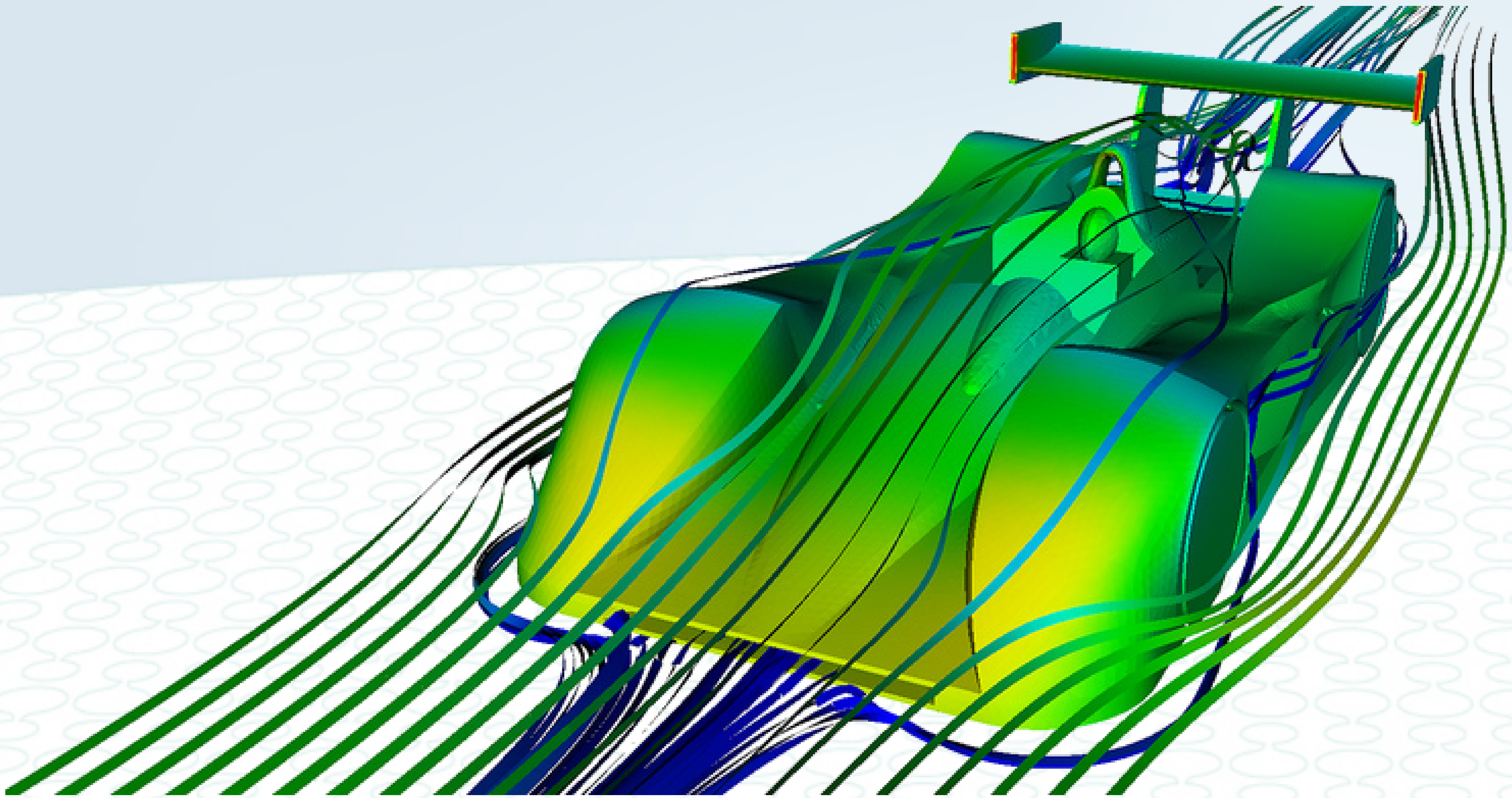


[Kitware]

Vector Field Visualization

M2S

J. Tierny

Previously

$$f : \mathcal{D} \rightarrow \mathbb{R}$$

Function

Definition Domain

Arrival Domain

[Attributes]

Previously

$$f : \mathcal{D} \rightarrow \mathcal{D}$$

Function

Definition Domain

Arrival Domain

<u>**Attributes**</u>

Previously

$$f : \mathcal{D} \rightarrow \mathcal{T}(\mathcal{D})$$

Function

Definition Domain

Arrival Domain

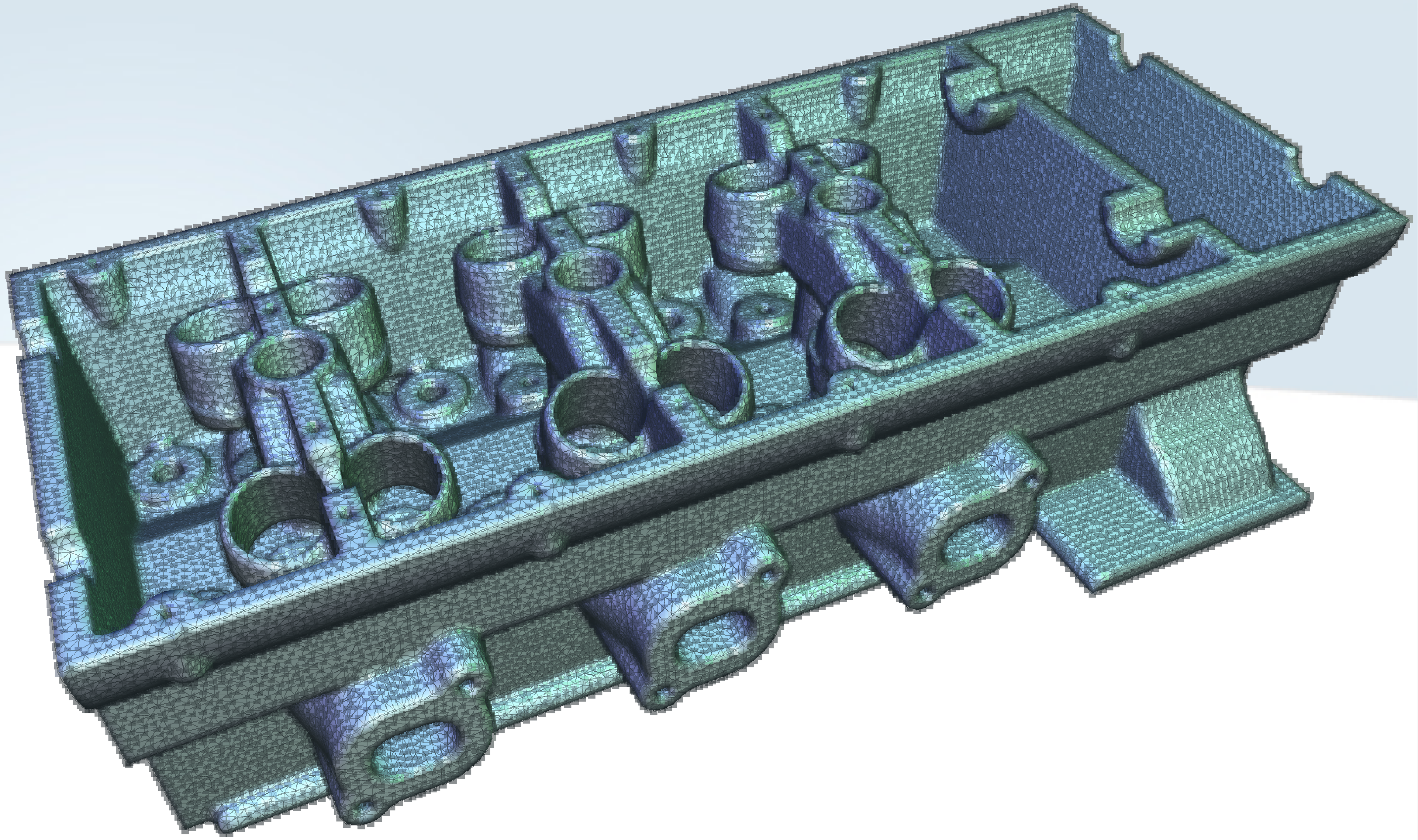
Attributes

In practice

- Given a domain $\mathcal{D}$

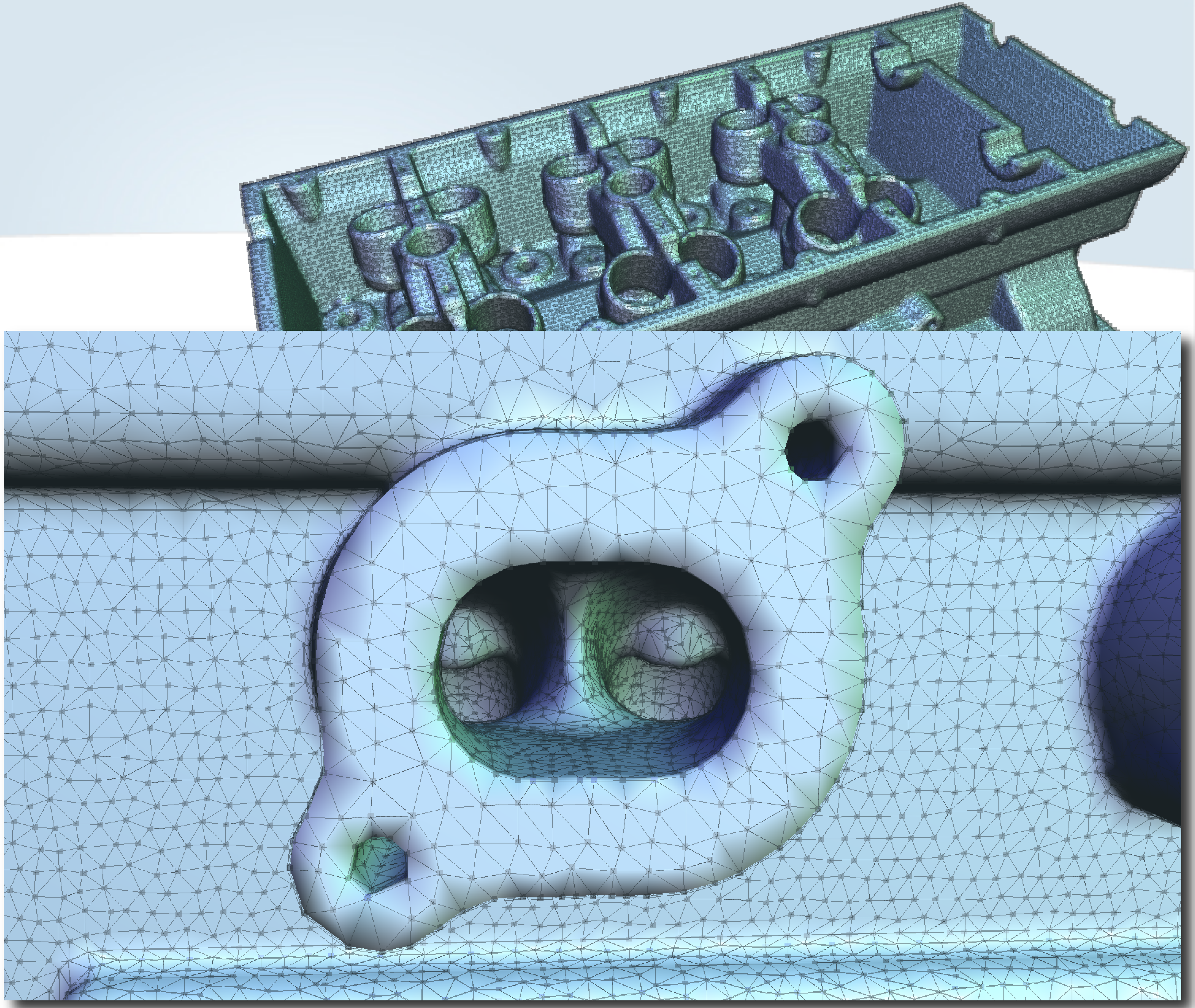
In practice

- Given a domain $\mathcal{D}$



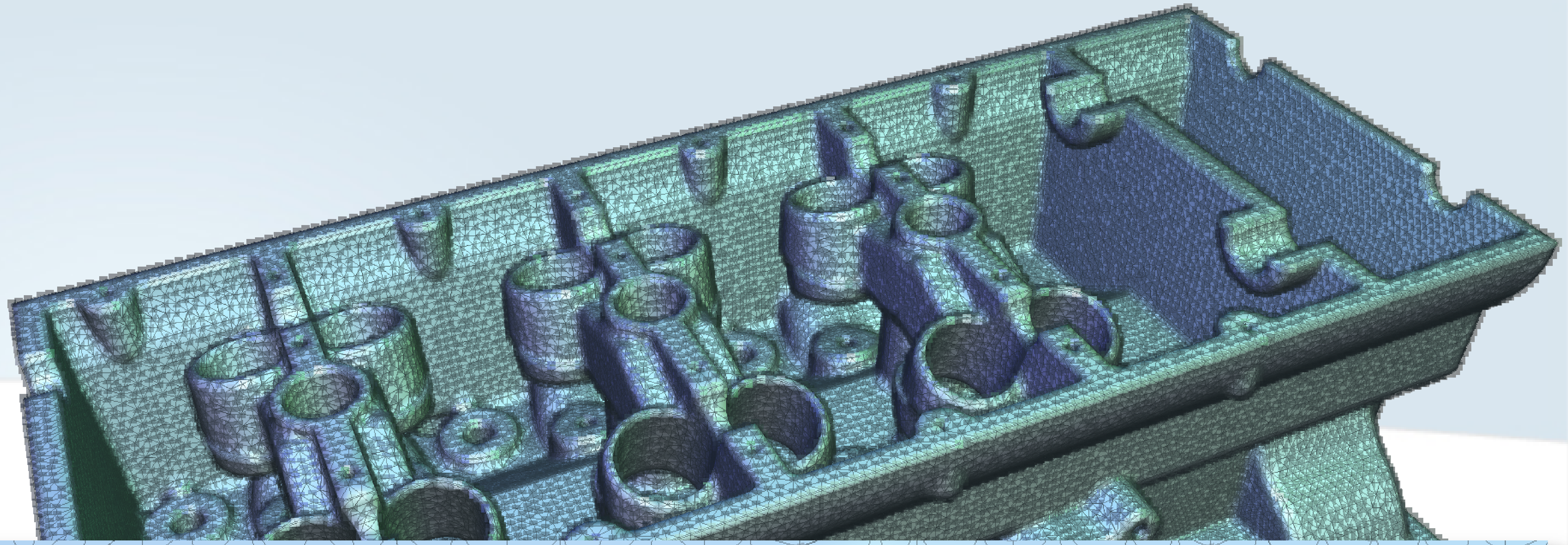
In practice

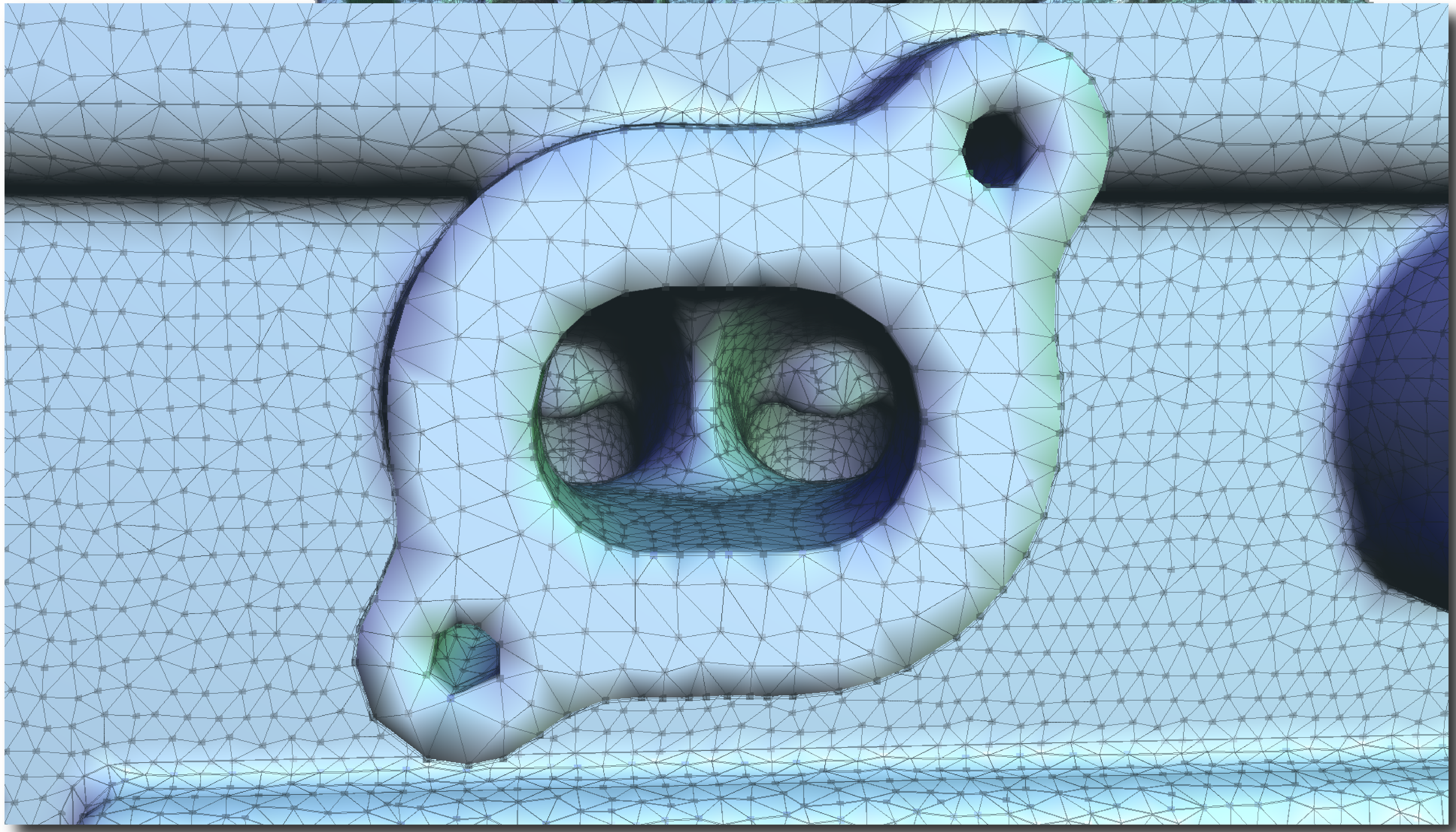
- Given a domain $\mathcal{D}$



In practice

- Given a domain $\mathcal{D}$
- For each vertex v





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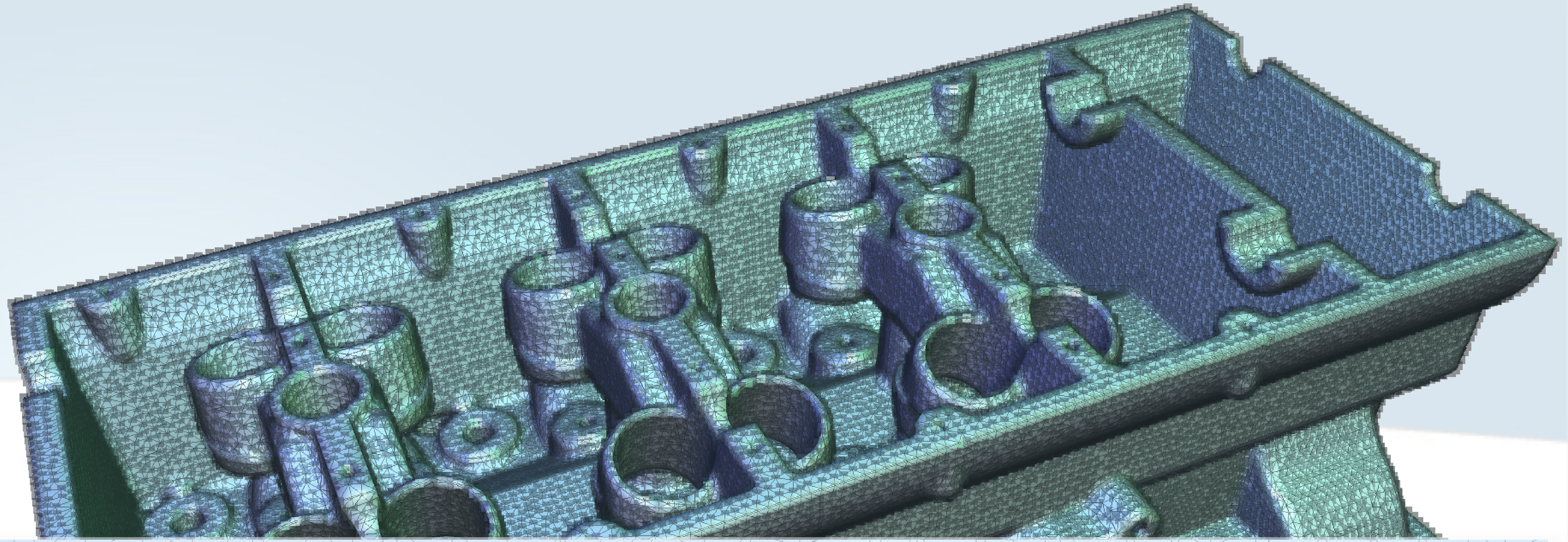
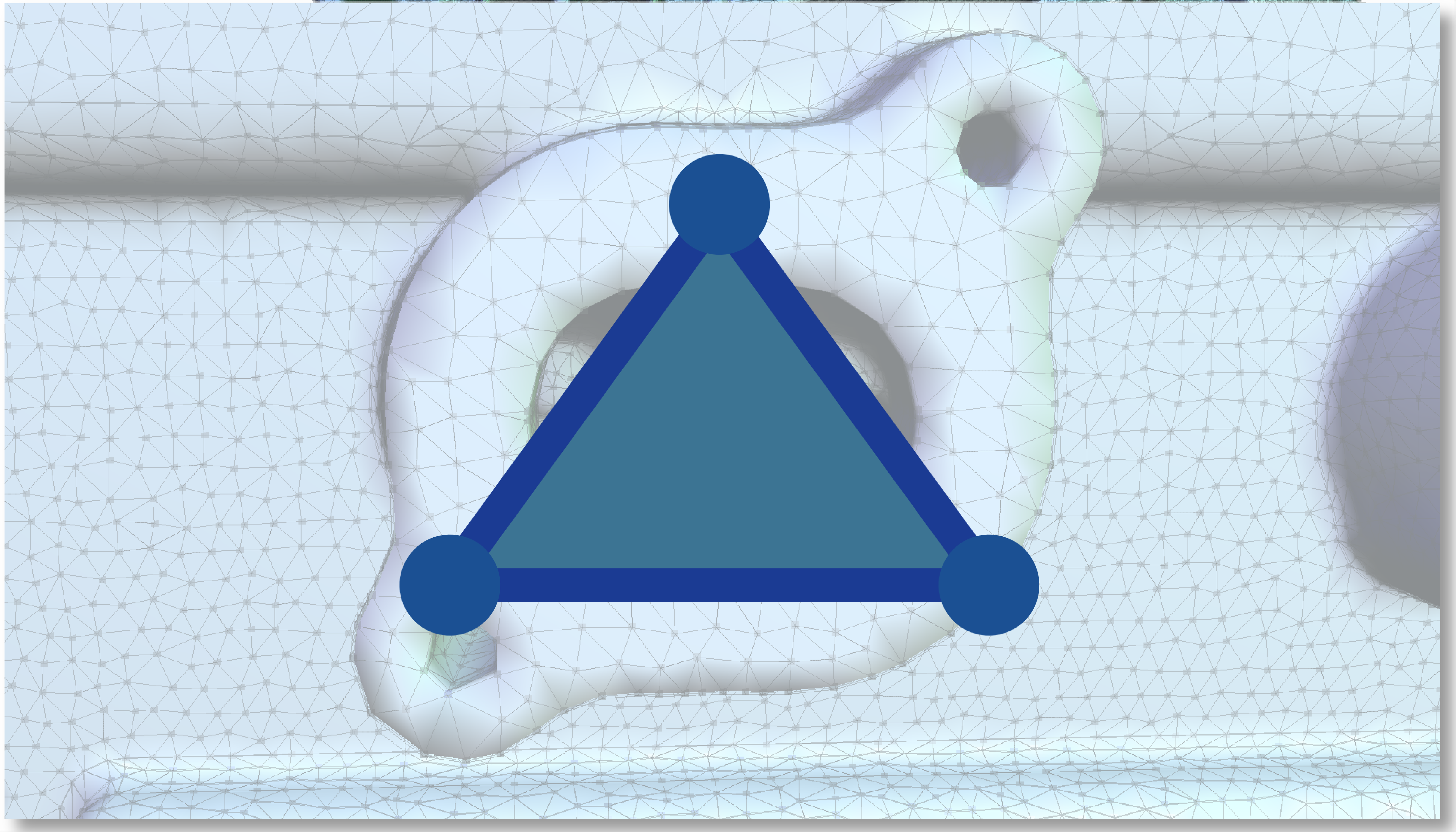
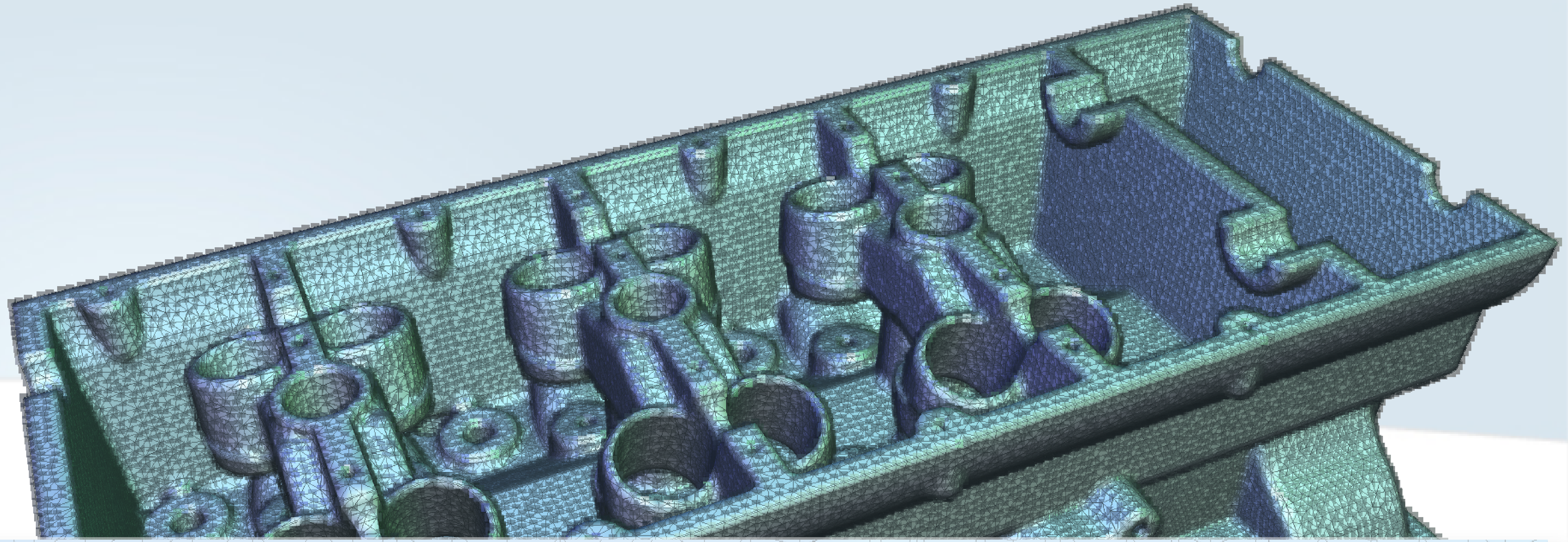
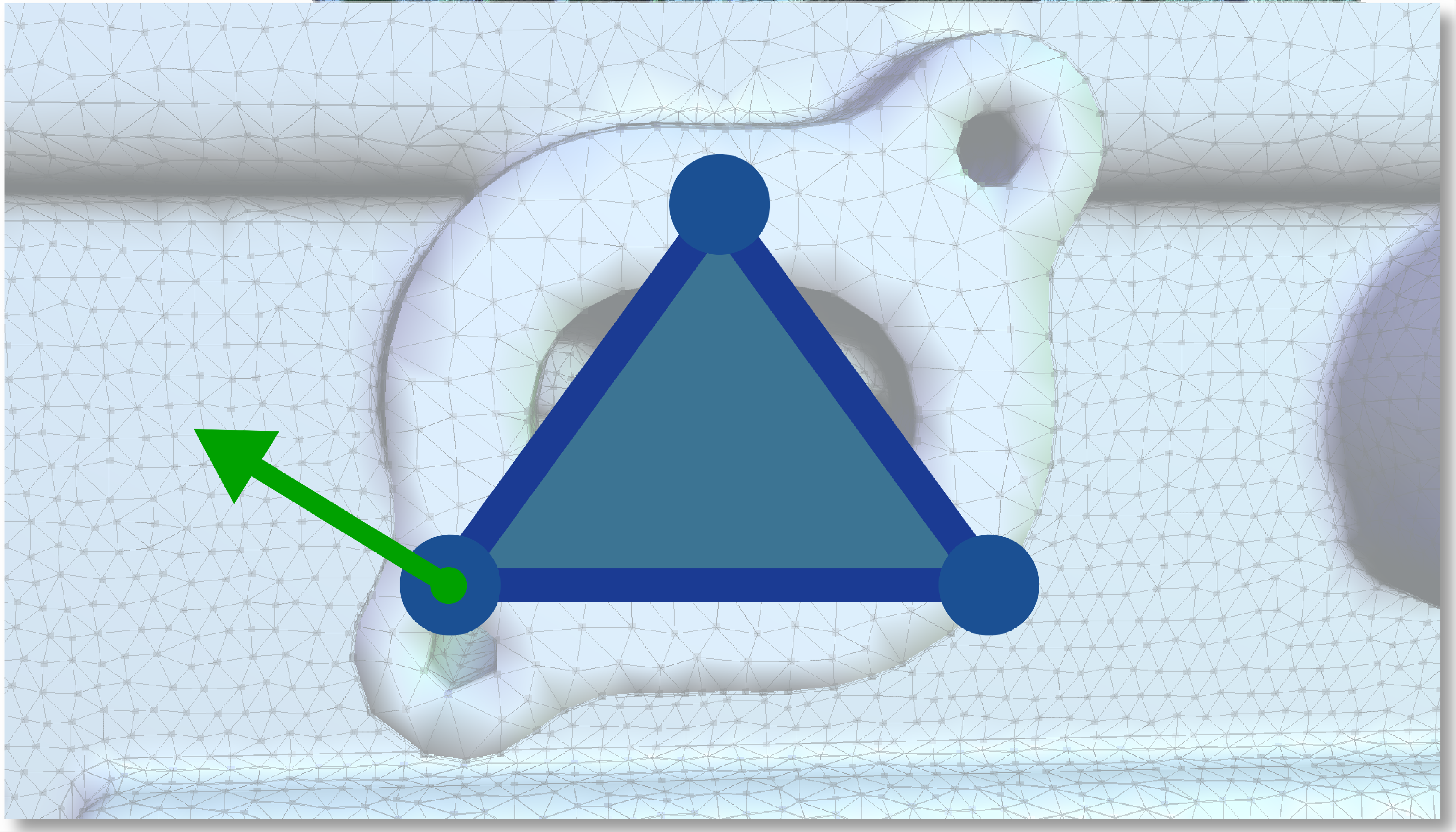




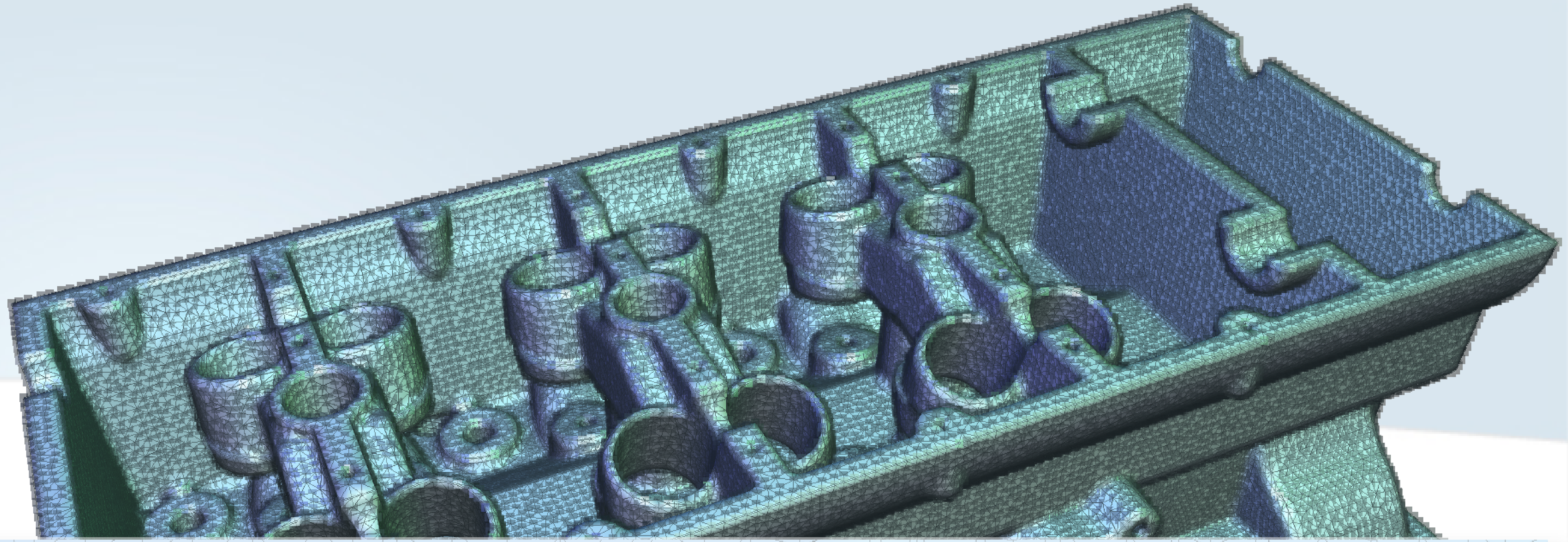
In practice

- Given a domain $\mathcal{D}$
- For each vertex v
- One vector $\vec{f}(v)$

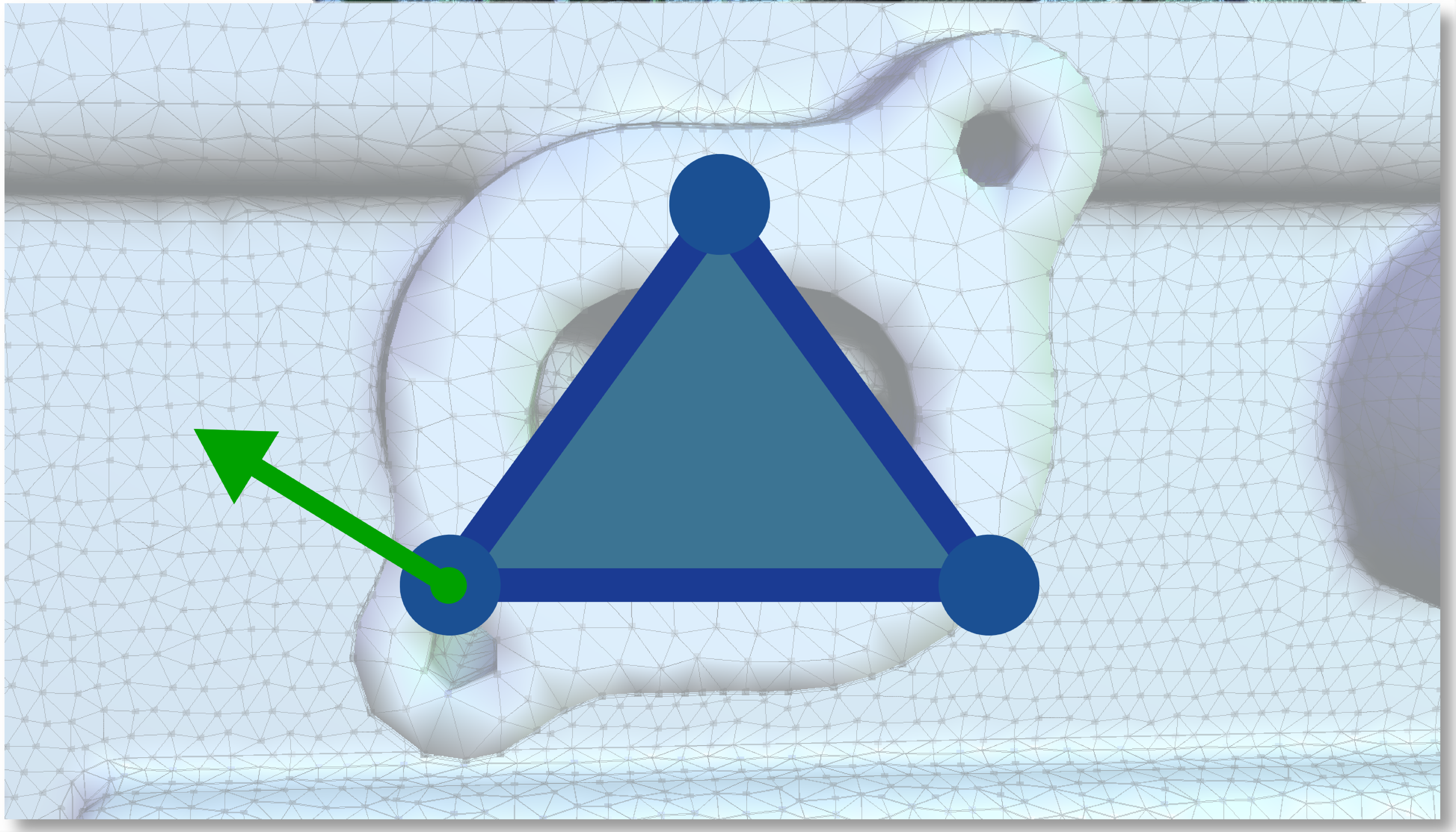




In practice

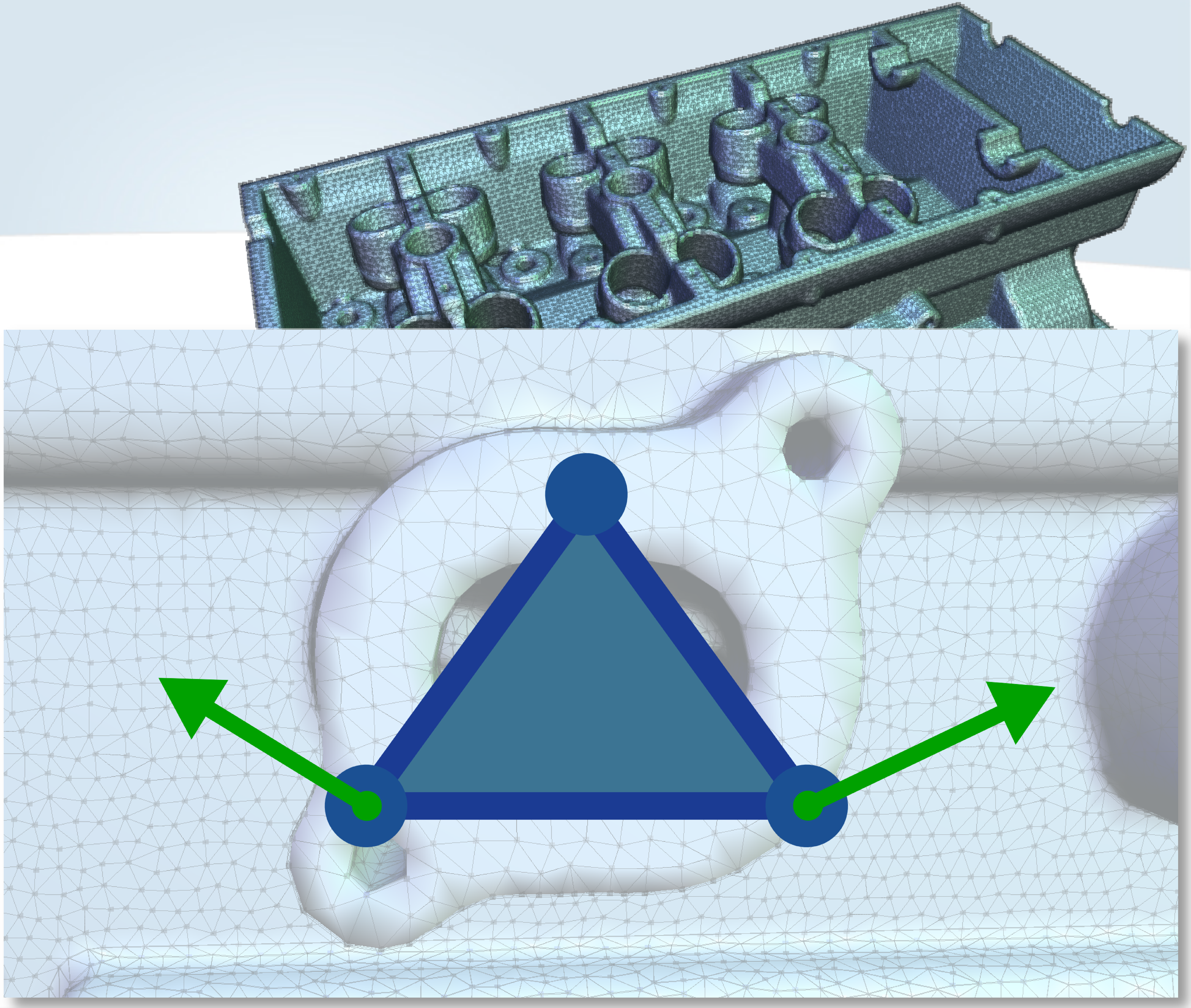


- Given a domain $\mathcal{D}$
- For each vertex v
- One vector $\vec{f}(v)$
  - *Coordinates in* $\mathbb{E}$



In practice

- Given a domain $\mathcal{D}$
- For each vertex v
- One vector $\vec{f}(v)$
  - Coordinates in $\mathbb{E}$
- Interpolation on the other simplices:



In practice

- Given a domain $\mathcal{D}$
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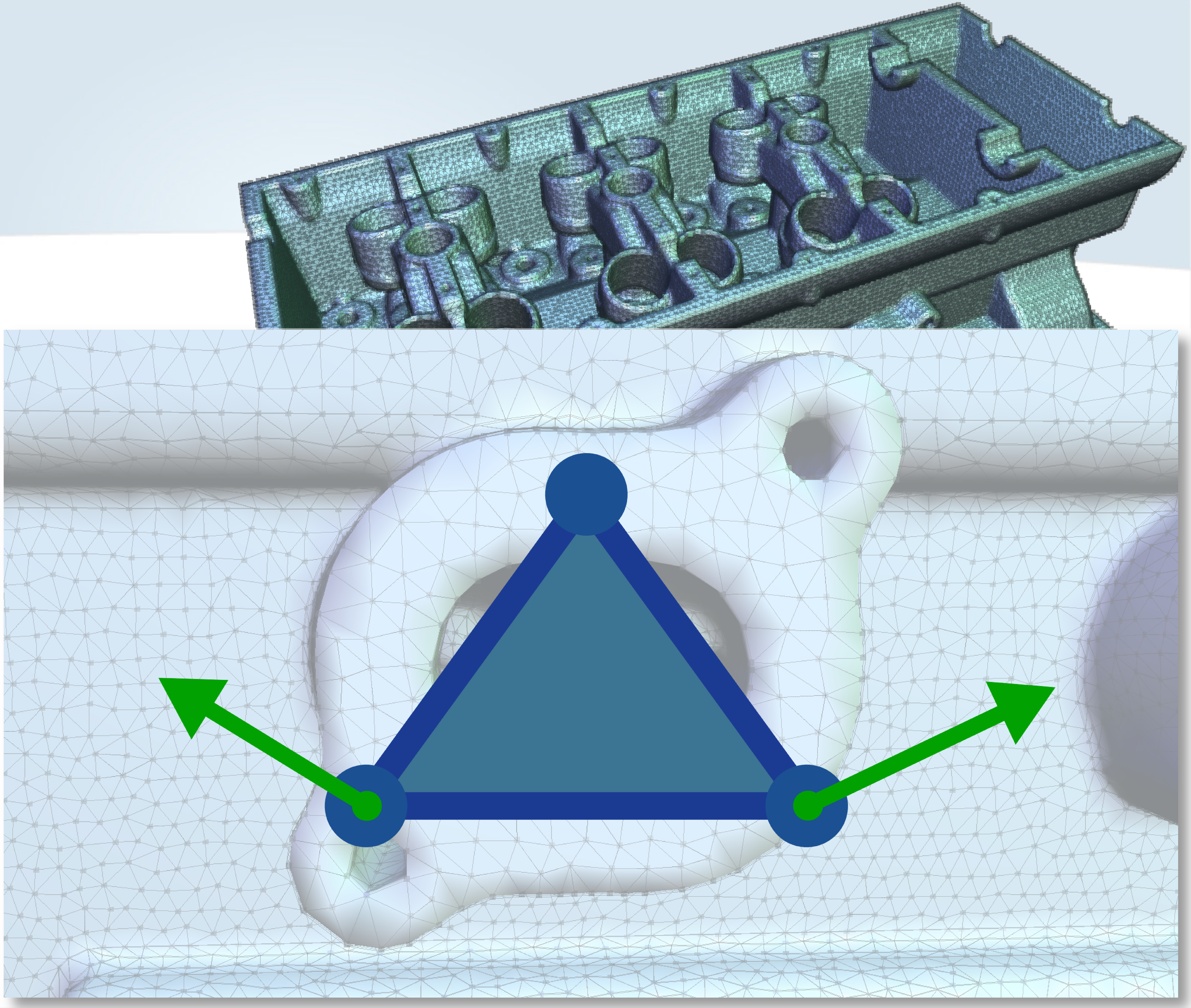

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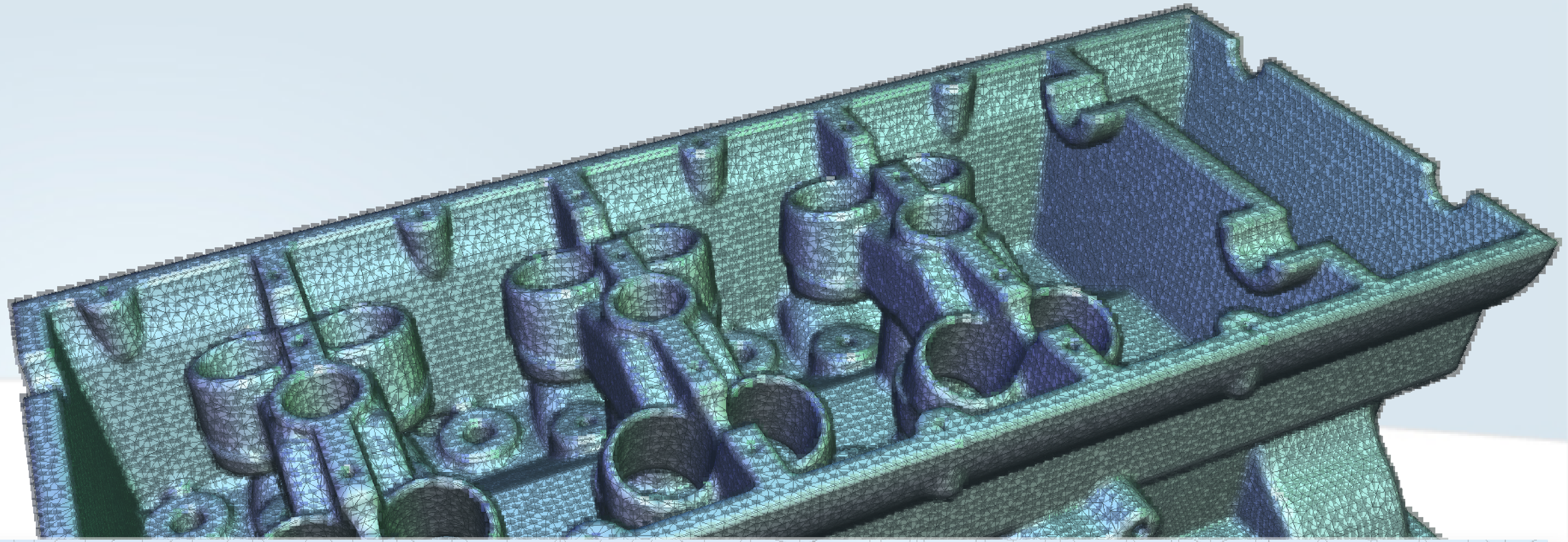

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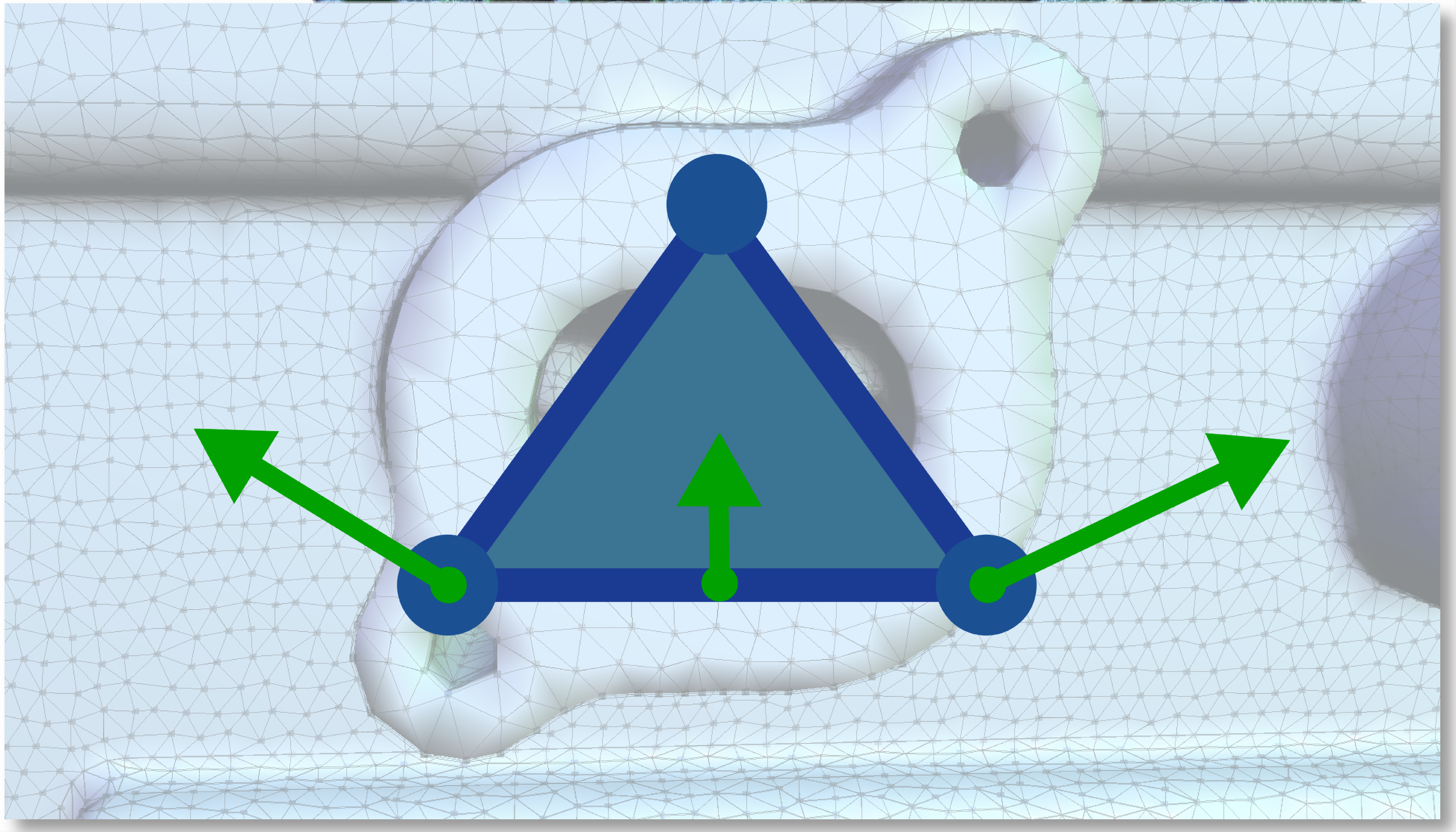

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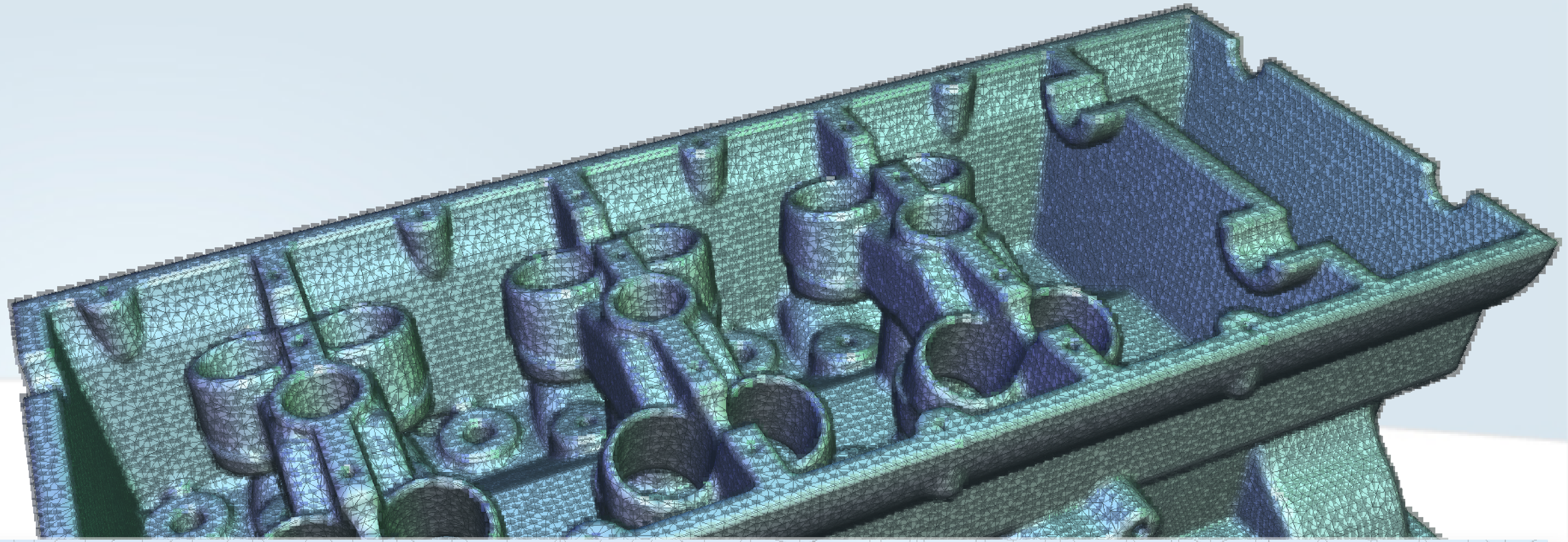

