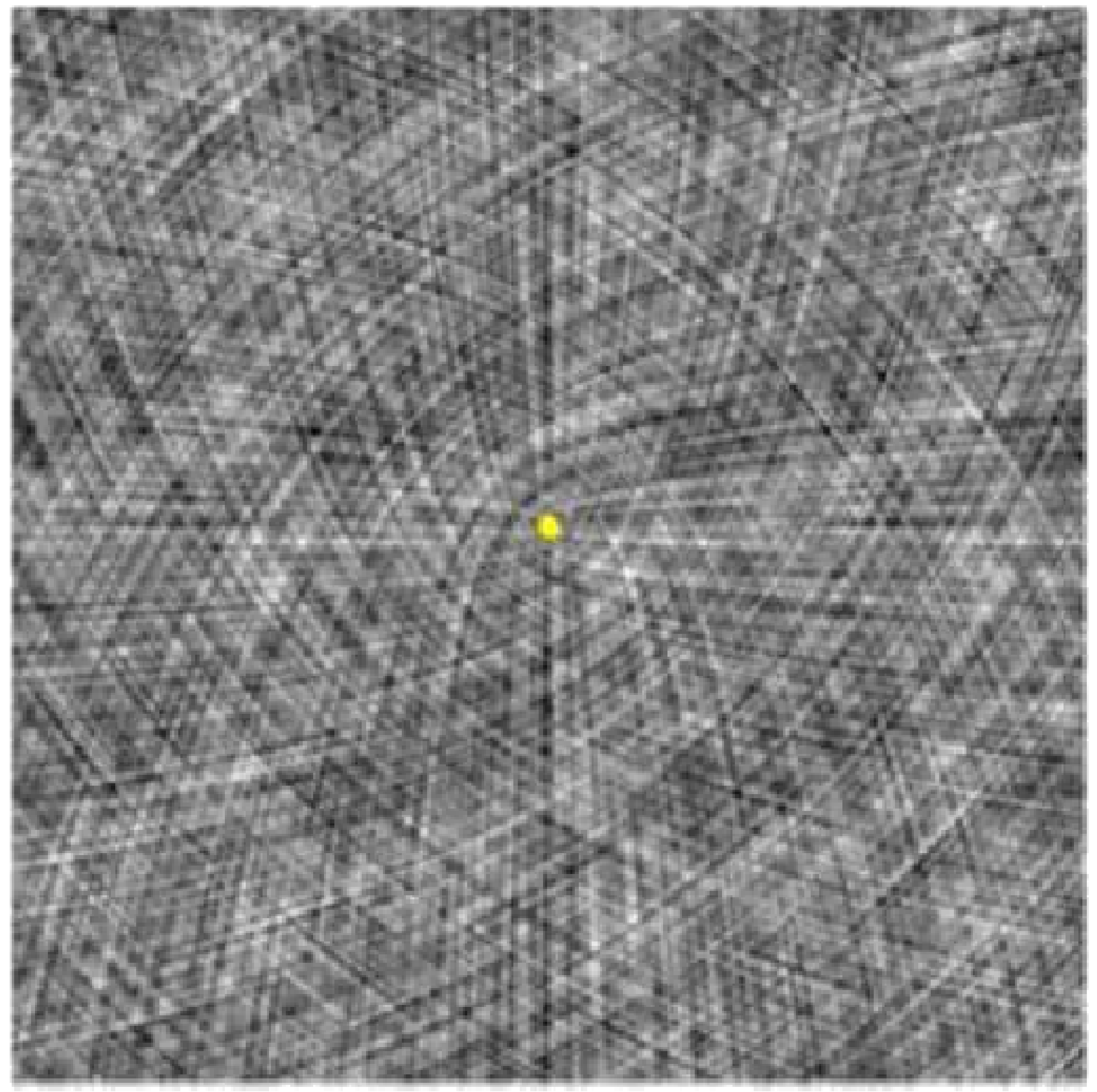

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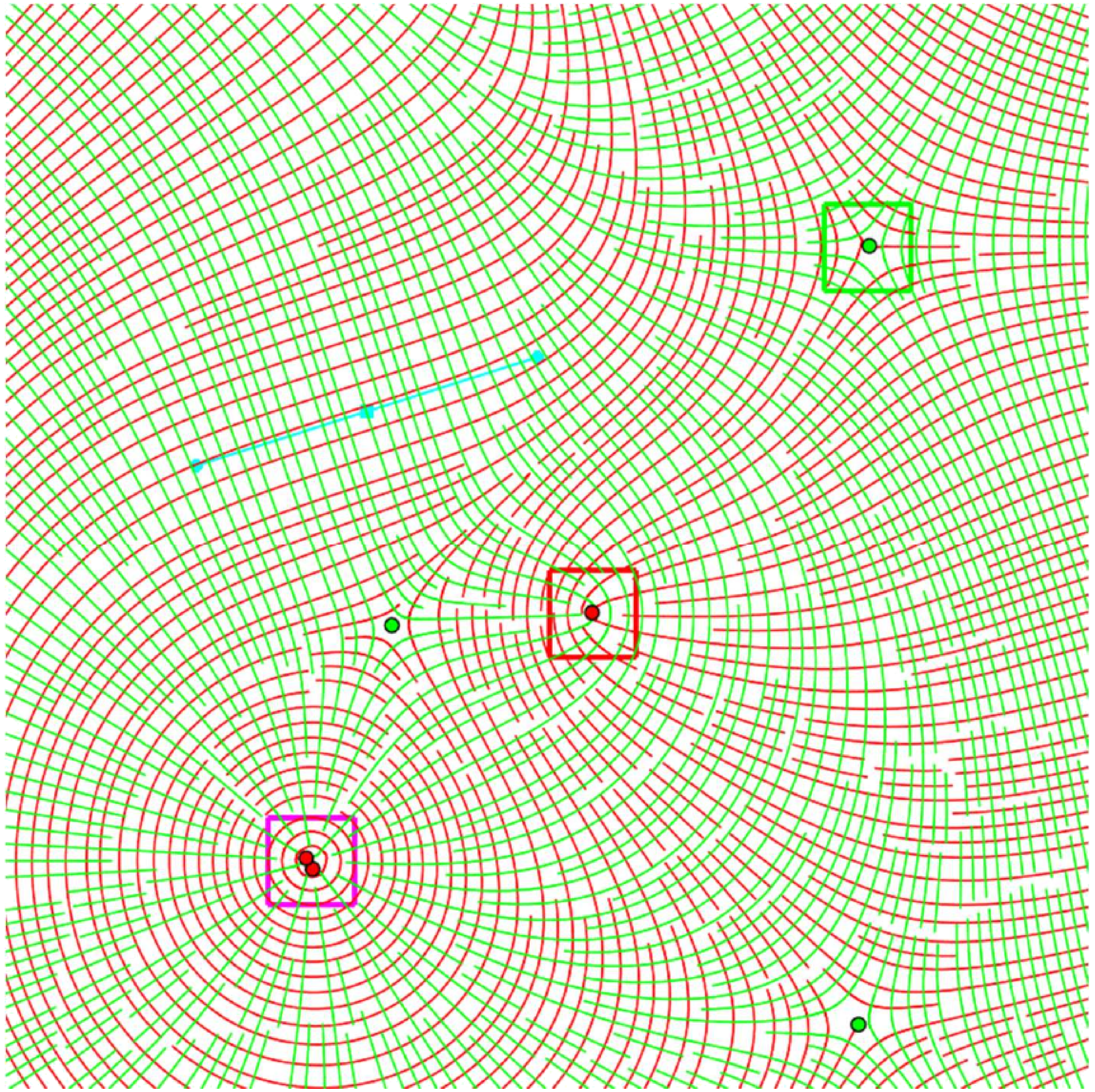
Critical points in tensor fields

- Usually called “degenerate points”



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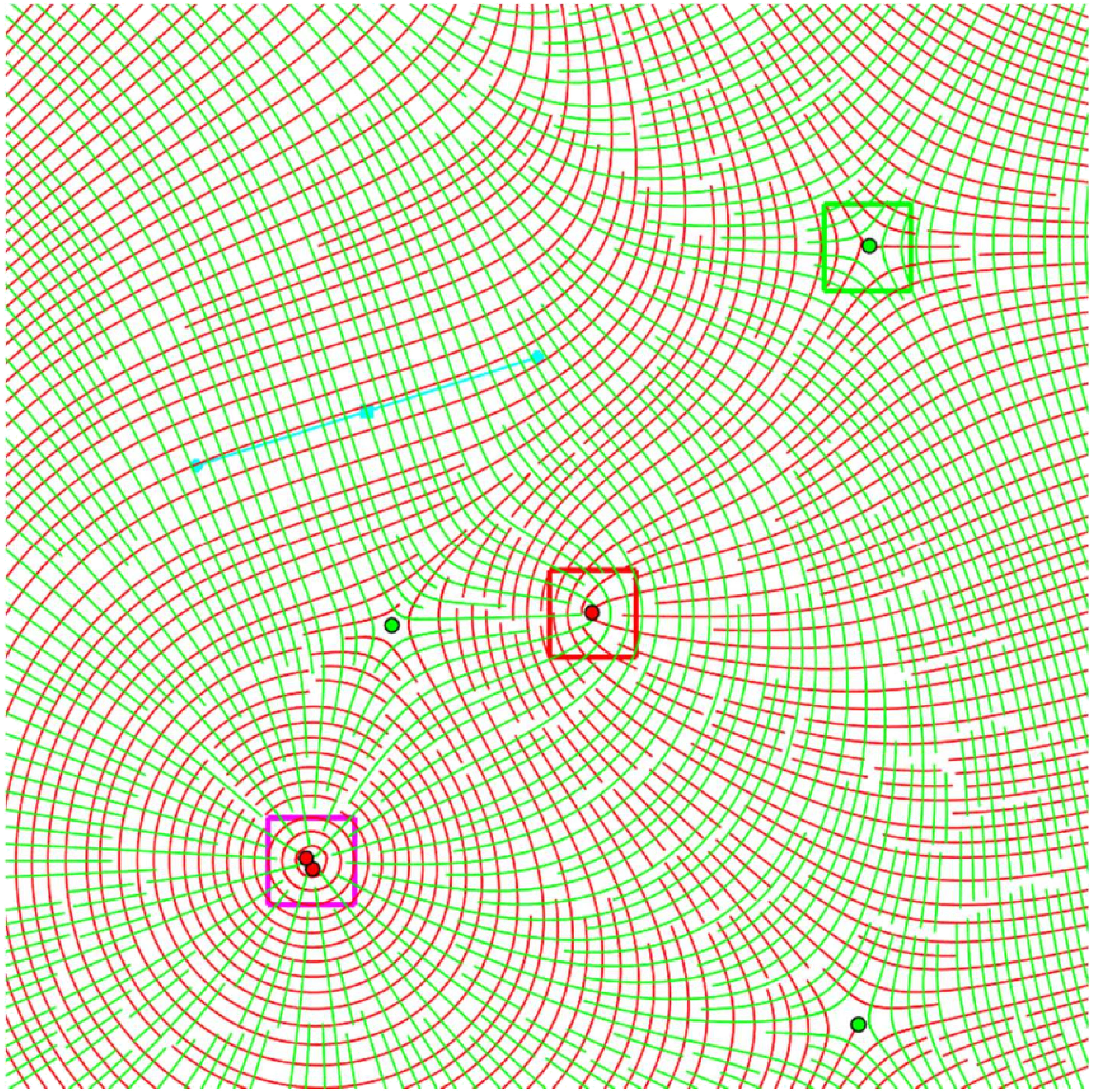

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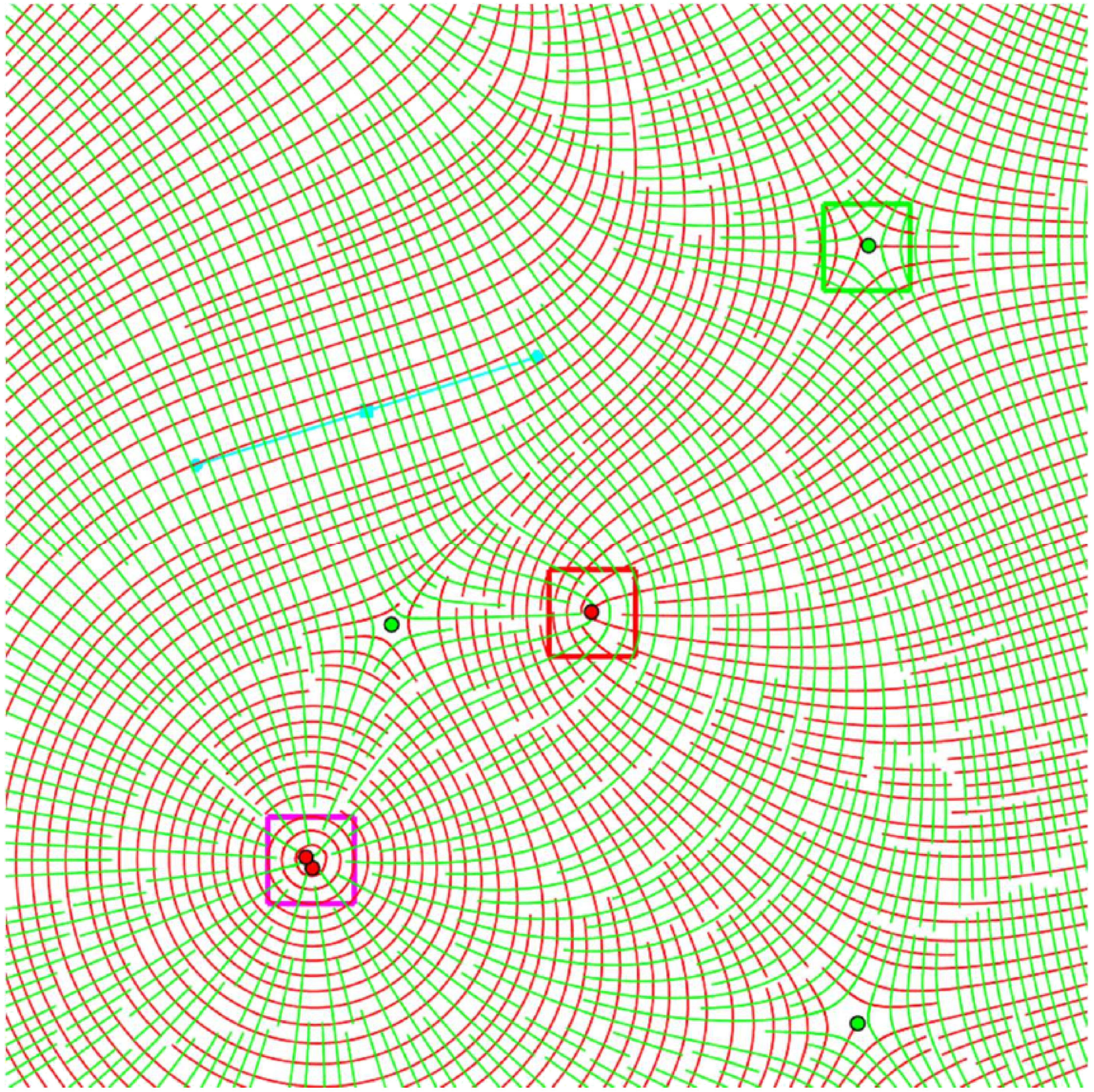
- Critical point classification



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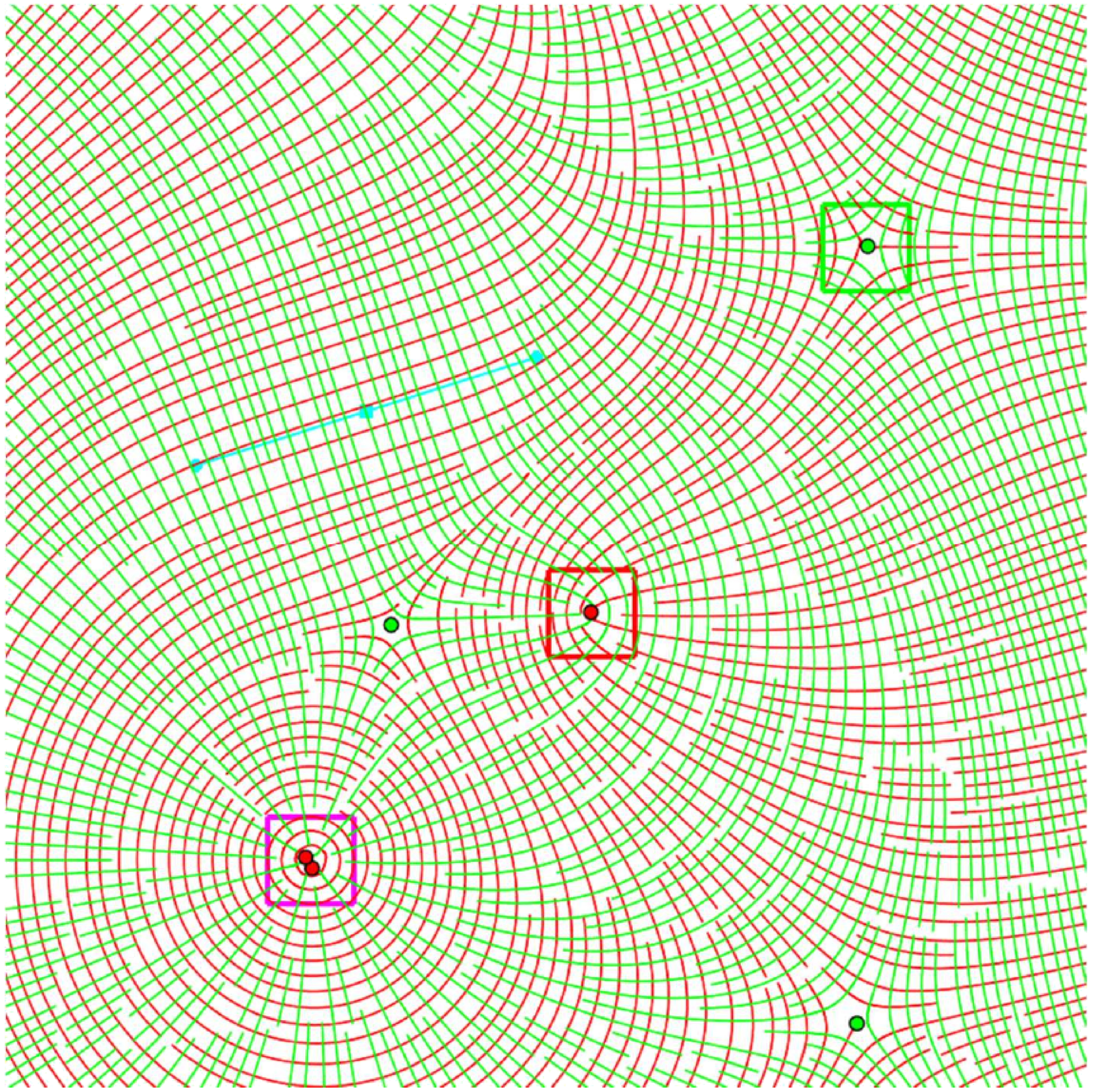

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Critical points in tensor fields

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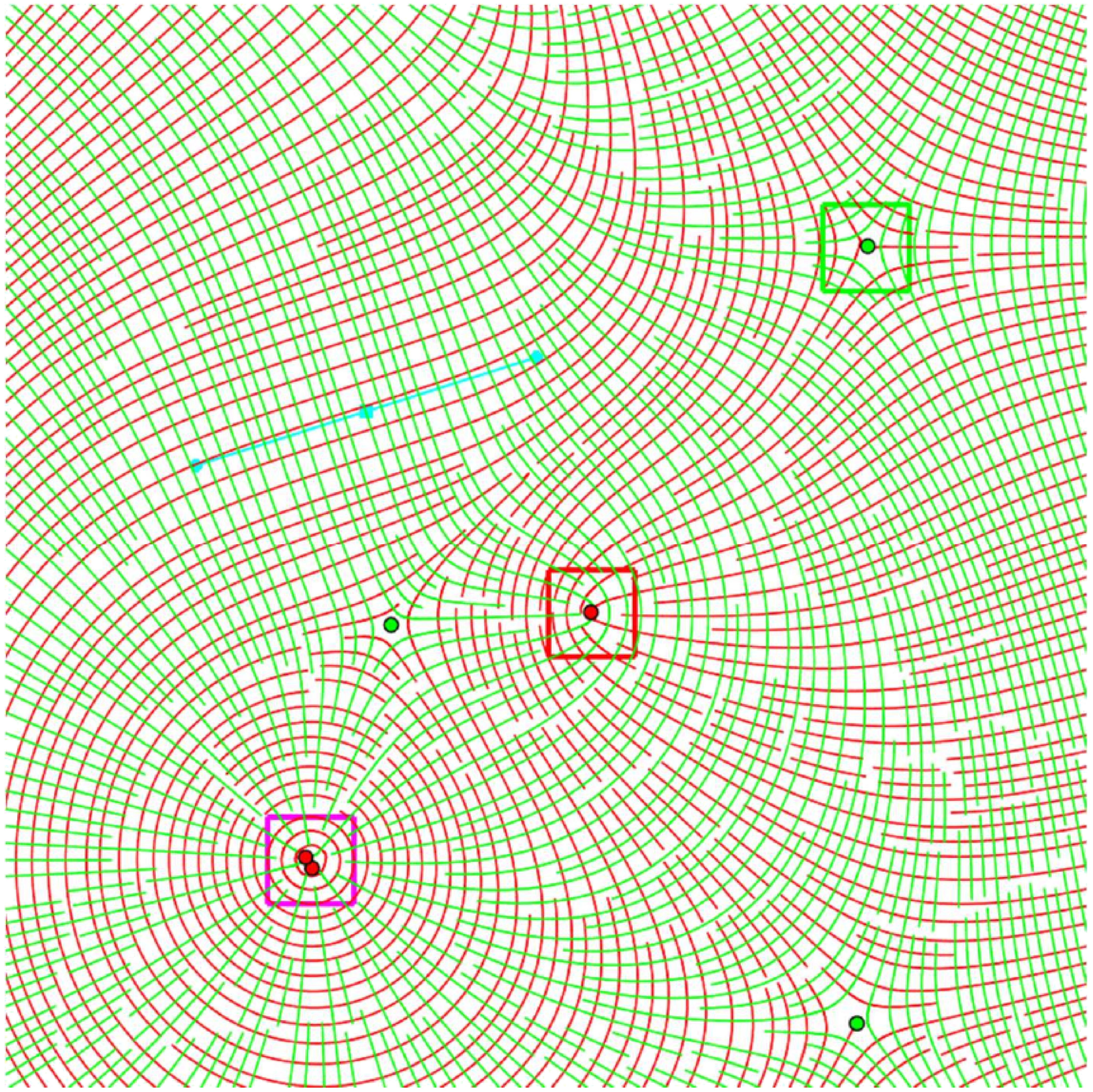
    - More types of critical points



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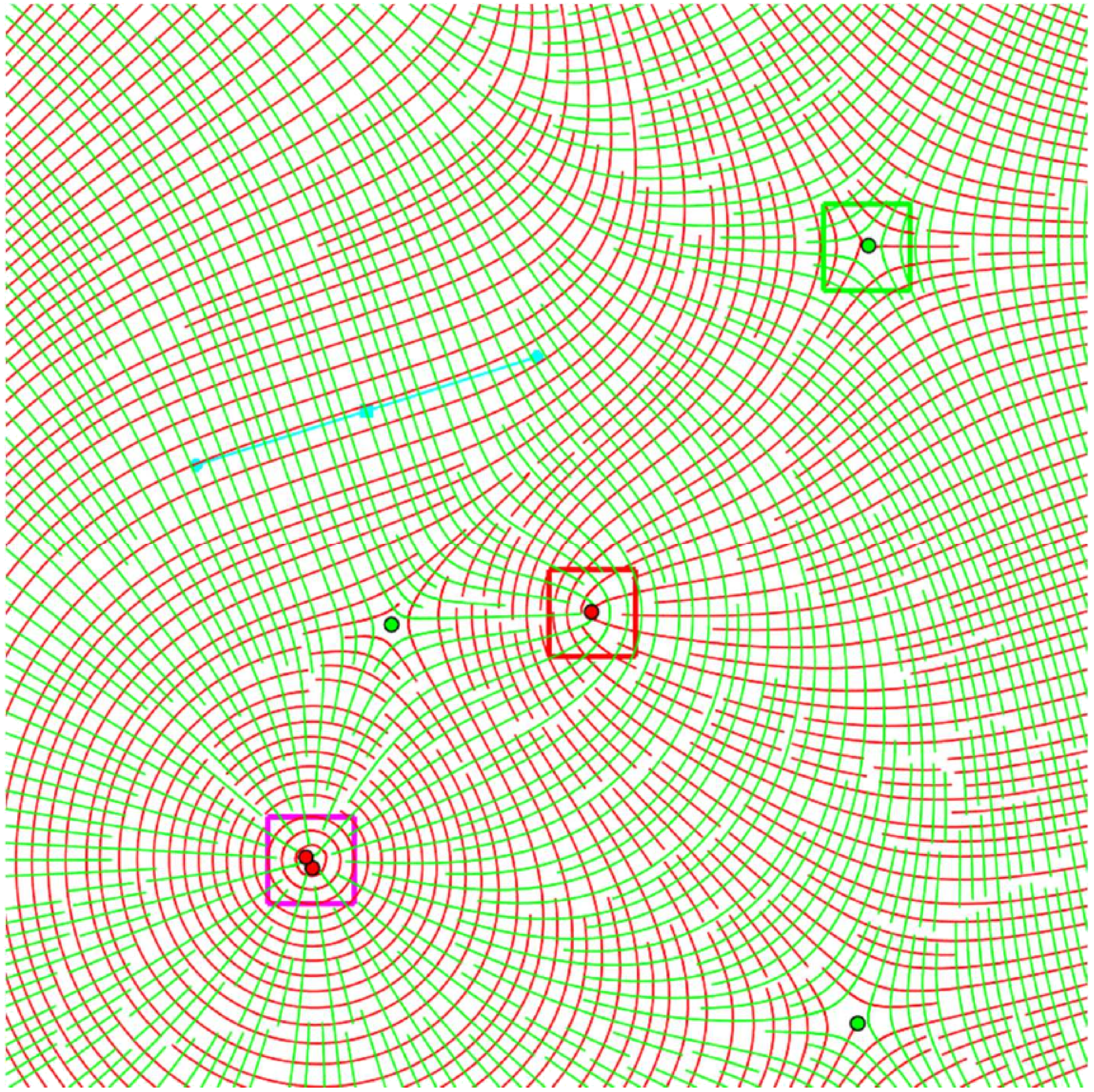

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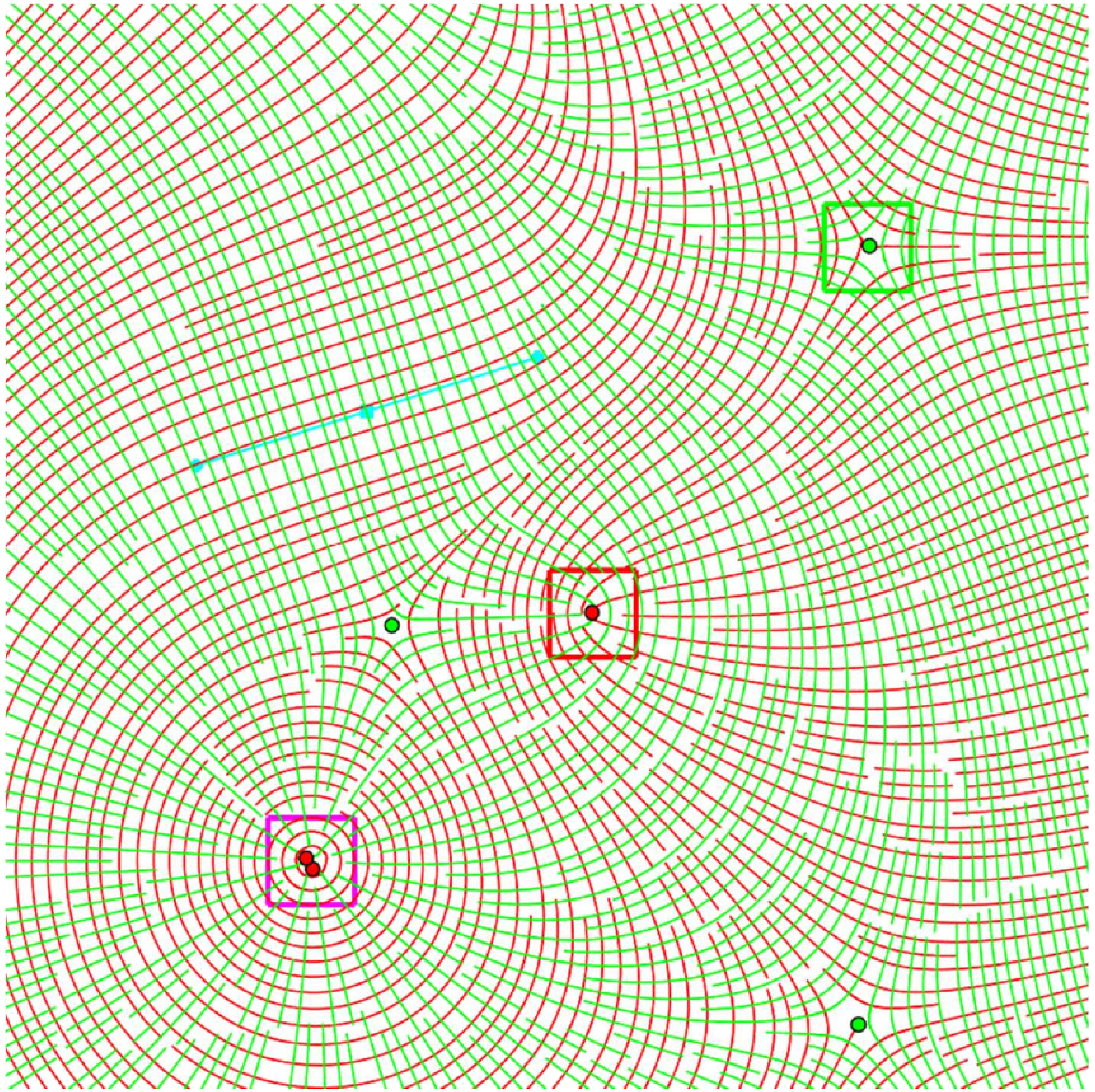
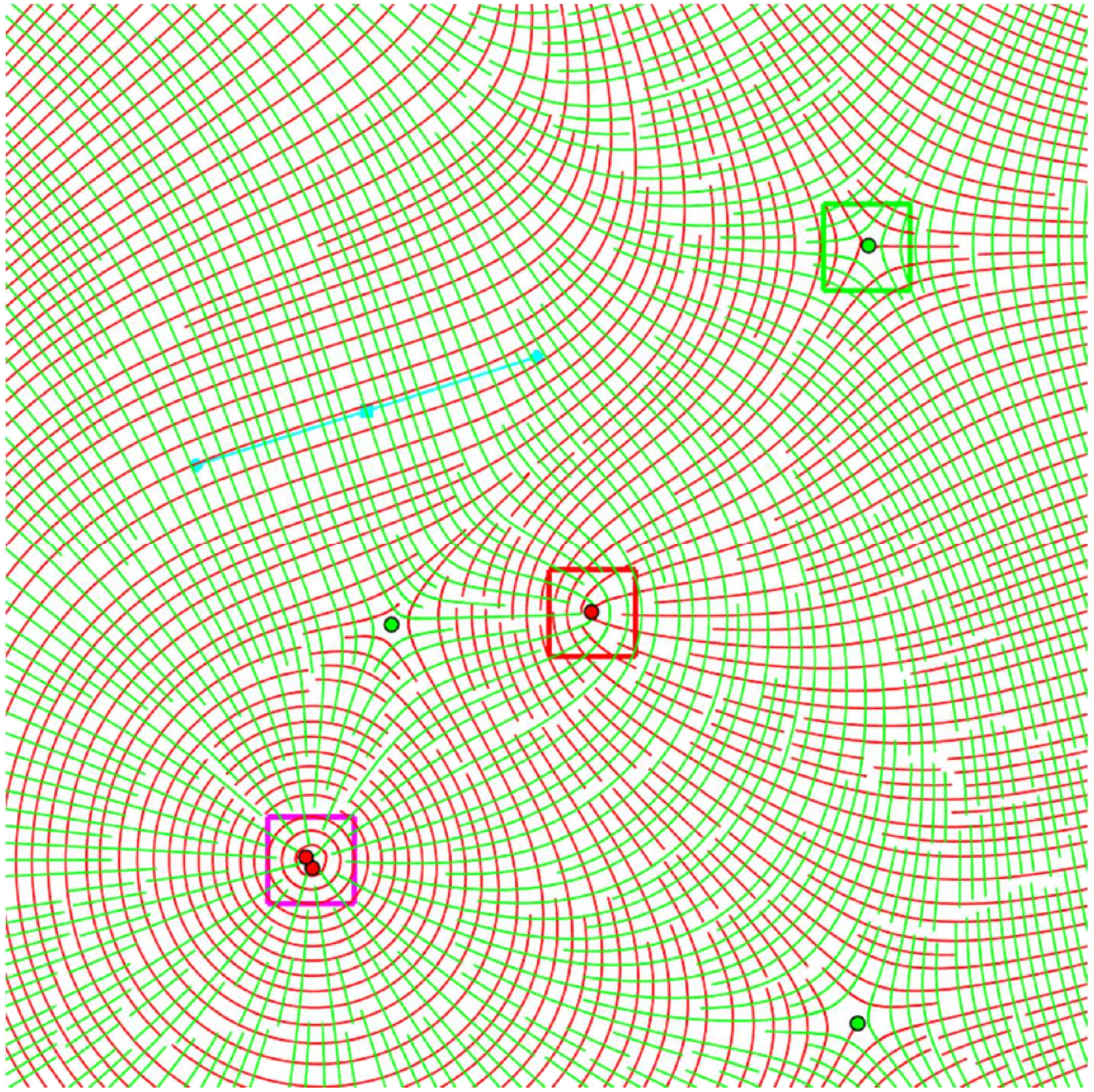


[Chen]

Critical points in tensor fields

- For instance
  - $f : \mathbb{R}^2 \to \mathbb{M}_{2\times 2}$



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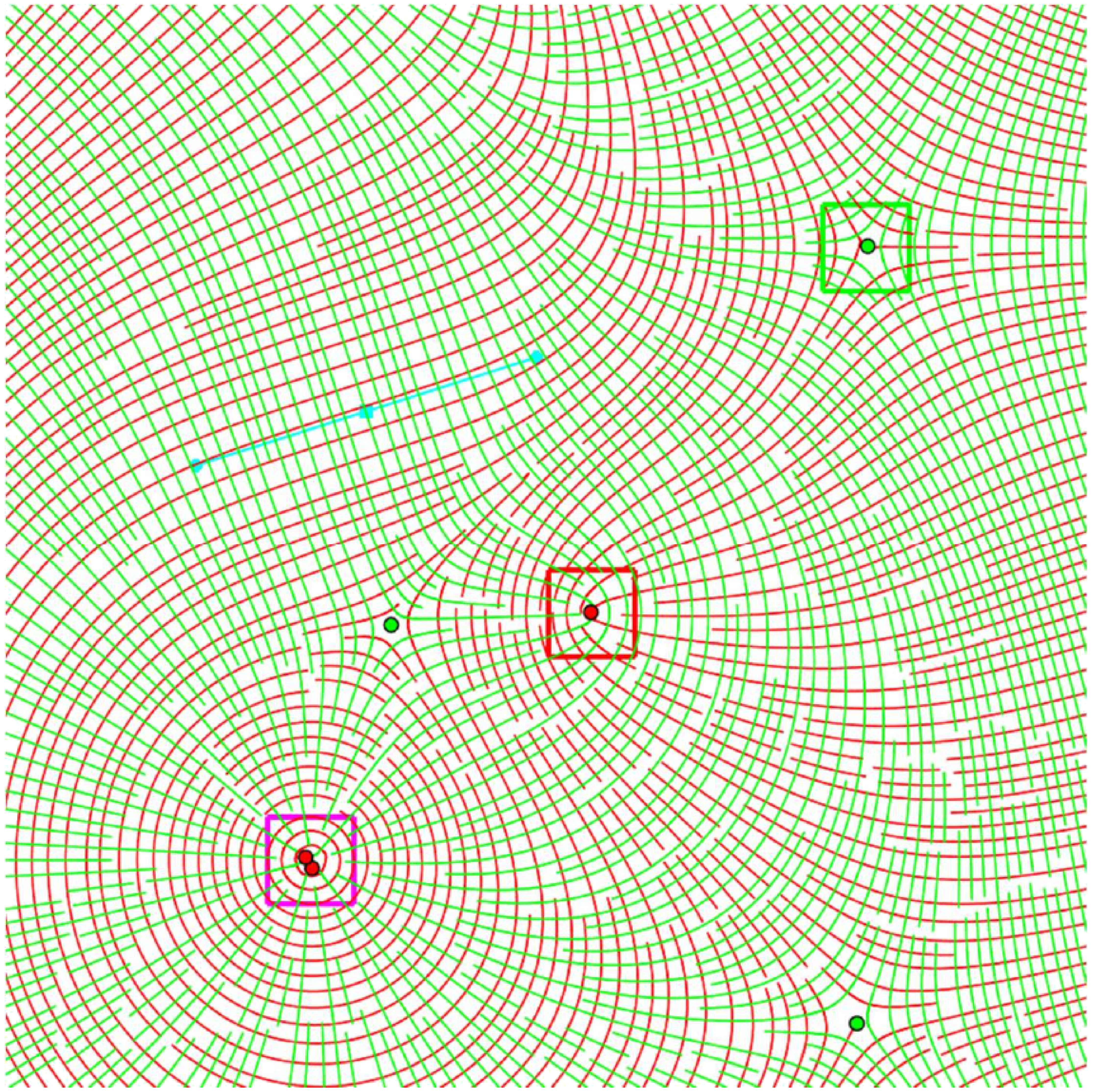

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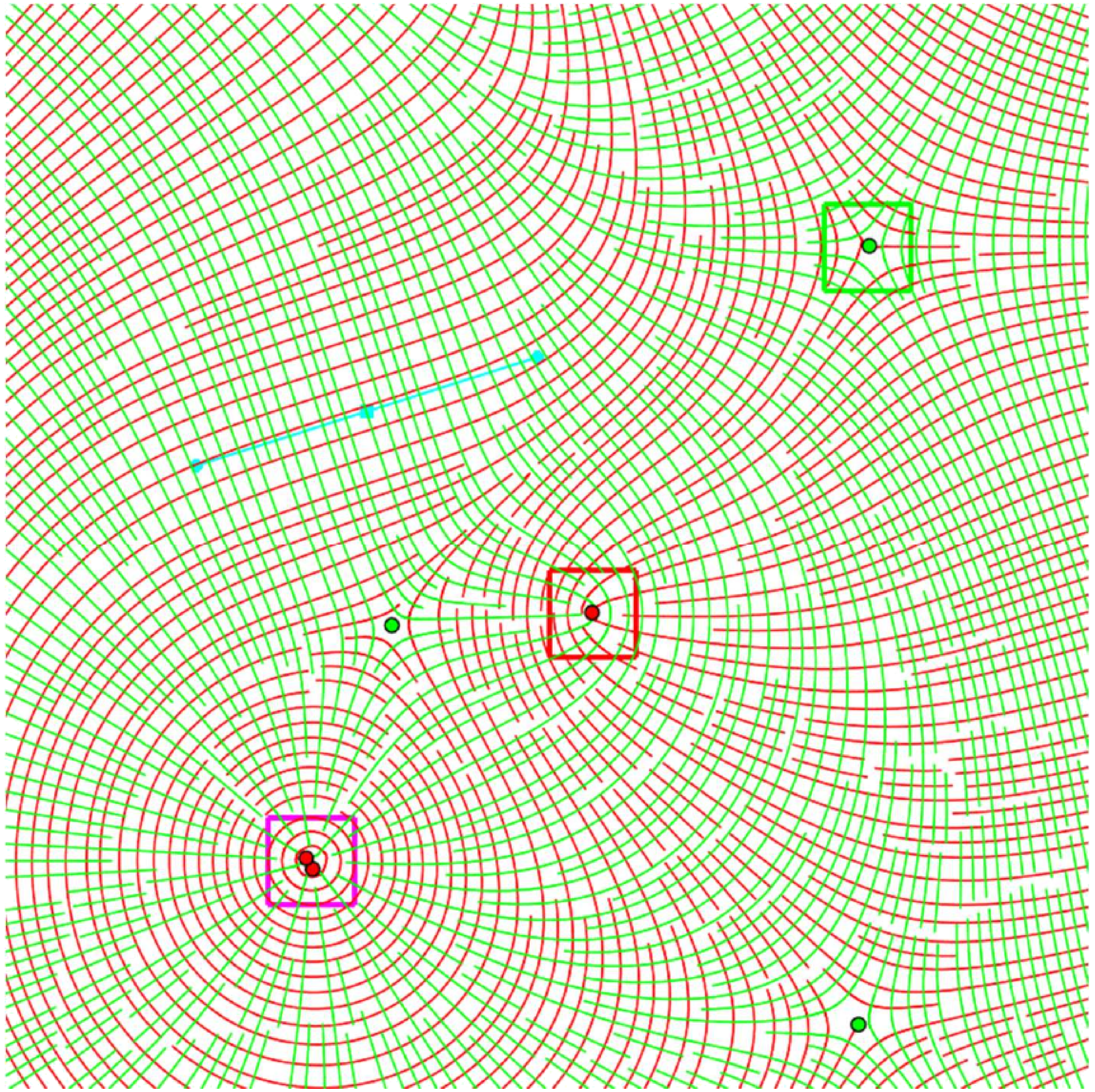

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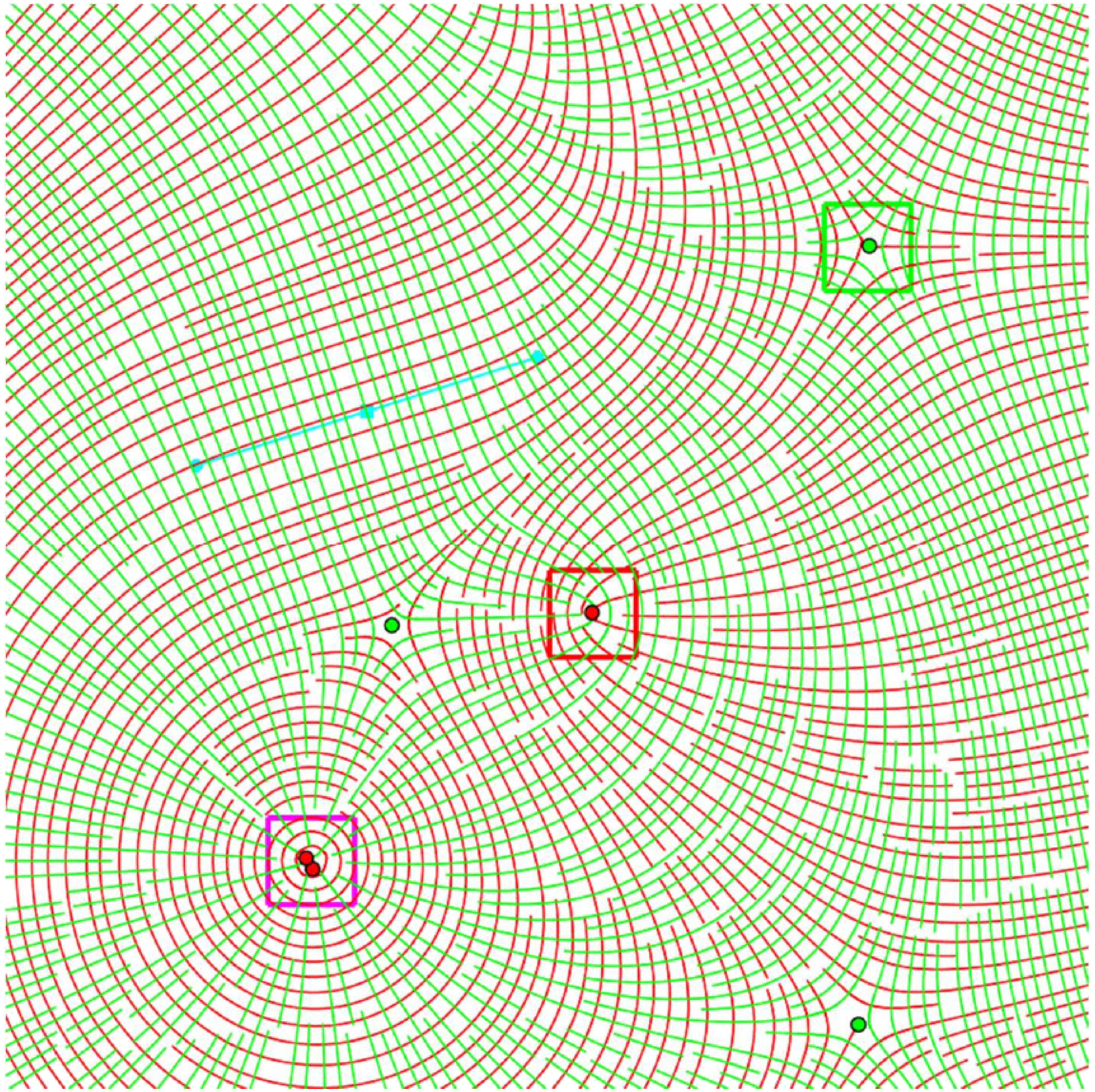

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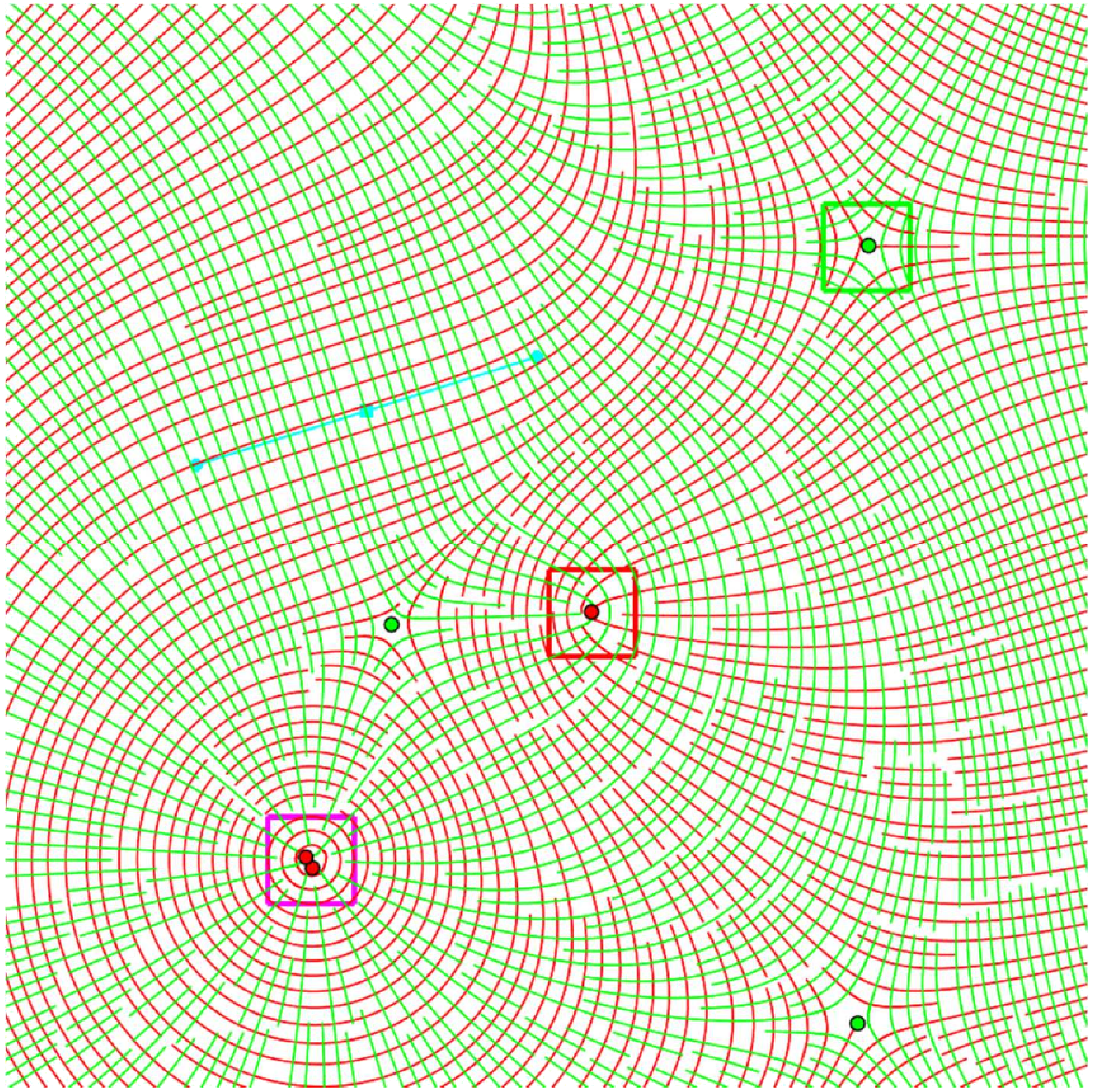
  - $det(\tilde{D}) < 0$
    - “Trisector”



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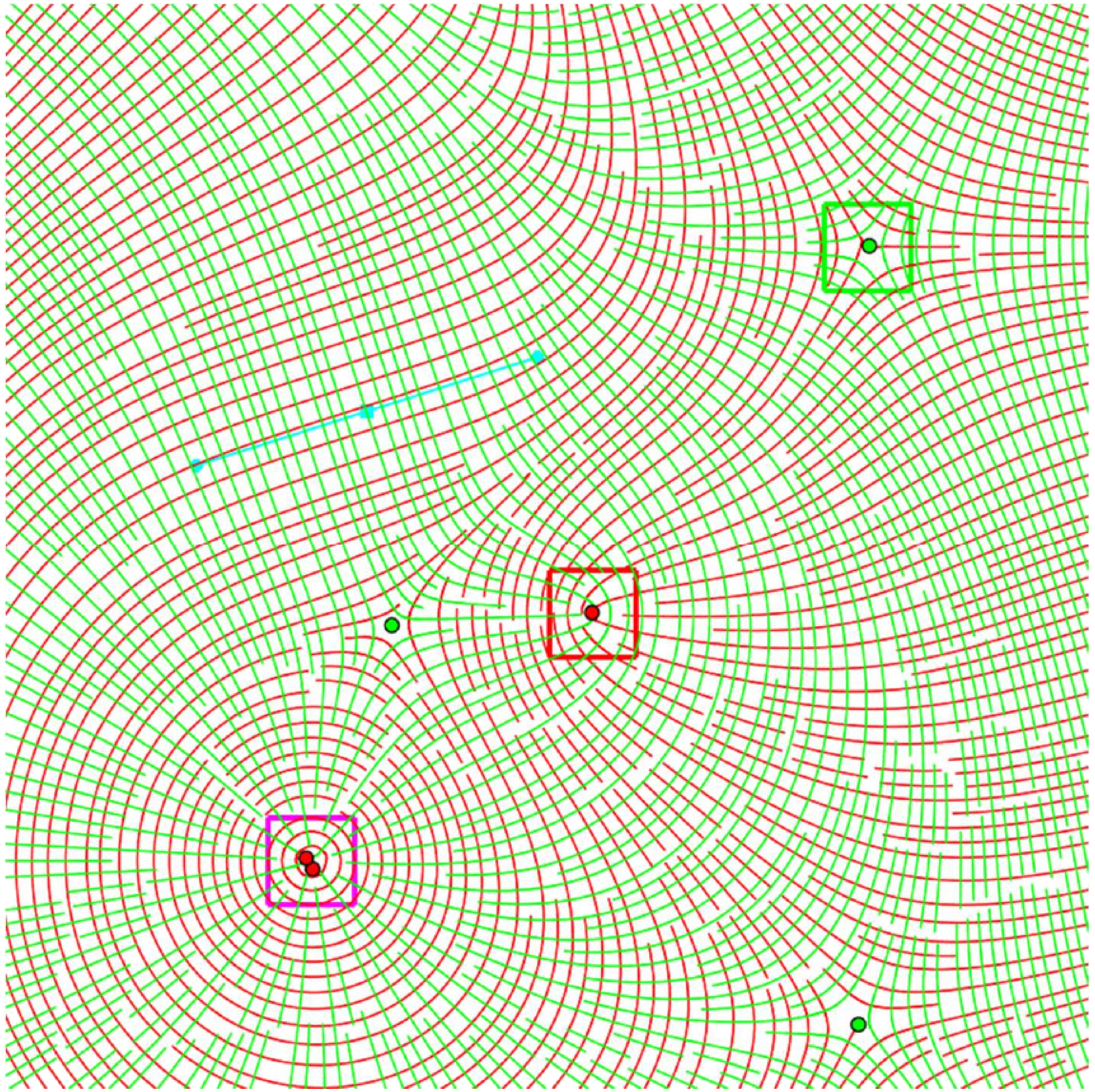

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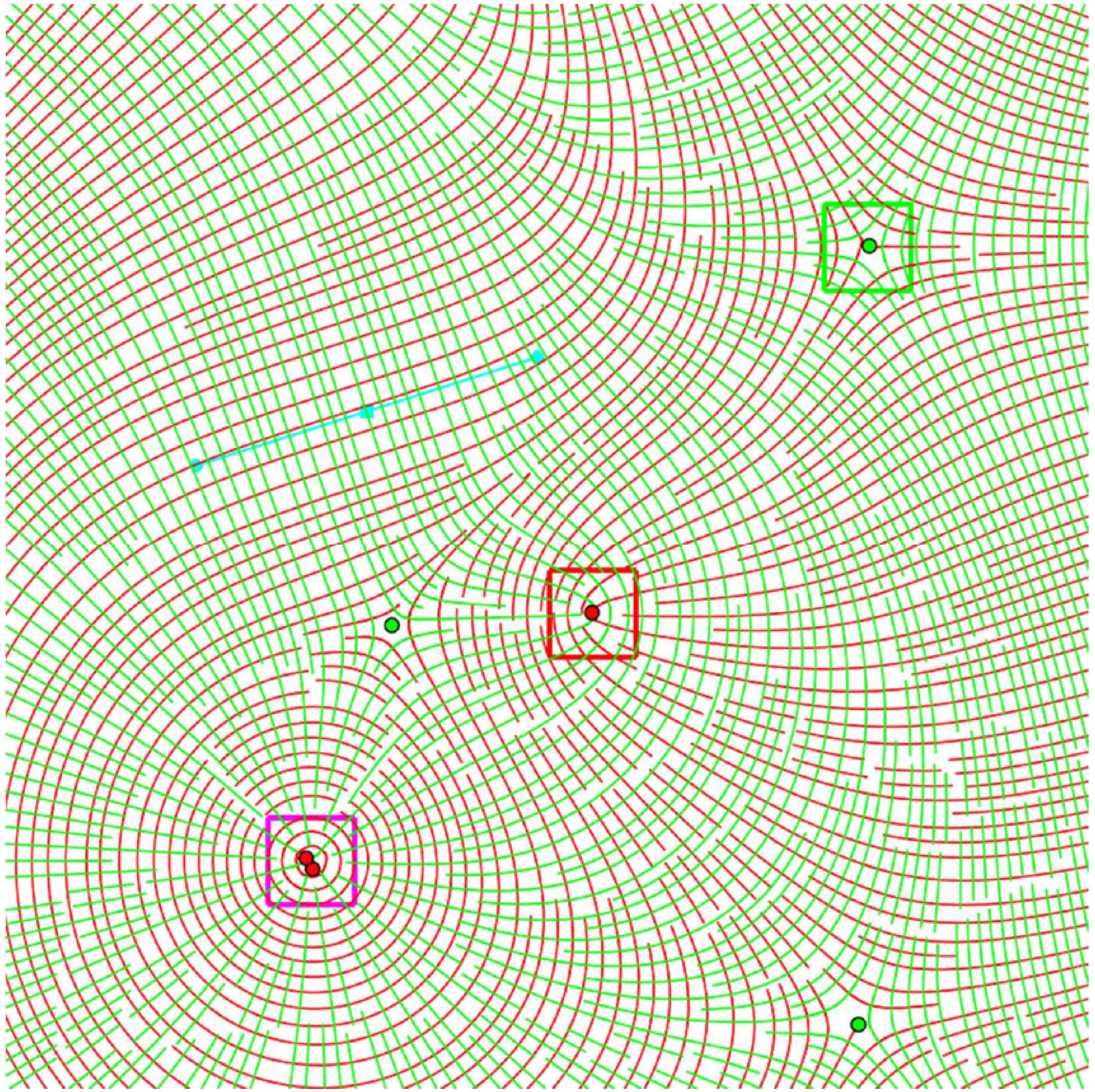

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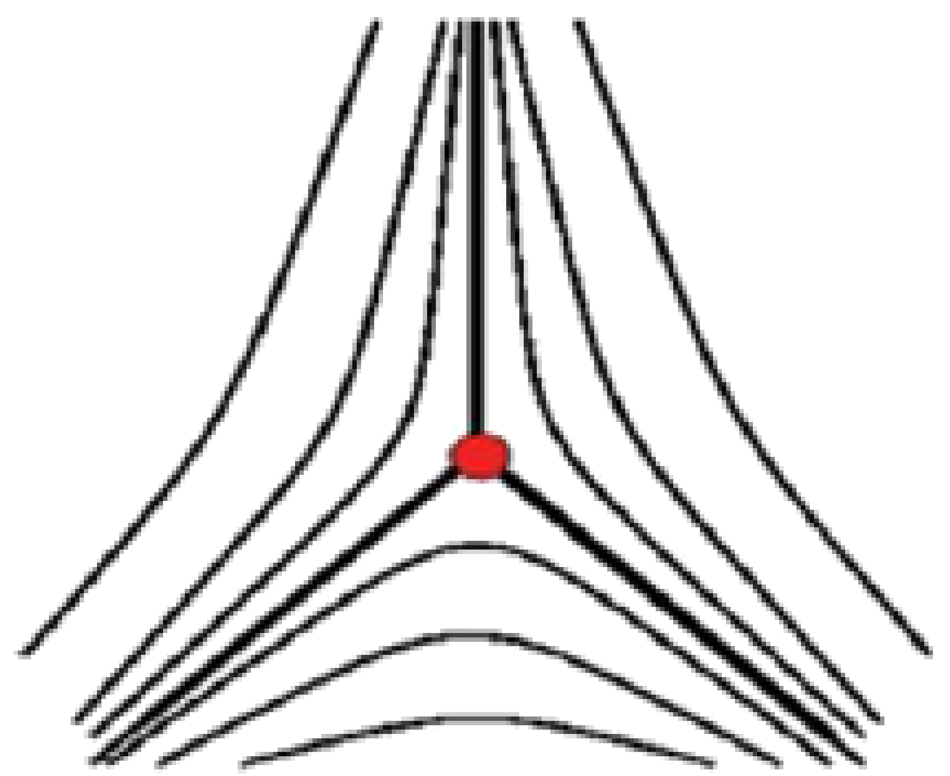
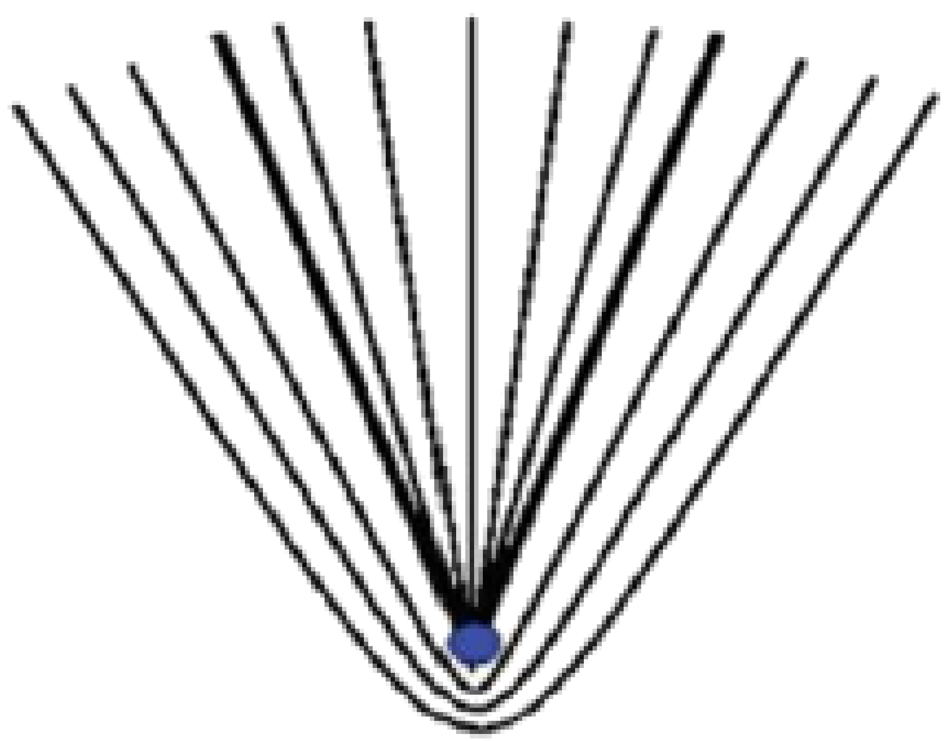
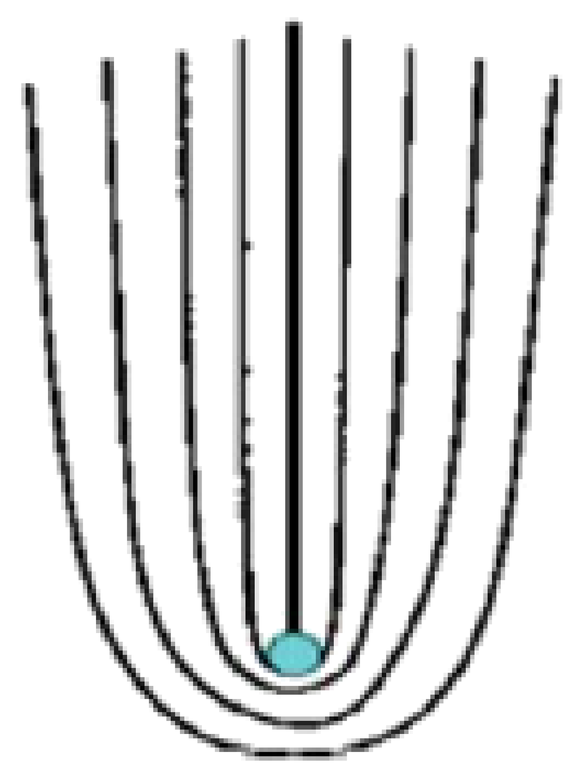


Trisector



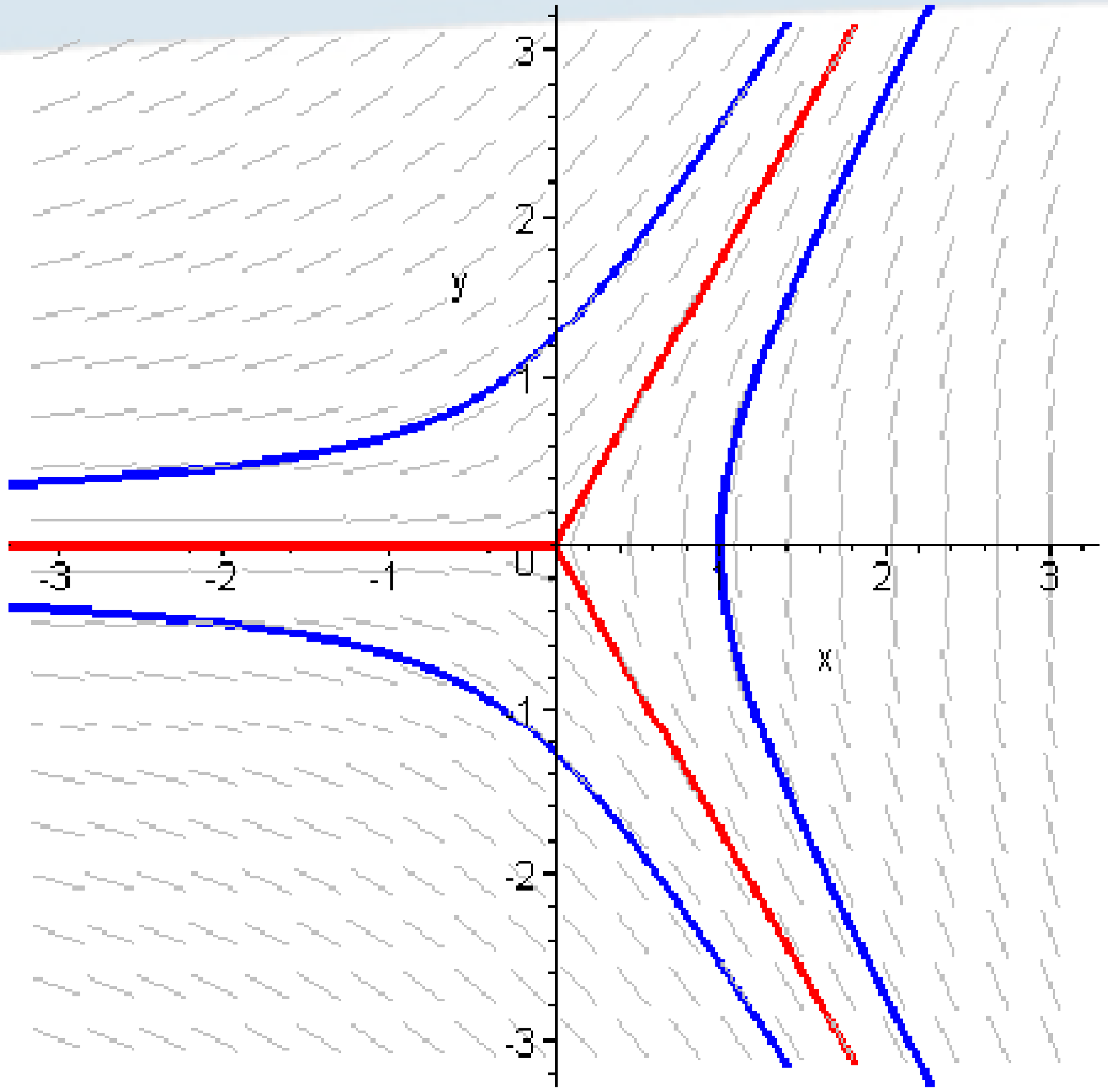
Wedge I



[Tricoche]

Wedge II

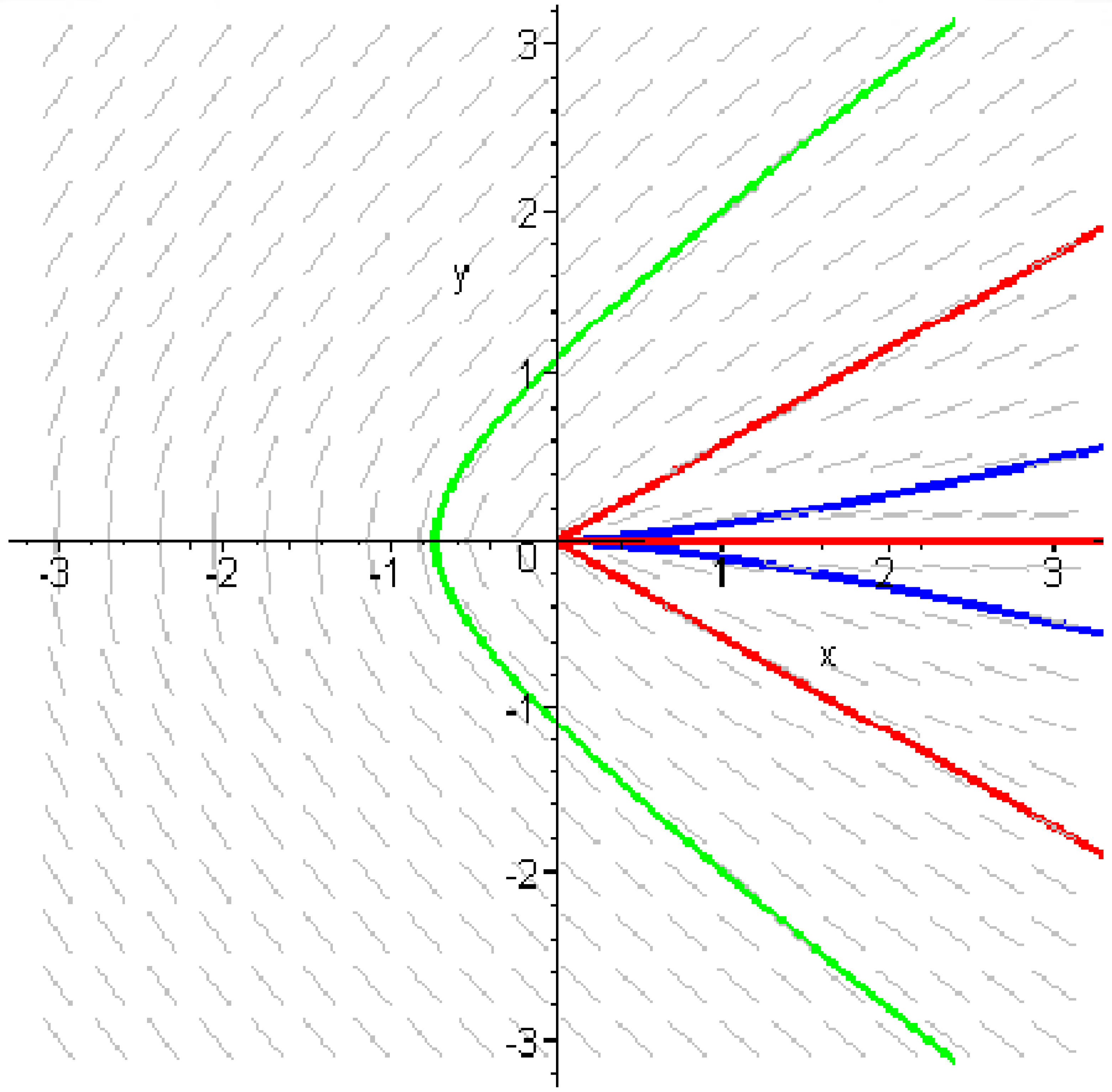
Critical points in tensor fields



trisector

$$\mathbf{T} = \begin{pmatrix} 1-2x & y \\ y & 1 \end{pmatrix}$$

$$\delta = -1$$



double wedge

$$\mathbf{T} = \begin{pmatrix} 1+2x/3 & y \\ y & 1 \end{pmatrix}$$

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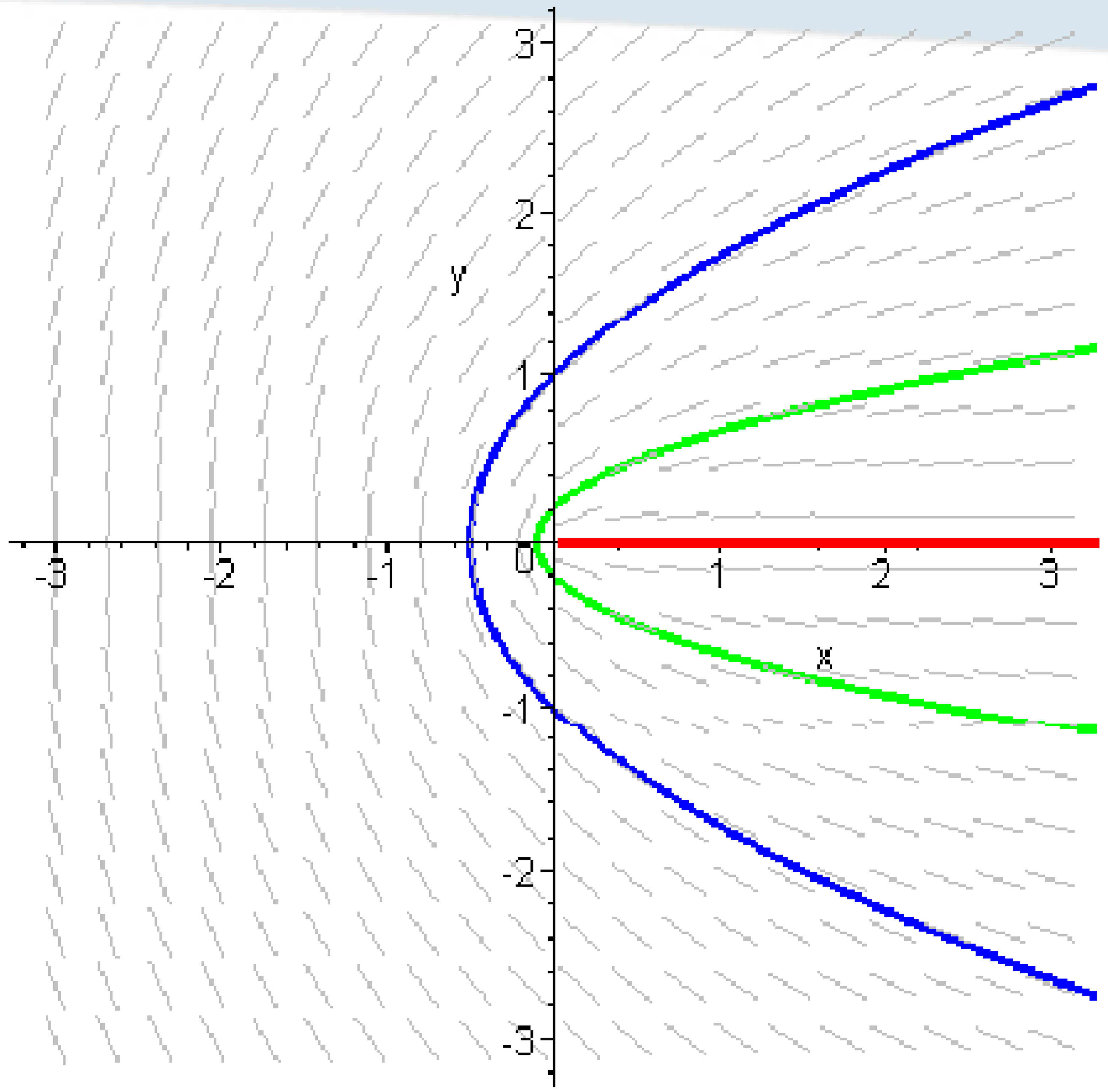

[Peikert]

single wedge

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High-order degenerate points

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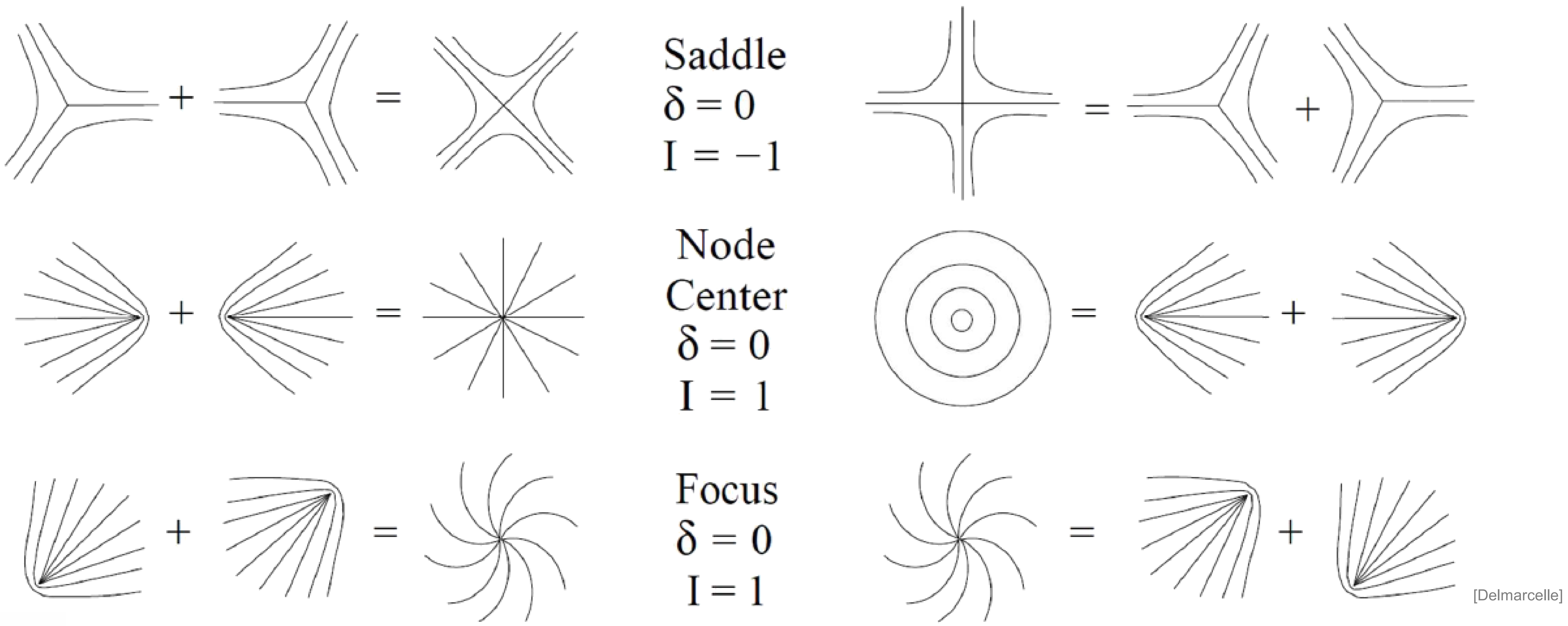

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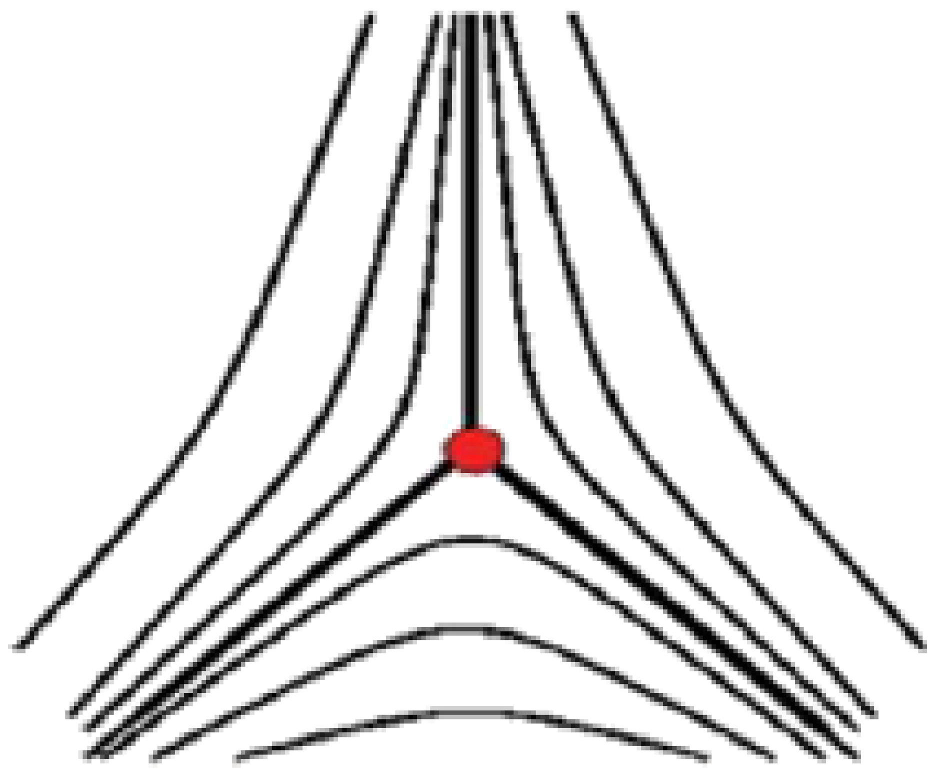

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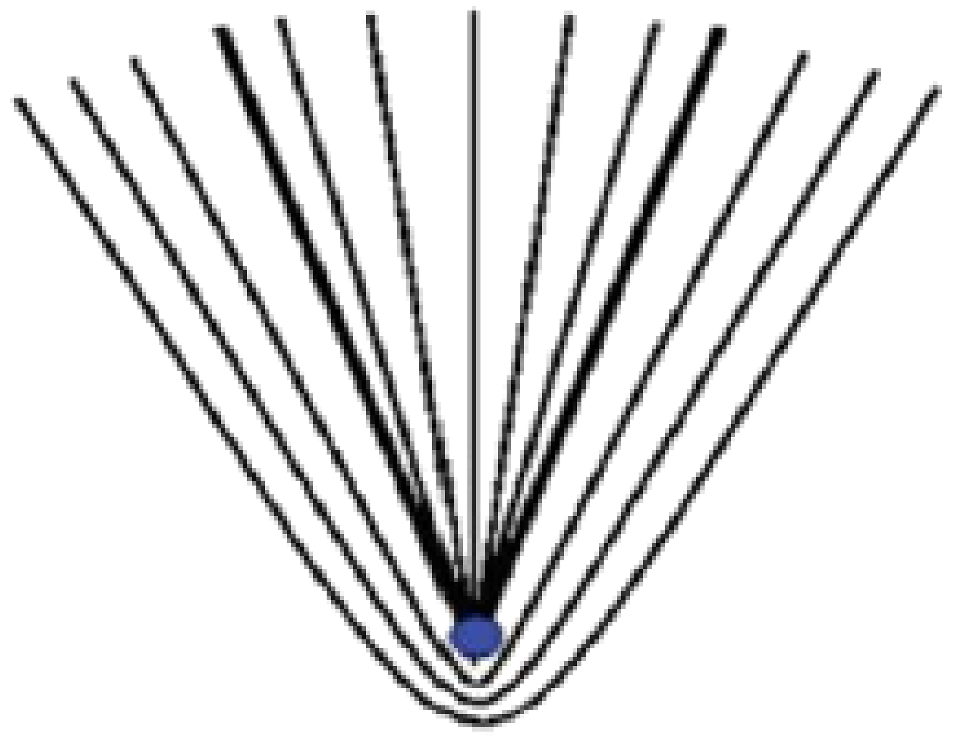

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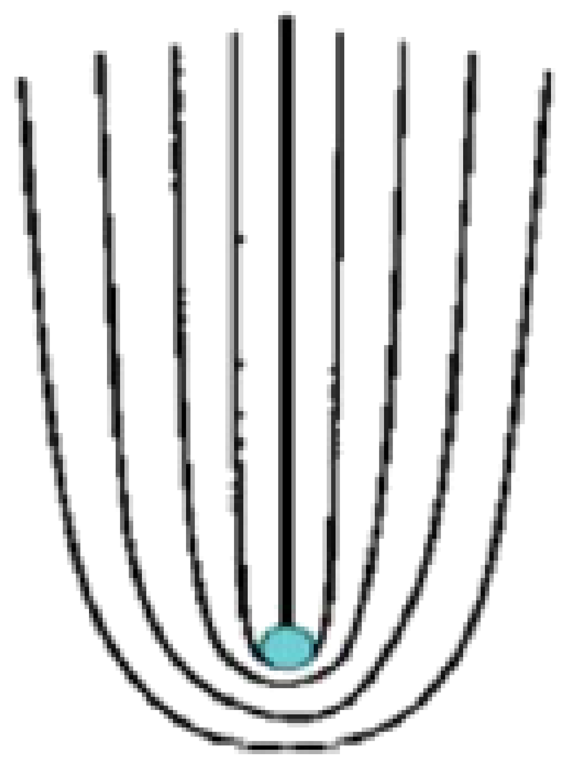

[Tricoche]

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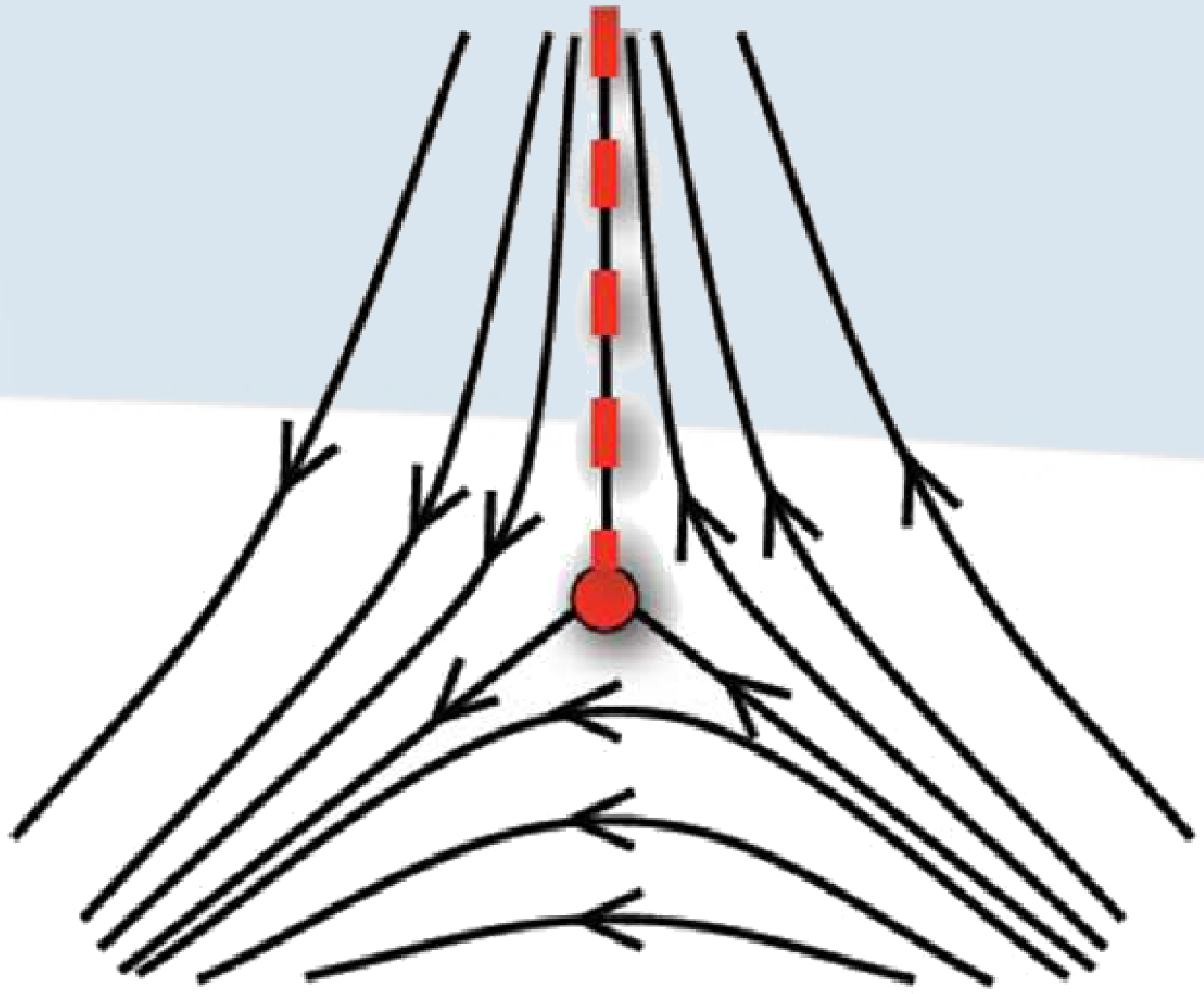

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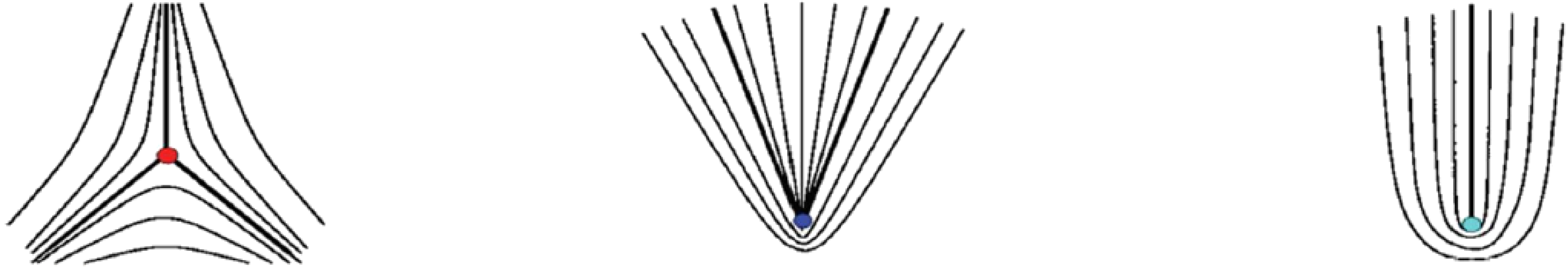

[Tricoche]

Trisector **Wedge I** **Wedge II**

Tensor field decomposition

- Now

Tensor field decomposition

- Now
  - What's the relation between the critical points?

Tensor field decomposition

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[Zhang]

Tensor field separatrices

- From the critical points

[Zhang]

Tensor field separatrices

- From the critical points
  - Hyper-streamline
    - Major eigenvalue

[Zhang]

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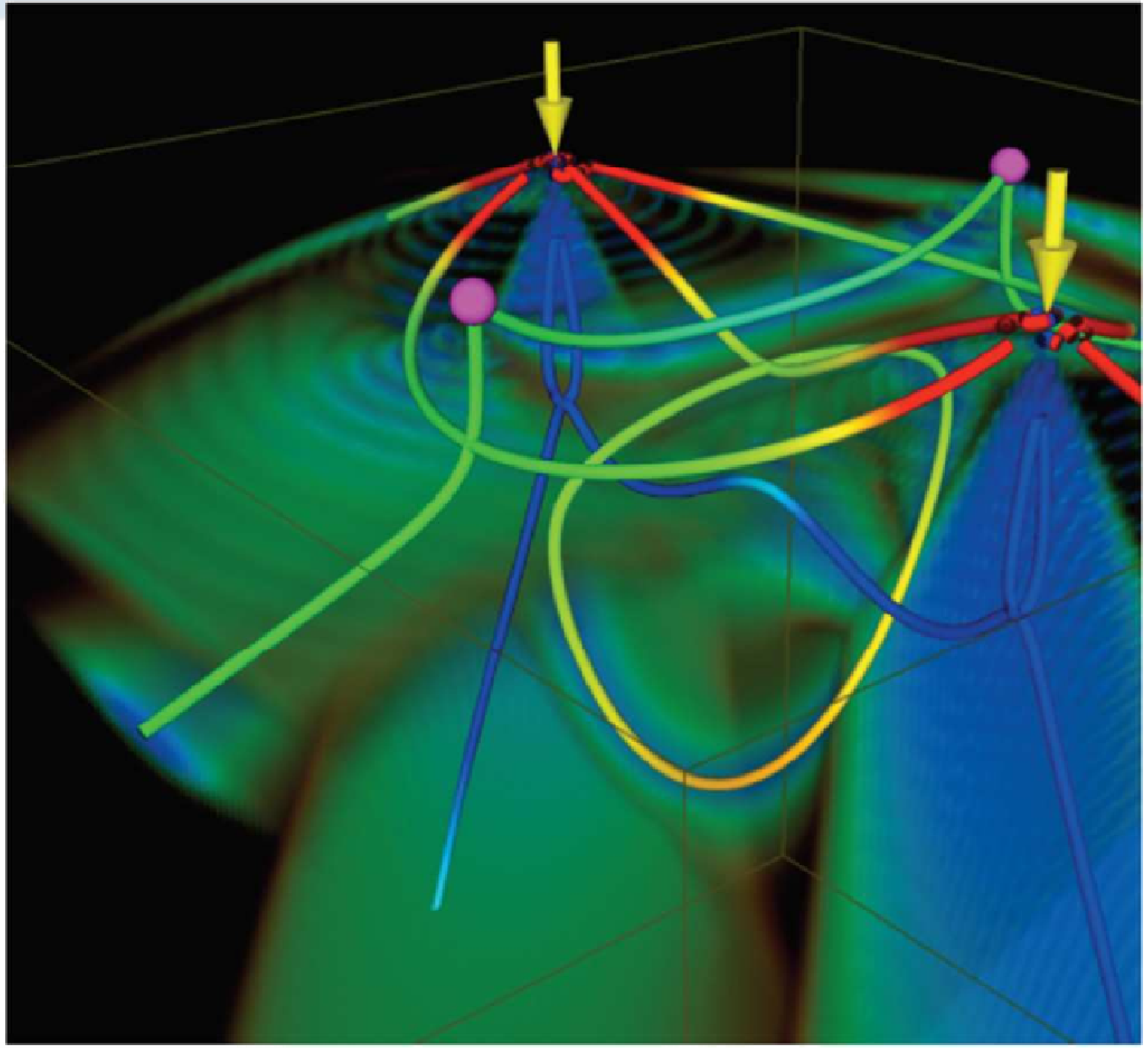
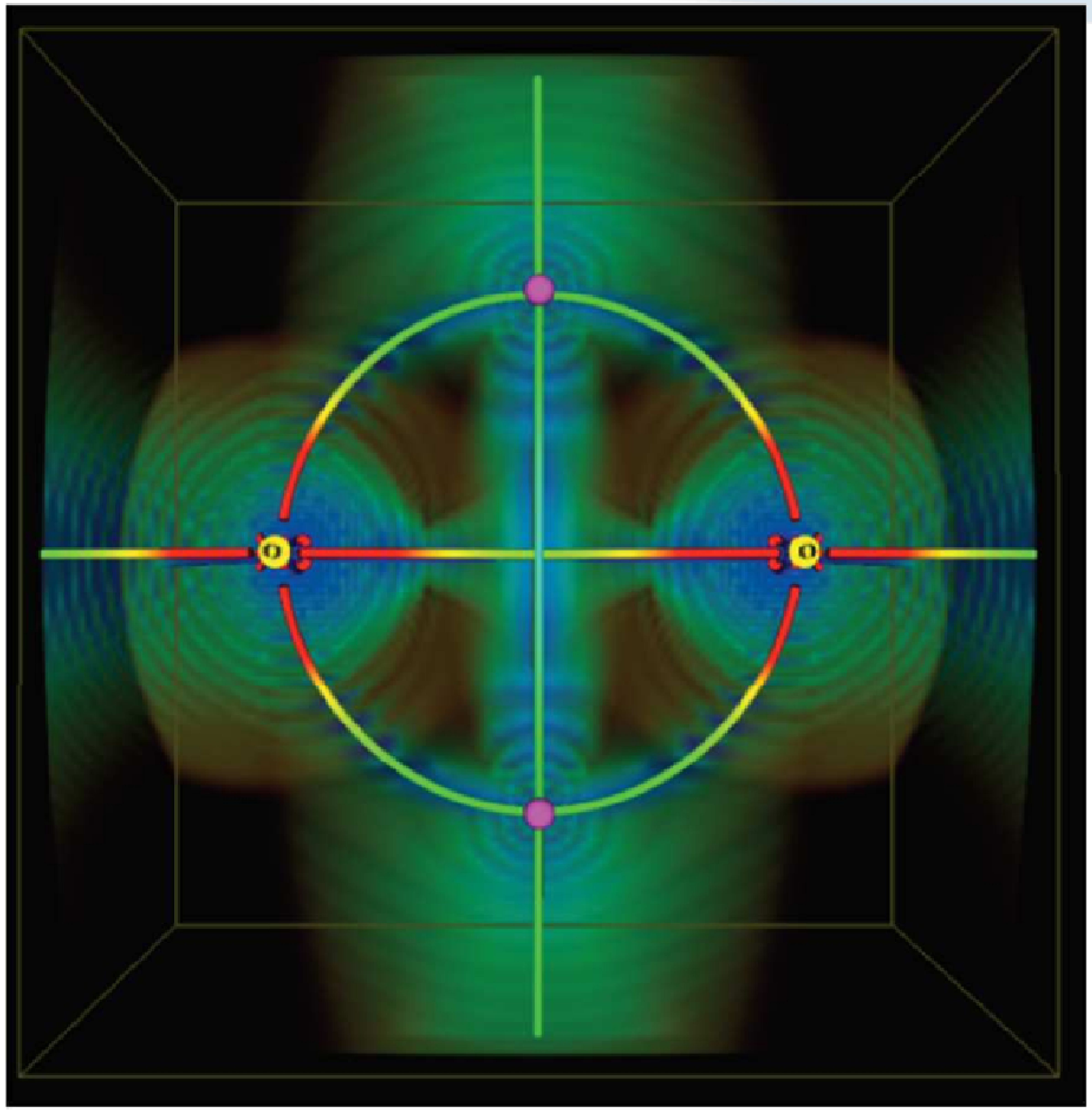
Tensor field decomposition

[Zhang]

Volumetric domains





[Zheng]

Tensor field topology 2.0

- Critical points detection and classification

[Zhang]

Tensor field topology 2.0

- Critical points detection and classification
  - Matrix diagonalization

[Zhang]

Tensor field topology 2.0

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[Zhang]

Tensor field topology 2.0

- Scalar field topology

$$\chi(\mathcal{D}) = \sum_{i=0}^{d} (-1)^i \mu_f^i$$

[Zhang]

Tensor field topology 2.0

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$$\chi(\mathcal{D}) = \sum_{j=1}^{n} index(p_j)$$

[Zhang]

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- Similar phenomenon for tensor fields

[Zhang]

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- Similar phenomenon for tensor fields
  - Unexplored territory

[Zhang]

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  - Asymmetric tensor fields: work in progress

Farewell

- Domain representations

Farewell

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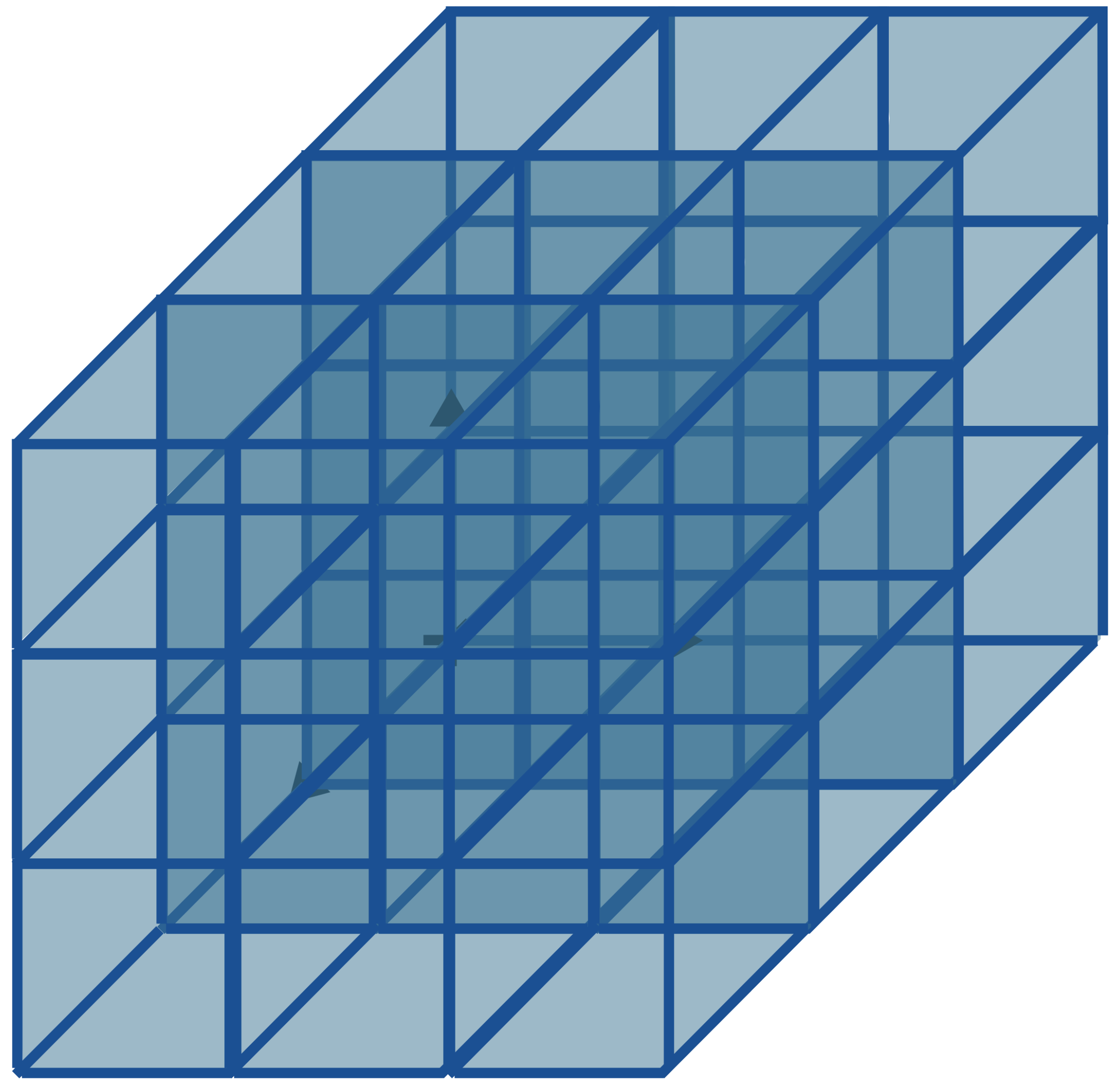

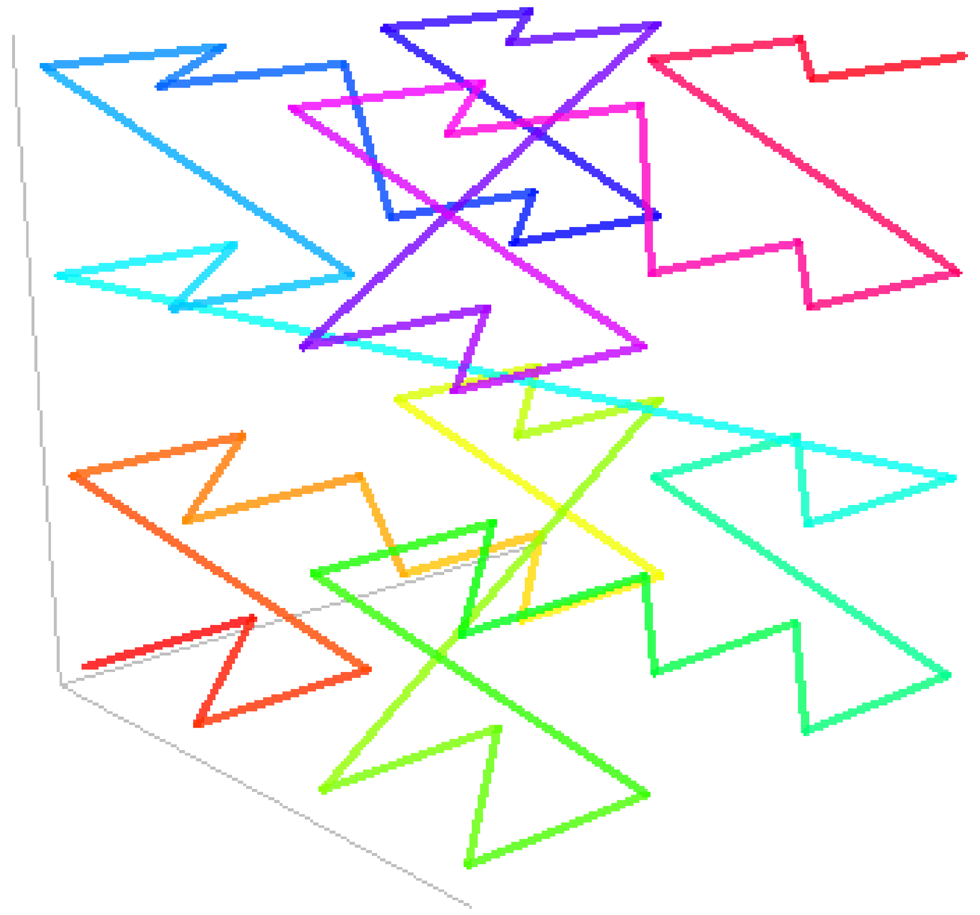

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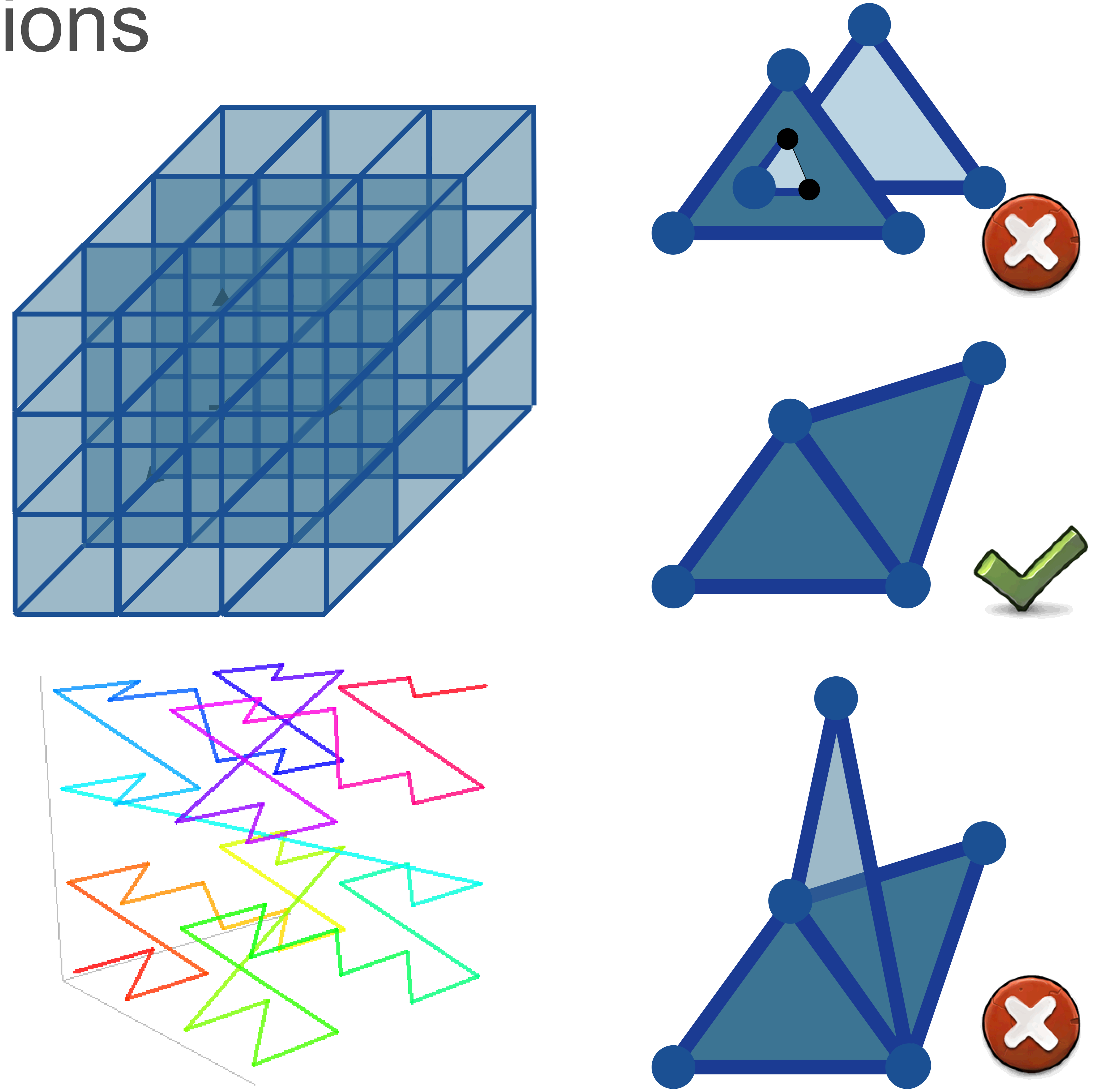