- Given a domain $\mathcal{D}$
- For each vertex v
- One vector $\vec{f}(v)$
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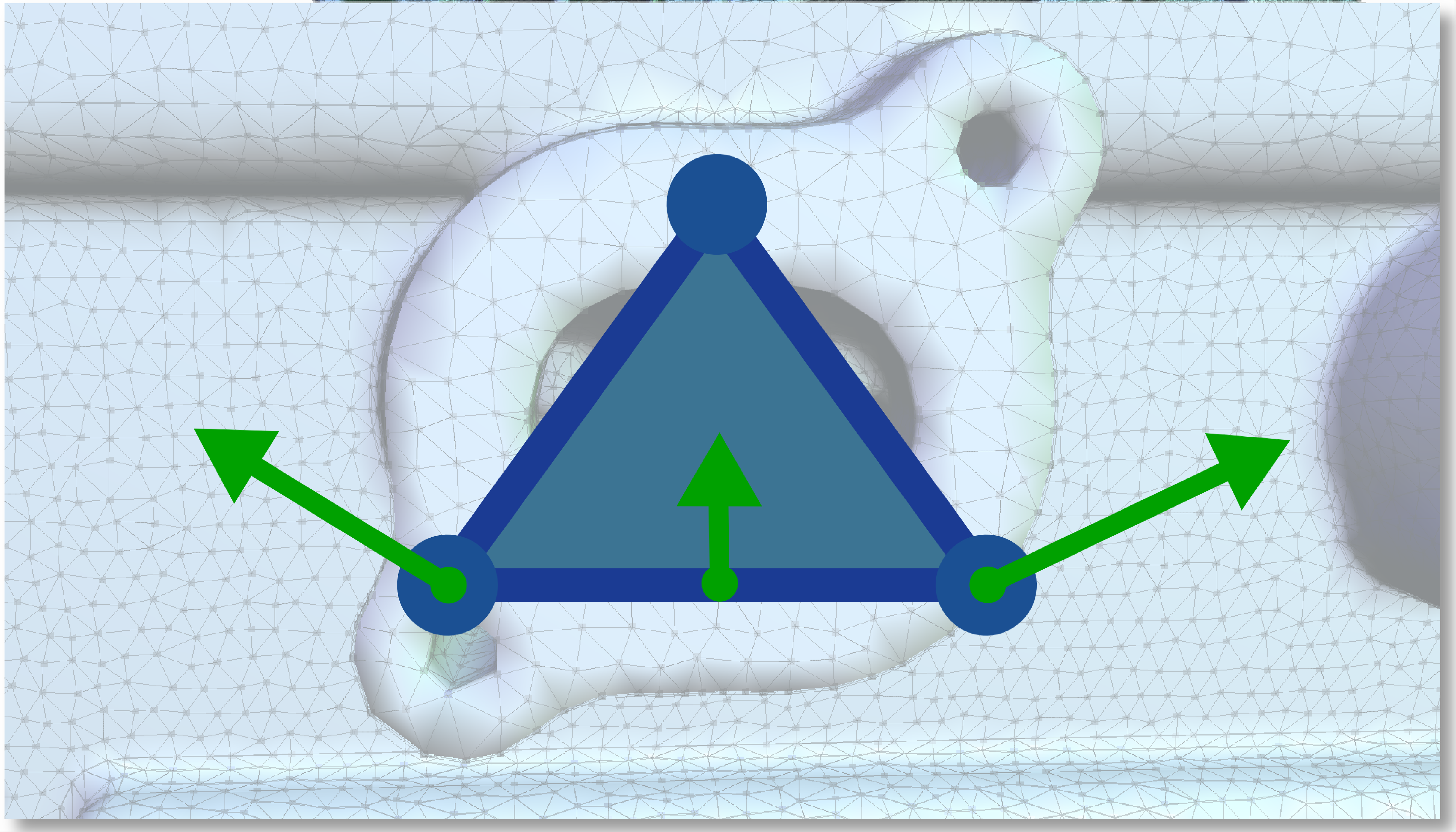
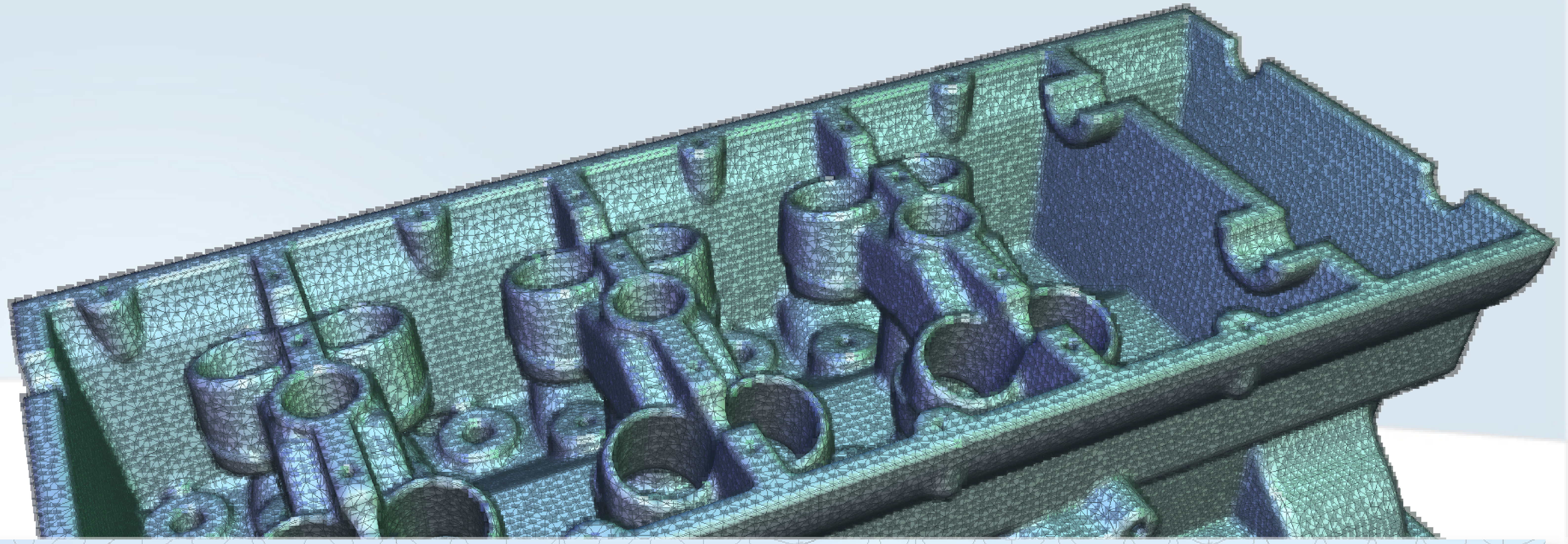
- Interpolation on the other simplices:
  - Coordinates
  - Magnitude/angle



In practice



- Given a domain $\mathcal{D}$
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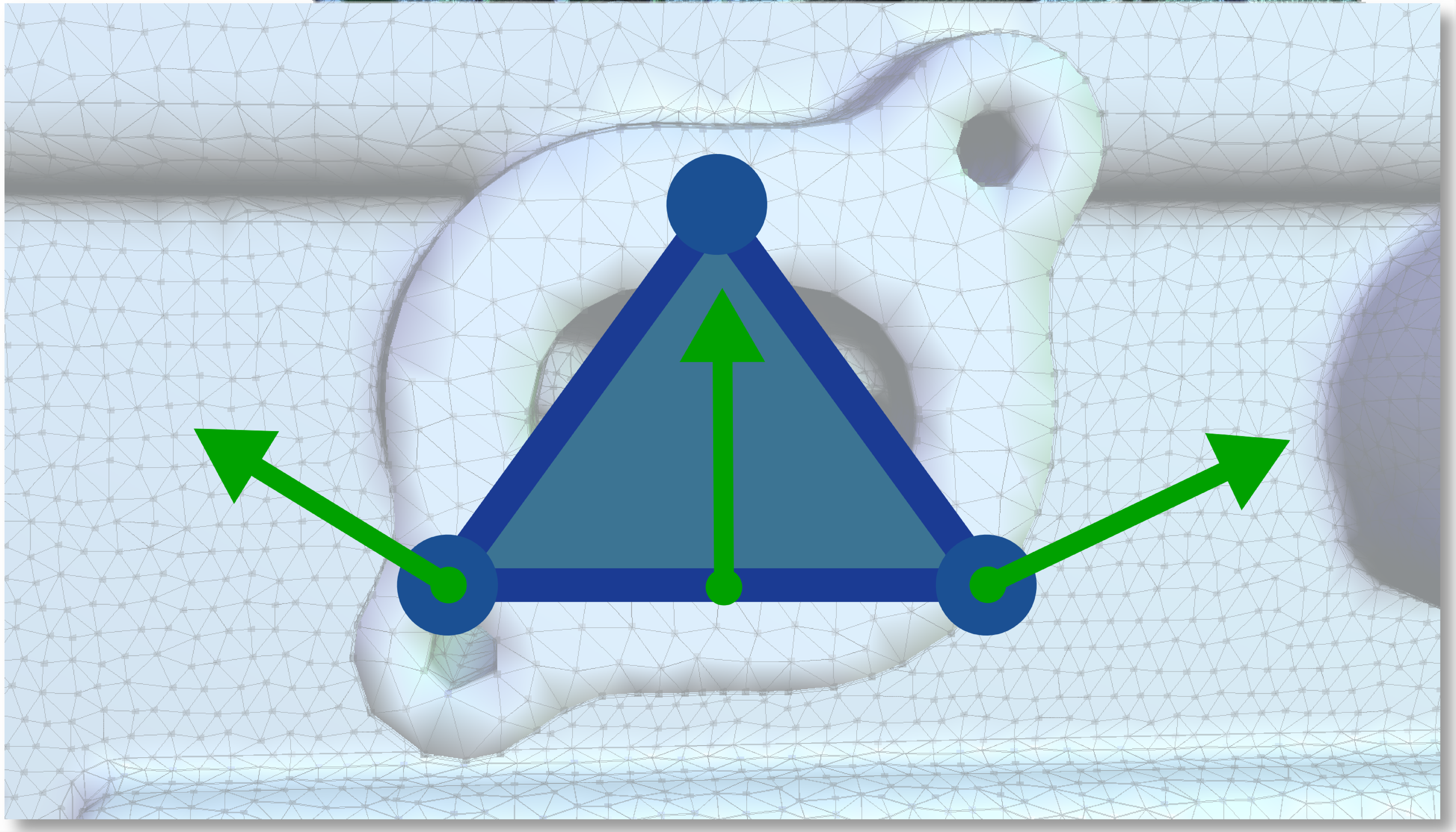
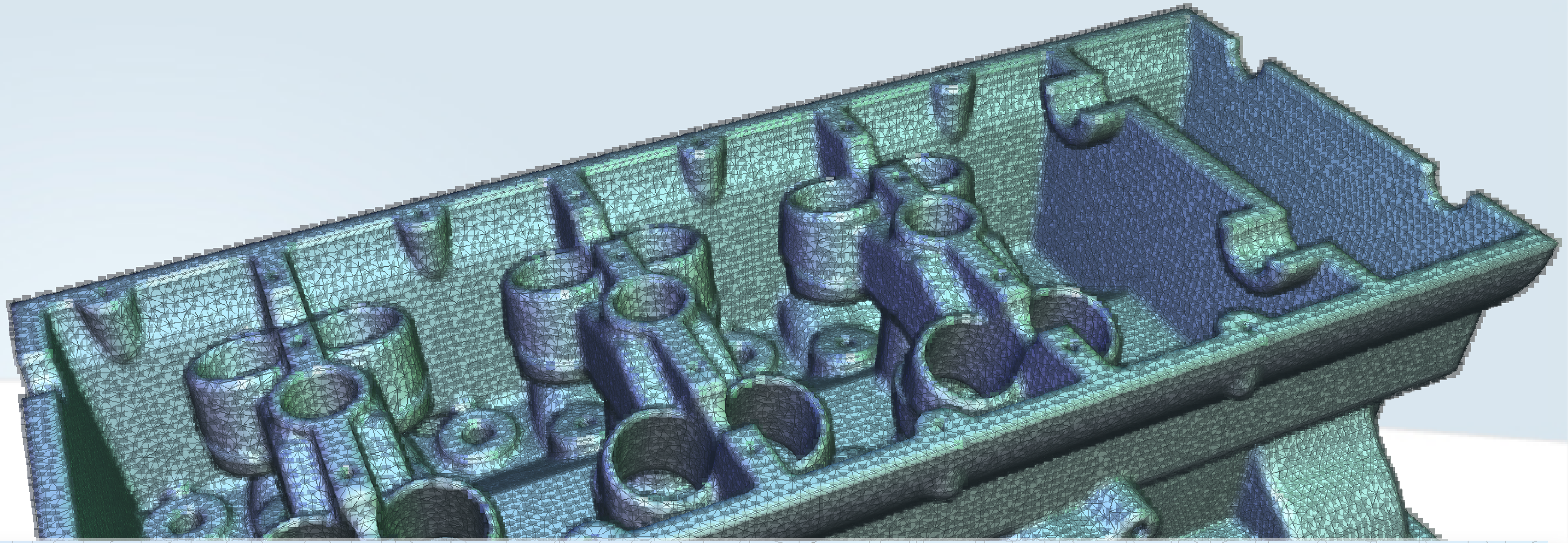


In practice



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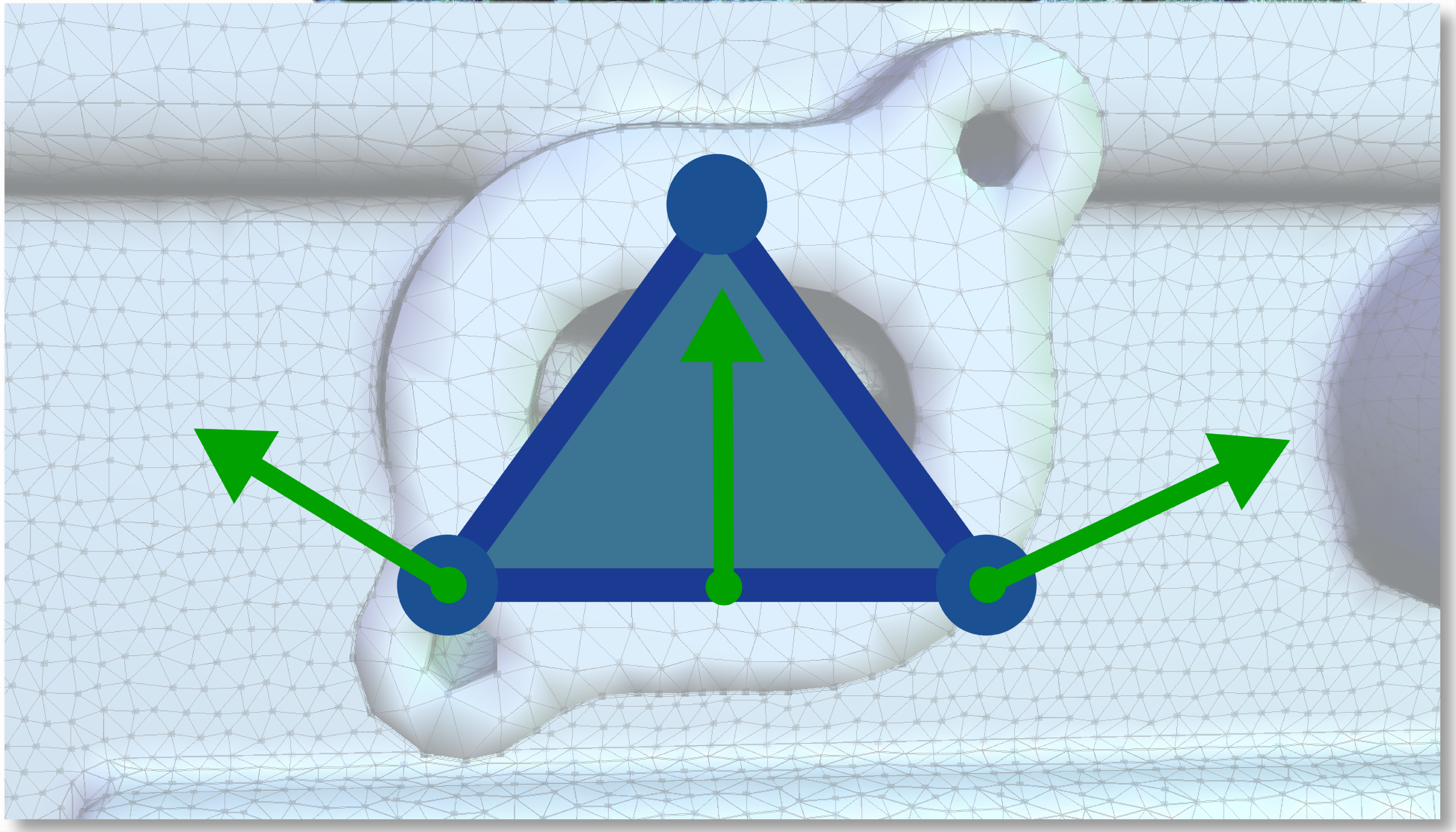

Why?

Why?

- Any problem which implies derivatives of scalar values

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- Computational fluid dynamics

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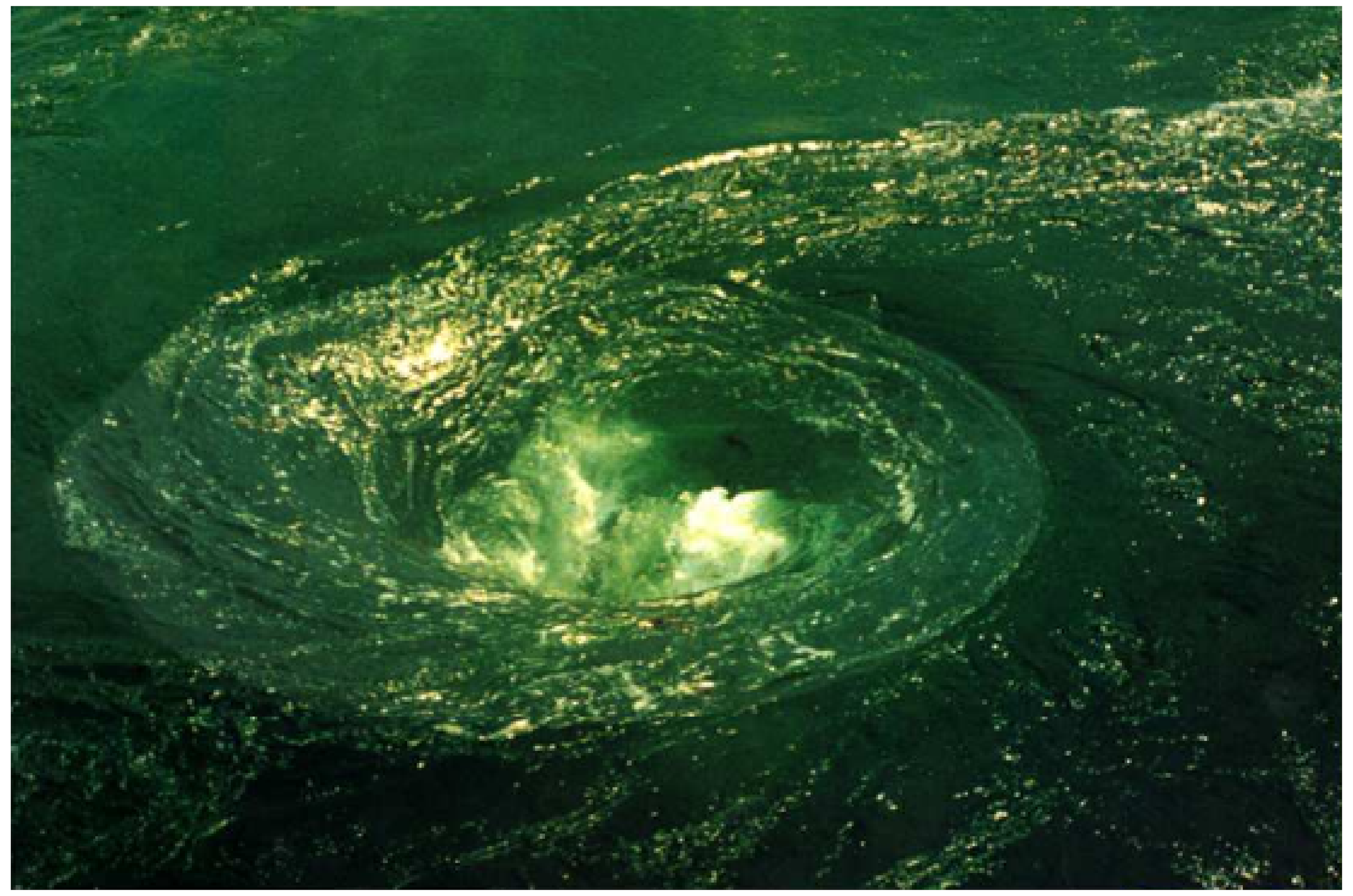

[Post et al. 2003]

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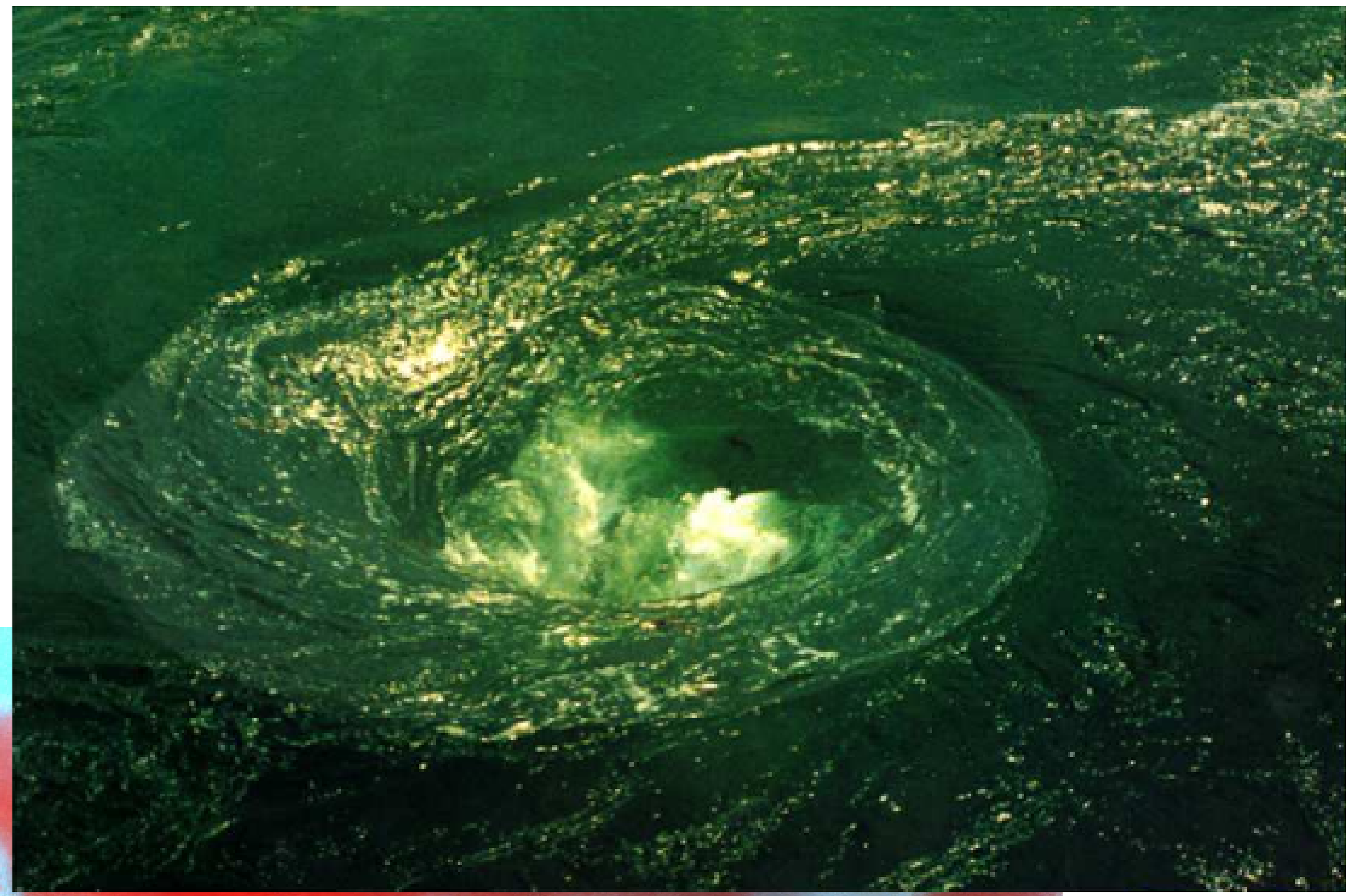
- Computational fluid dynamics



[Post et al. 2003]

[Wikipedia]

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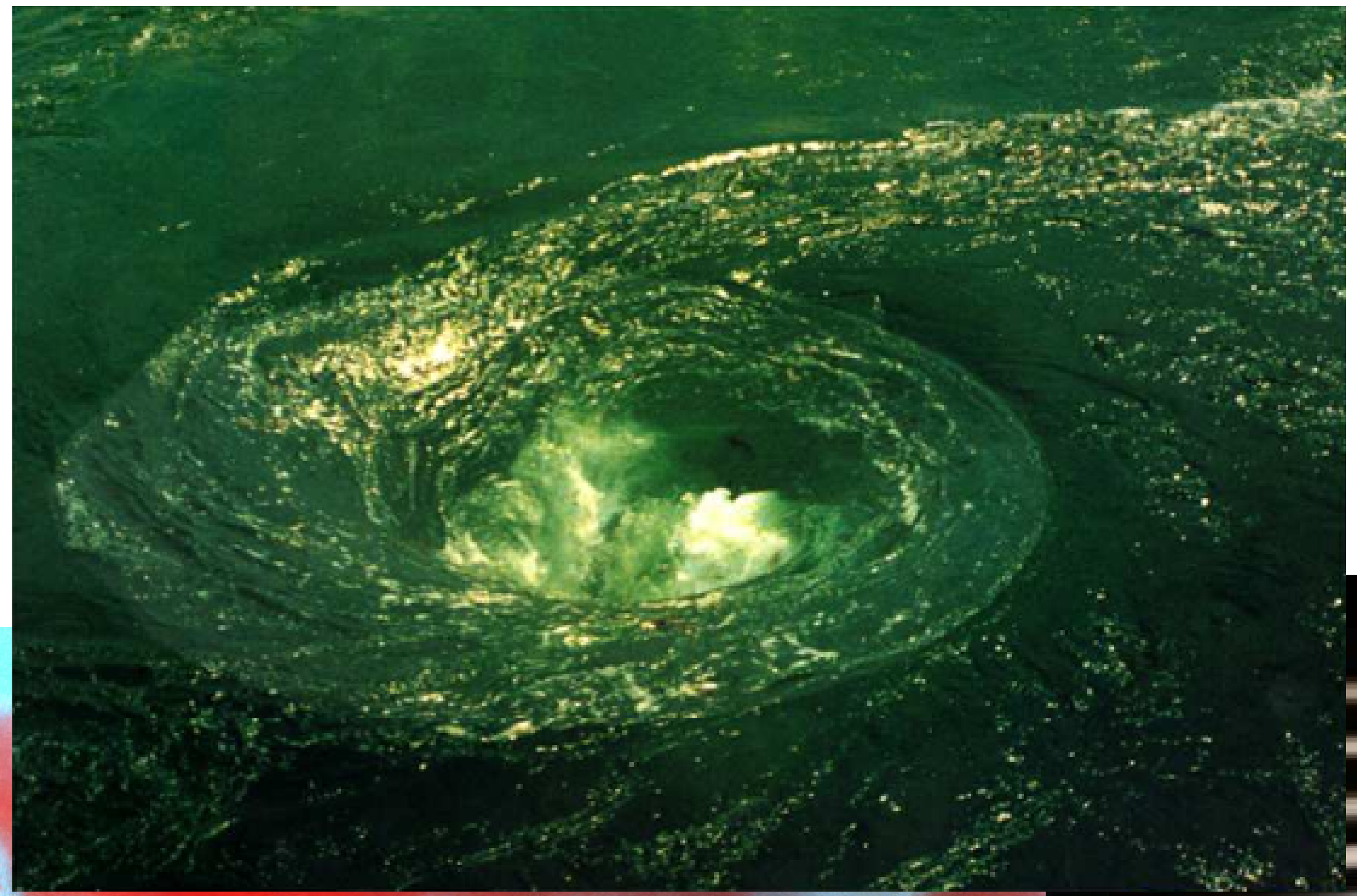

[Post et al. 2003]

[www.speedhunter.com]

[Wikipedia]

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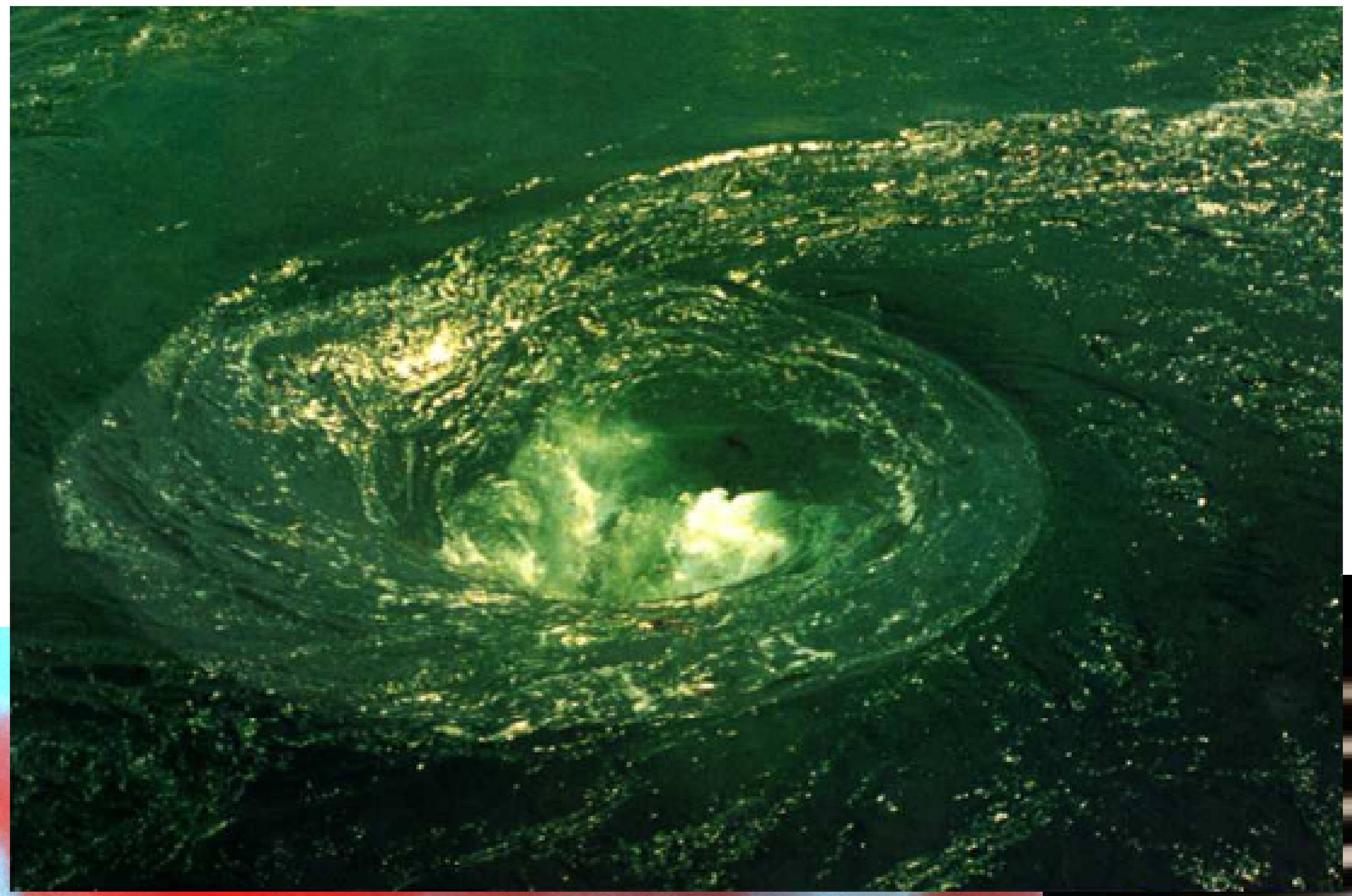
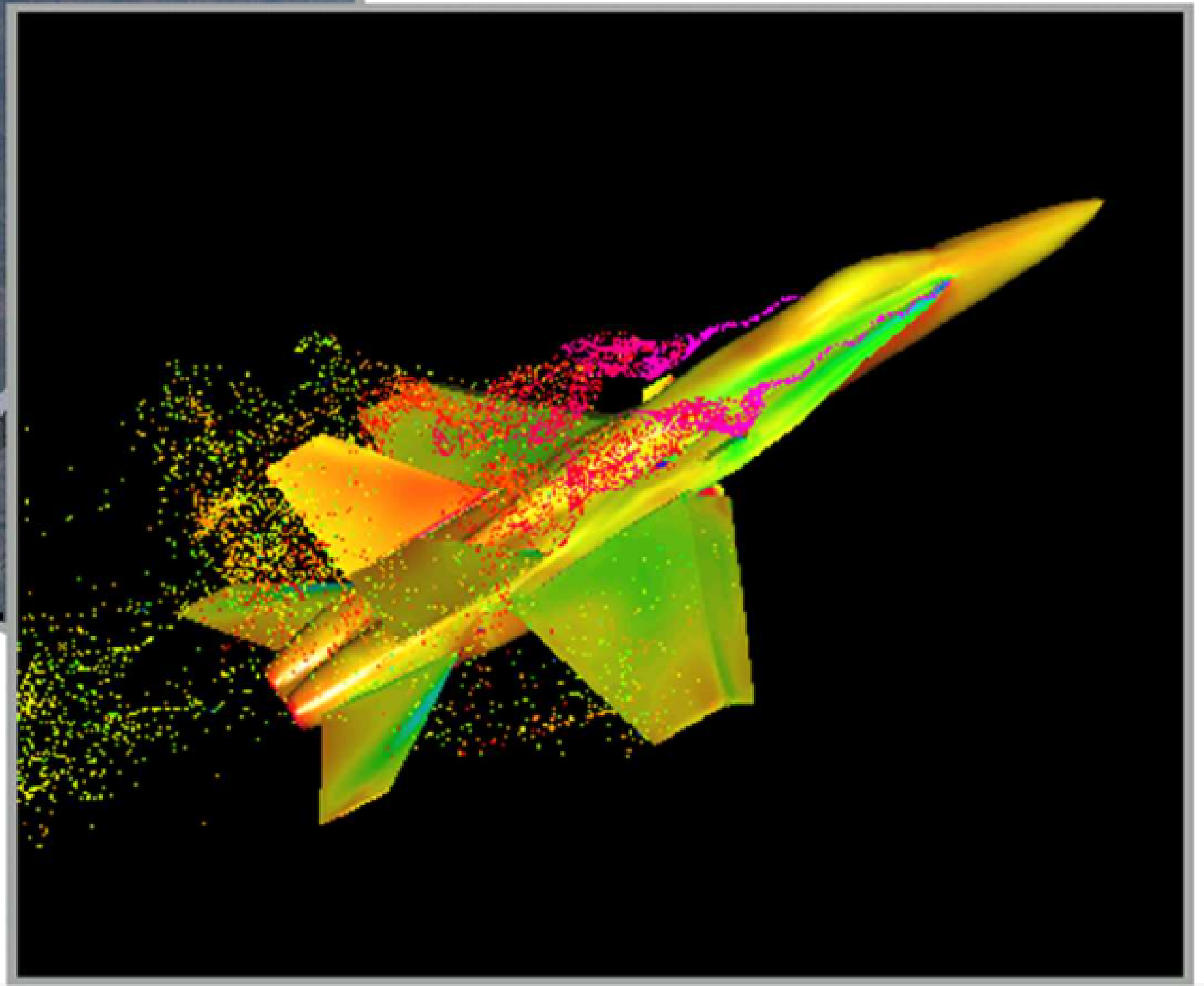


[Post et al. 2003]

[www.speedhunter.com]

[Chen]



[Wikipedia]

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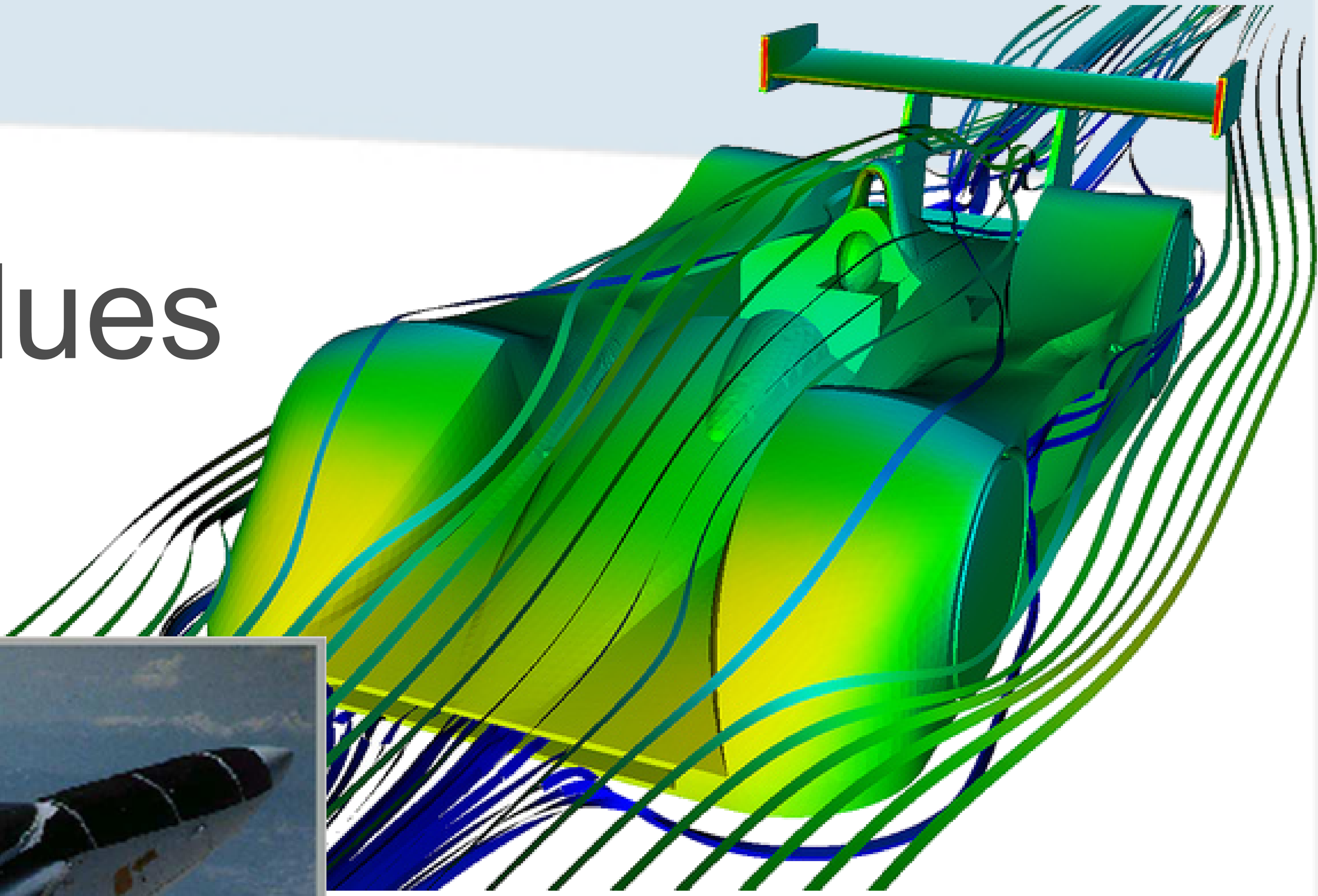
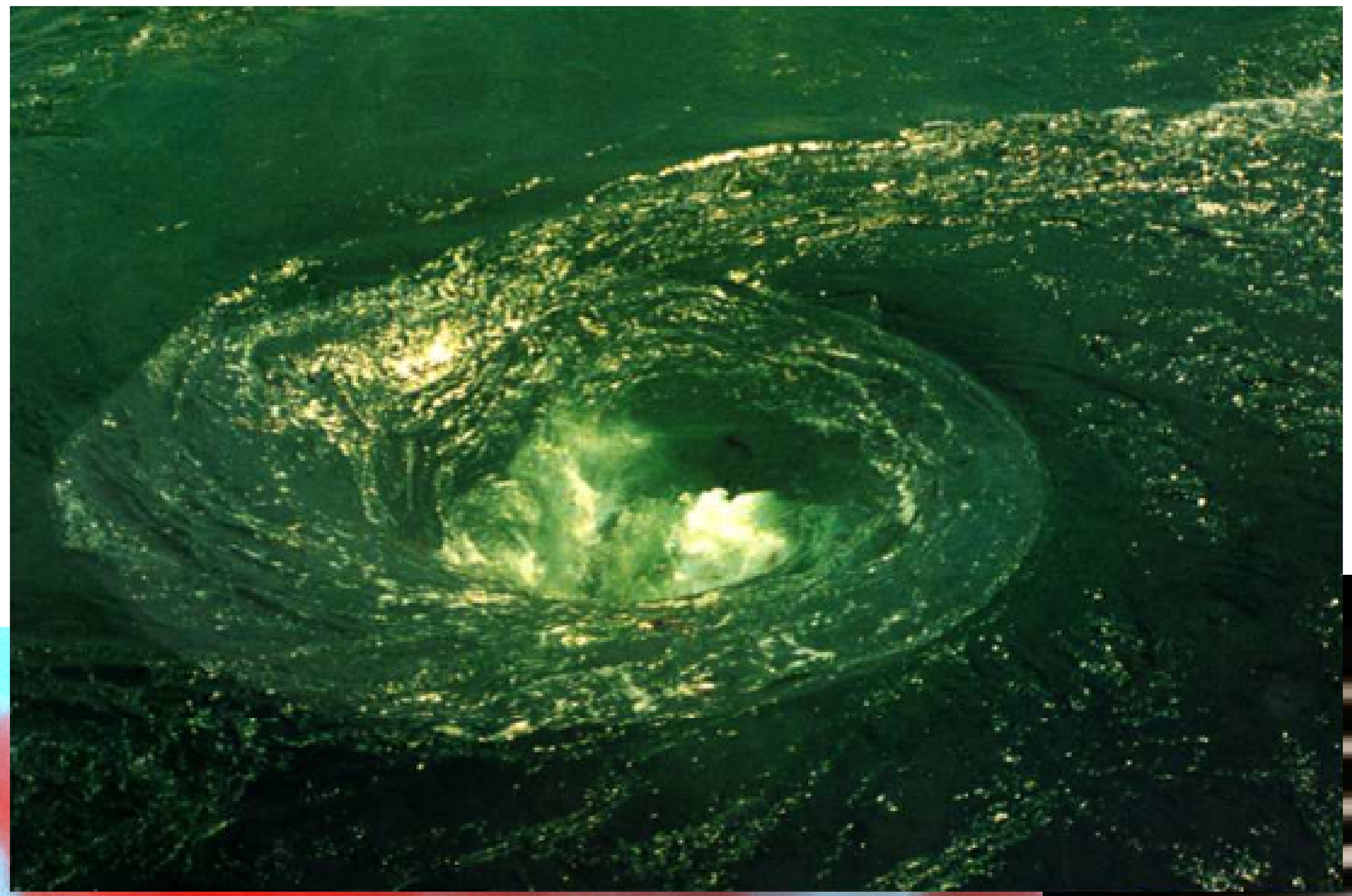


[Kitware]



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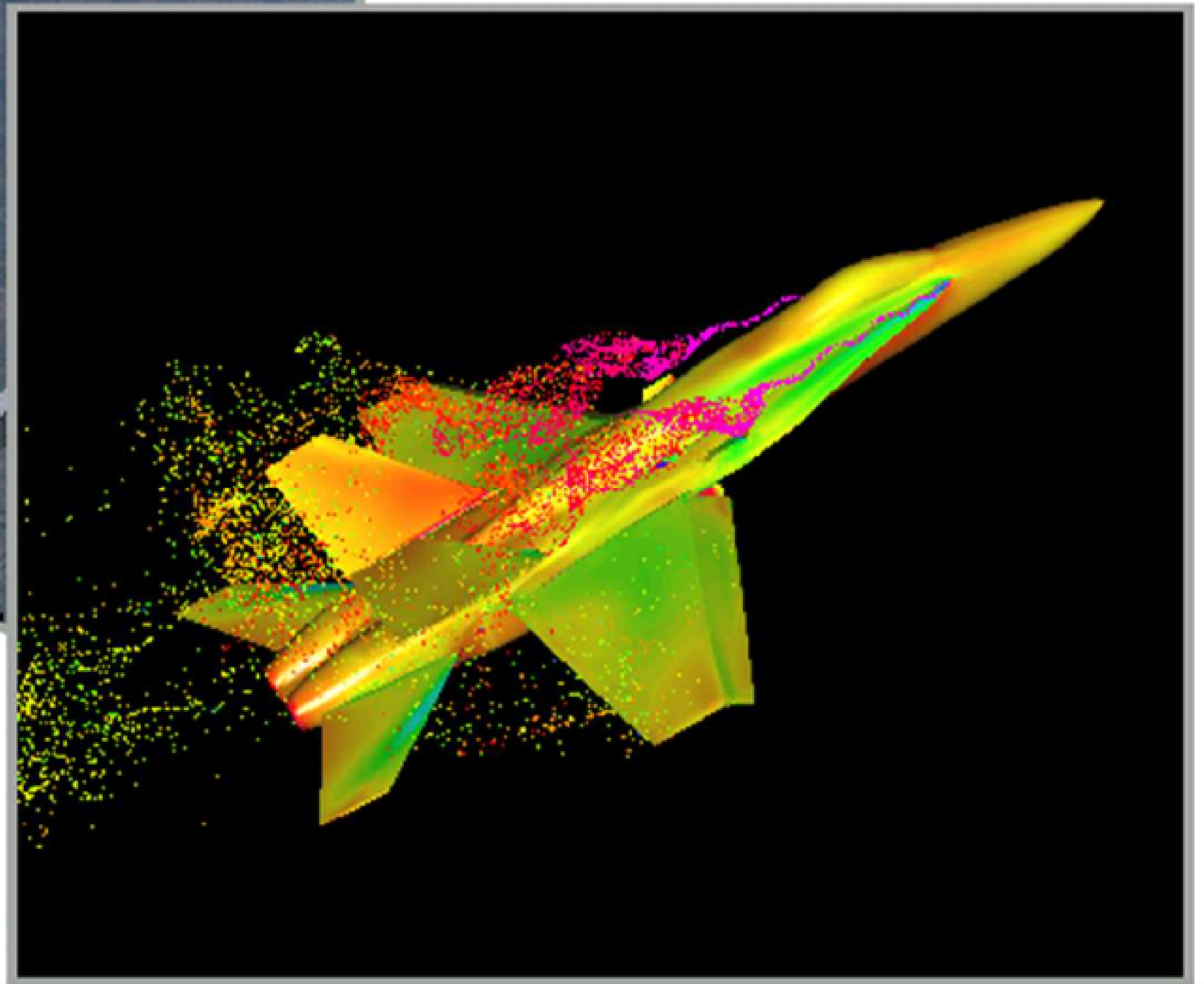
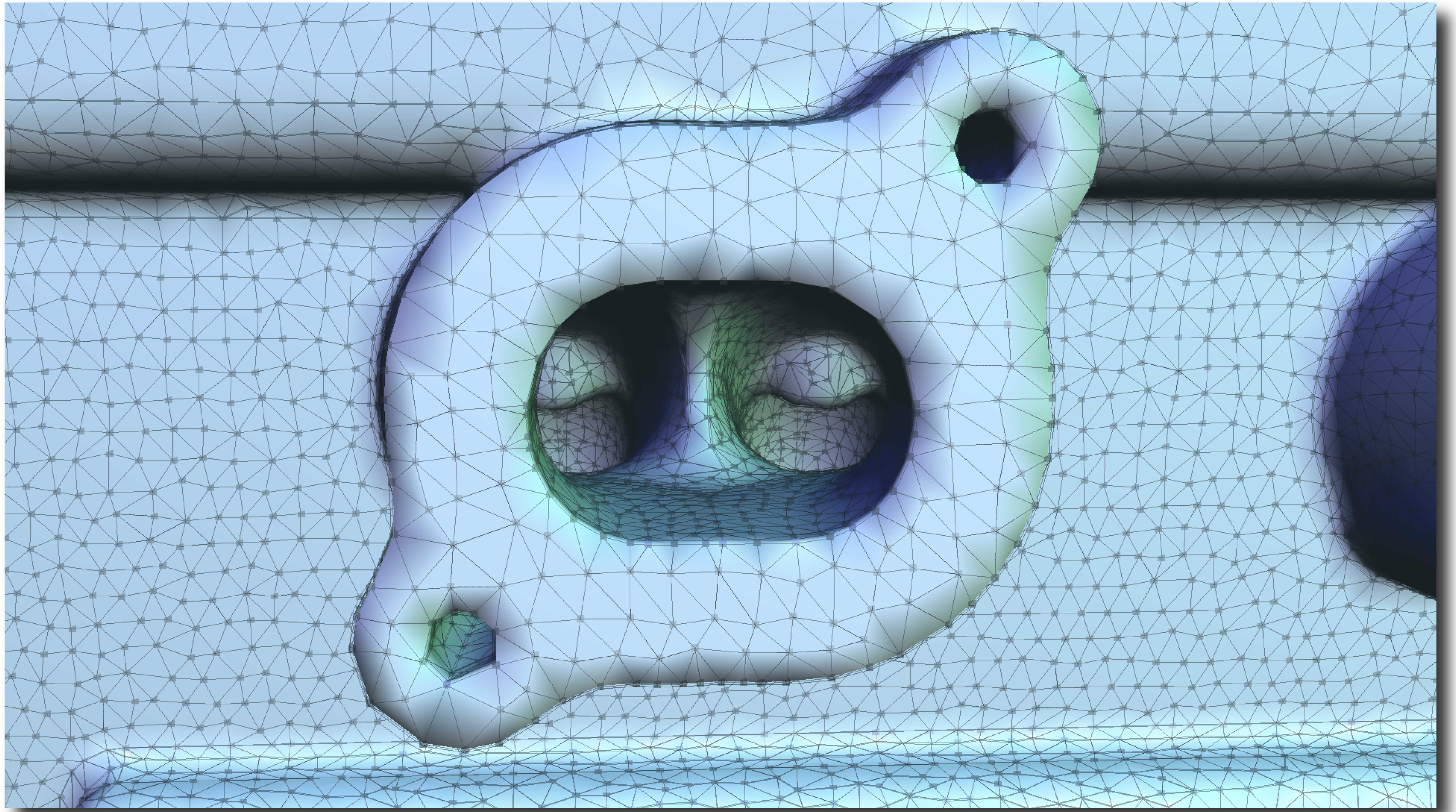


What is there to visualize?