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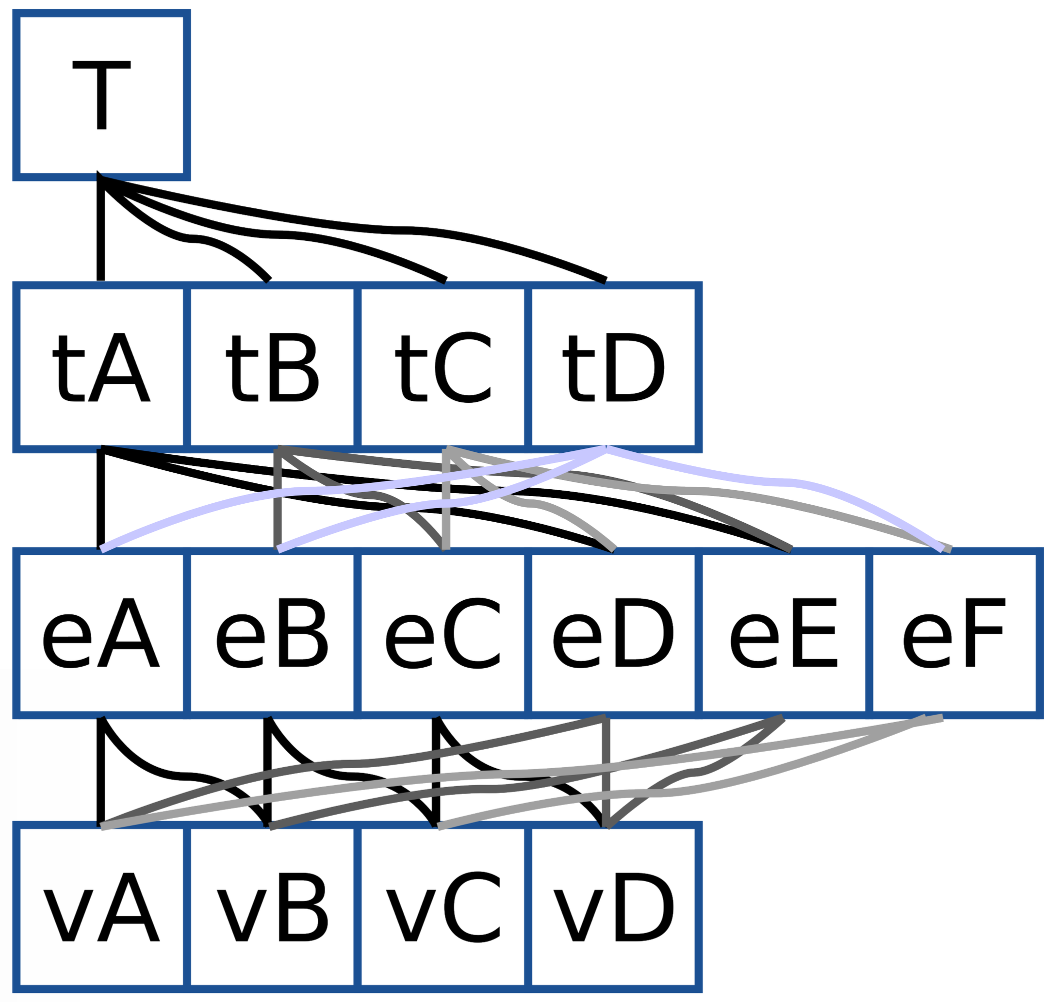
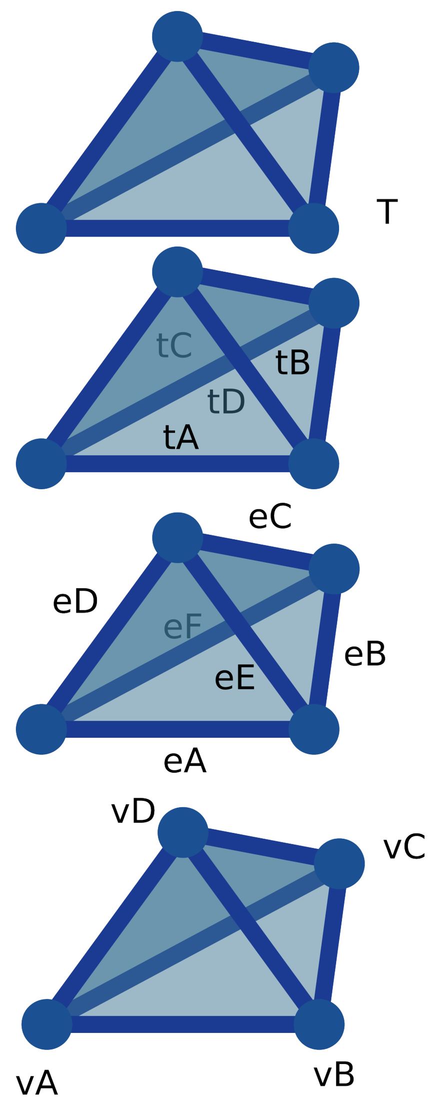
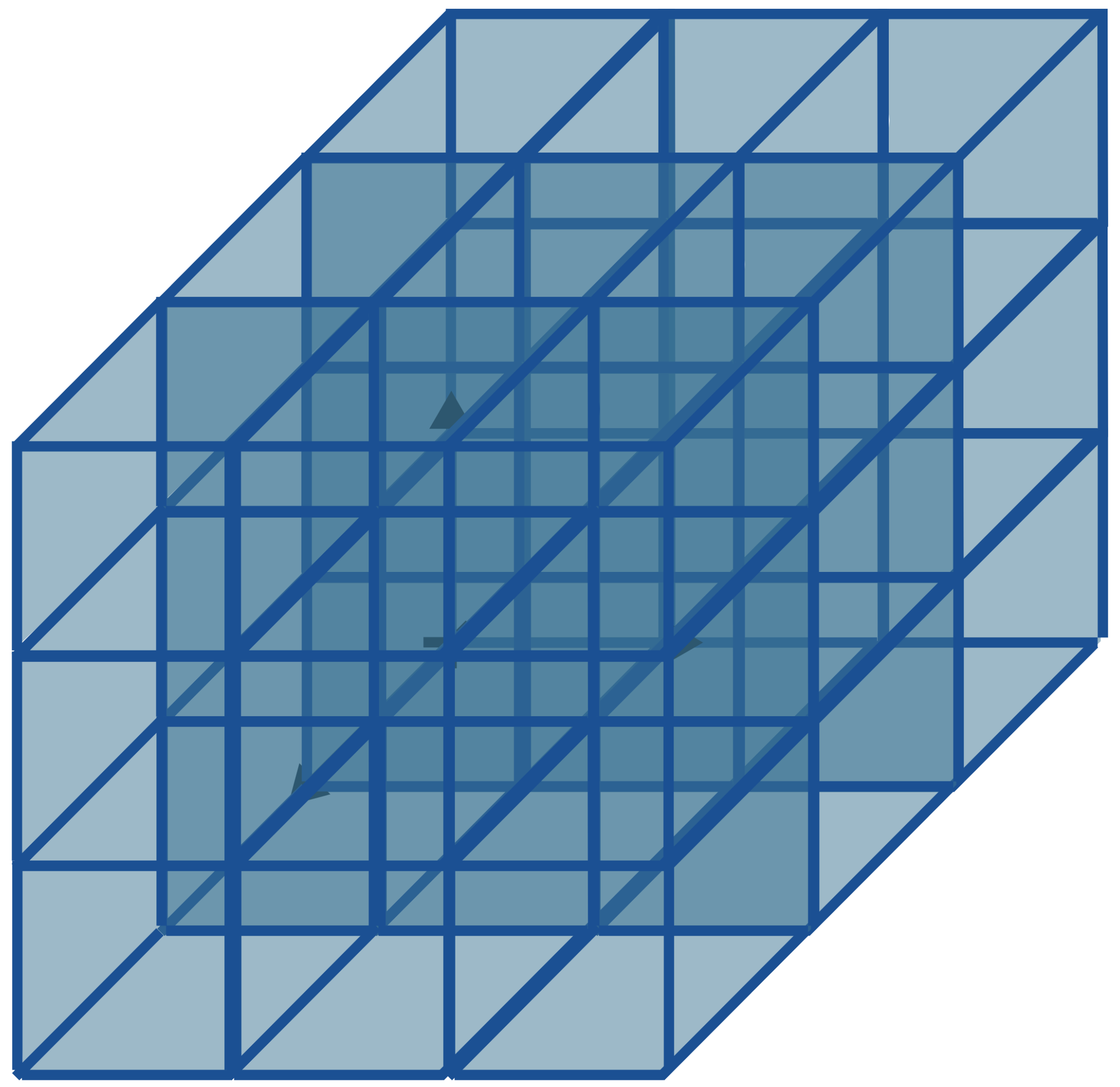
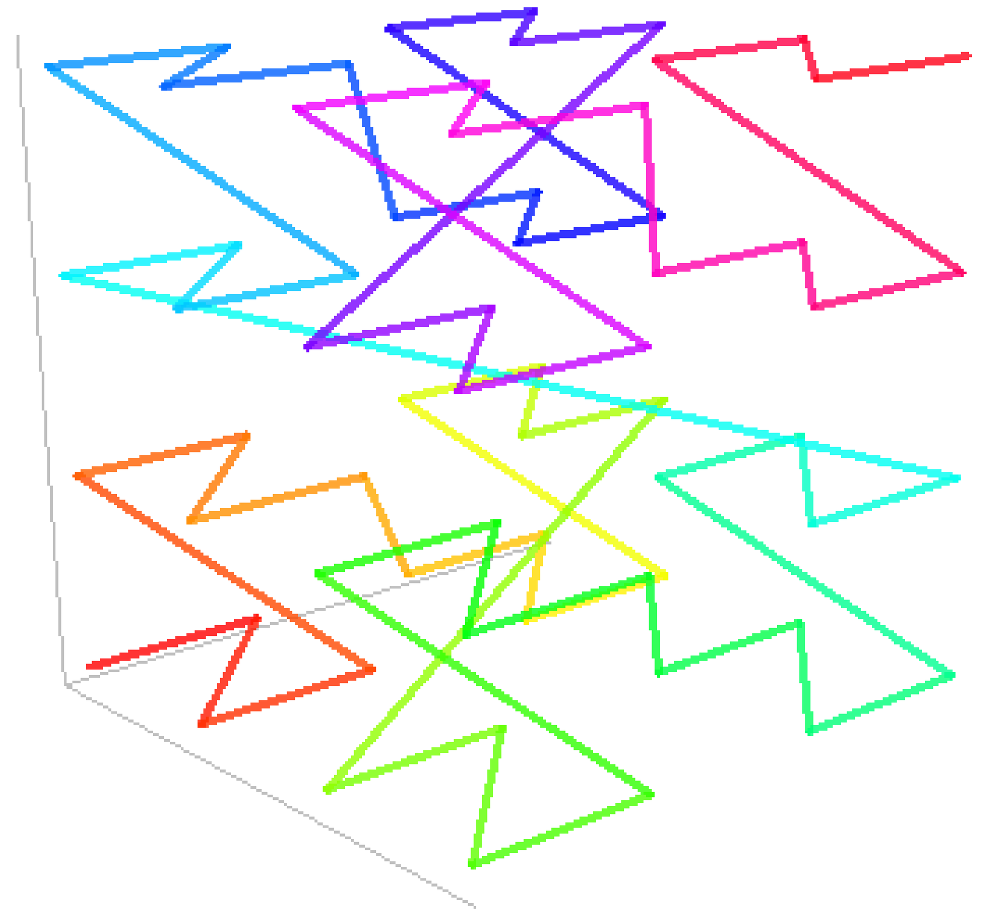
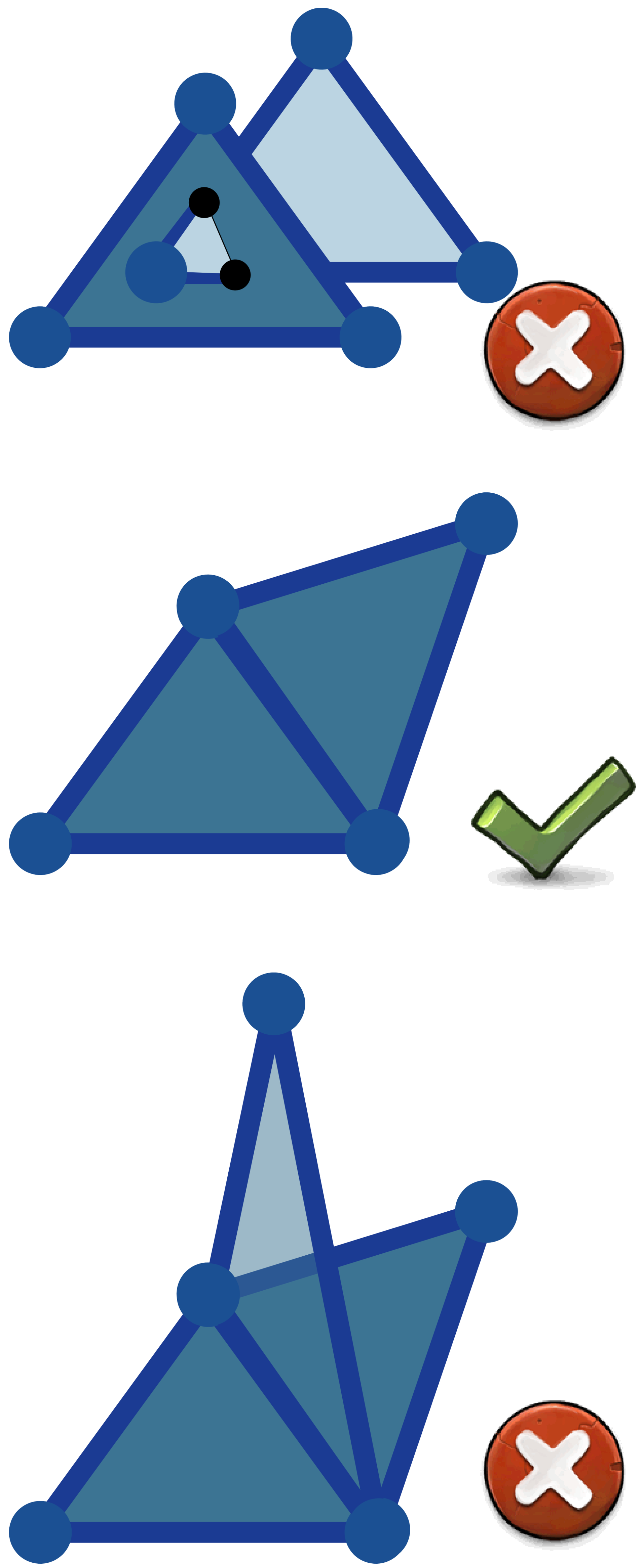




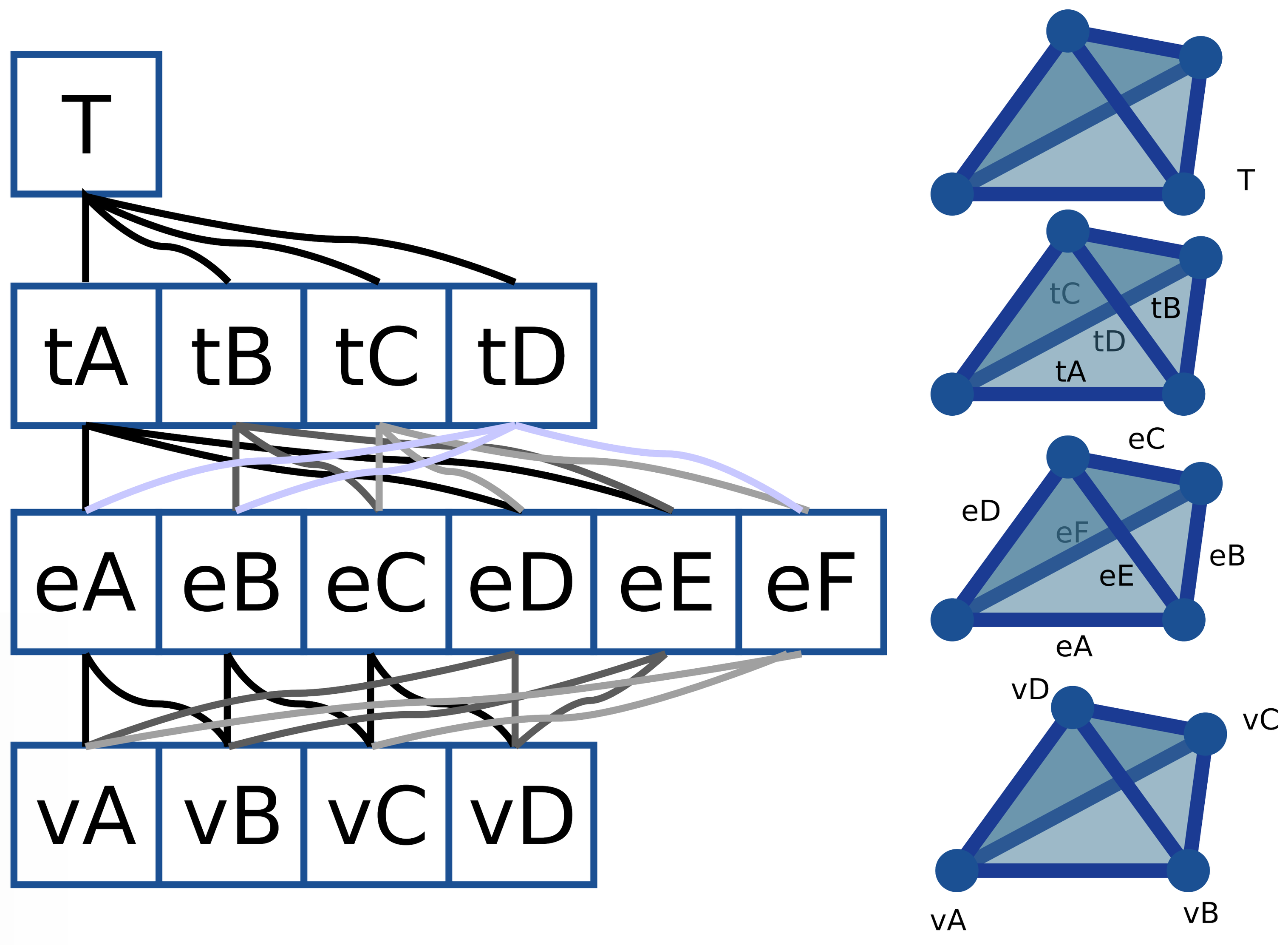


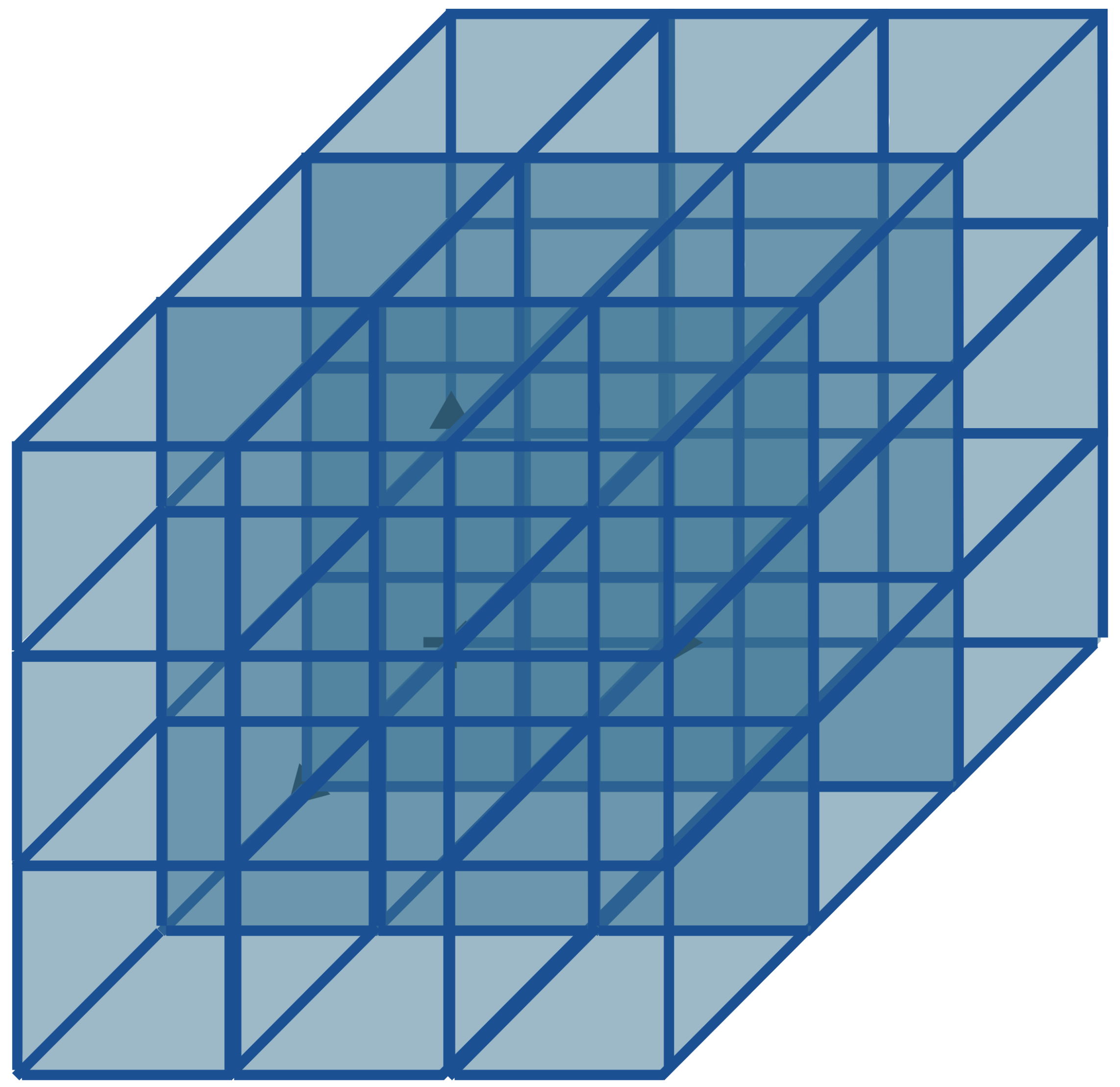


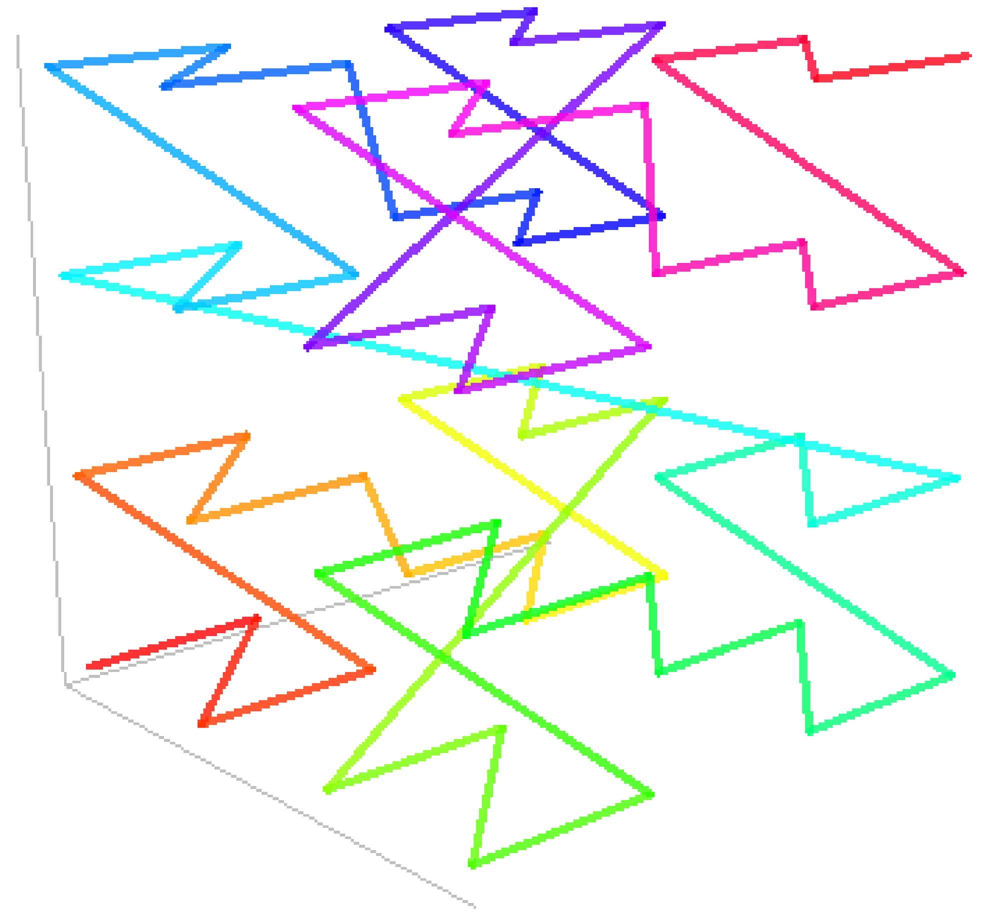


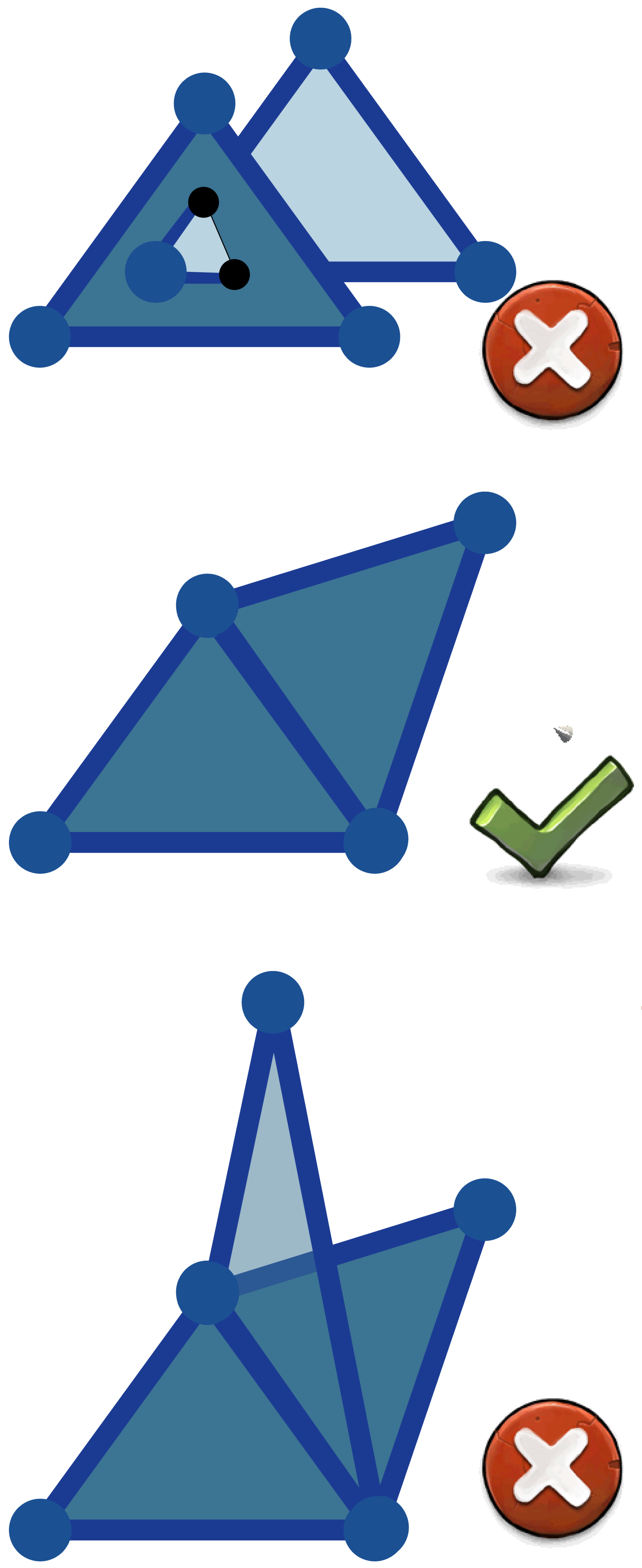
Farewell

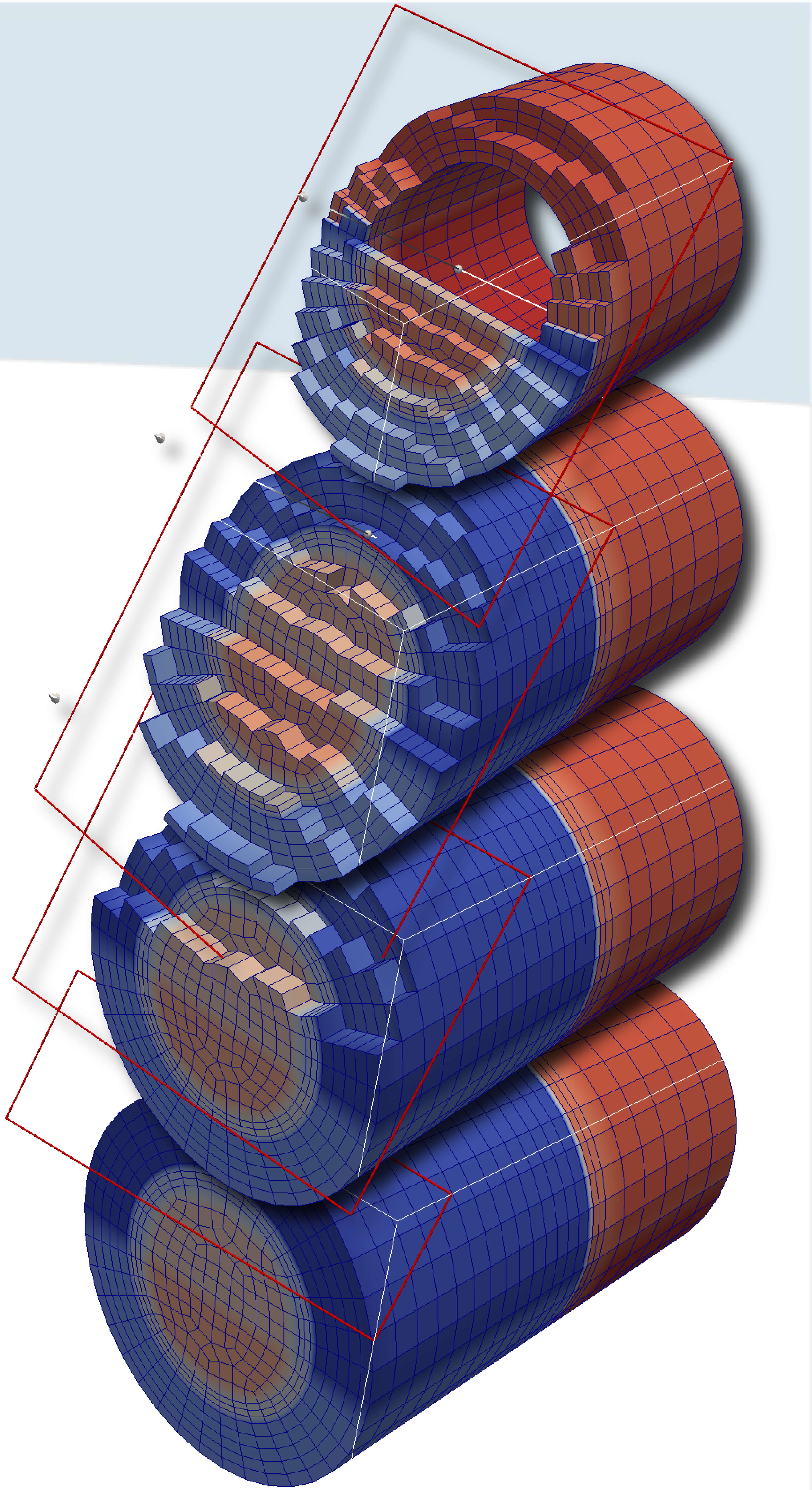
- Domain representations











Farewell

- Scalar fields

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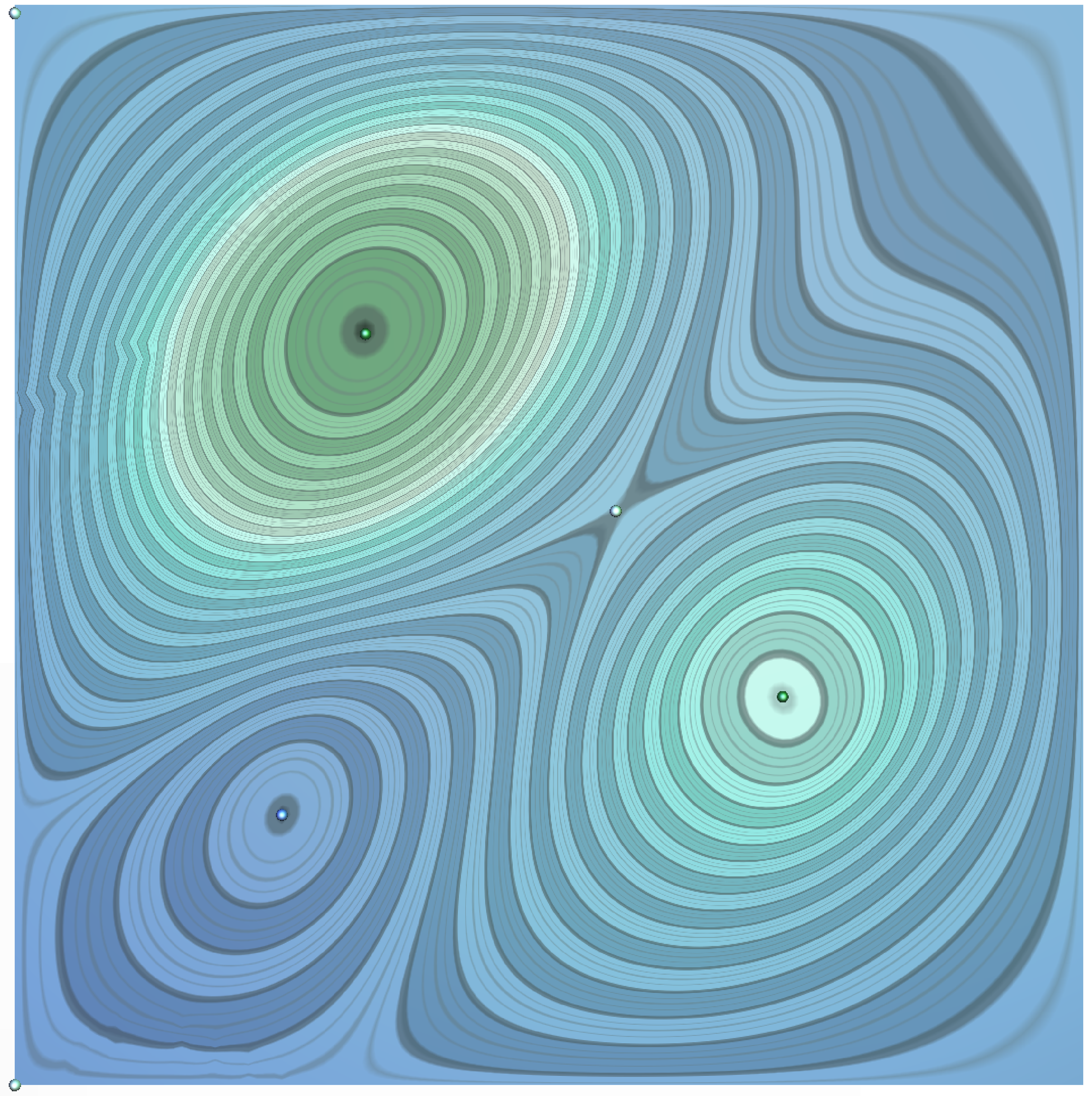
- Scalar fields



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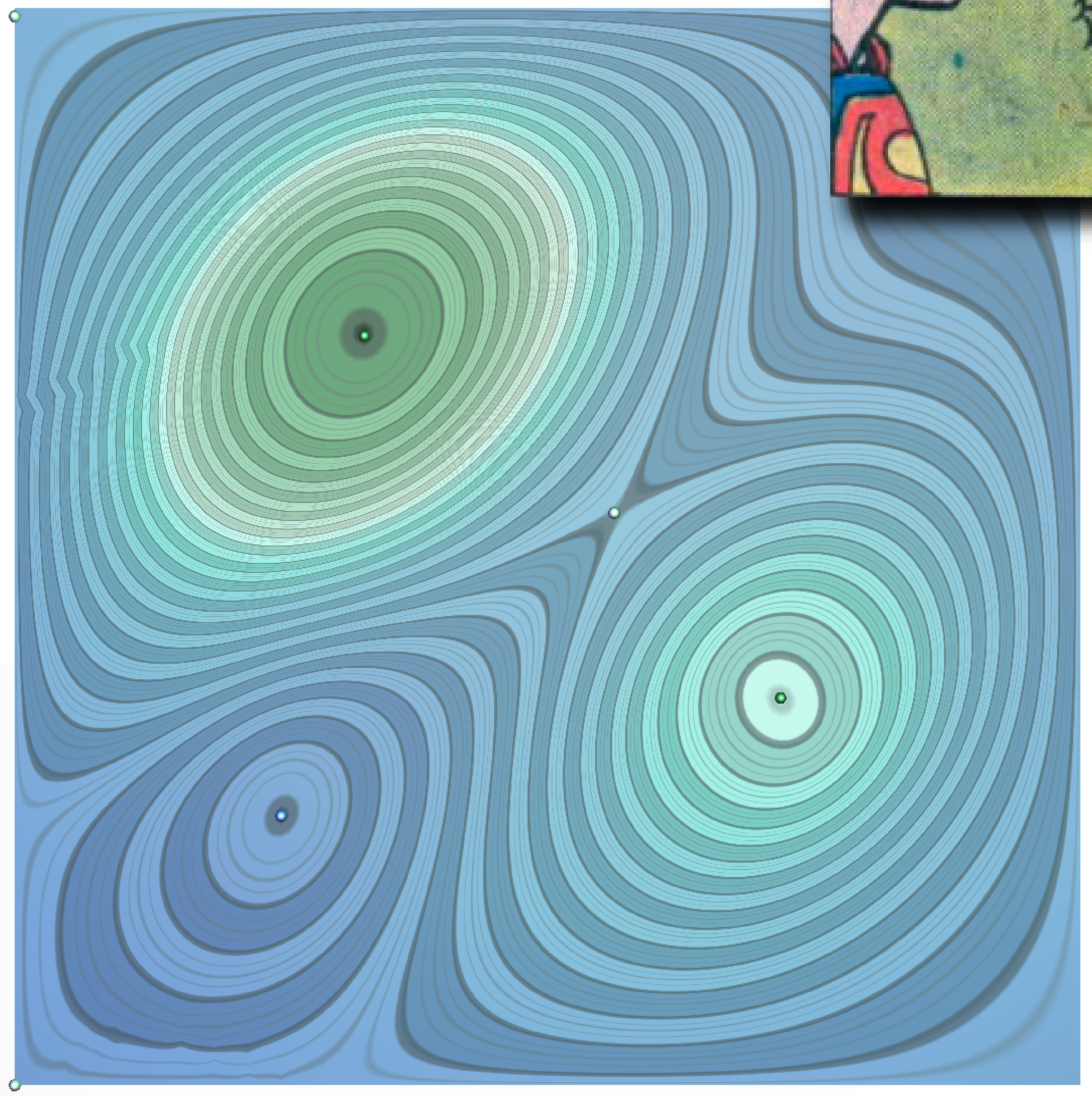

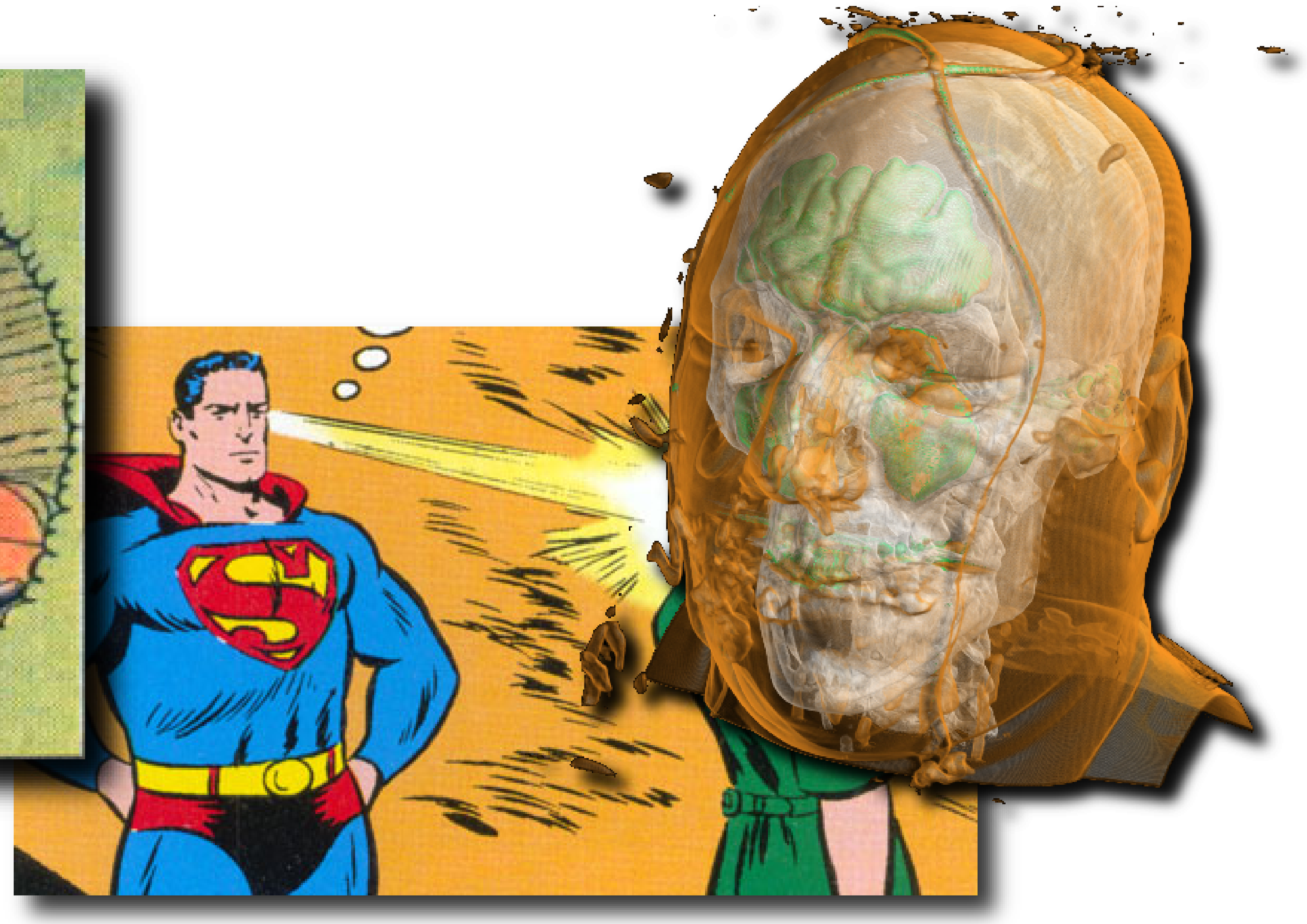

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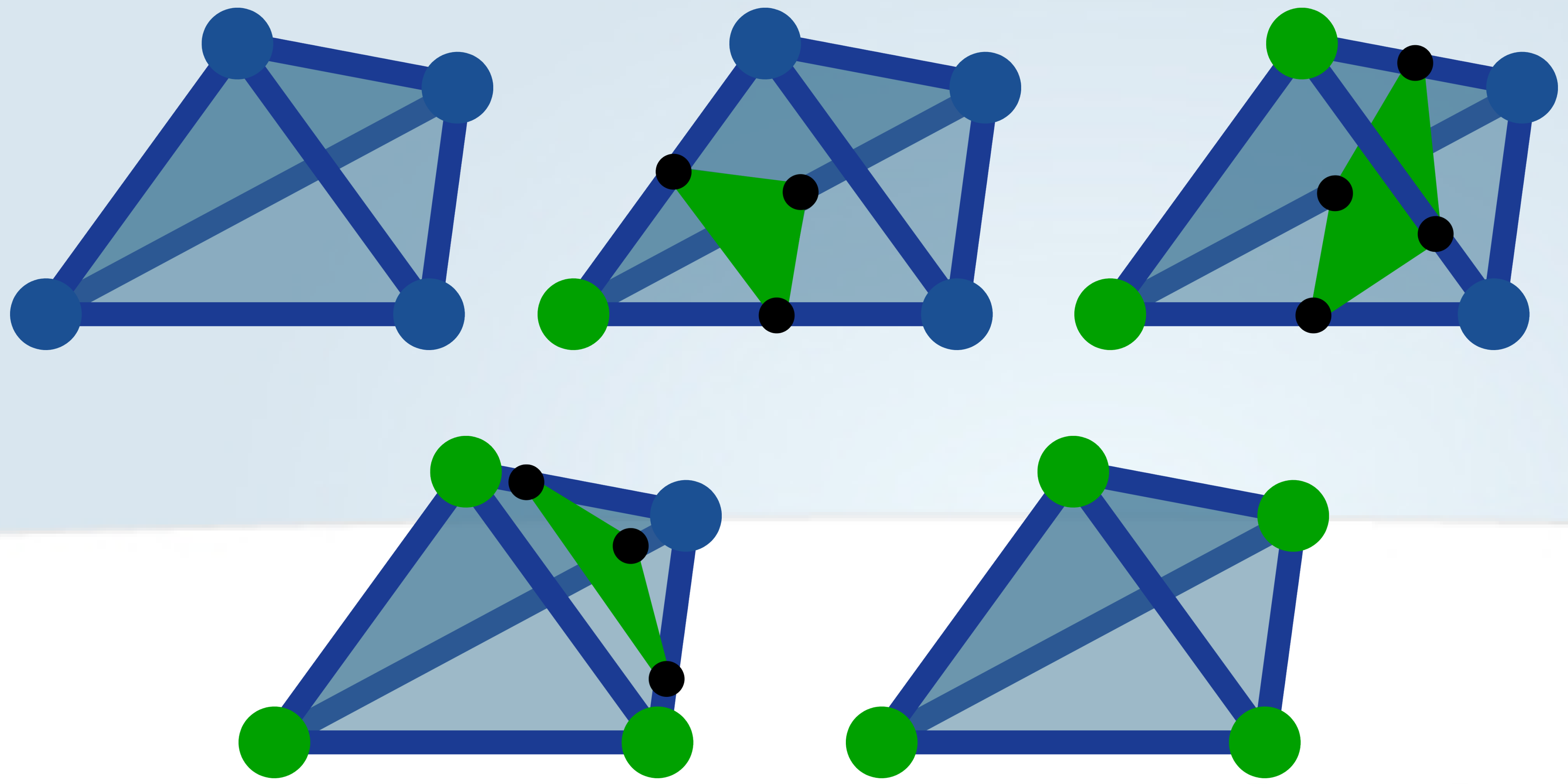

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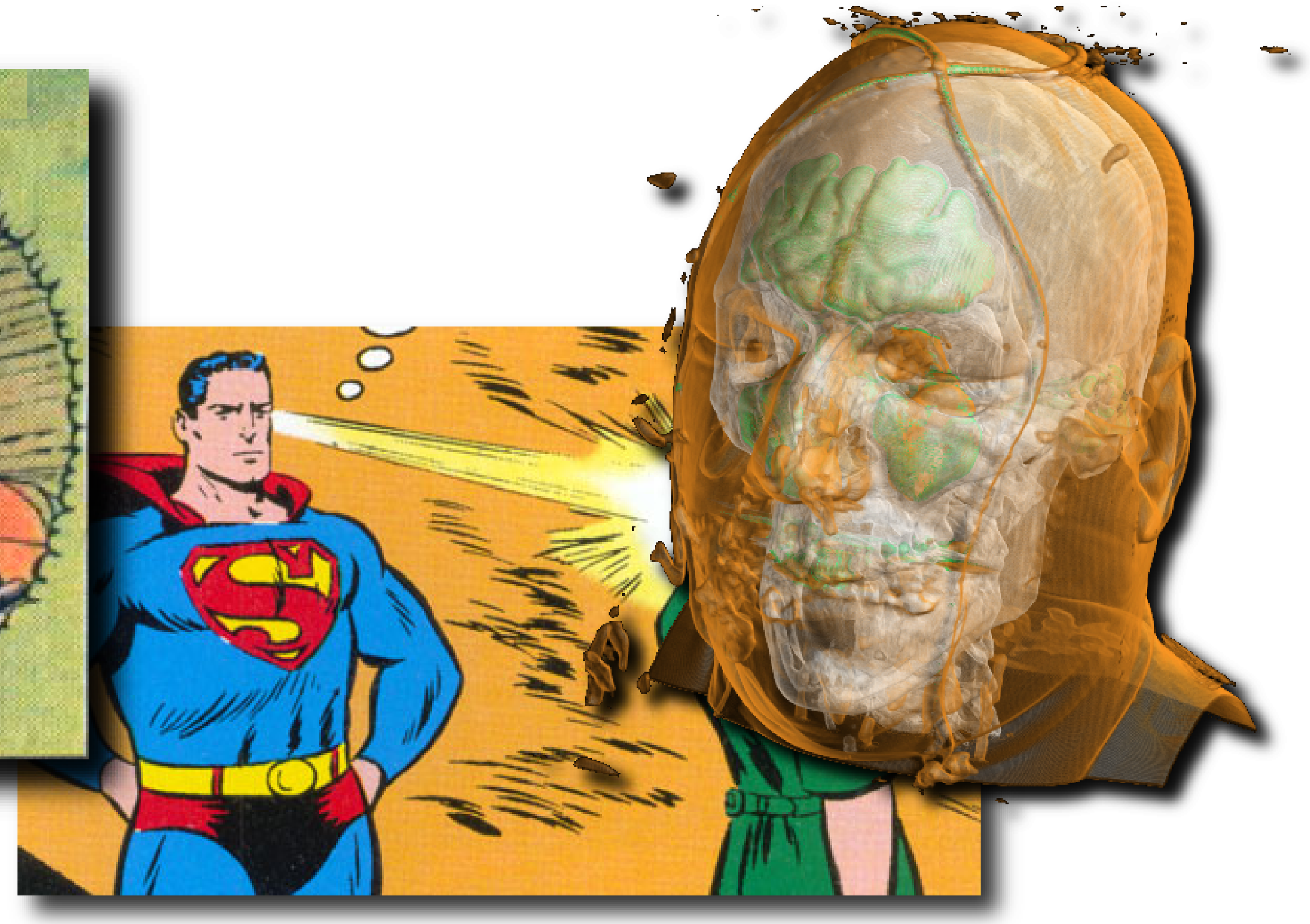

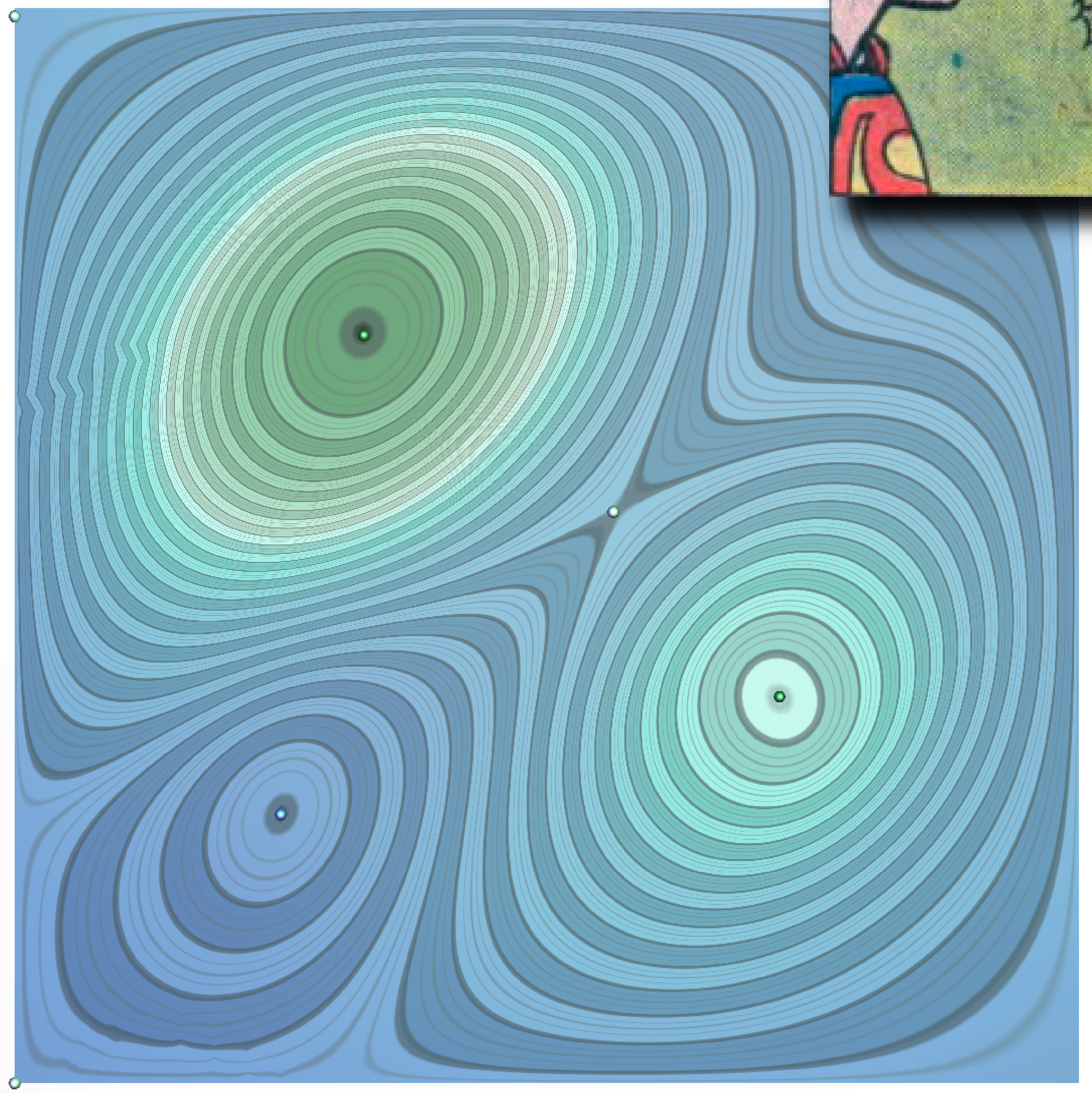

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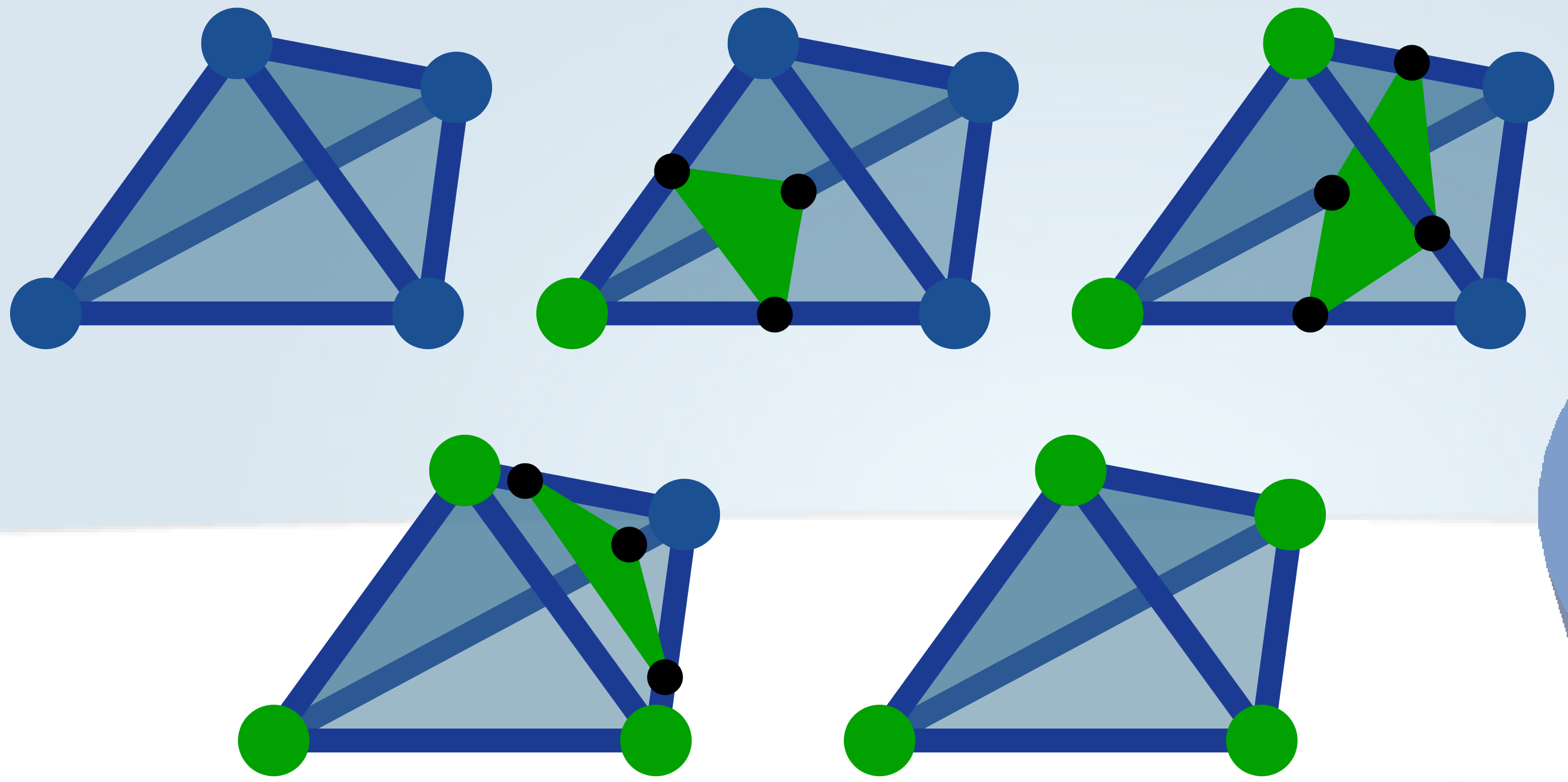

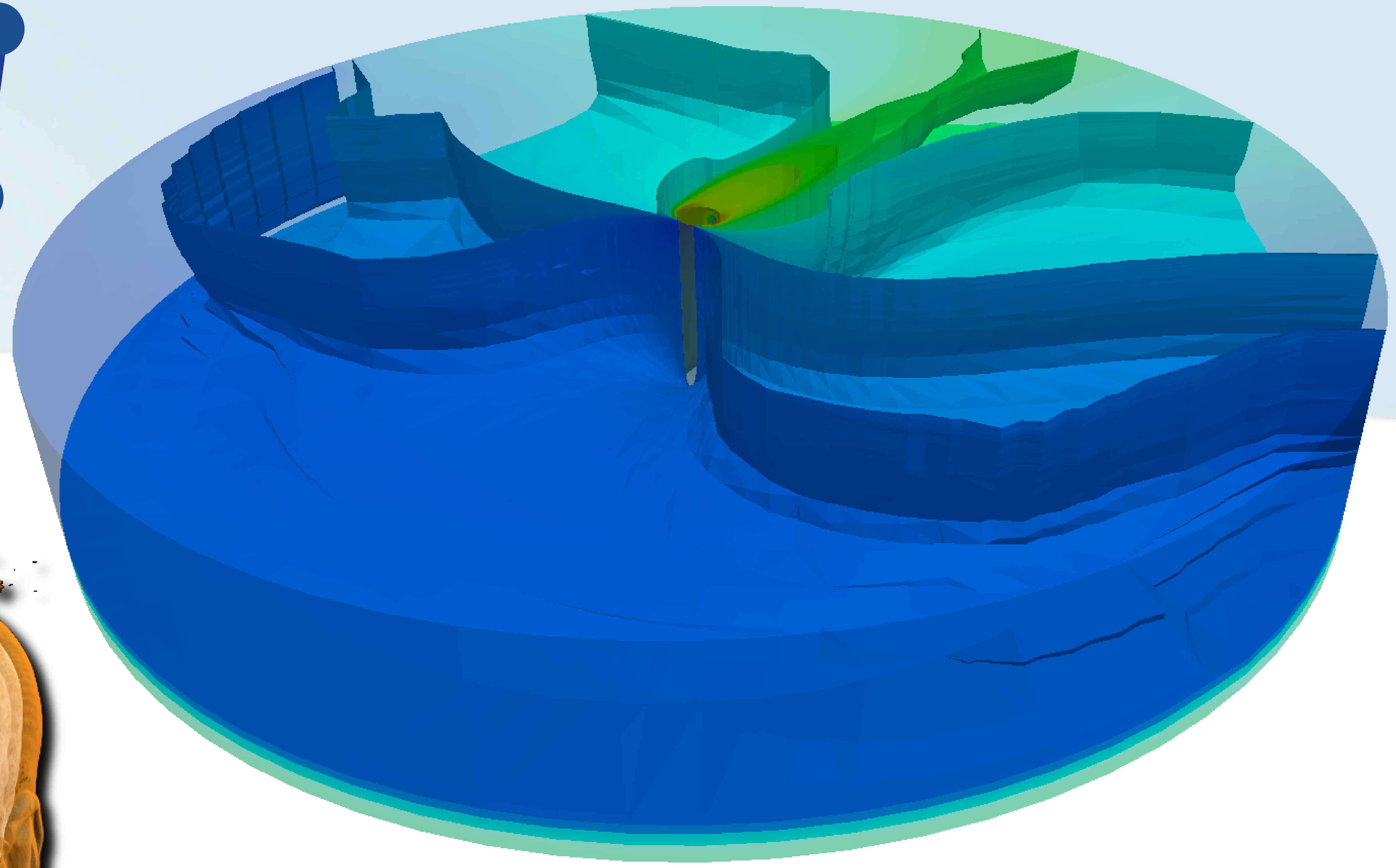

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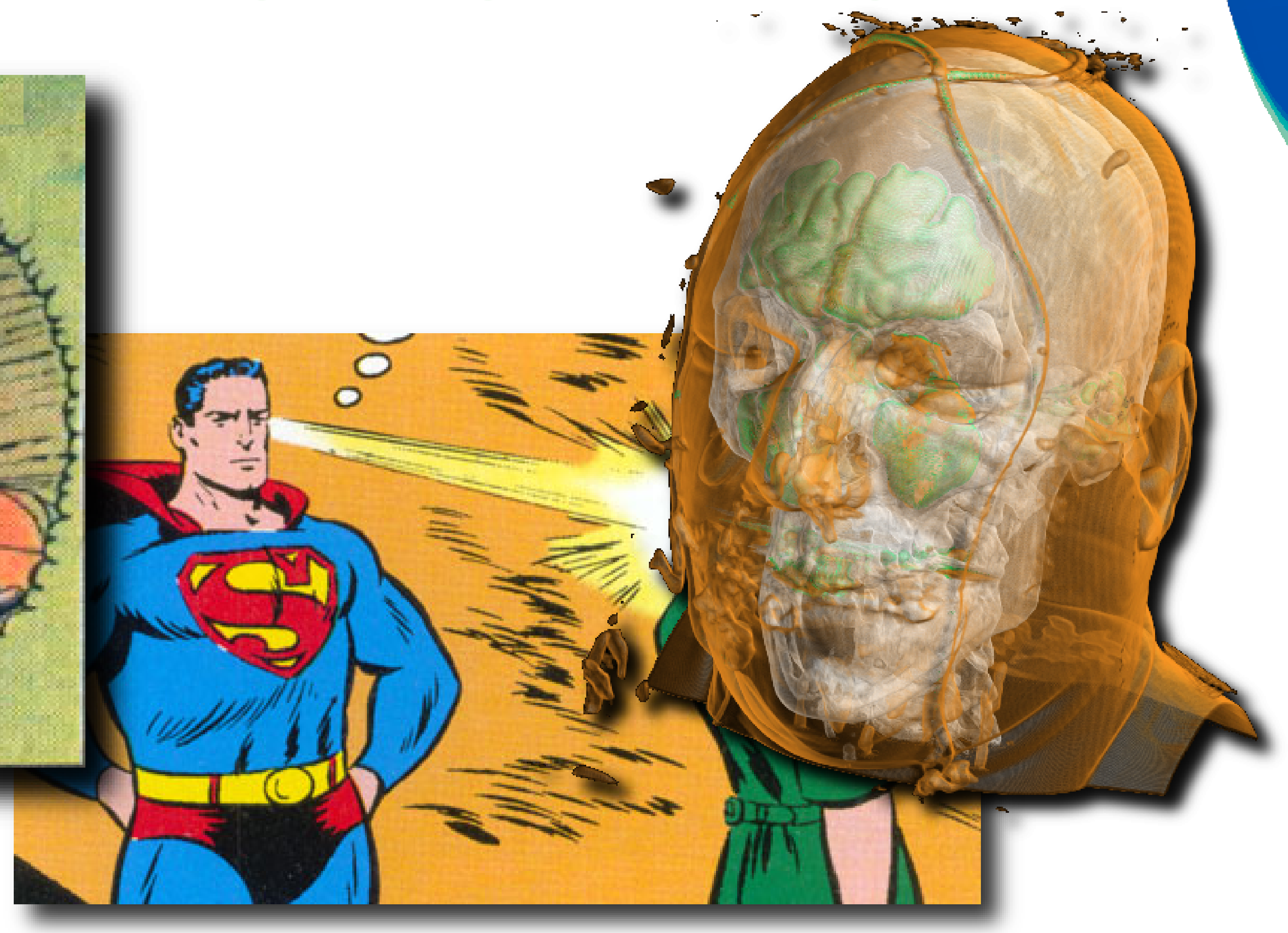

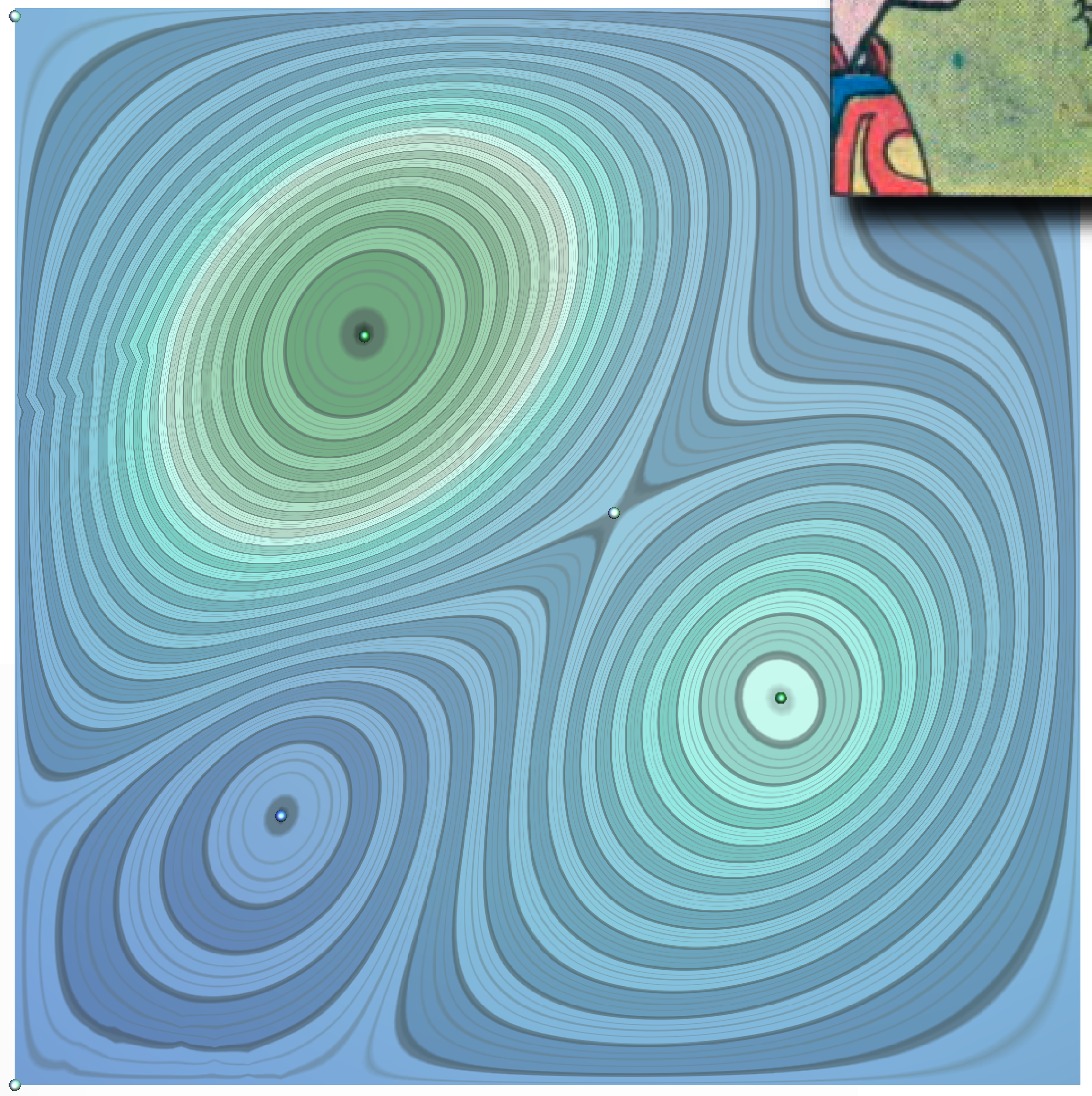

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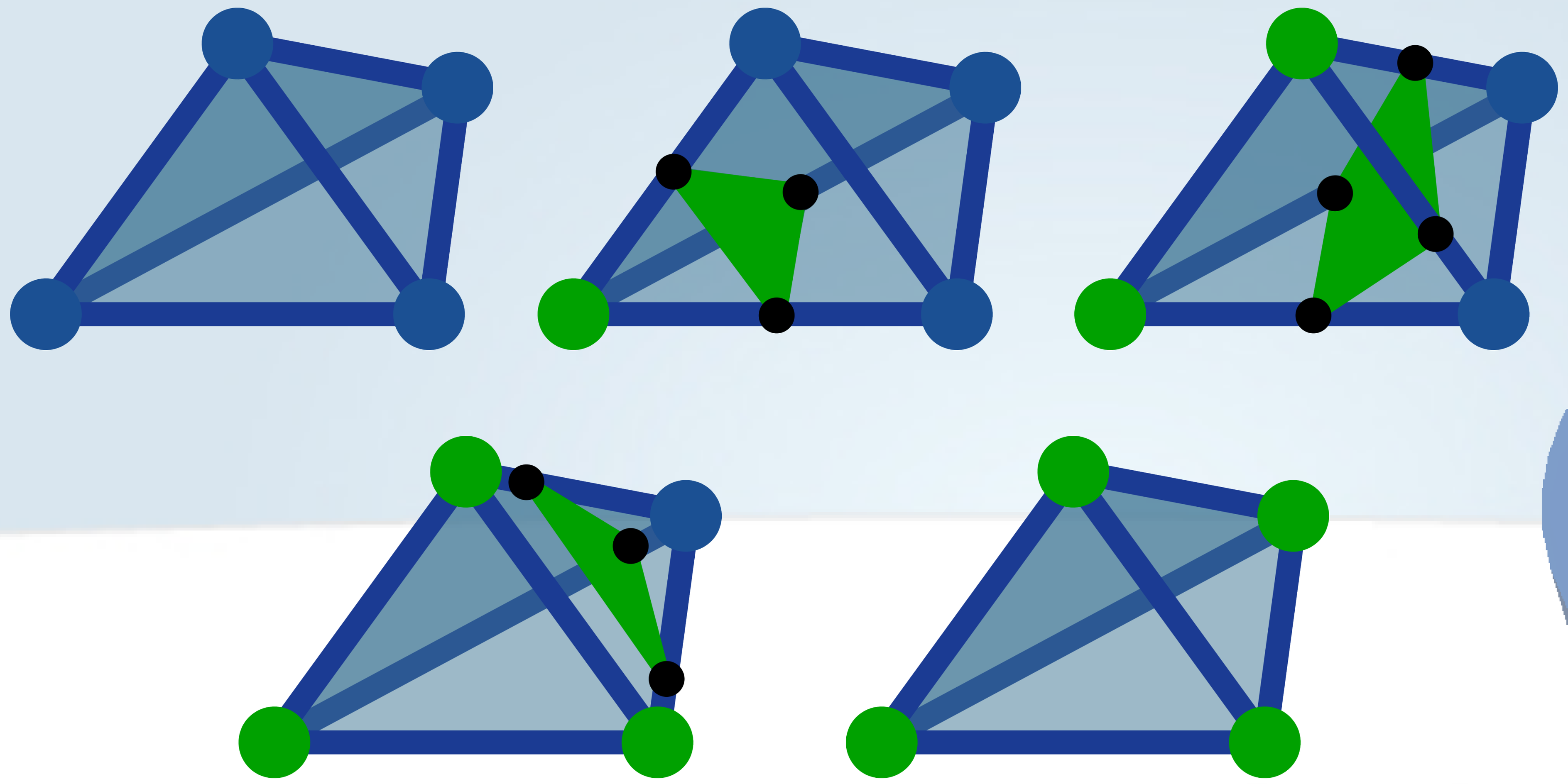

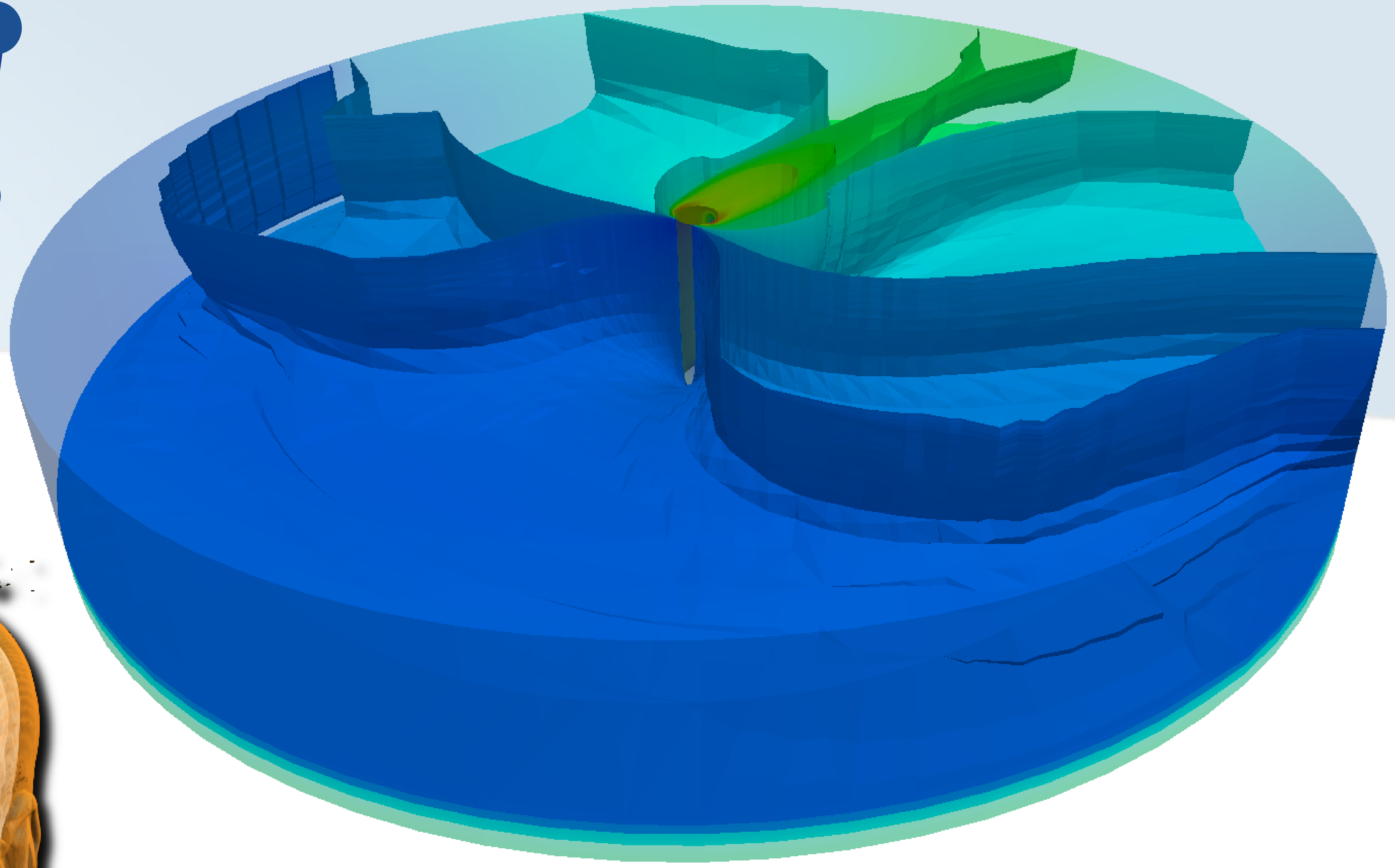

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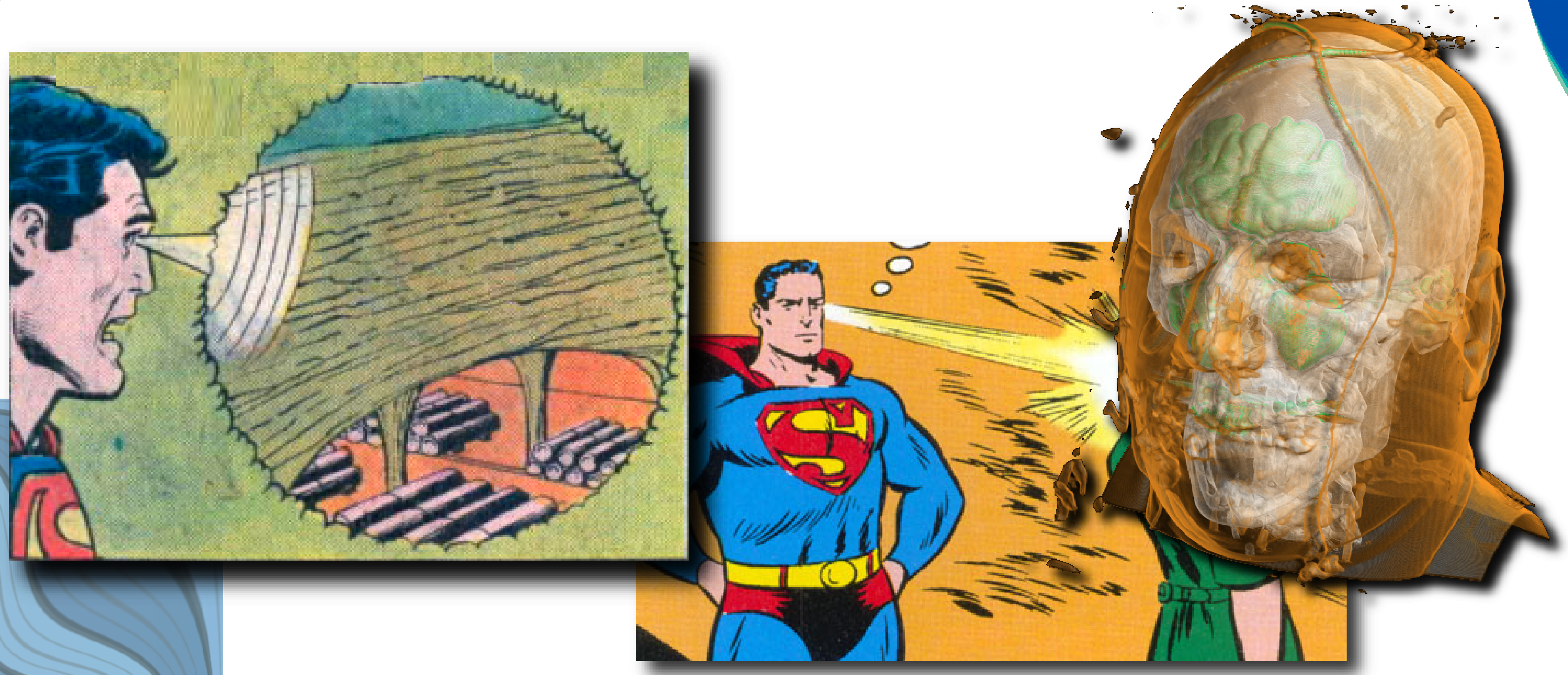

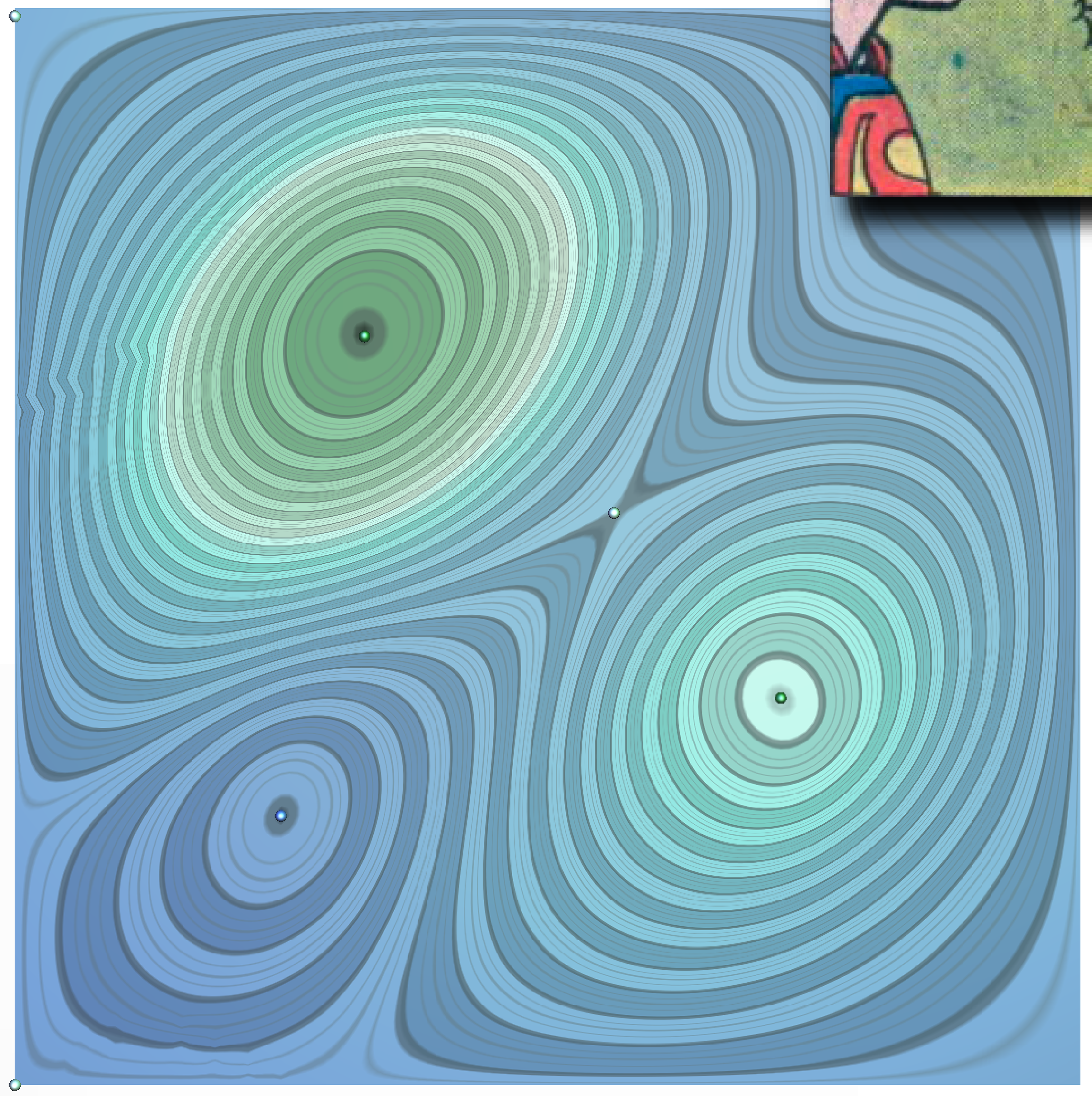

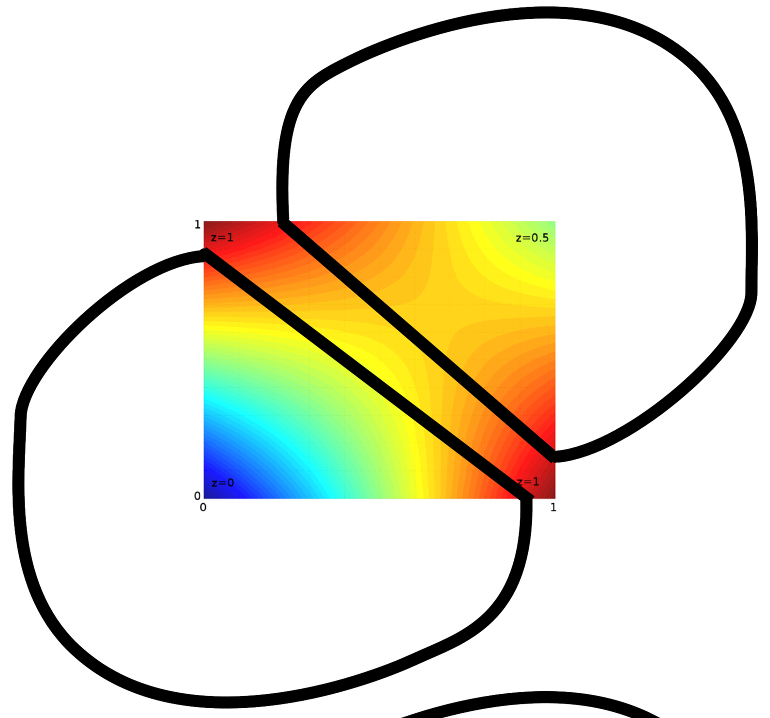

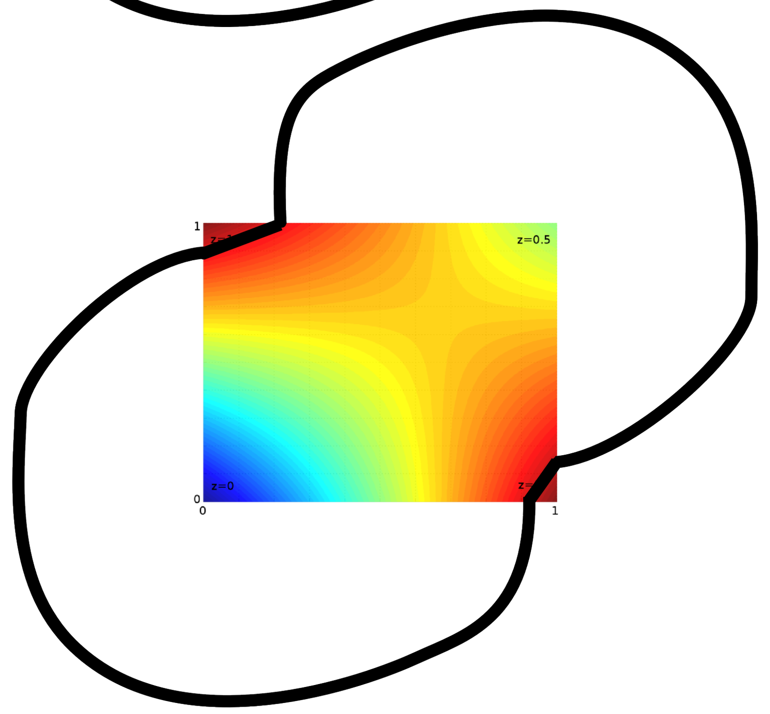

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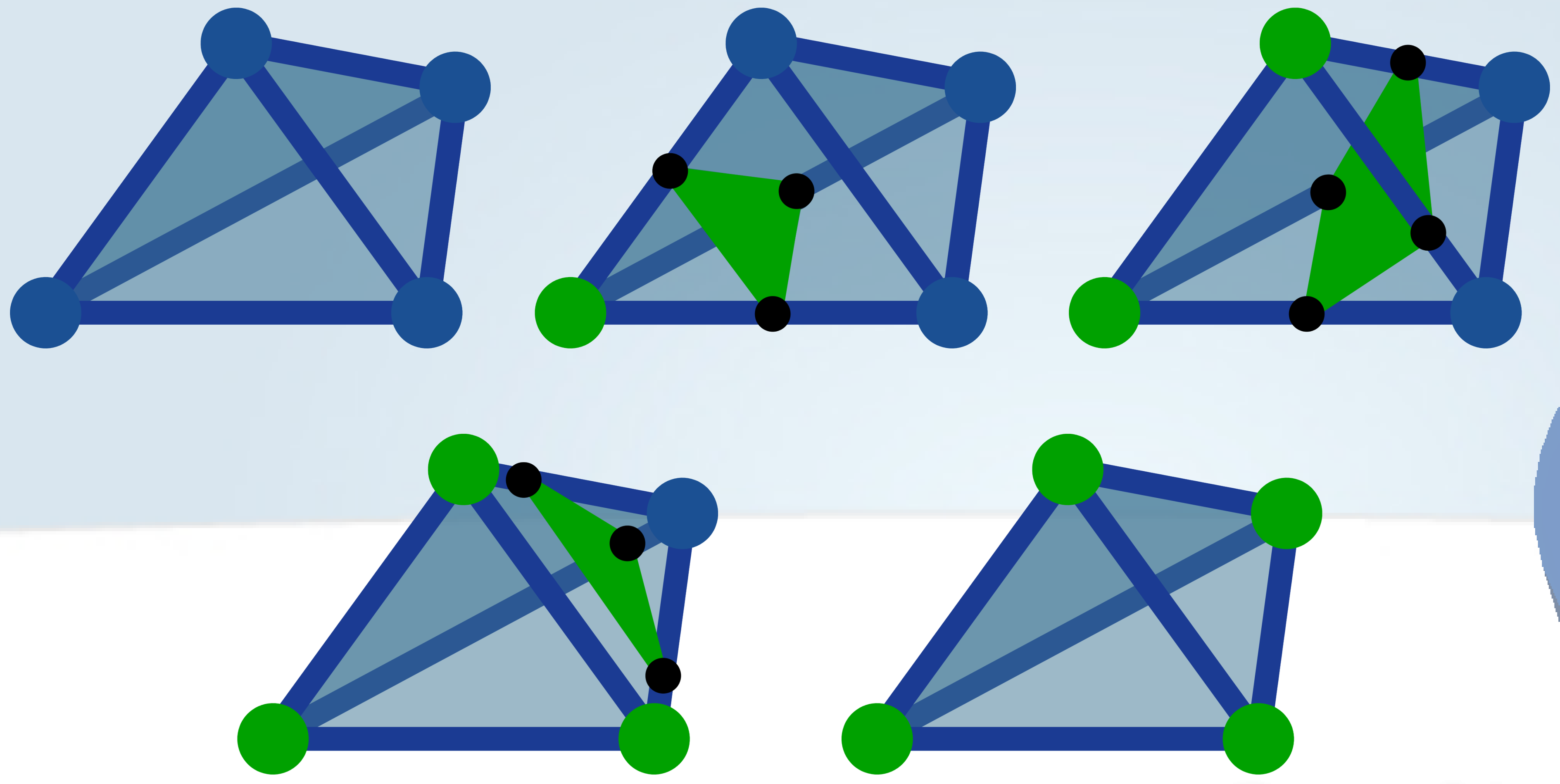
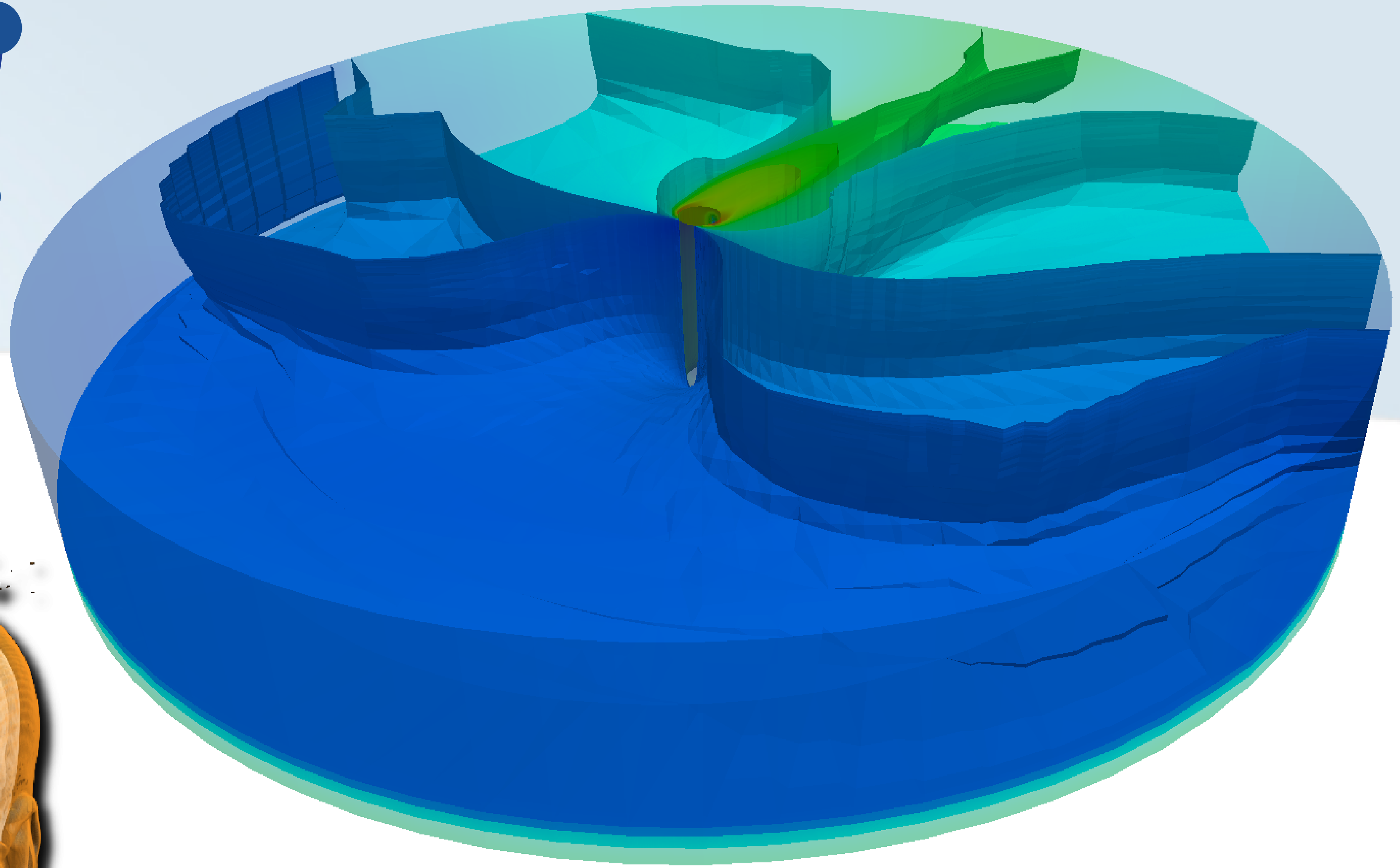

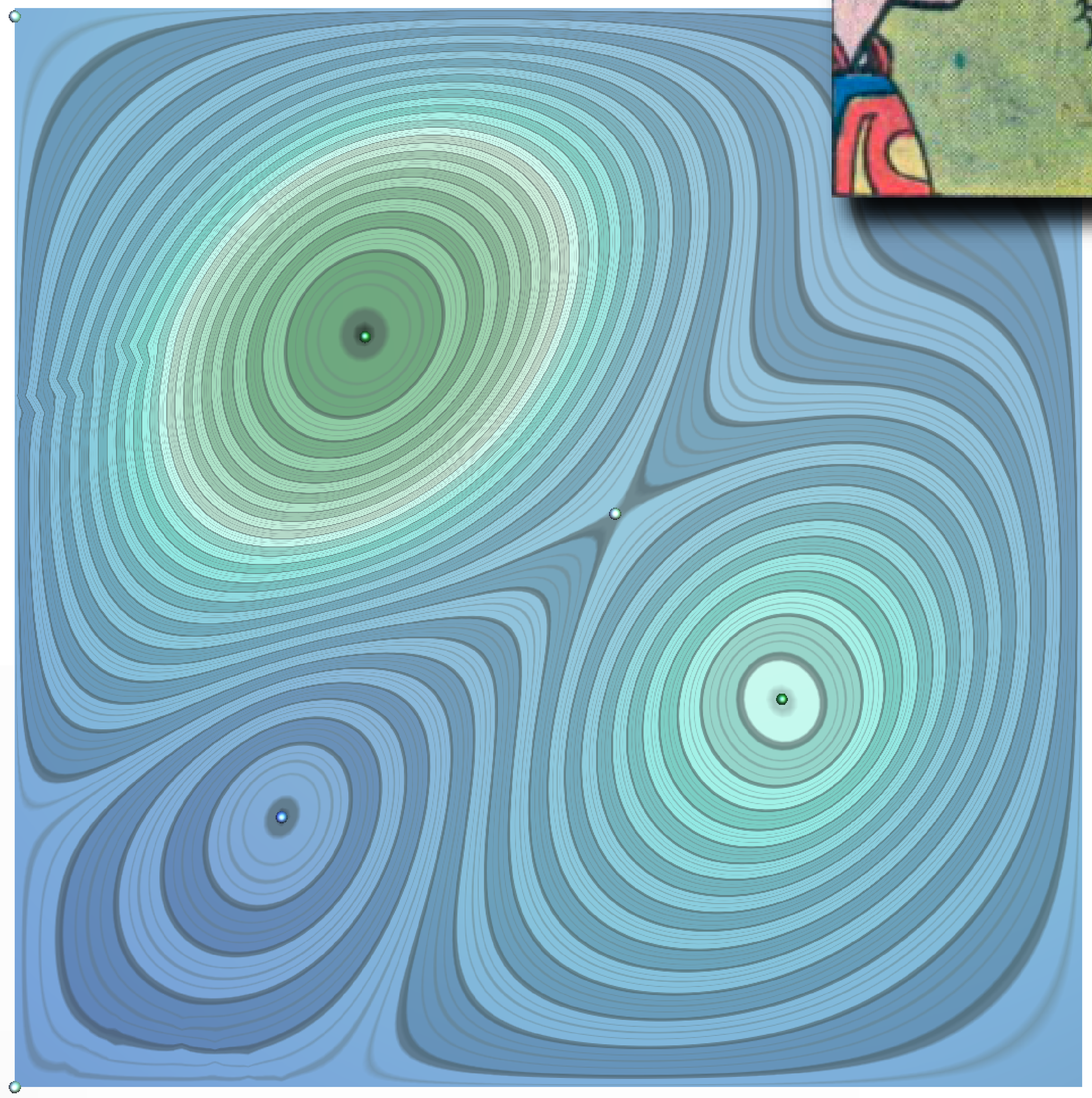
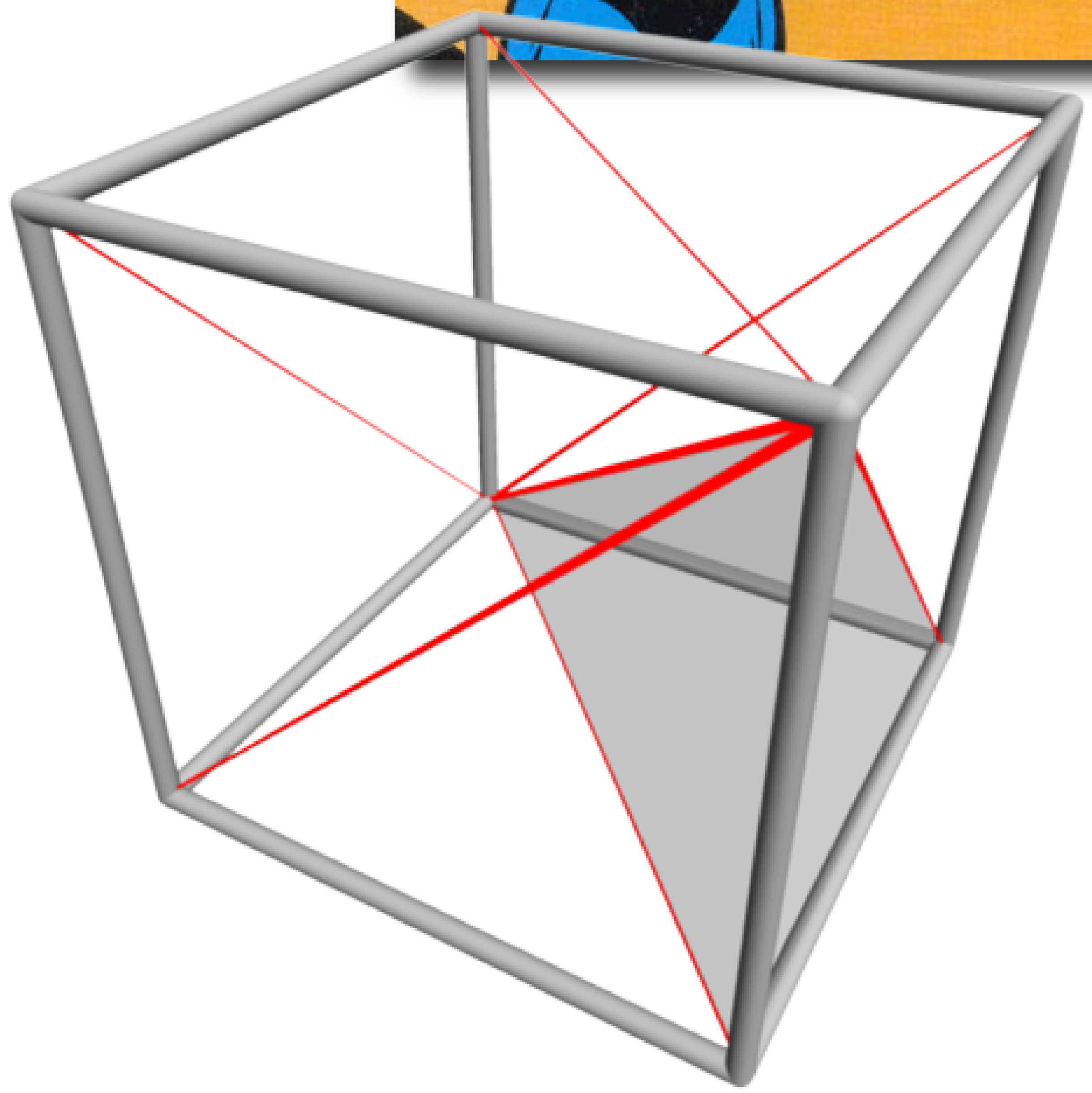


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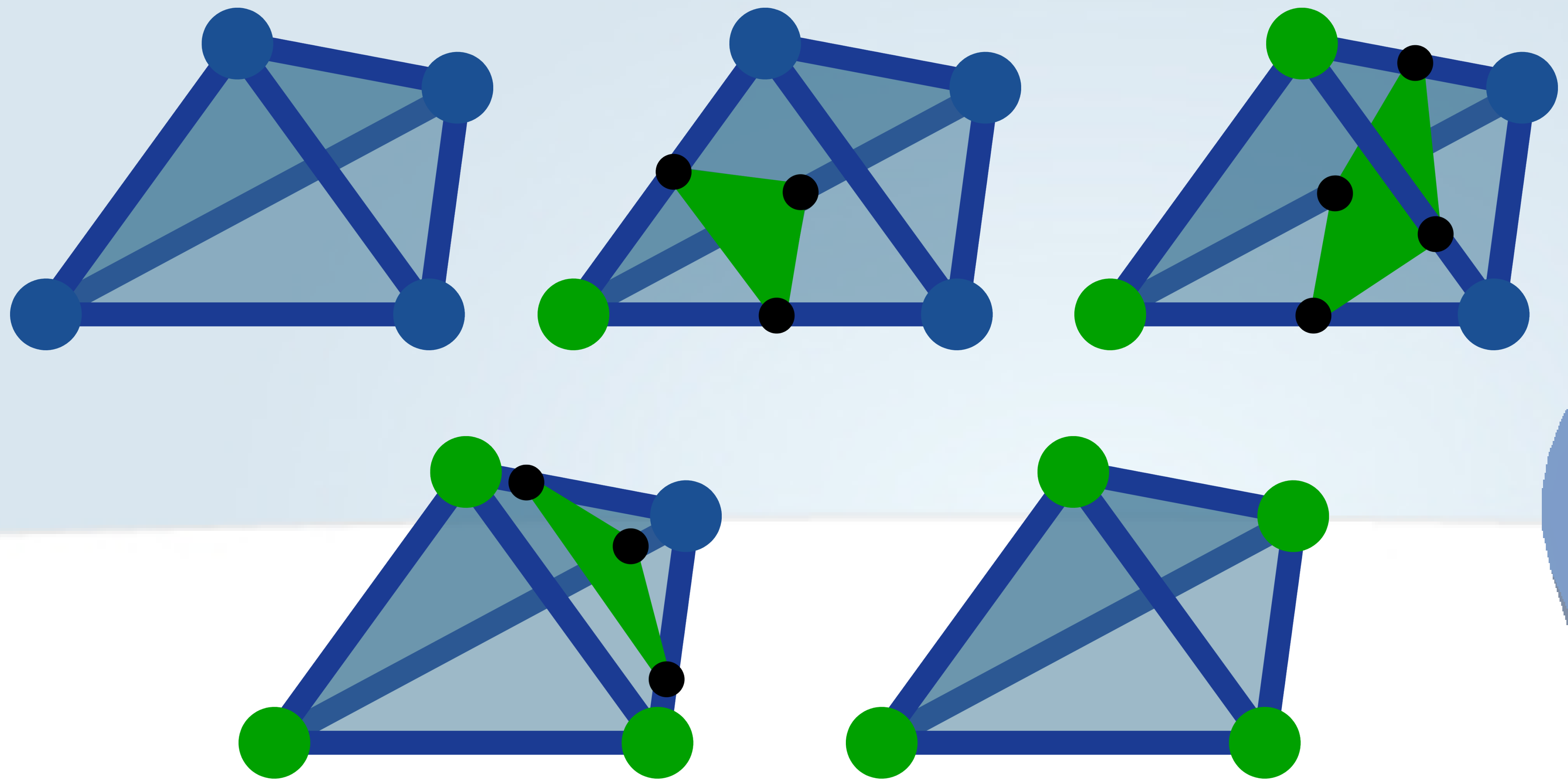