- The simple way: glyphs



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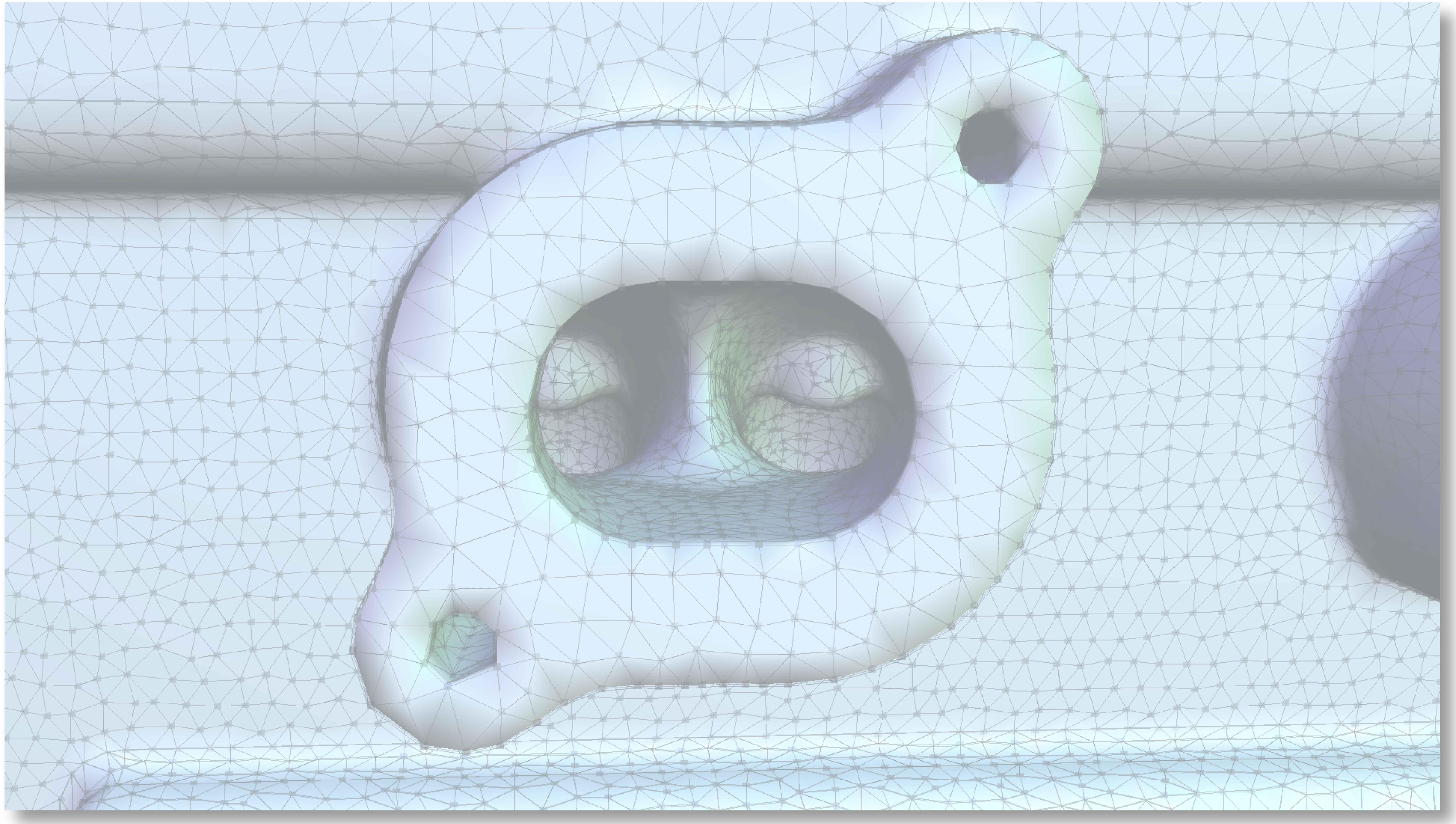

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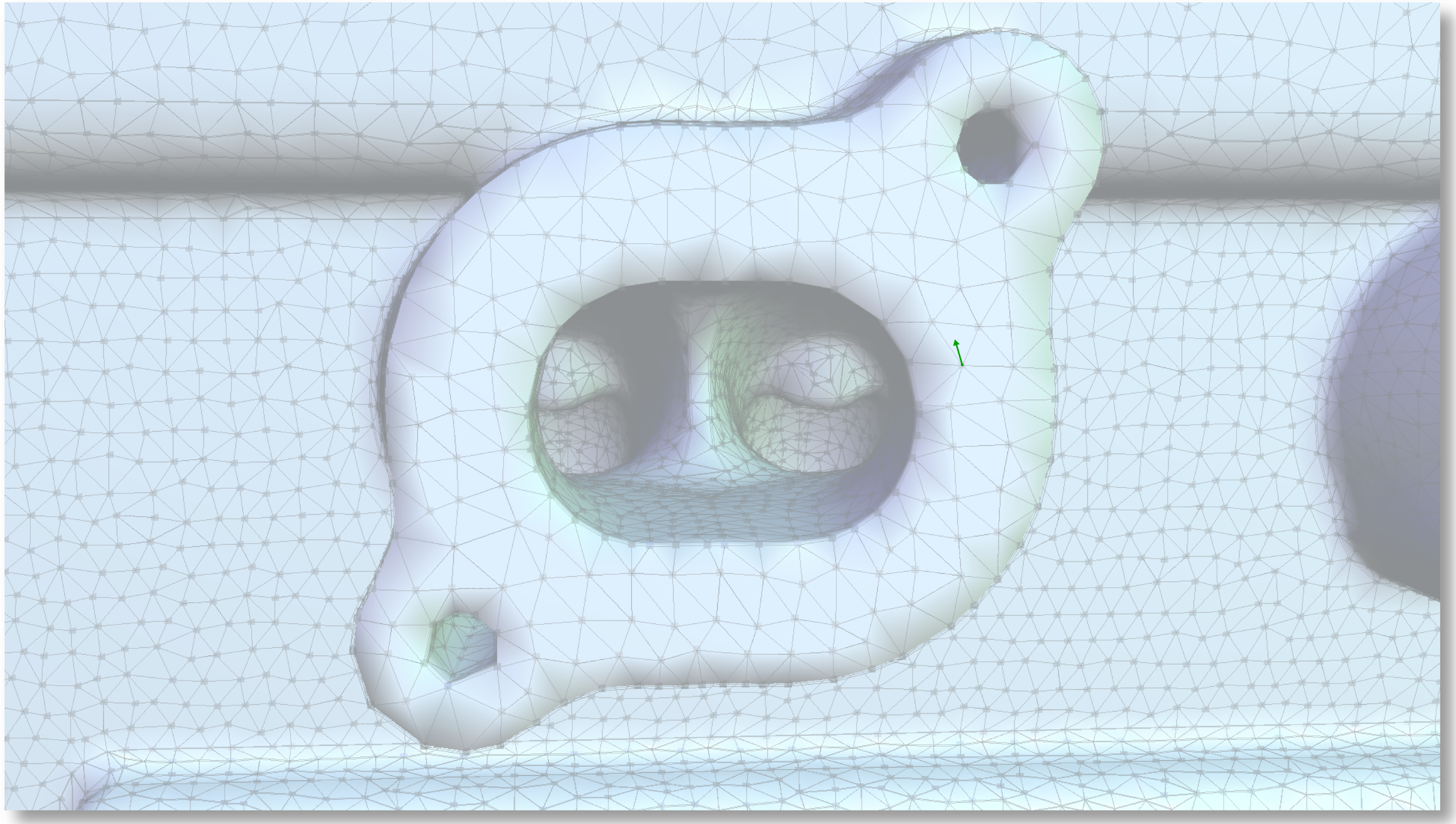

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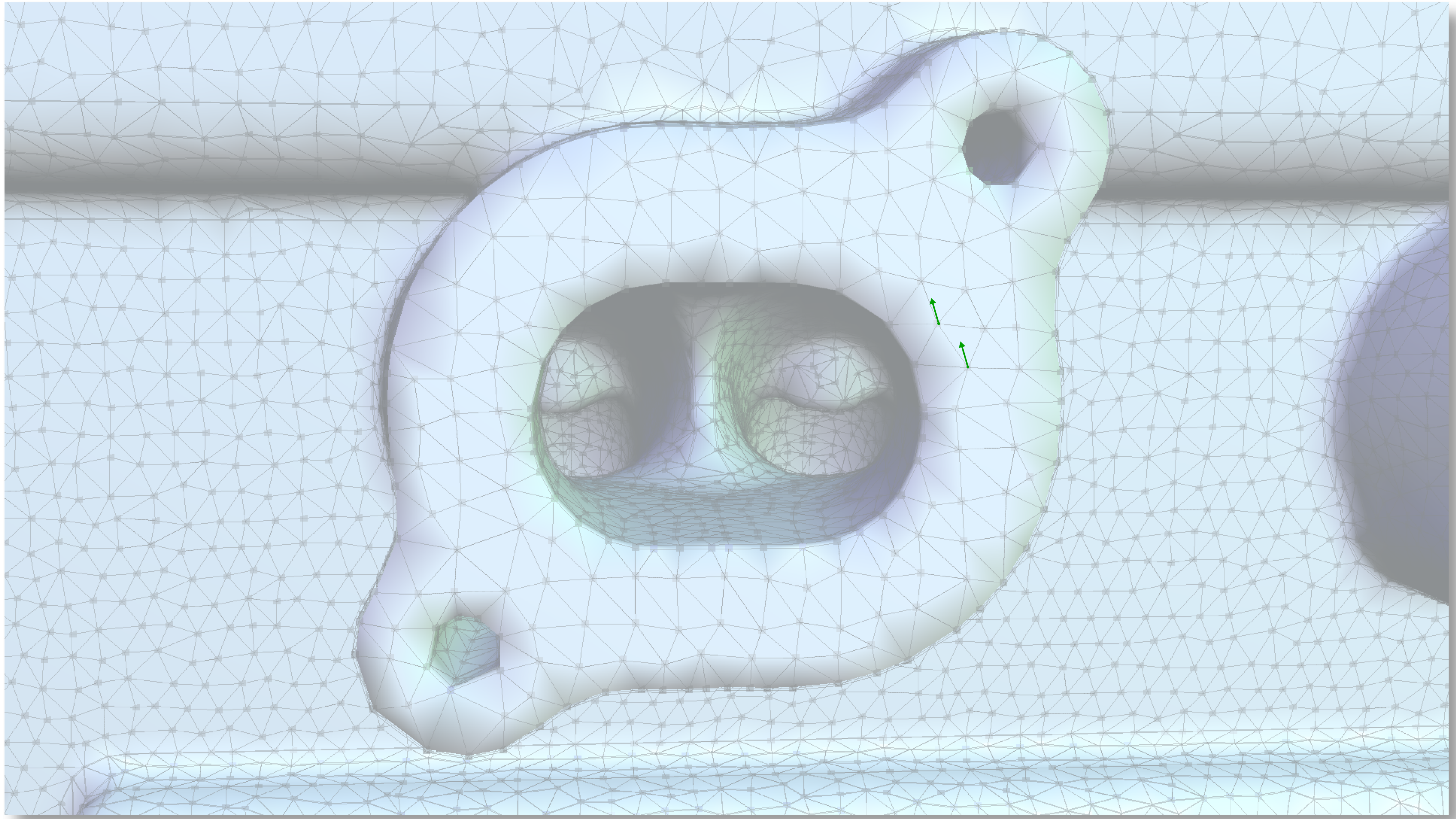

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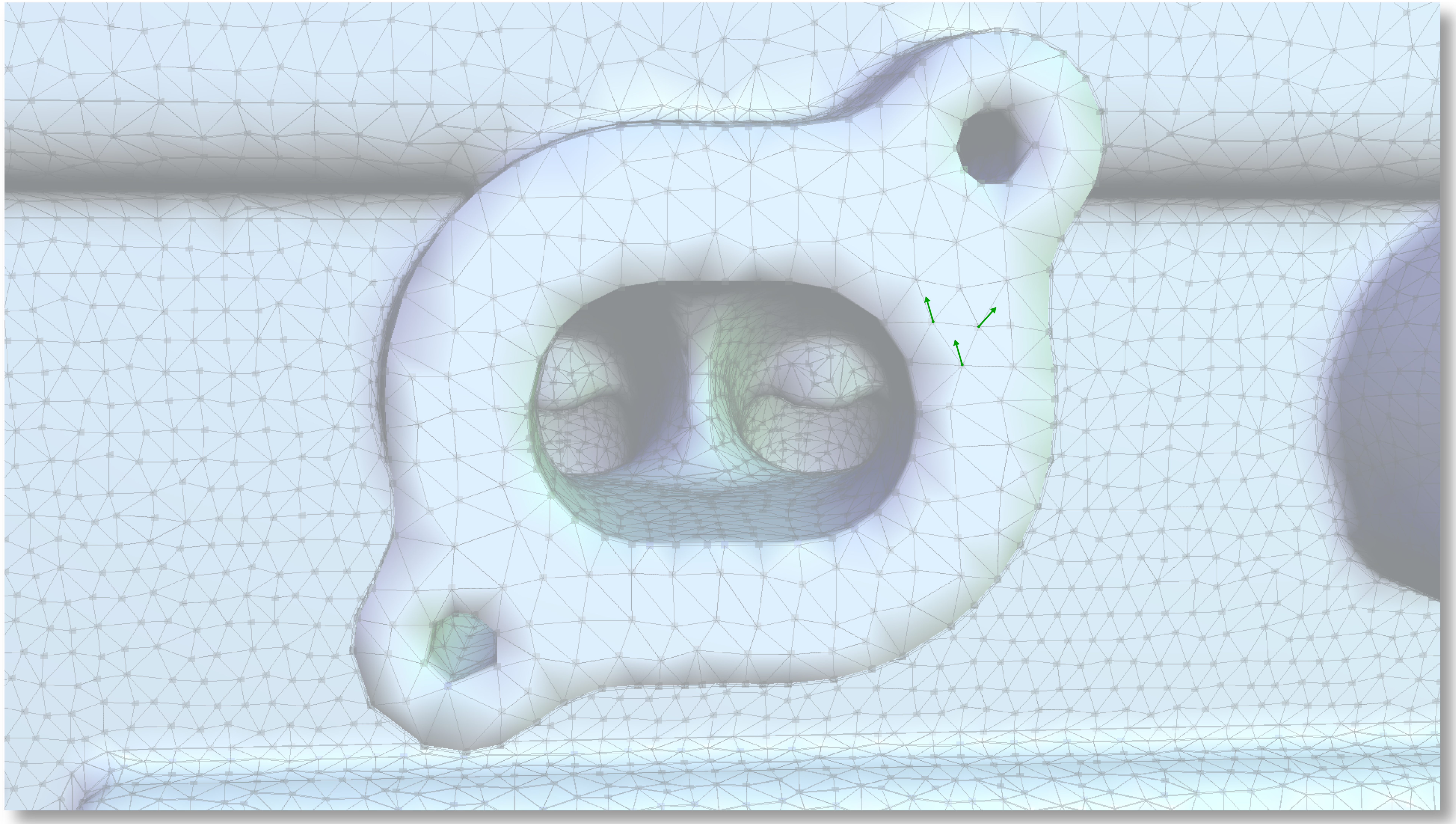

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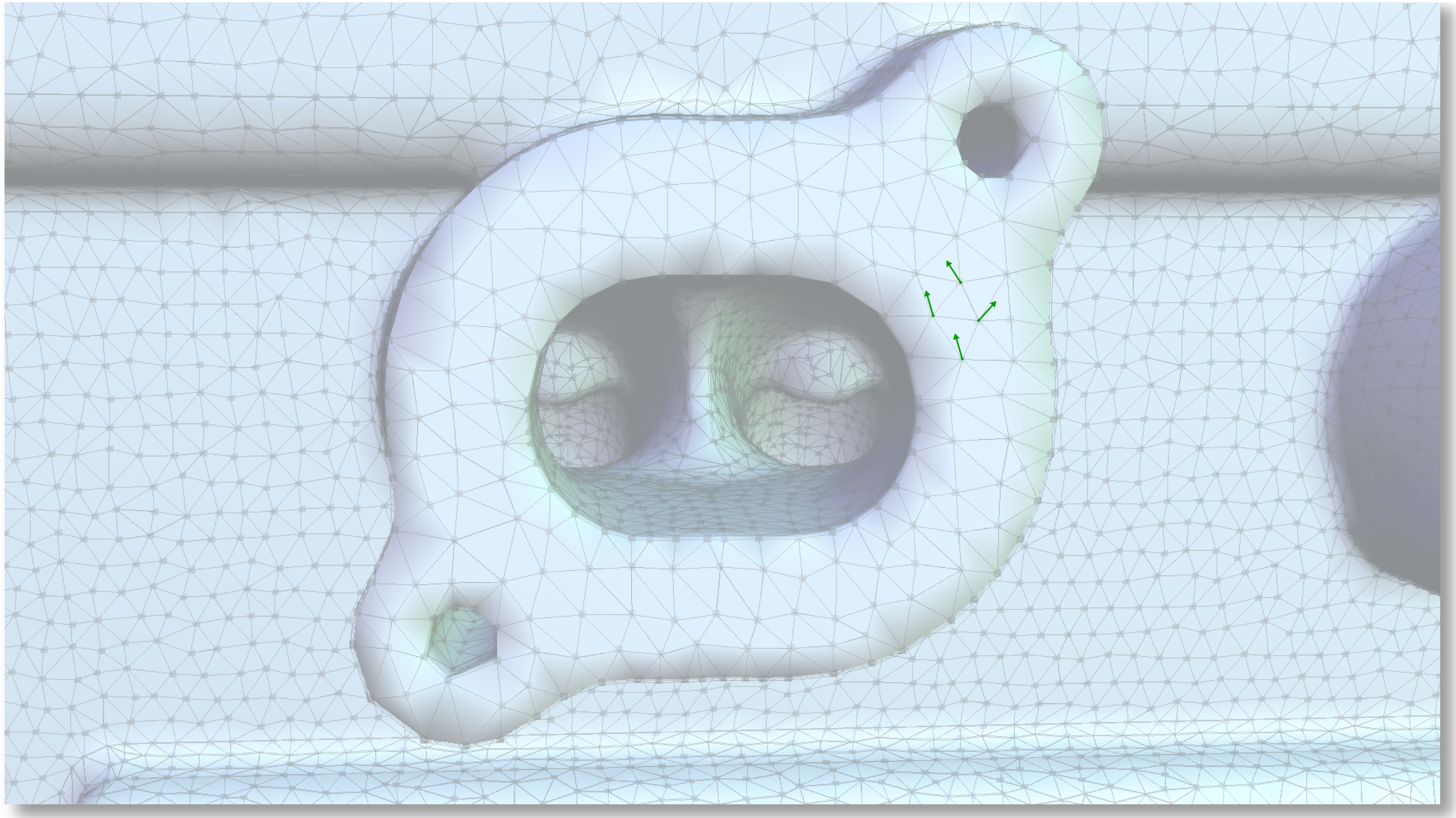

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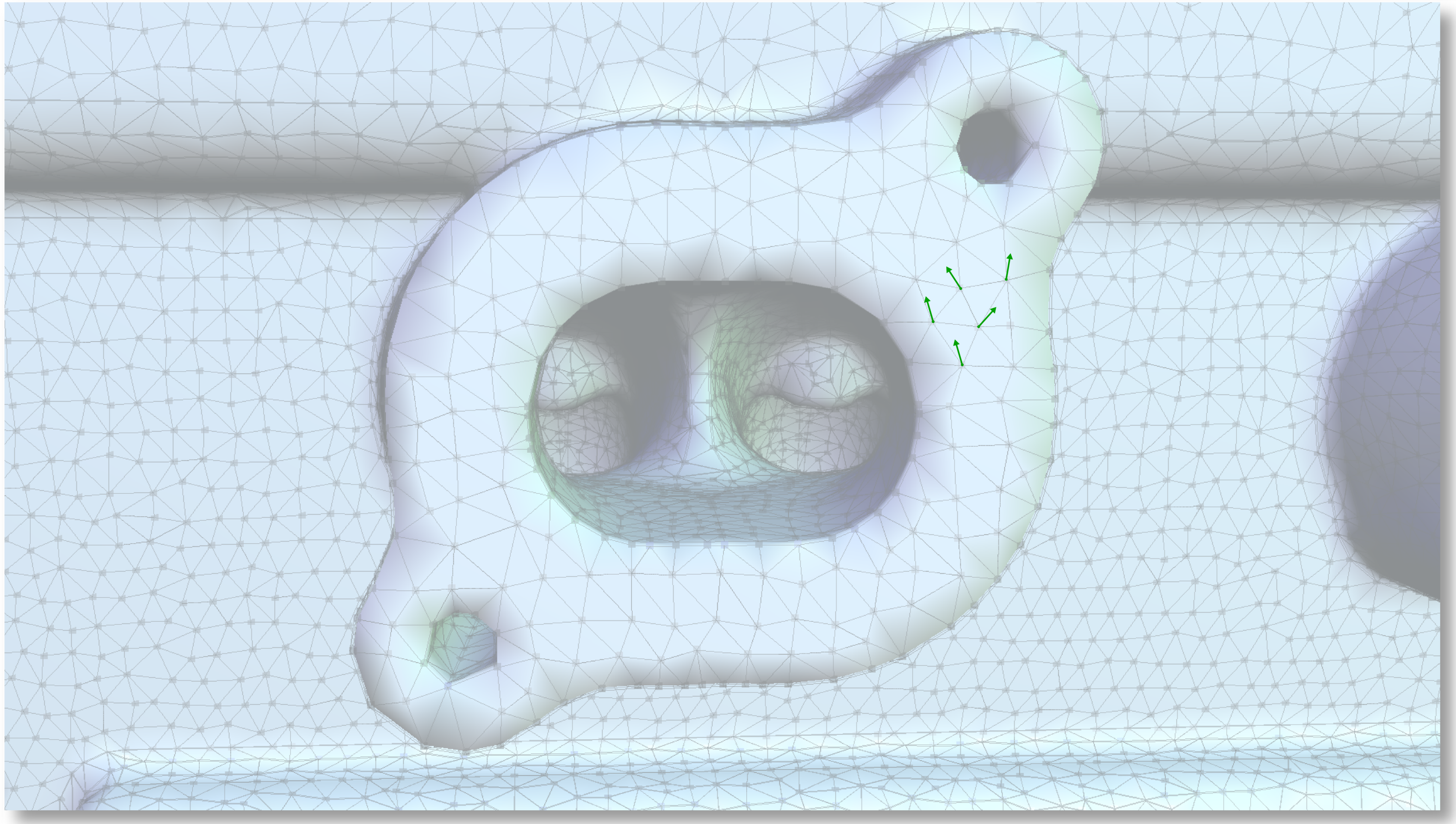
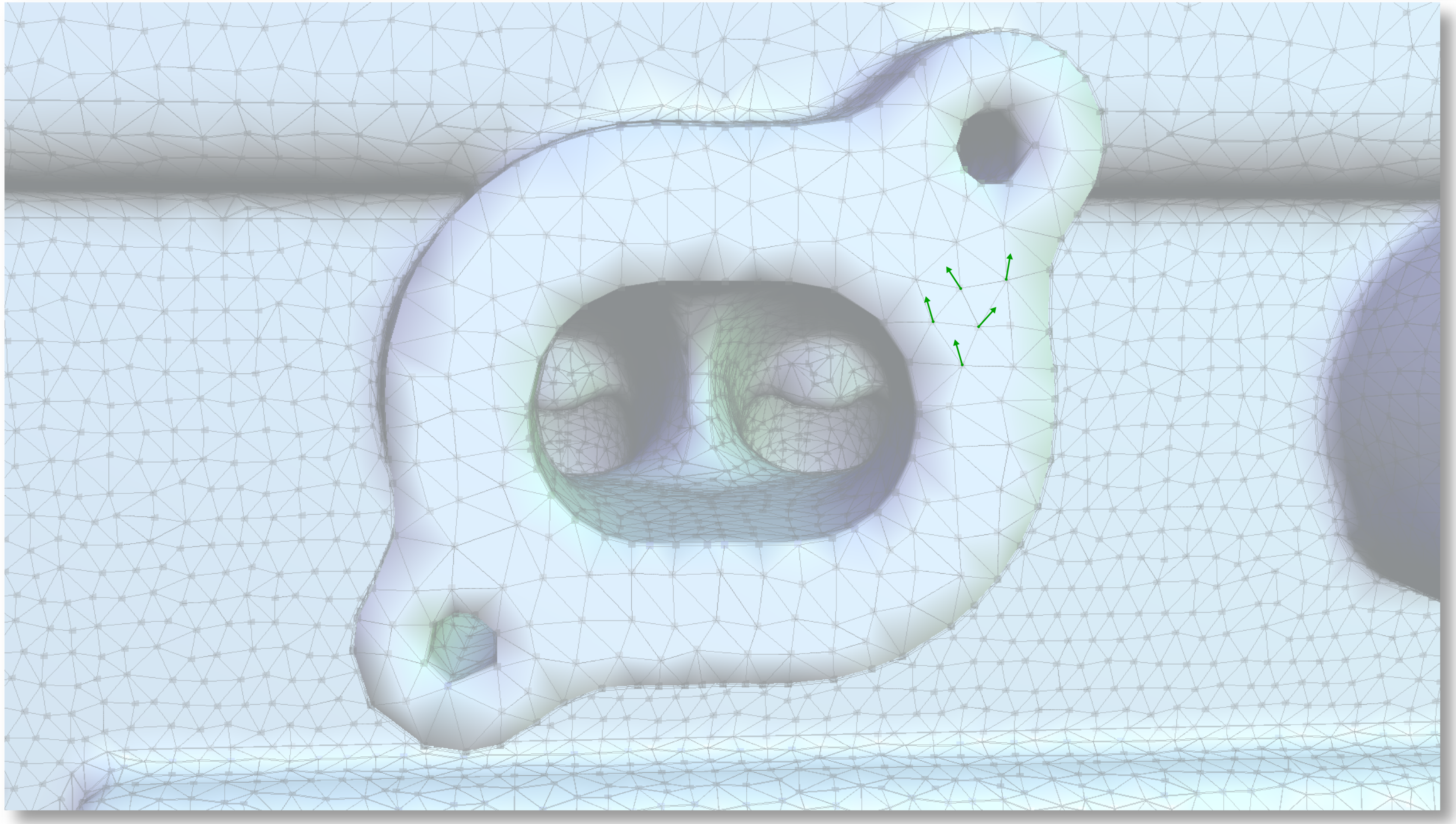


What is there to visualize?

- The simple way: glyphs
- Data overload



What is there to visualize?

- The simple way: glyphs
- Data overload
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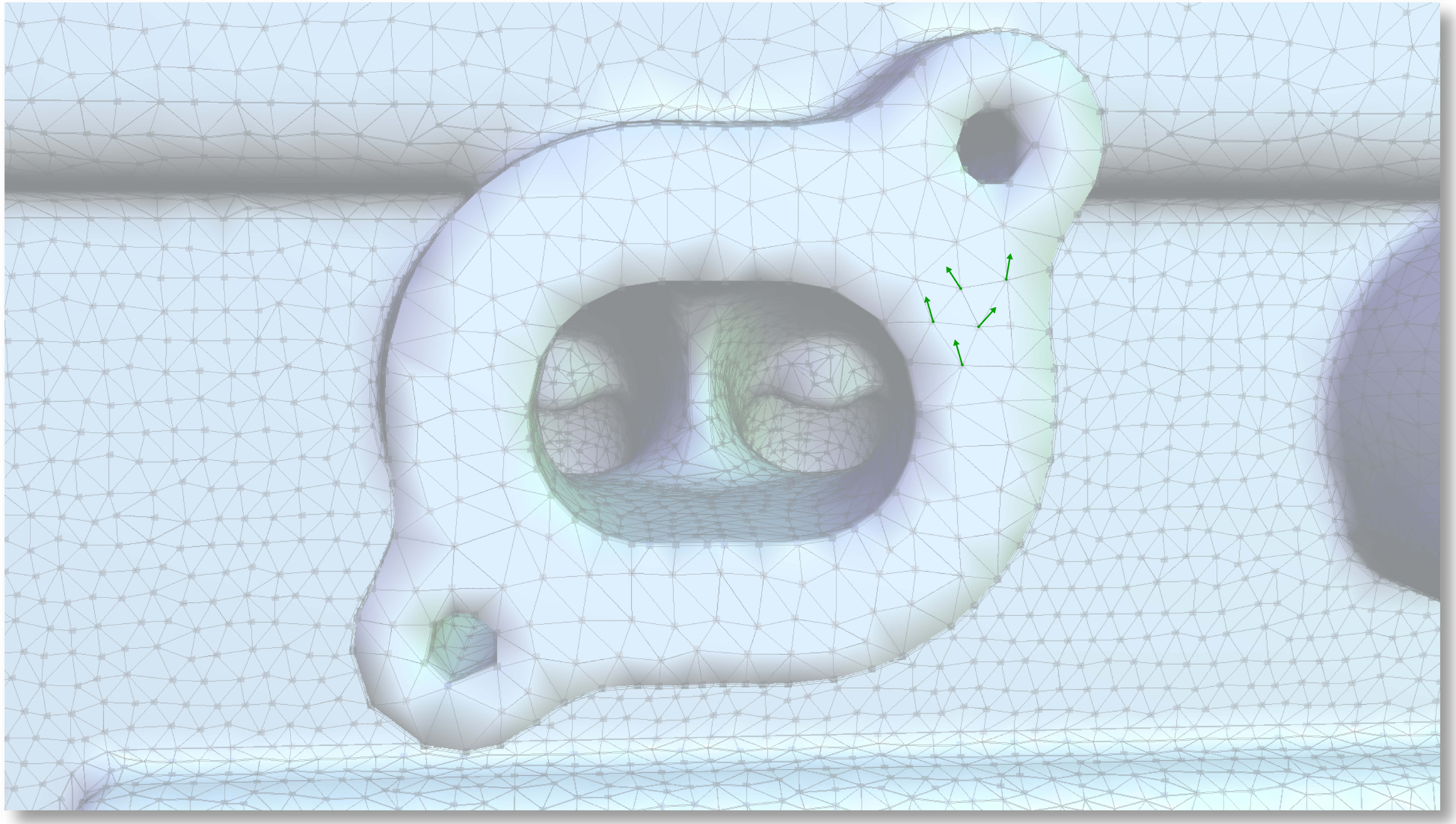

What is there to visualize?

- The simple way: glyphs
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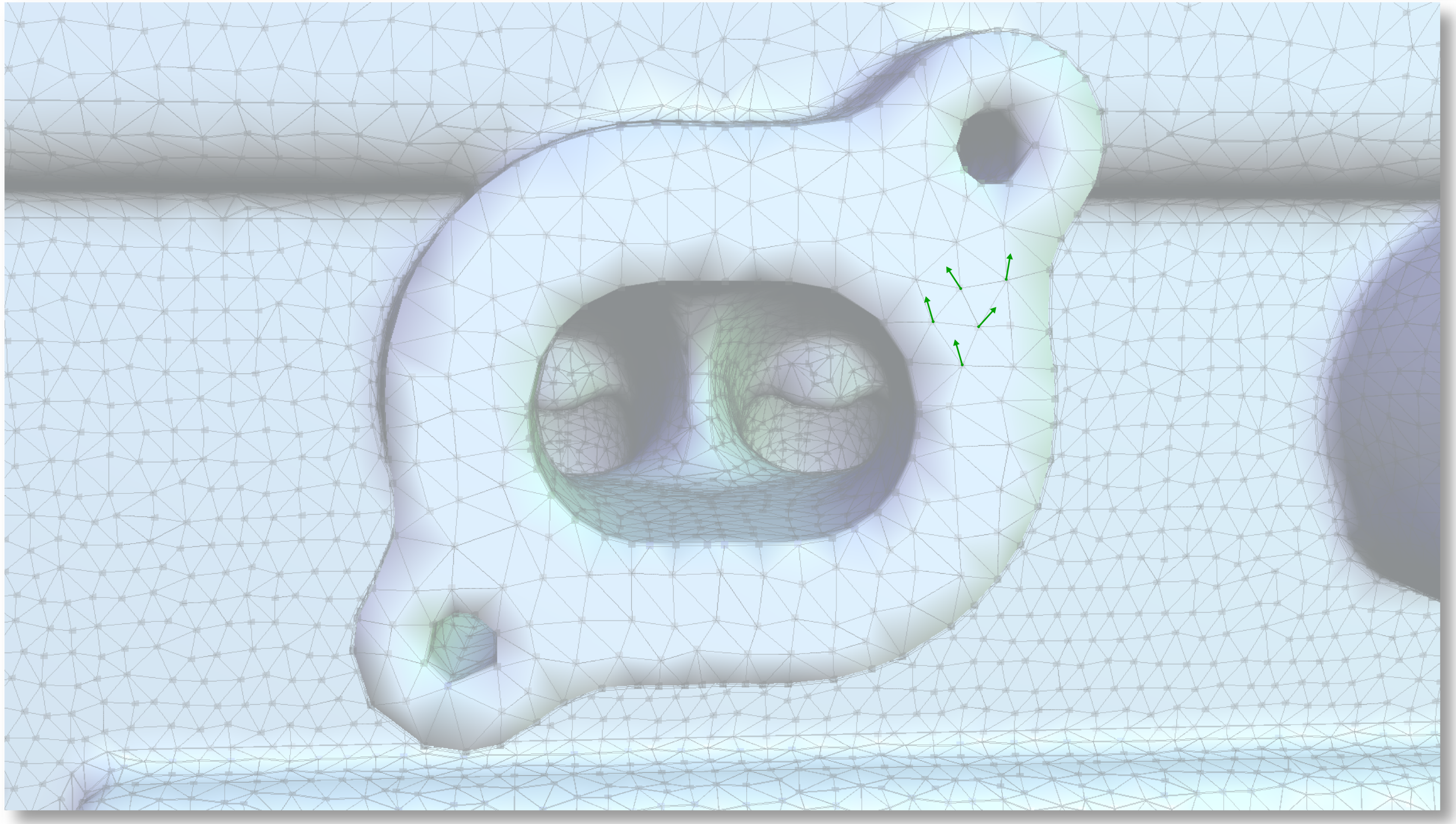
- Small scale details



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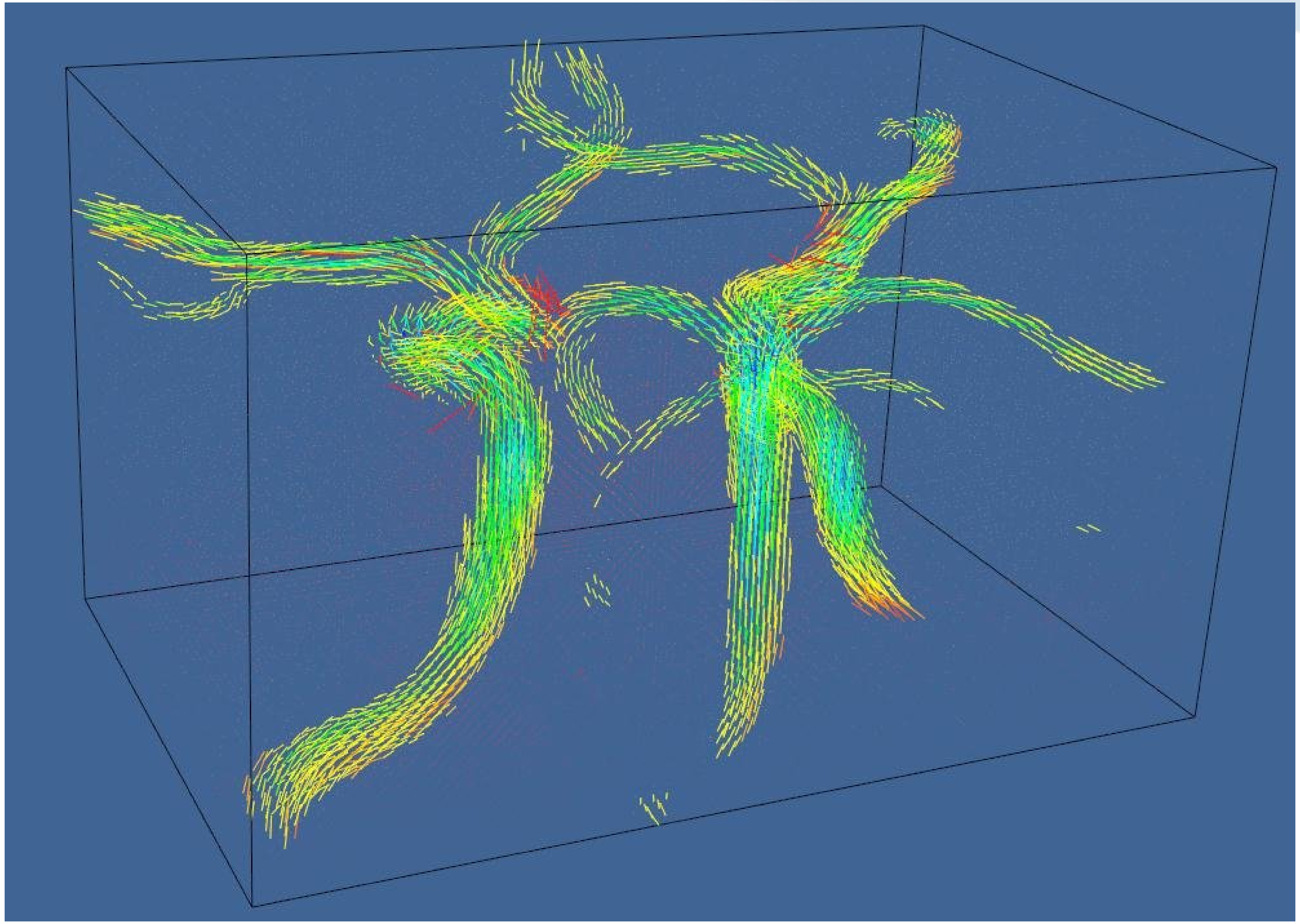

[Kitware]


Kitware ParaView 3.6.2
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Tools
Help
Time: 0
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Pipeline Browser
builtin:
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StreamTracer1
Glyph2
GenerateTubes1
Glyph3
disk_out_ref_surface.vtp
Object Inspector
Properties
Display
Information
Root
Element Blocks (1)
Unnamed block ID: 1 Type: HEX8 Size: 747...
Face Blocks (3)
Edge Blocks (4)
Interpolation
Gouraud
Material
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Point size
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Line width
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Edge Style
Set Edge Color...
Lighting
Specular Intensity
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The Visualization Toolkit Book Images
Dolphin
NOTES - KWrite
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What is there to visualize?

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- Getting inspiration from nature

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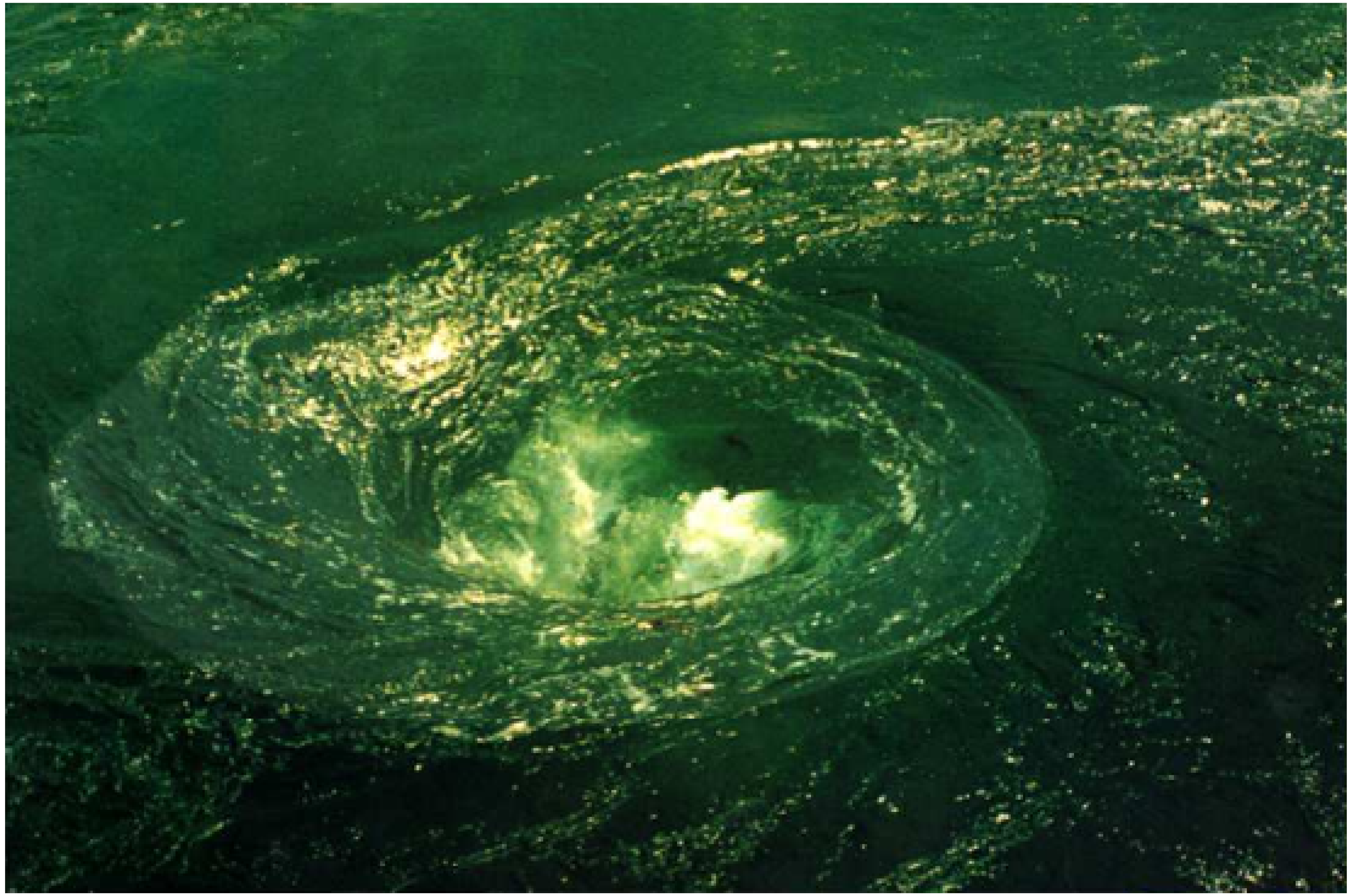
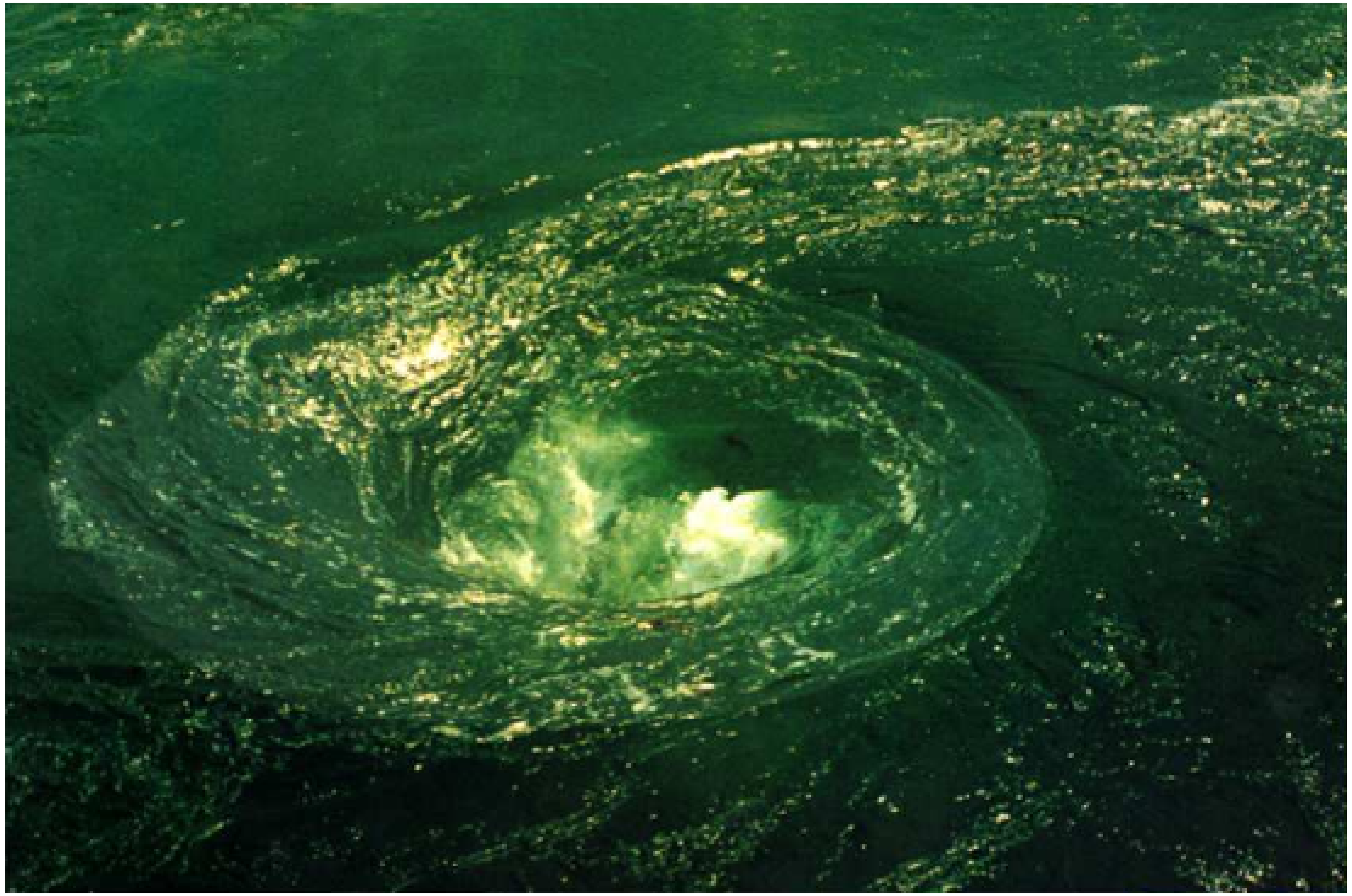


[Post et al. 2003]

What is there to visualize?

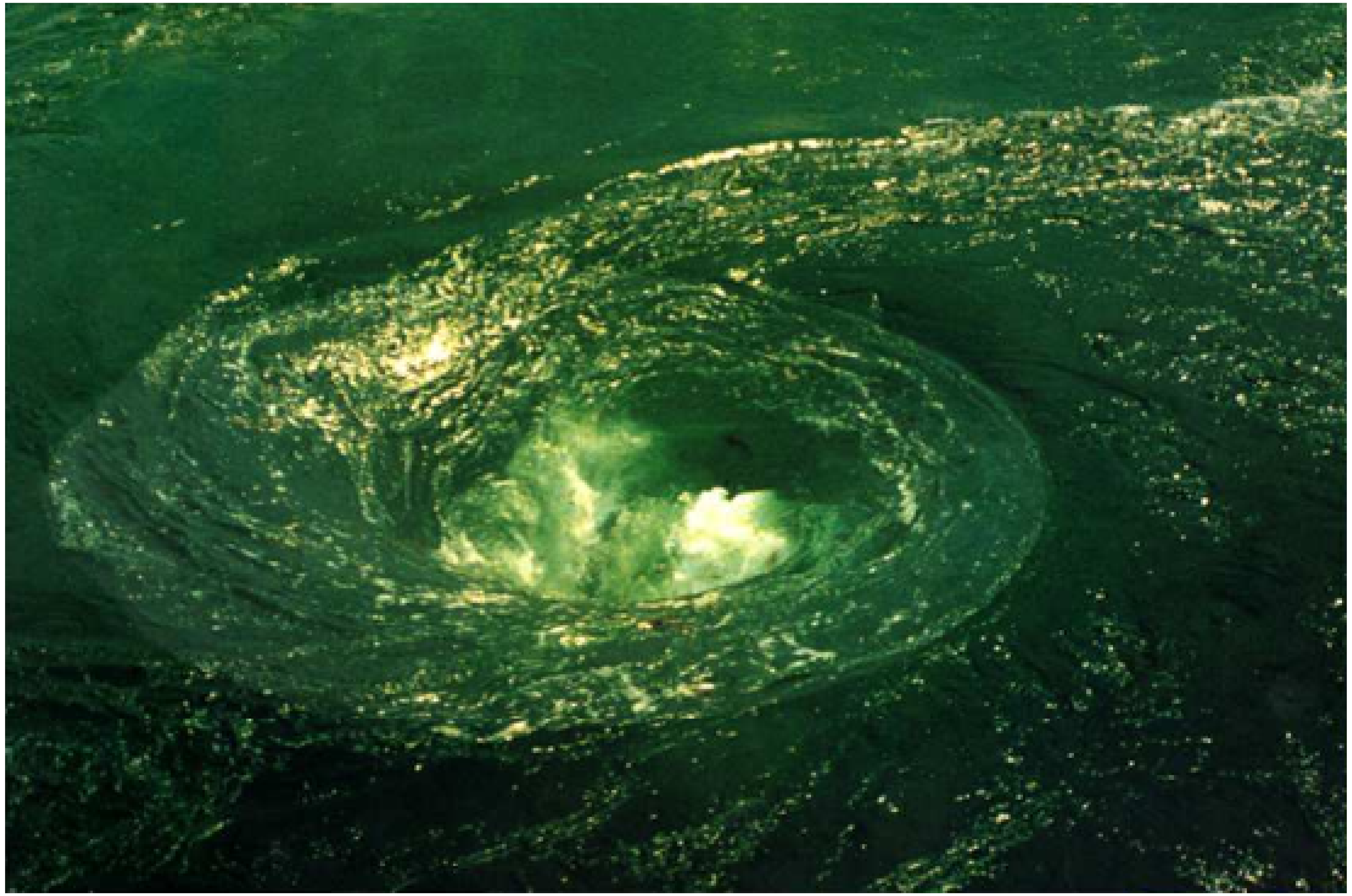
- Getting inspiration from nature
  - Global visualization



[Post et al. 2003]

What is there to visualize?

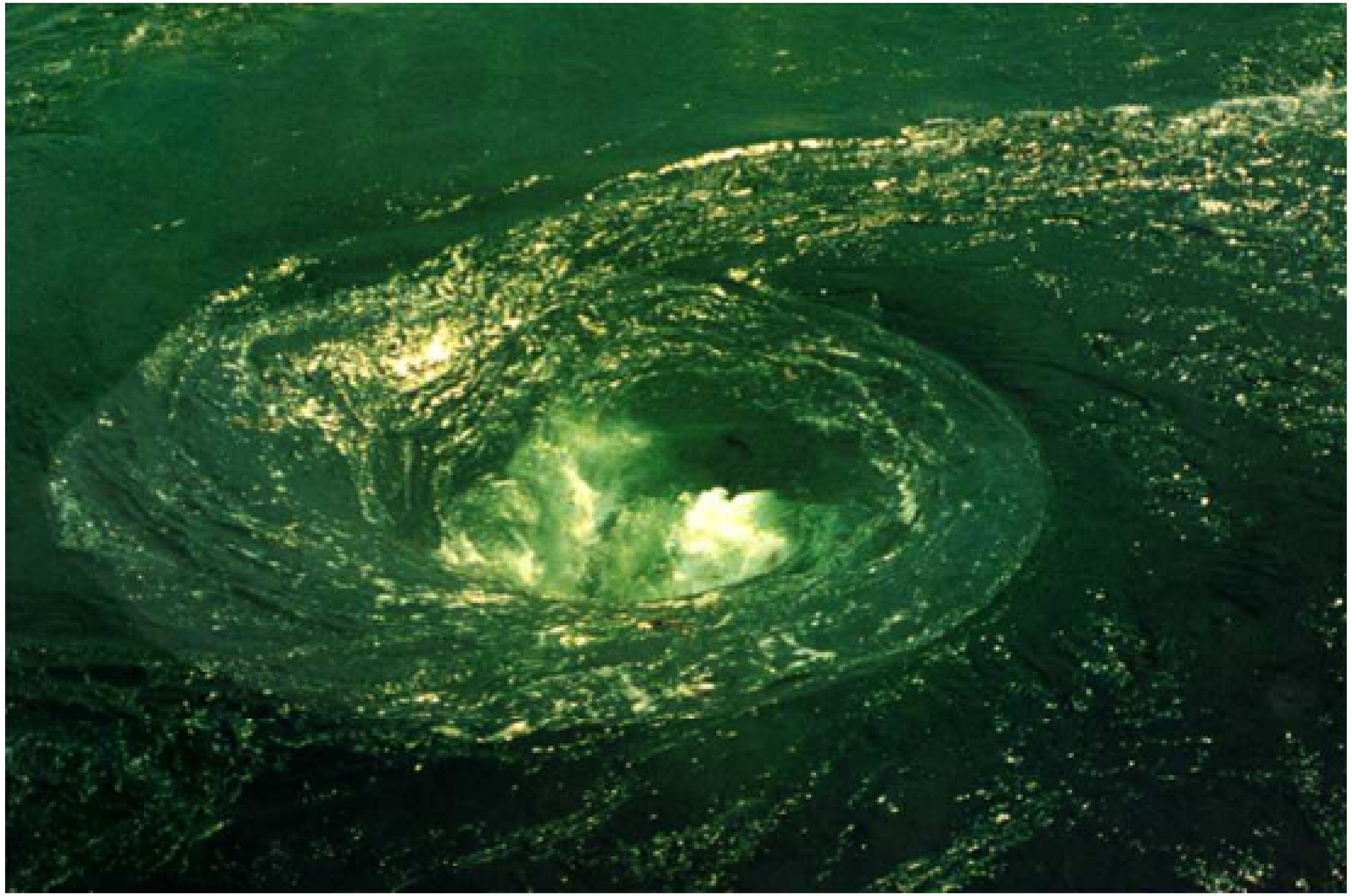
- Getting inspiration from nature
  - Global visualization
  - Implicit representations



[Post et al. 2003]

What is there to visualize?

- Getting inspiration from nature
  - Global visualization
  - Implicit representations
  - *Line Integral Convolution*

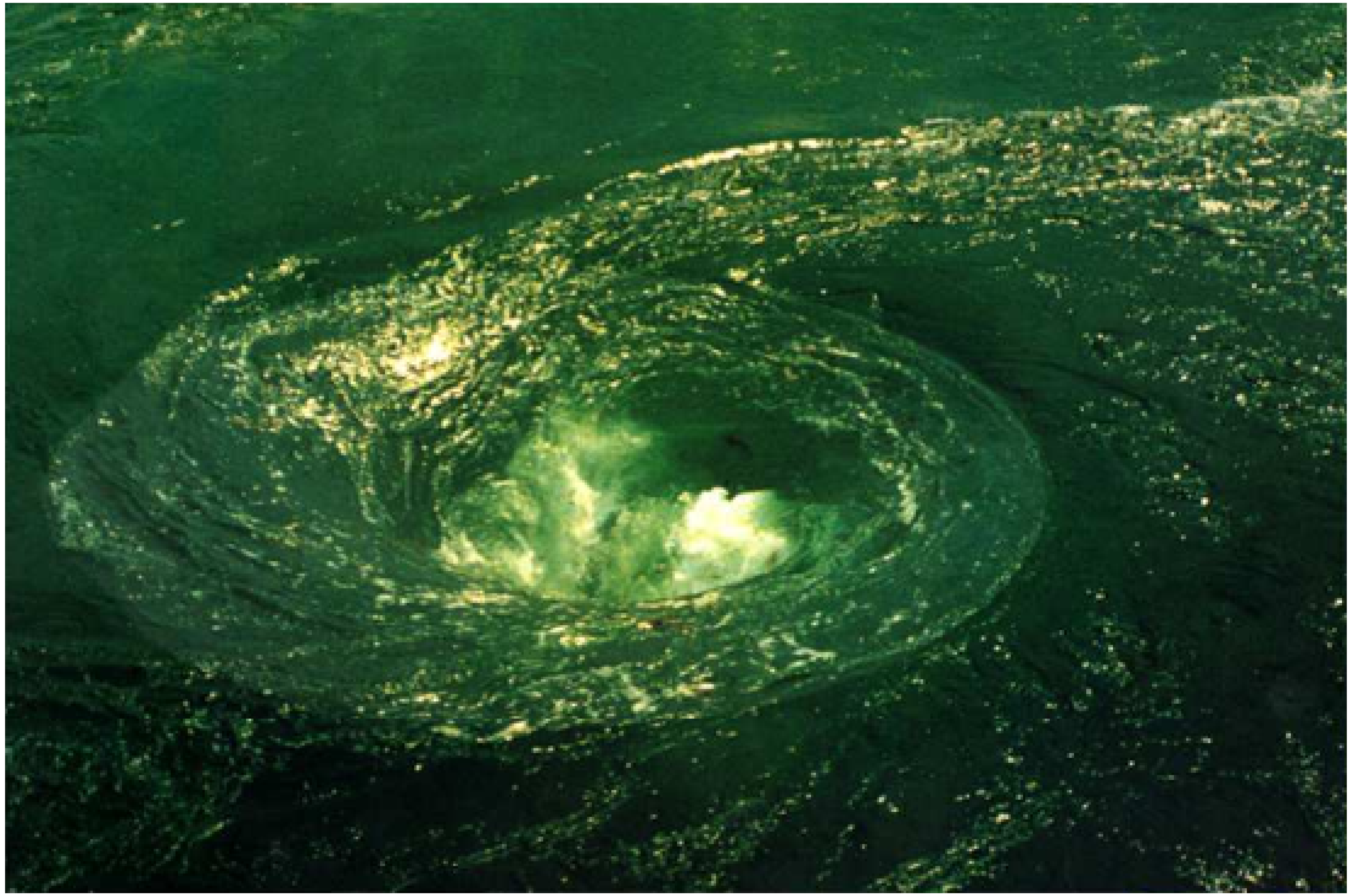


[Post et al. 2003]

What is there to visualize?

- Getting inspiration from nature
  - Global visualization
  - Implicit representations
  - *Line Integral Convolution*

- Analogy to scalar fields:
  - Stripped color maps
  - Volume rendering



[Post et al. 2003]

What is there to visualize?

- Getting inspiration from engineering sciences

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[www.speedhunter.com]

What is there to visualize?

- Getting inspiration from engineering sciences
  - Localized visualization

[www.speedhunter.com]

What is there to visualize?

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  - Localized visualization
  - Explicit representations

[www.speedhunter.com]

What is there to visualize?

- Getting inspiration from engineering sciences
  - Localized visualization
  - Explicit representations
  - *Stream lines and surfaces*

[www.speedhunter.com]

What is there to visualize?

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  - *Stream lines and surfaces*

[www.speedhunter.com]

- Analogy to scalar fields:
  - Isocontours
  - Isosurfaces

What is there to visualize?