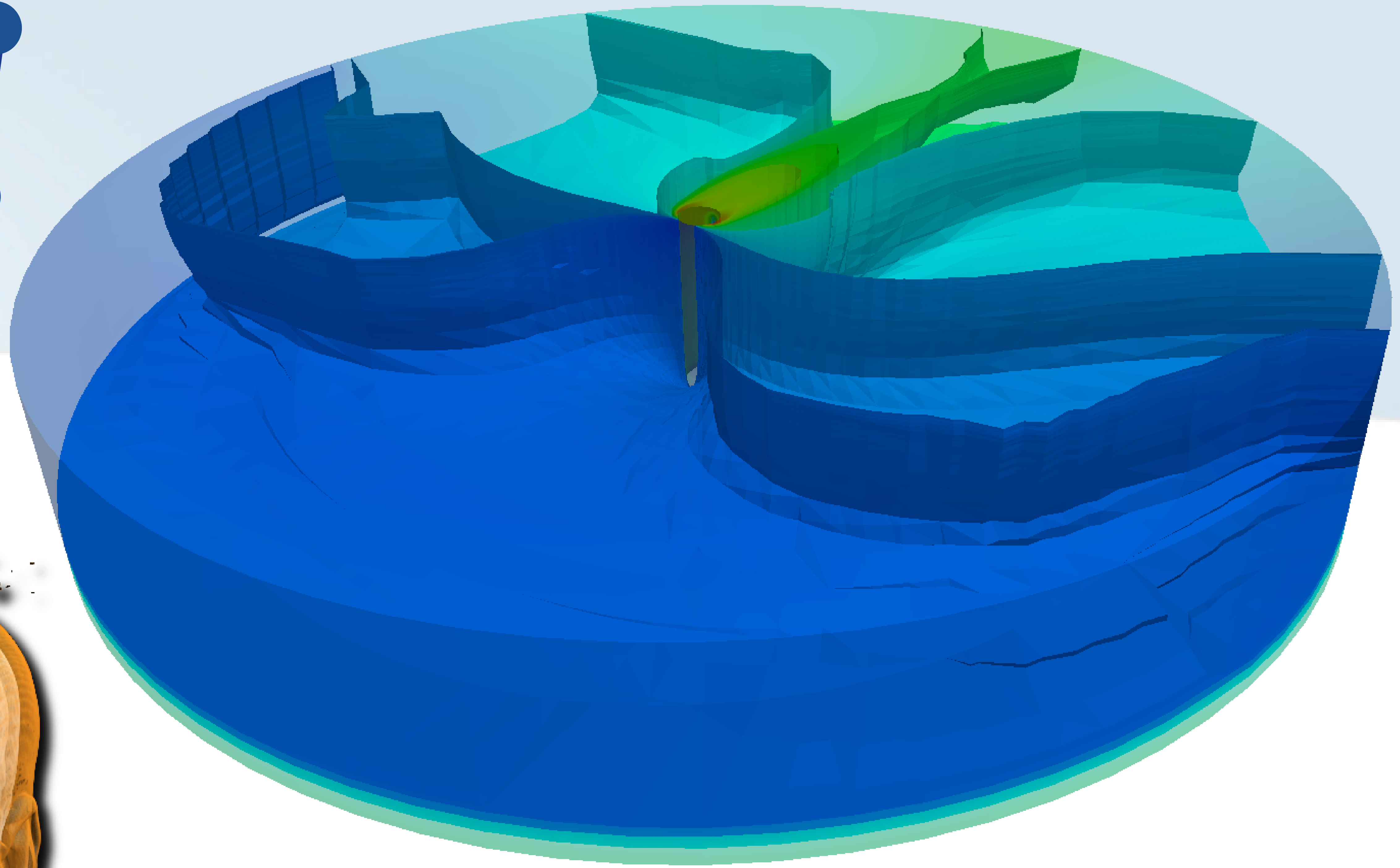

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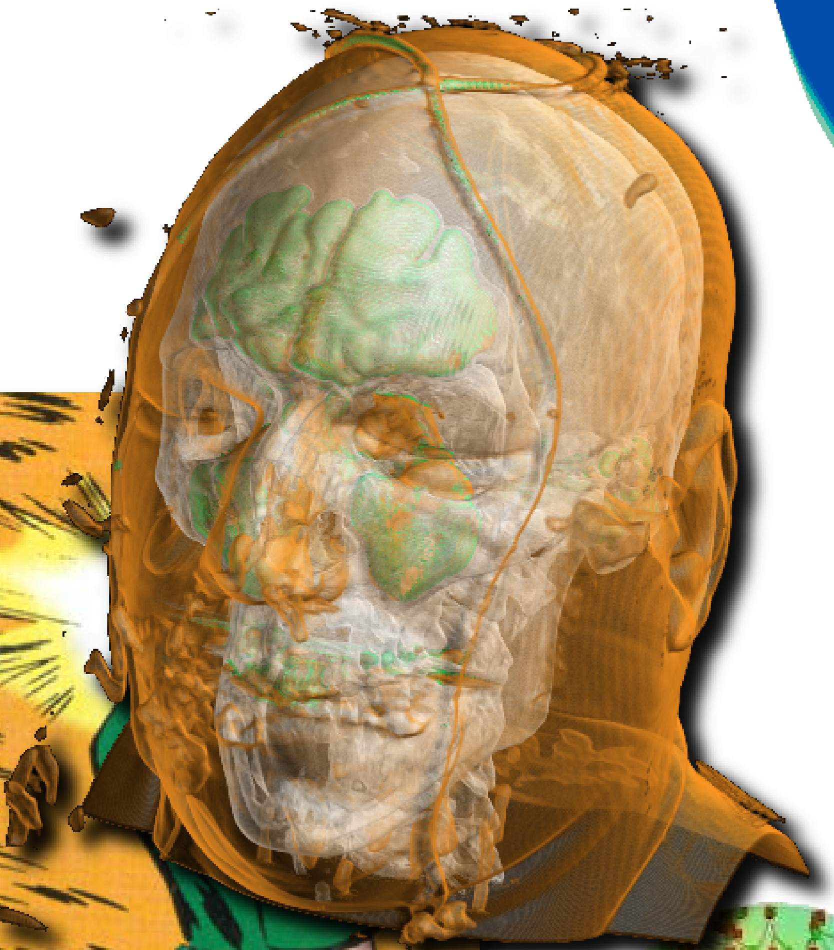

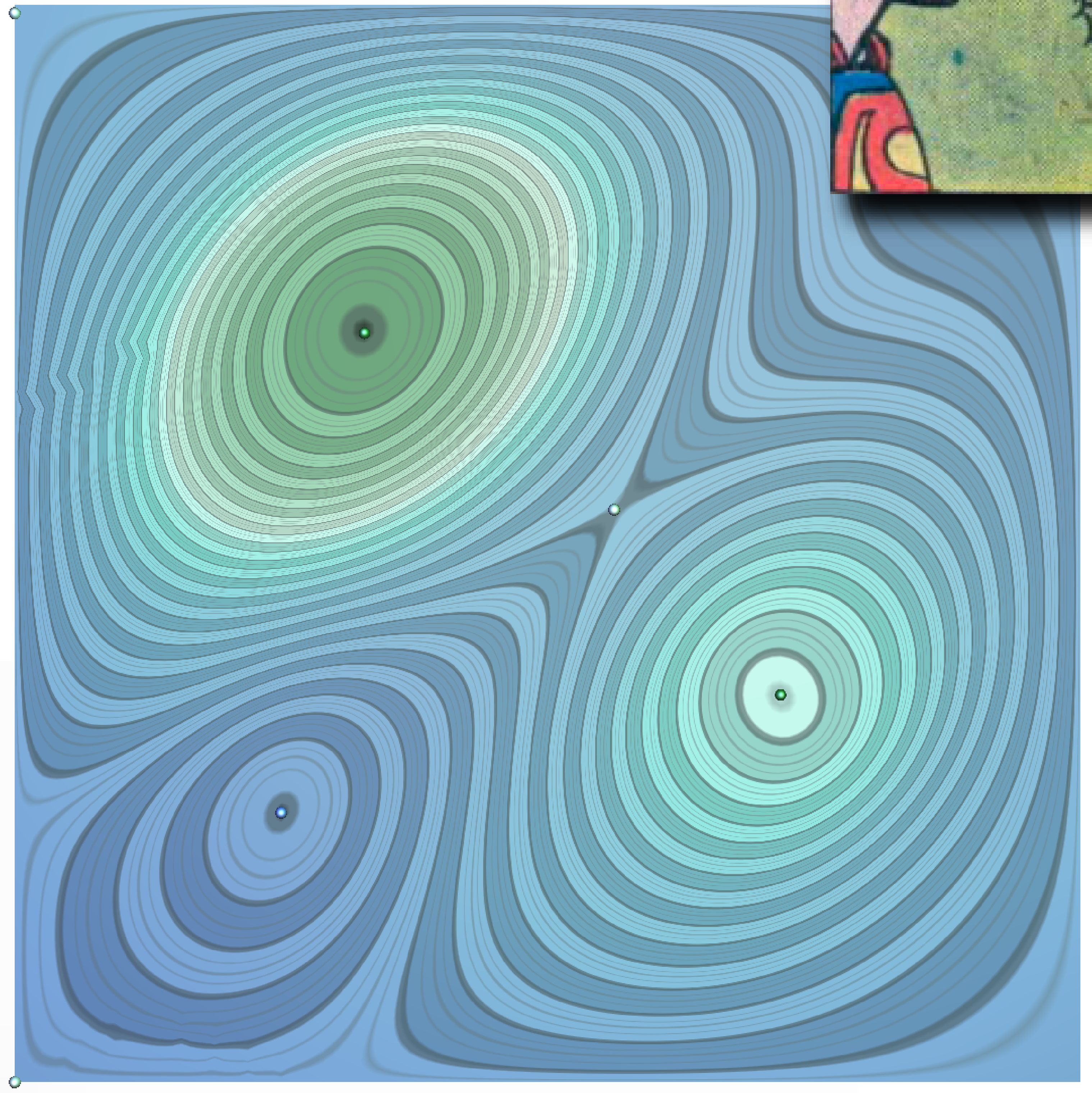

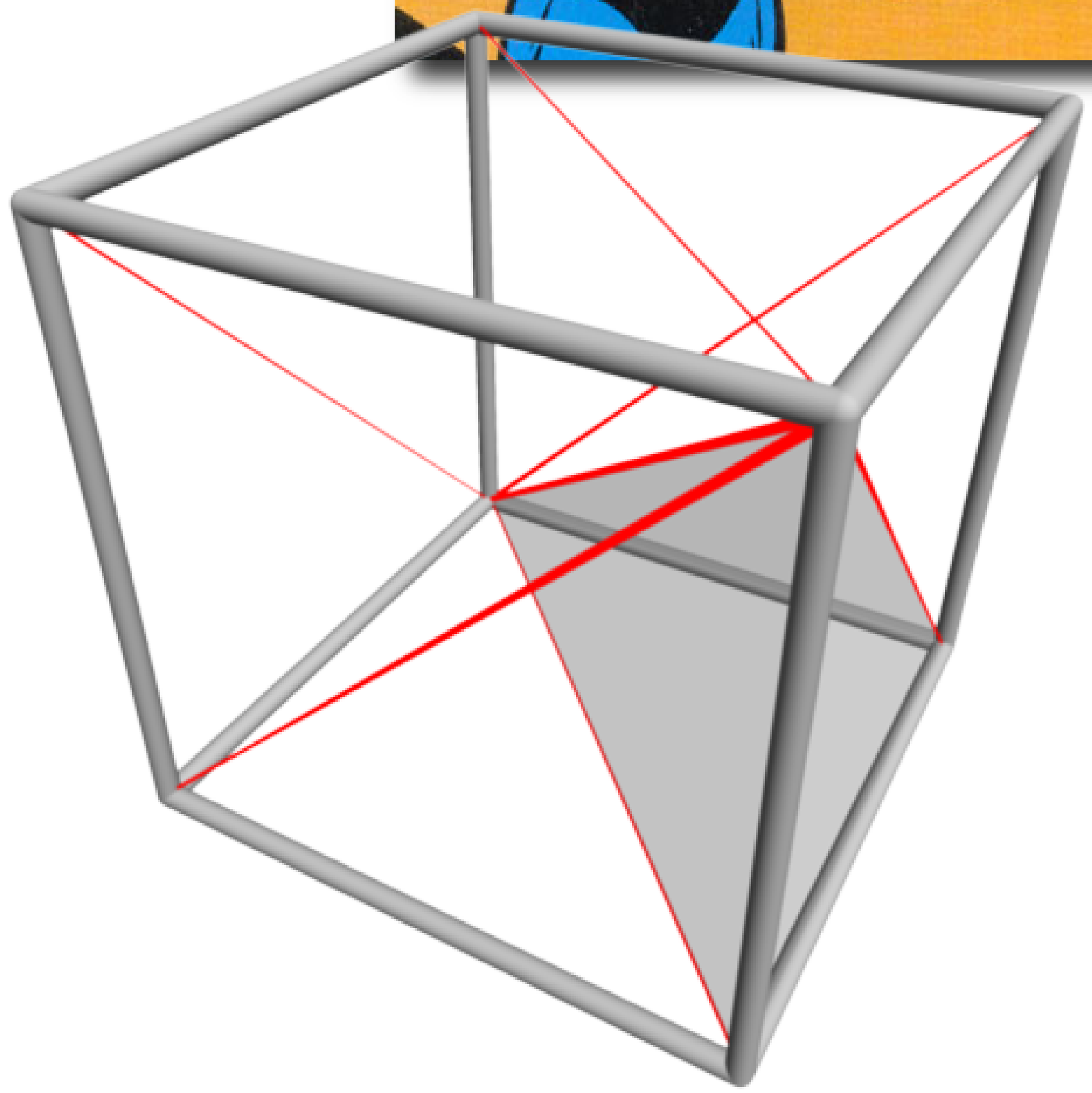

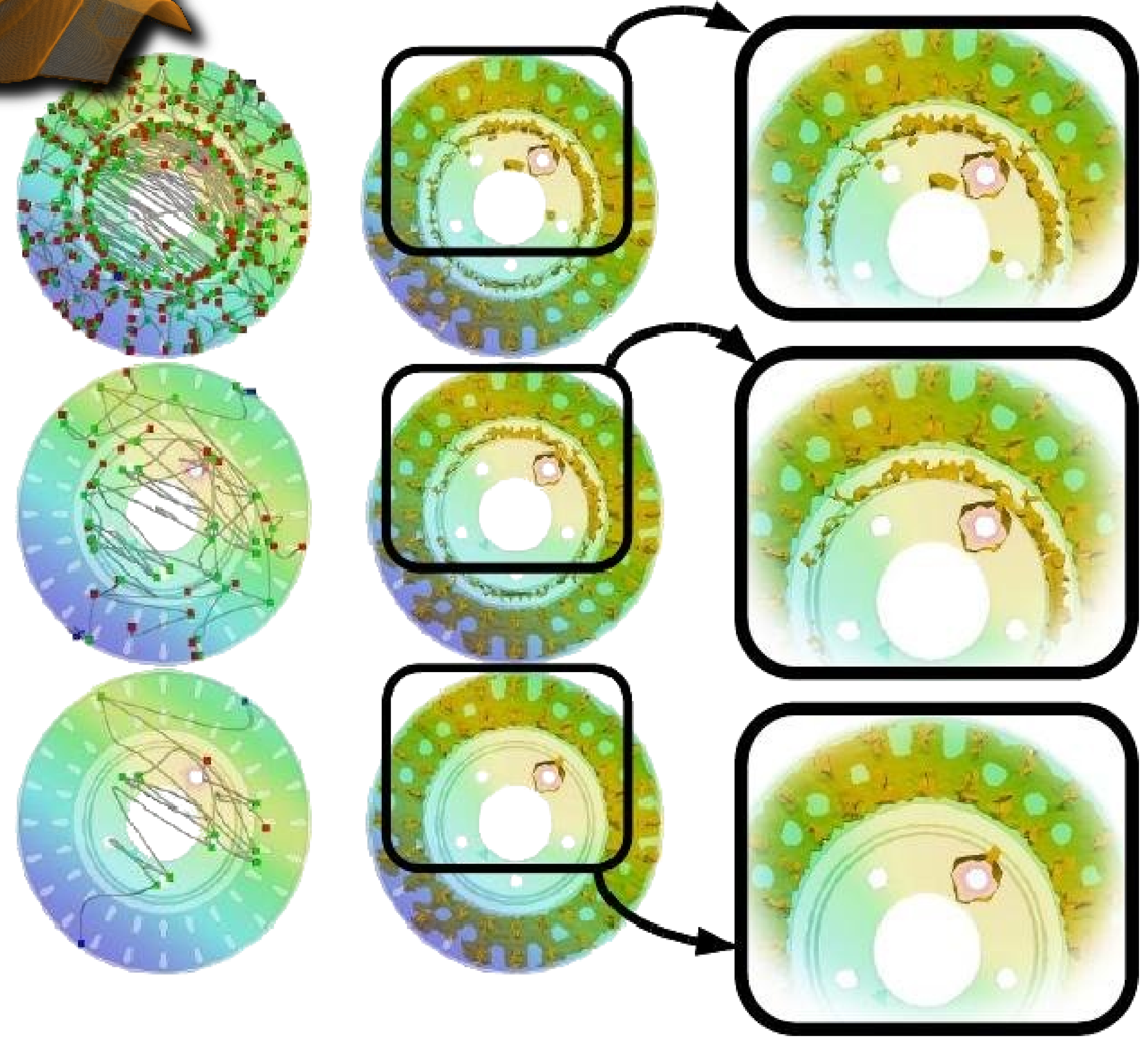

Farewell

- Vector fields

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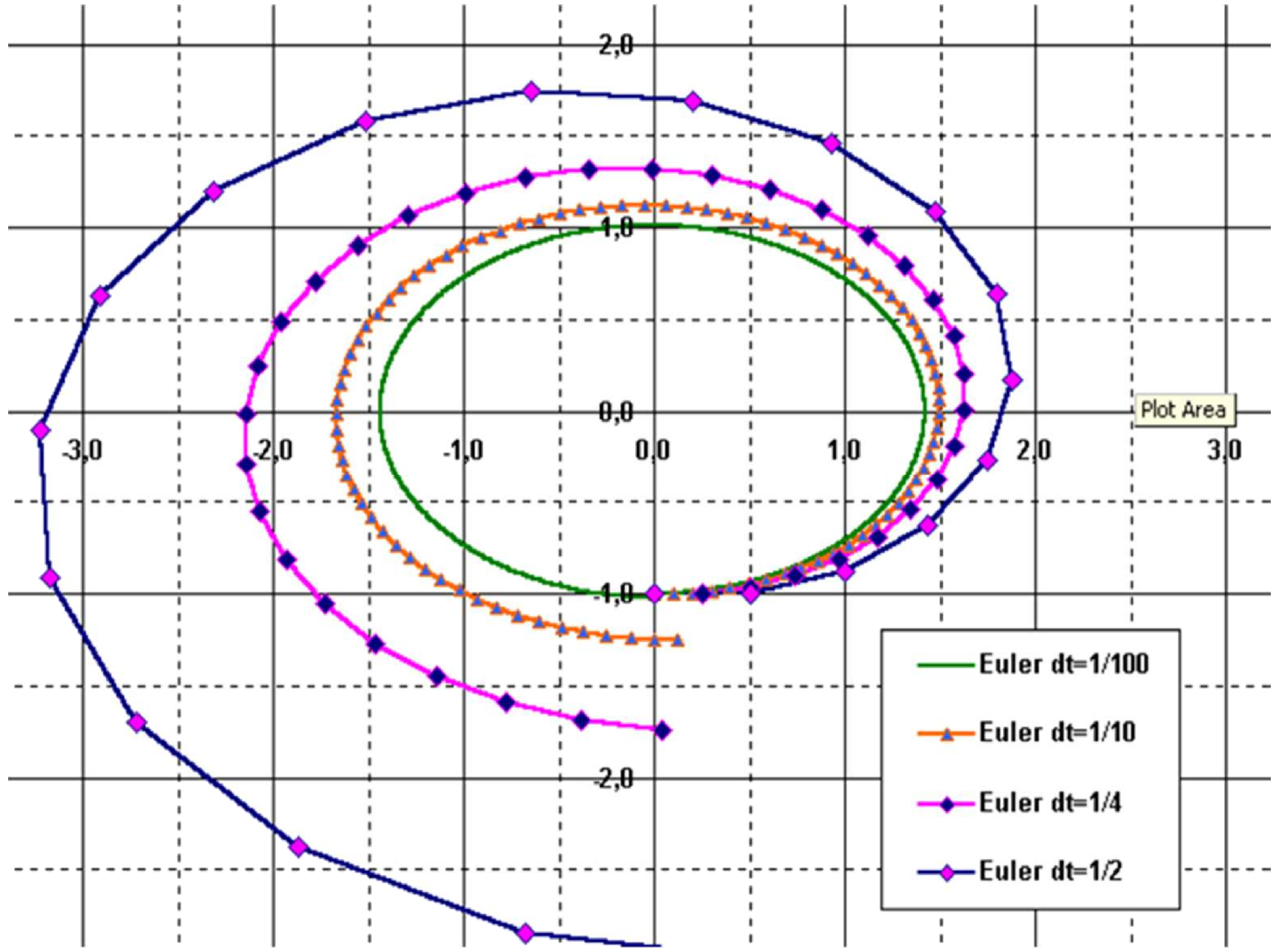
- Vector fields



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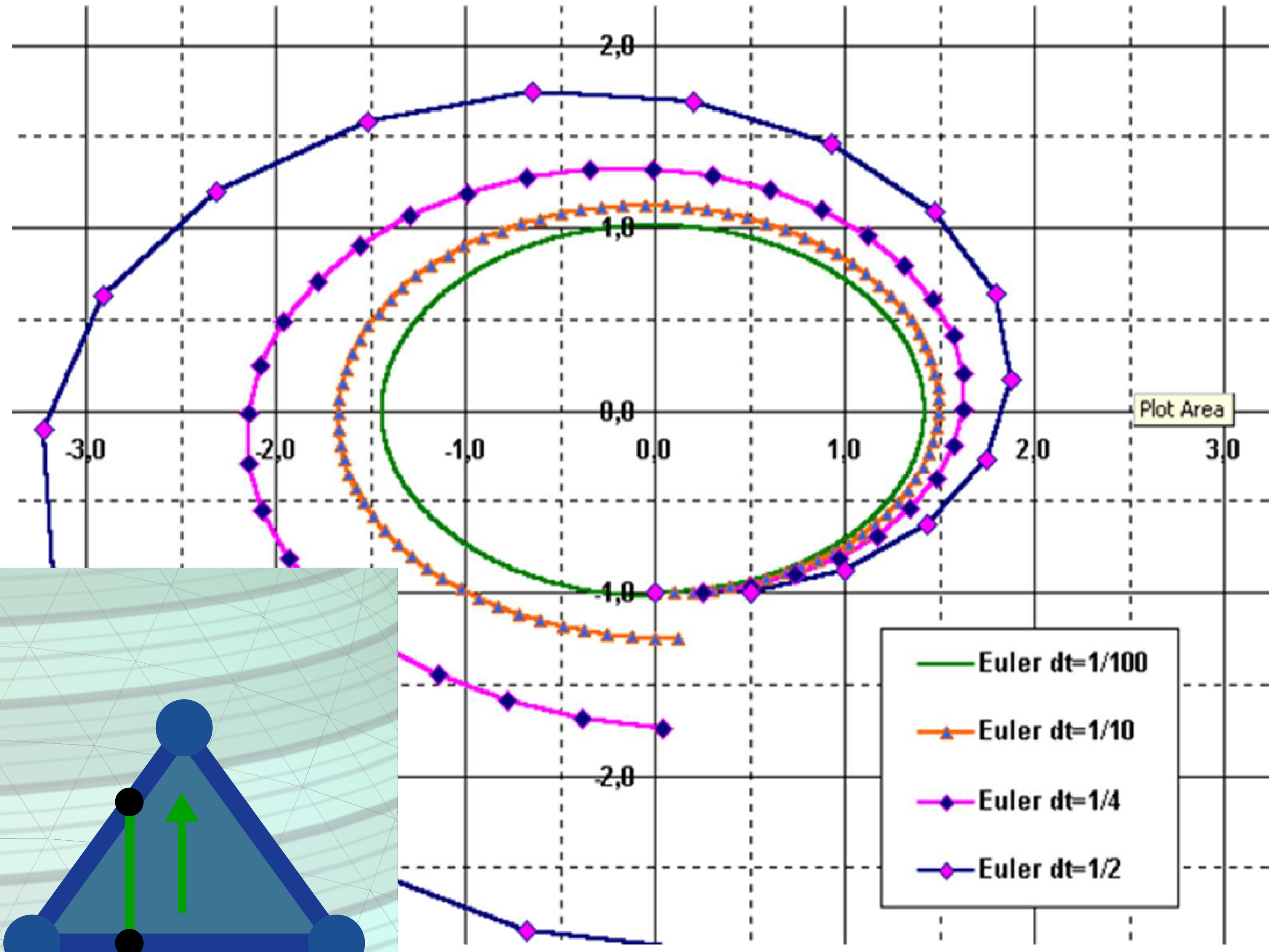

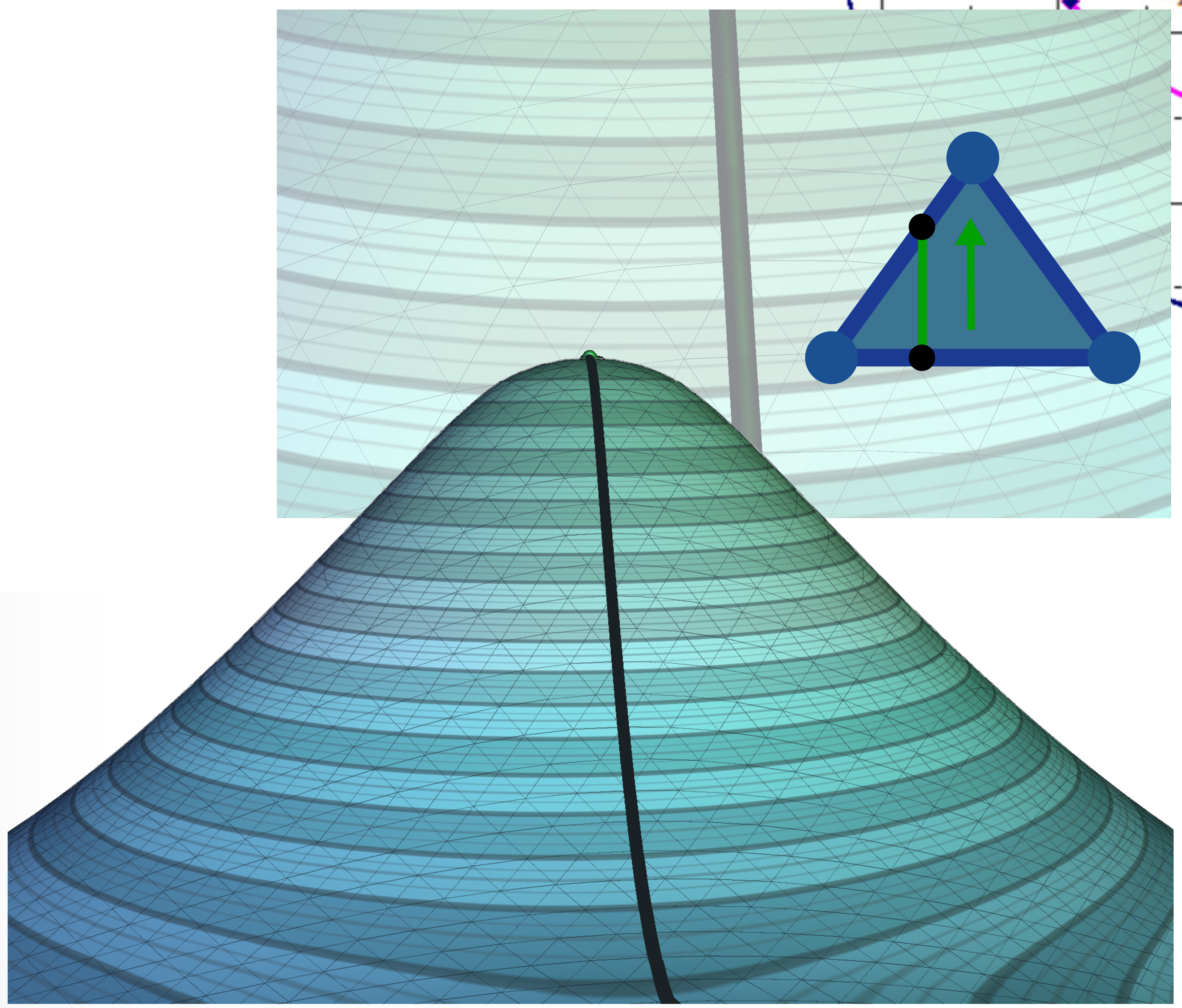

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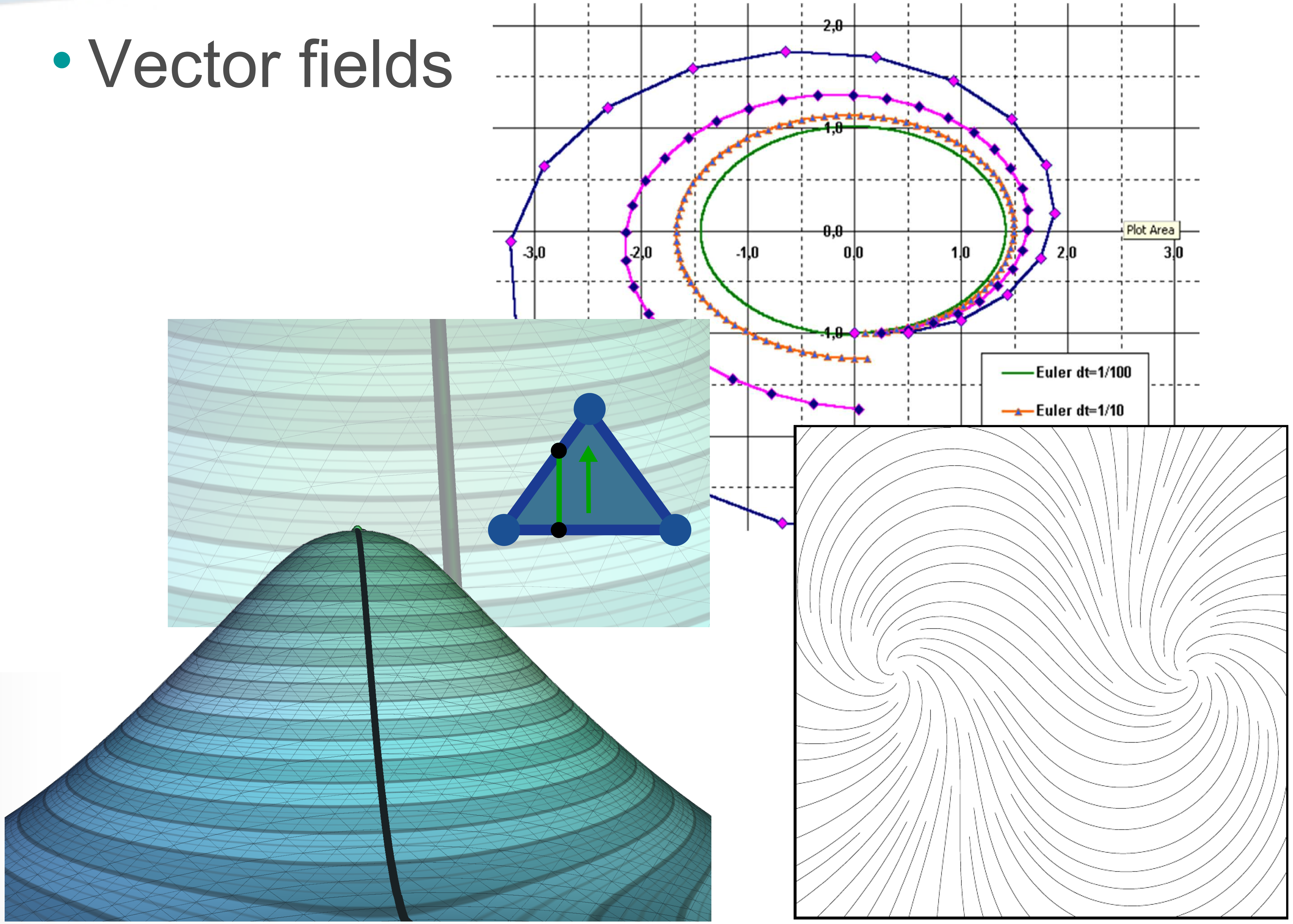

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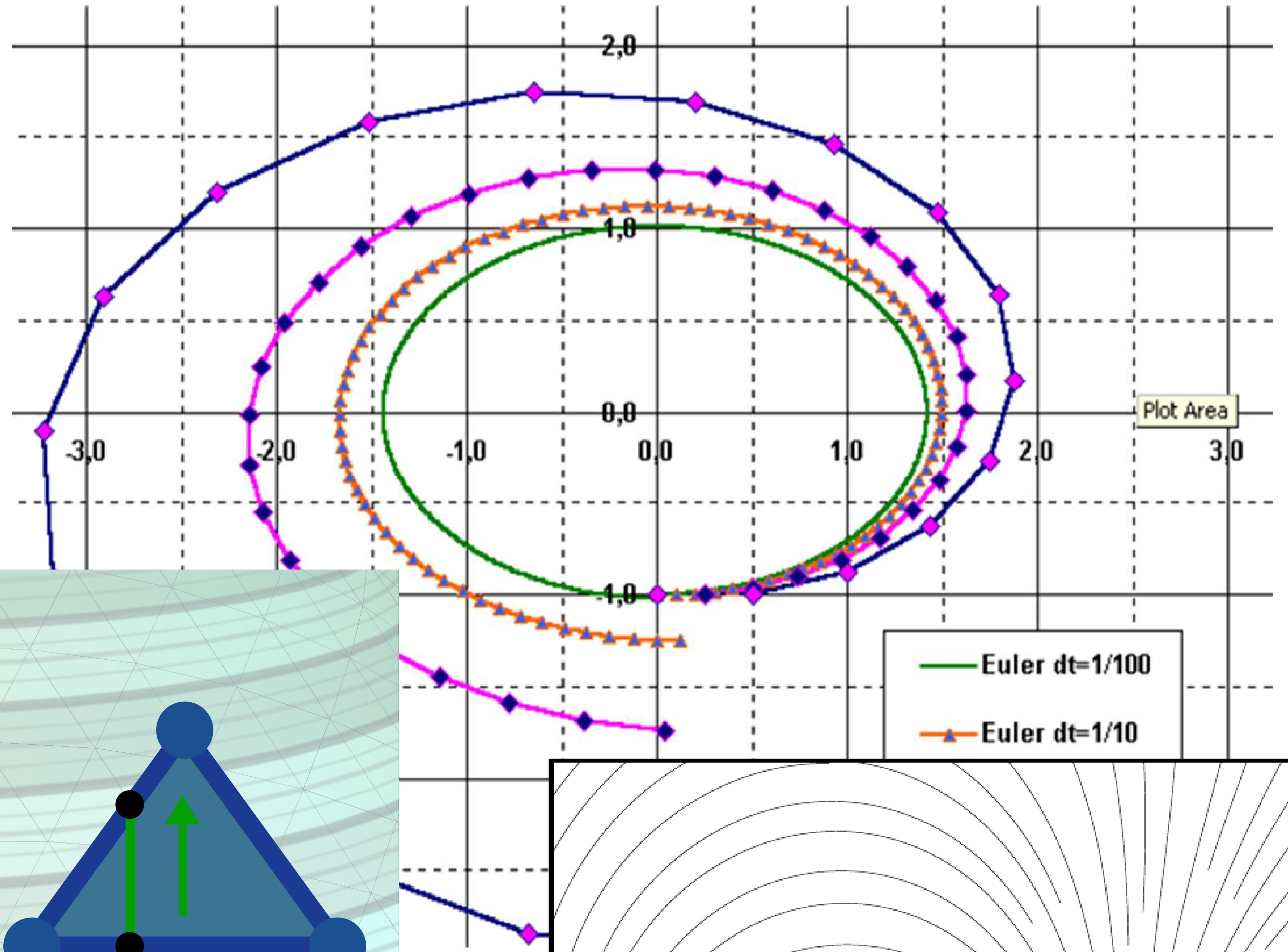

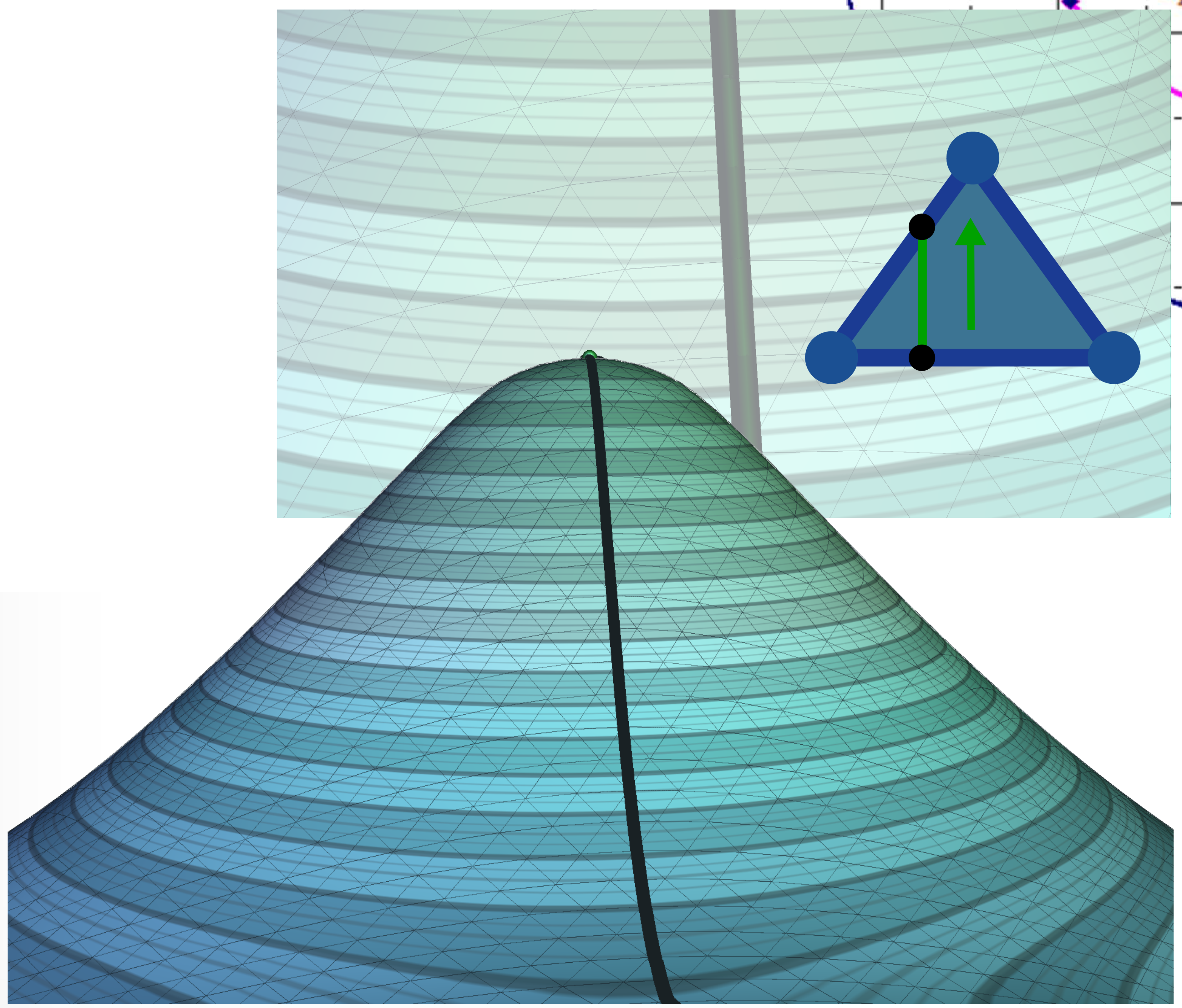

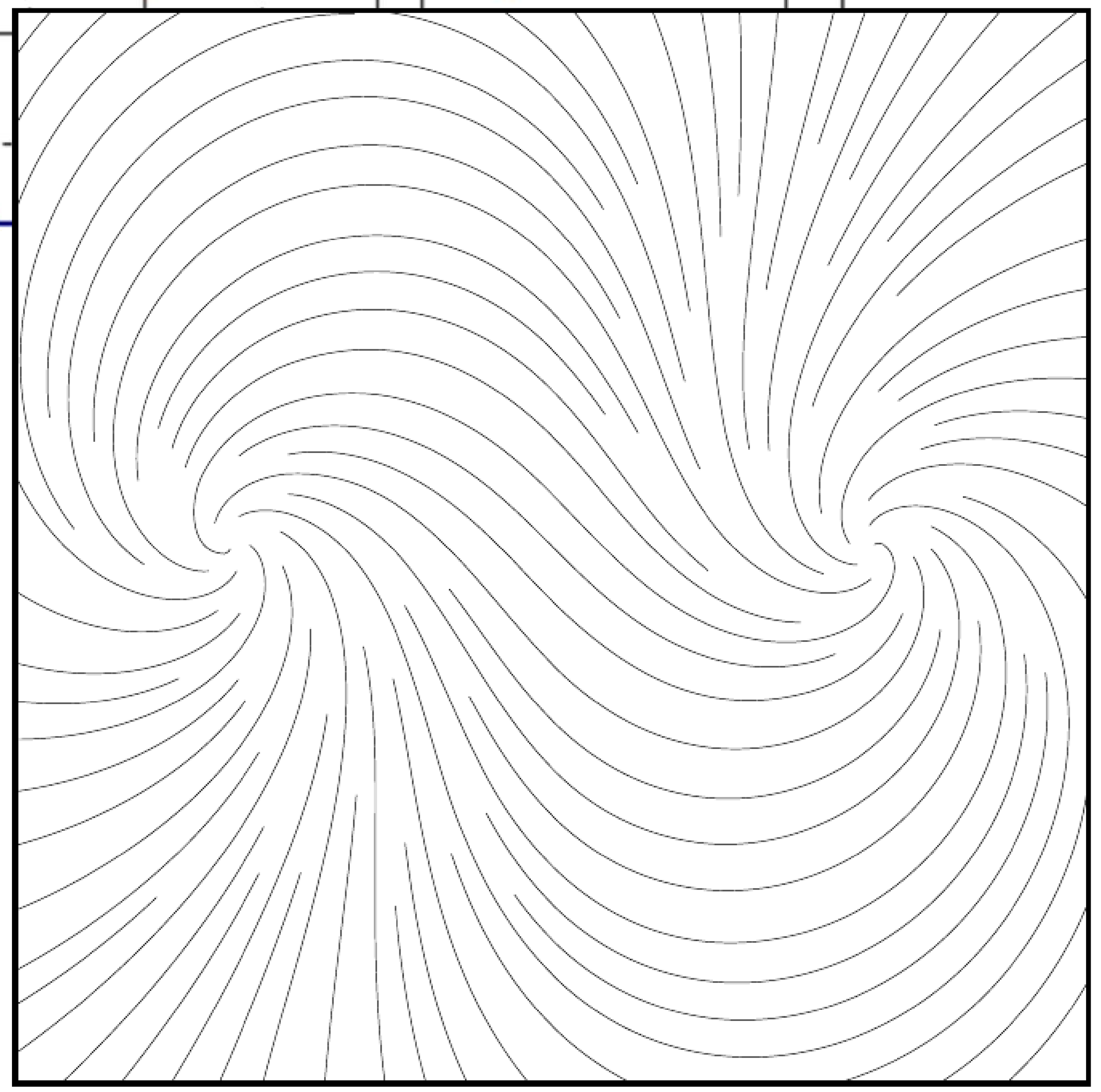

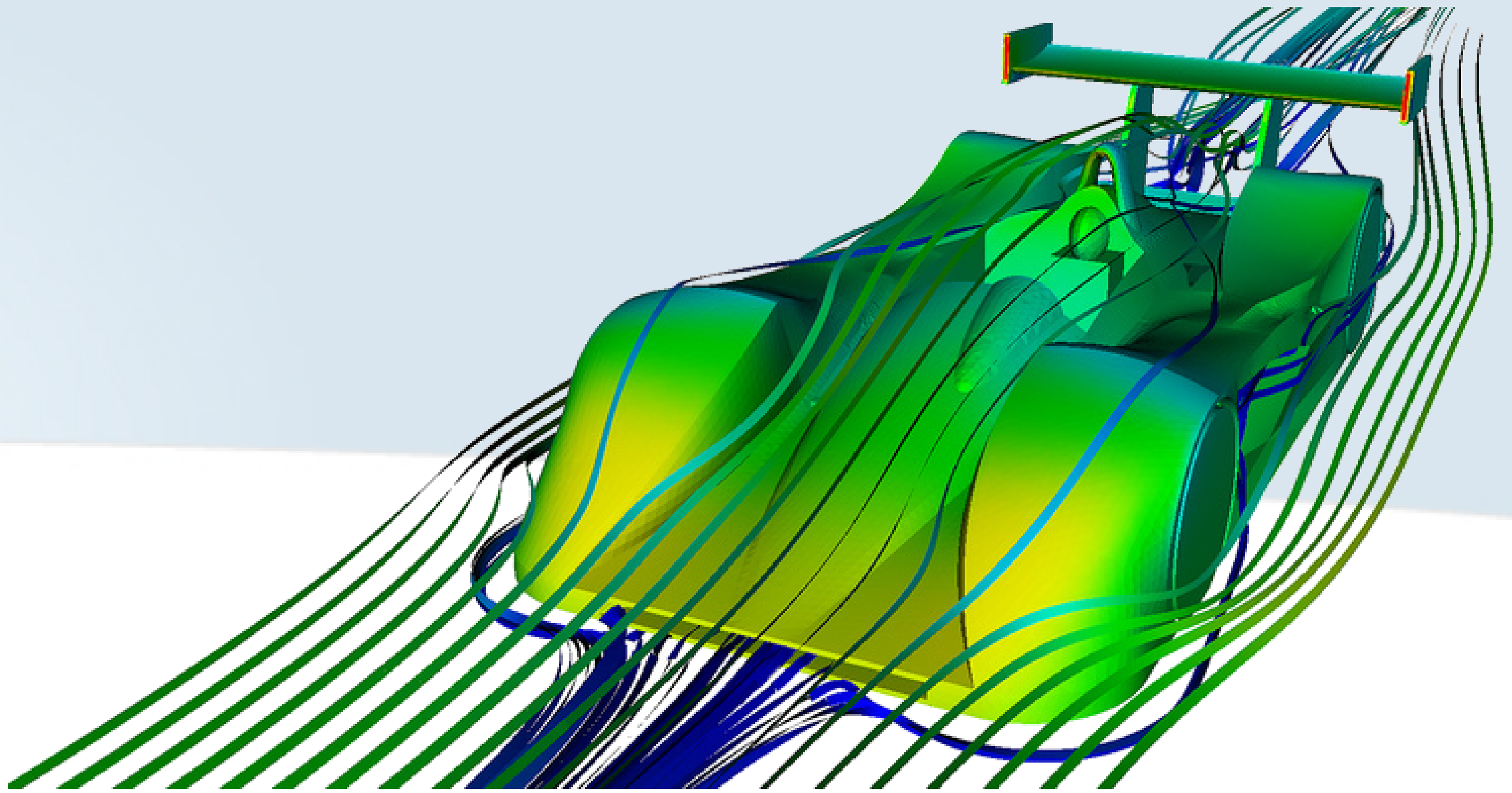

[Kitware]

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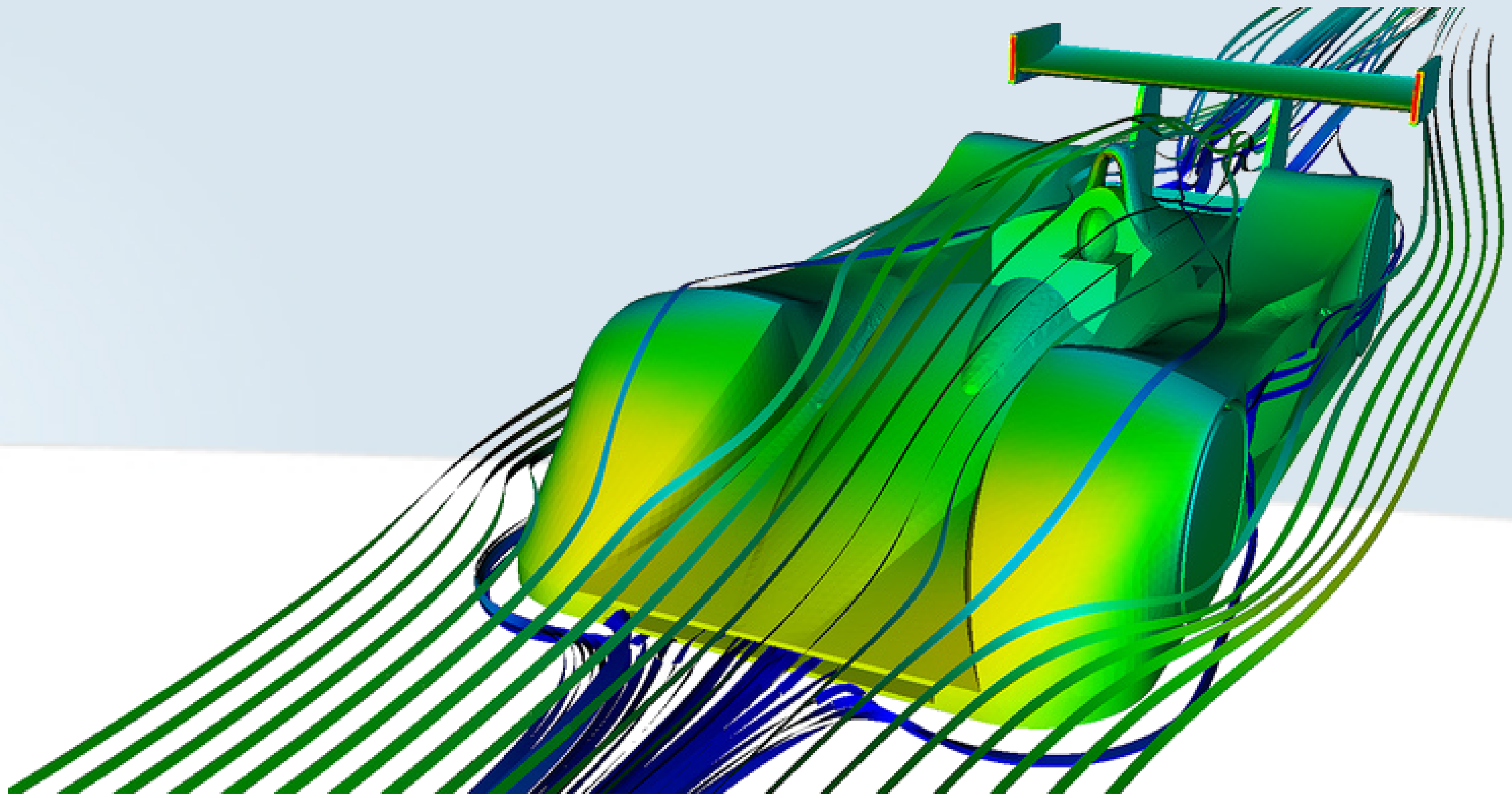

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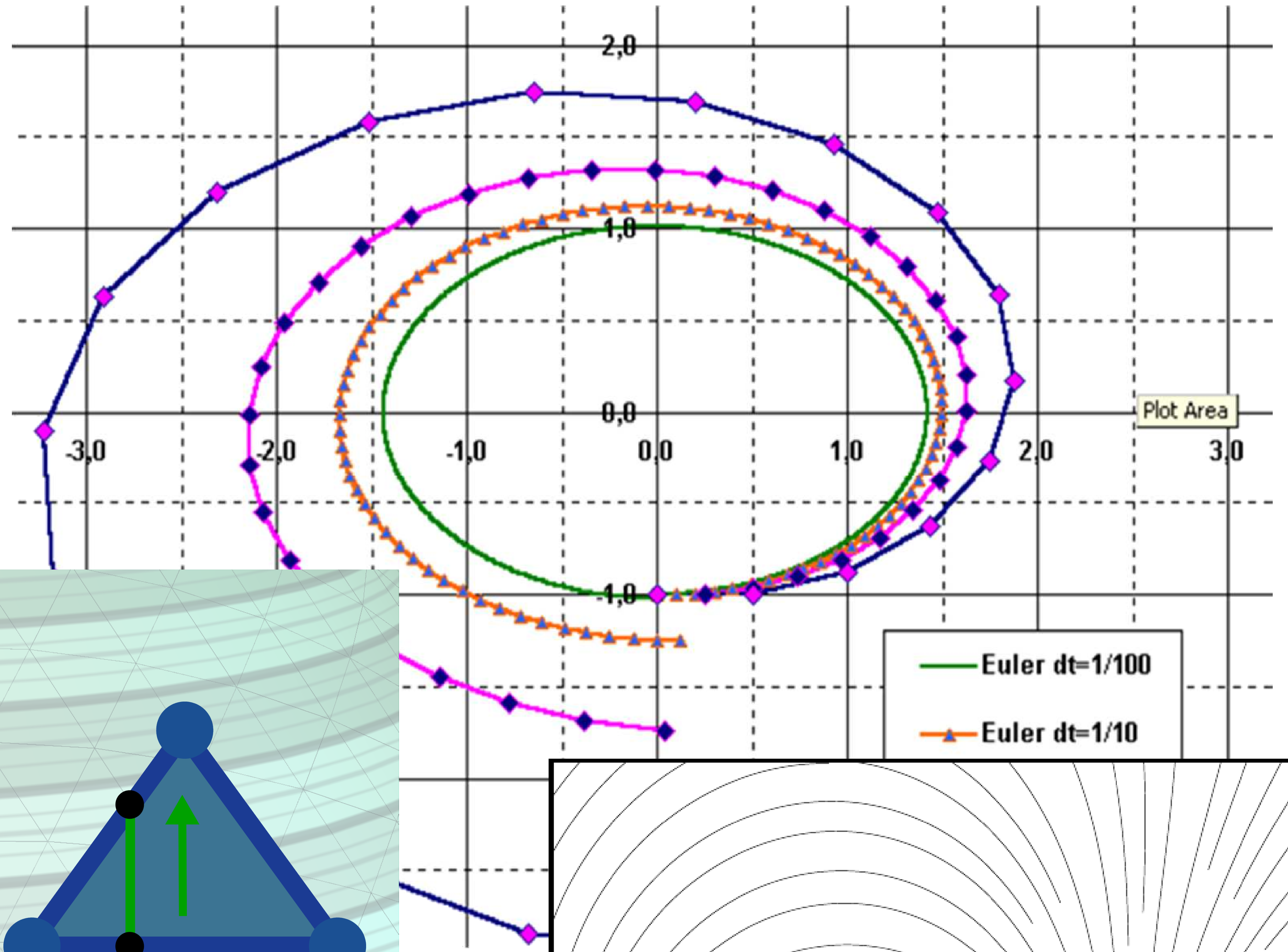