- Understanding the structure of the data

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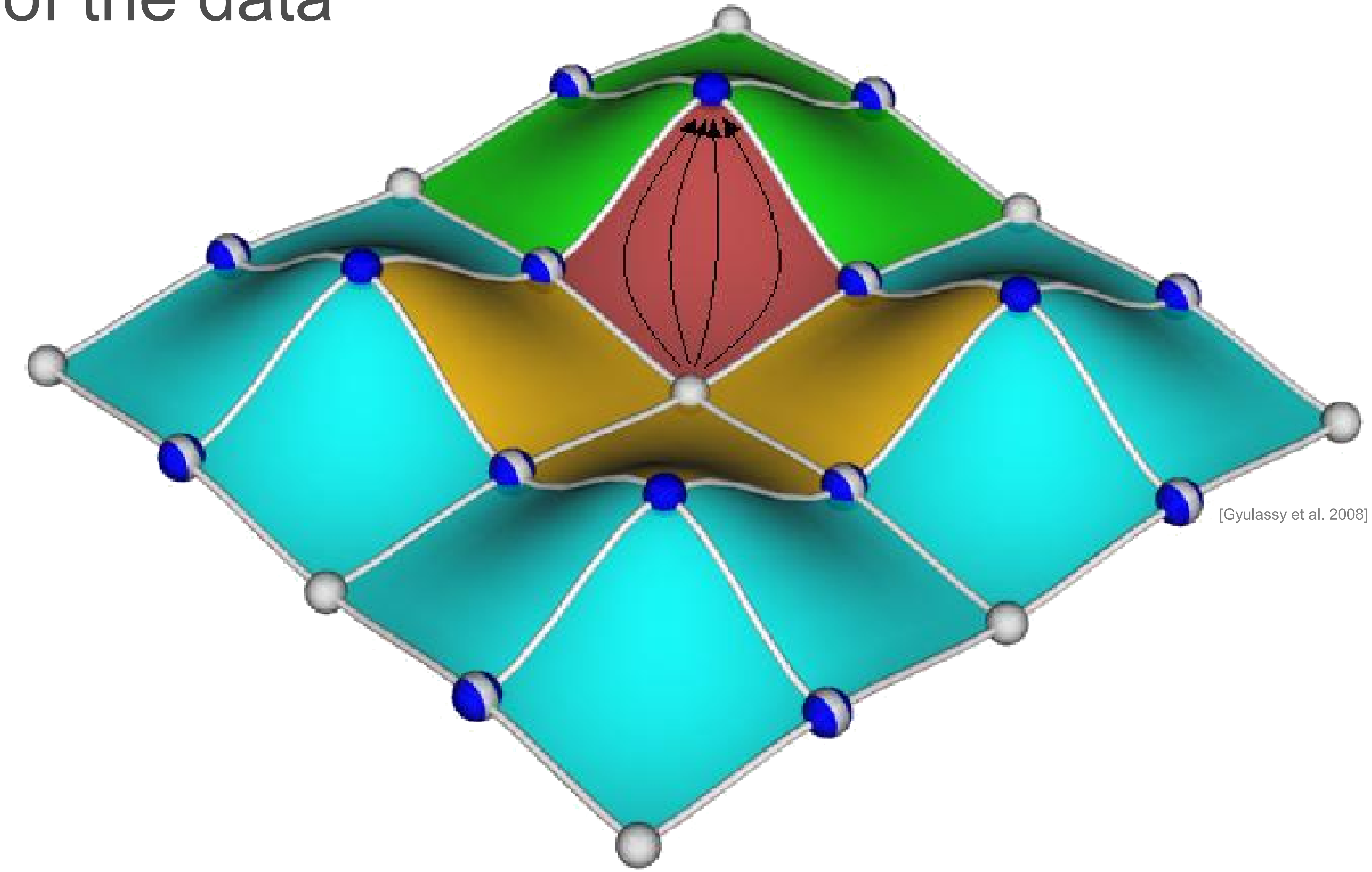
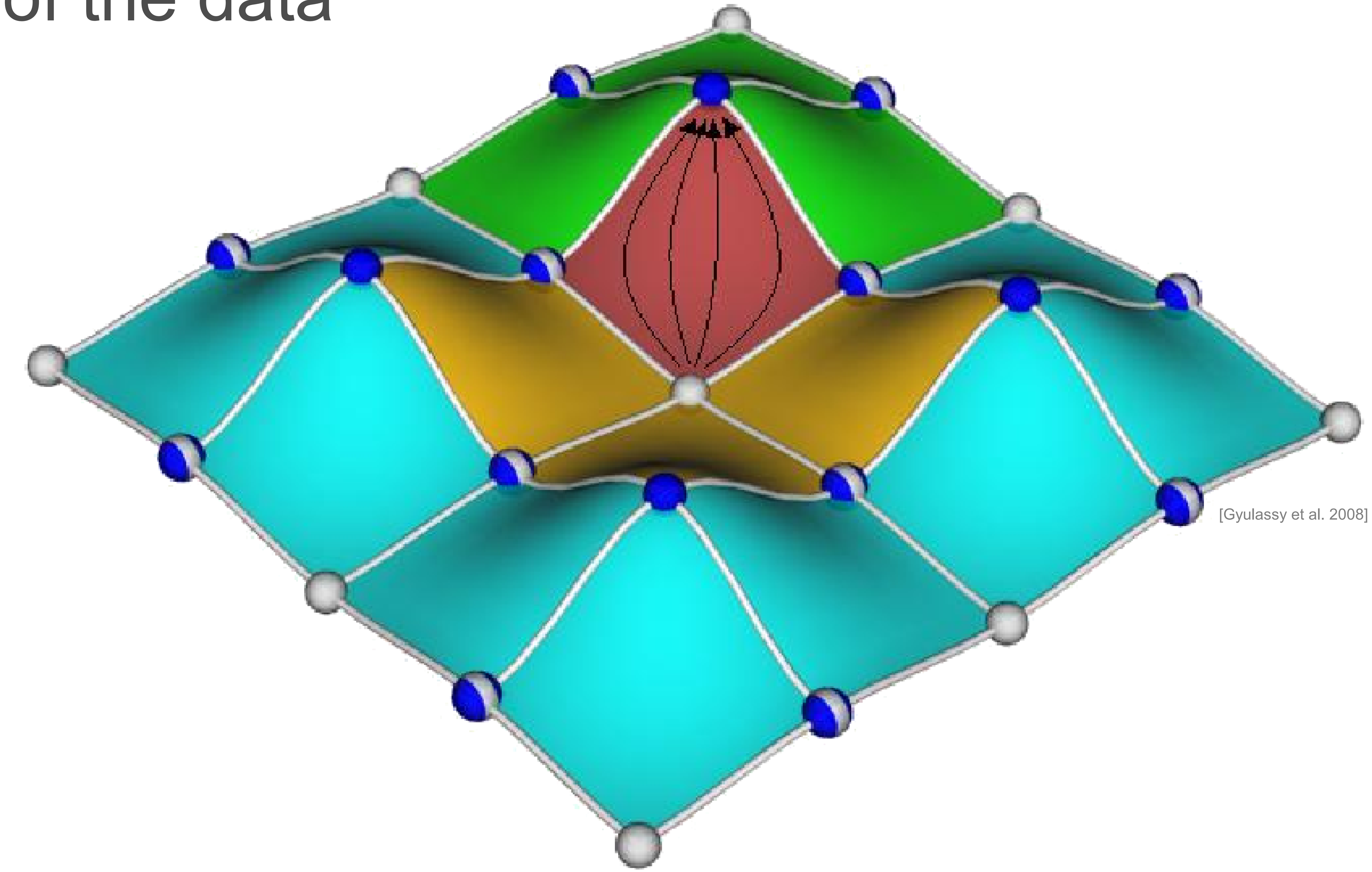
- Understanding the structure of the data



[Gyulassy et al. 2008]

What is there to visualize?

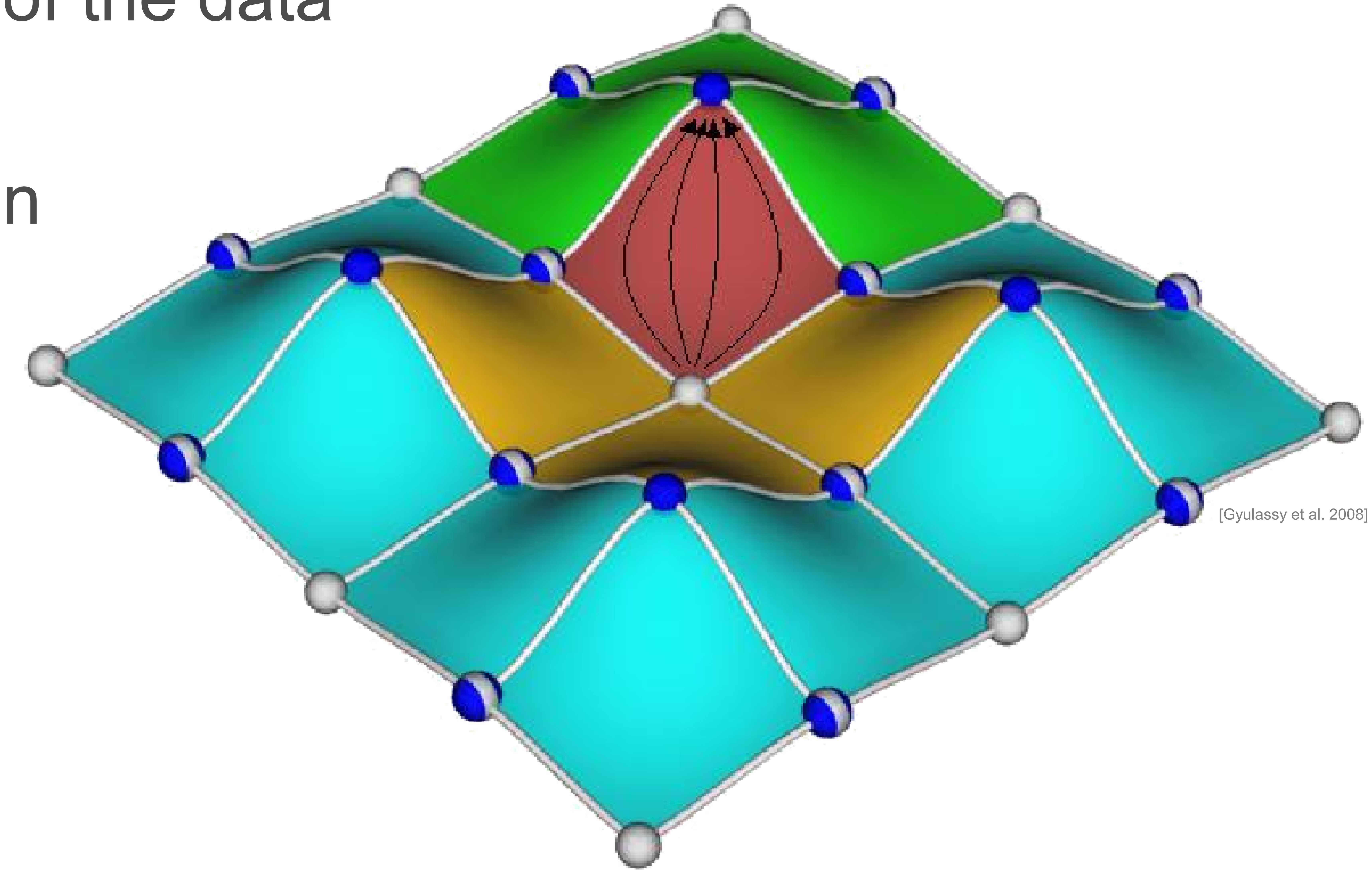
- Understanding the structure of the data
  - Vector field topology



[Gyulassy et al. 2008]

What is there to visualize?

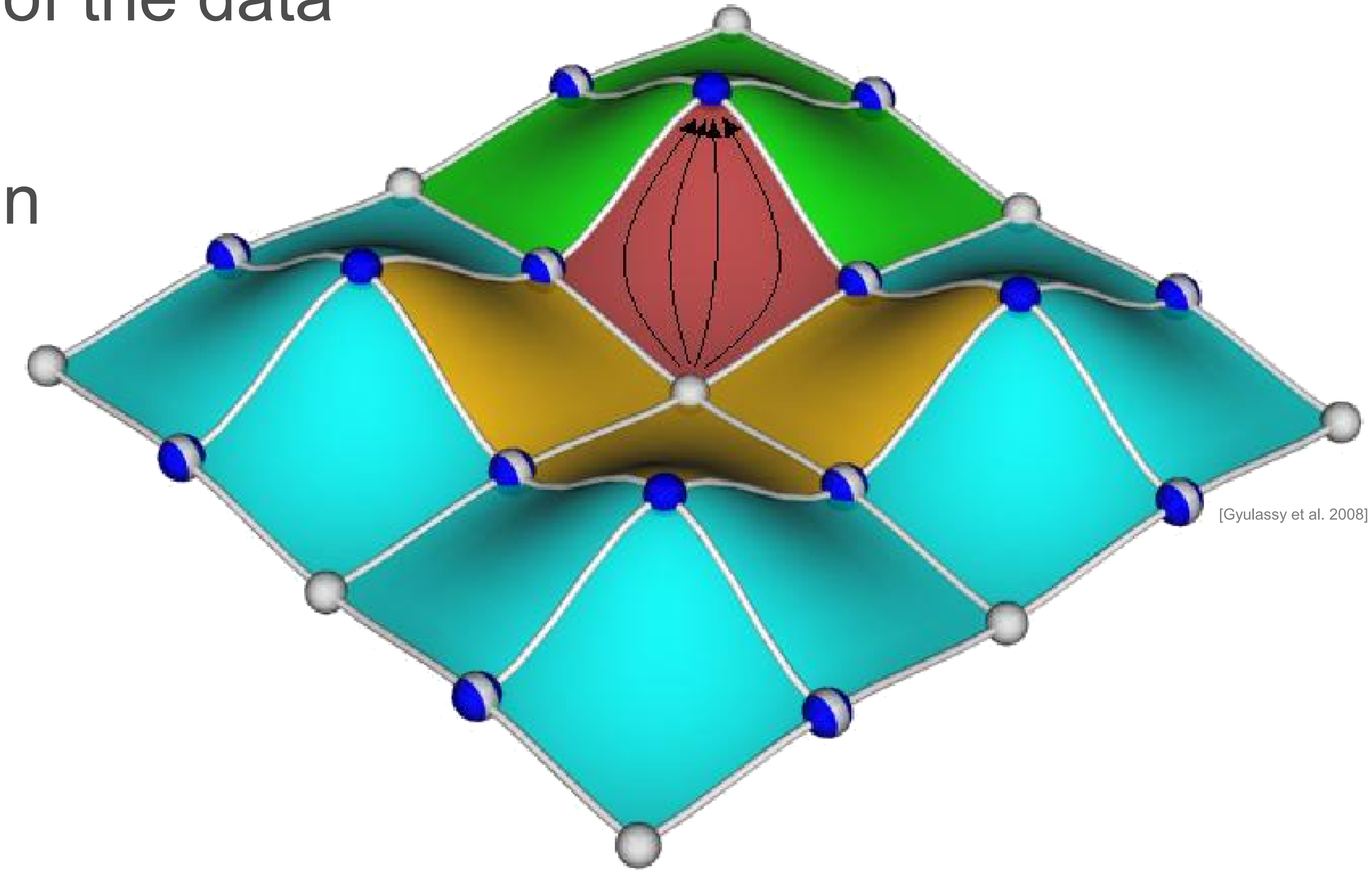
- Understanding the structure of the data
  - Vector field topology
  - Segmentation of the domain



[Gyulassy et al. 2008]

What is there to visualize?

- Understanding the structure of the data
  - Vector field topology
  - Segmentation of the domain
    - Homogeneous behavior

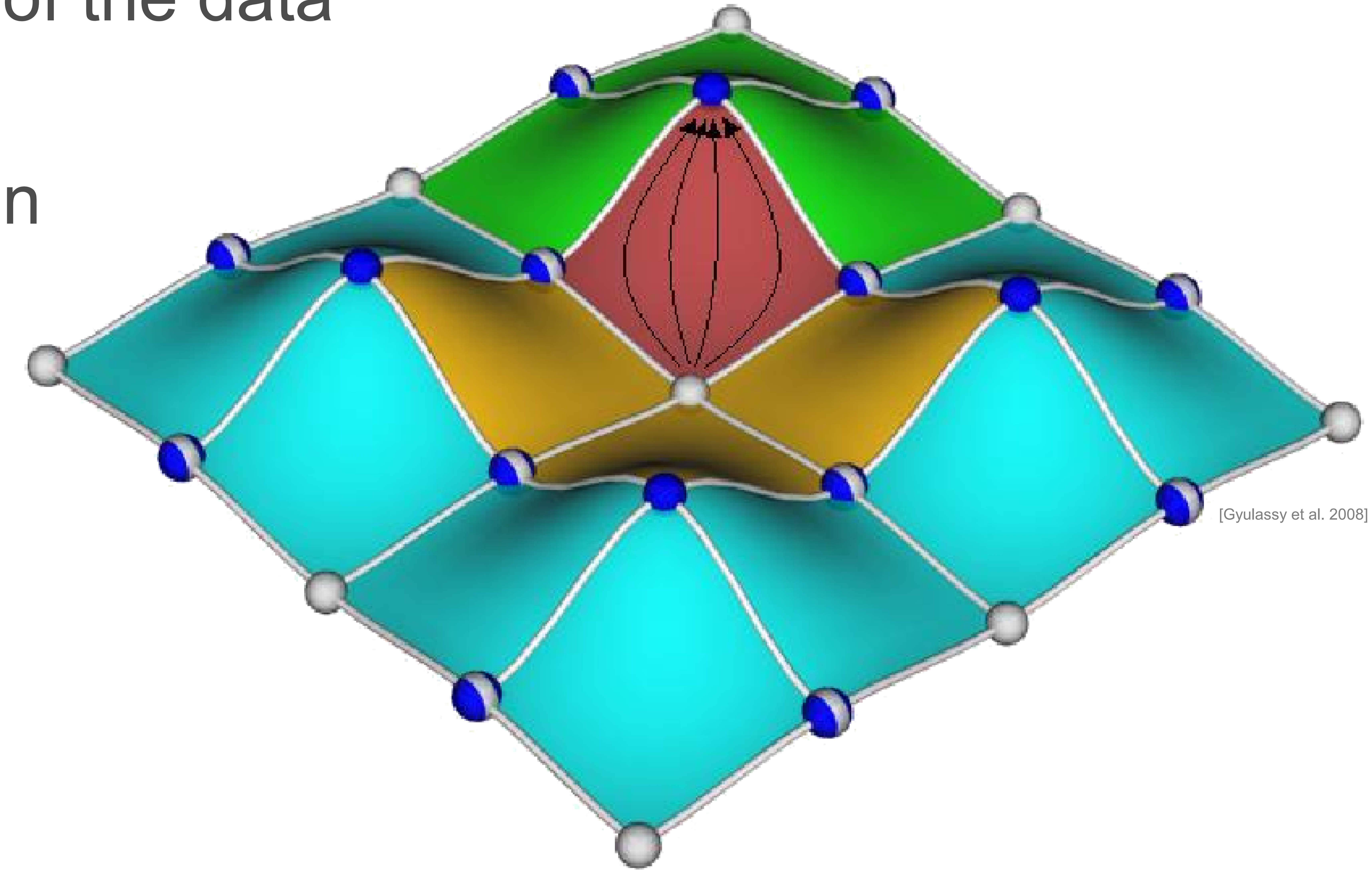


[Gyulassy et al. 2008]

What is there to visualize?

- Understanding the structure of the data
  - Vector field topology
  - Segmentation of the domain
    - Homogeneous behavior

- Analogy to scalar fields
  - Critical points
  - Reeb graphs



[Gyulassy et al. 2008]

Summary

Summary

- Steady vector fields

Summary

- Steady vector fields
  - Integral curves

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    - Integration techniques, seeding

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  - Integral surfaces
  - Line integral convolution
  - Derived scalar fields
  - Vector field topology
    - Gradient fields
    - Arbitrary fields

Steady vector fields

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- Point to point mapping to the tangent space of $\mathcal{D}$

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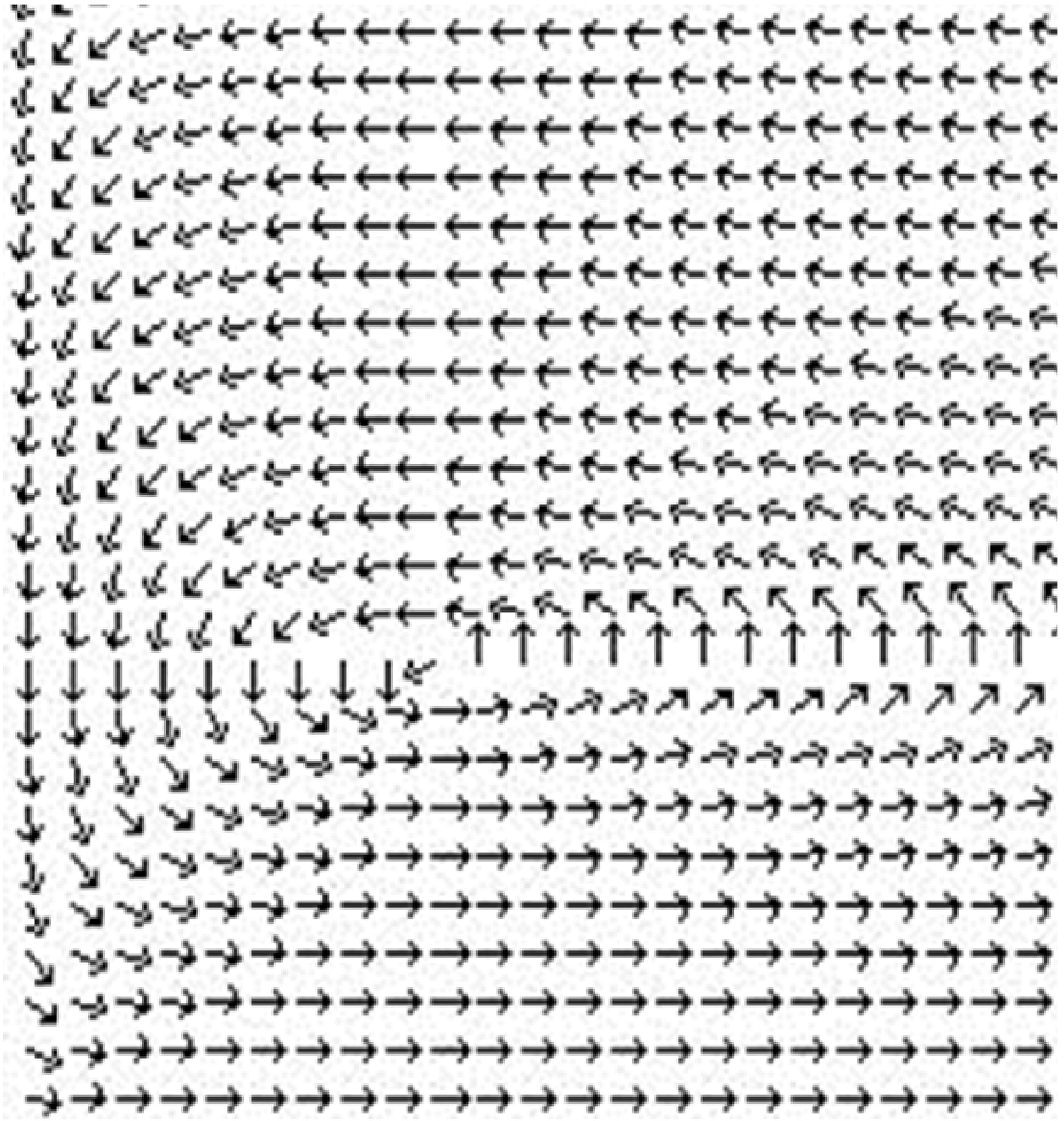

[Chen]

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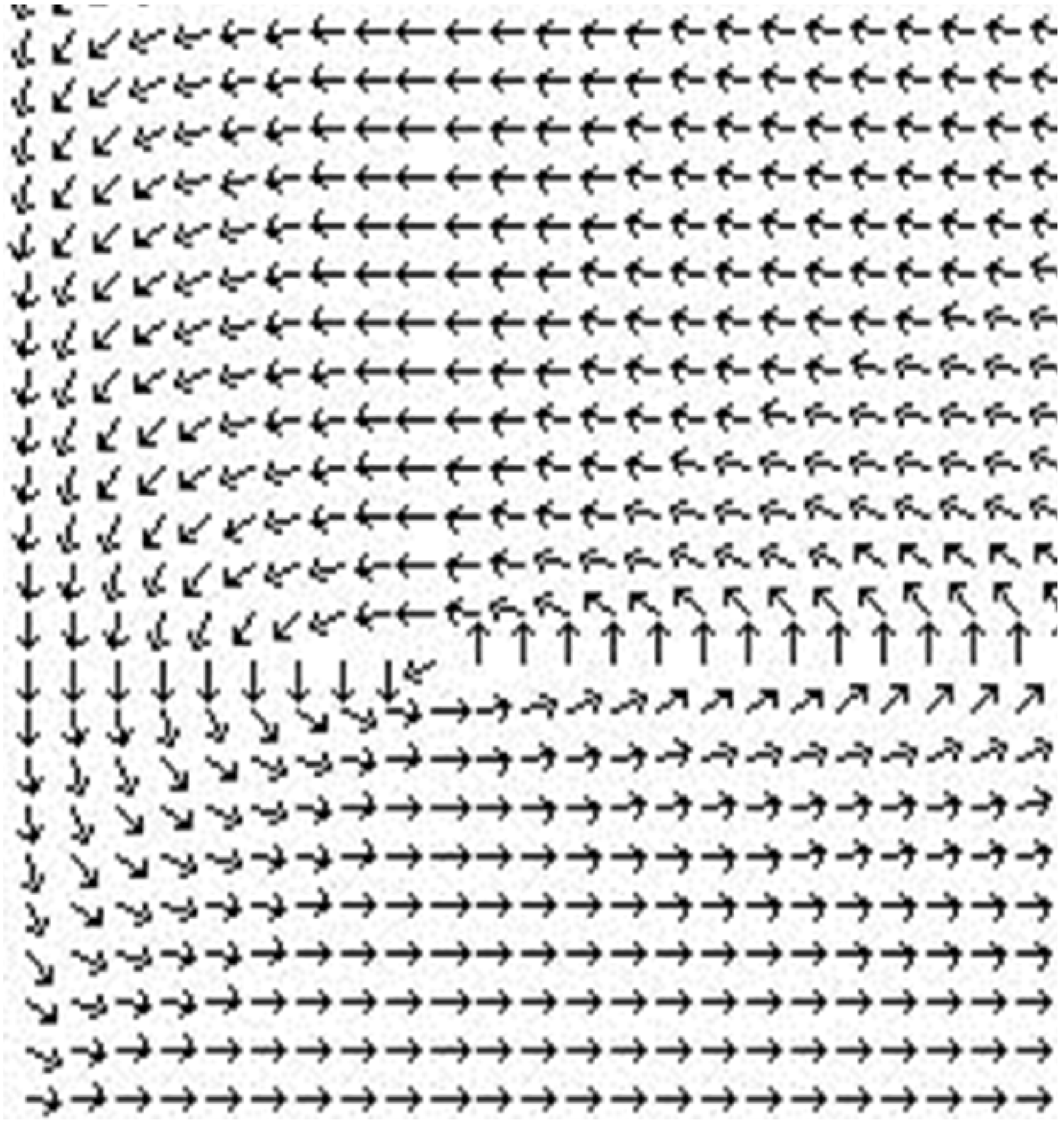

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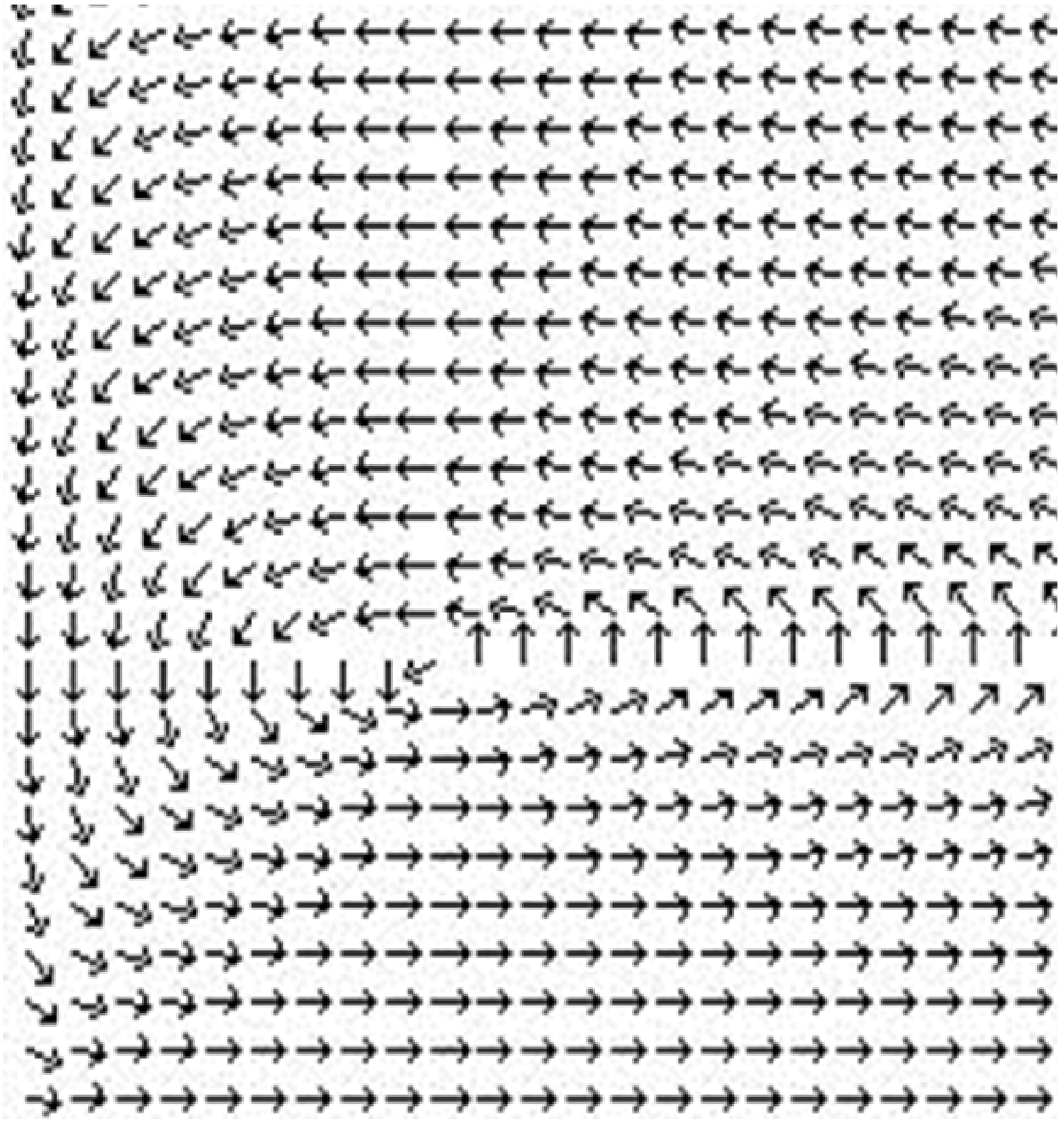

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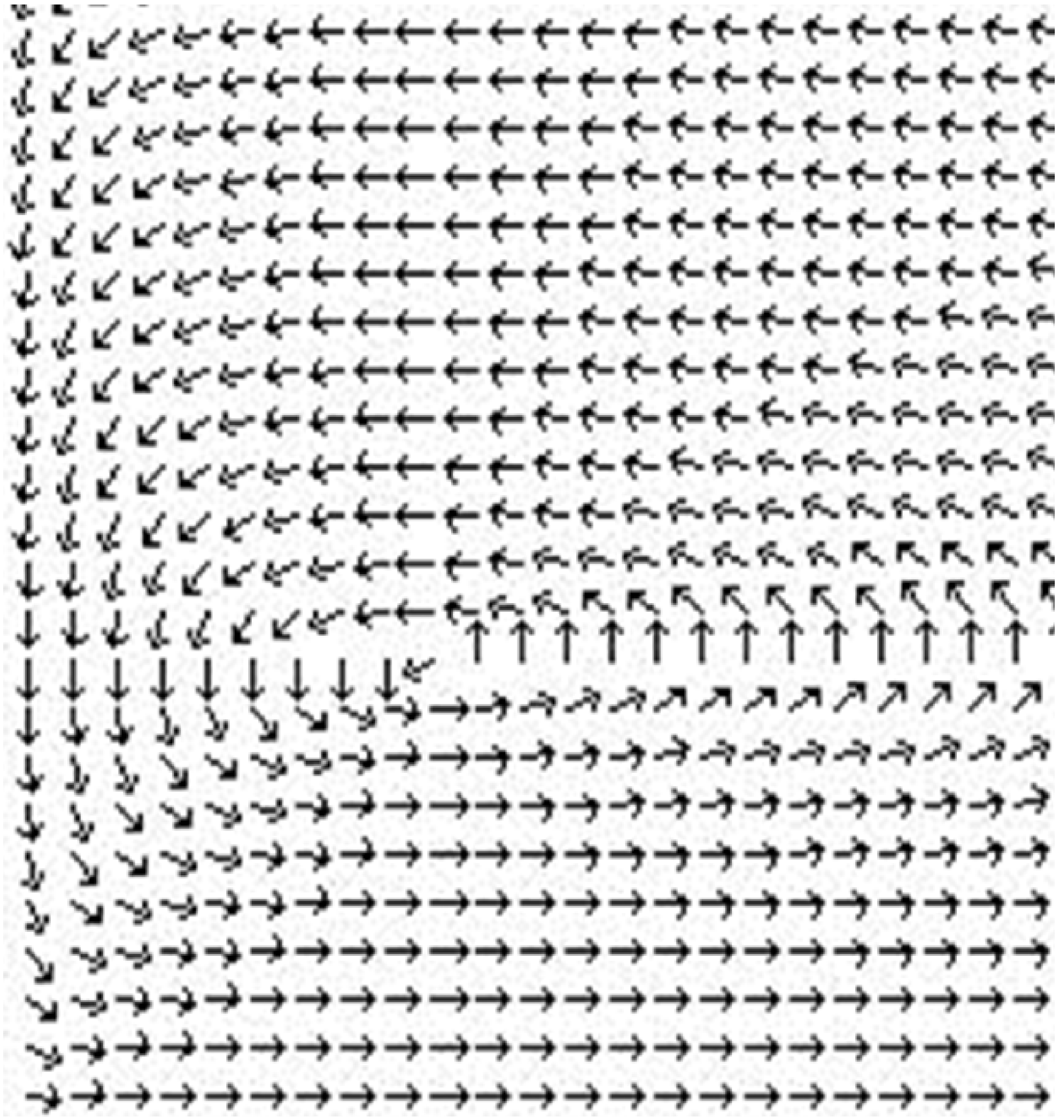

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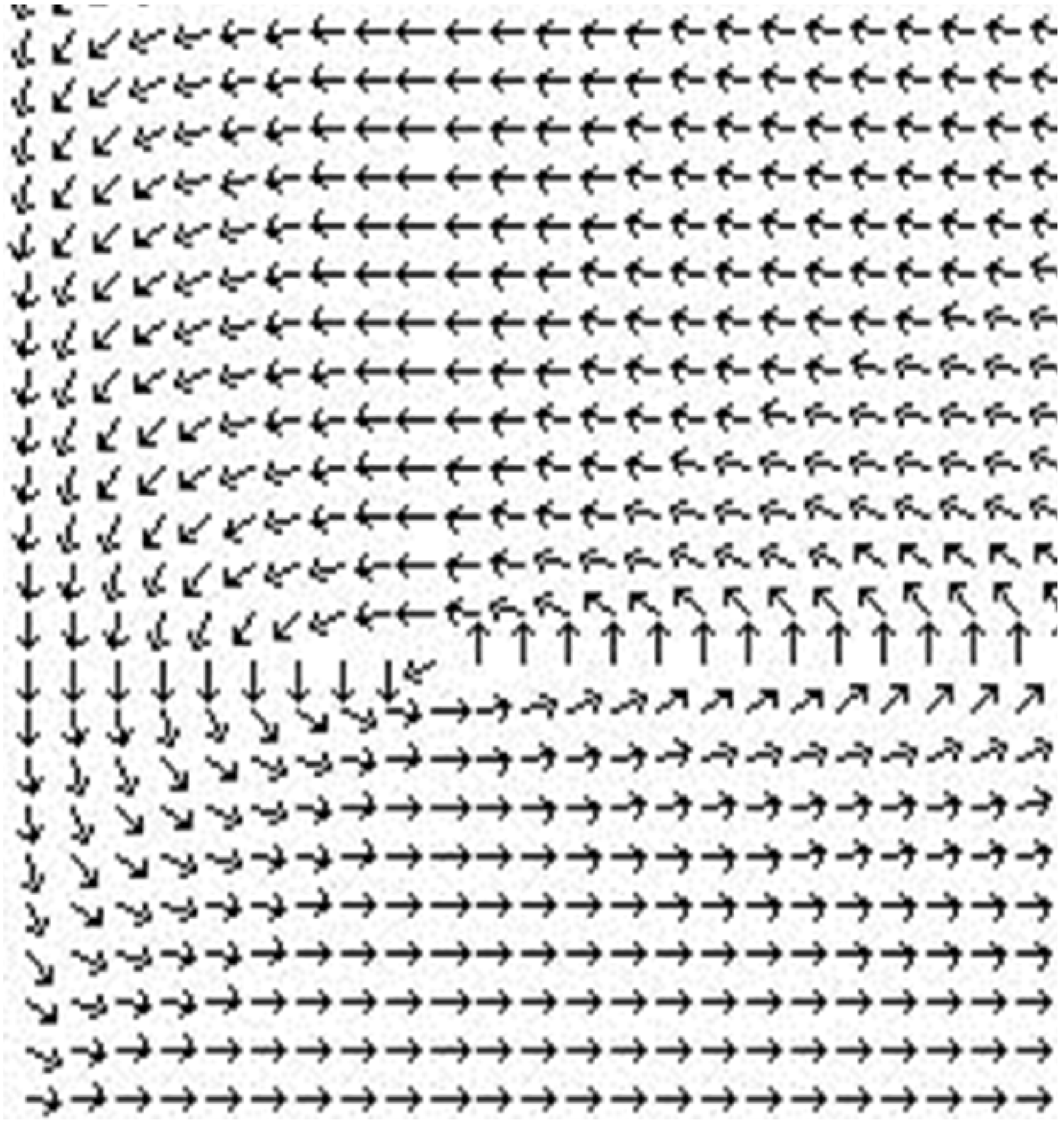
  - Gradient field, example
    - $g : \mathbb{R}^n \rightarrow \mathbb{R}$



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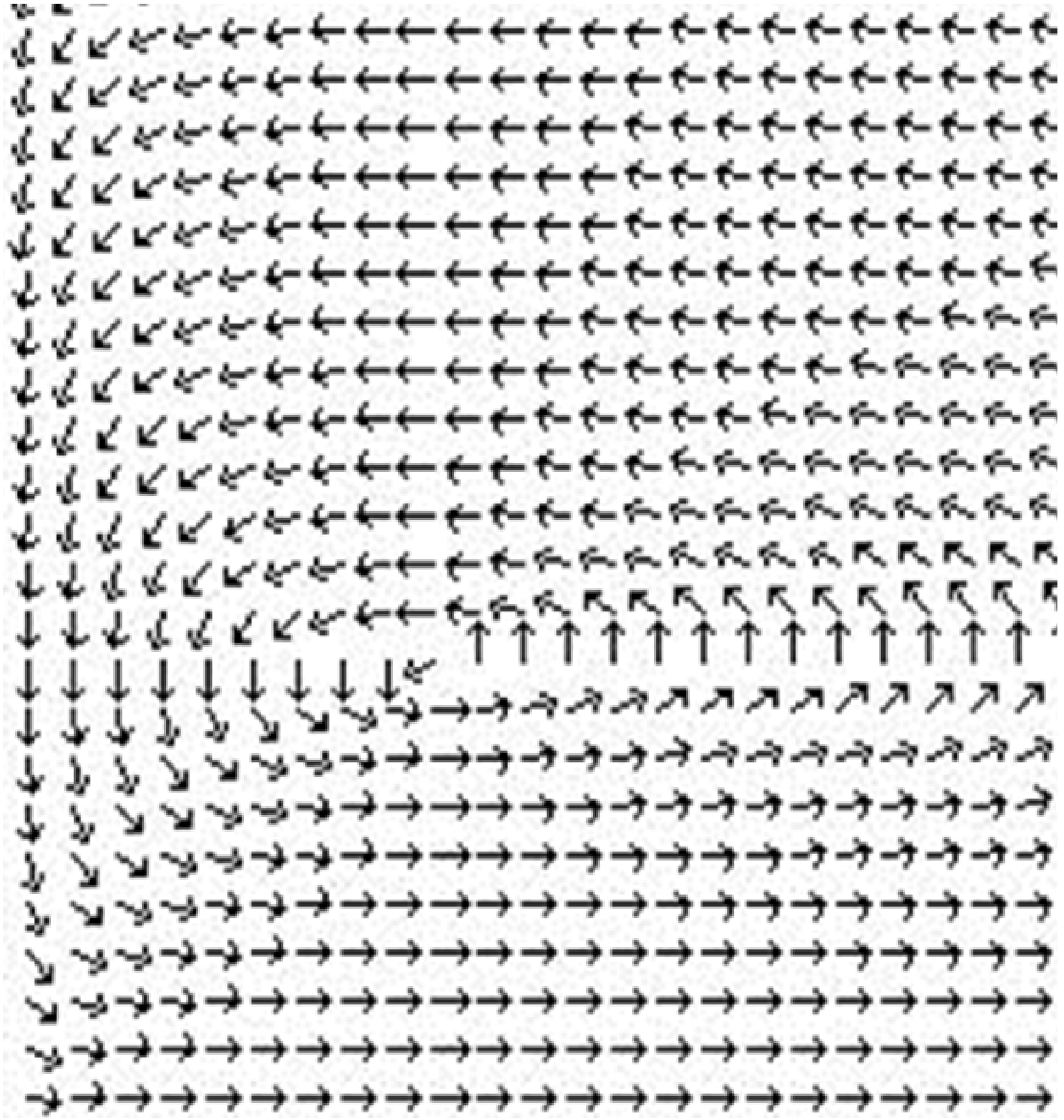

[Chen]

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- What is there to visualize?

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[www.speedhunter.com]

Steady vector fields

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[www.speedhunter.com]

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[www.speedhunter.com]

Streamlines

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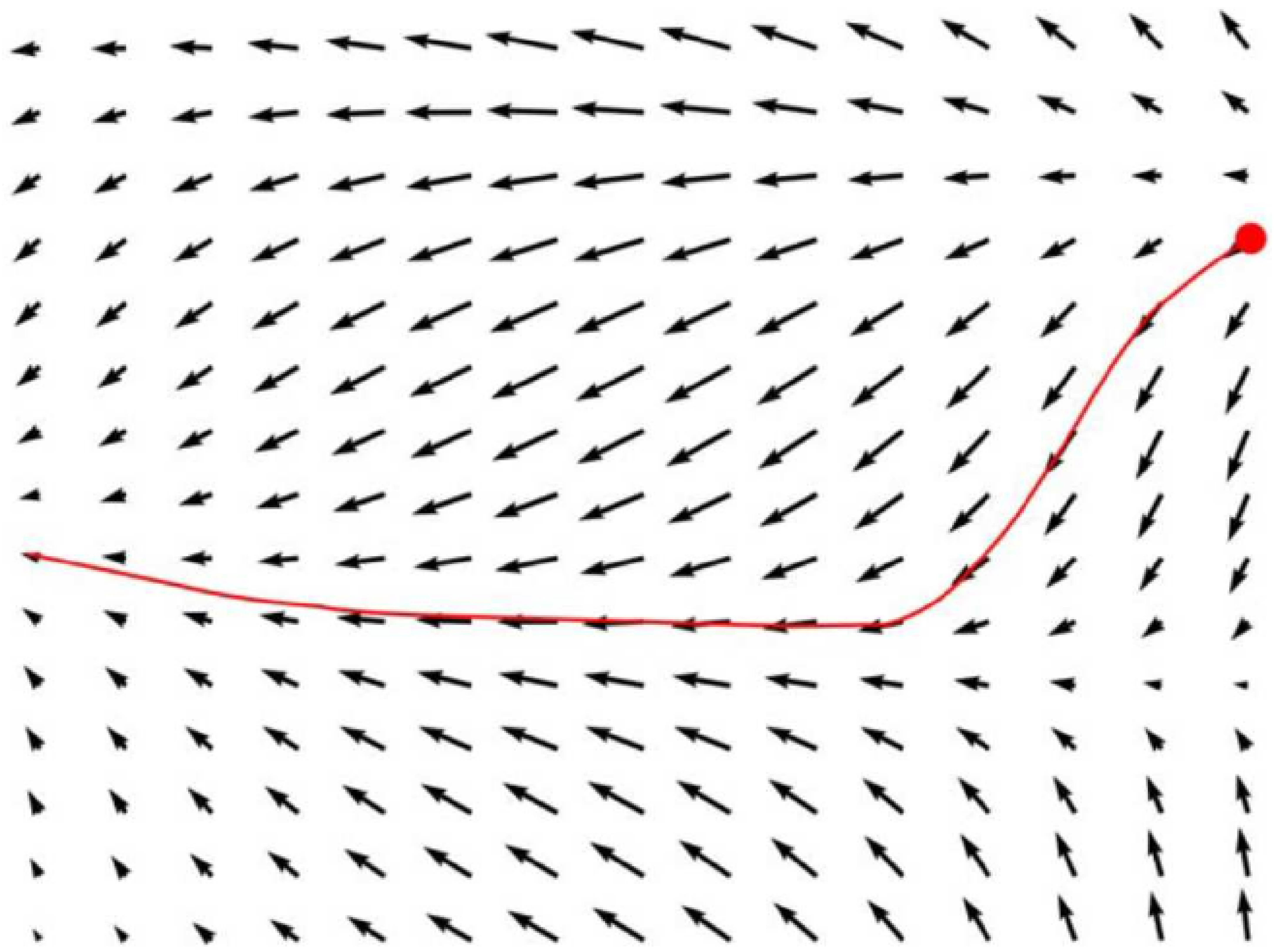

[Chen]

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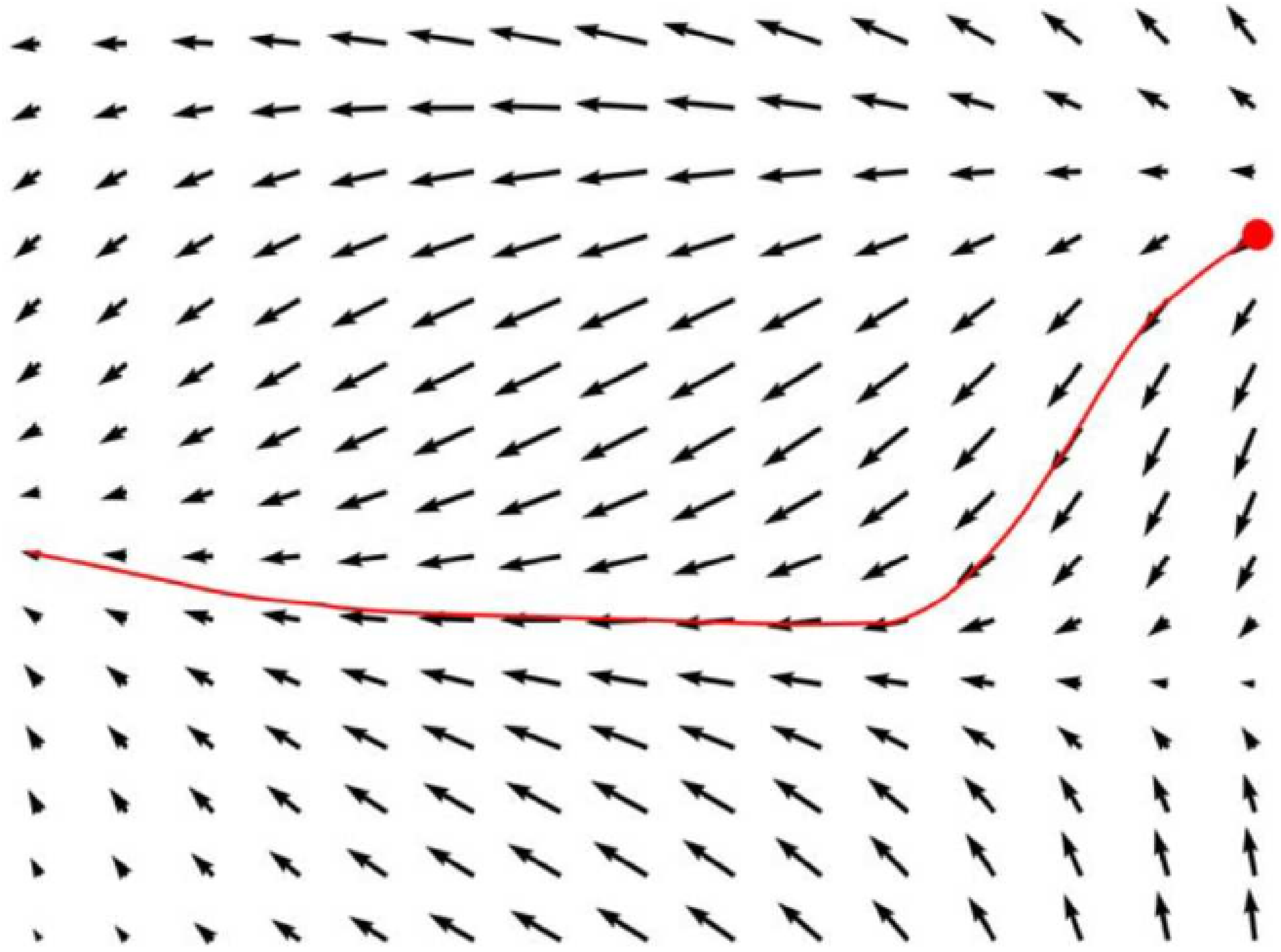

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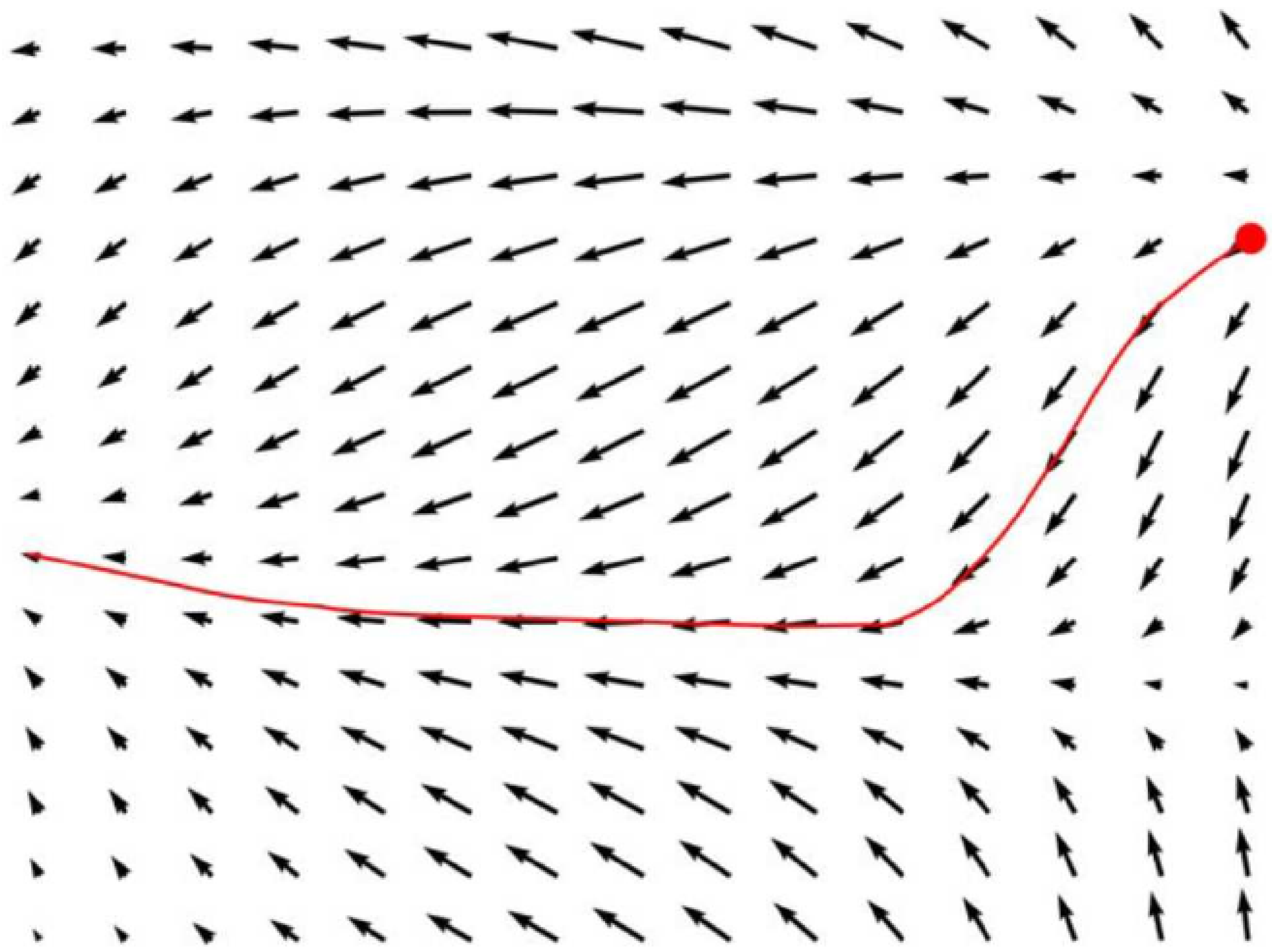

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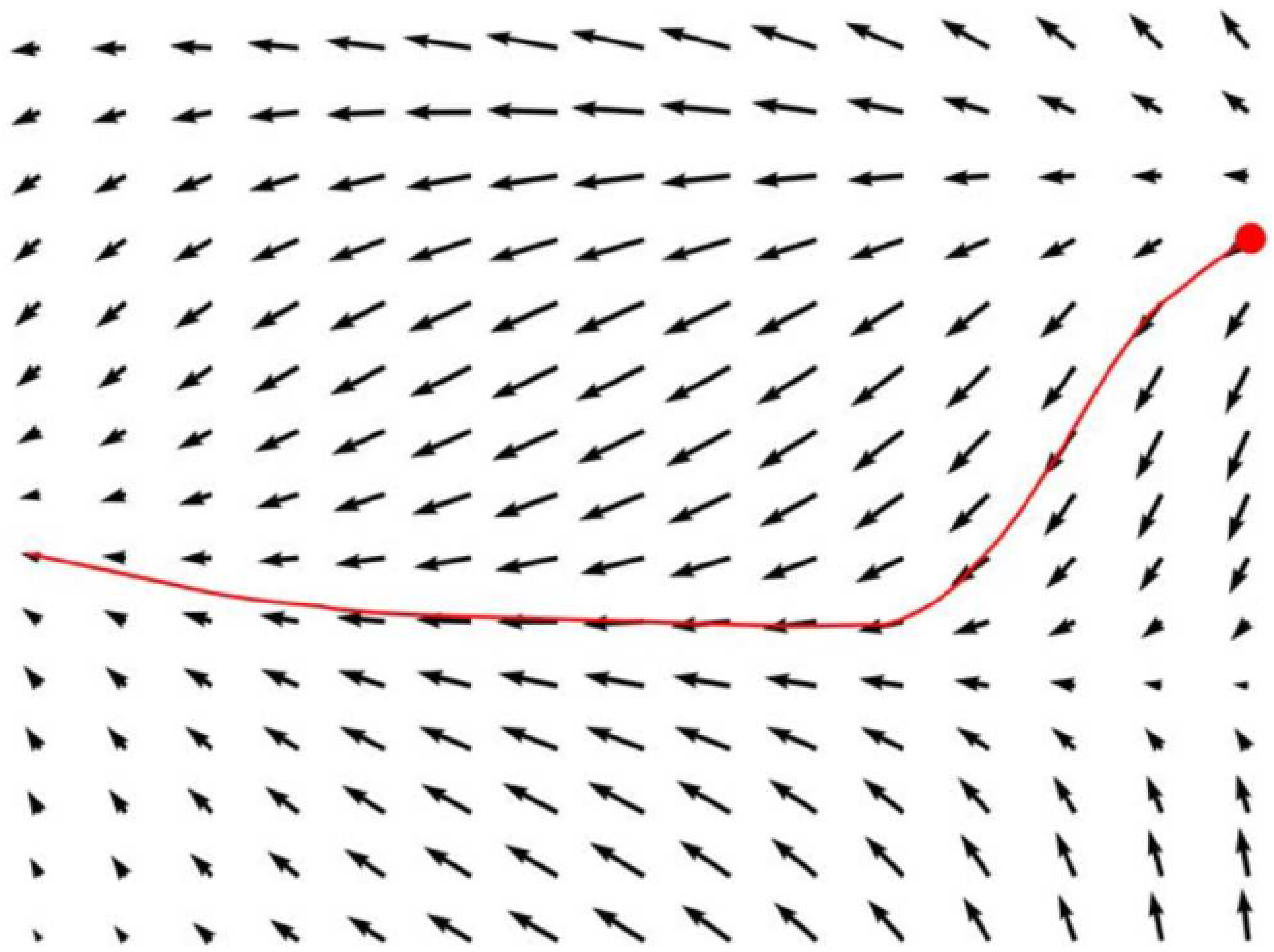

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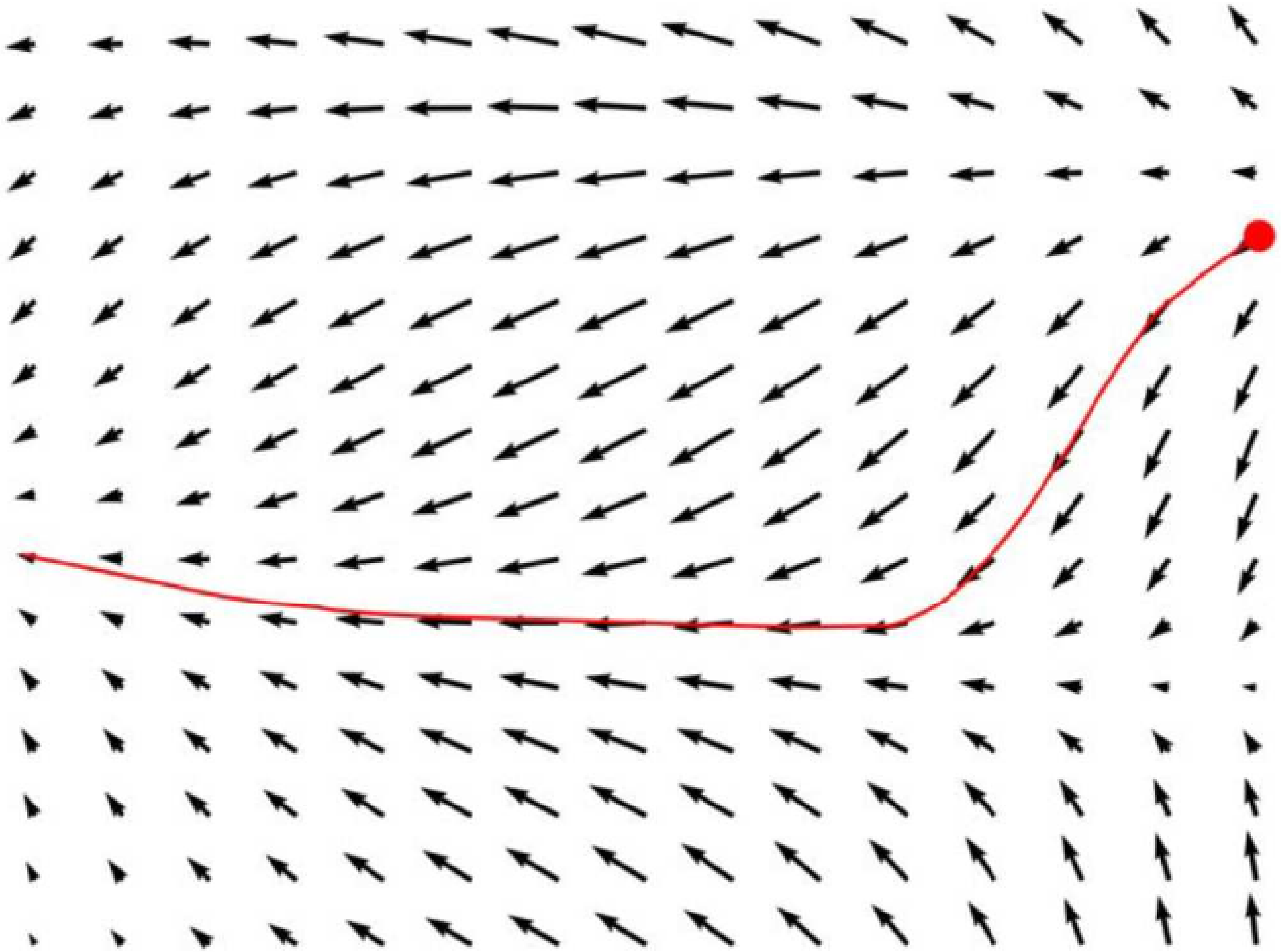

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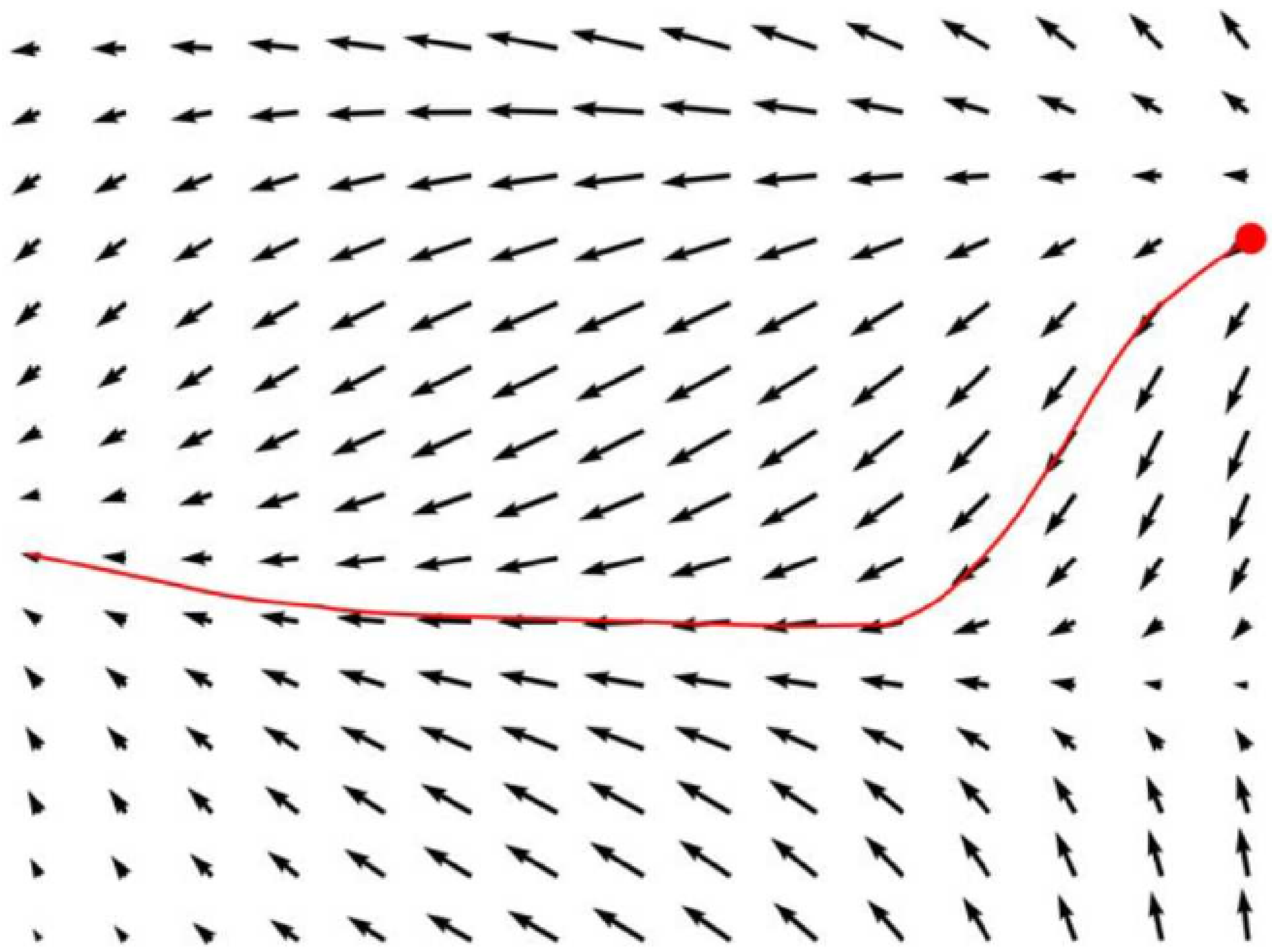

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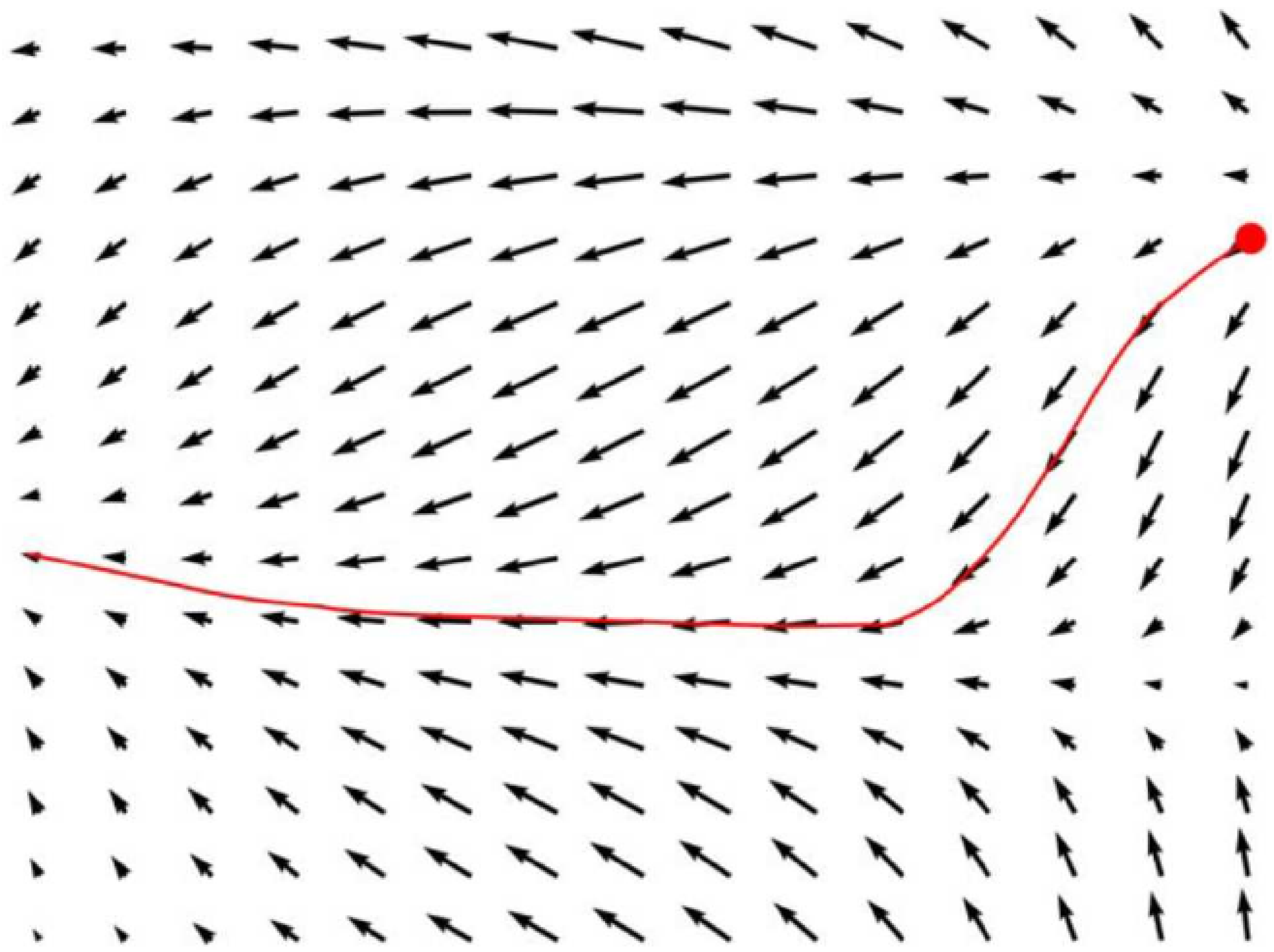

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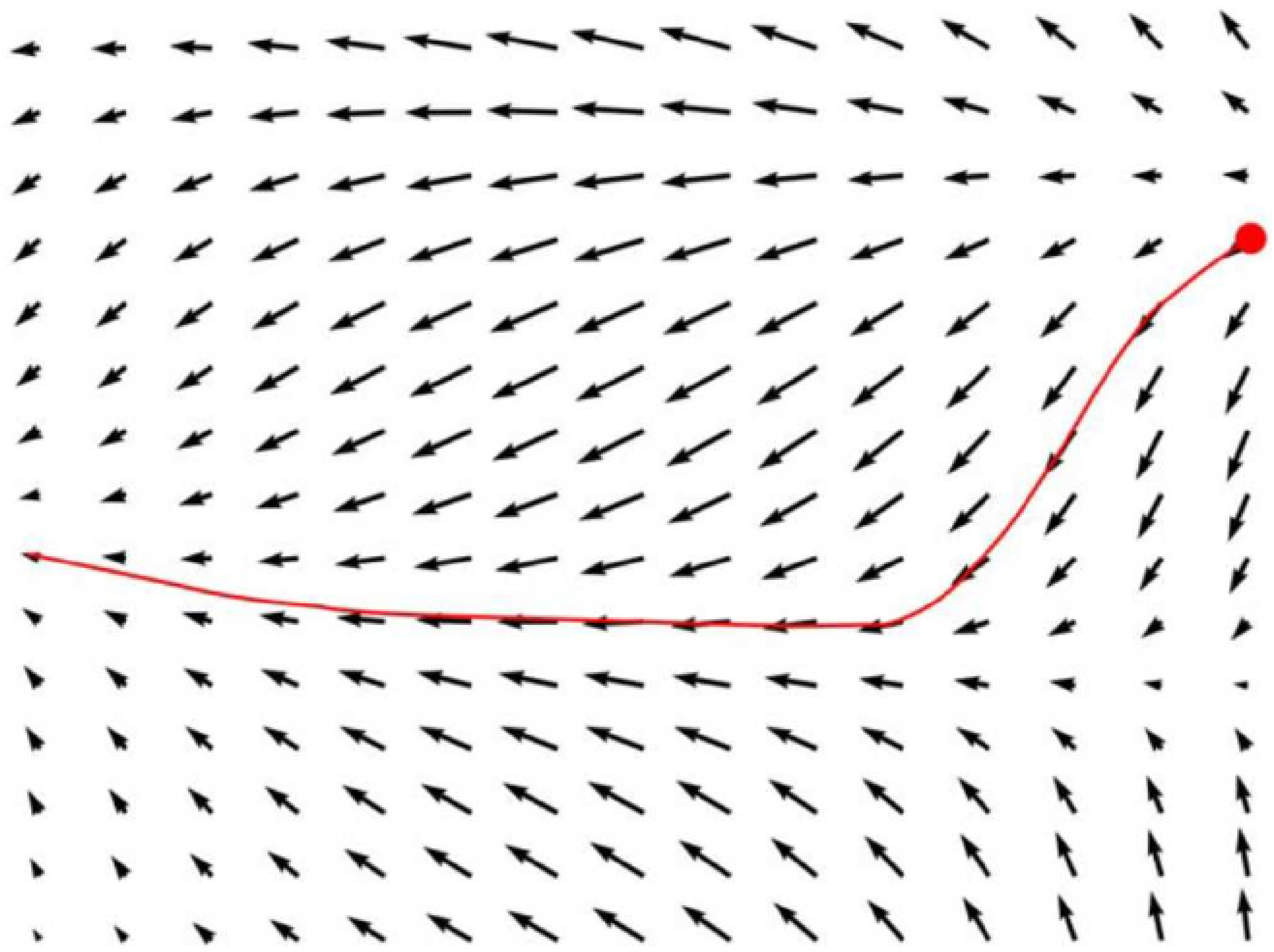

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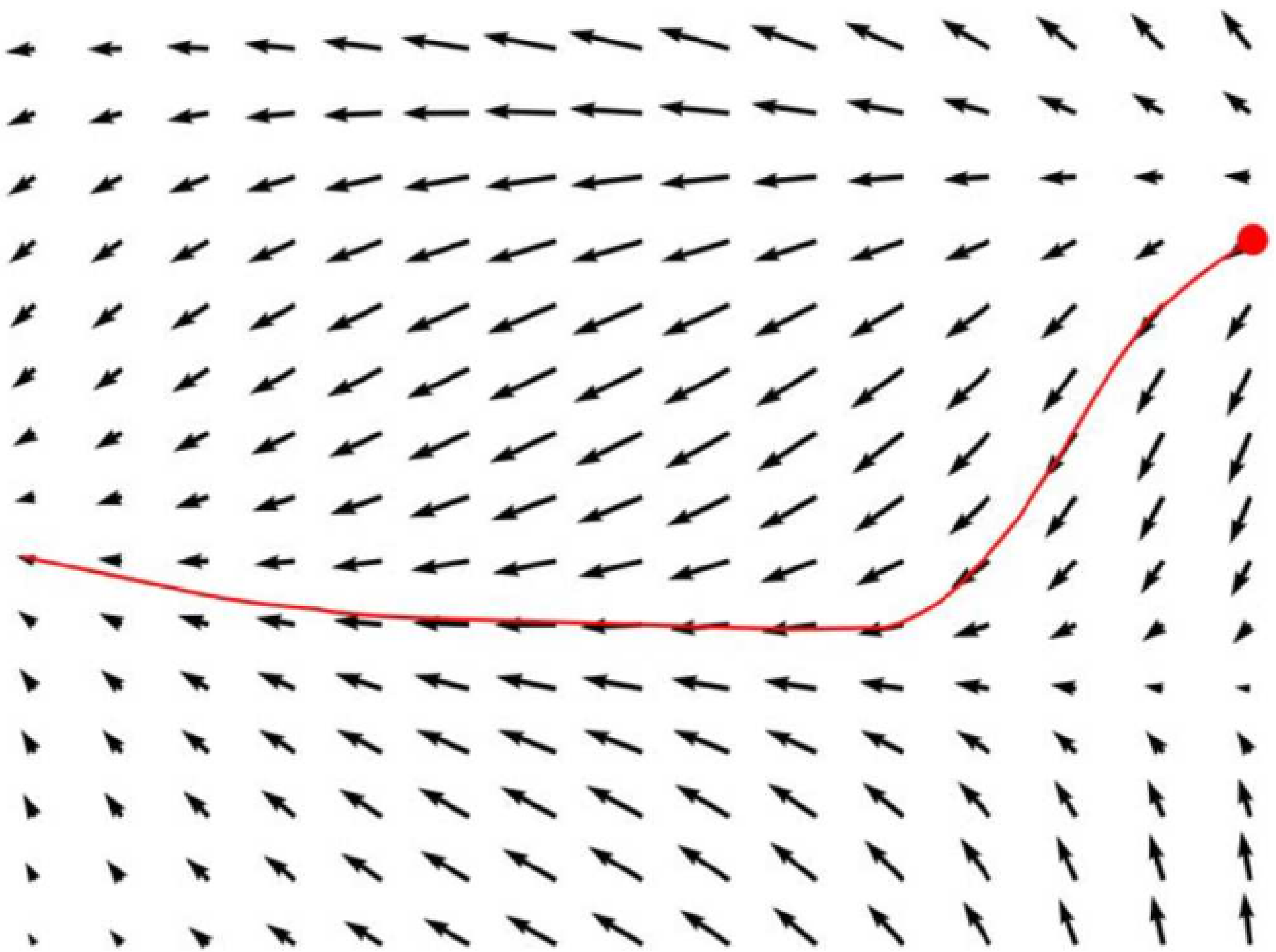

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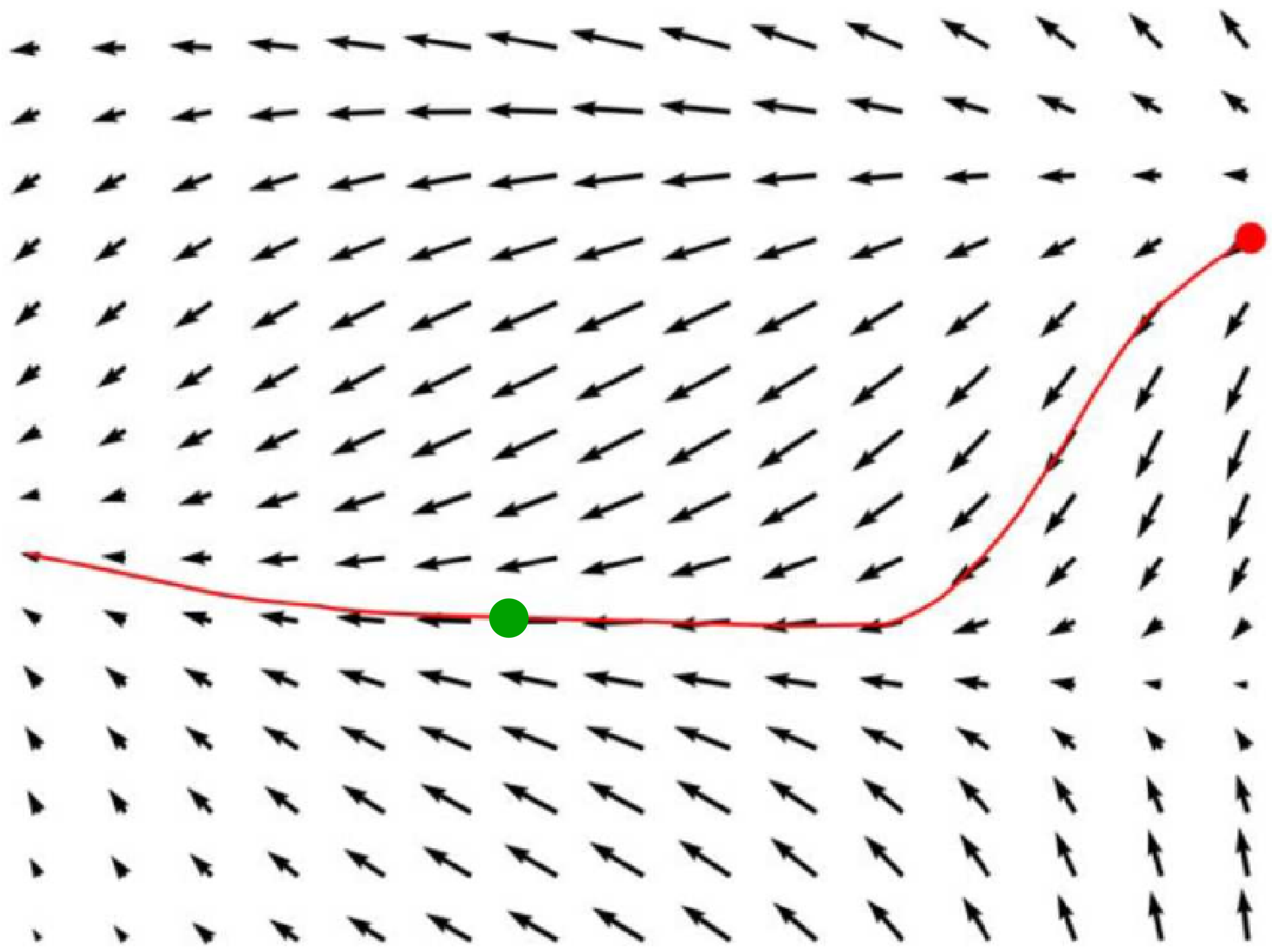

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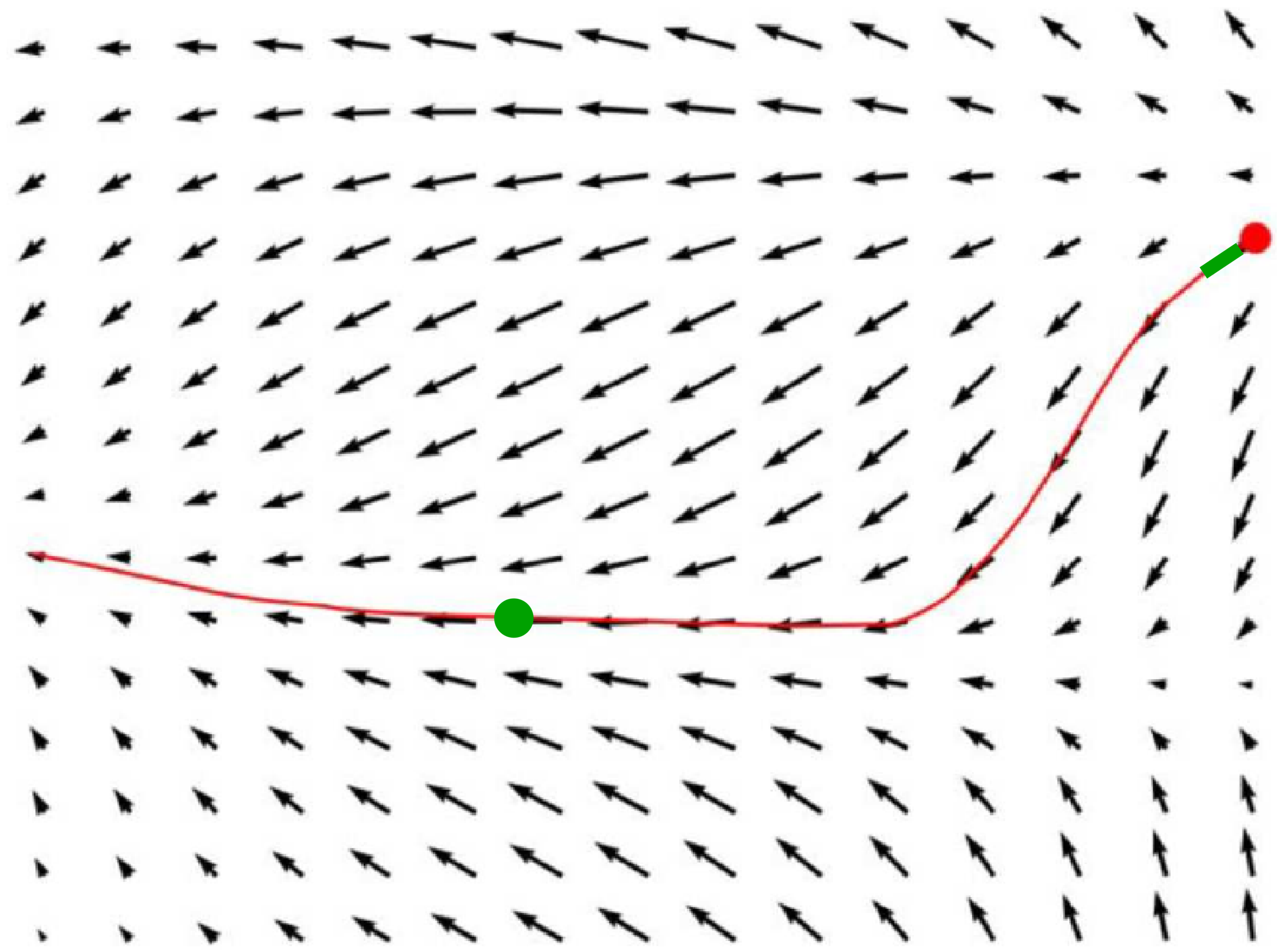

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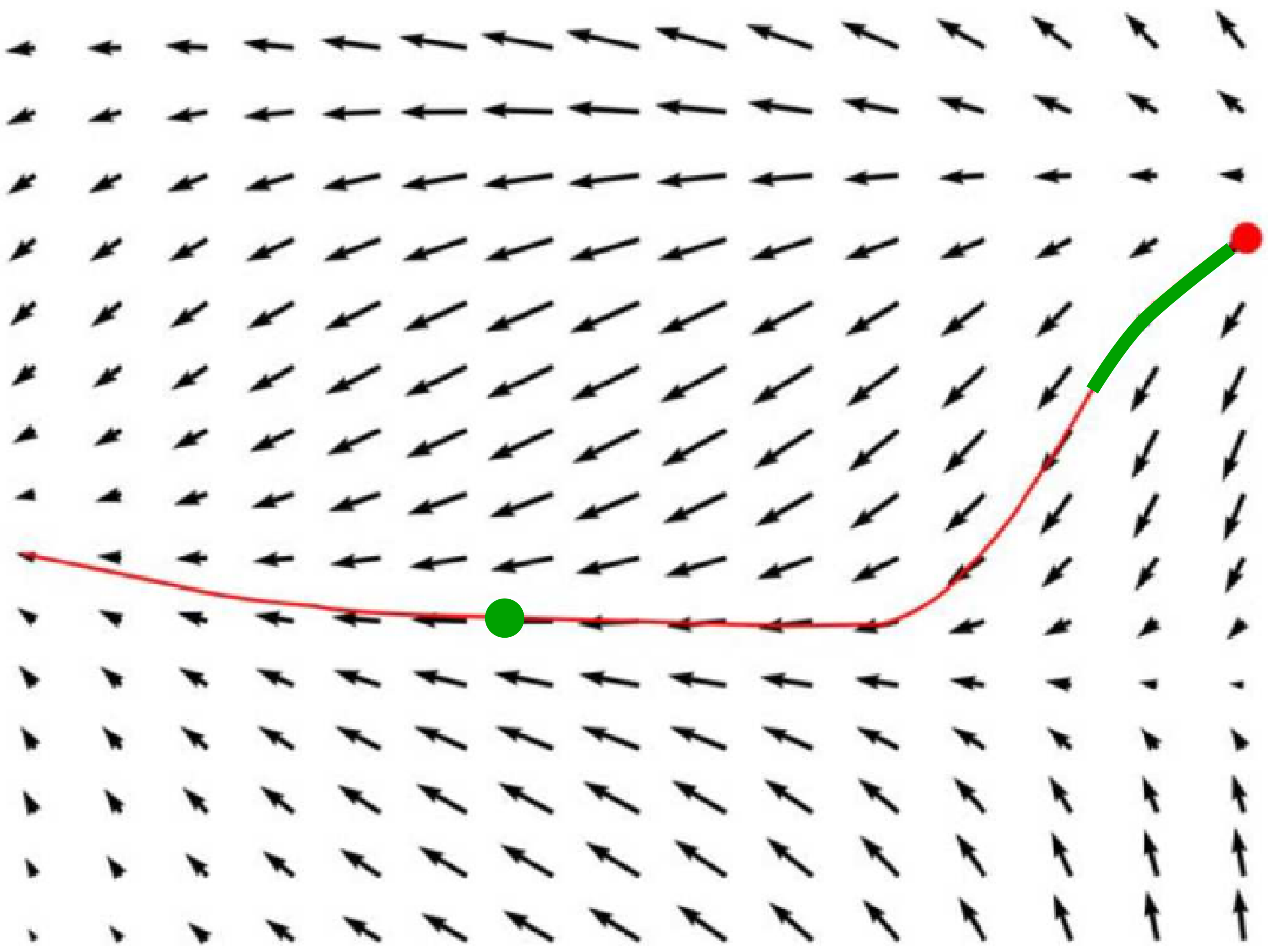

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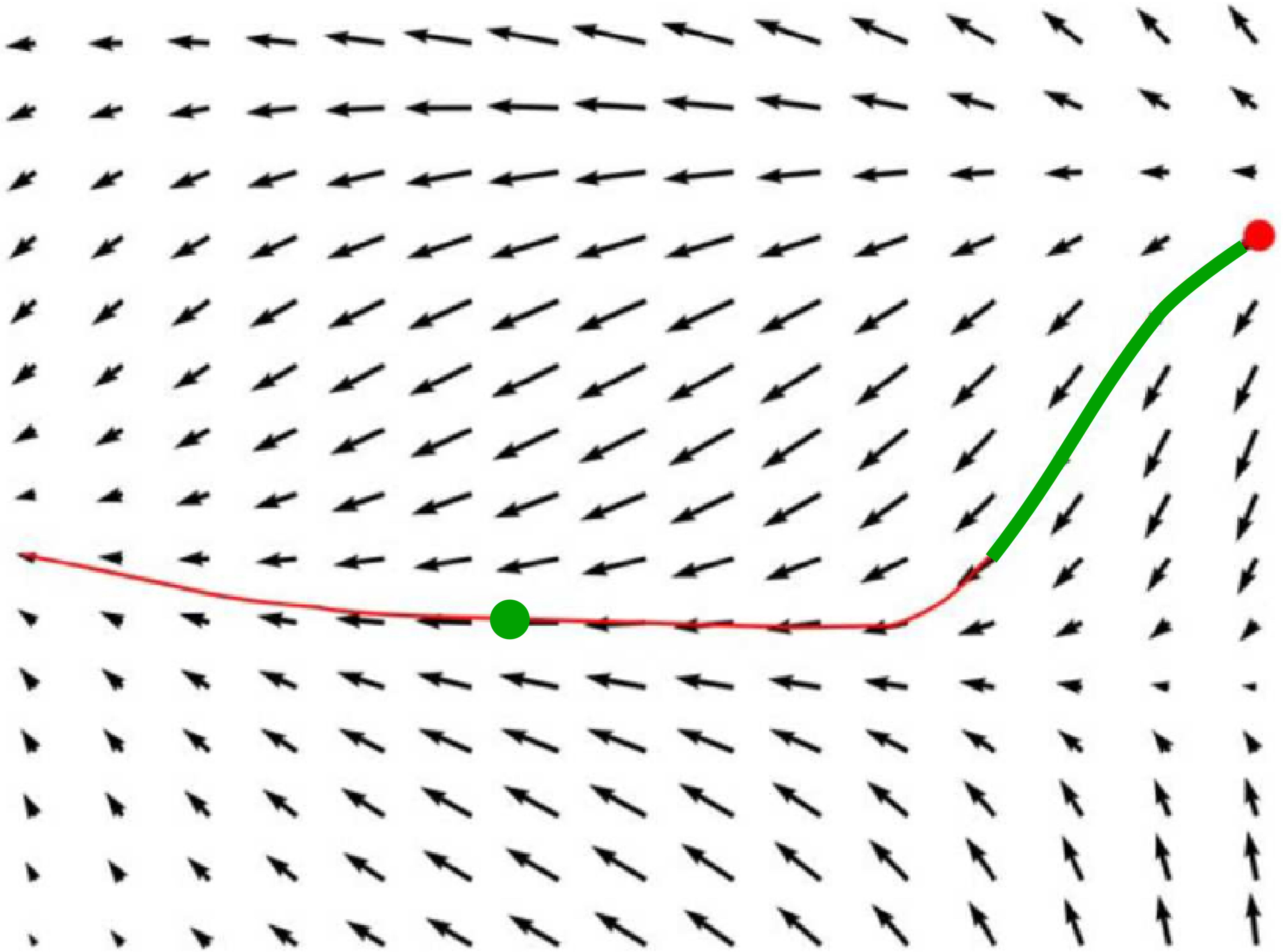

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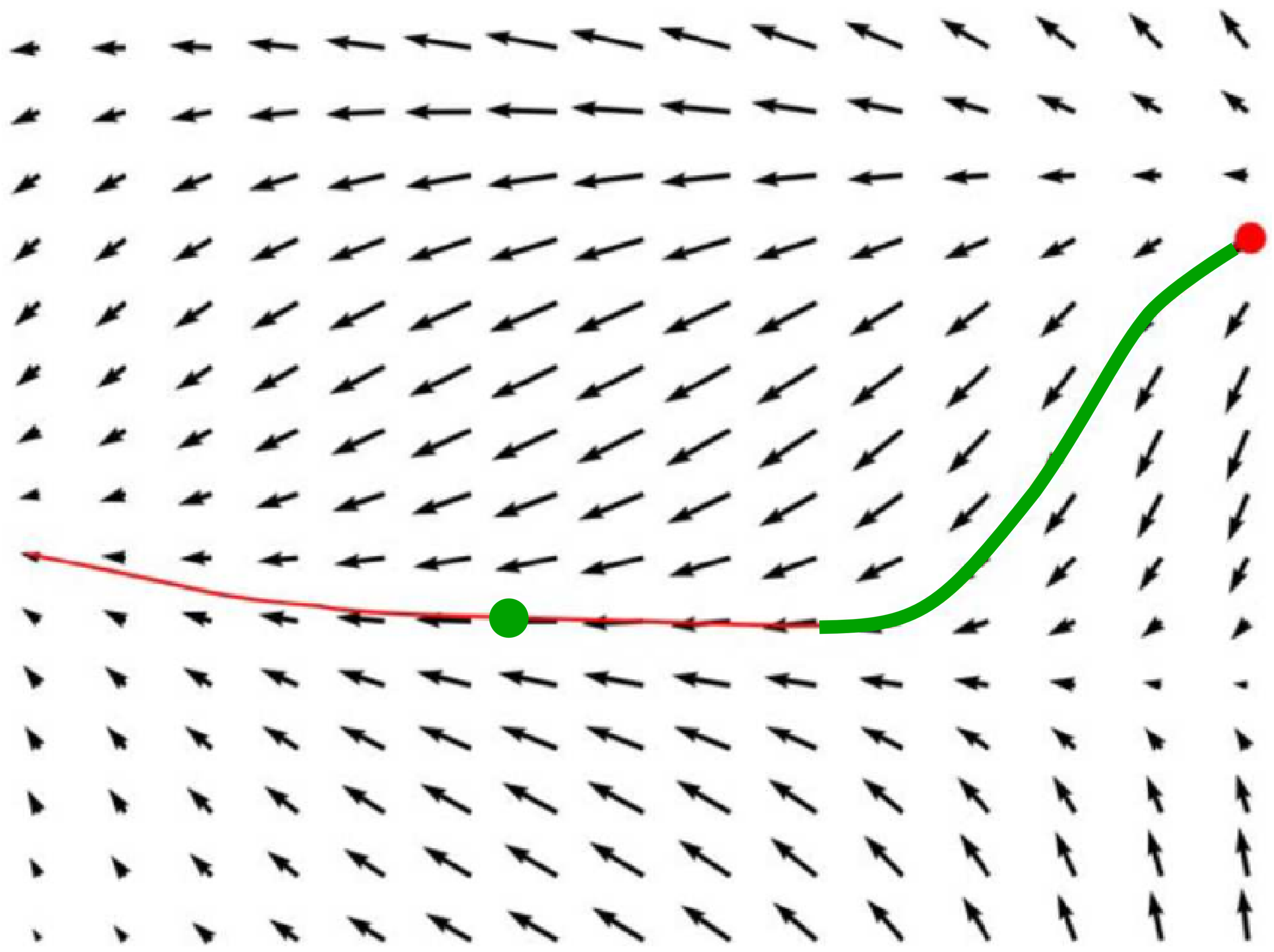

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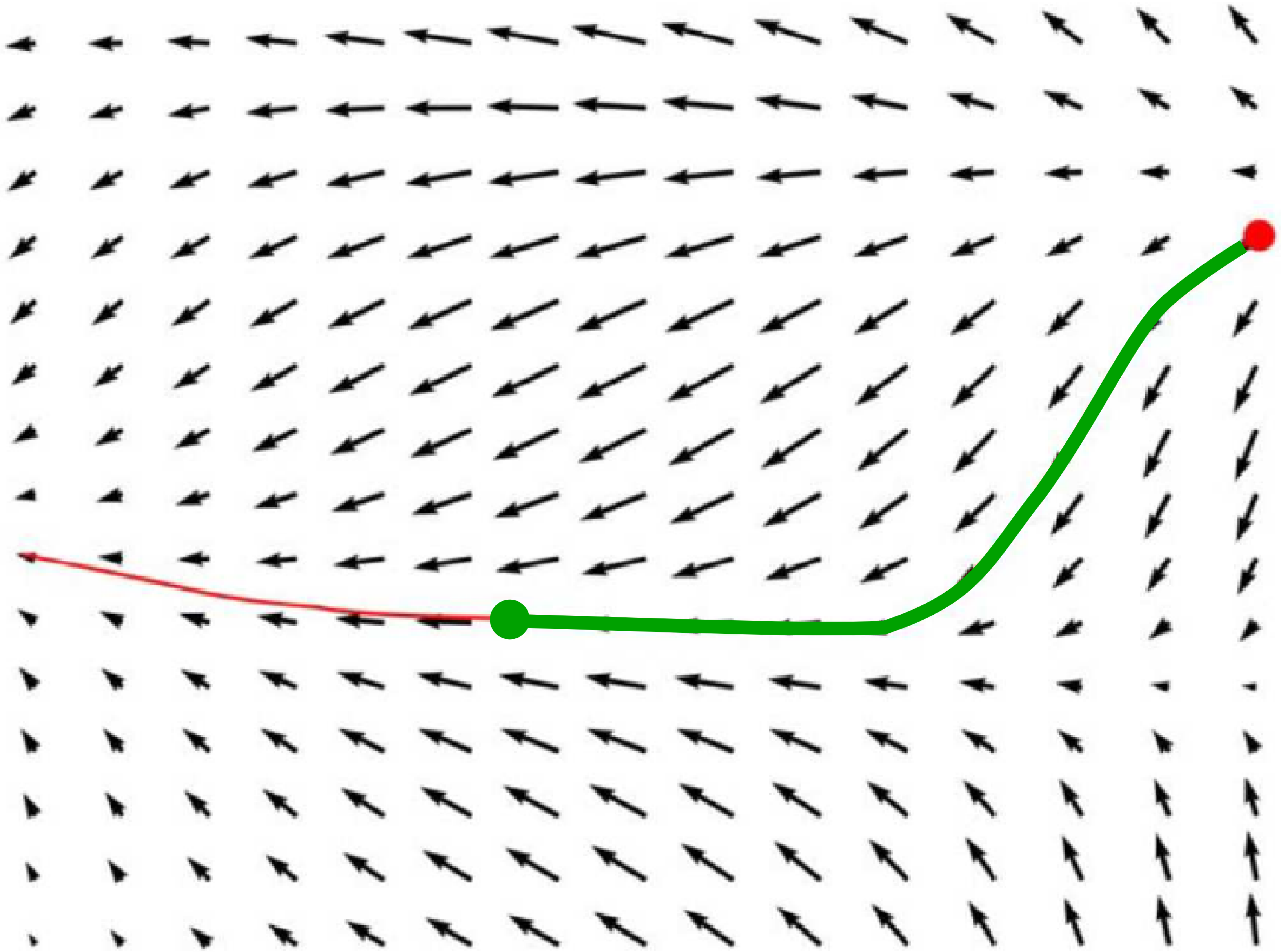

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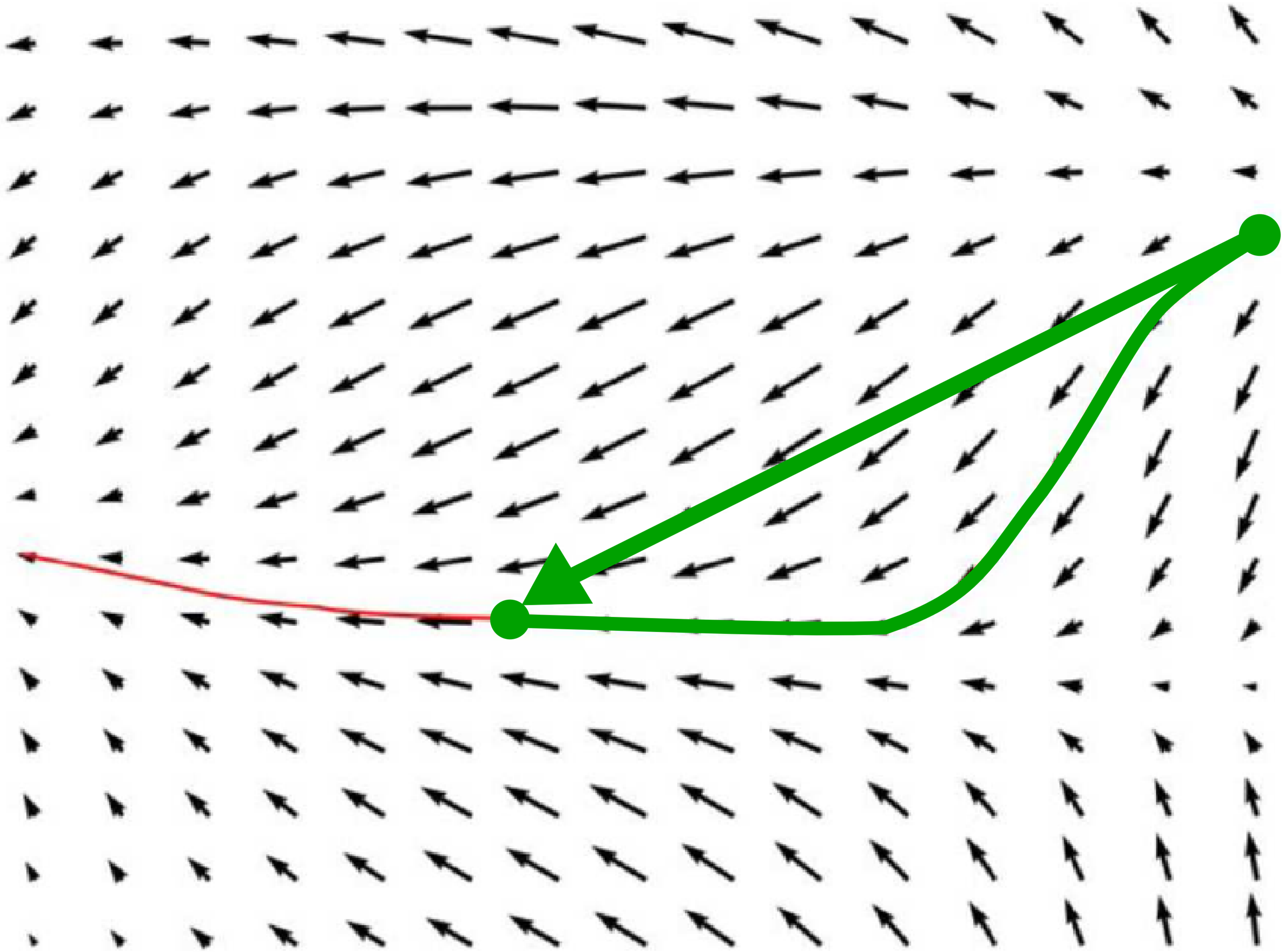

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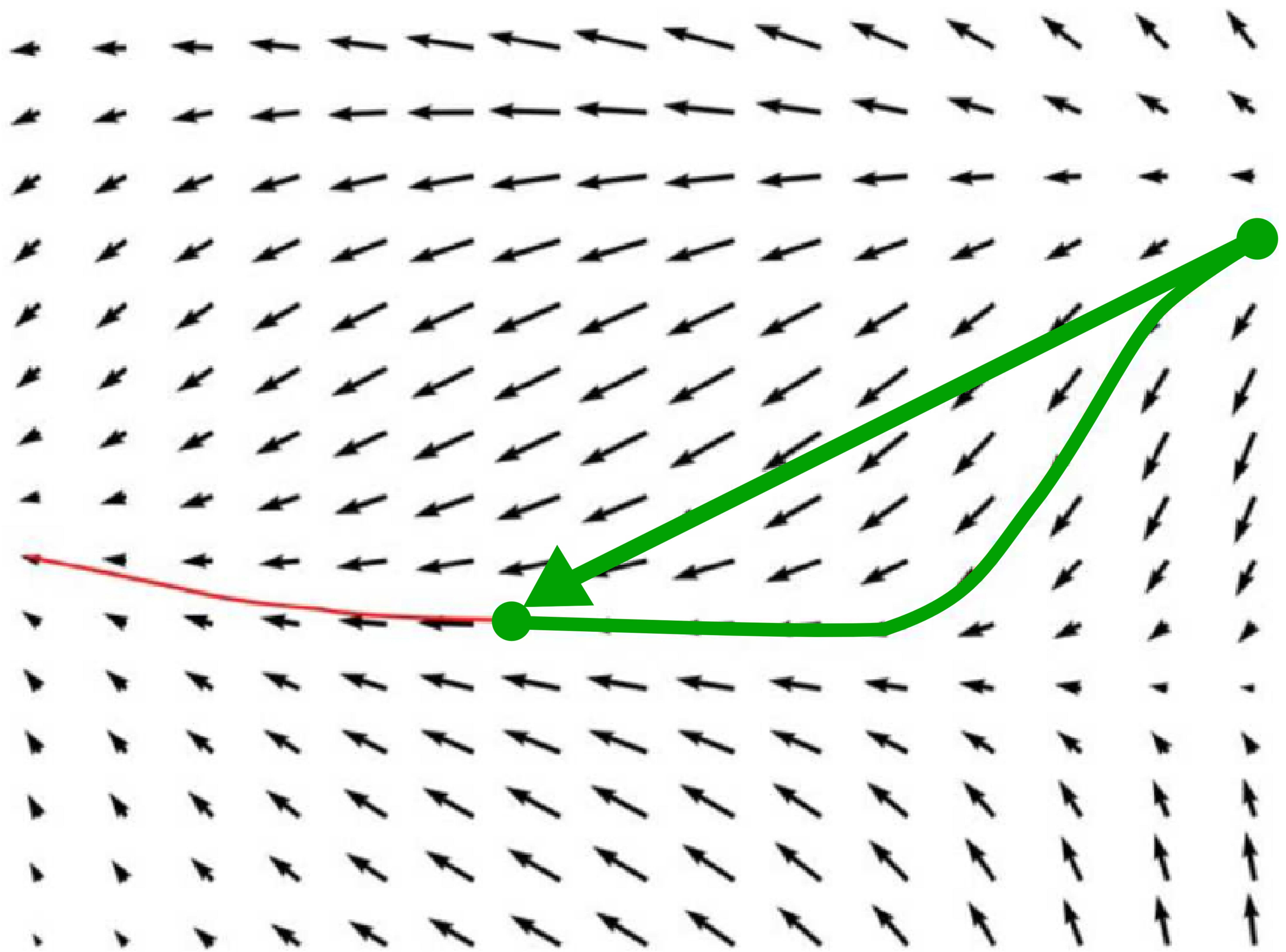
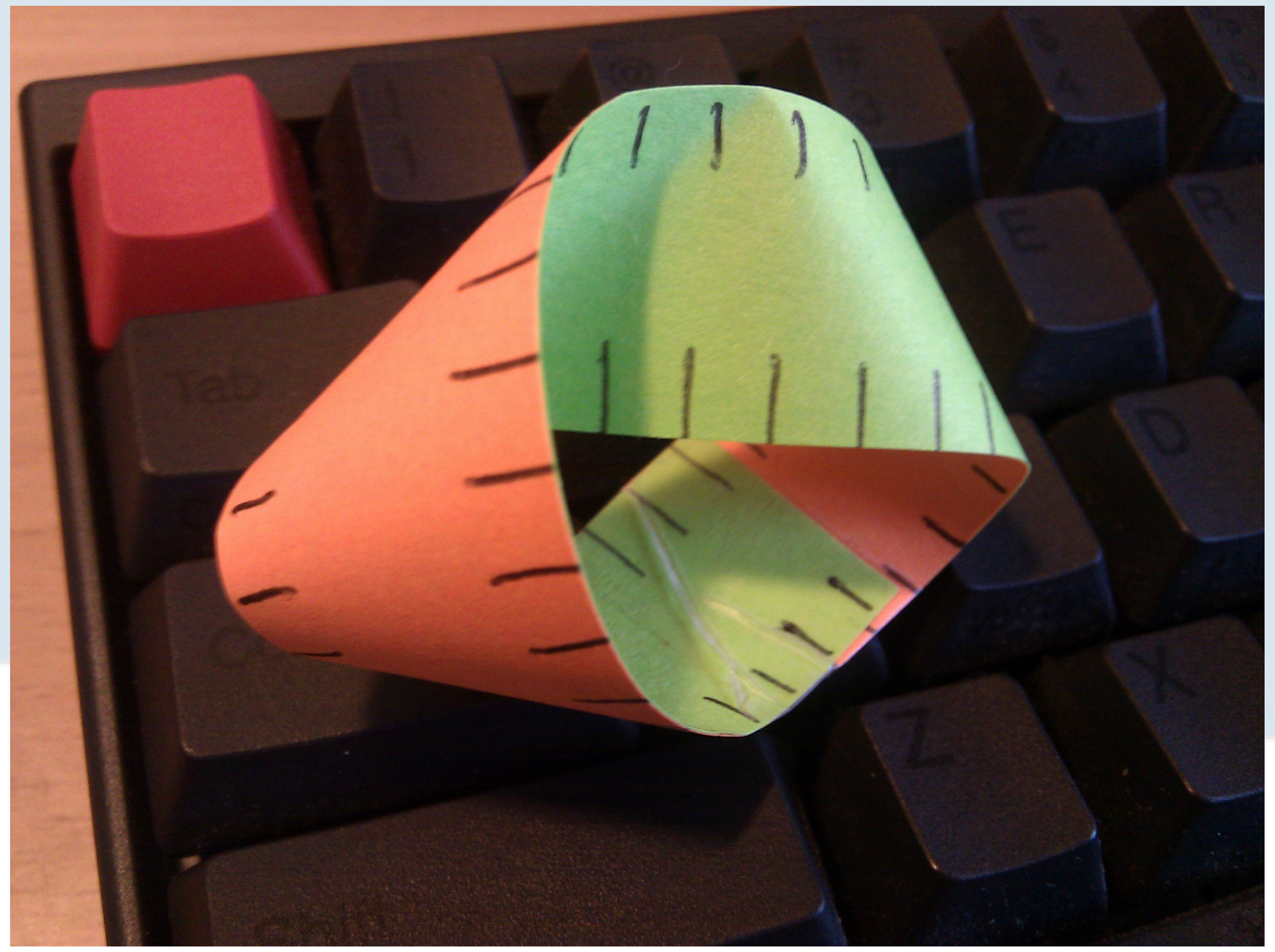


[Chen]