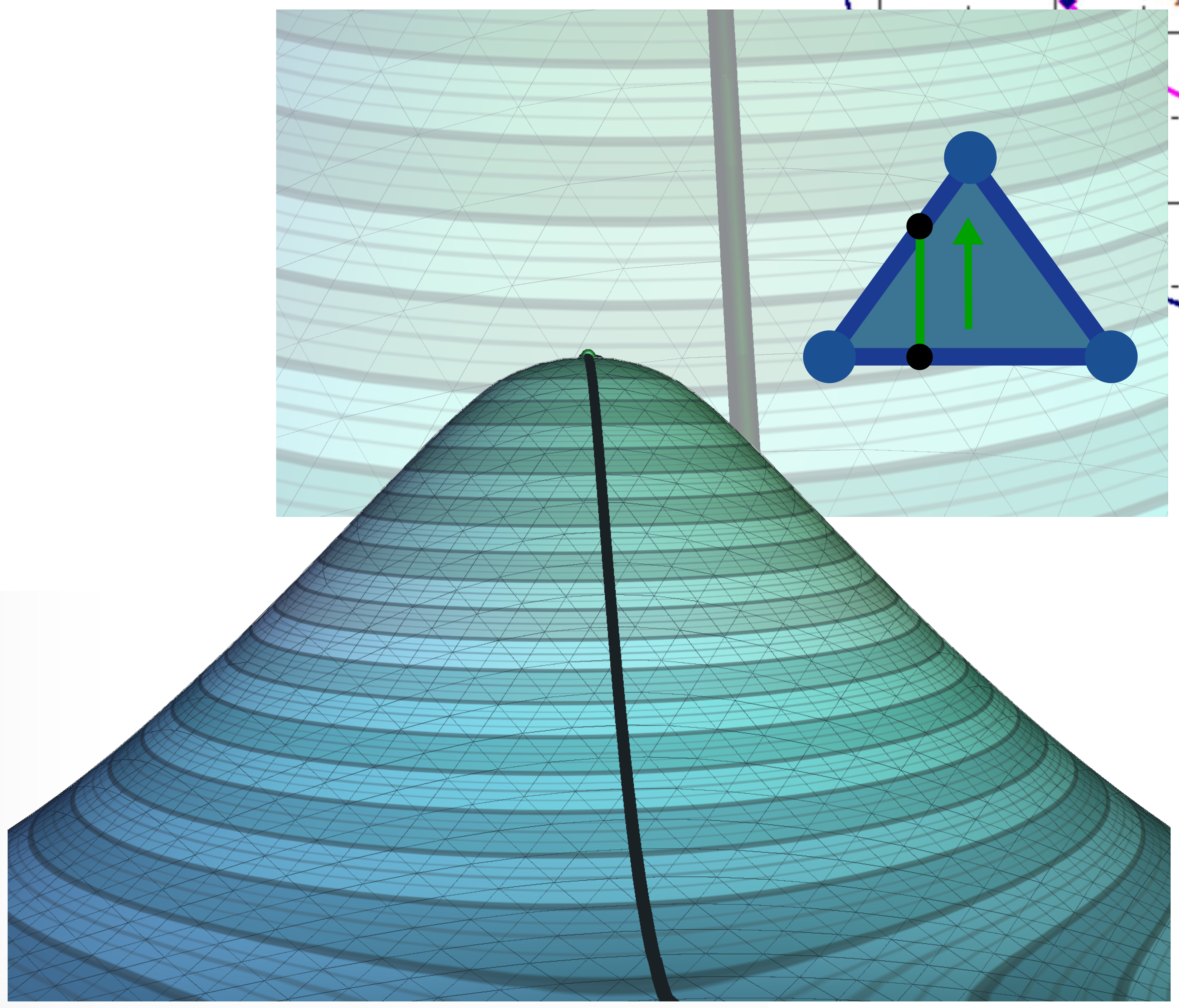

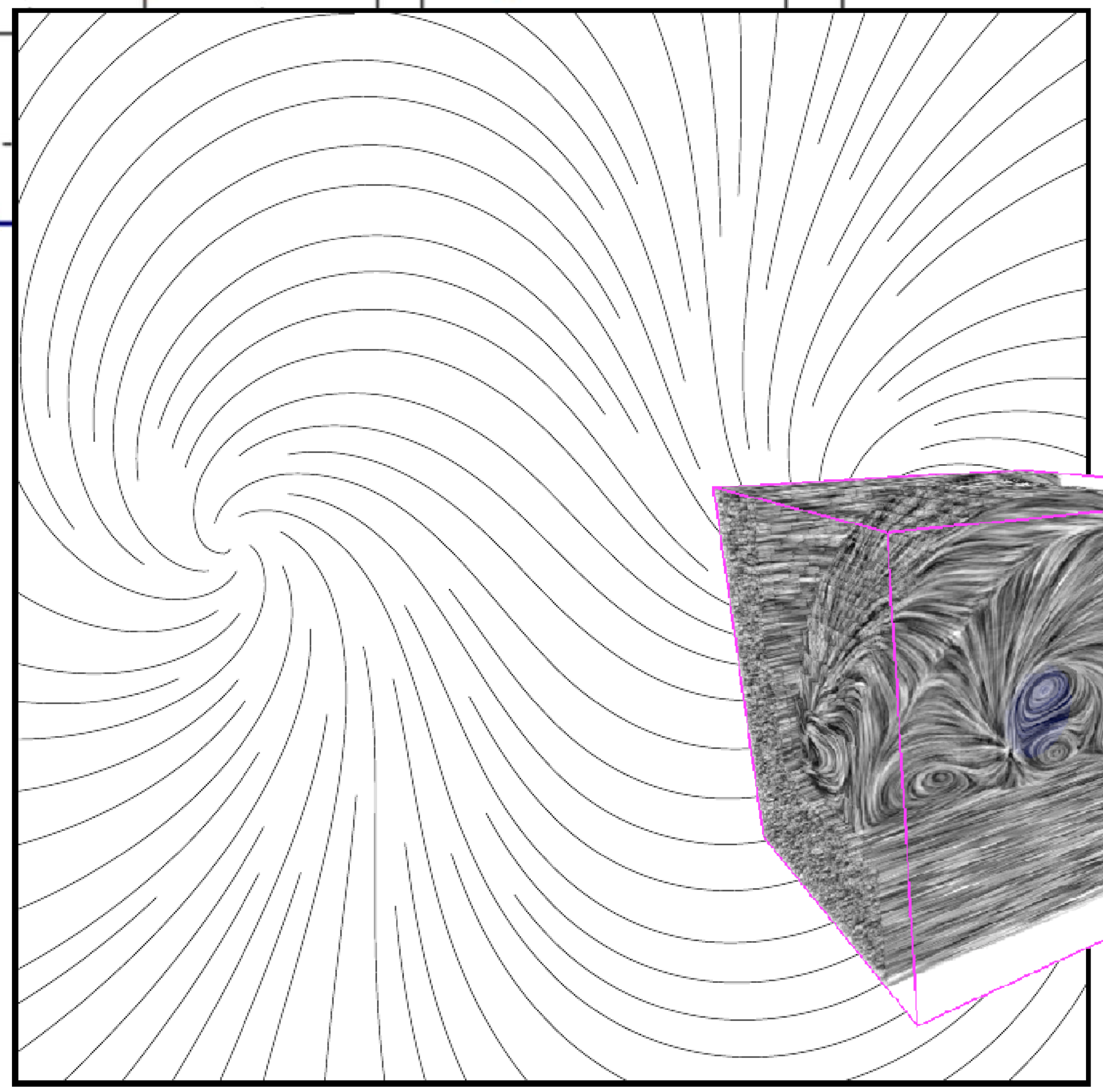

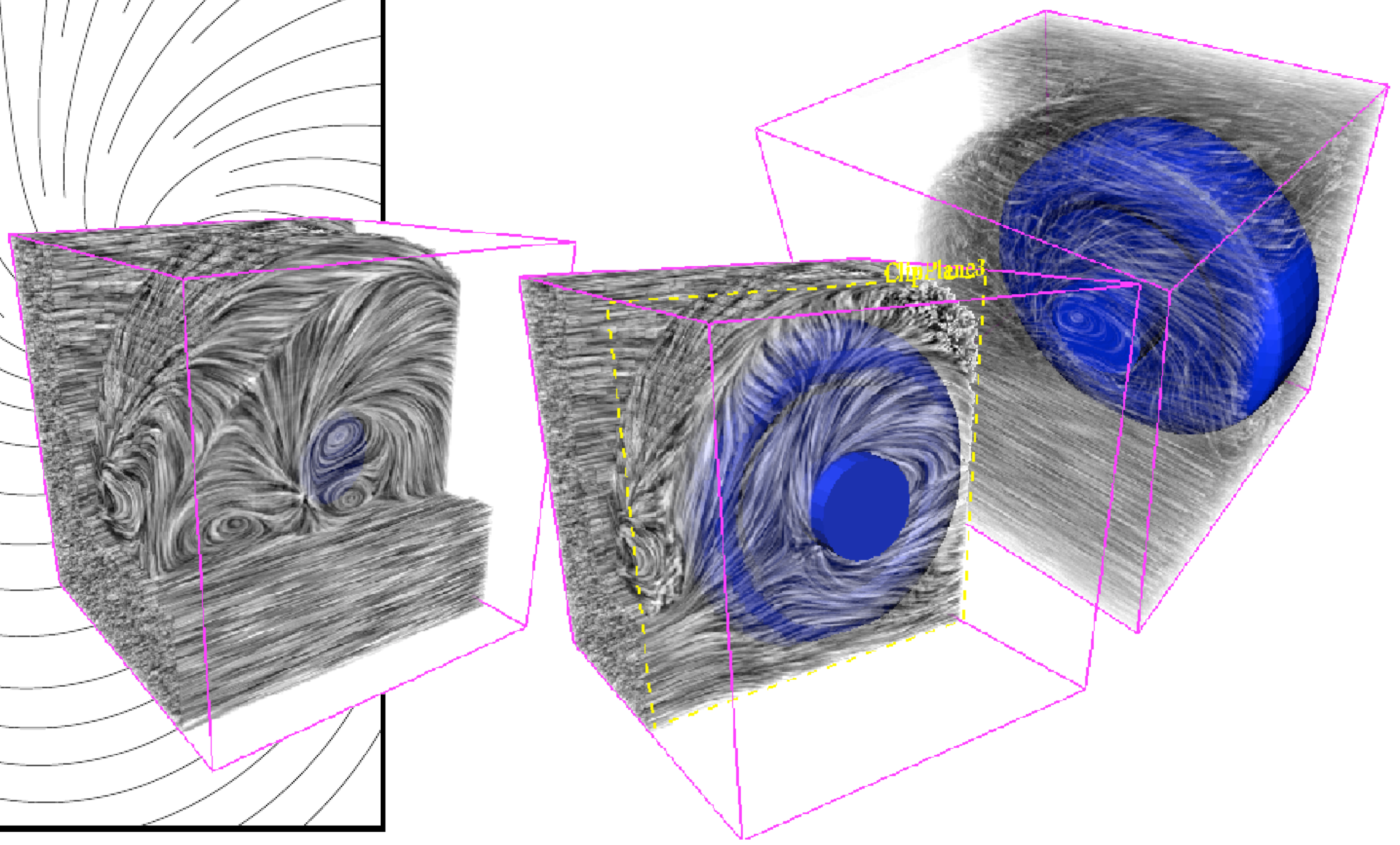

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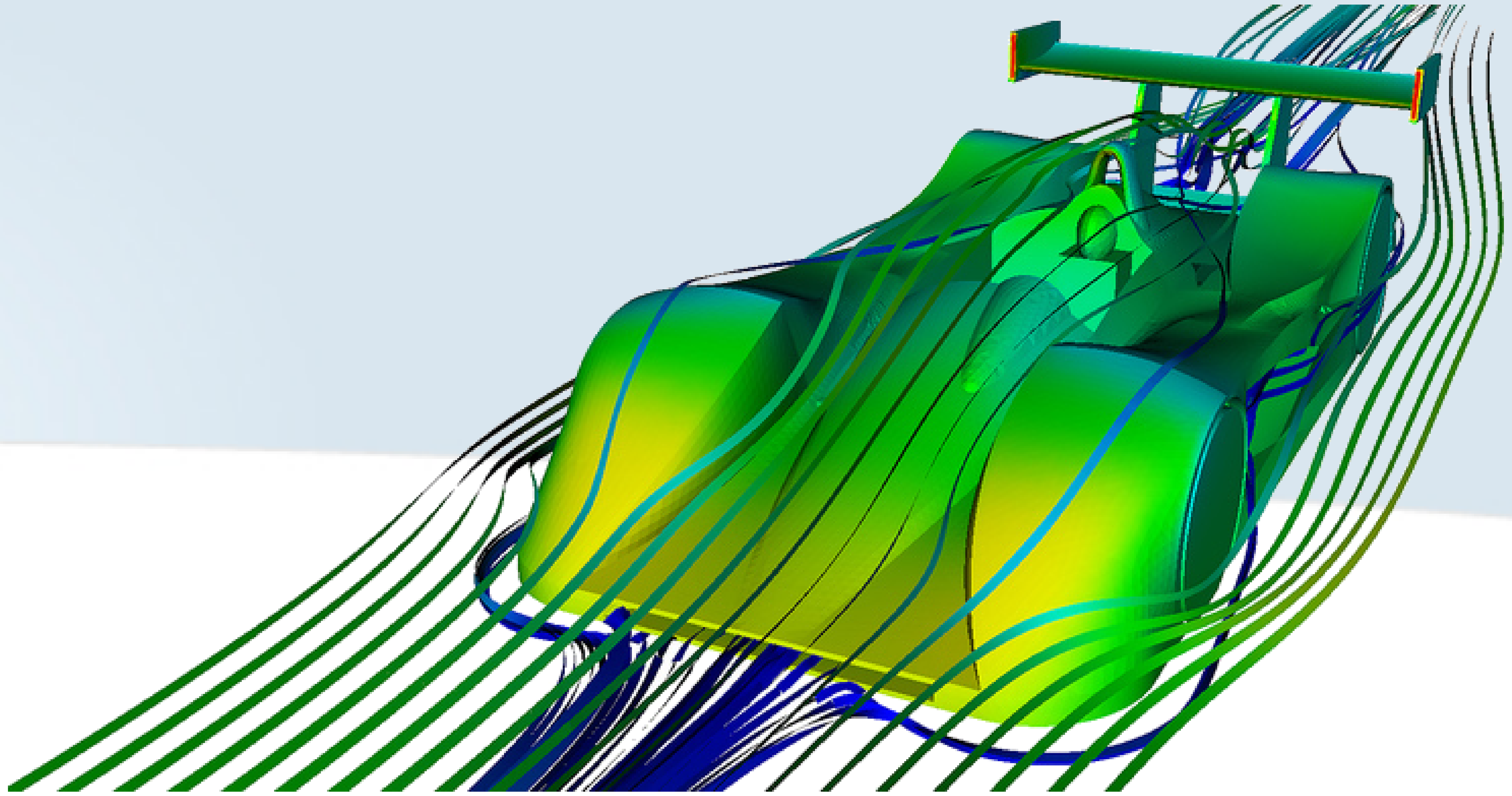

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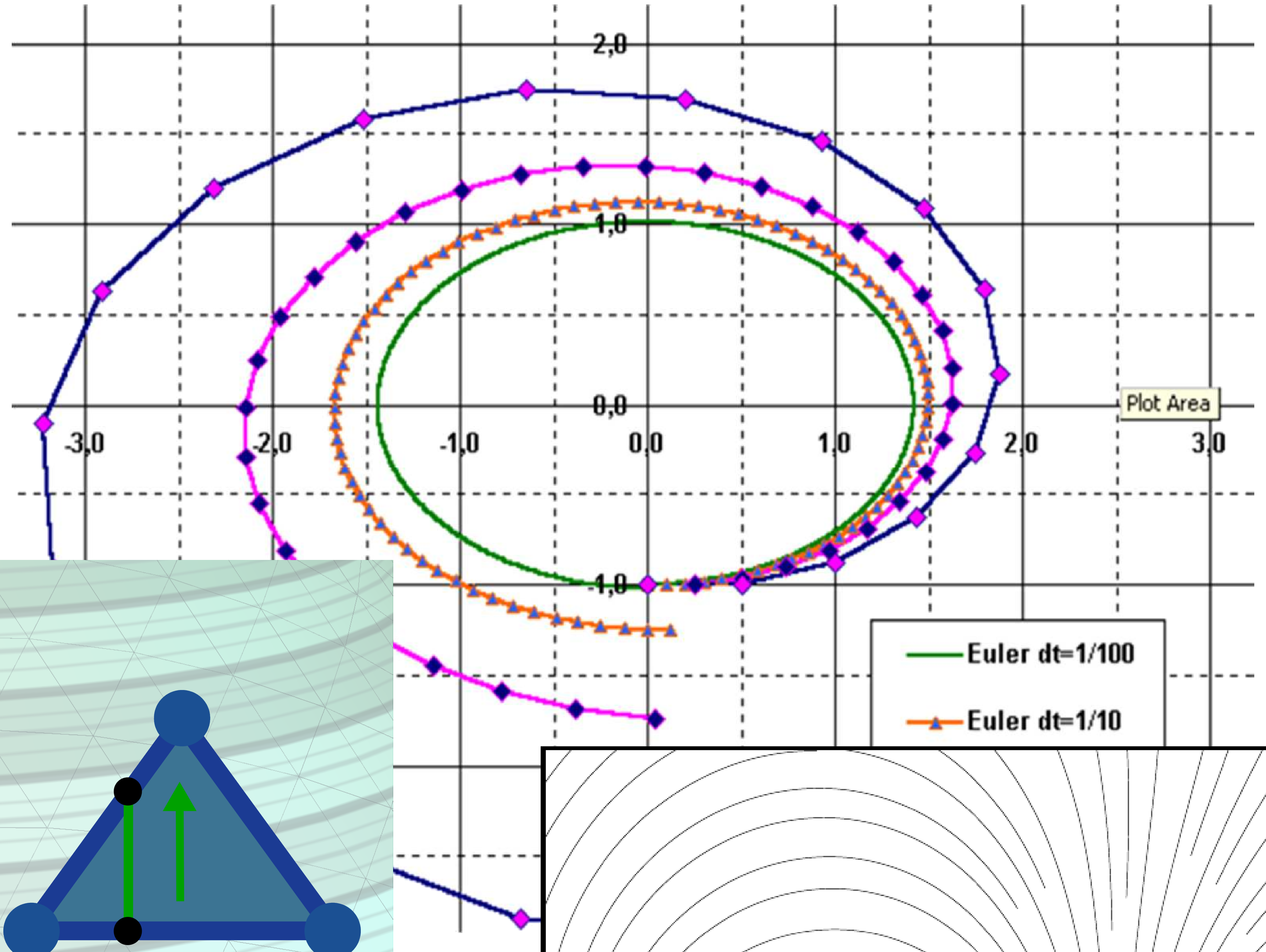

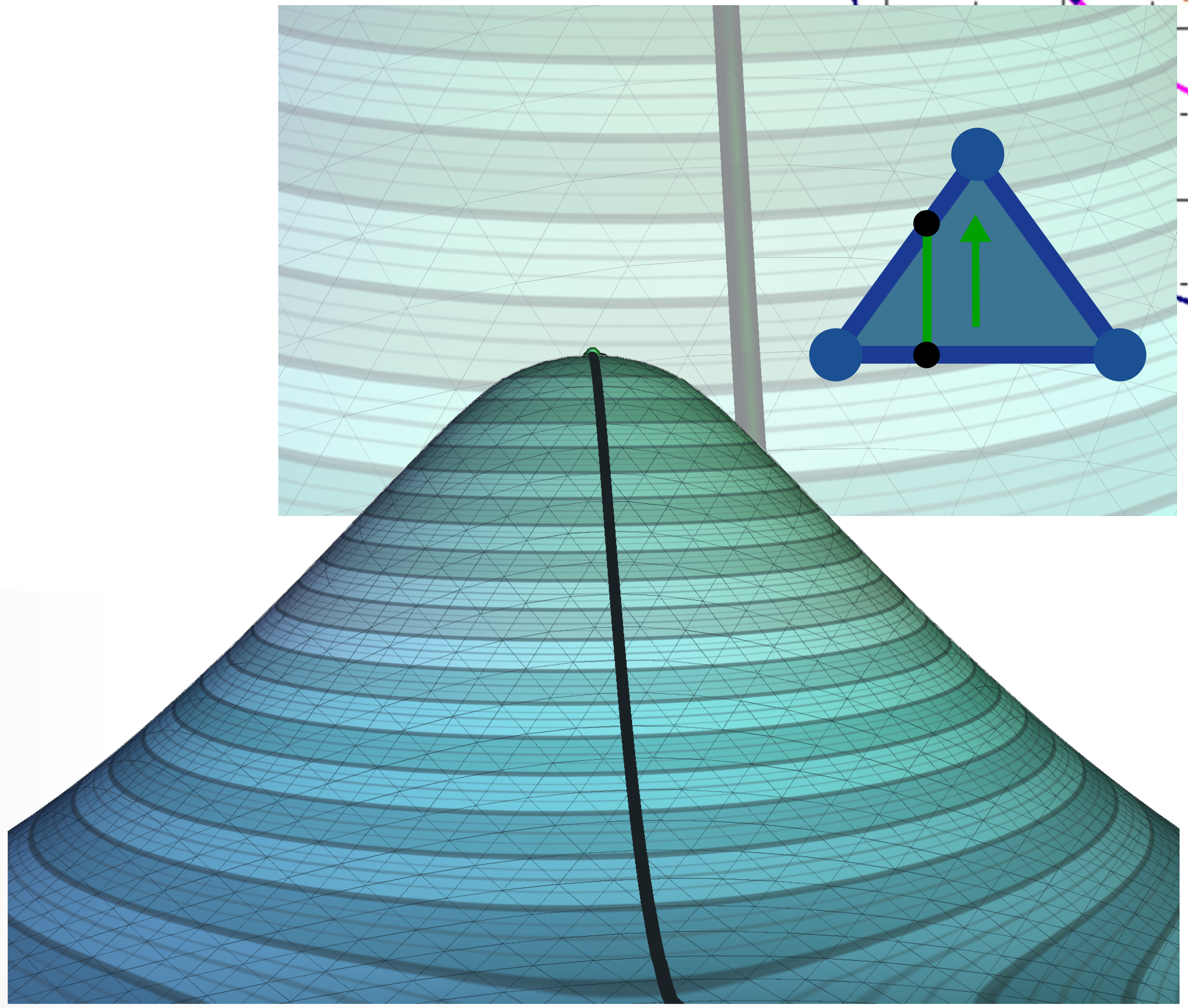

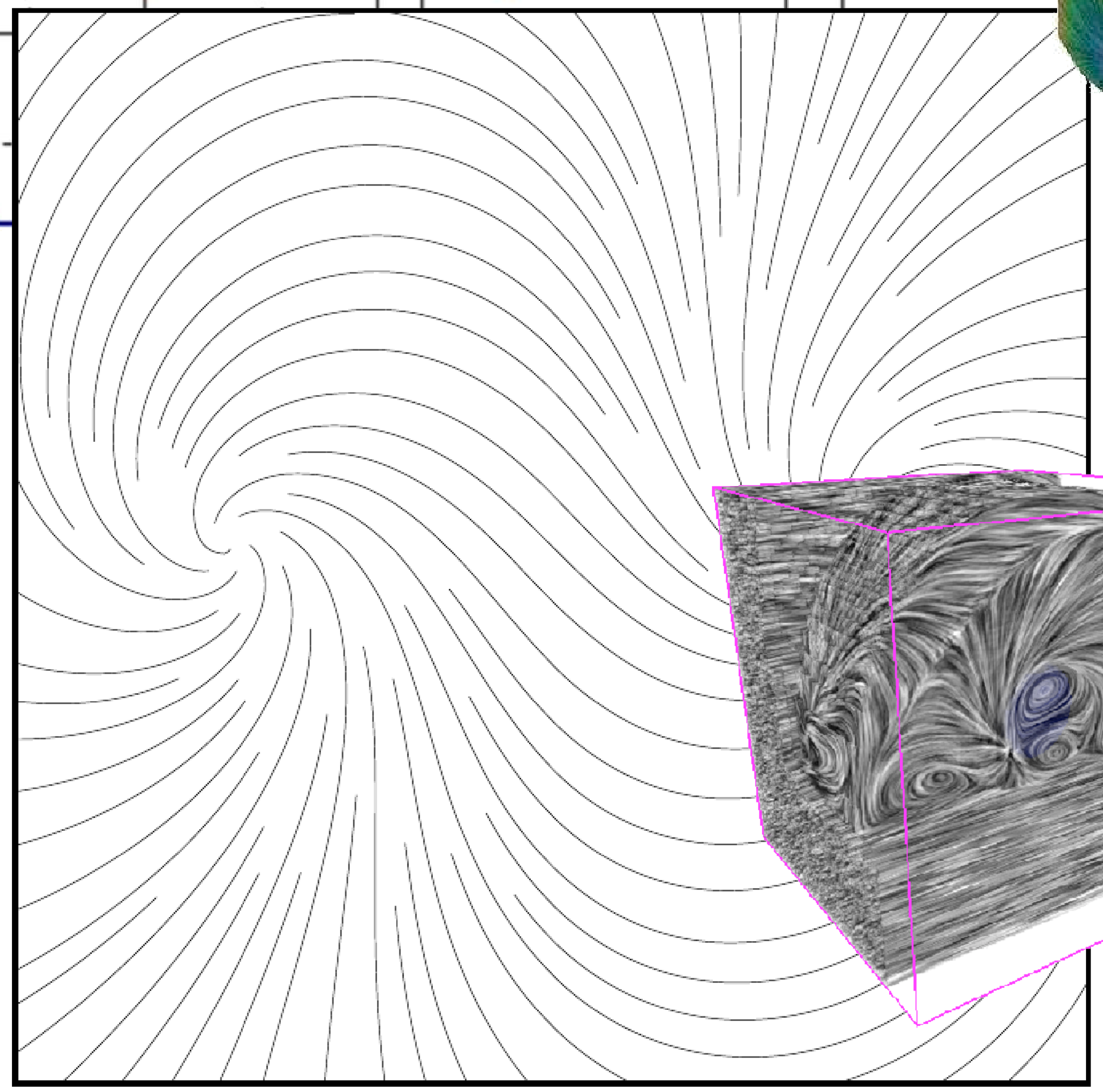

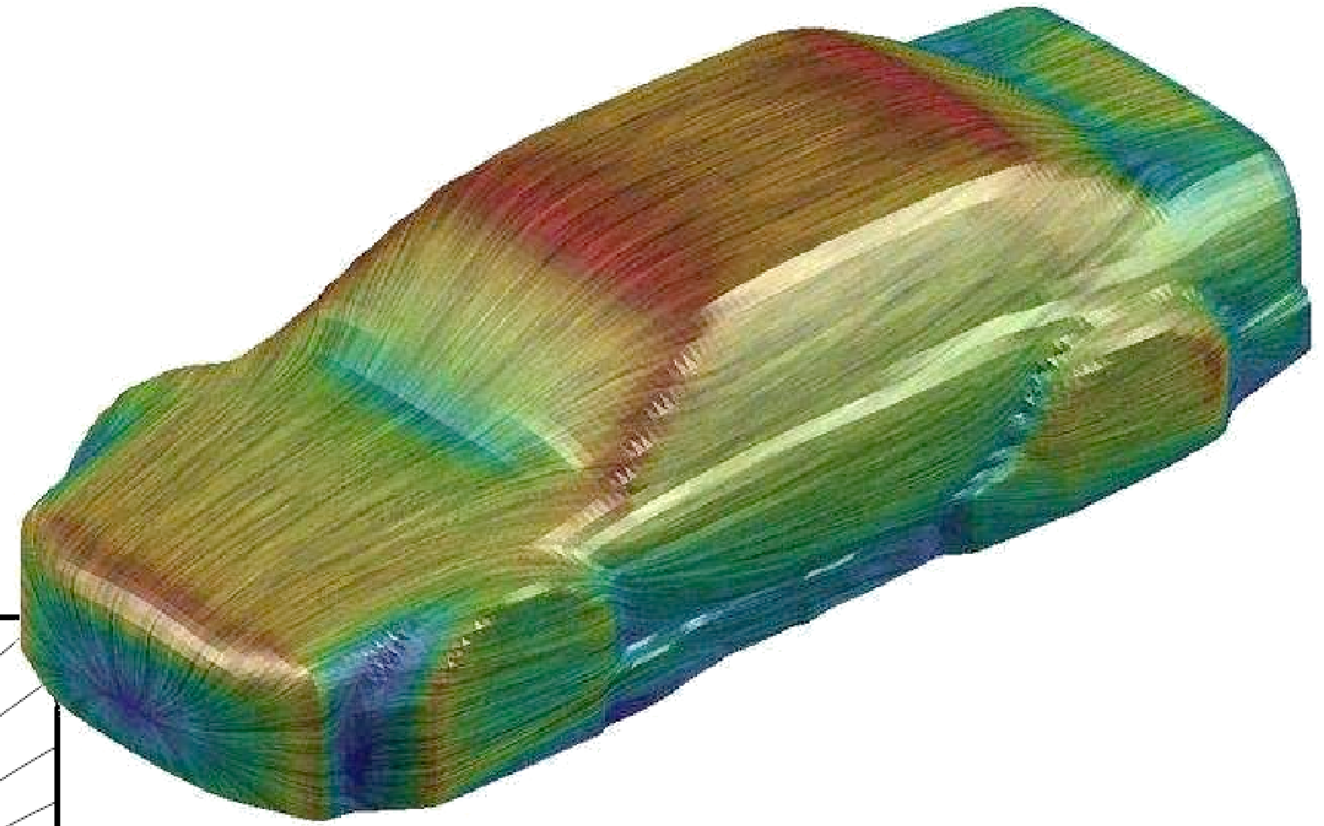

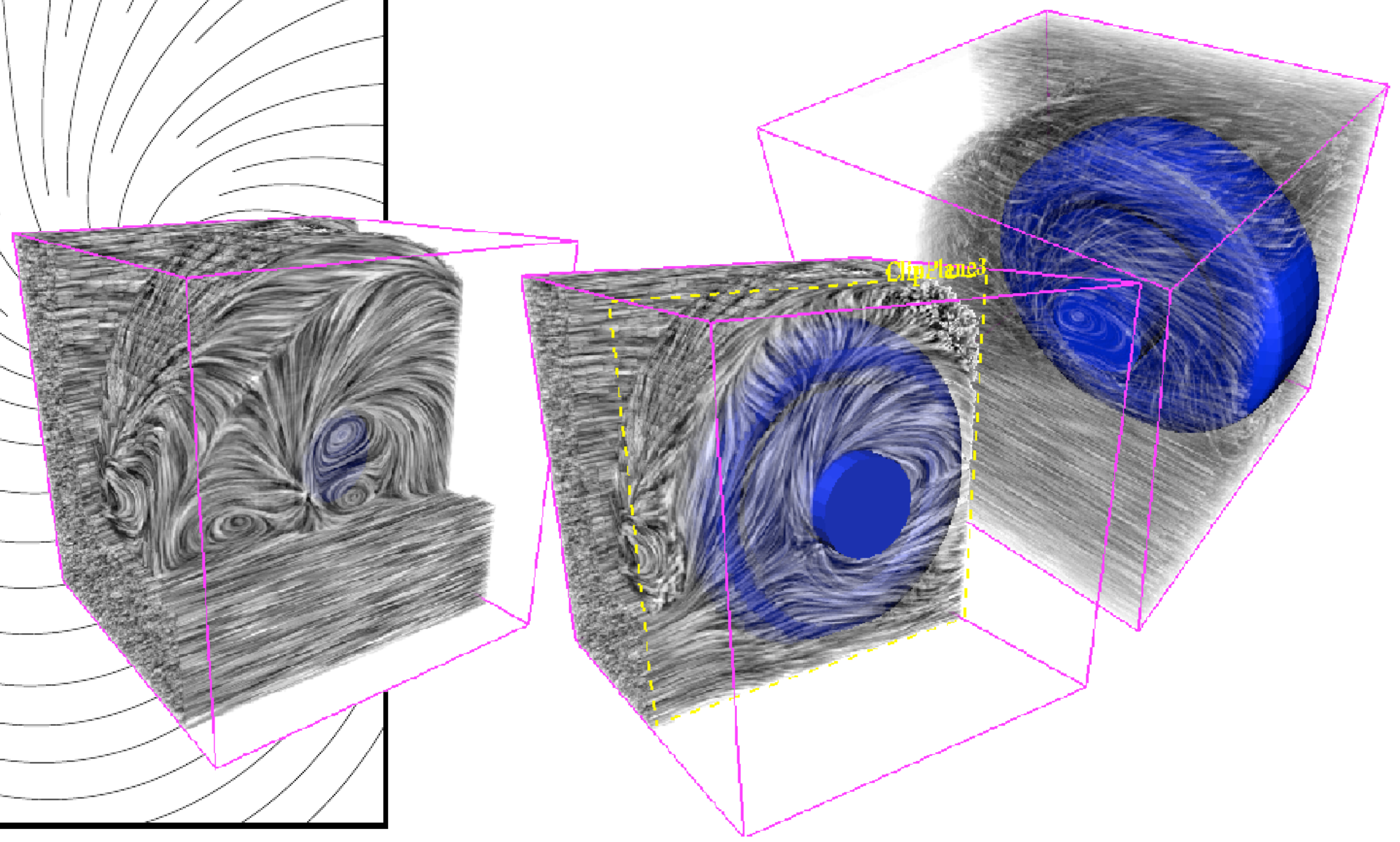

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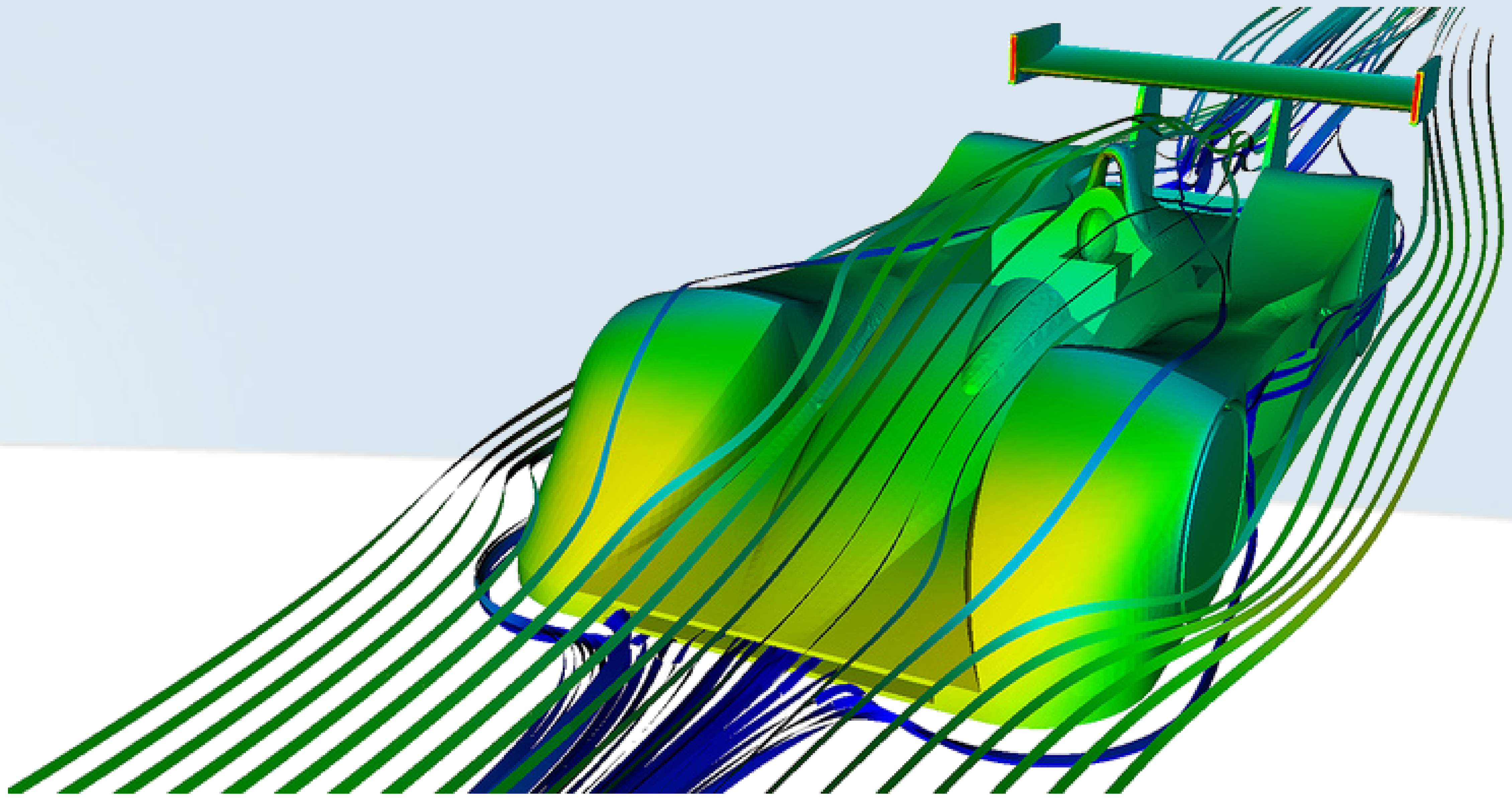
- Vector fields



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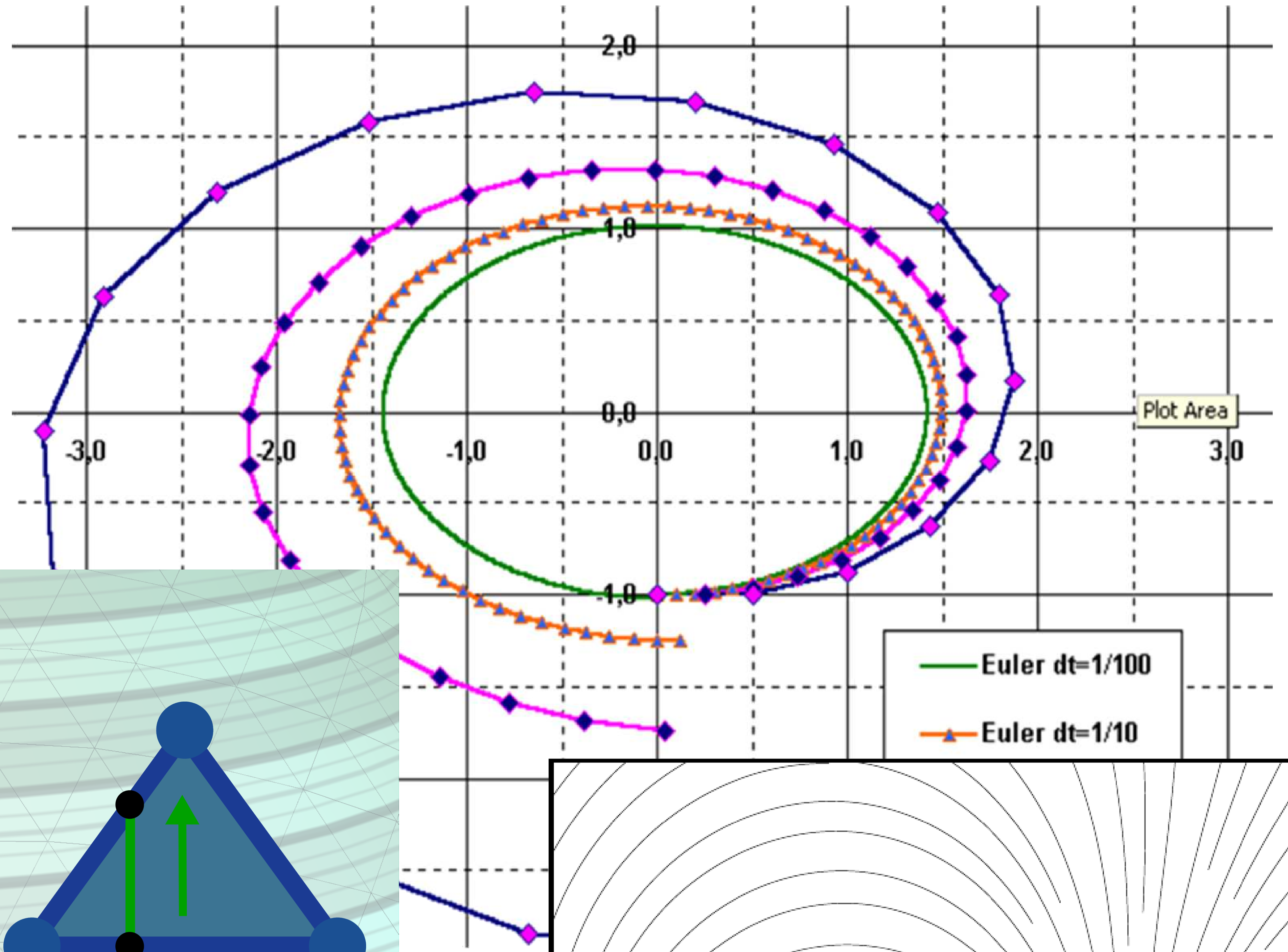
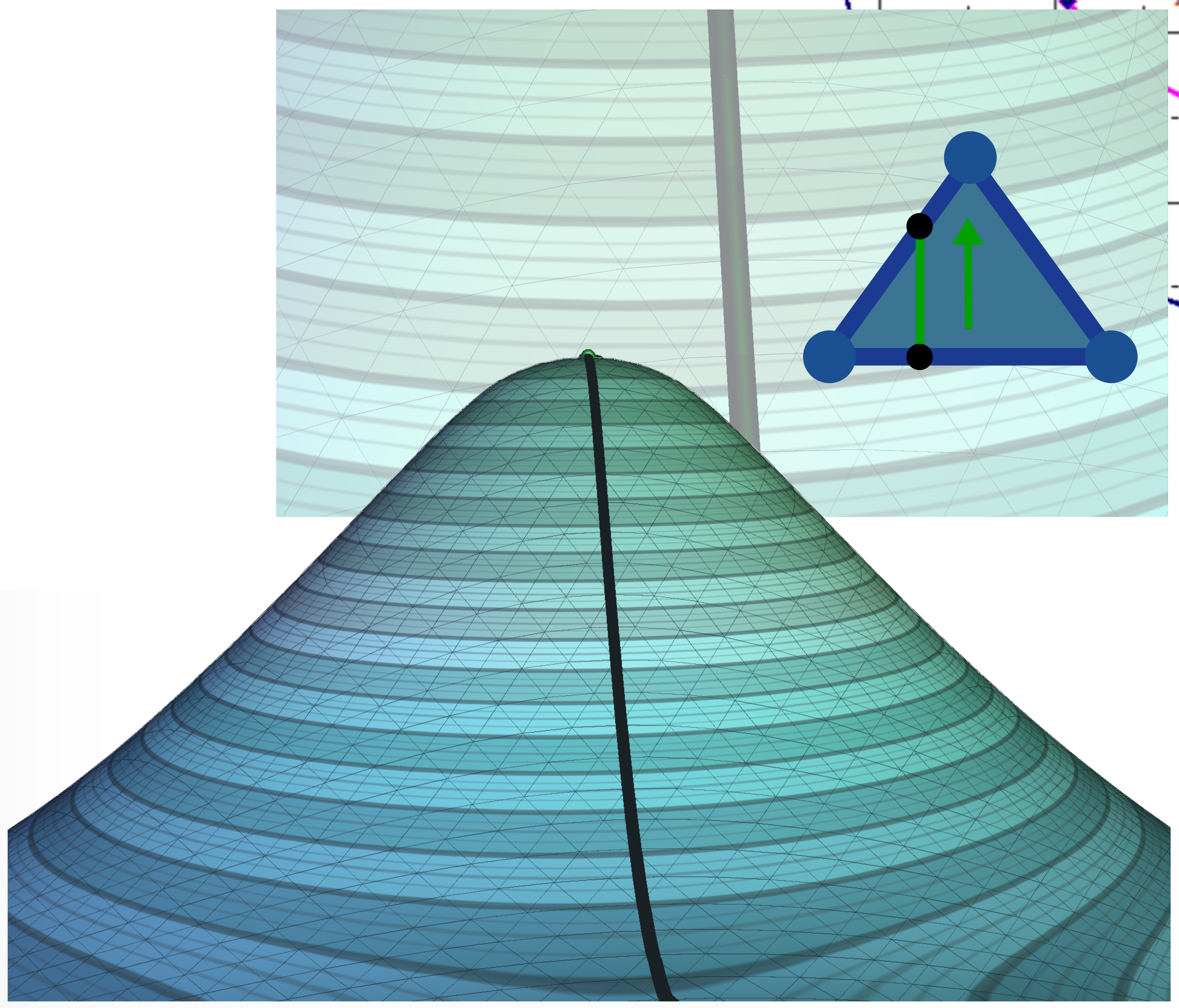
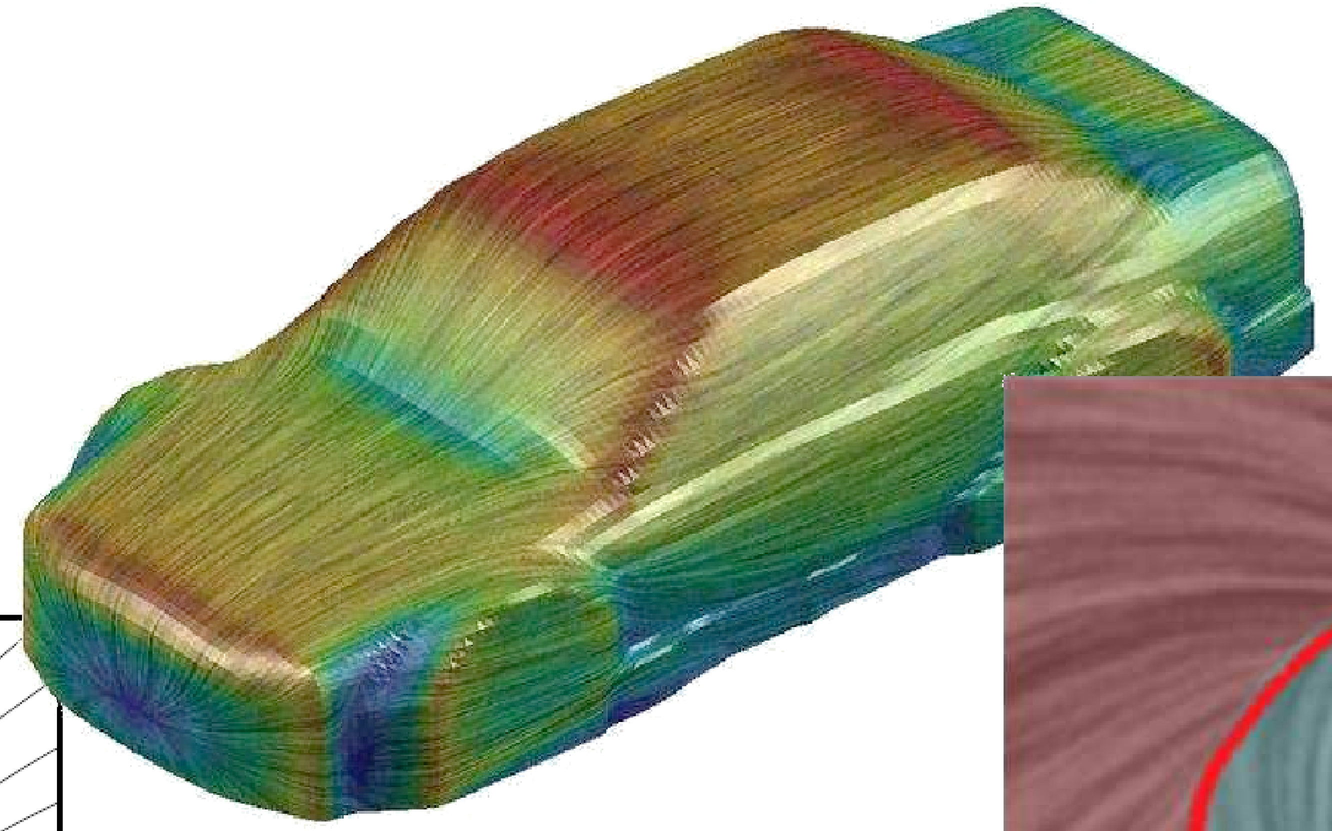
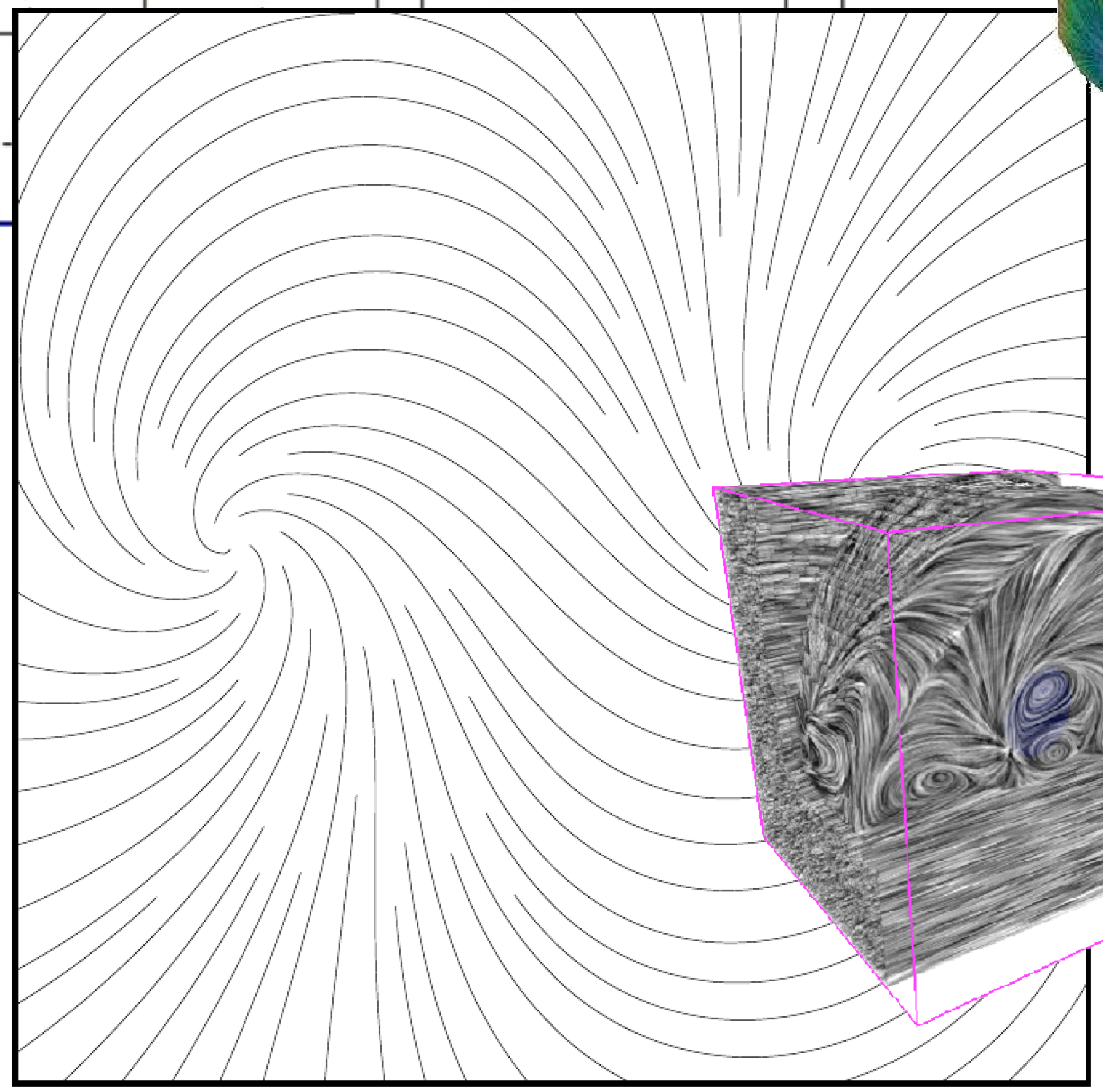
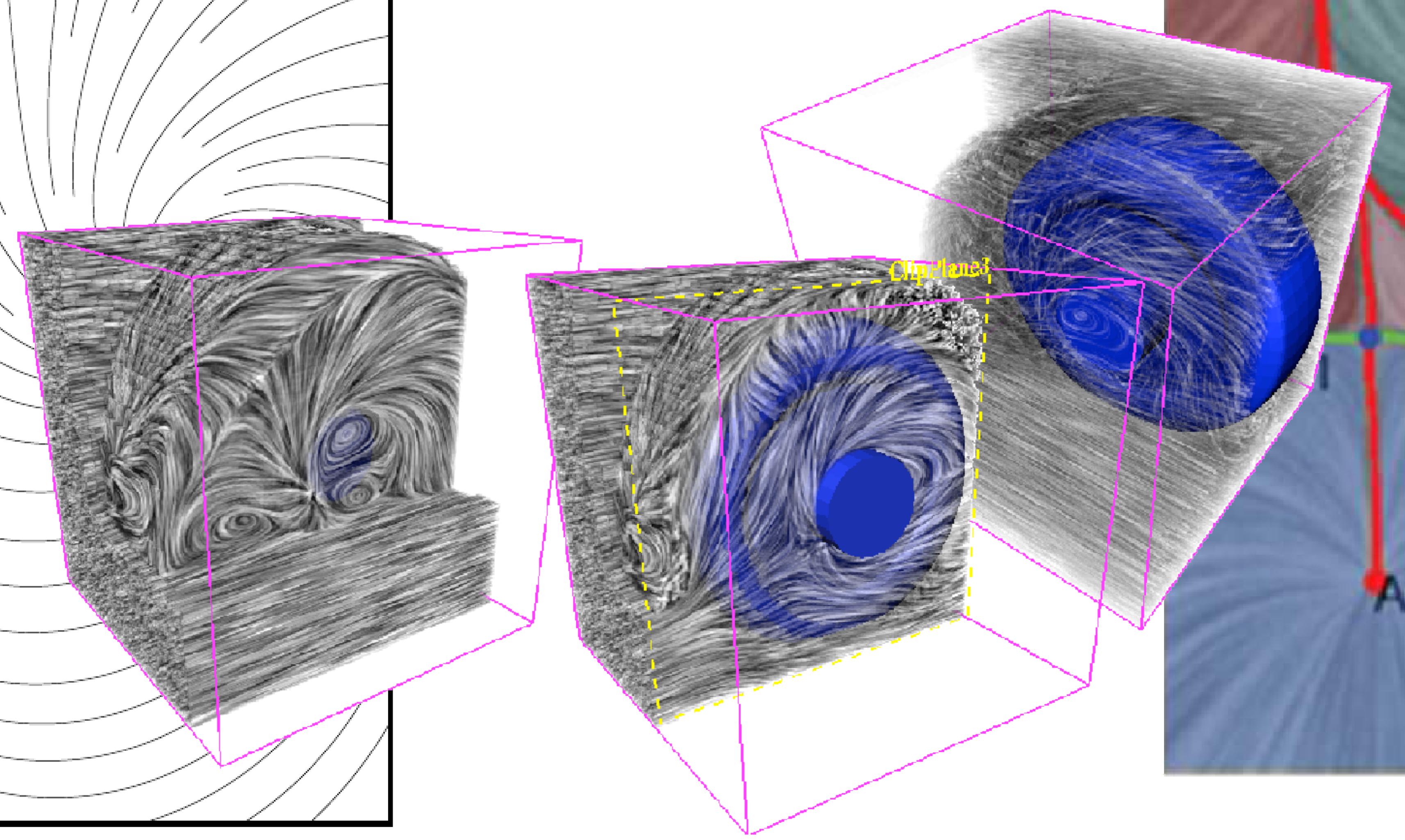
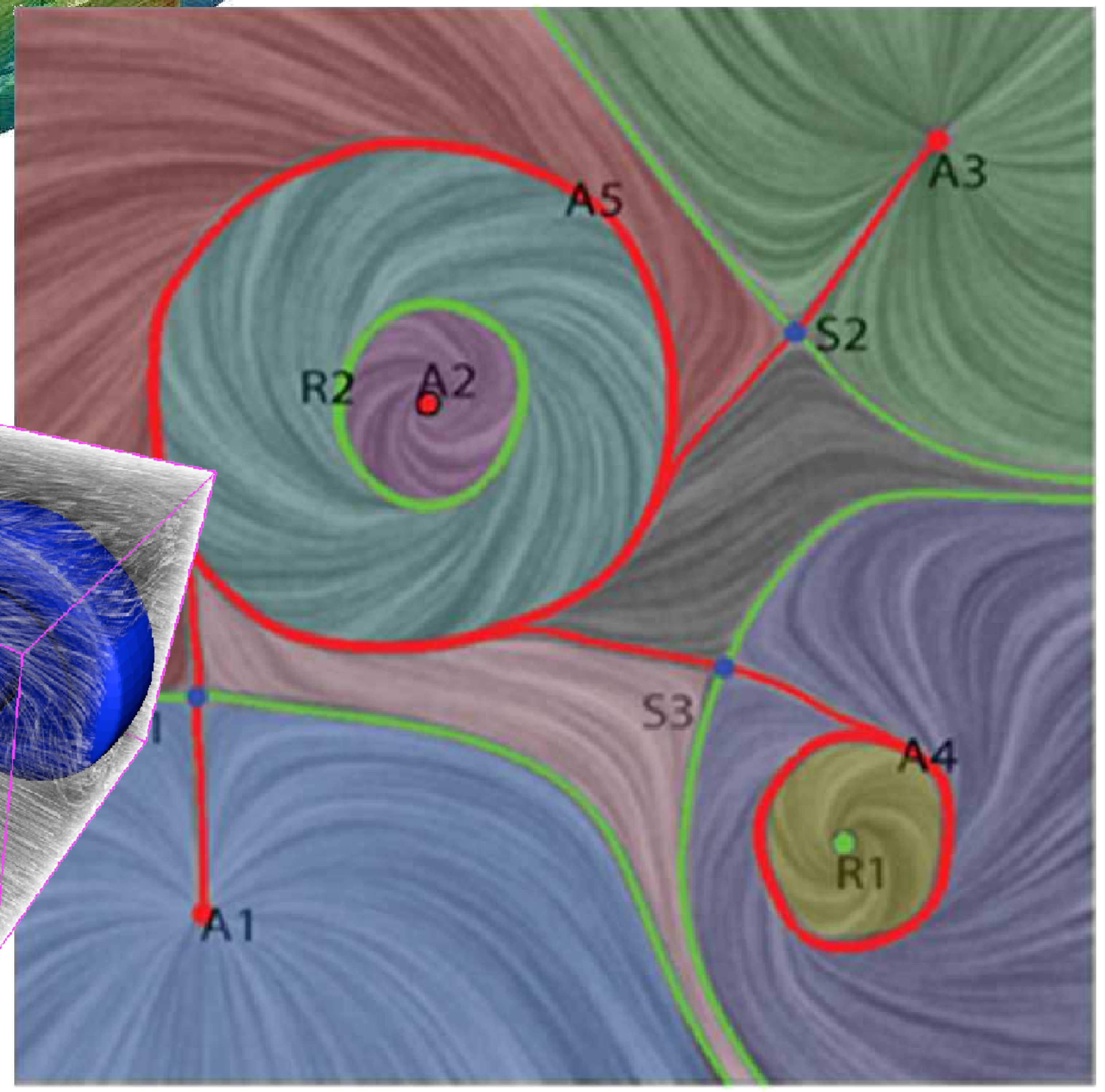










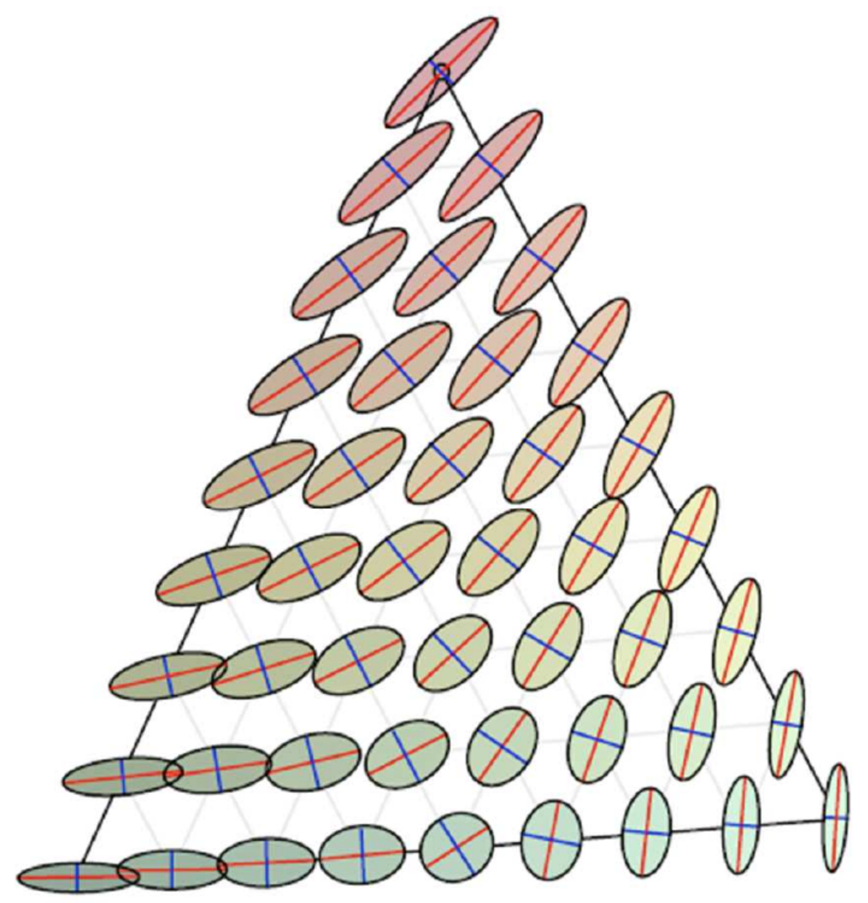


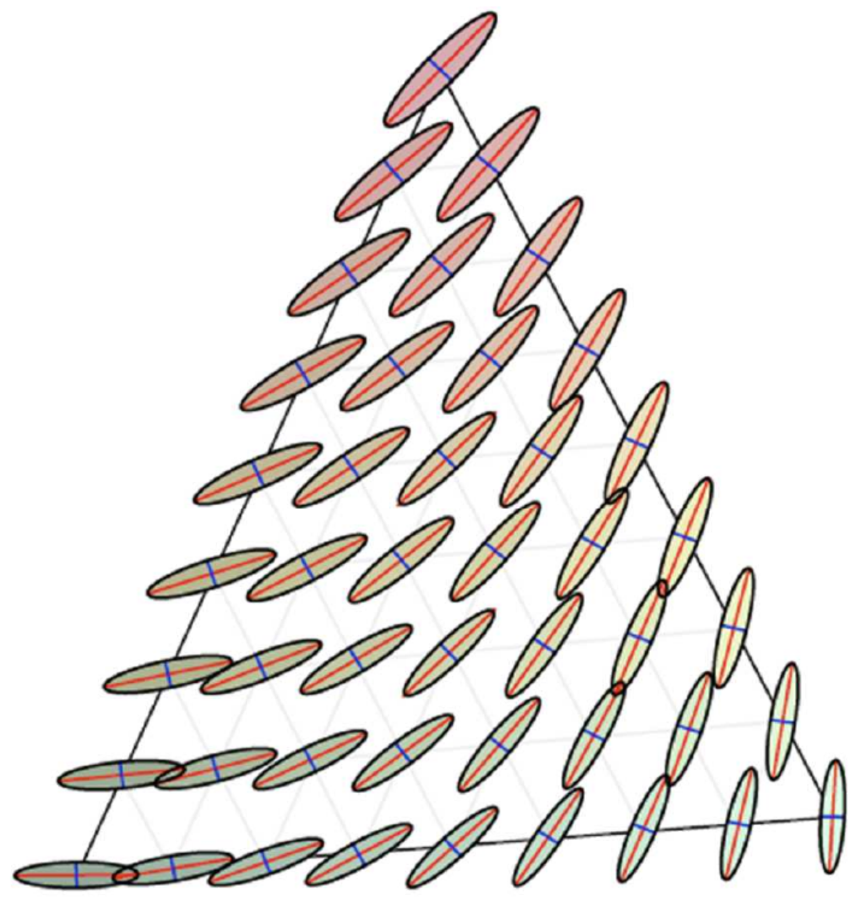
Farewell

- Tensor Fields

Farewell

- Tensor Fields





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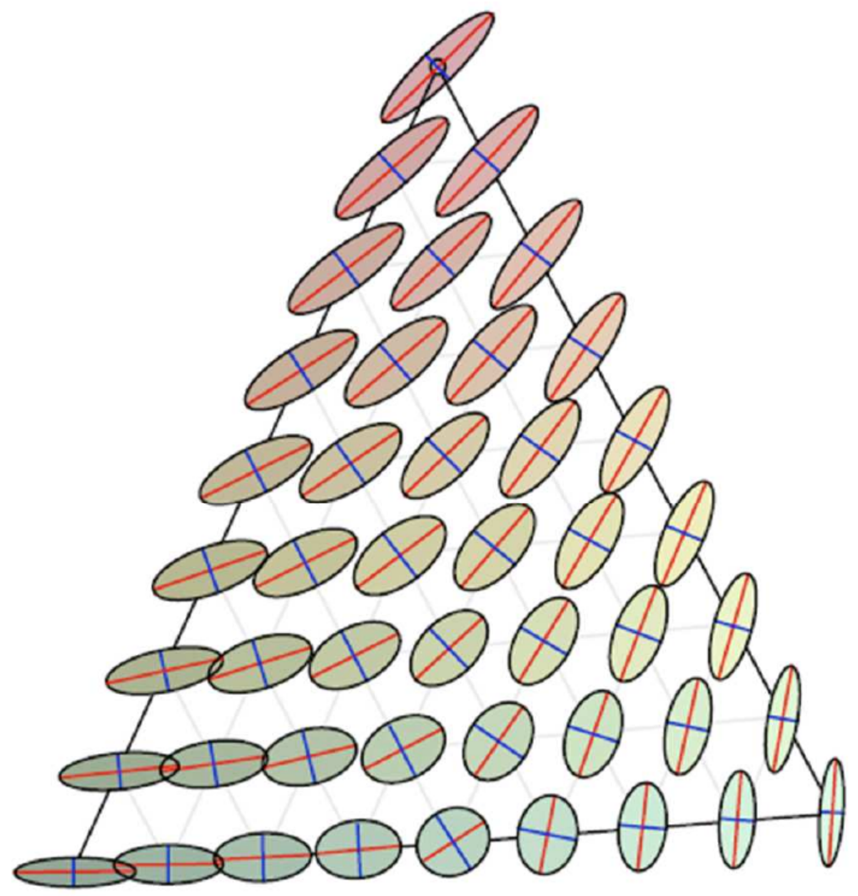
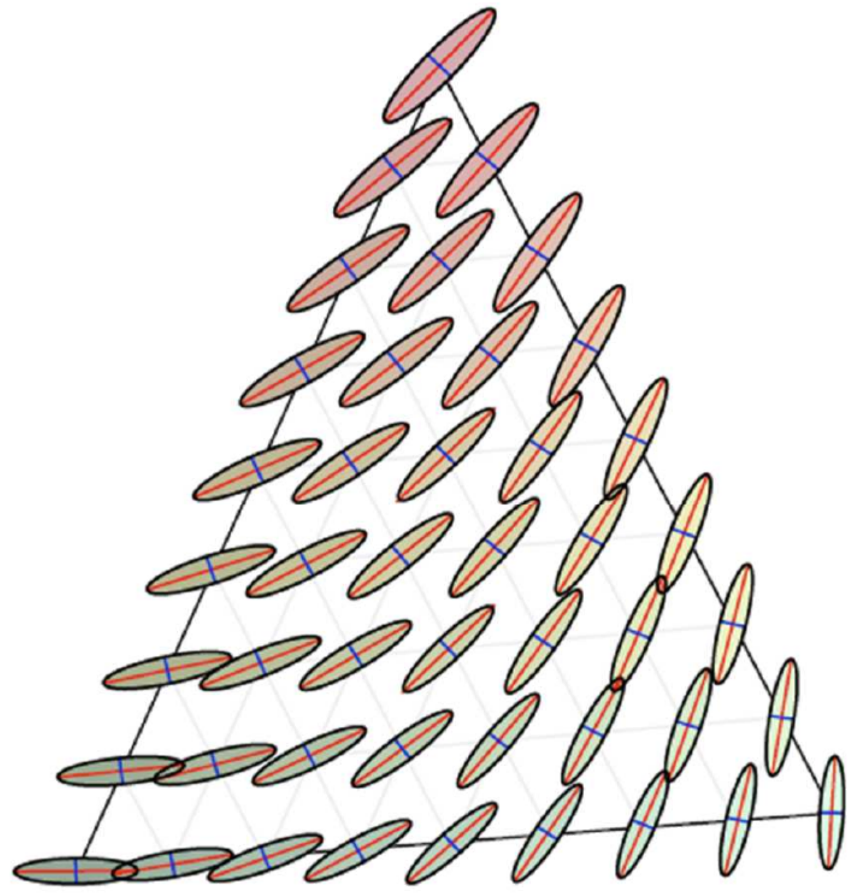
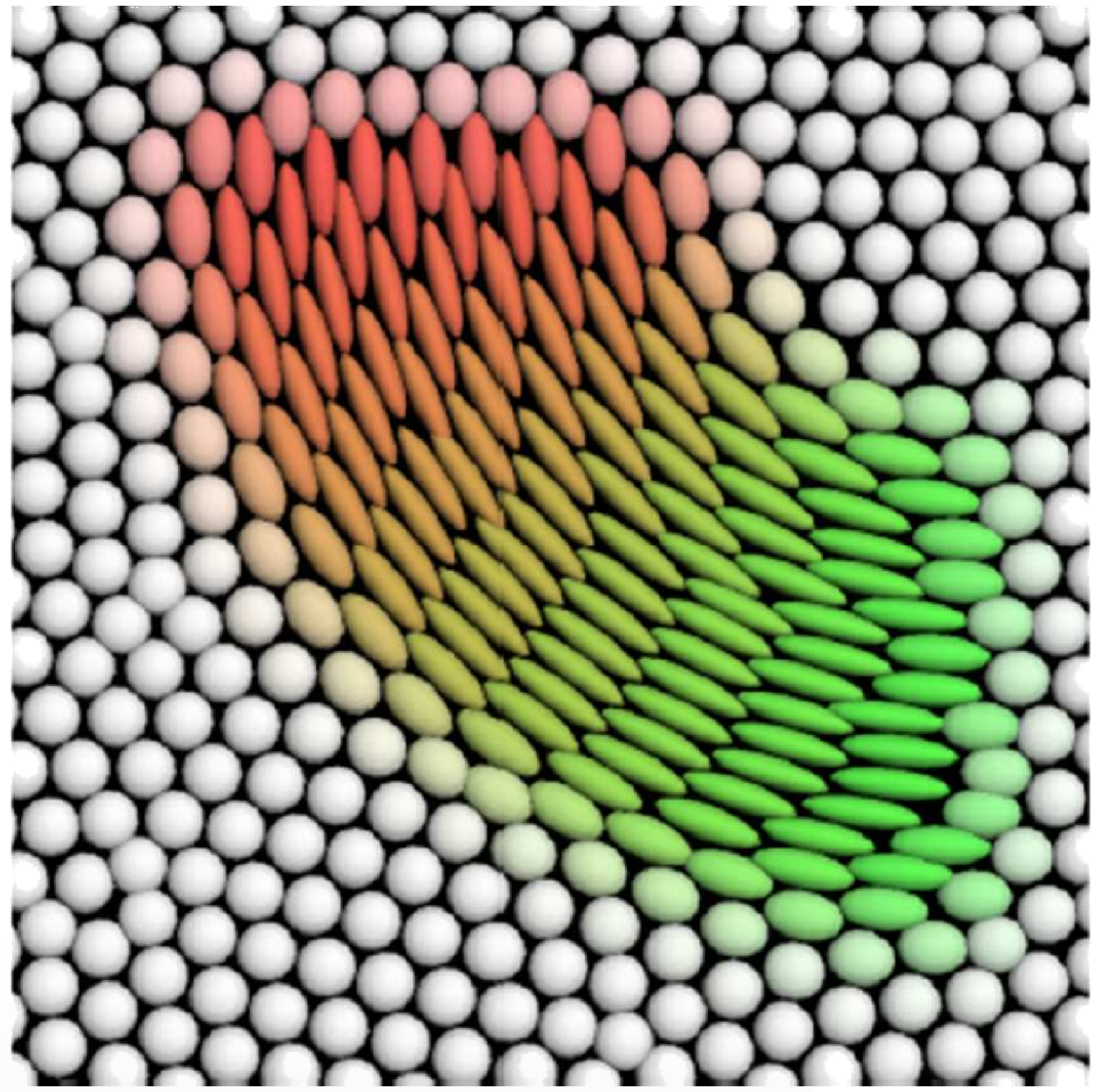
- Tensor Fields







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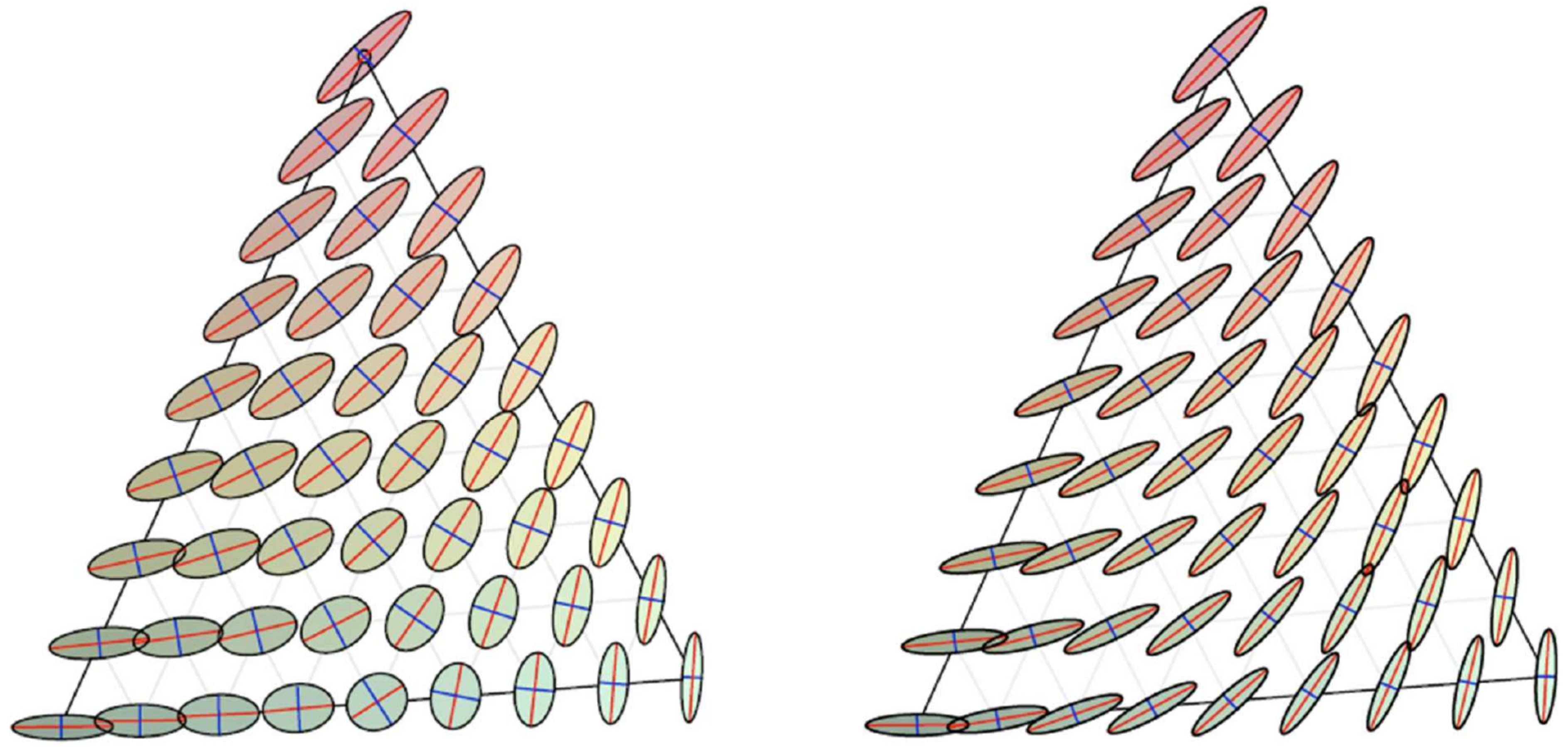
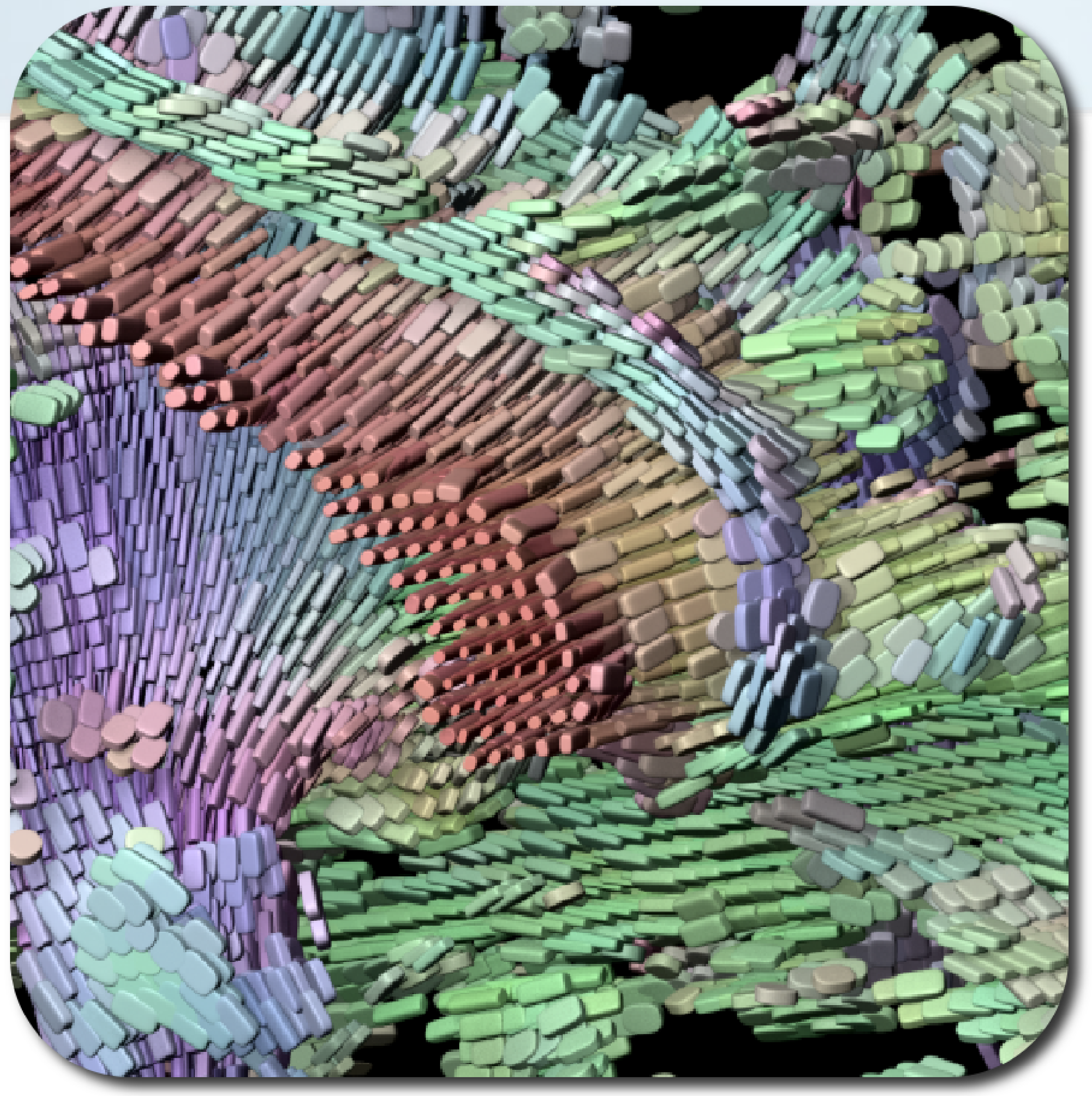
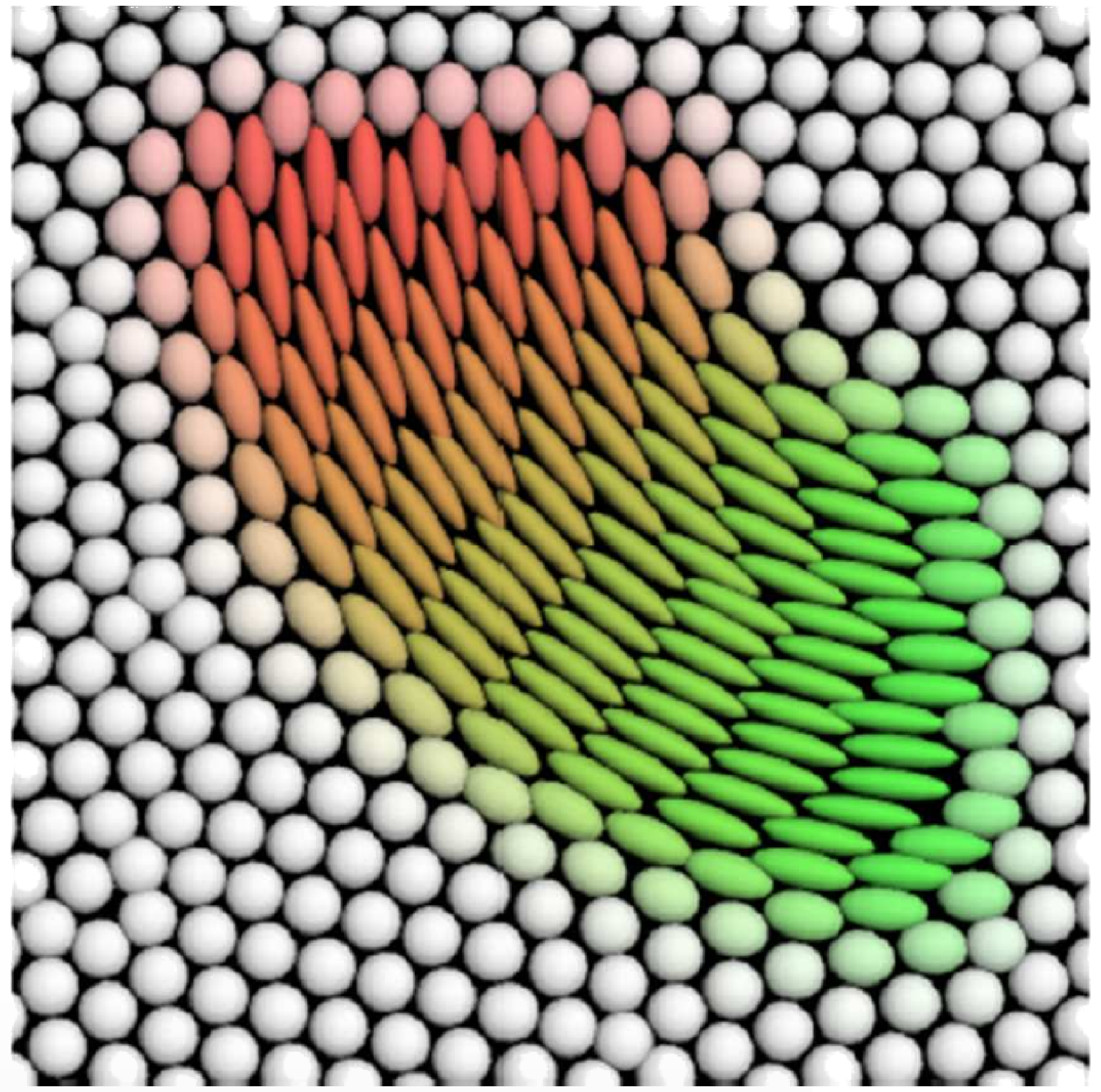
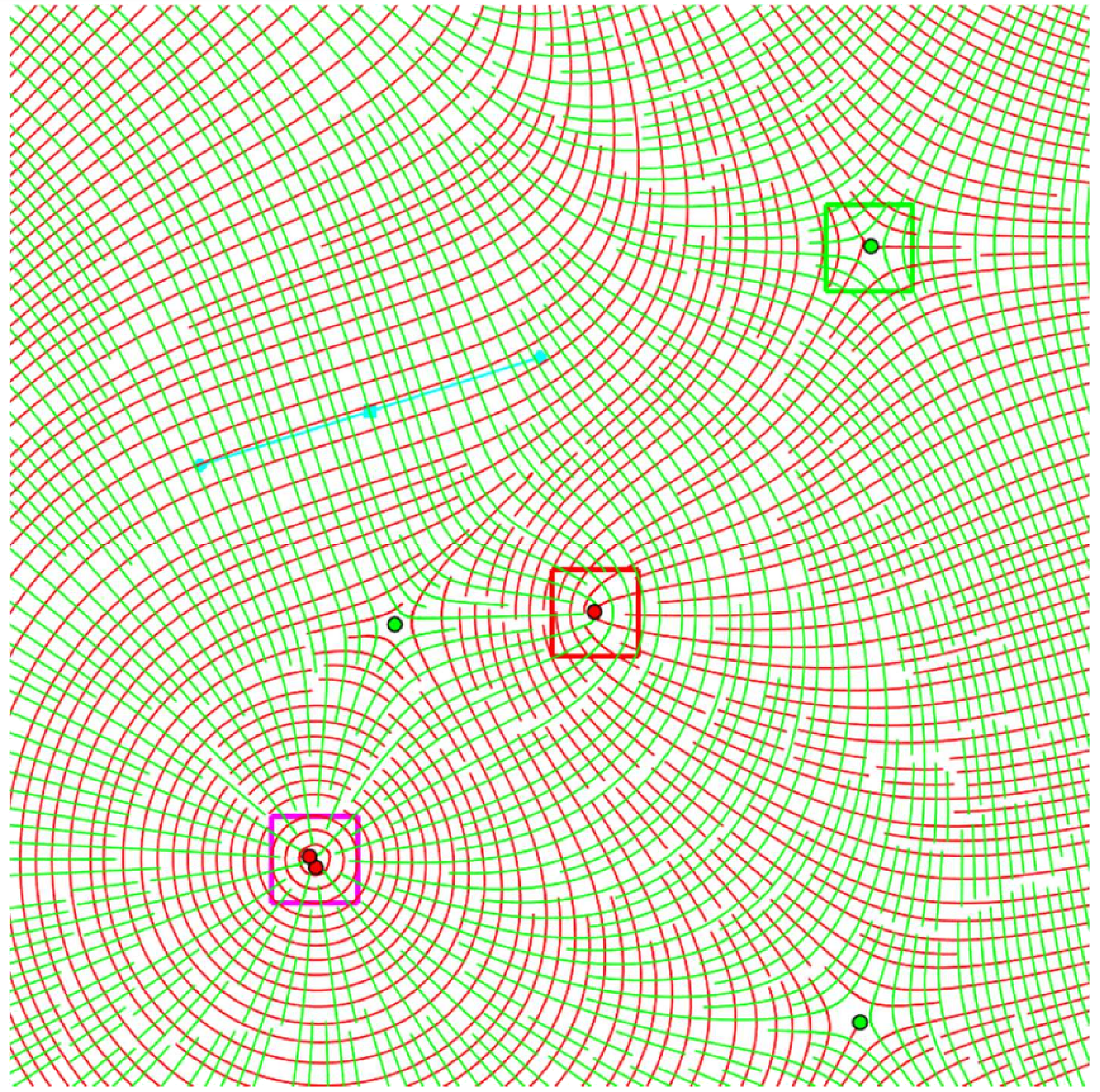
- Tensor Fields









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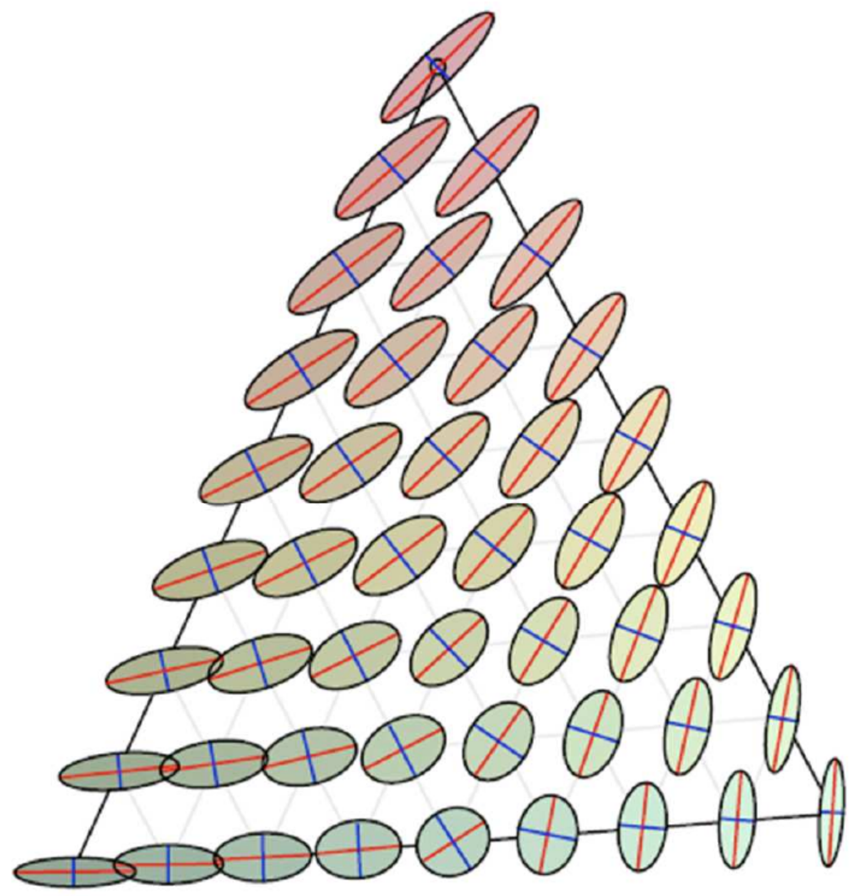
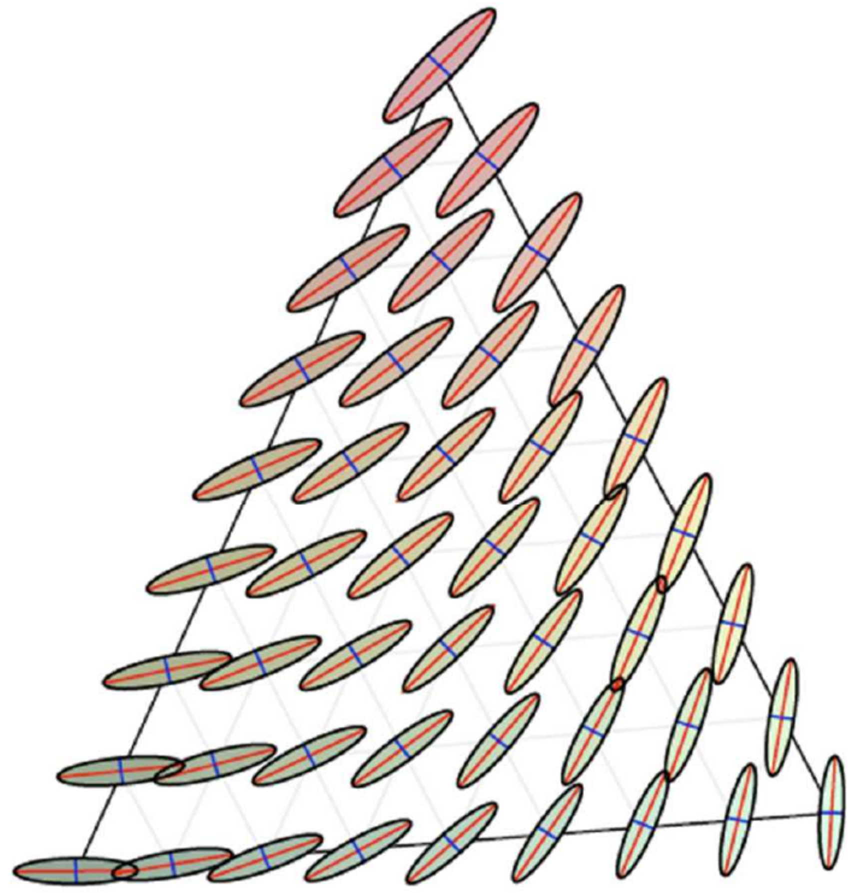
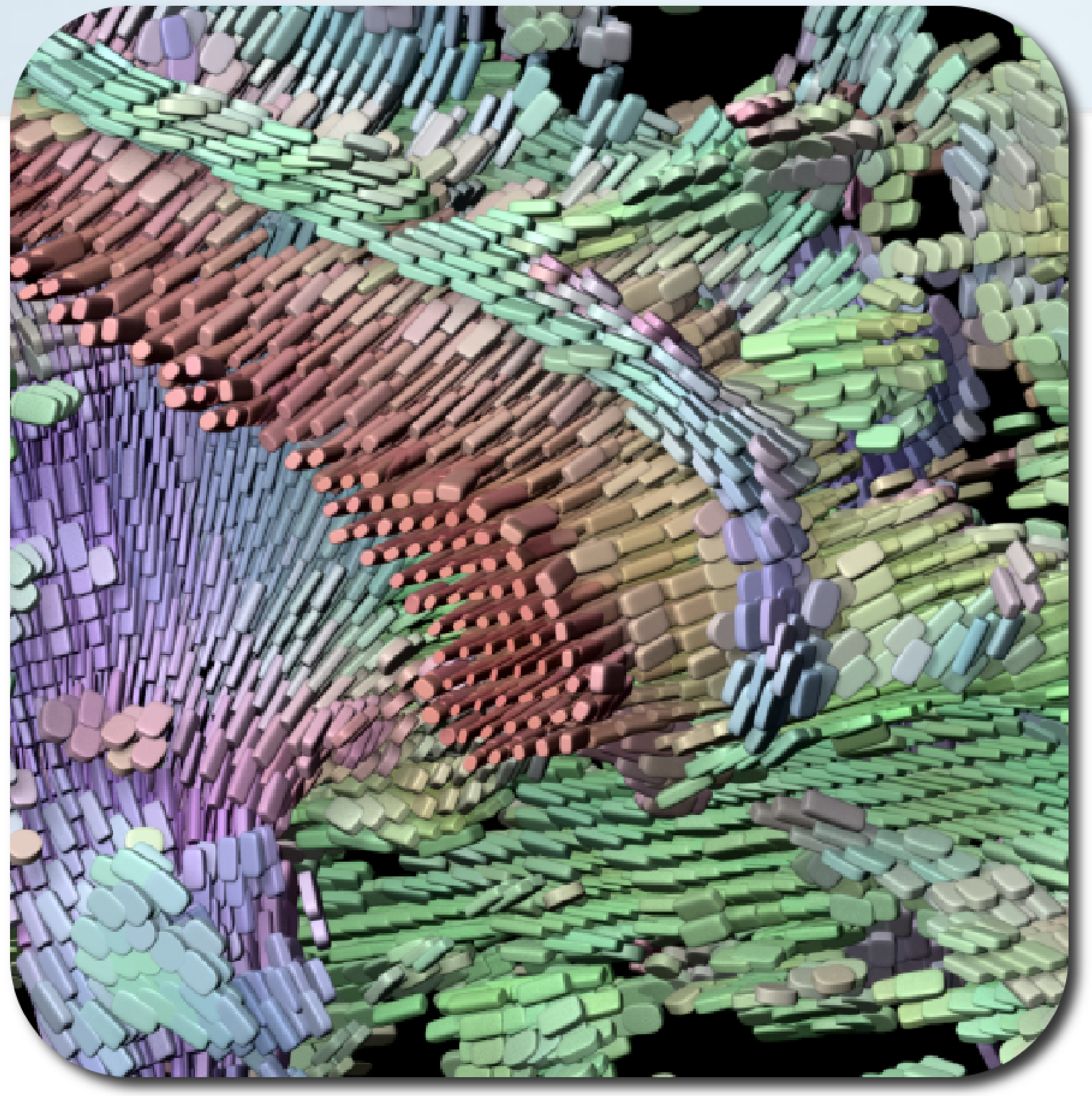
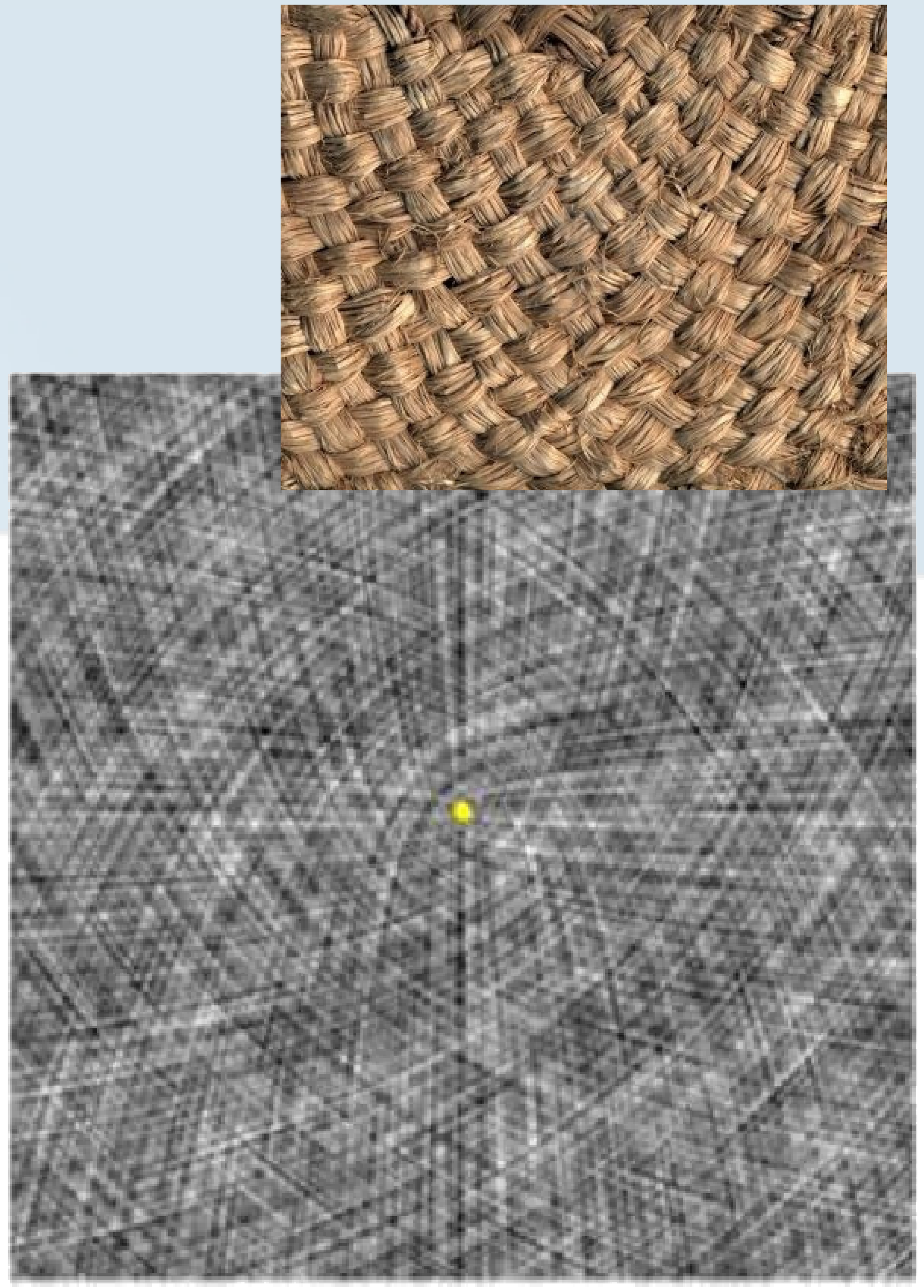
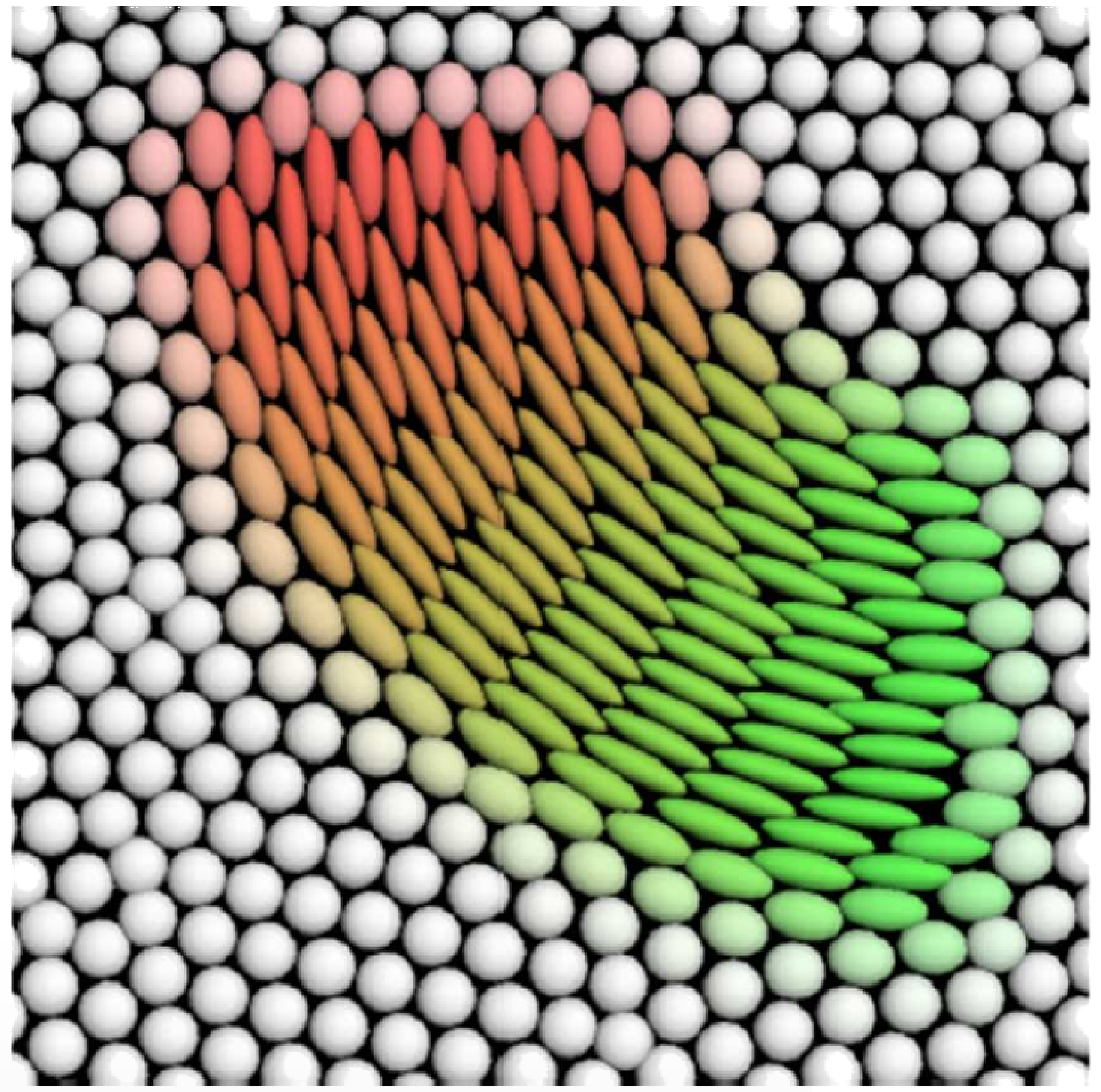
- Tensor Fields