Streamlines on a computer



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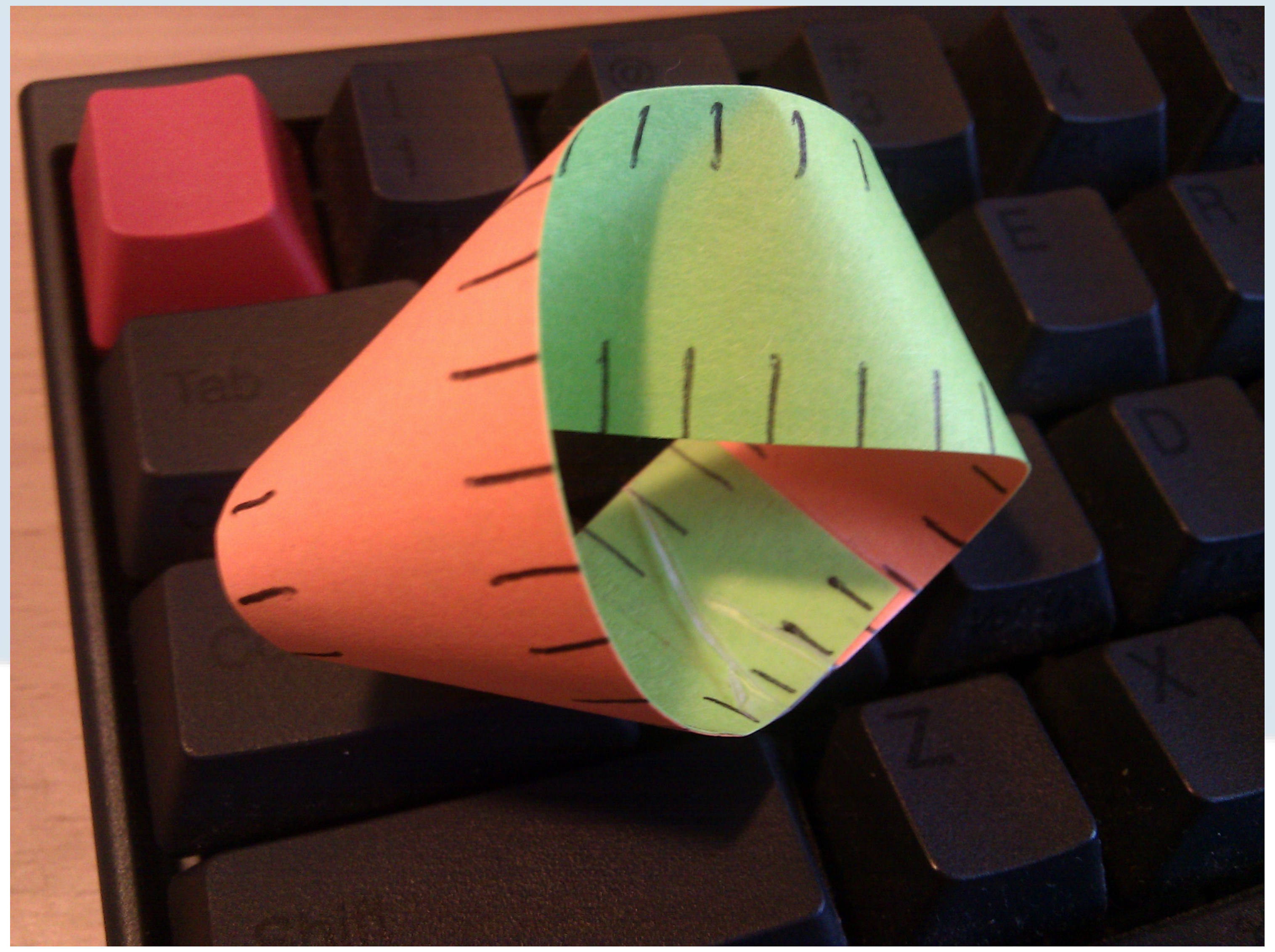
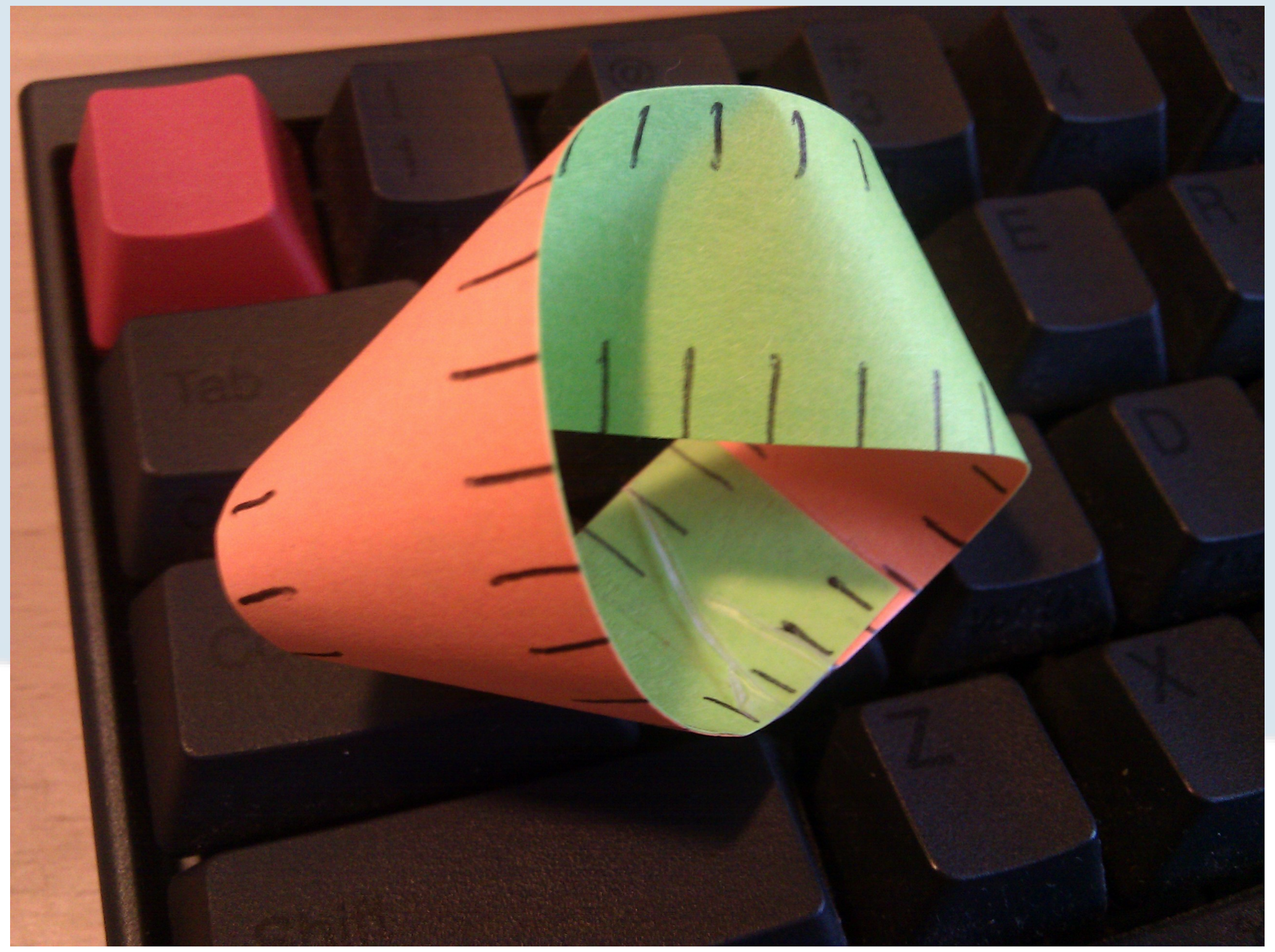


- Numerical integration of the ODE

Streamlines on a computer



- Numerical integration of the ODE
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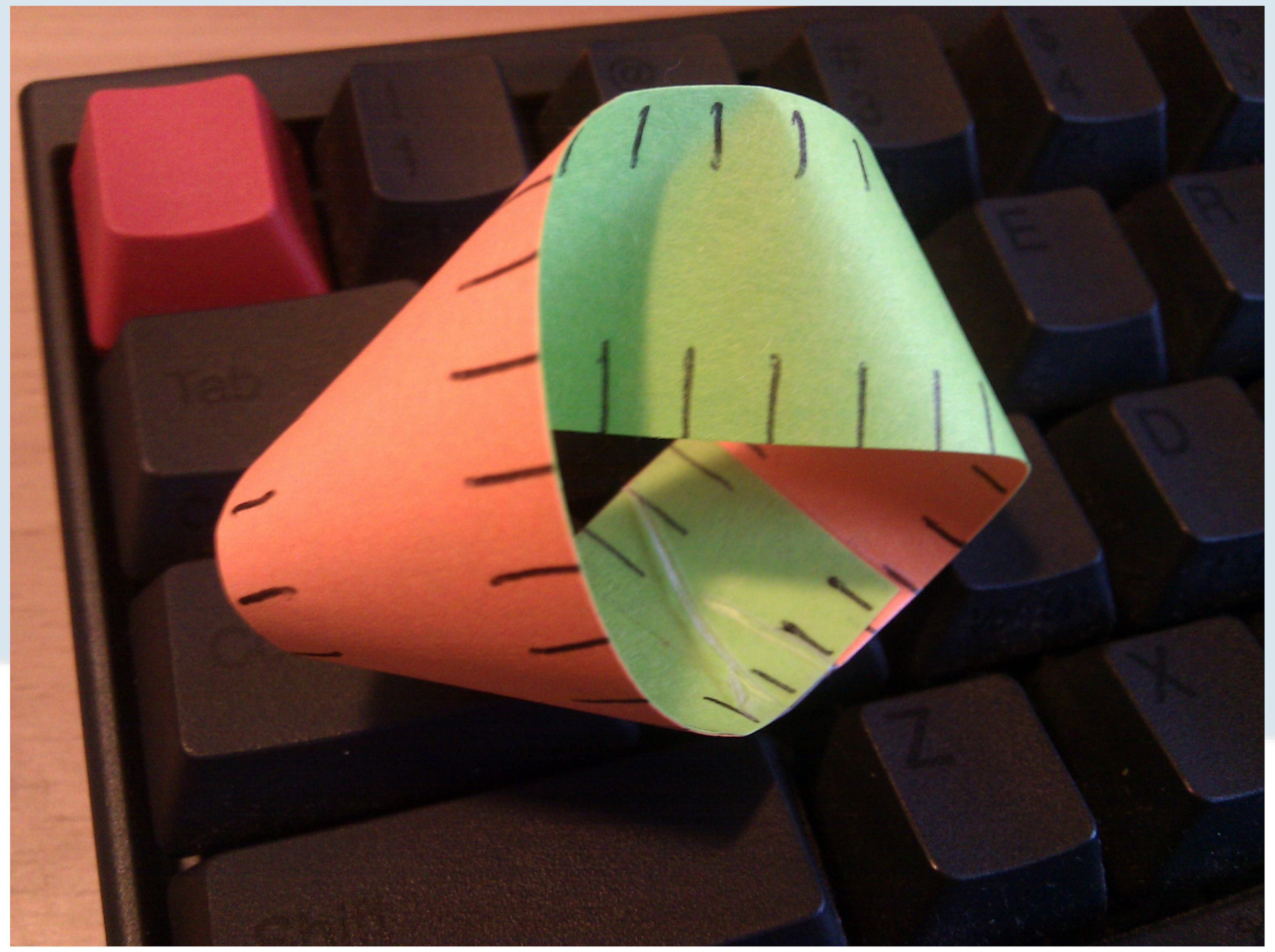

- Numerical integration of the ODE
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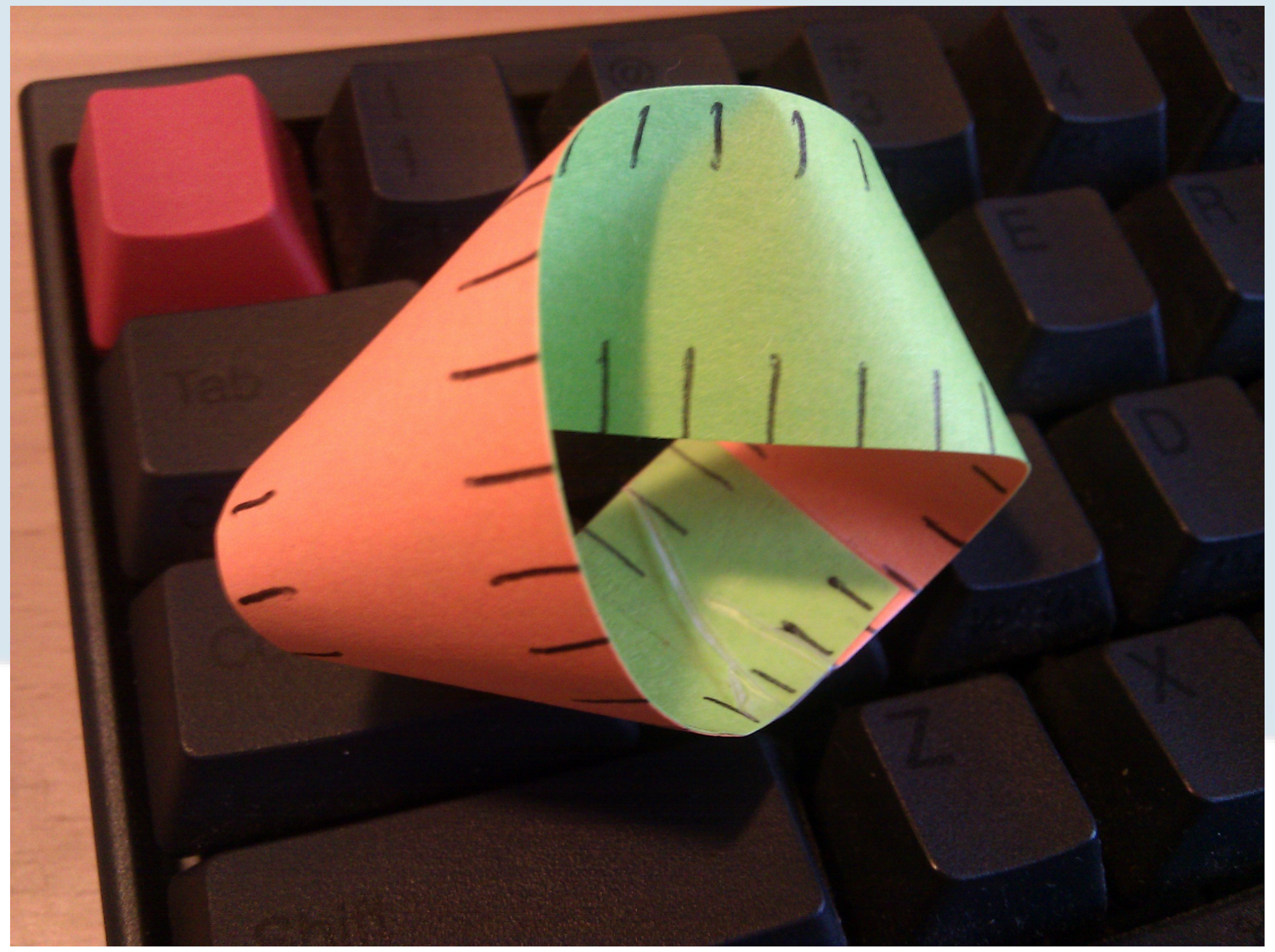
Streamlines on a computer



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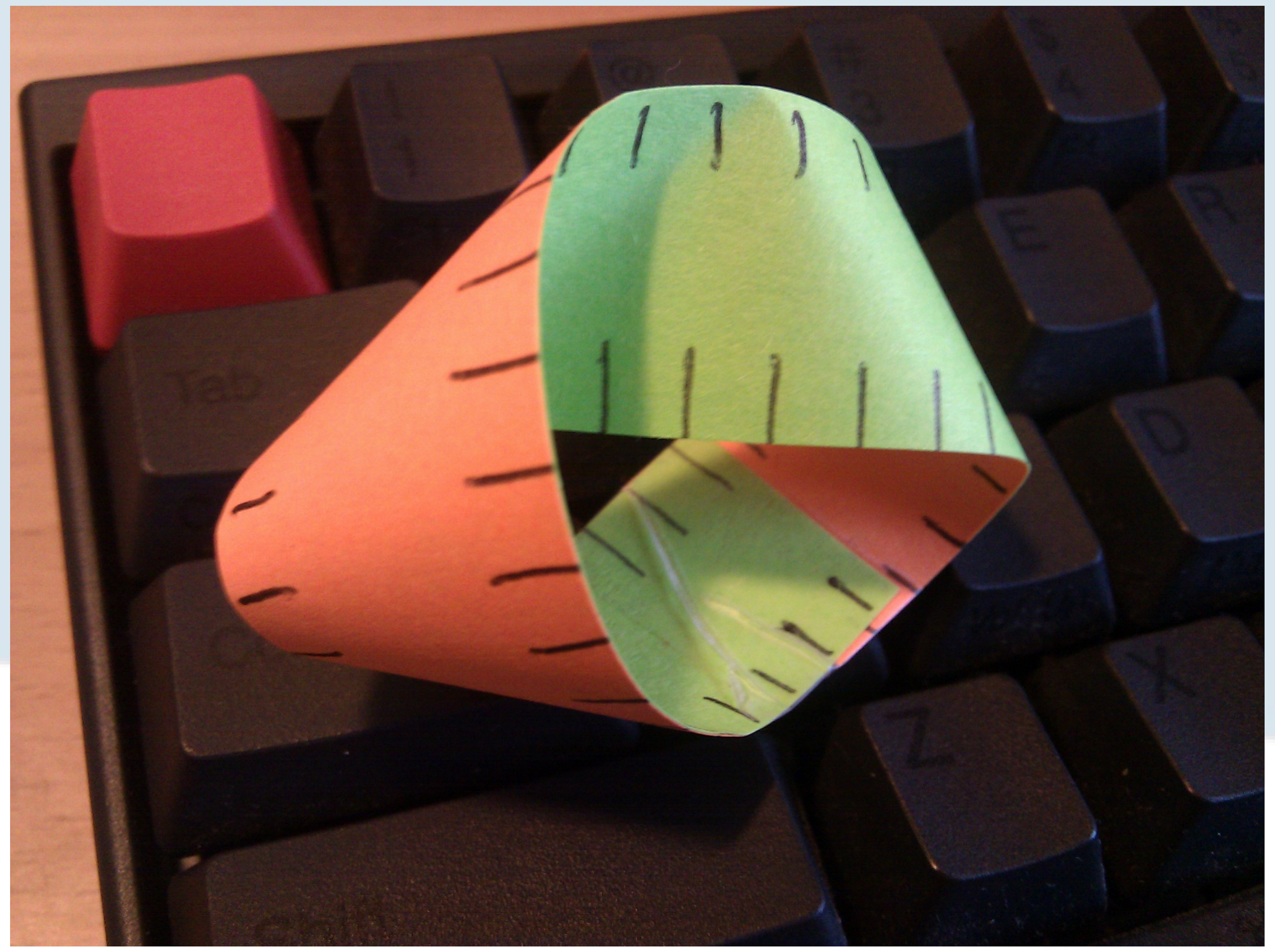

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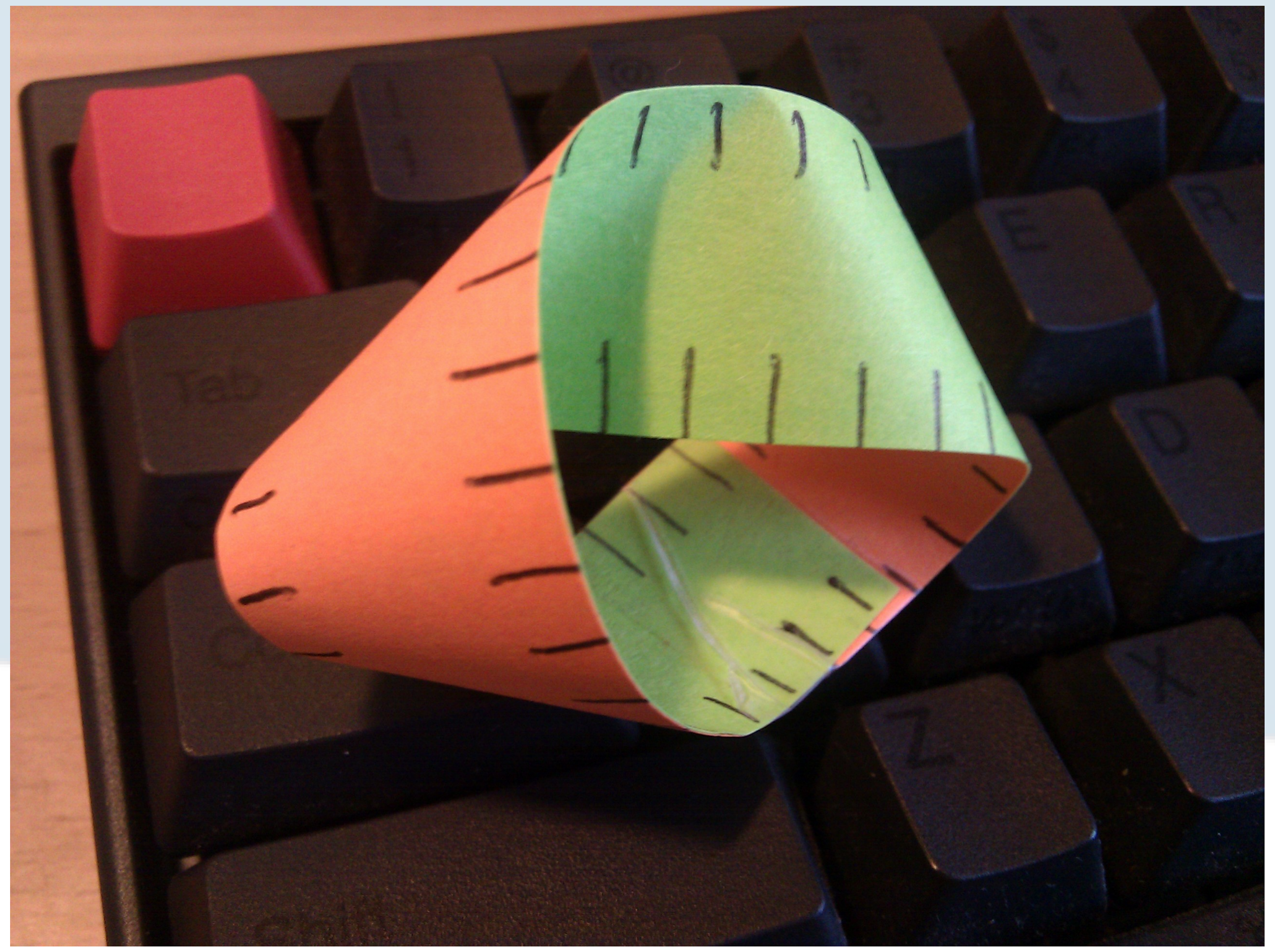

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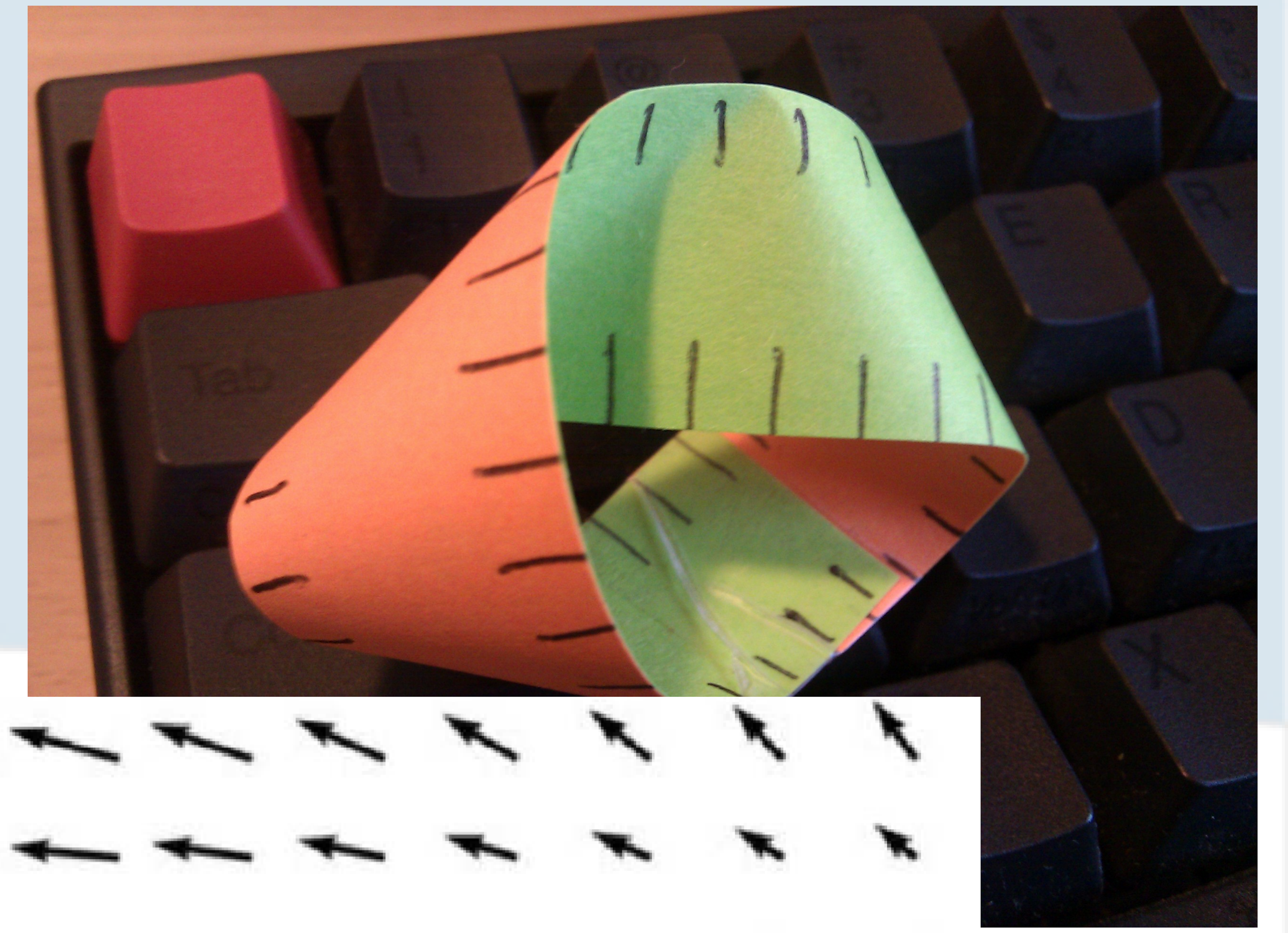

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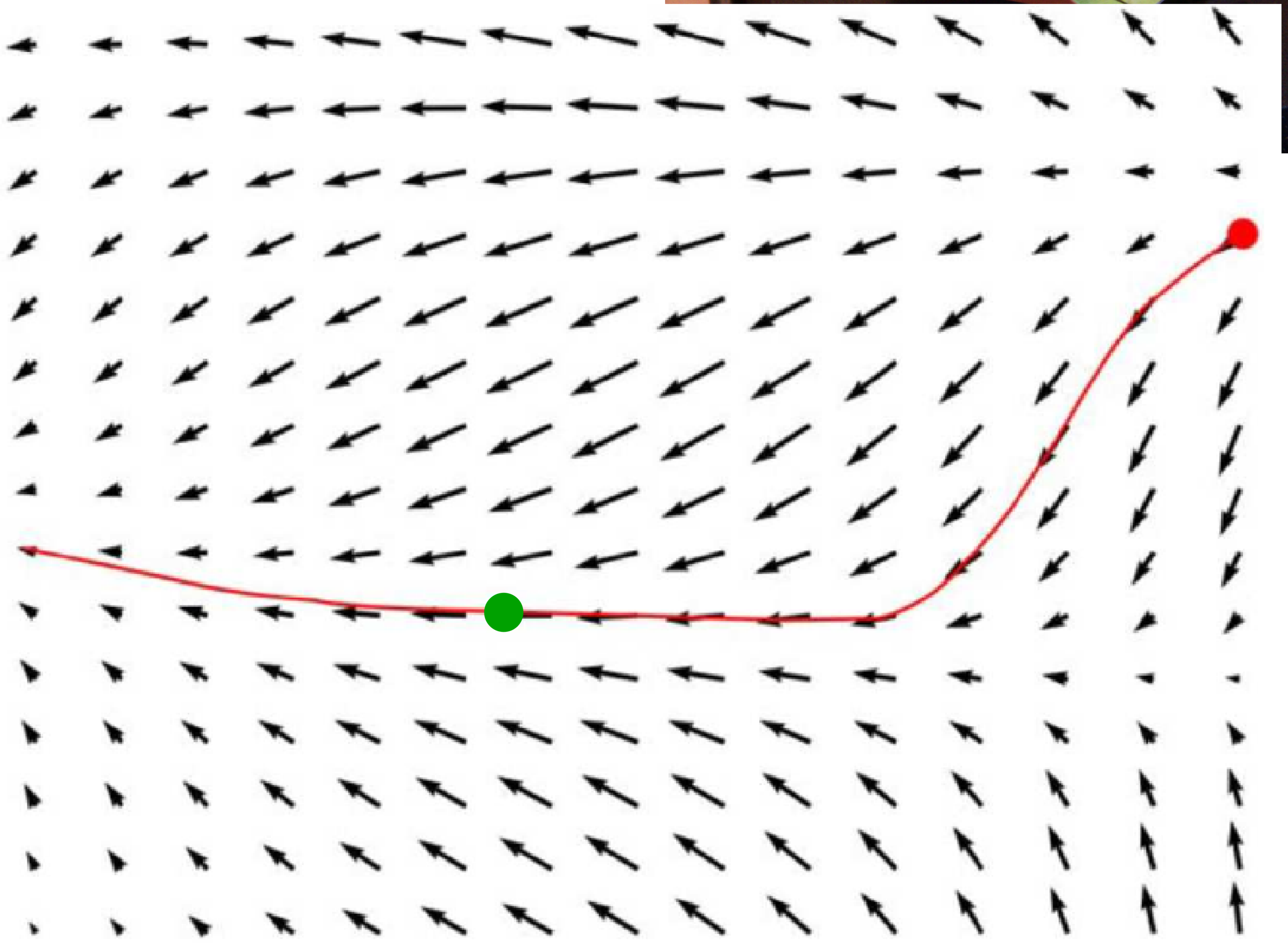

[Chen]

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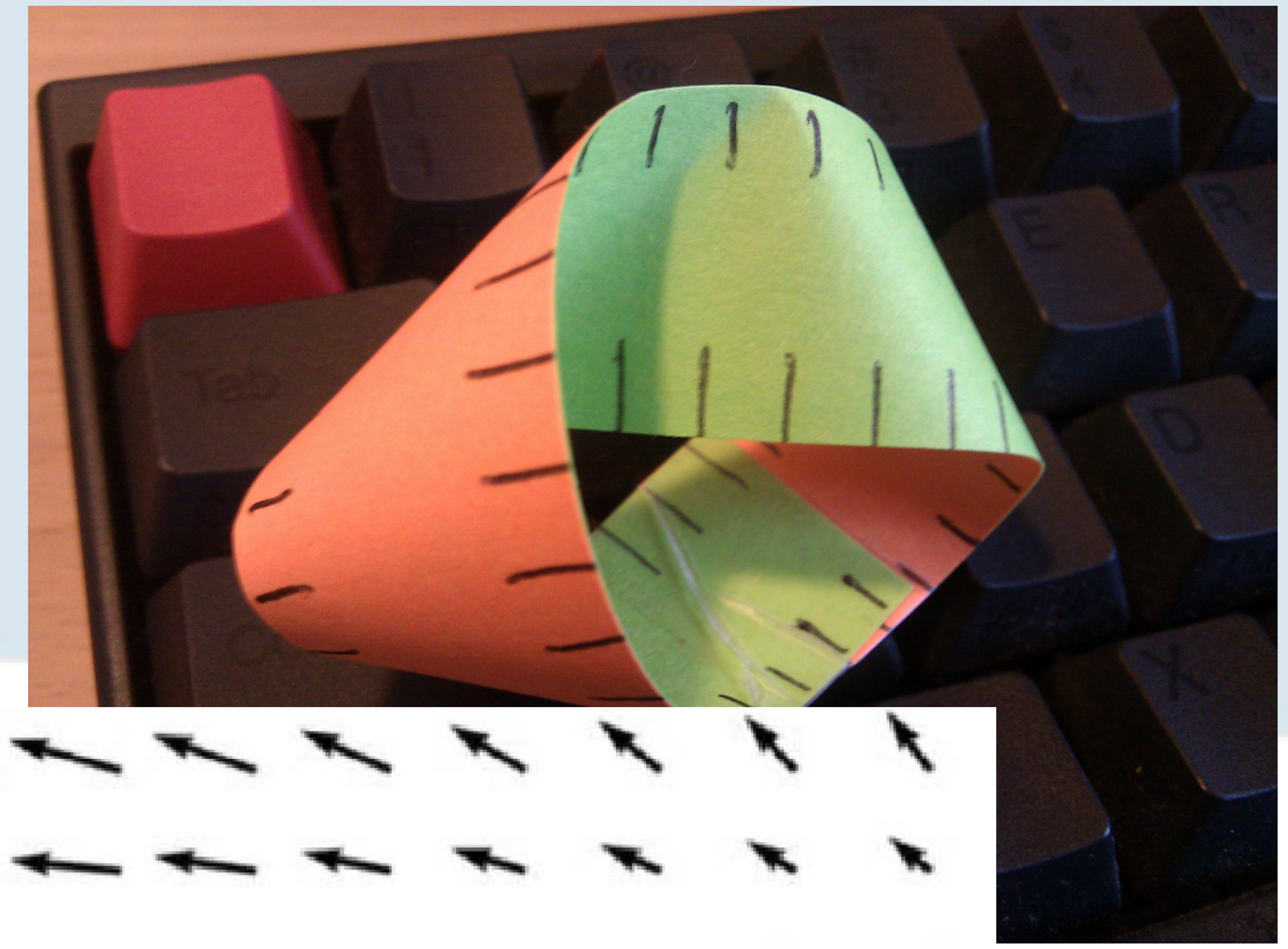

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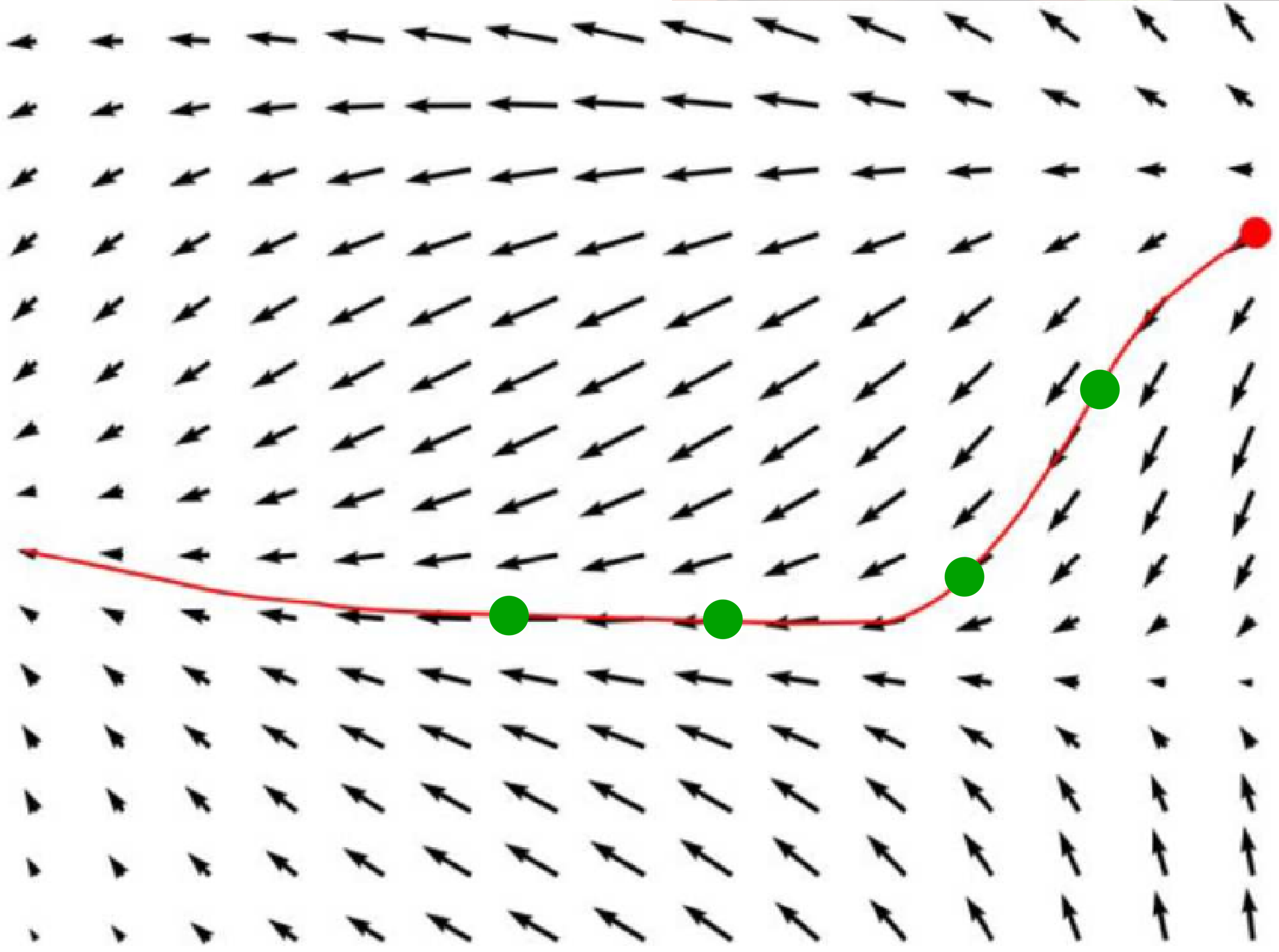

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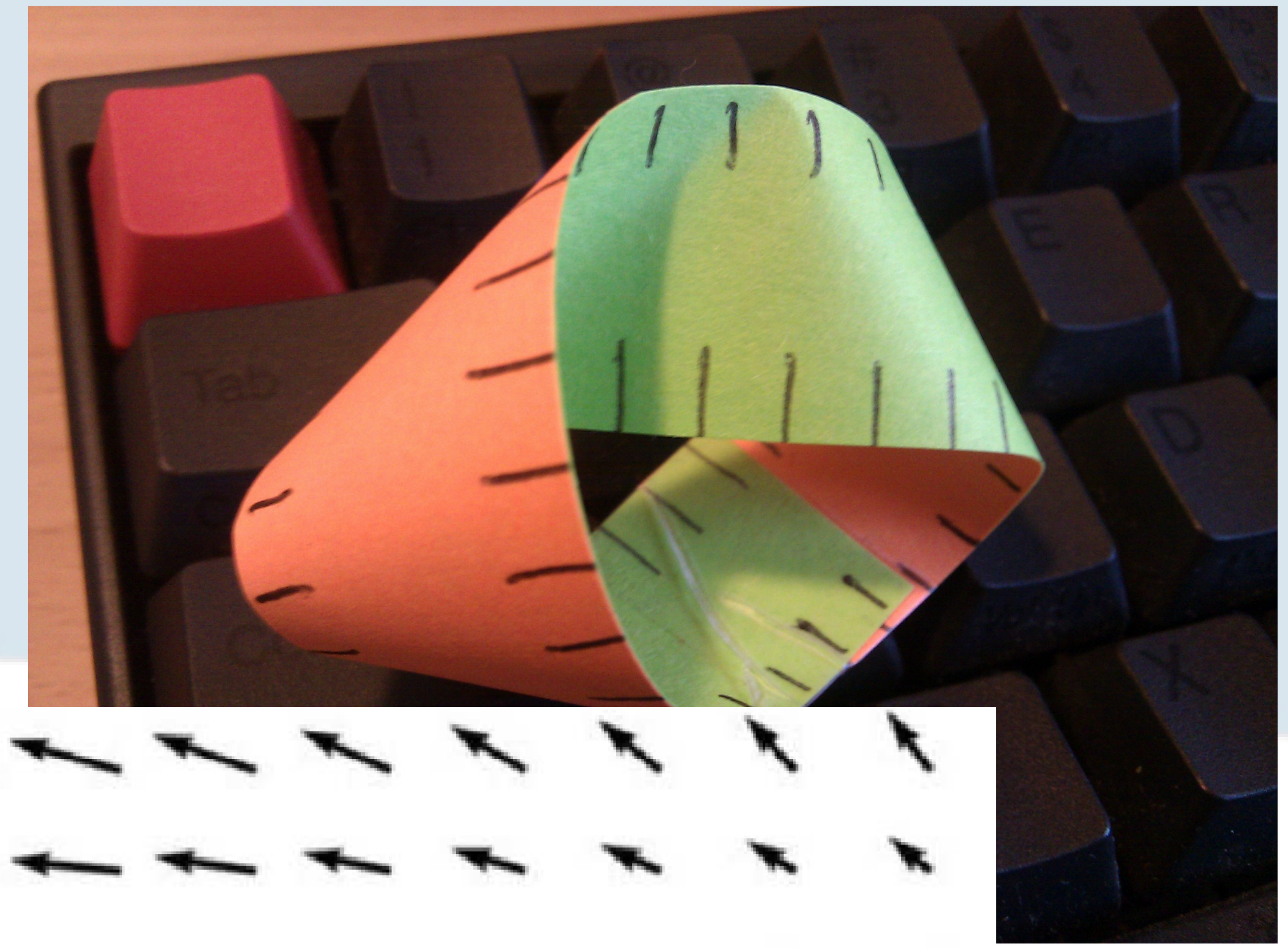

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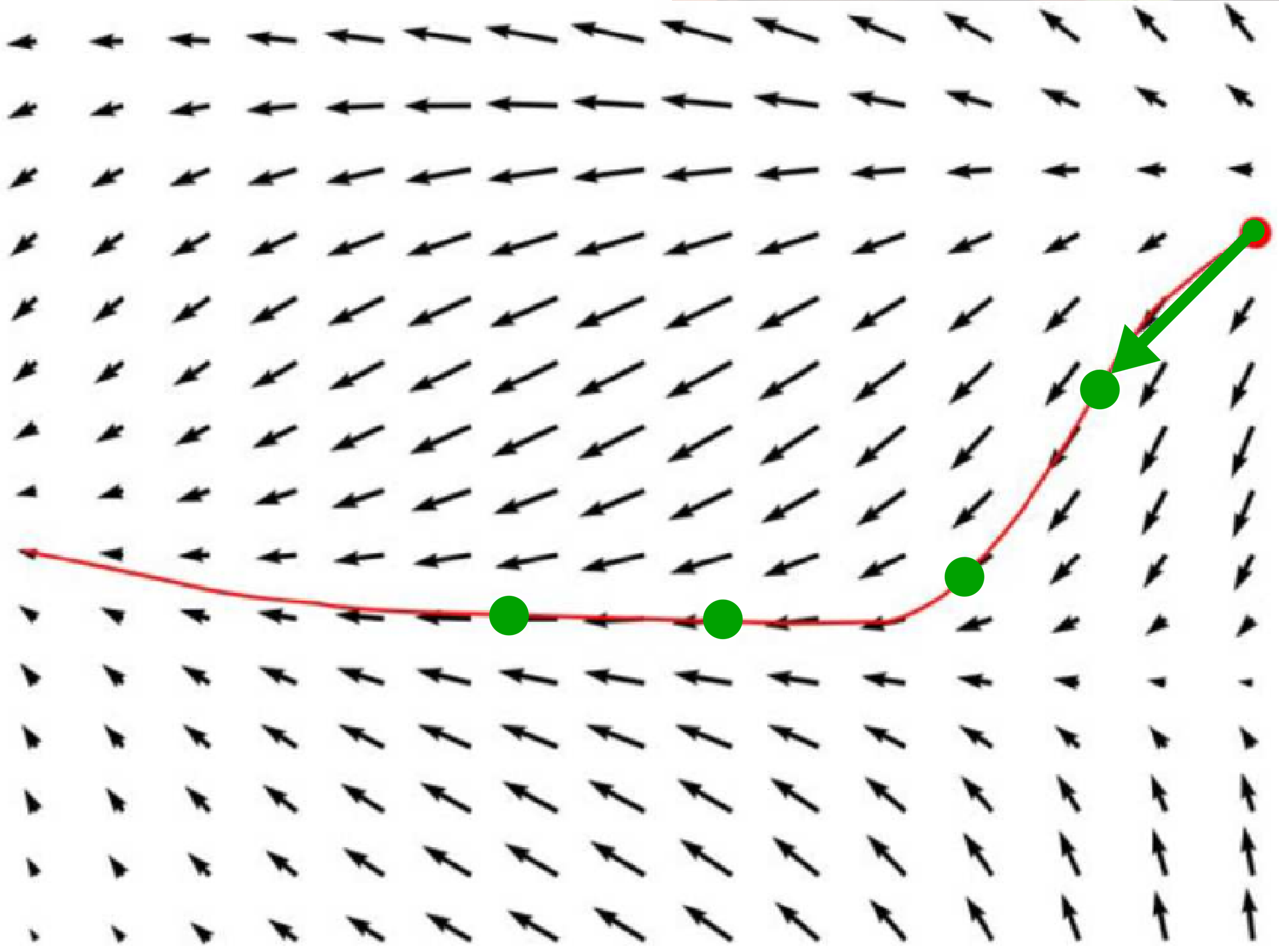

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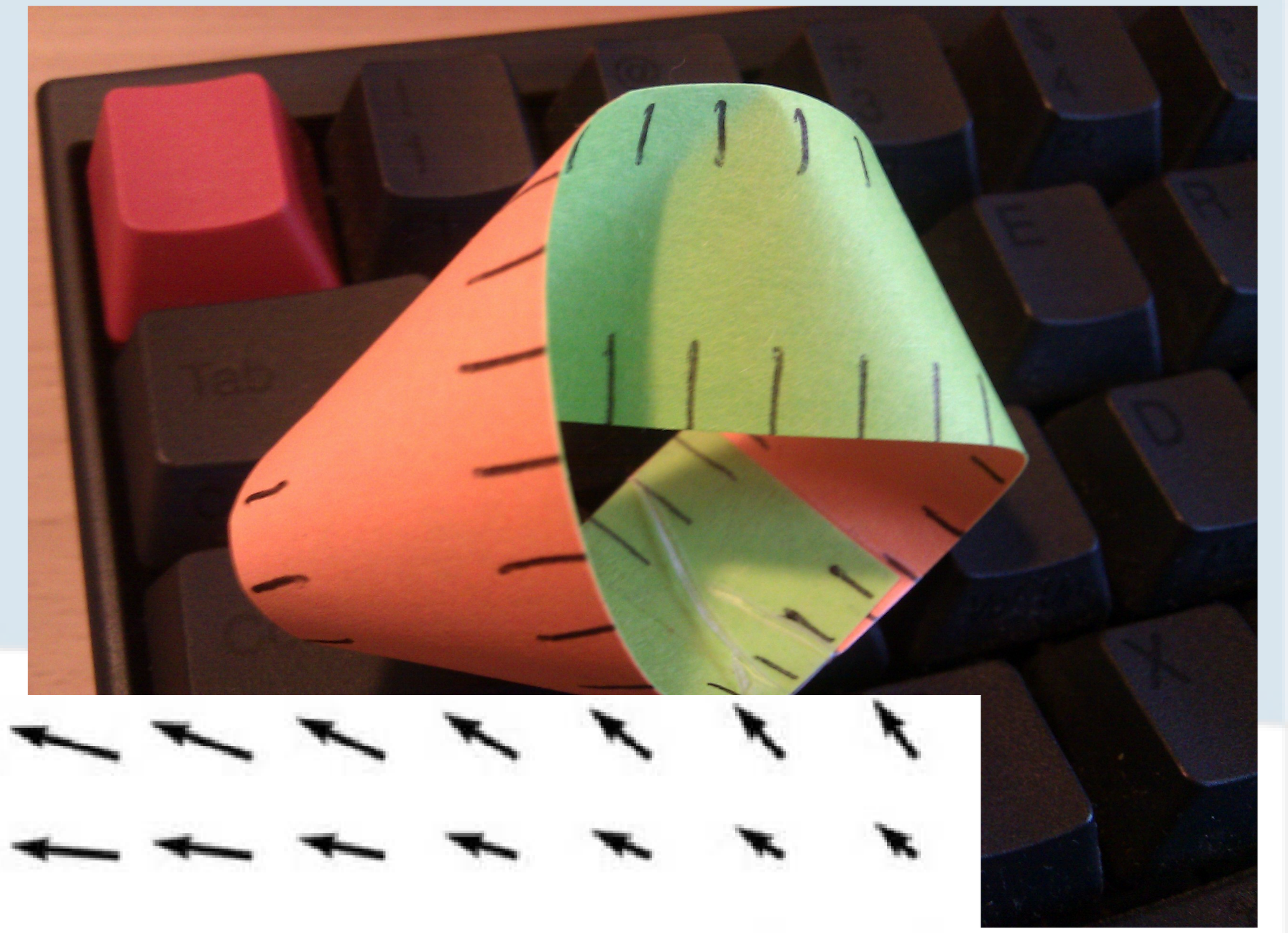

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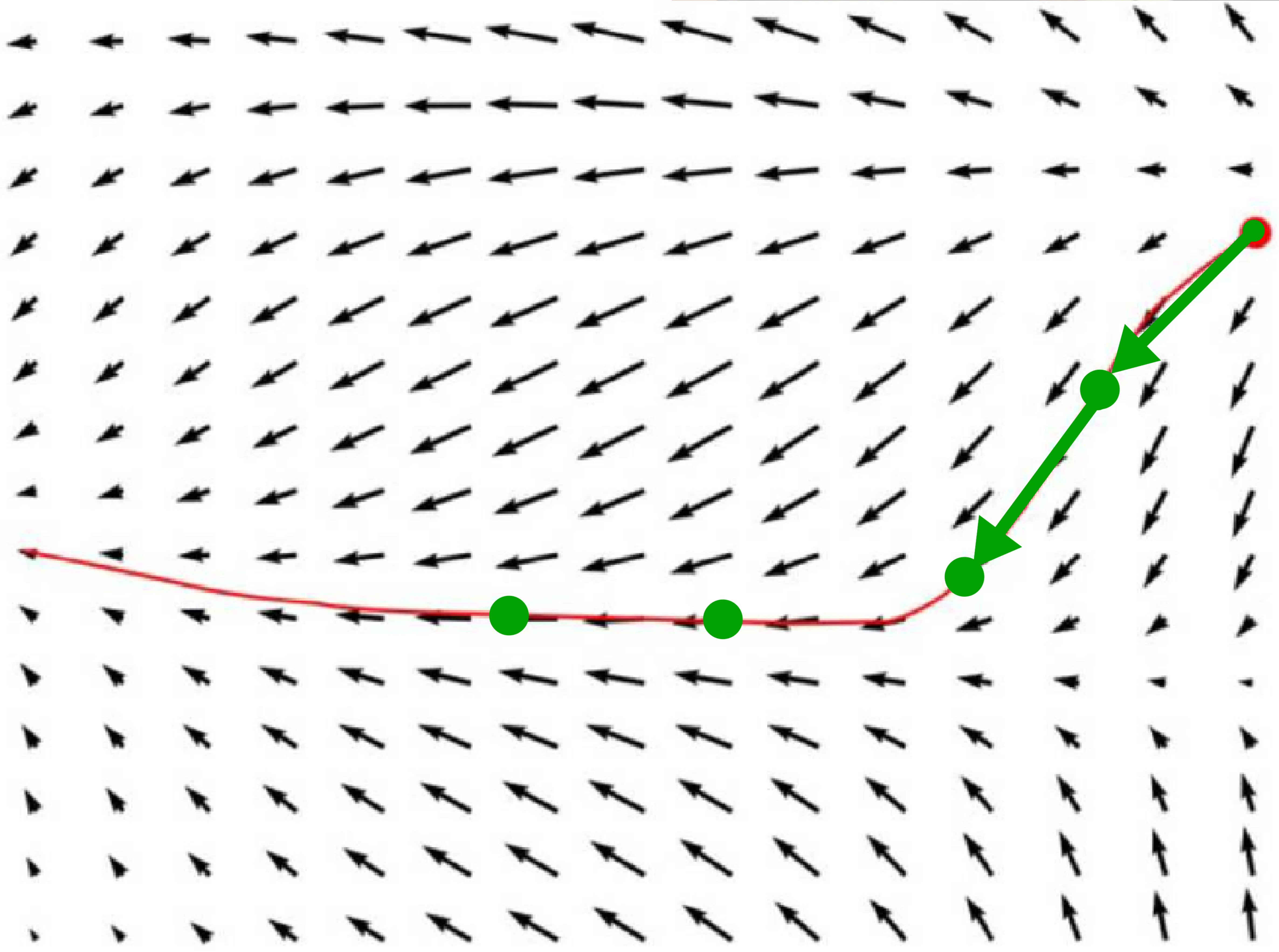
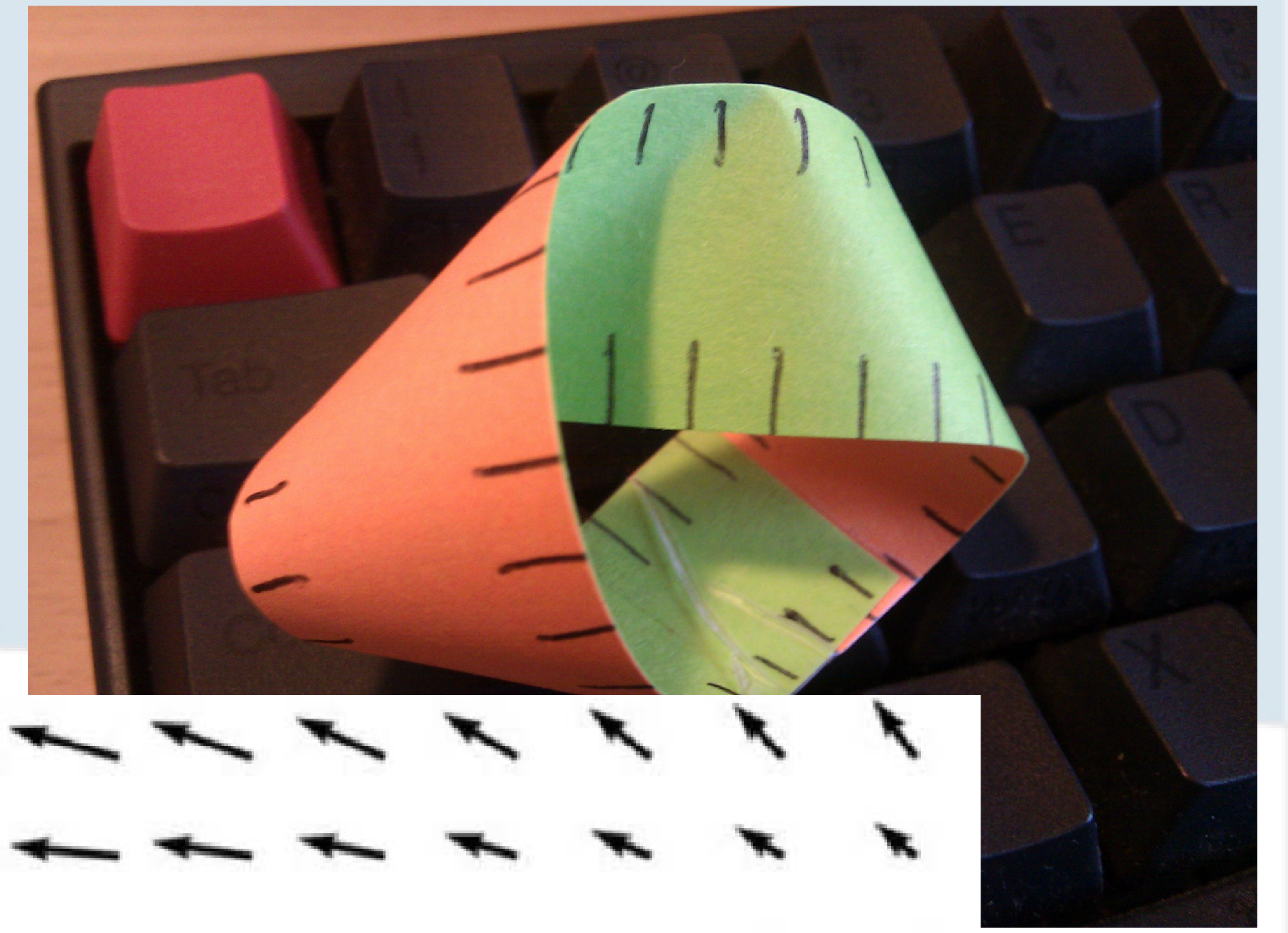


[Chen]

Streamlines on a computer



- Numerical integration of the ODE
  - **Euler method**
  - Runge-Kutta
    - Higher oder approximations
- Discretization
  - $c : \mathcal{C} \to [0, 1]$
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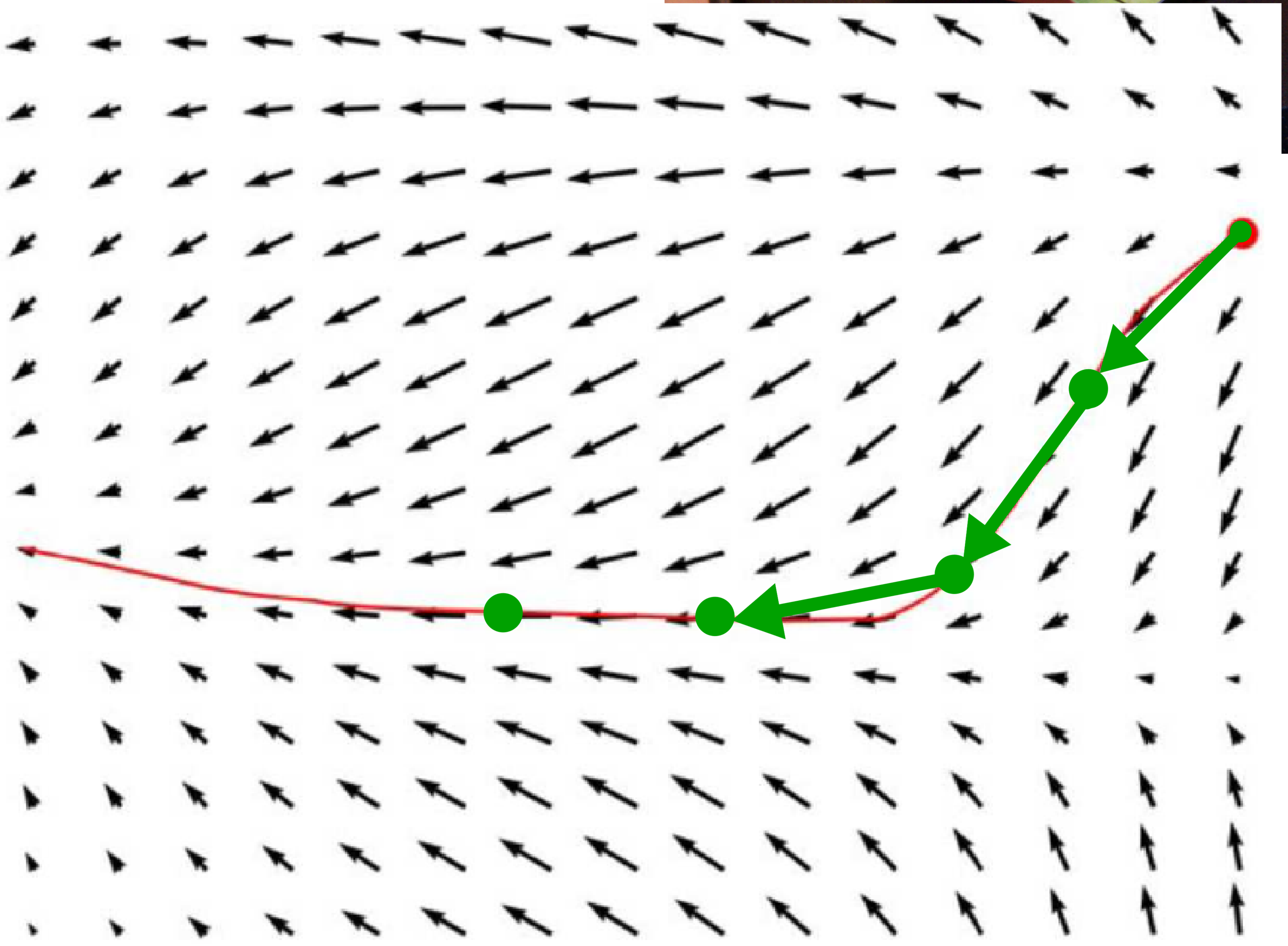

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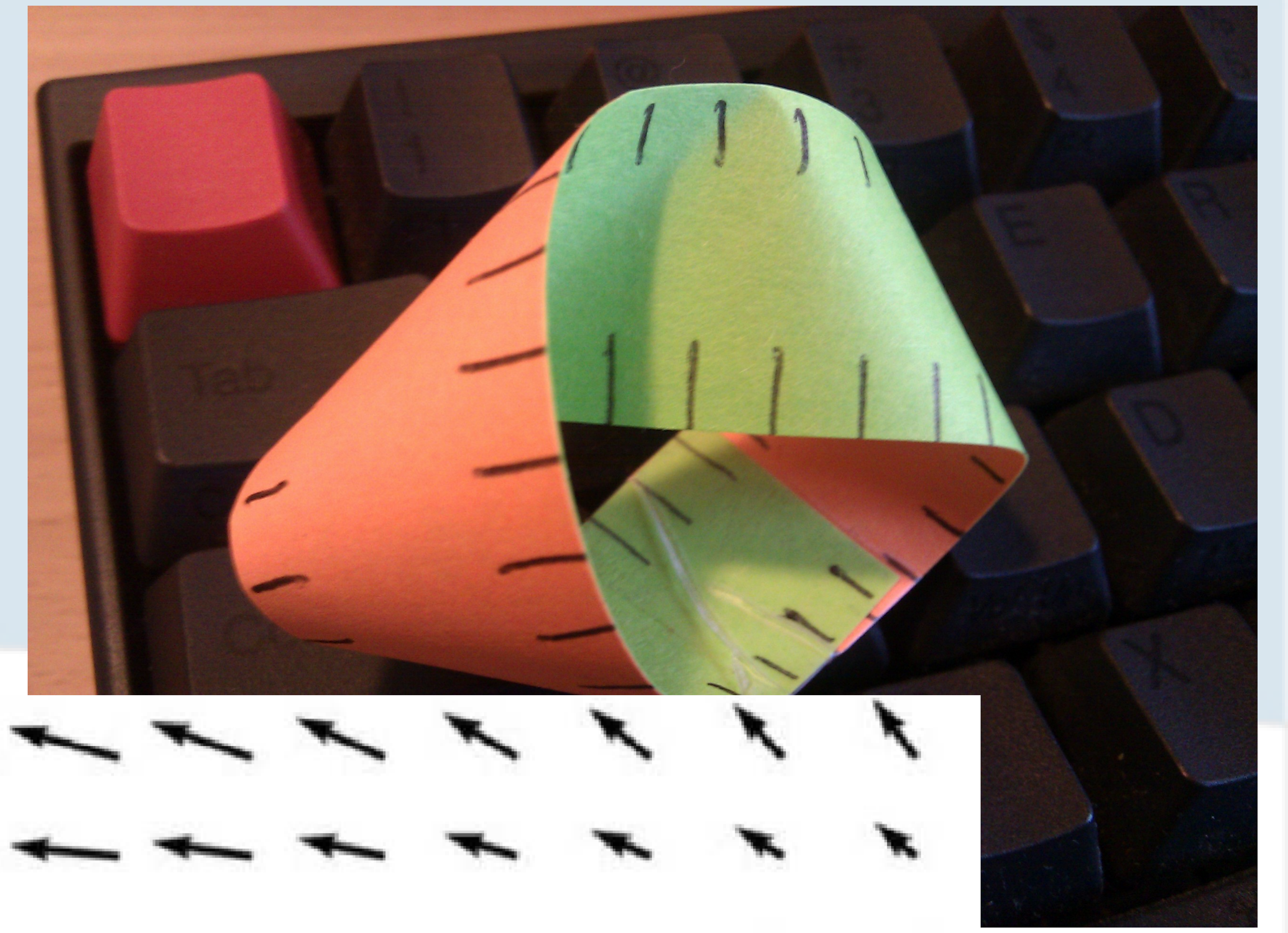

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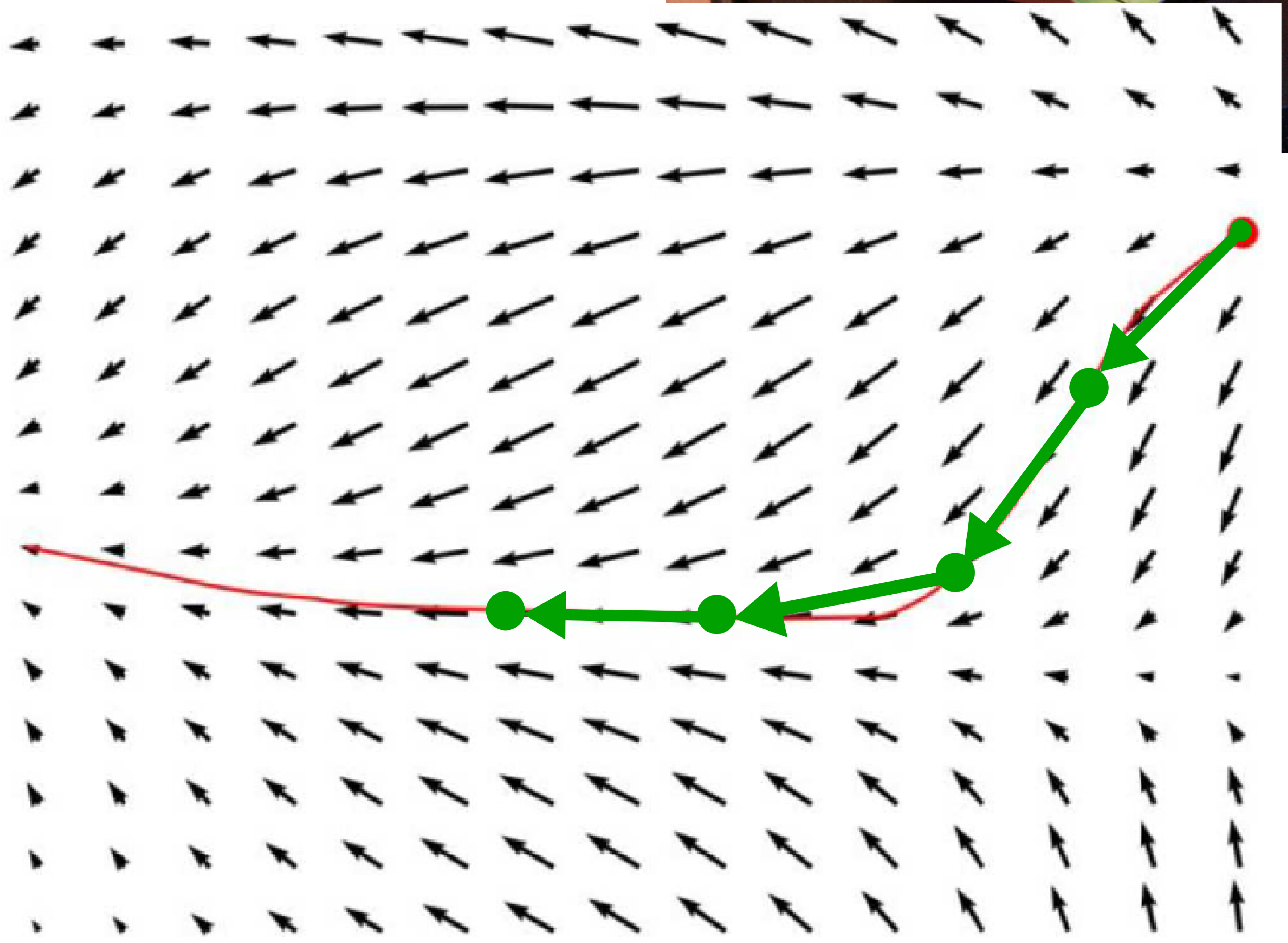

[Chen]

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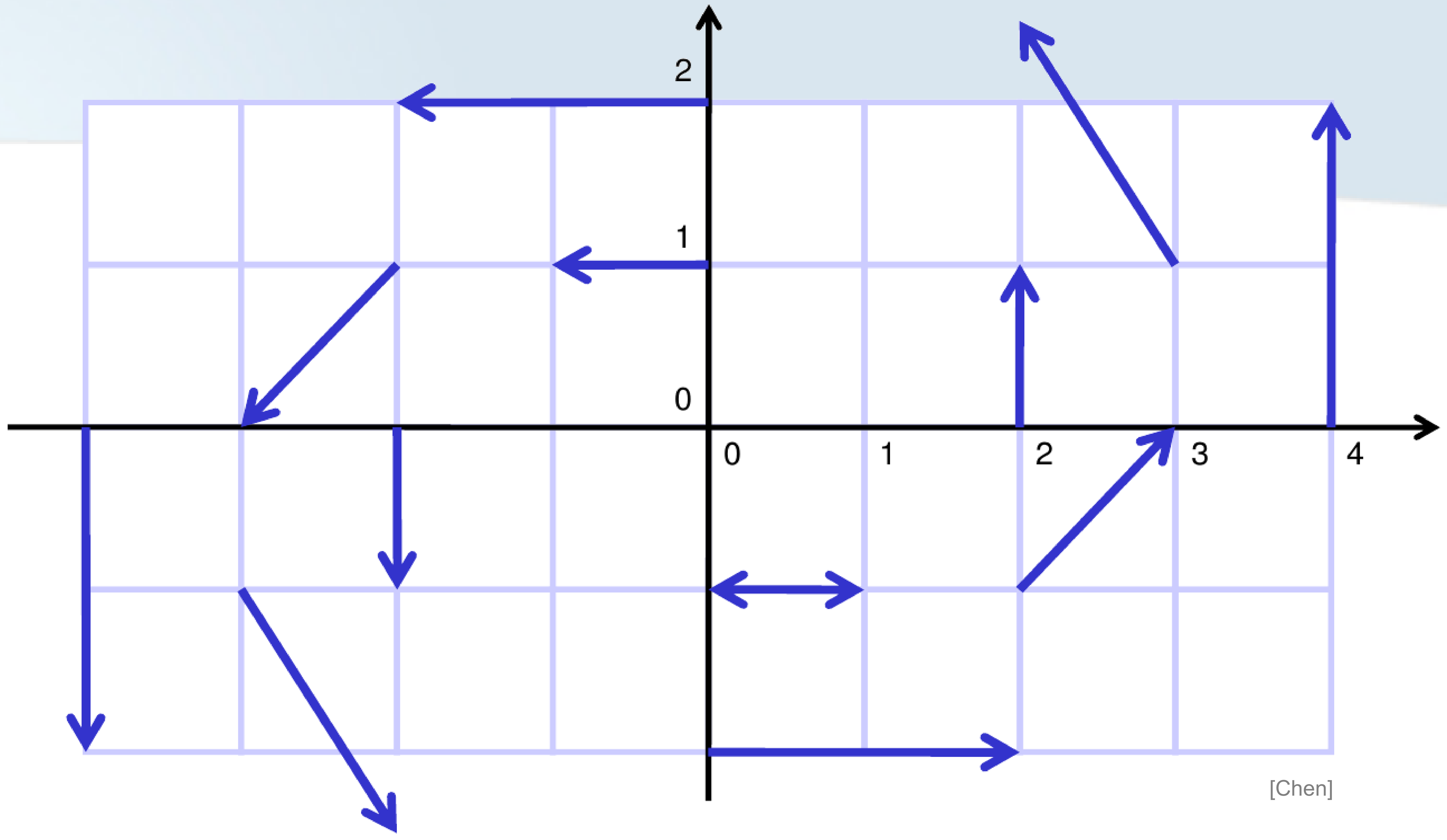

[Chen]

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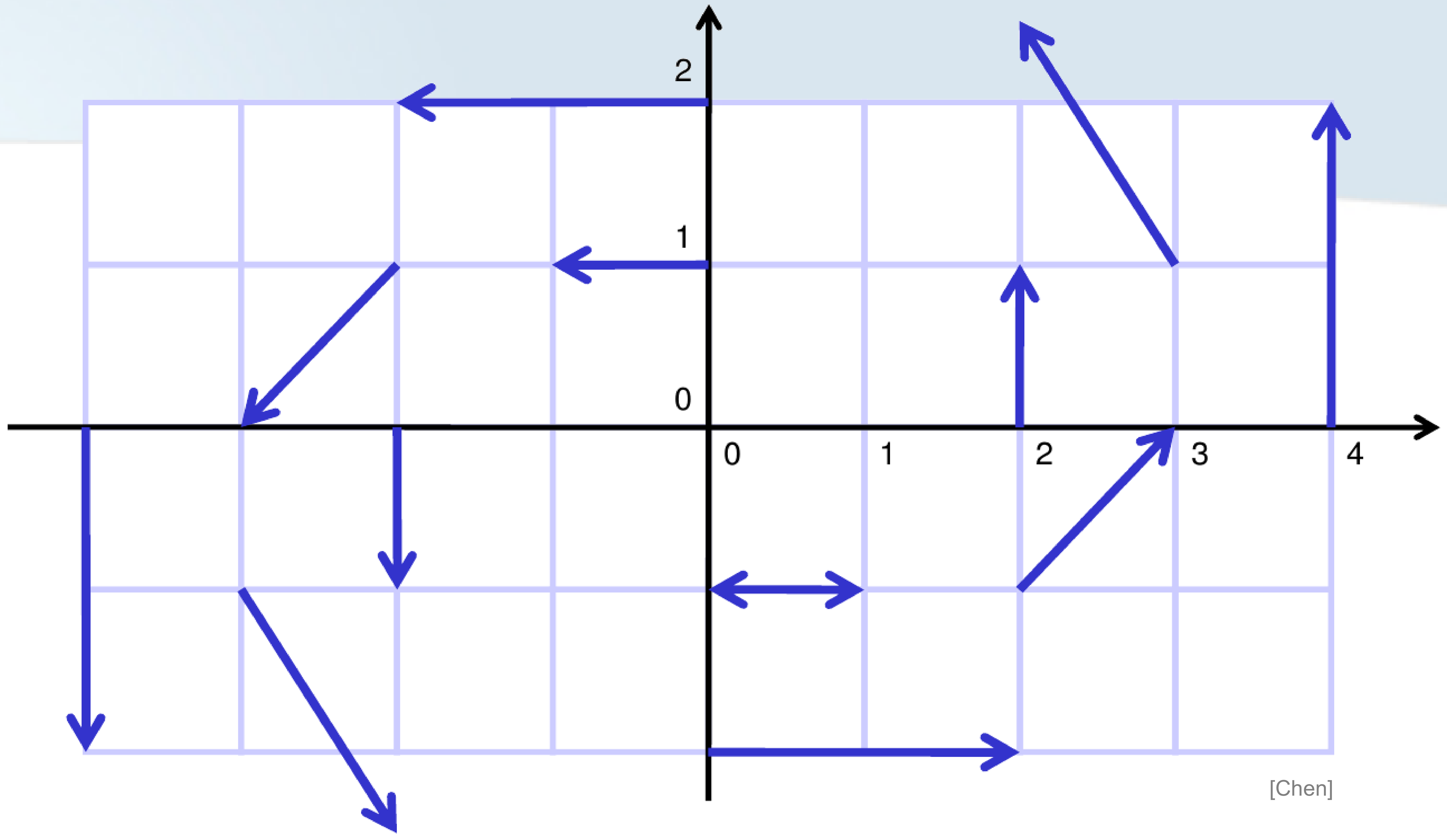

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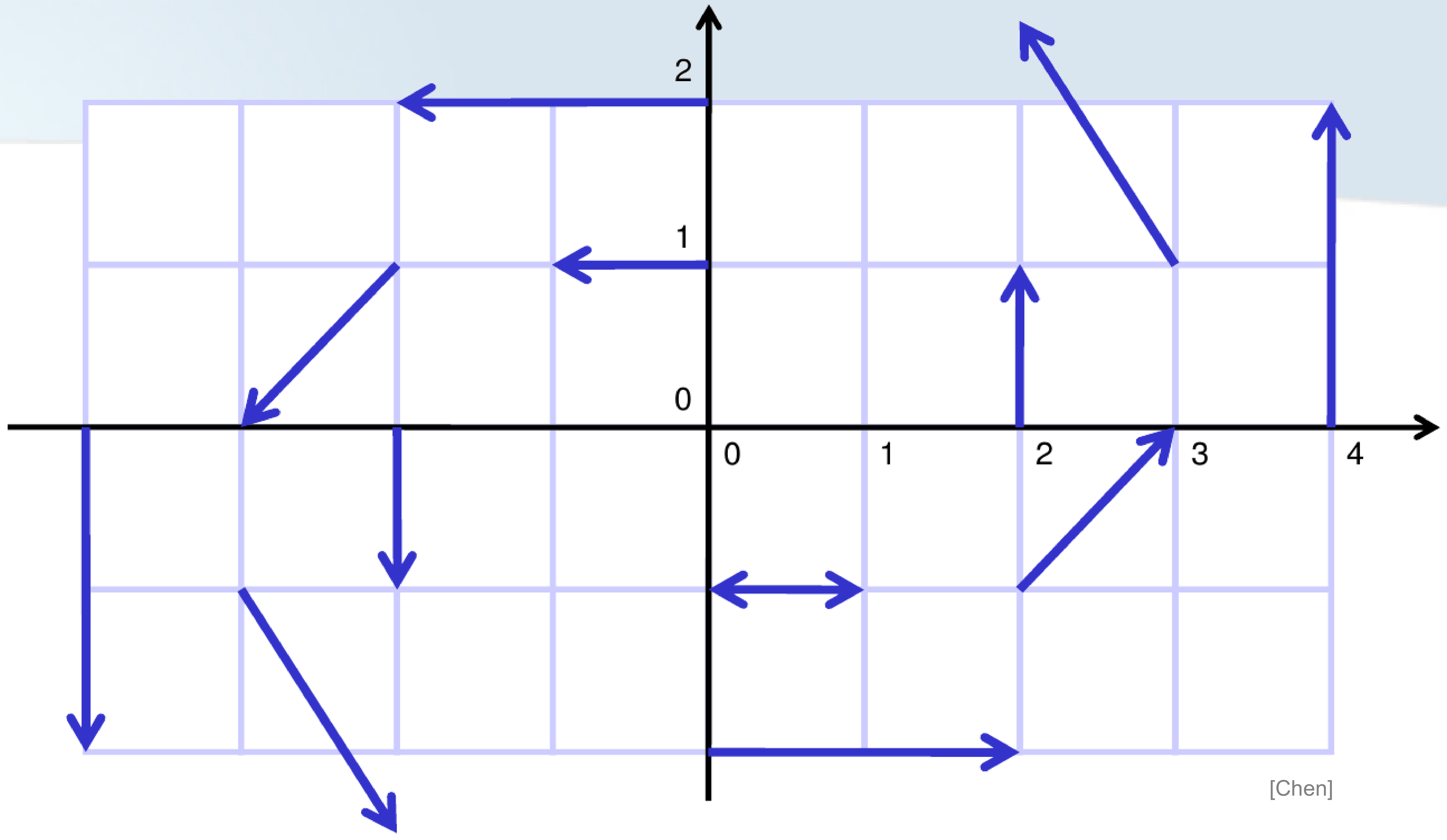

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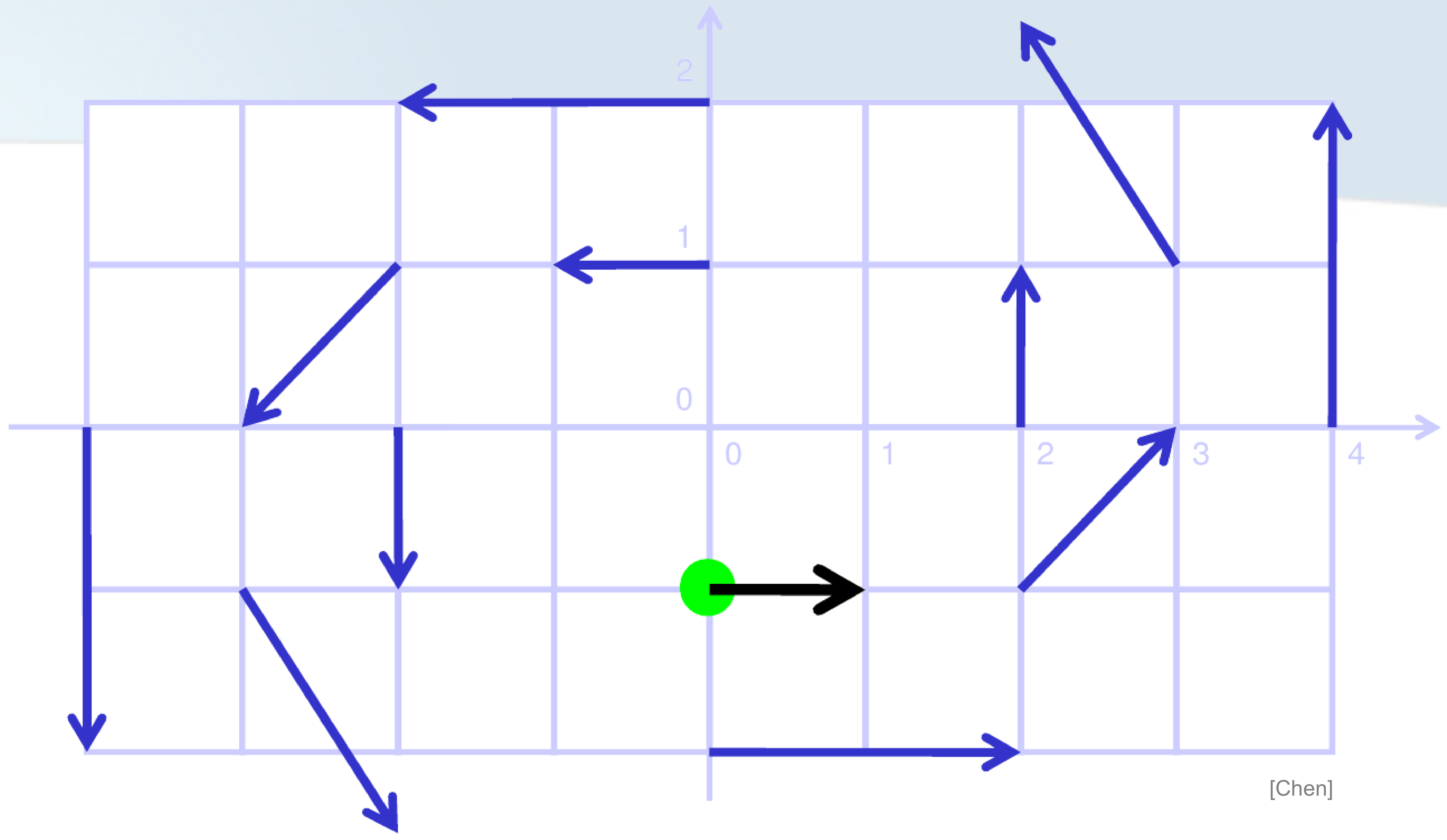

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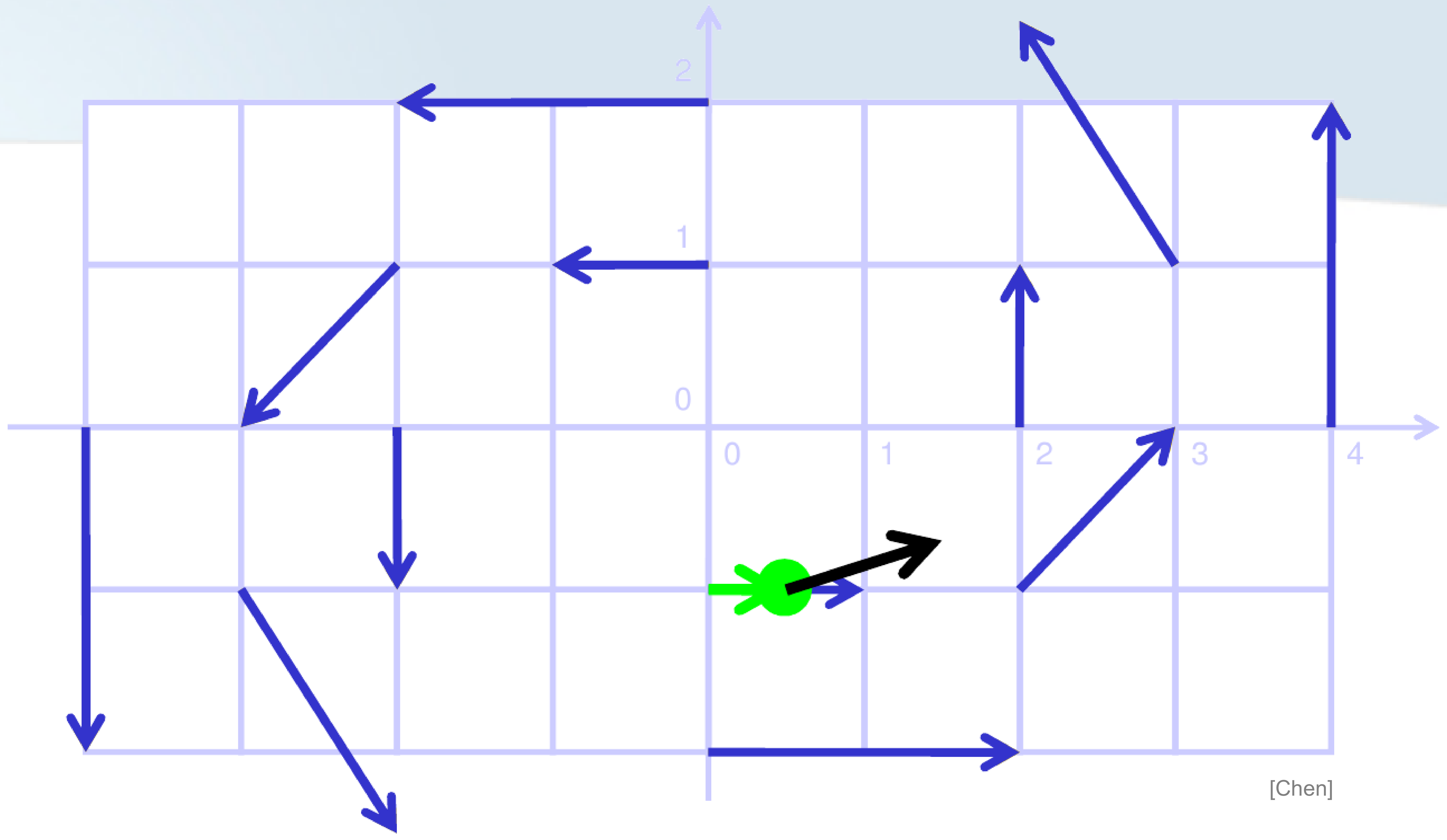

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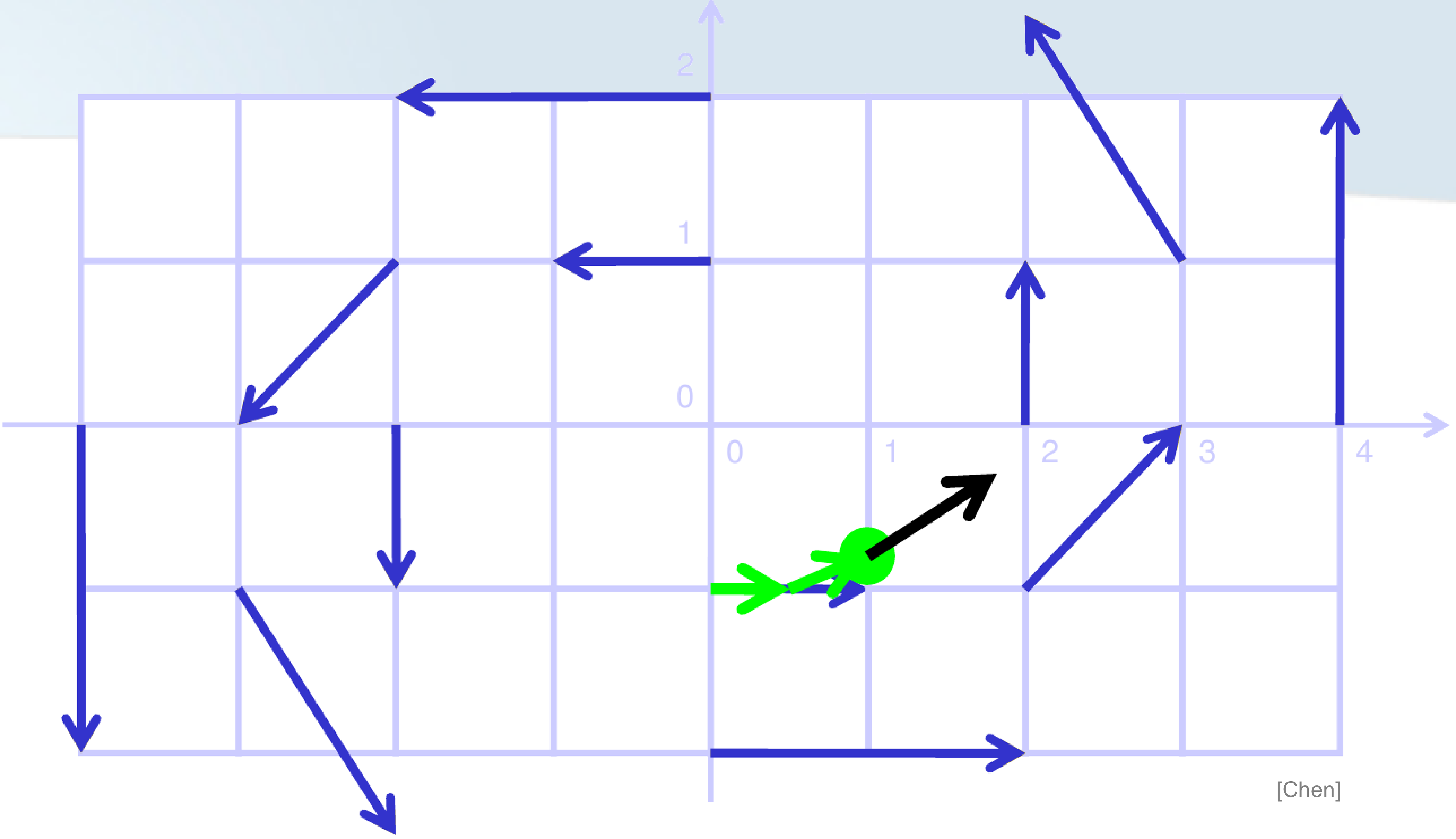
Streamlines on a computer



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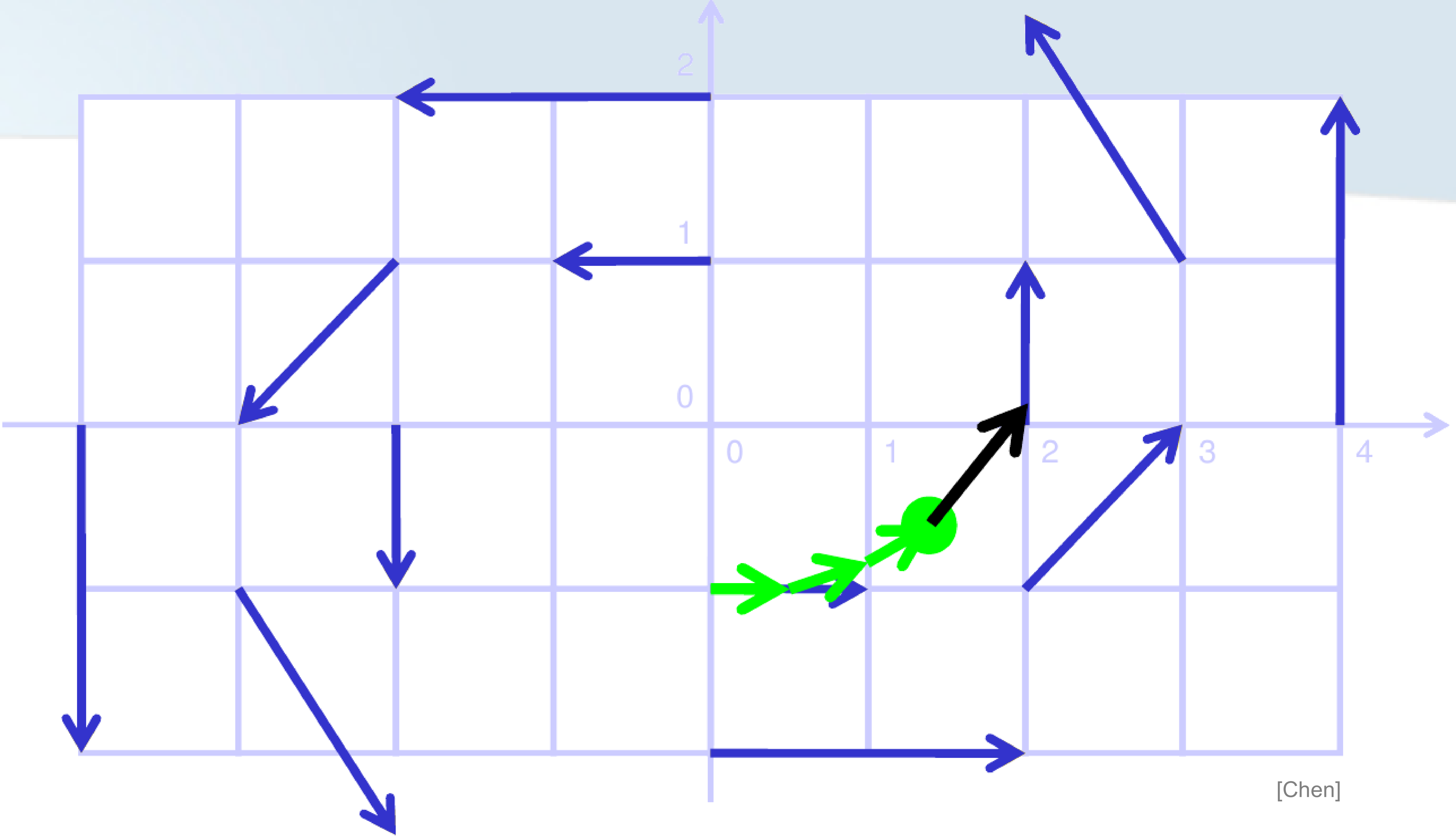

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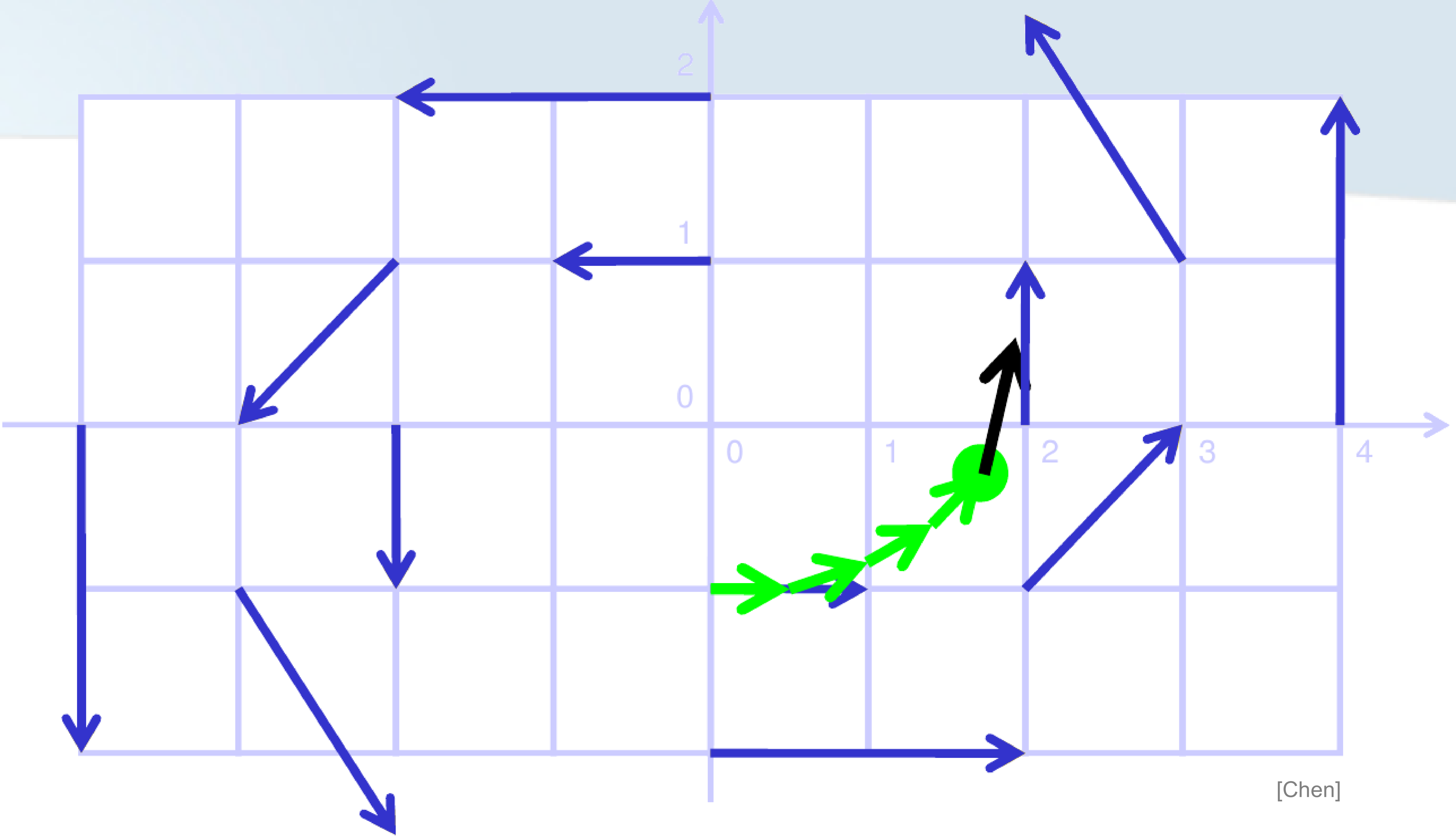

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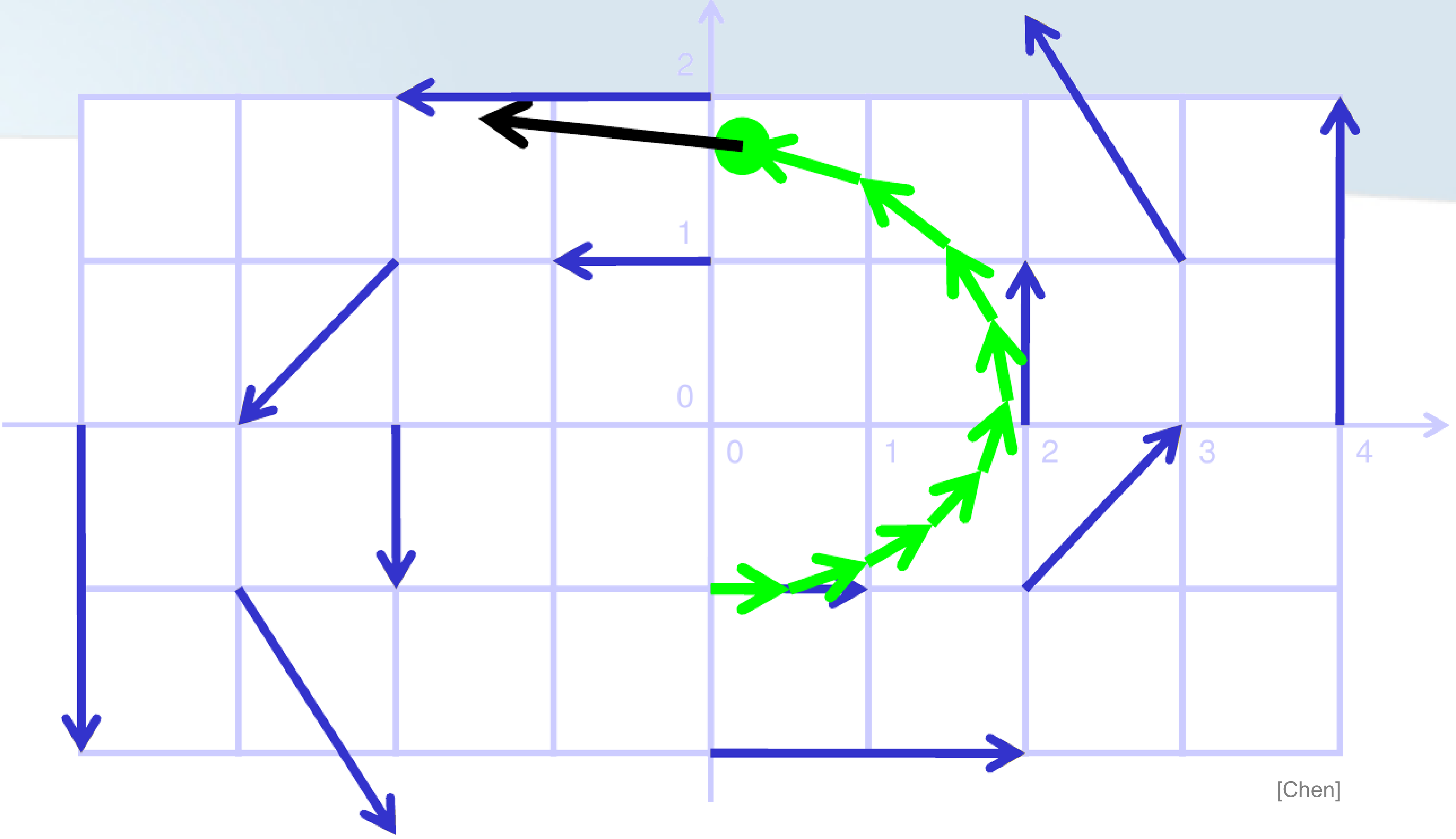

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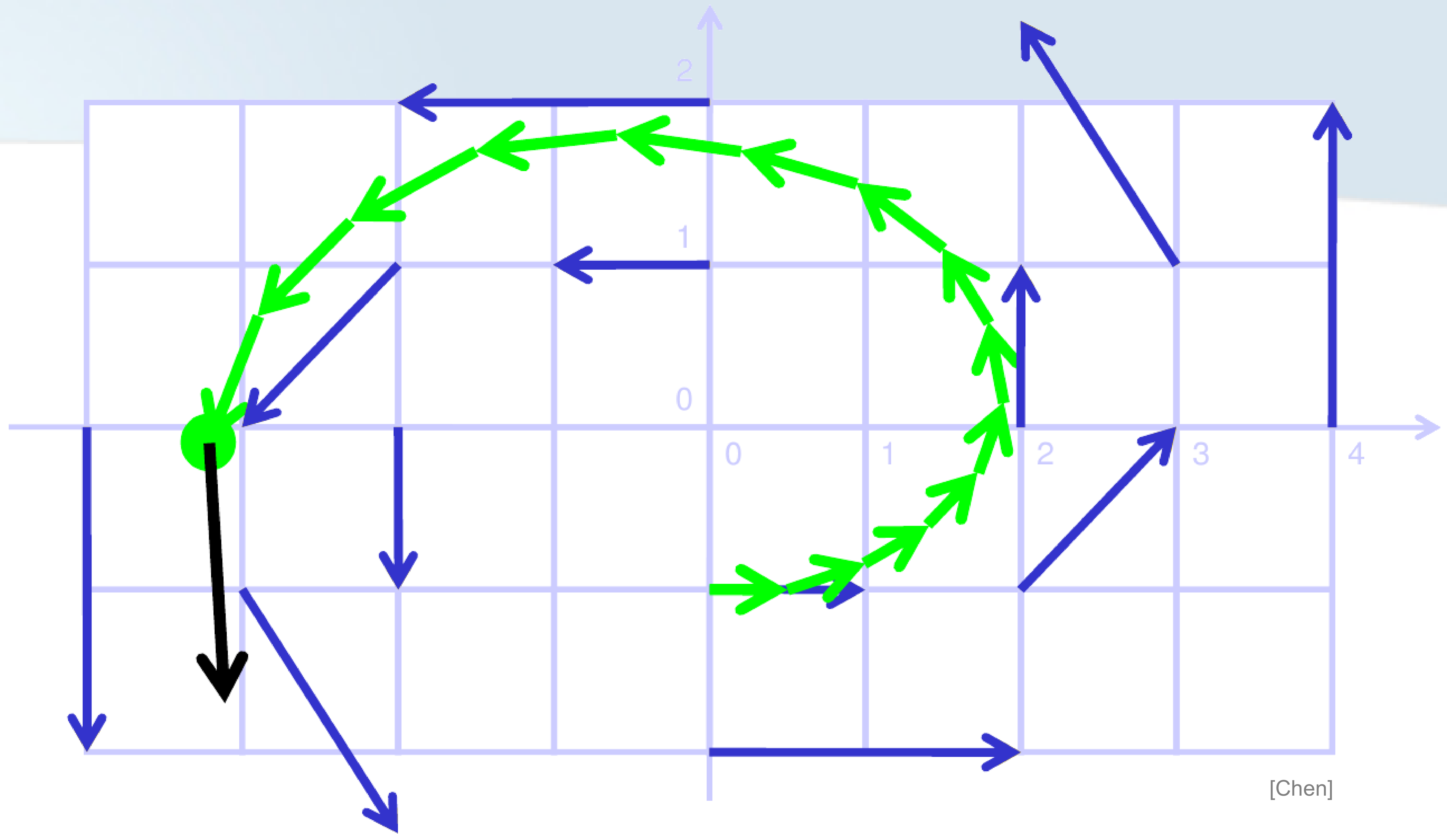

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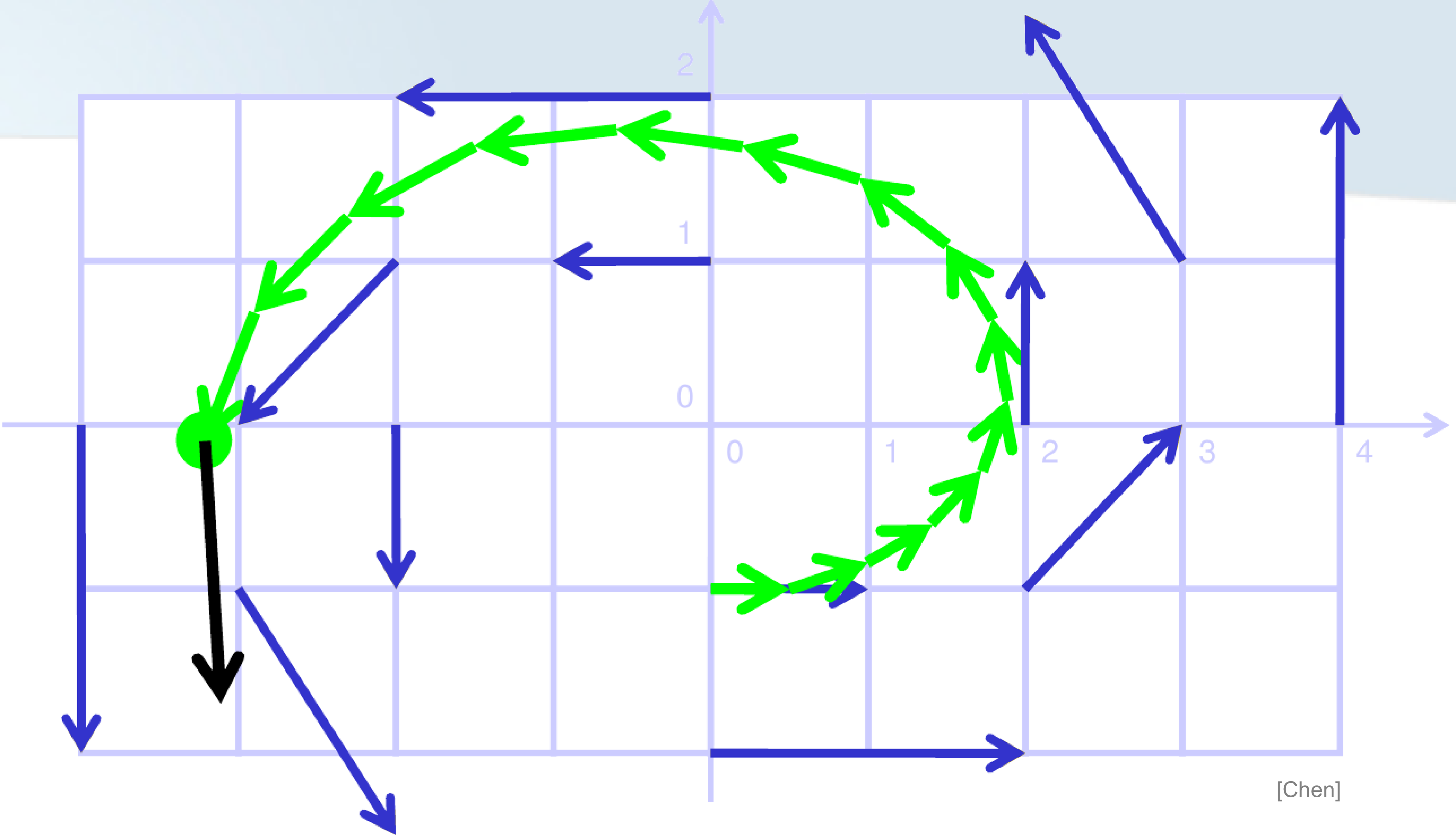

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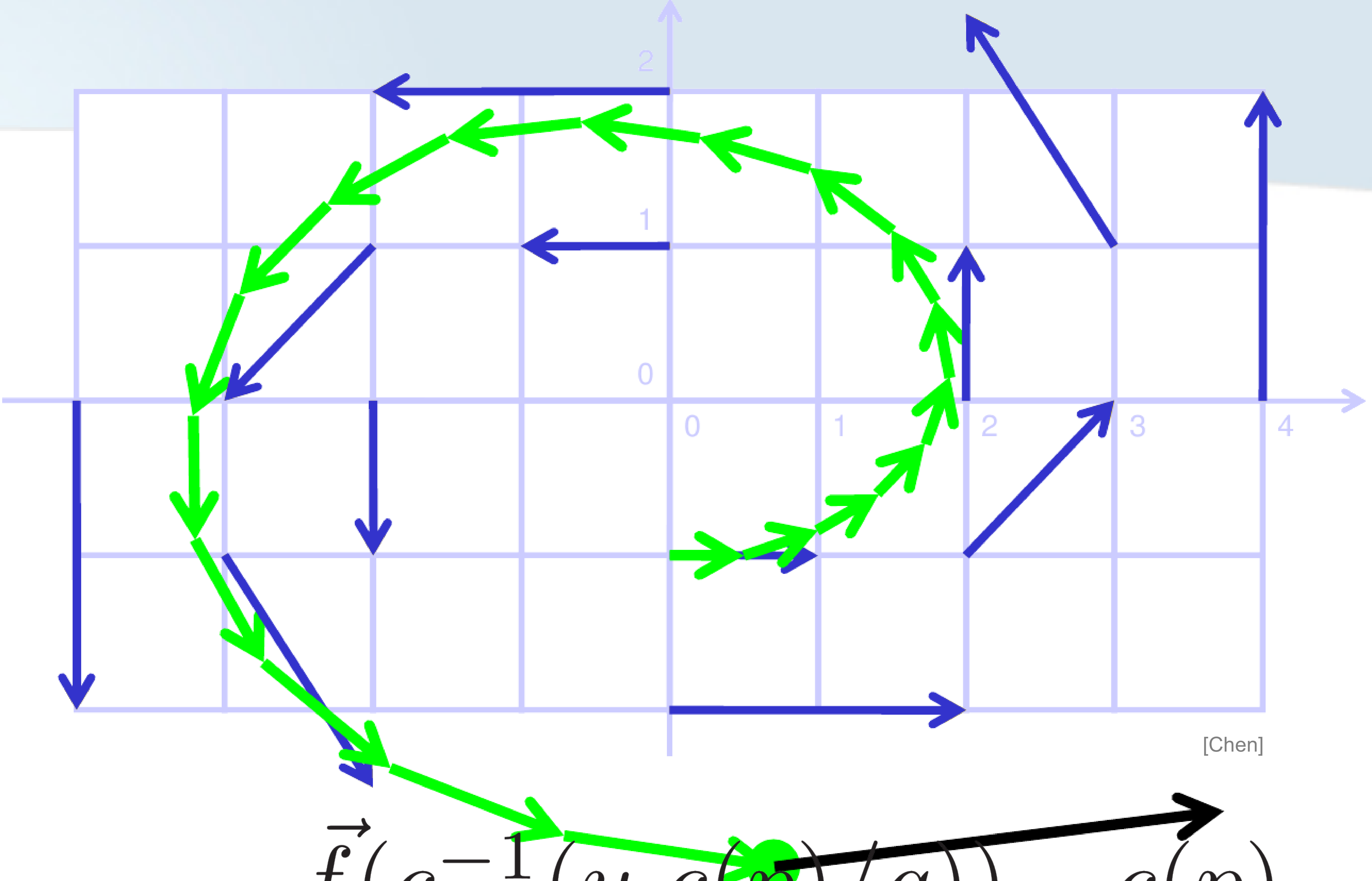
Streamlines on a computer