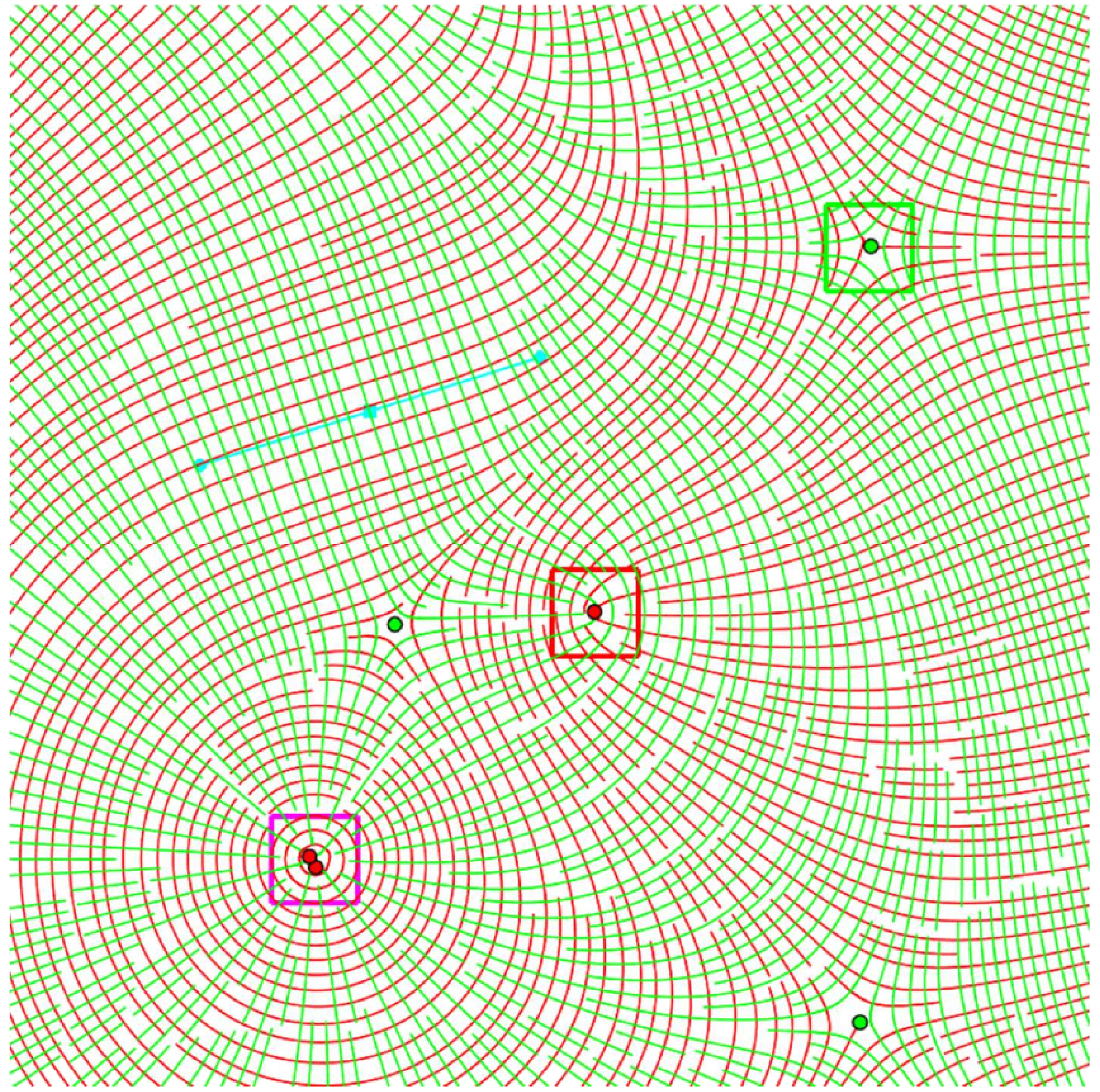






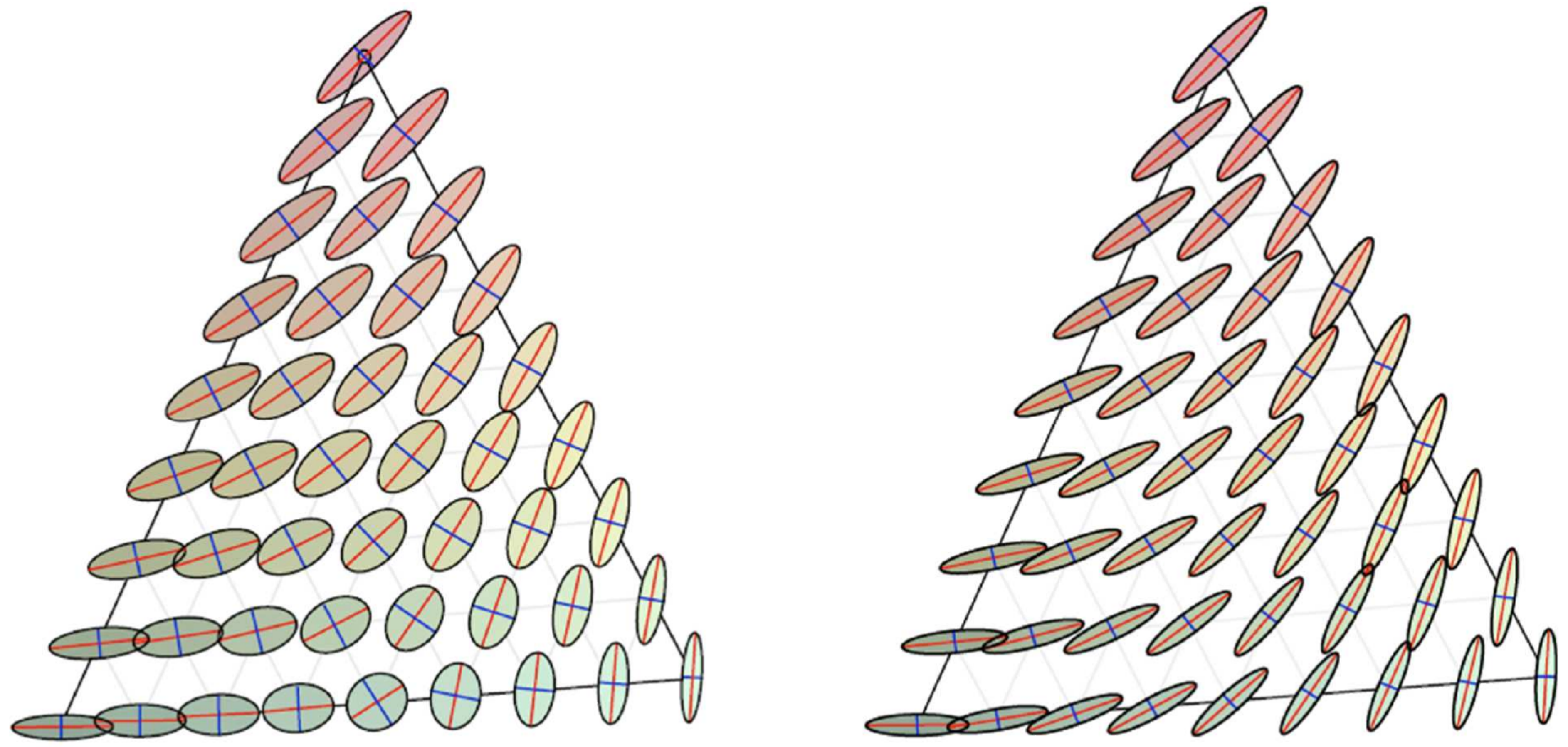


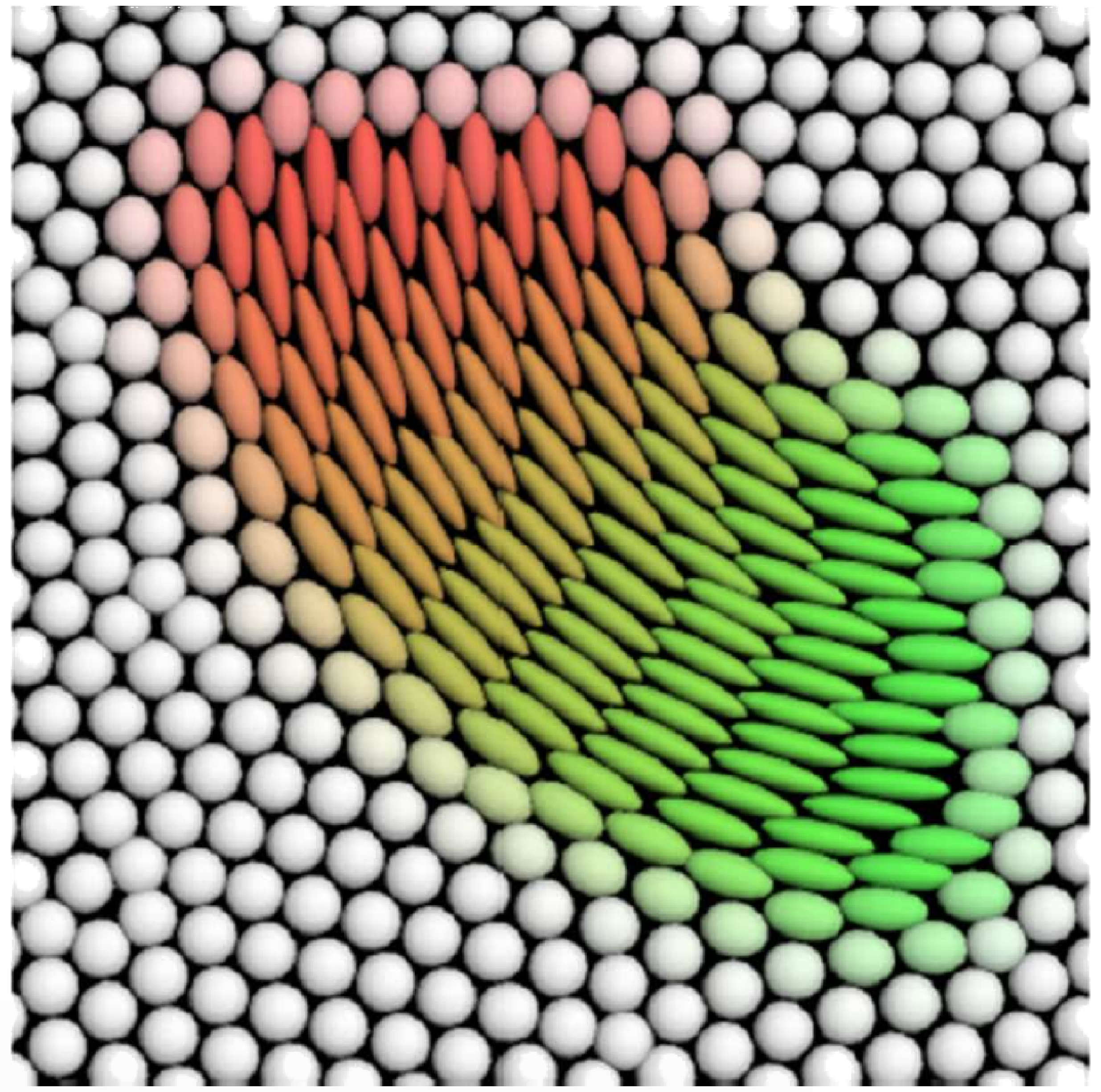


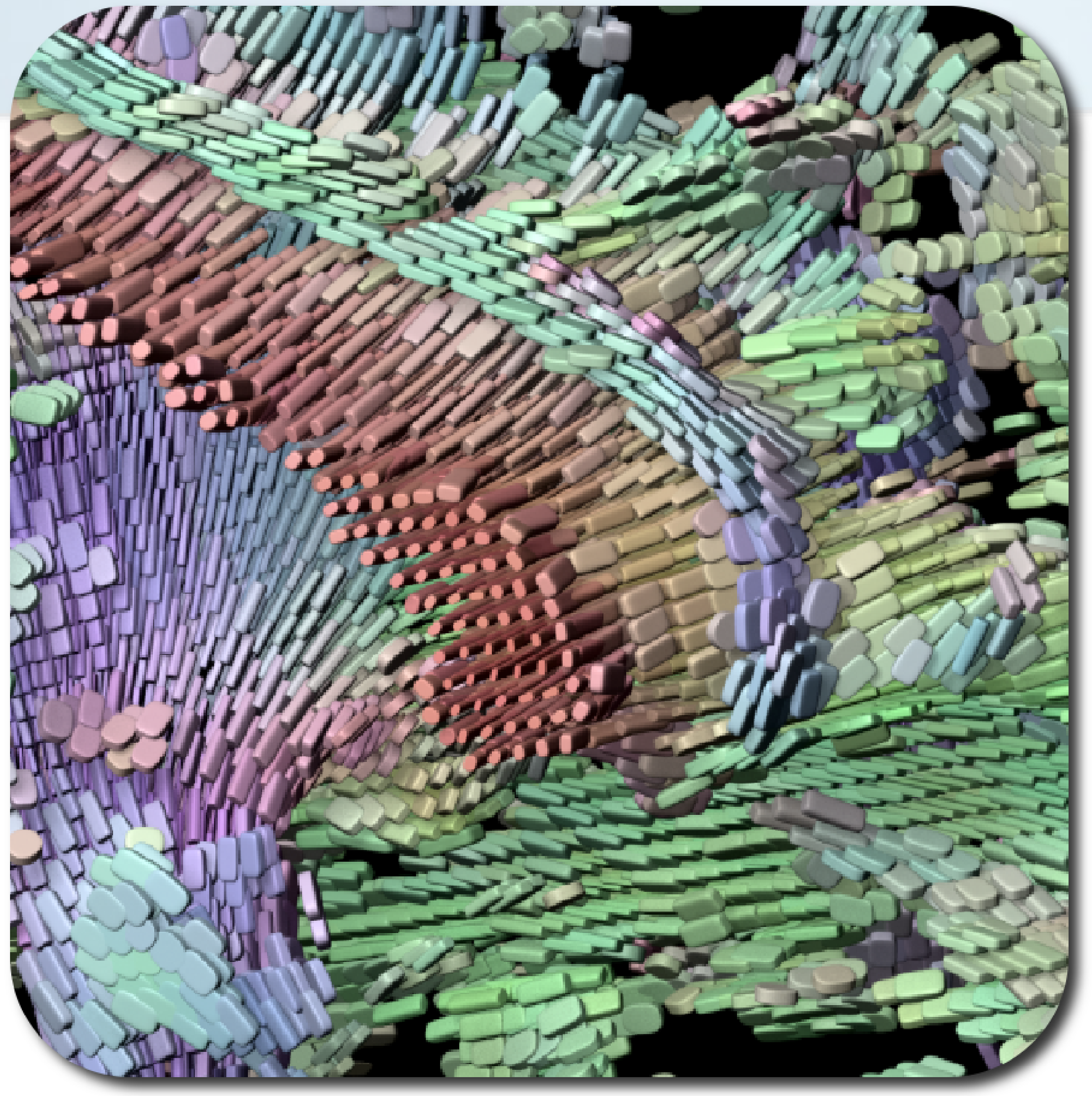


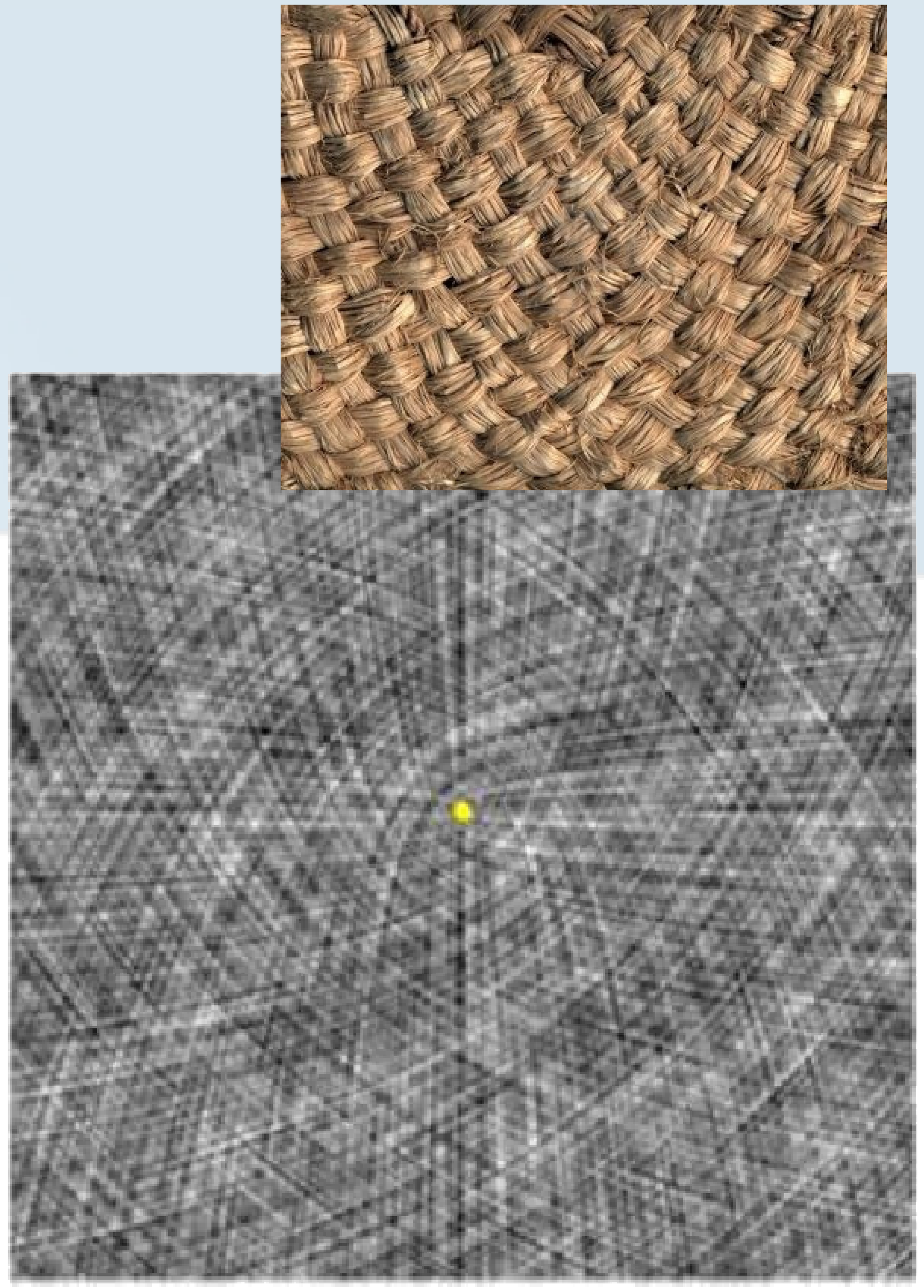
Farewell

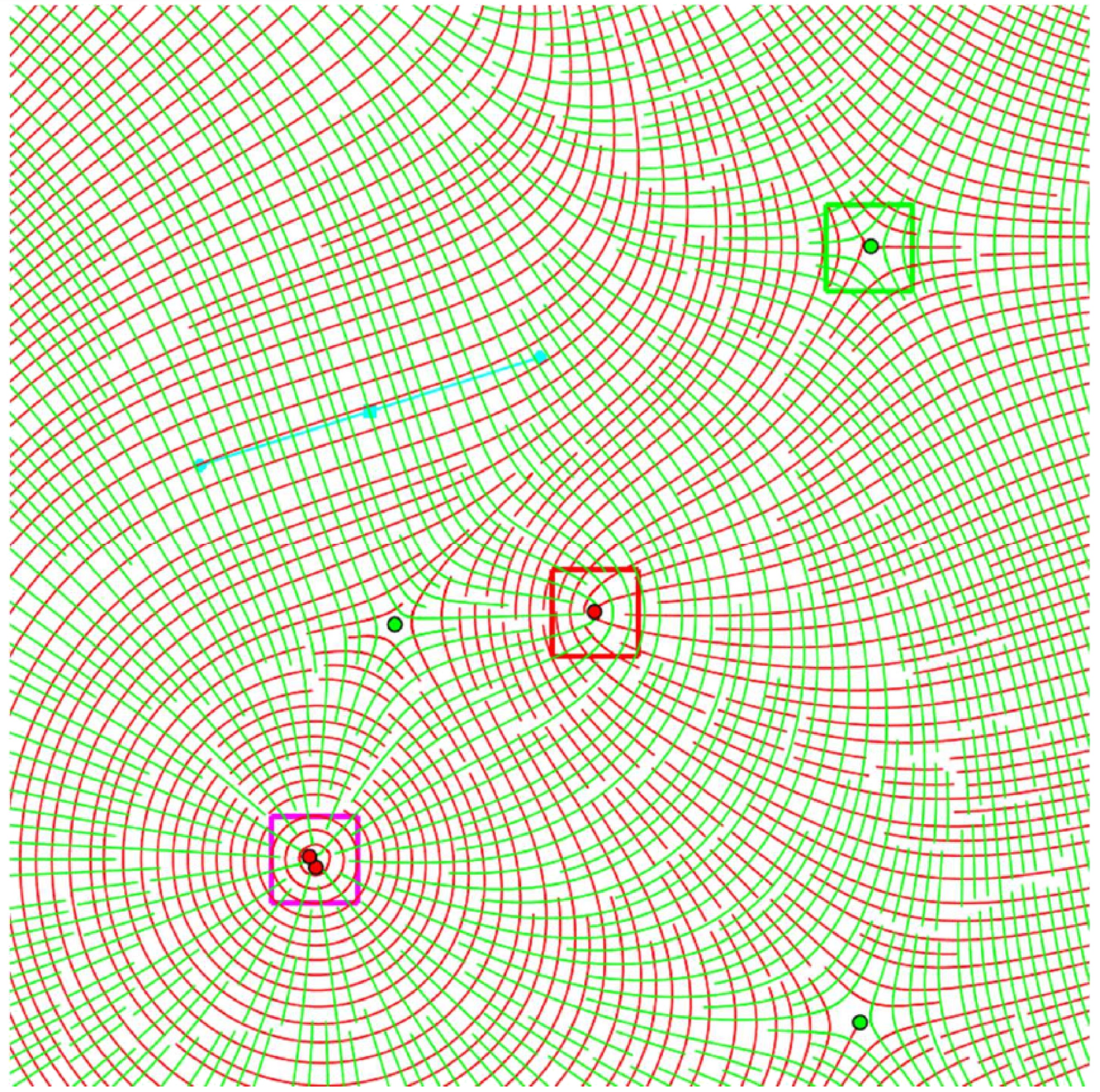
- Tensor Fields

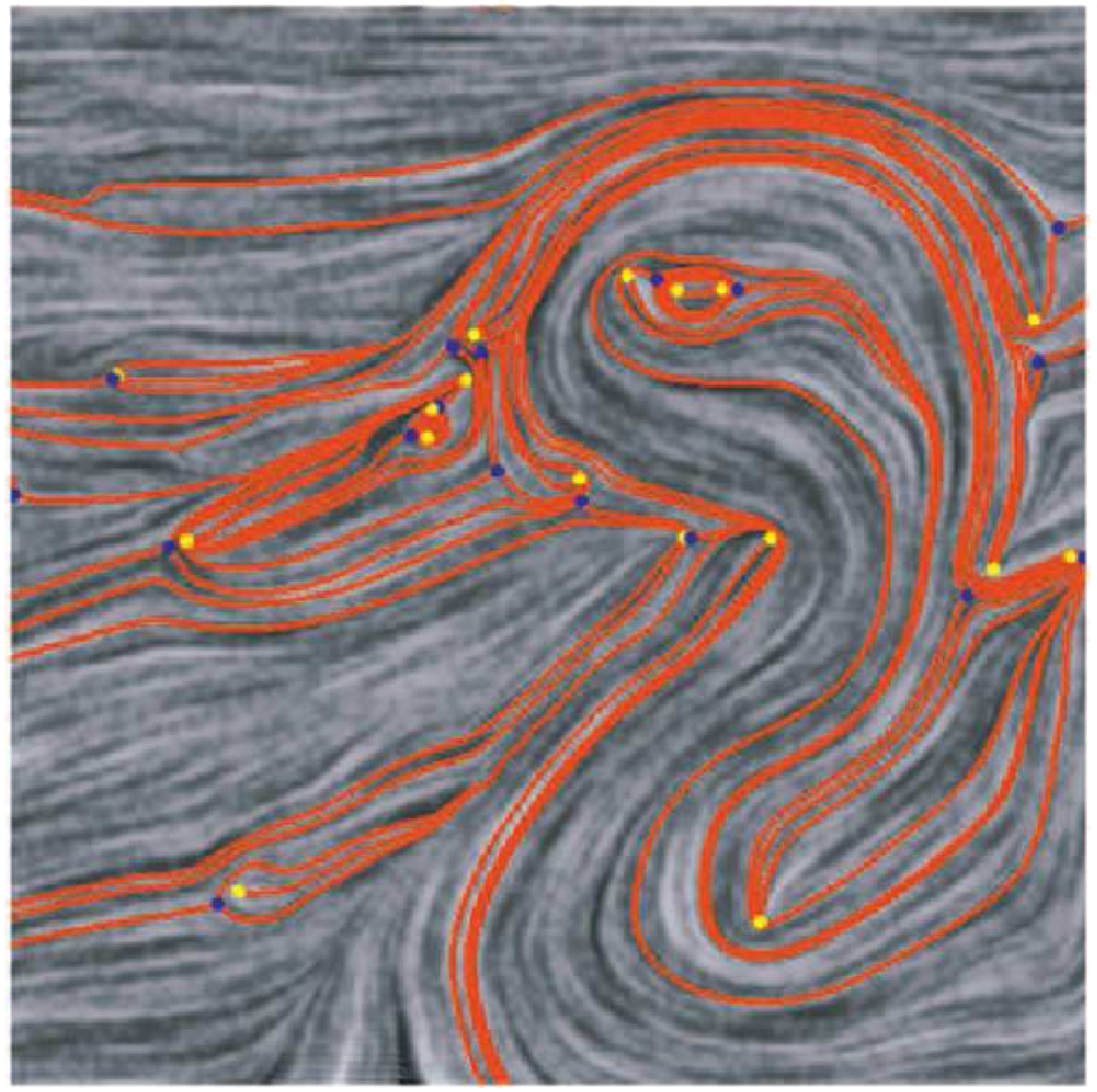










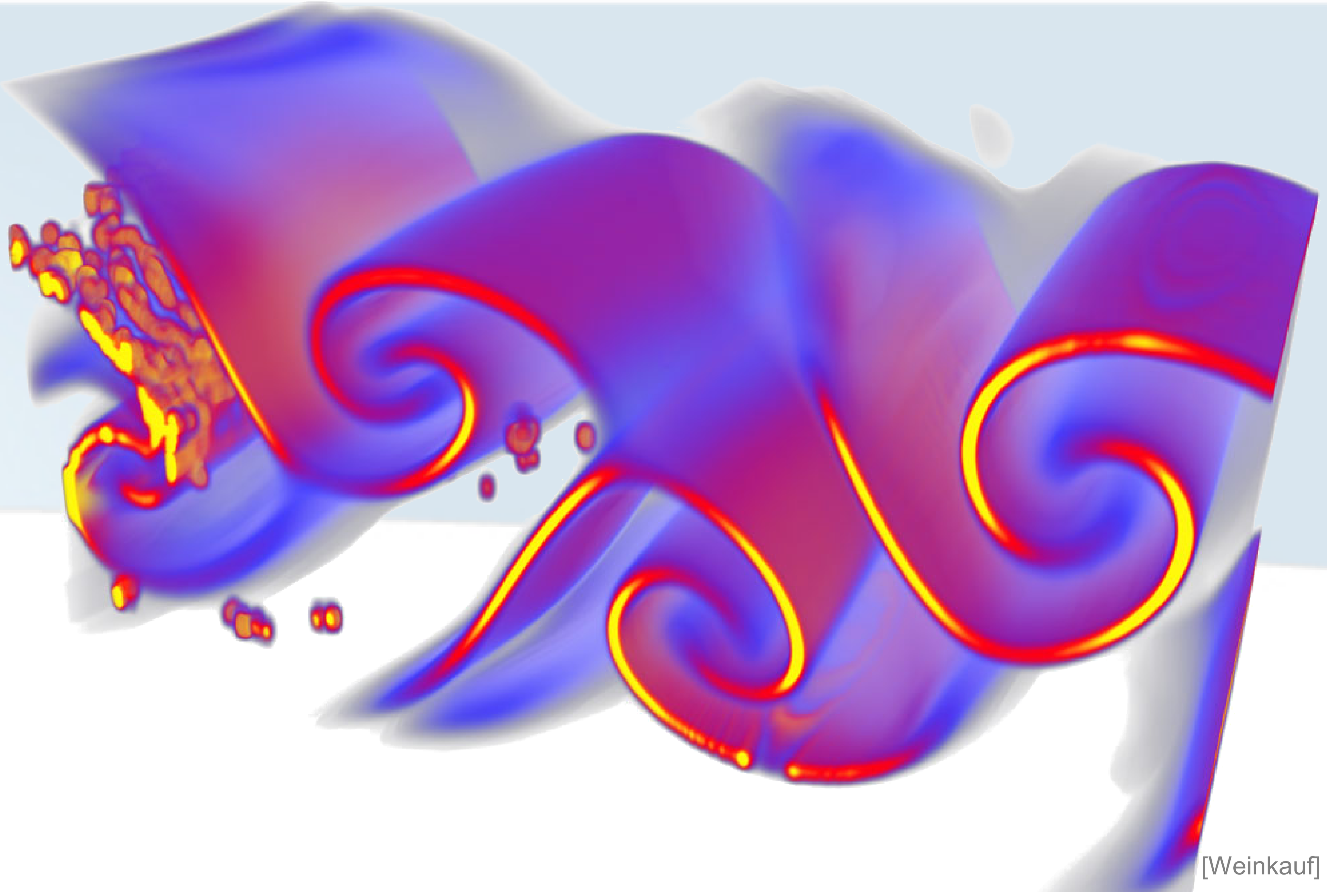


The awful truth

The awful truth

- Simulations usually have a temporal component

The awful truth


[Weinkauf]

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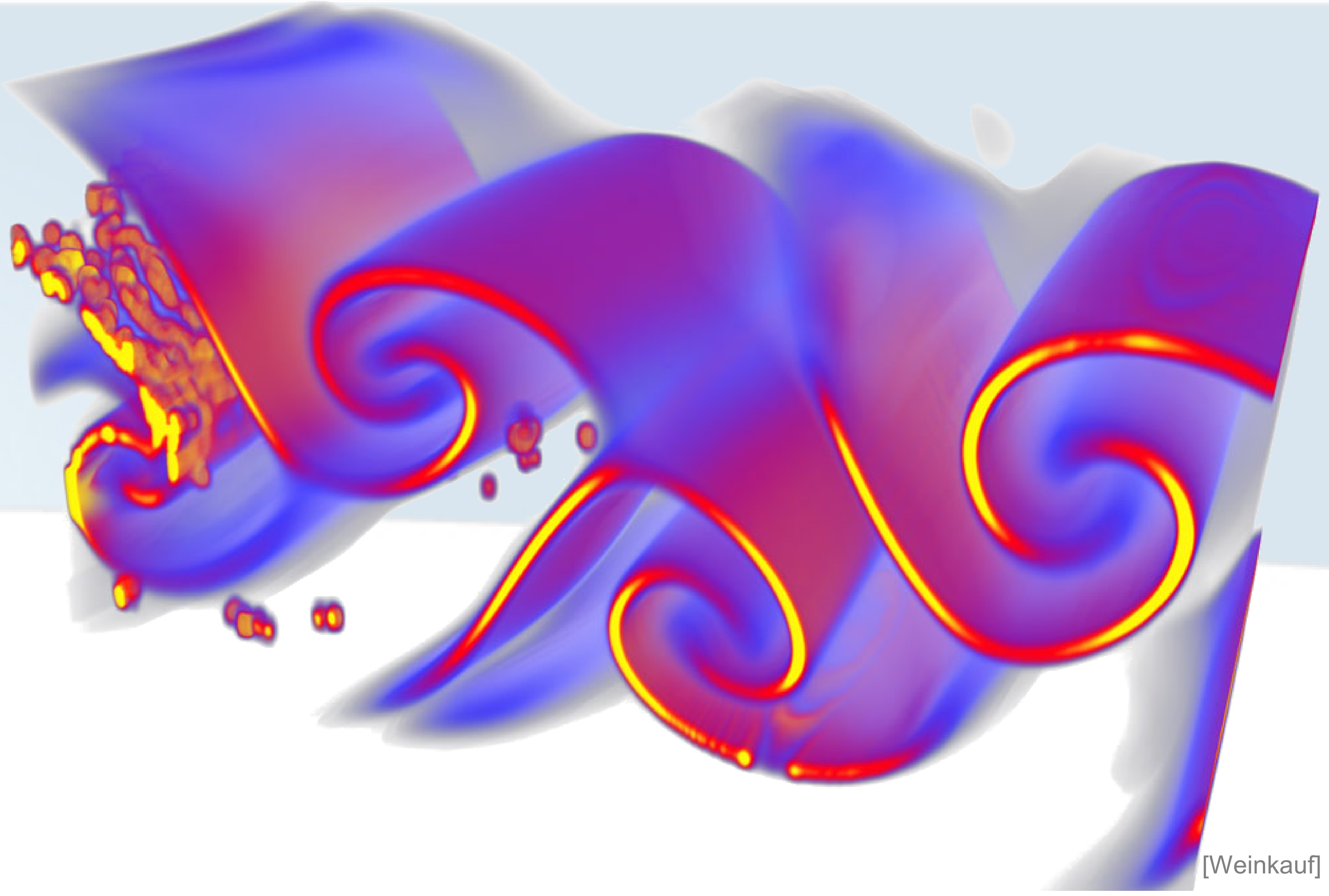
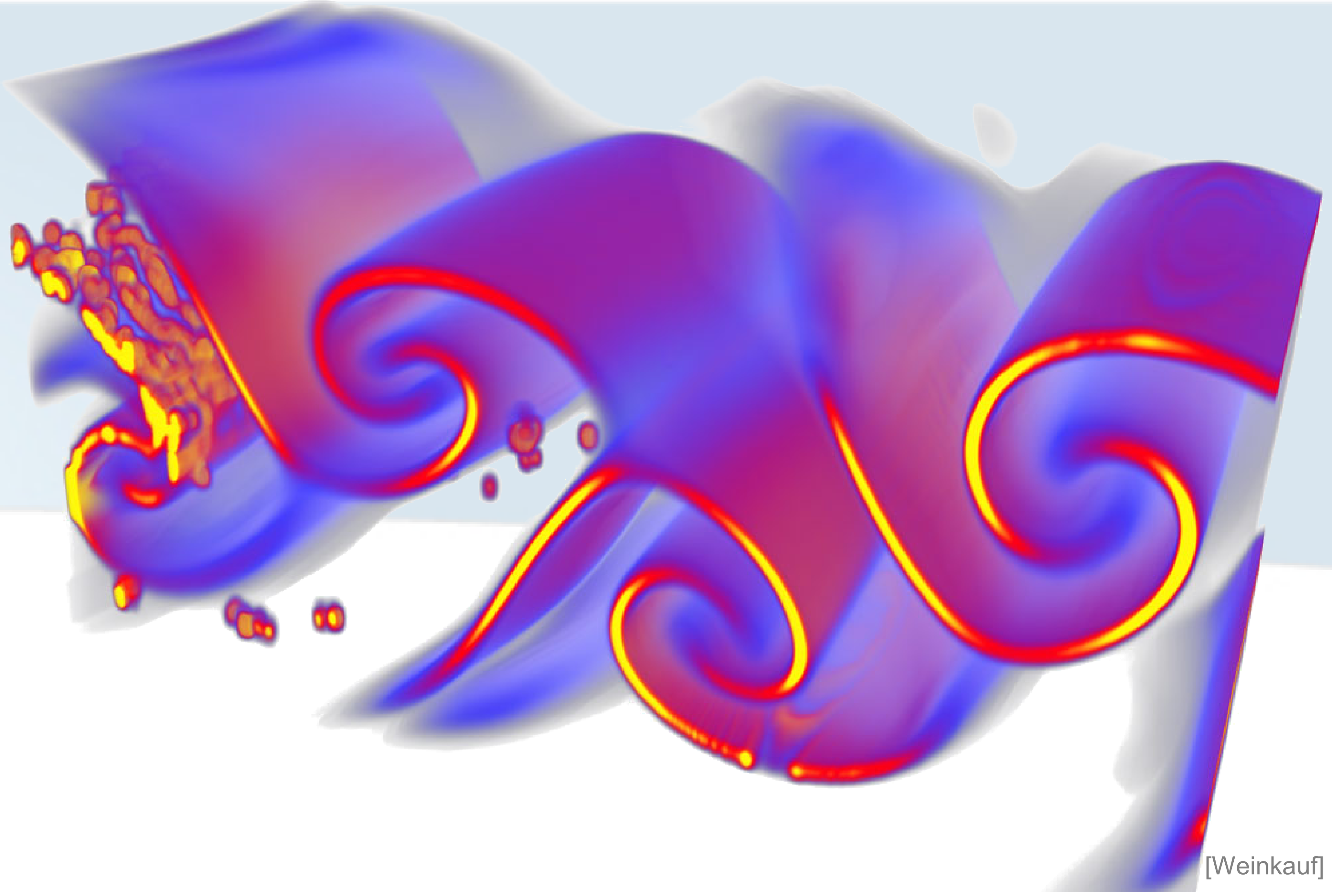
The awful truth



- Simulations usually have a temporal component

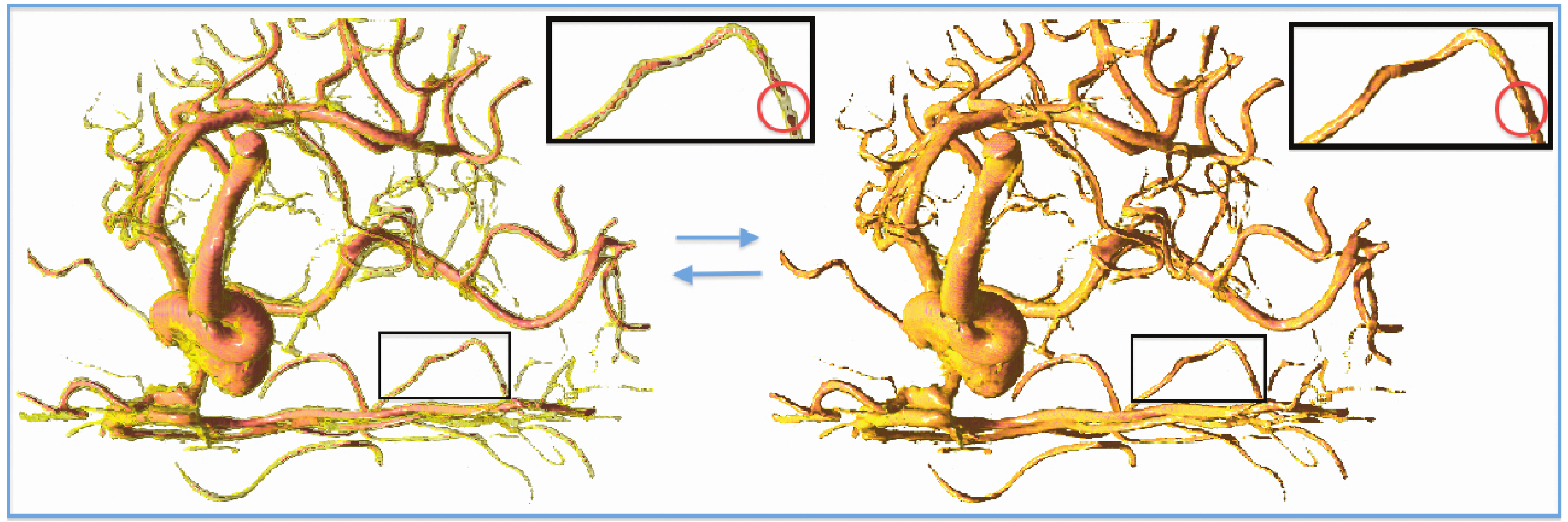
- Simulations often come uncertainty evaluation

The awful truth



[Weinkauf]

- Simulations usually have a temporal component



[Fout]

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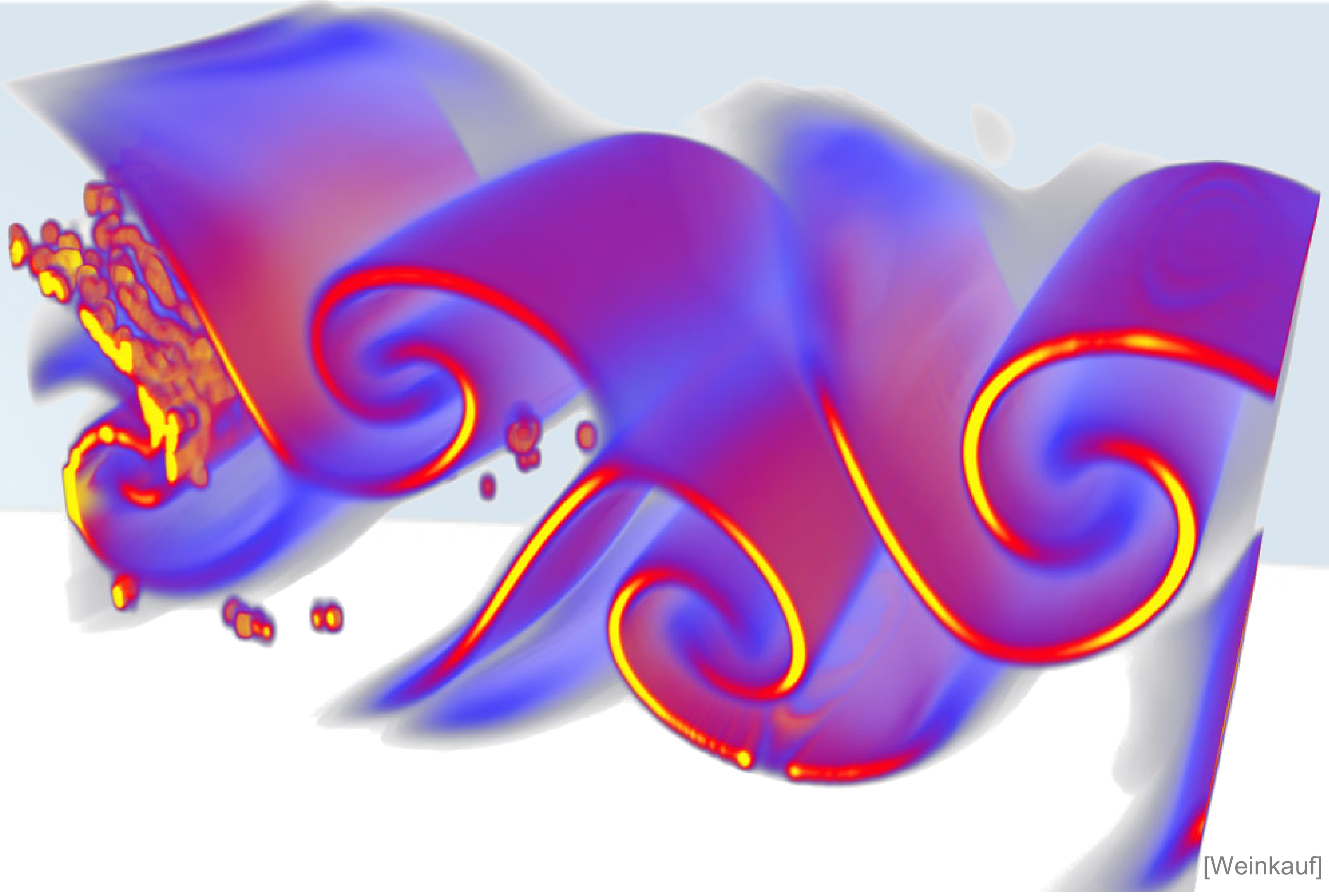
The awful truth


[Weinkauf]

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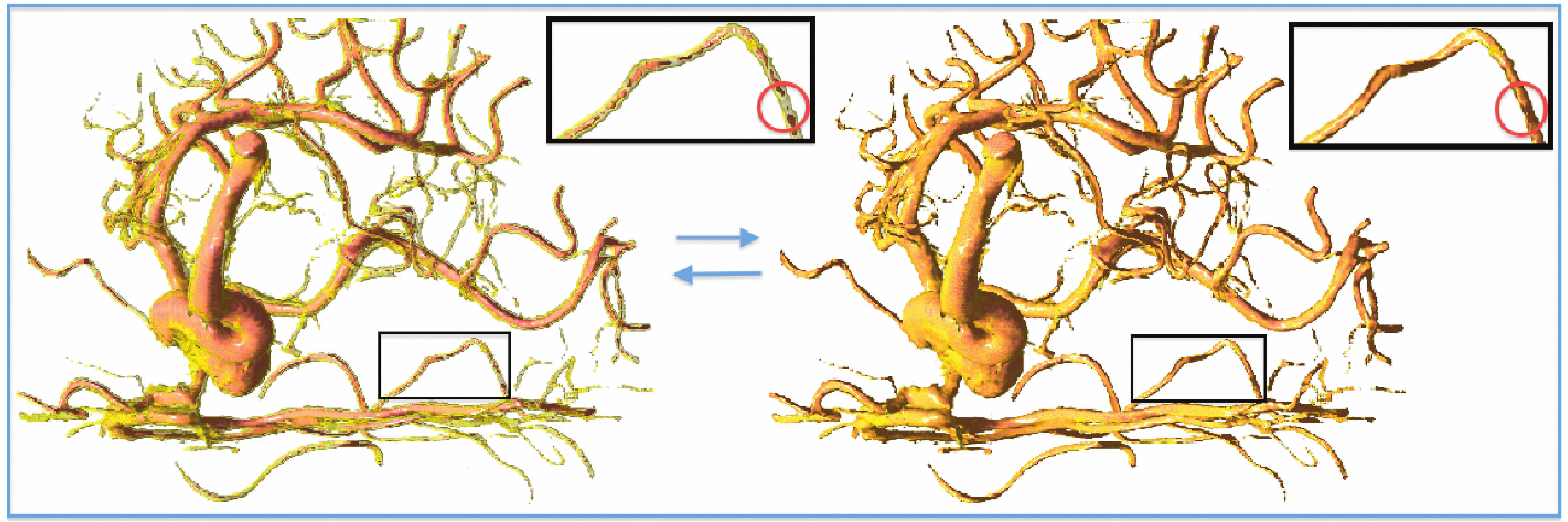
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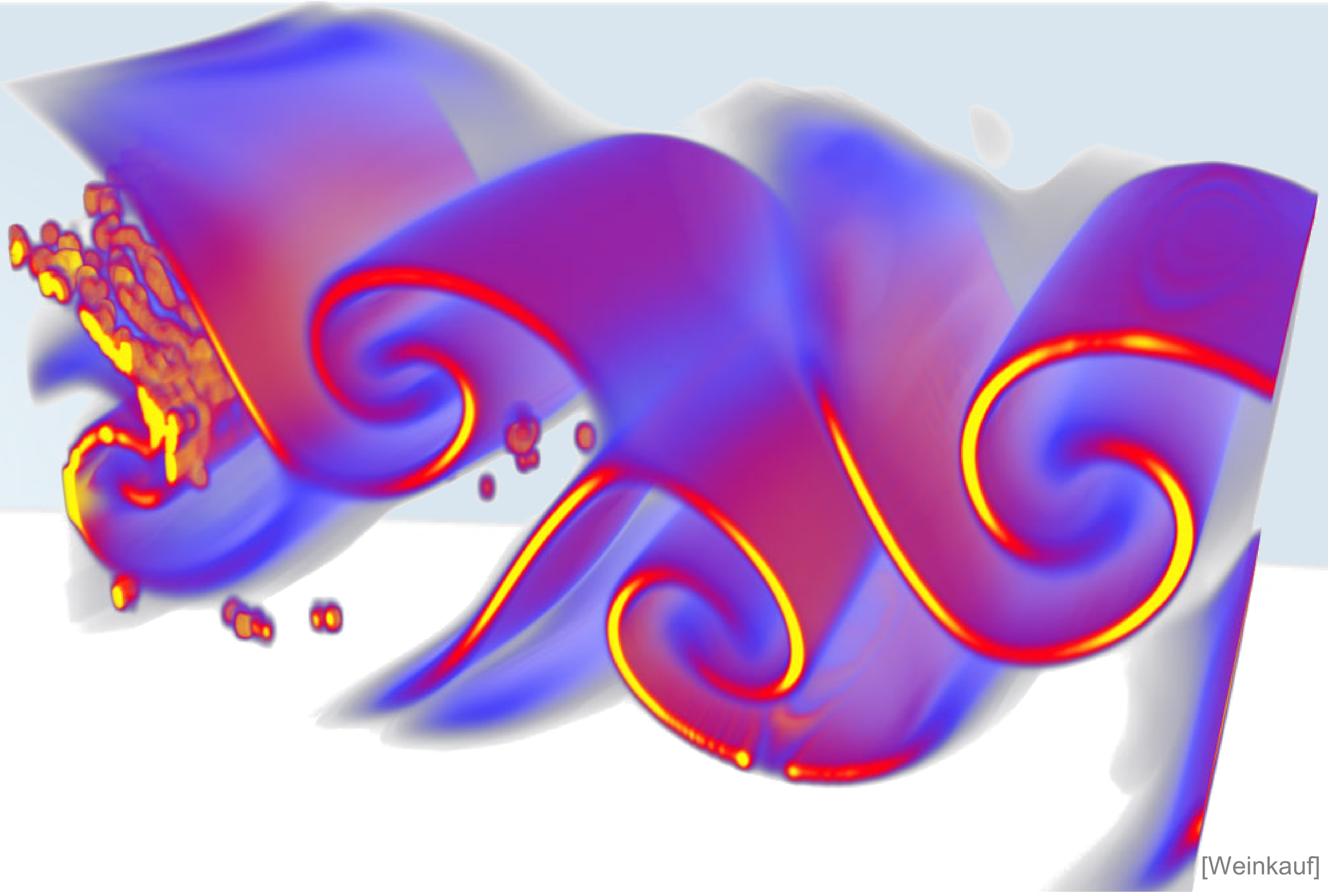
- Simulations often come uncertainty evaluation

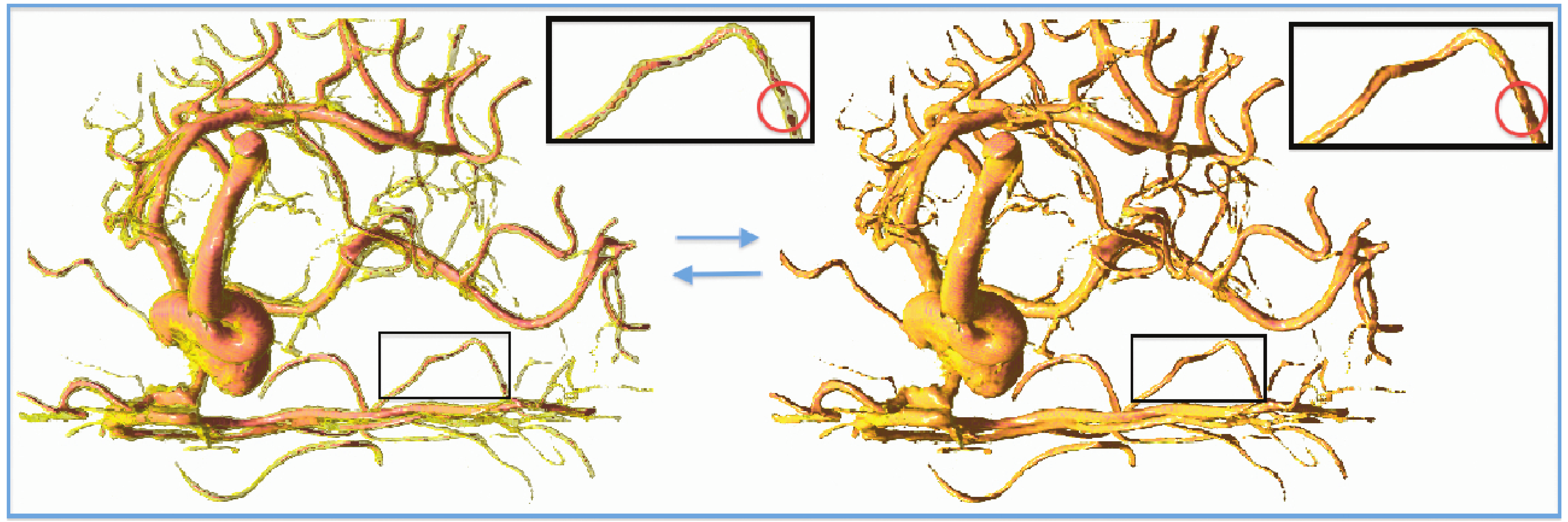
- Simulations often yield several fields per data-set

The awful truth



[Weinkauf]

- Simulations usually have a temporal component



[Fout]

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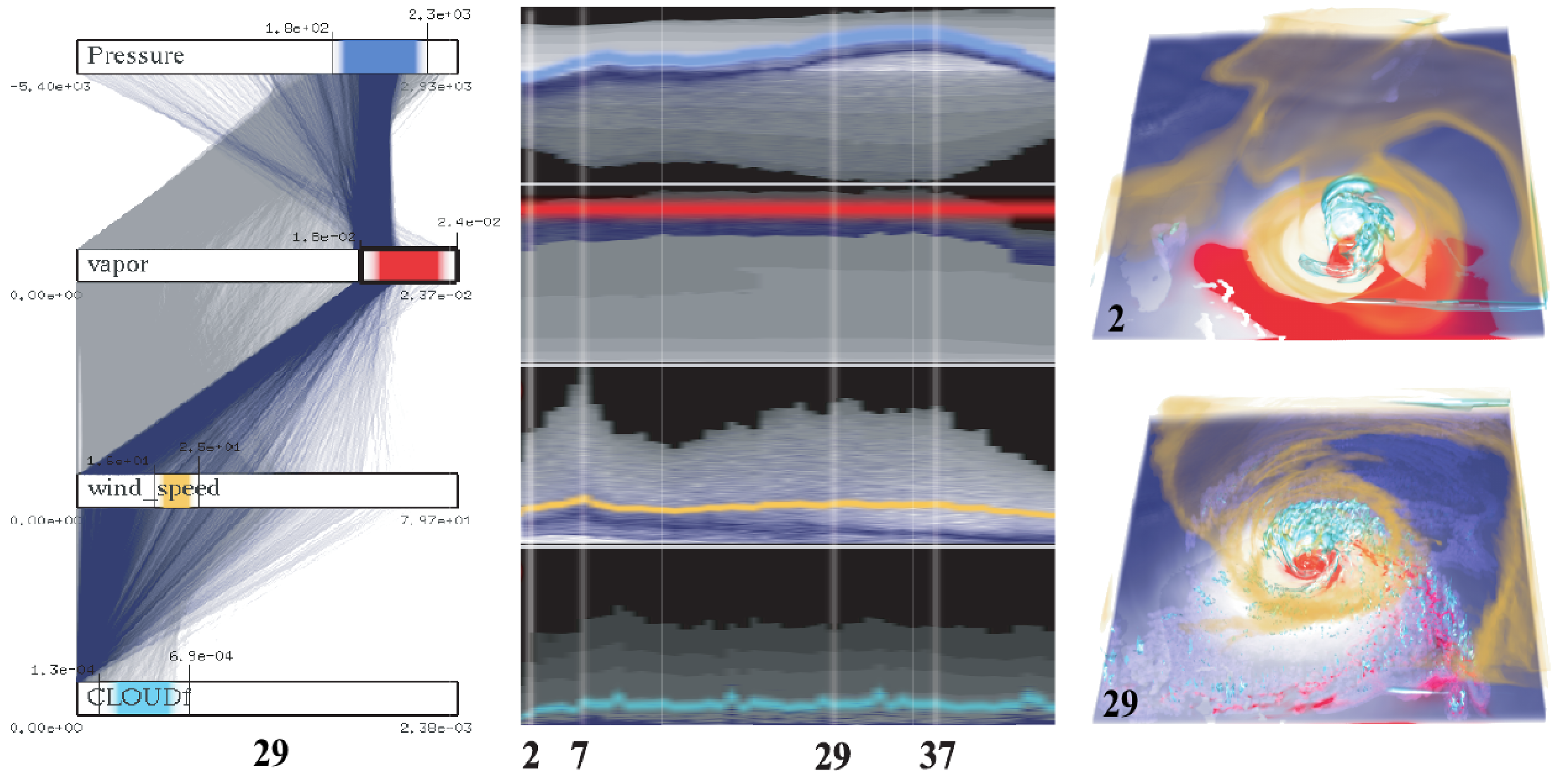

[Akiba]

- Simulations often yield several fields per data-set

Questions?

Acknowledgments

- Many thanks to
  - Guoning Chen, University of Houston