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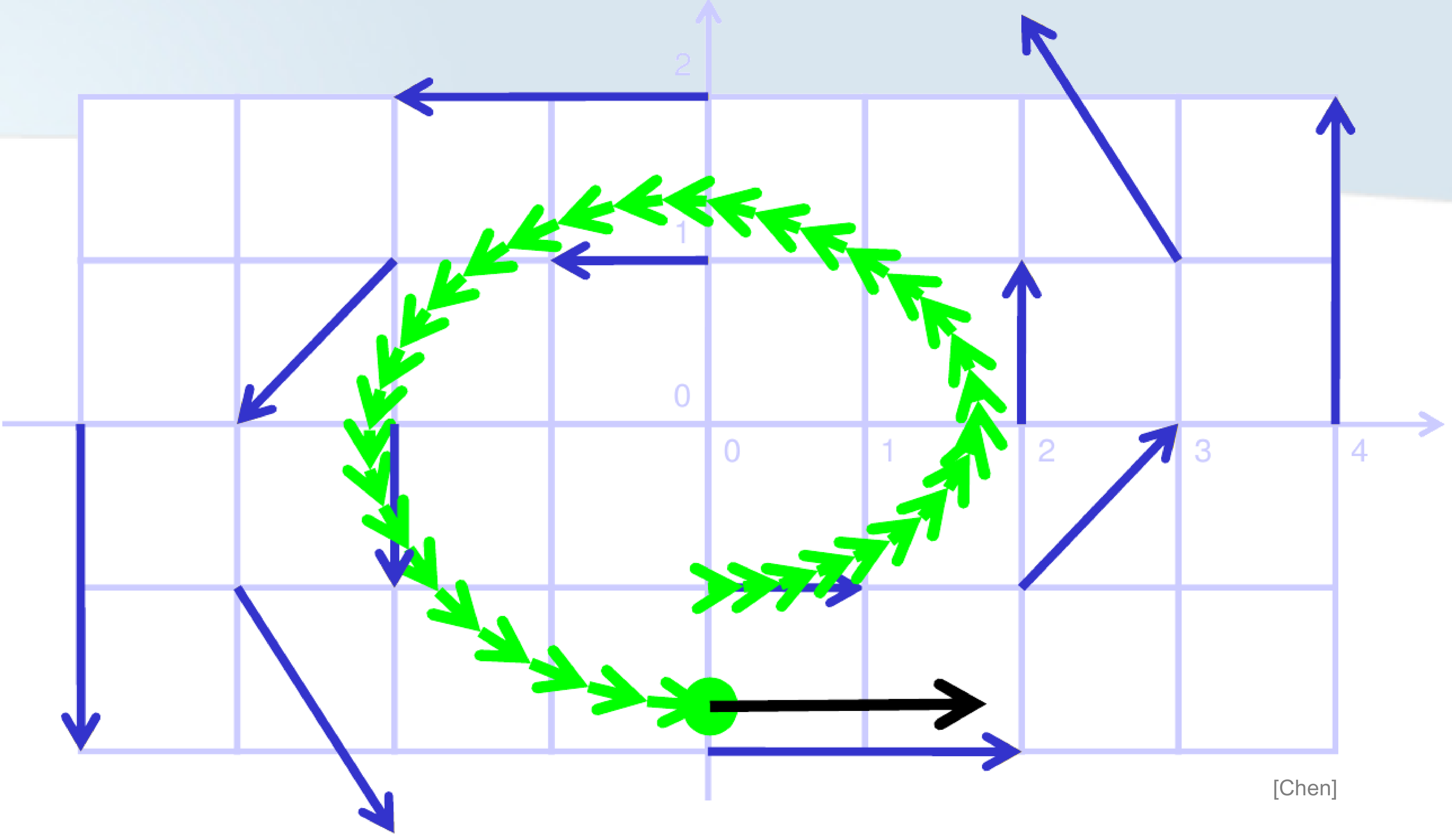

[Chen]

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Streamlines on a computer

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Streamlines on a computer

- Euler integration algorithm
  - Fixed step size
  - Ratio of the magnitude

Streamlines on a computer

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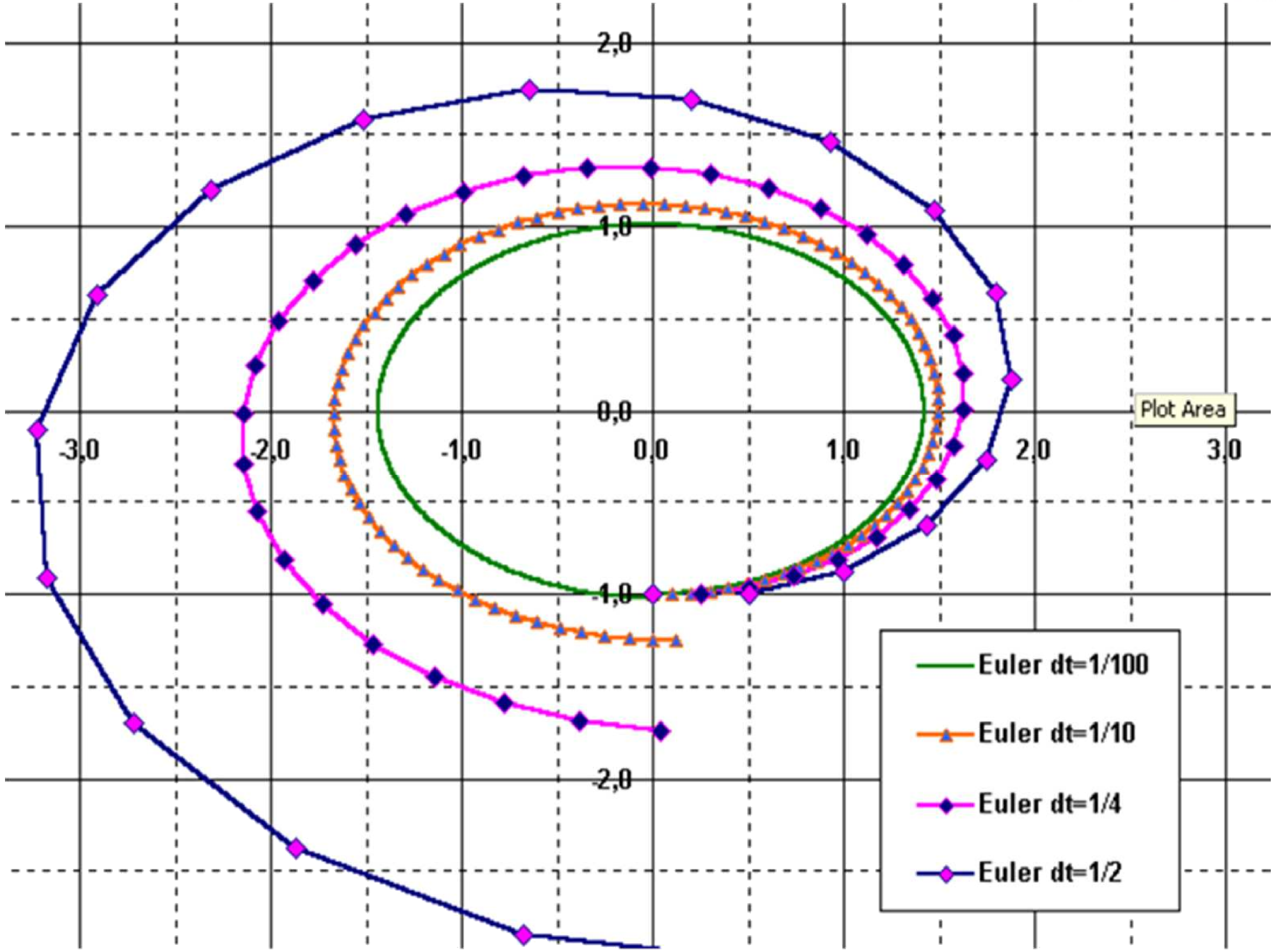
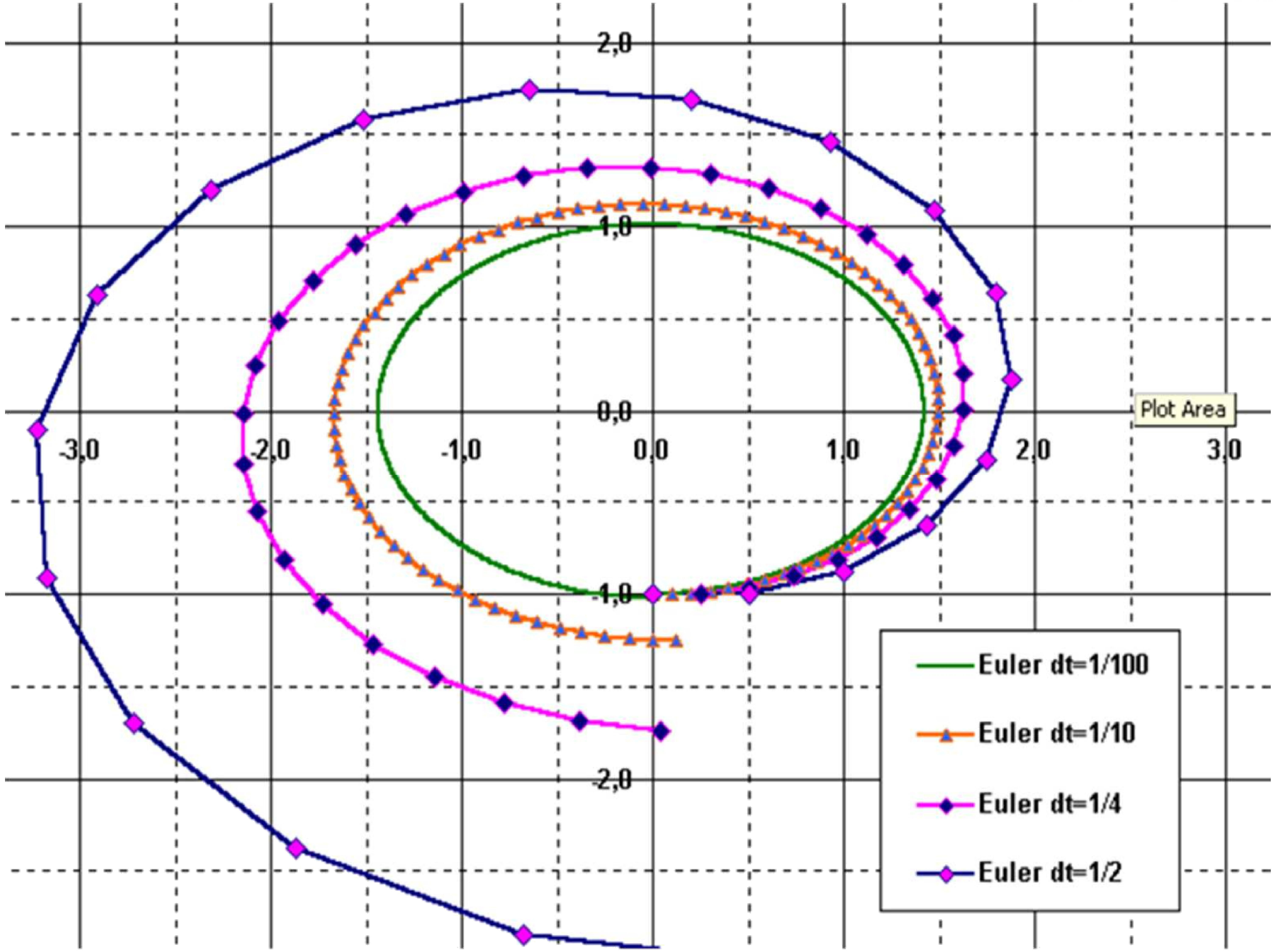


[Chen]

Streamlines on a computer

- Euler integration algorithm
  - Fixed step size
  - Ratio of the magnitude
- Trade-off
  - Sampling
  - Approximation quality



[Chen]

Special case

- Gradient fields on PL-manifolds

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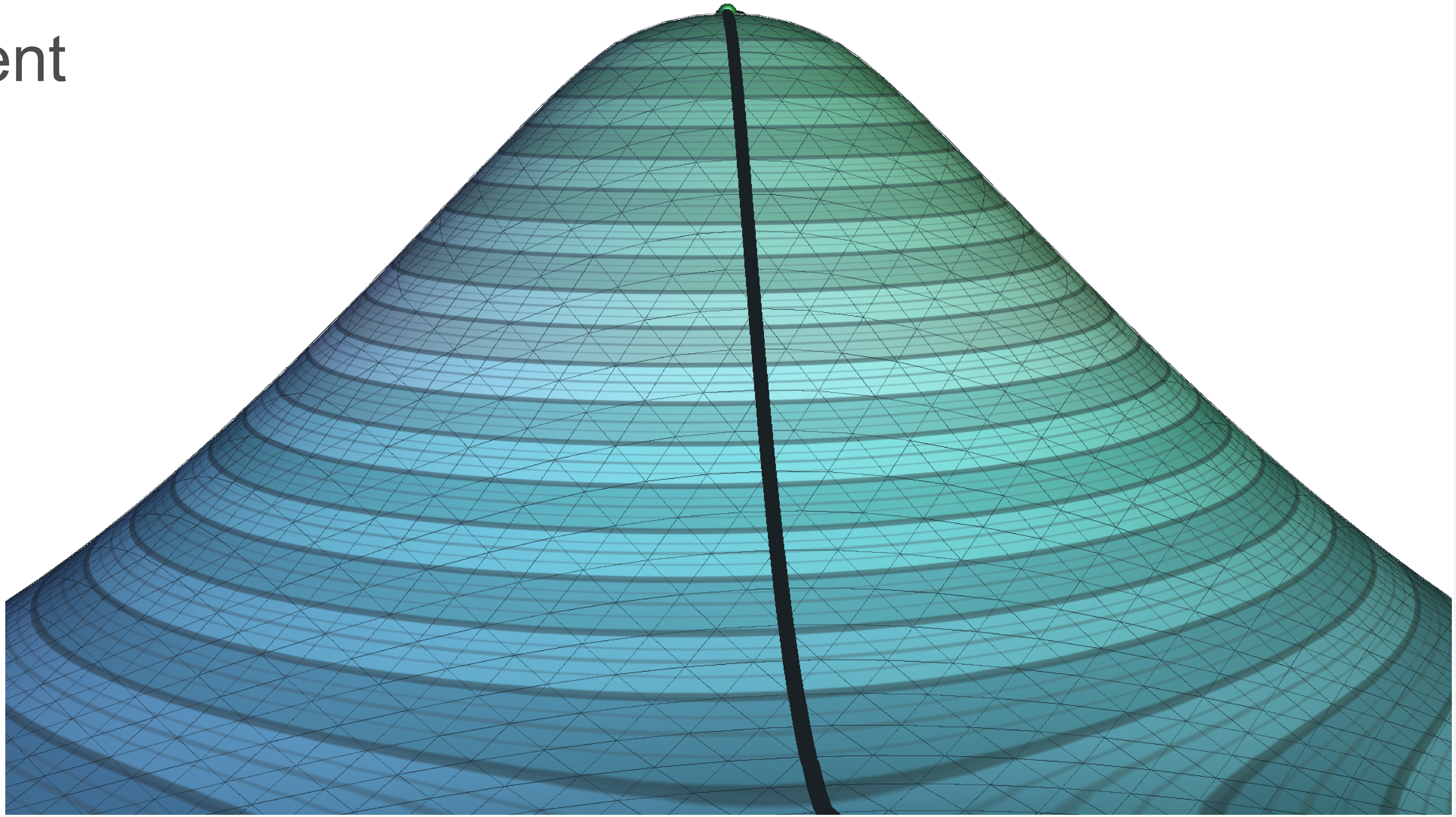
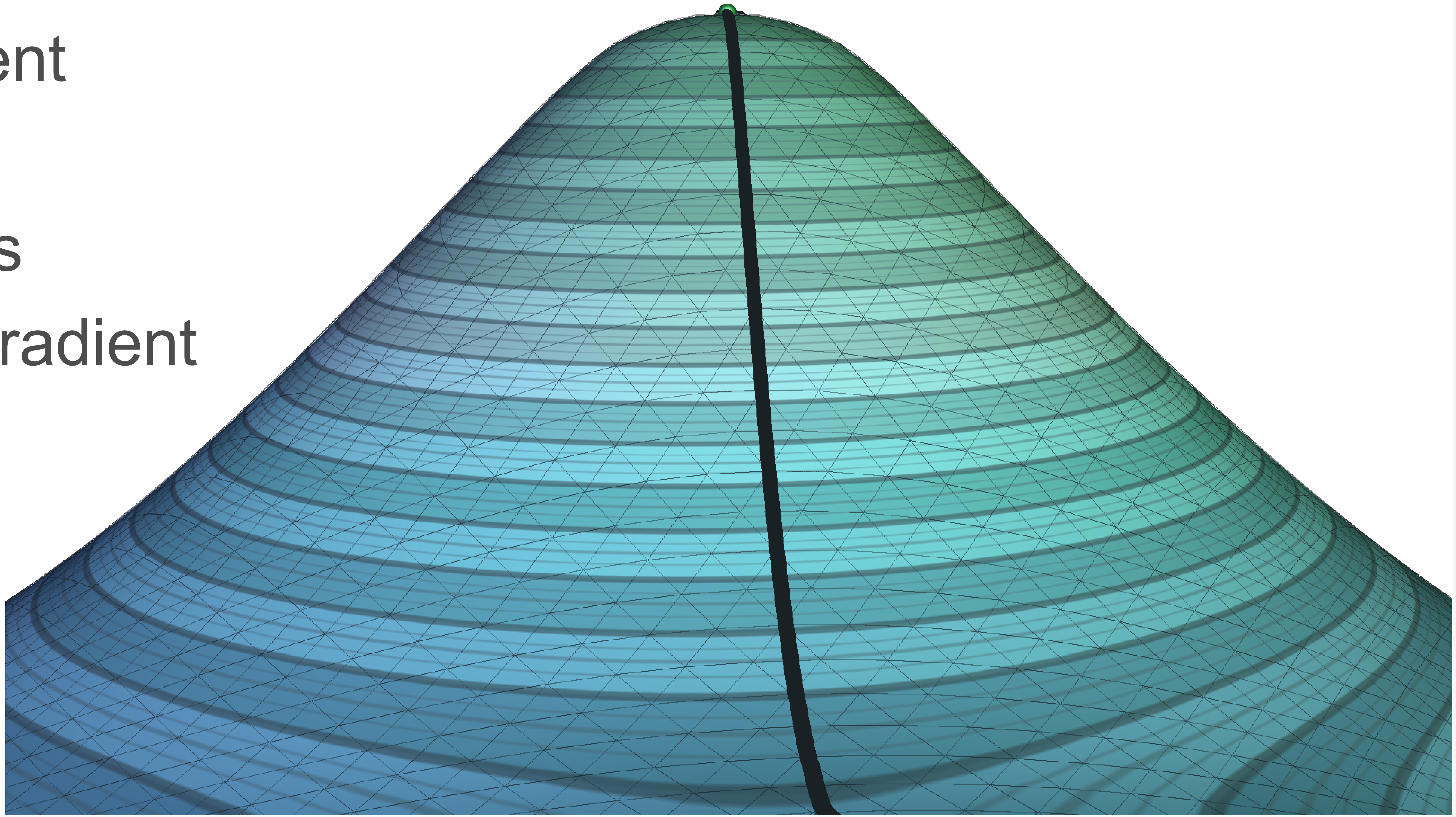


Special case

- Gradient fields on PL-manifolds
  - Path of steepest ascent
- Barycentric coordinates
  - Piecewise constant gradient



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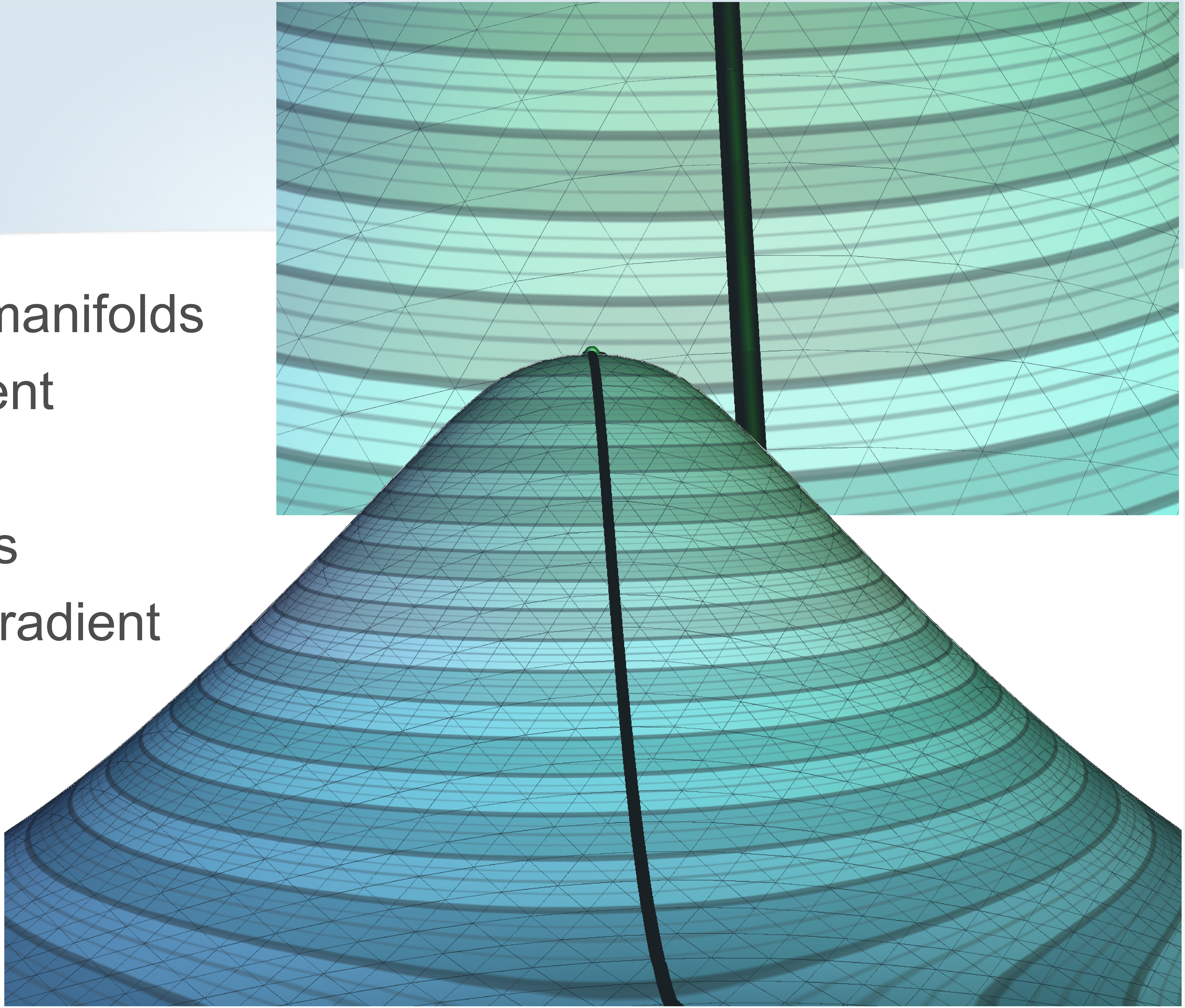

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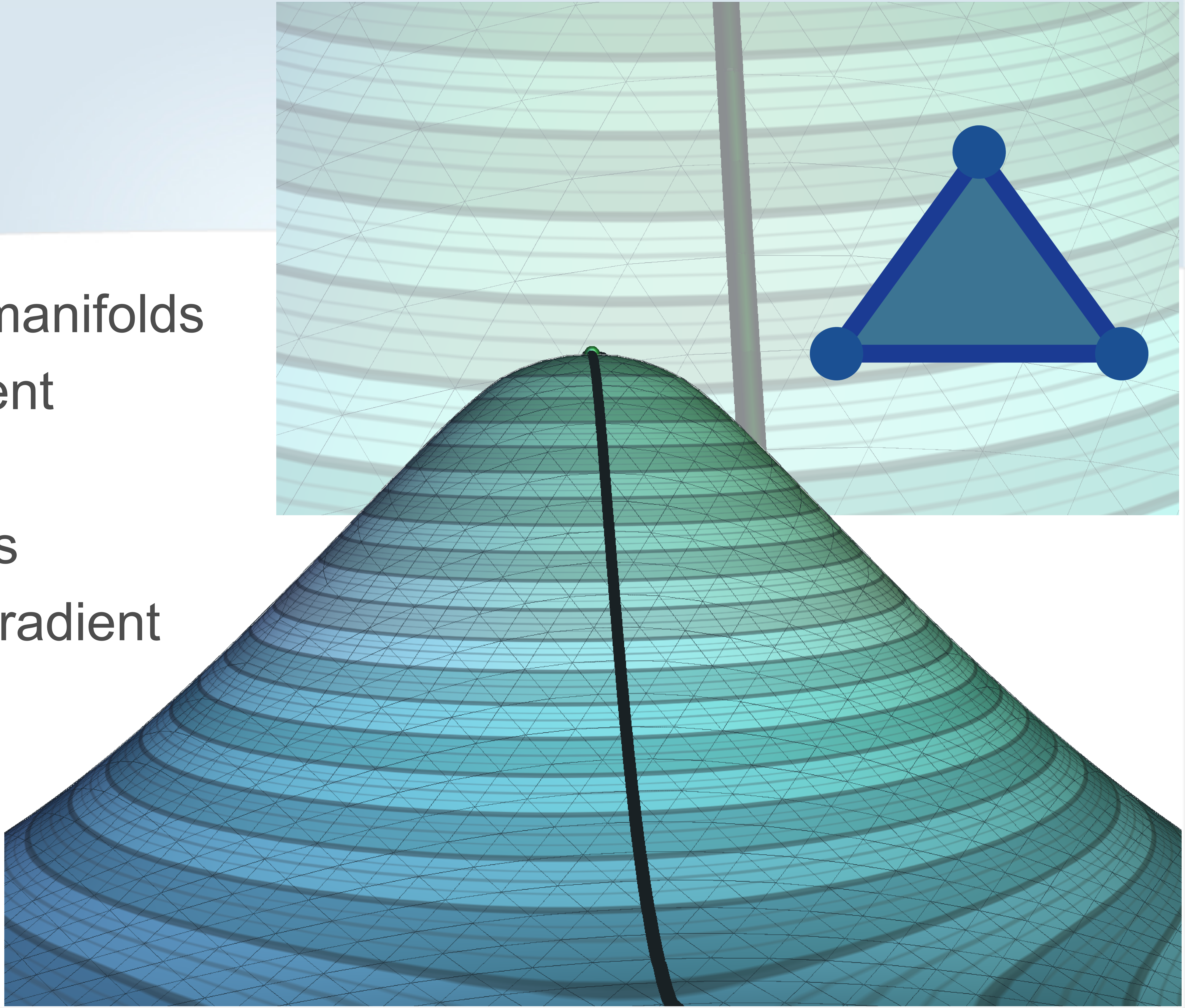

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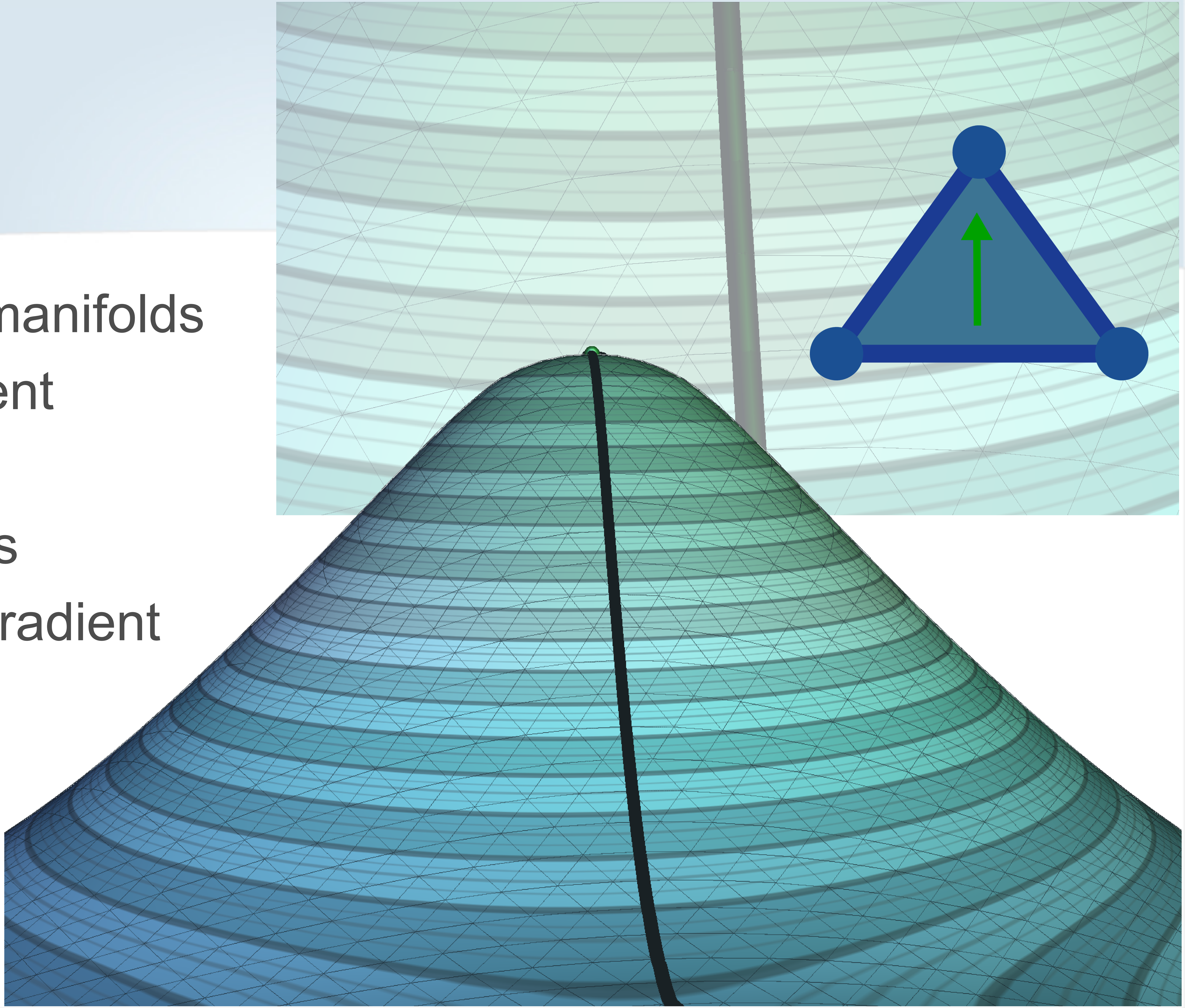

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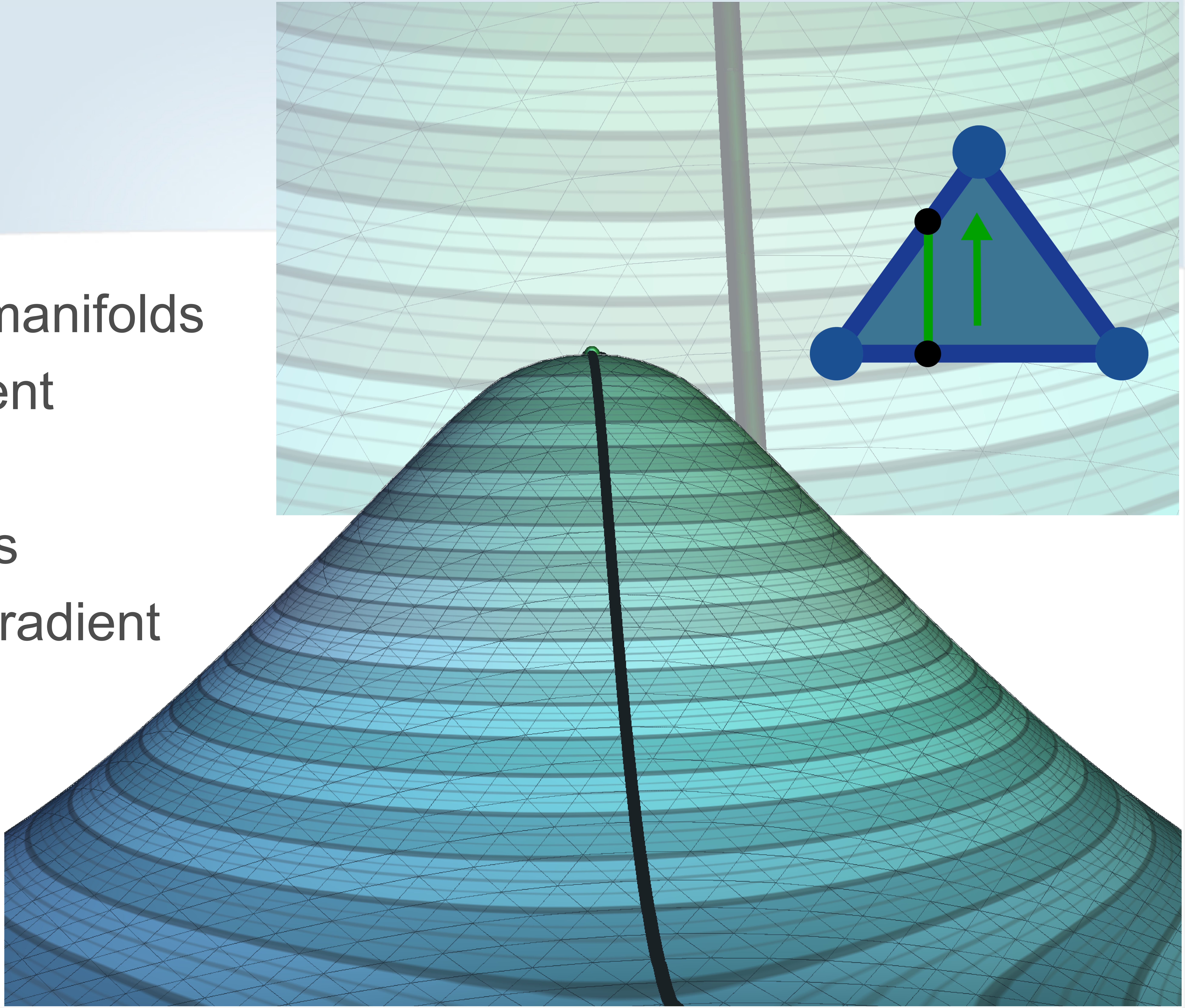

Special case

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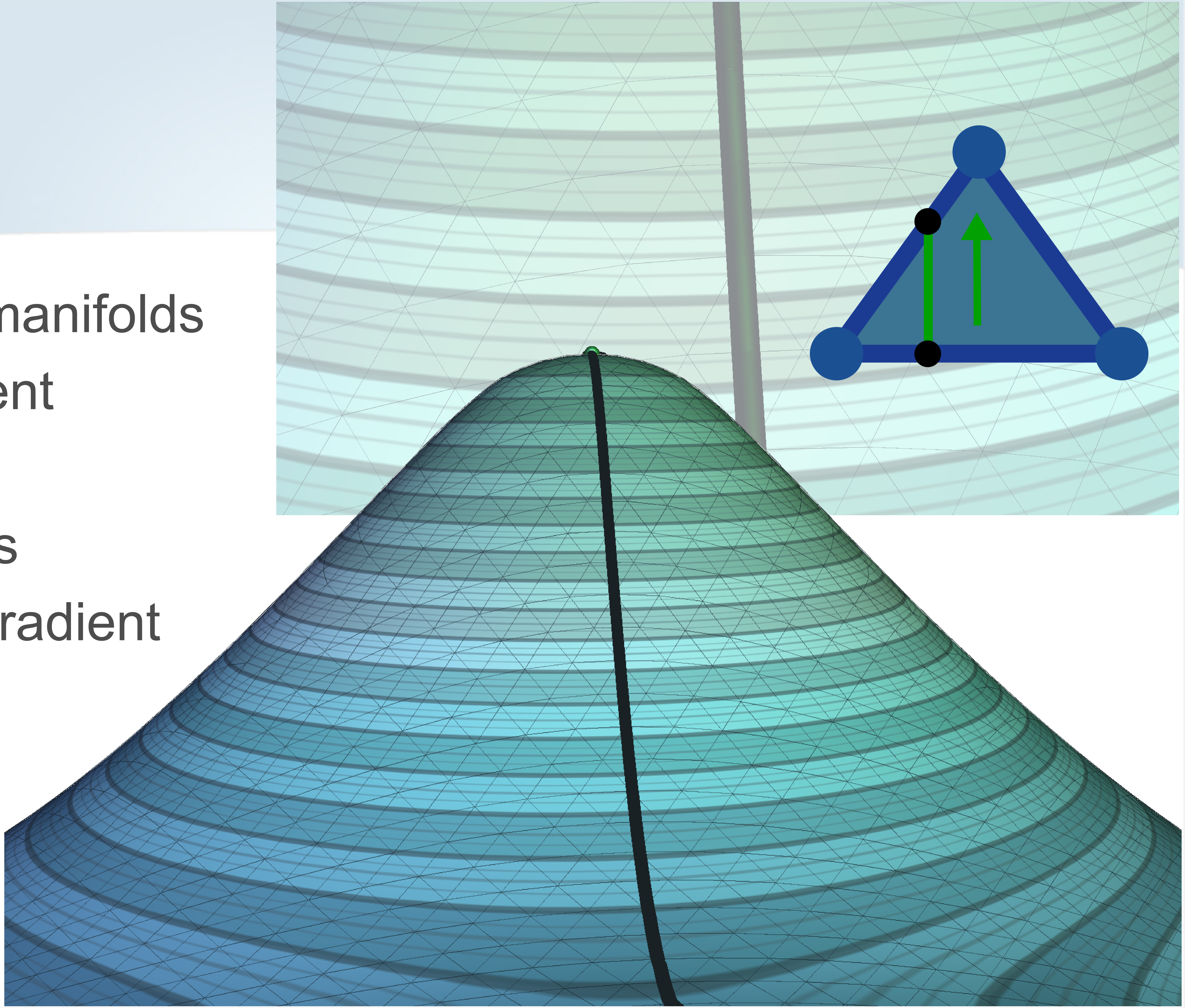
  - Nearly no ambiguity



Special case

- Gradient fields on PL-manifolds
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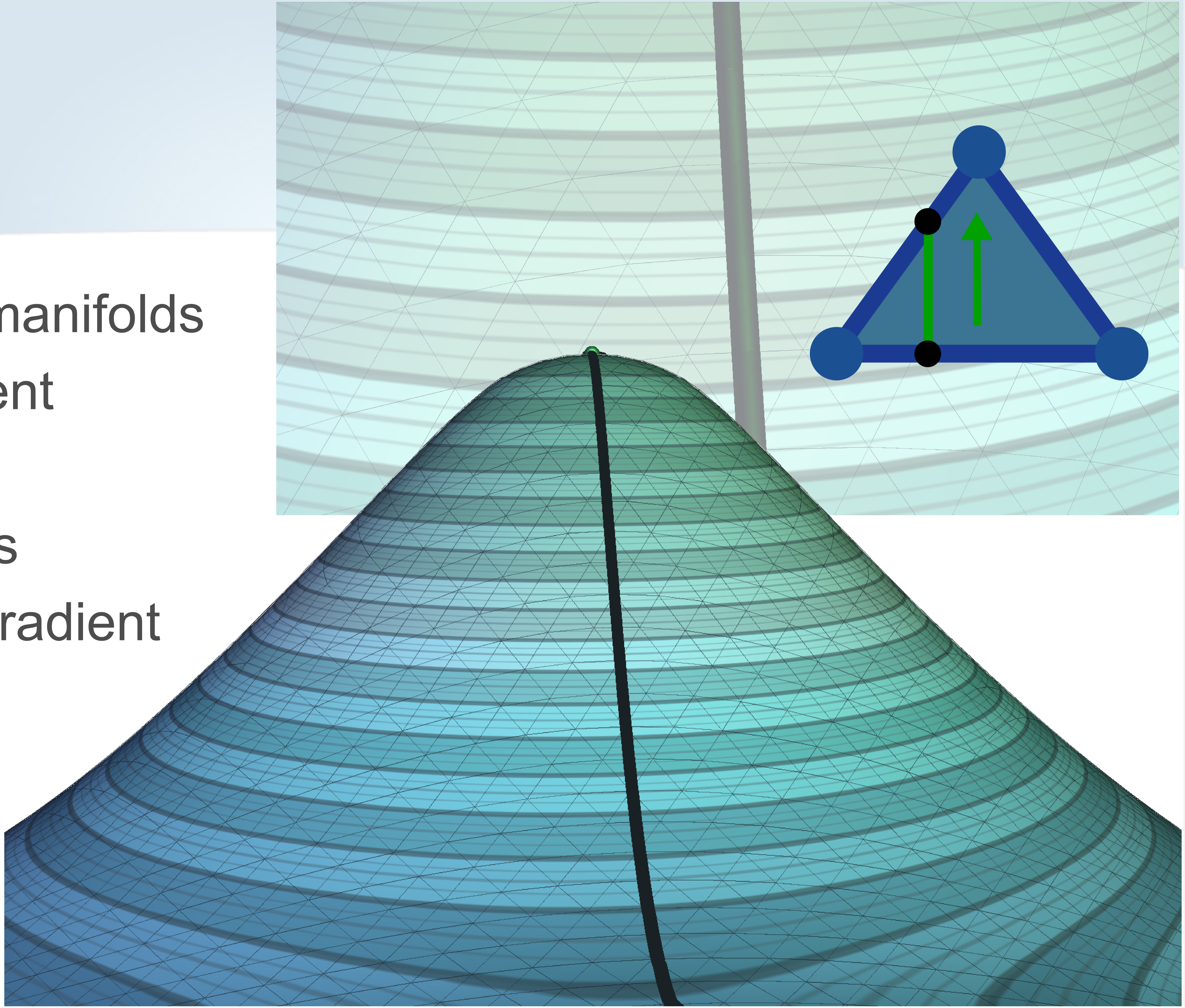
  - No integration error
  - Nearly no ambiguity
- Primal/dual meshes



Seeding streamlines

- Now, given a seed point

Seeding streamlines

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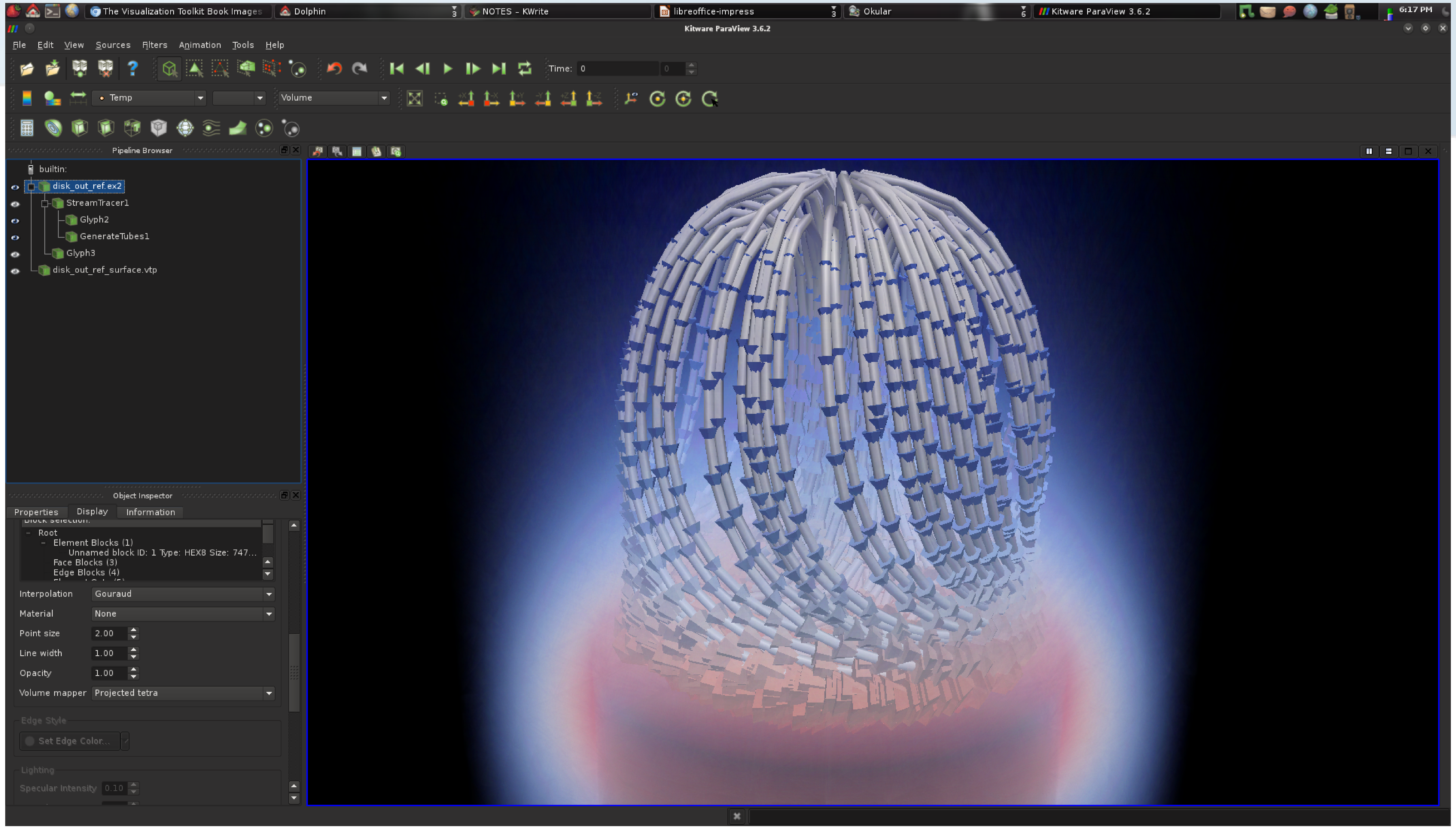
  - We can extract a streamline

Seeding streamlines



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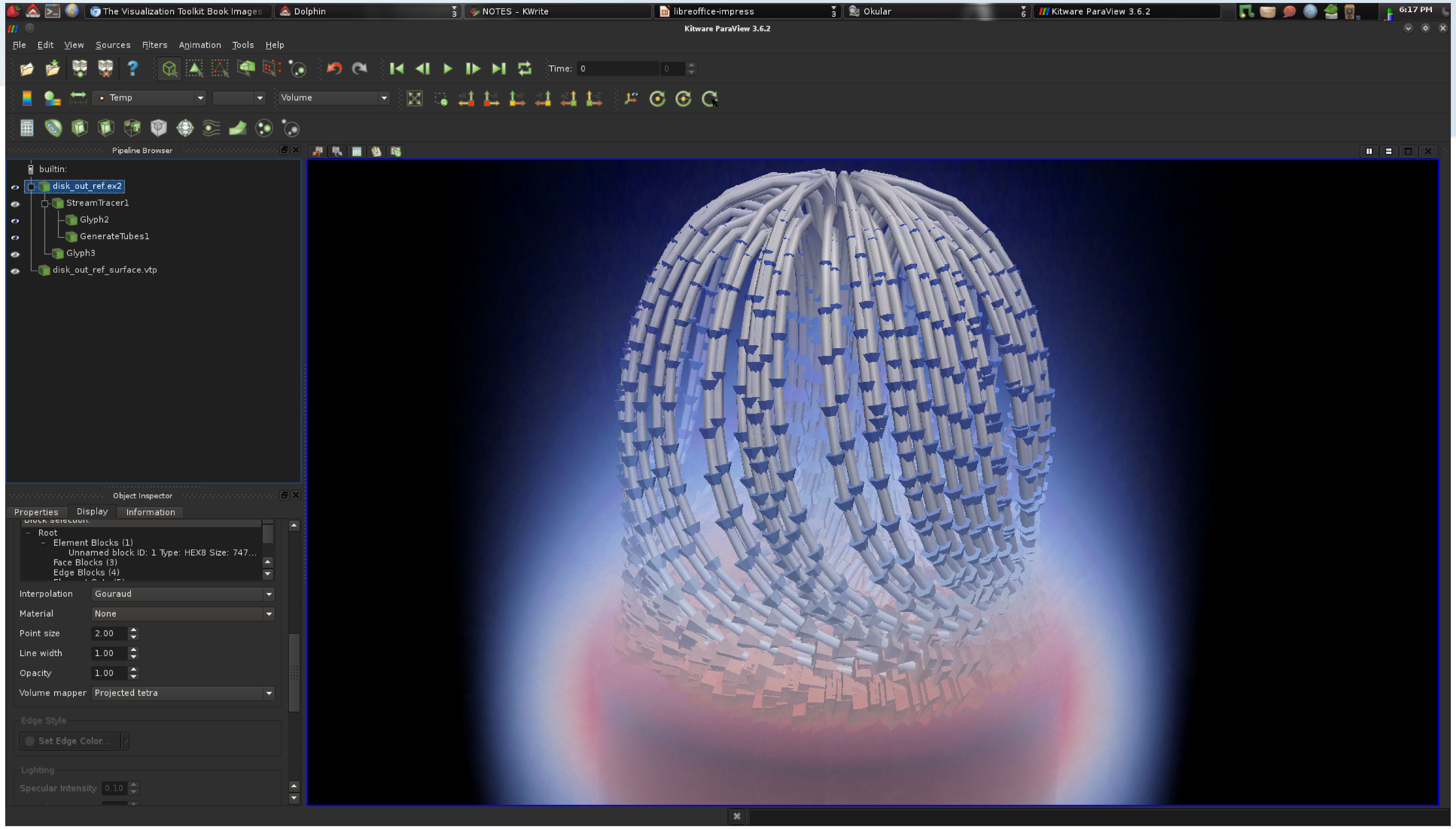

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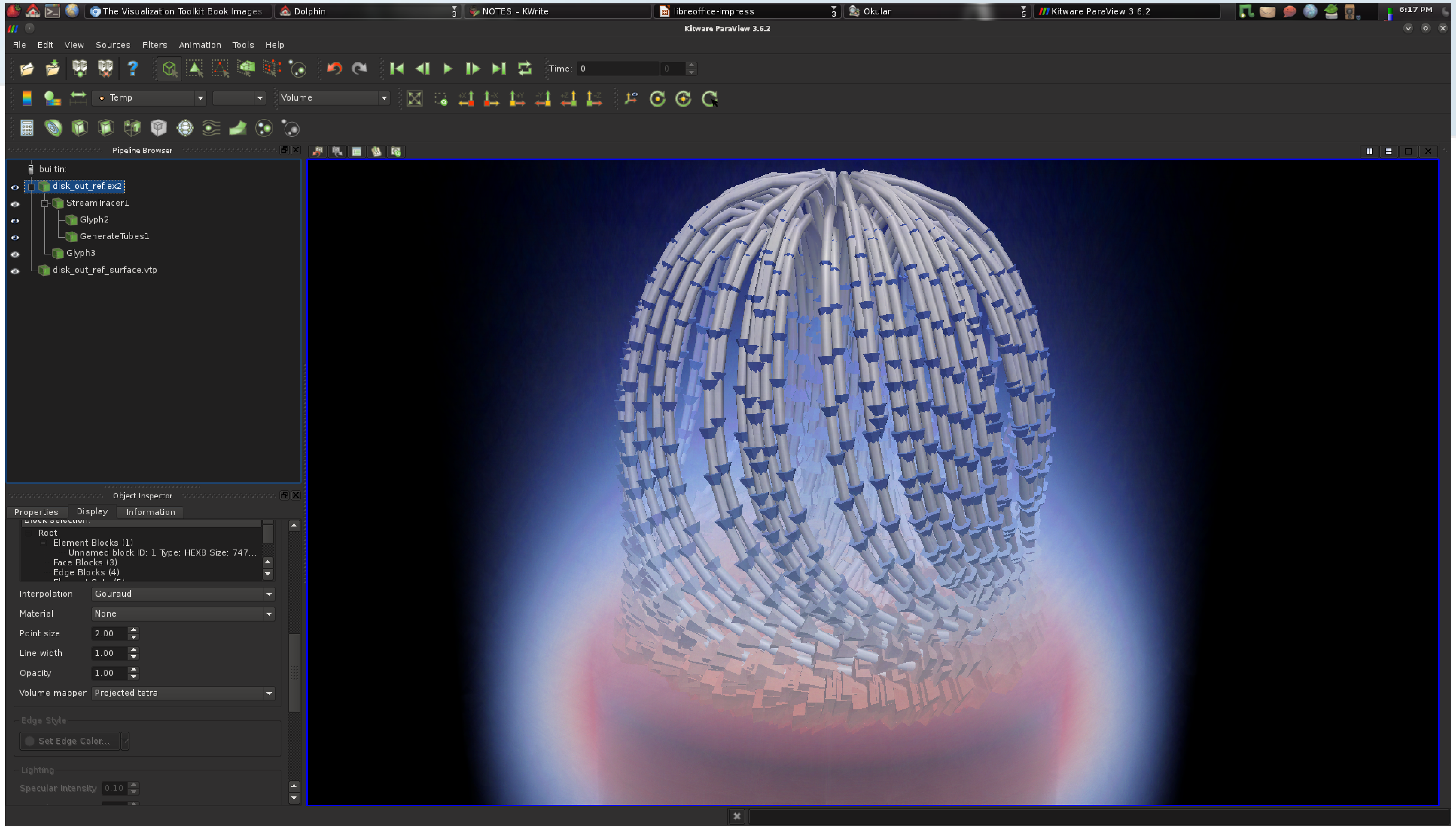

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- How to choose seeds?
  - Avoid data overload

Seeding streamlines

- For example,

Seeding streamlines

- For example,
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Seeding streamlines

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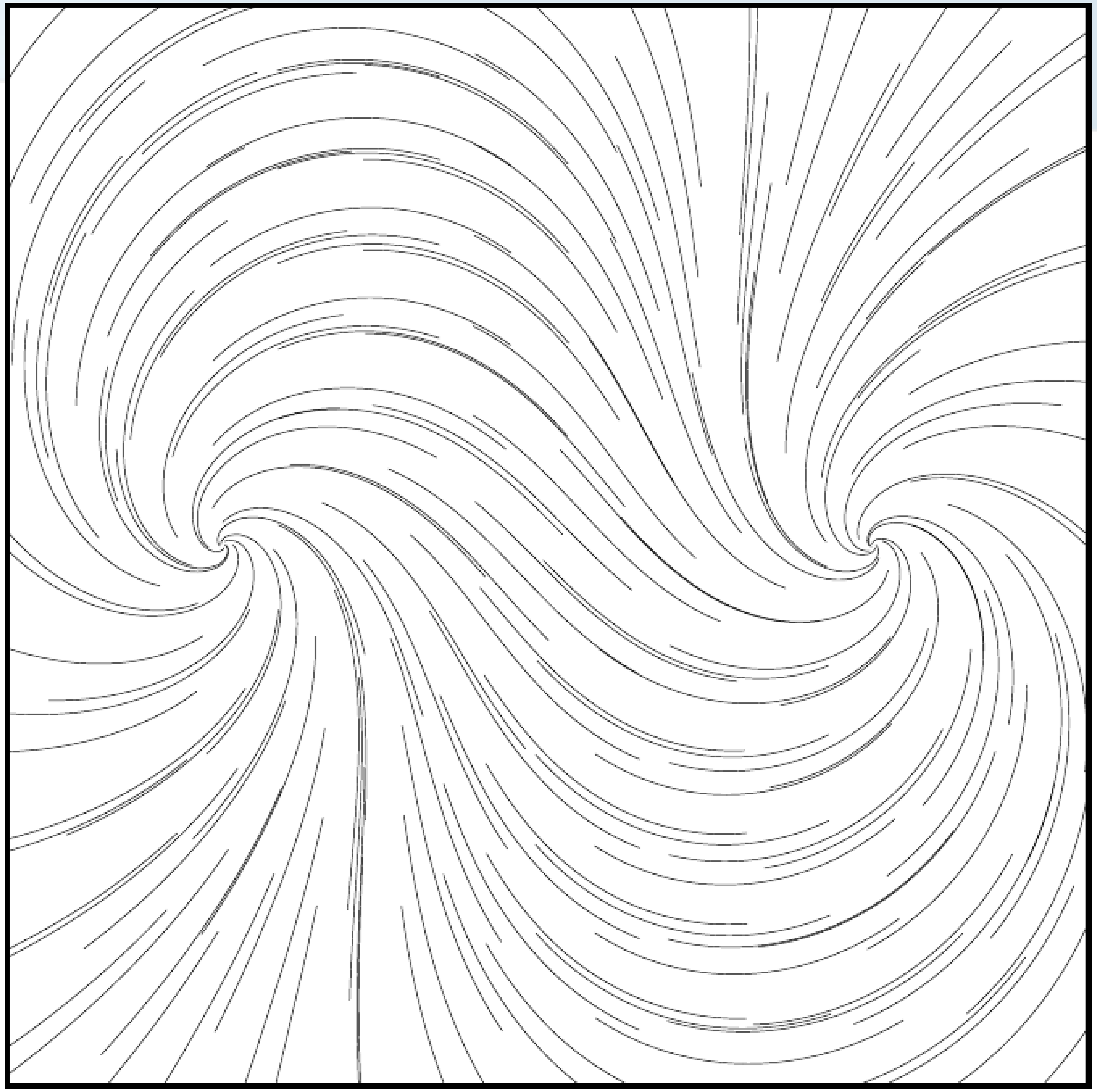
- Results



[Jobard et al. 97]

Seeding streamlines

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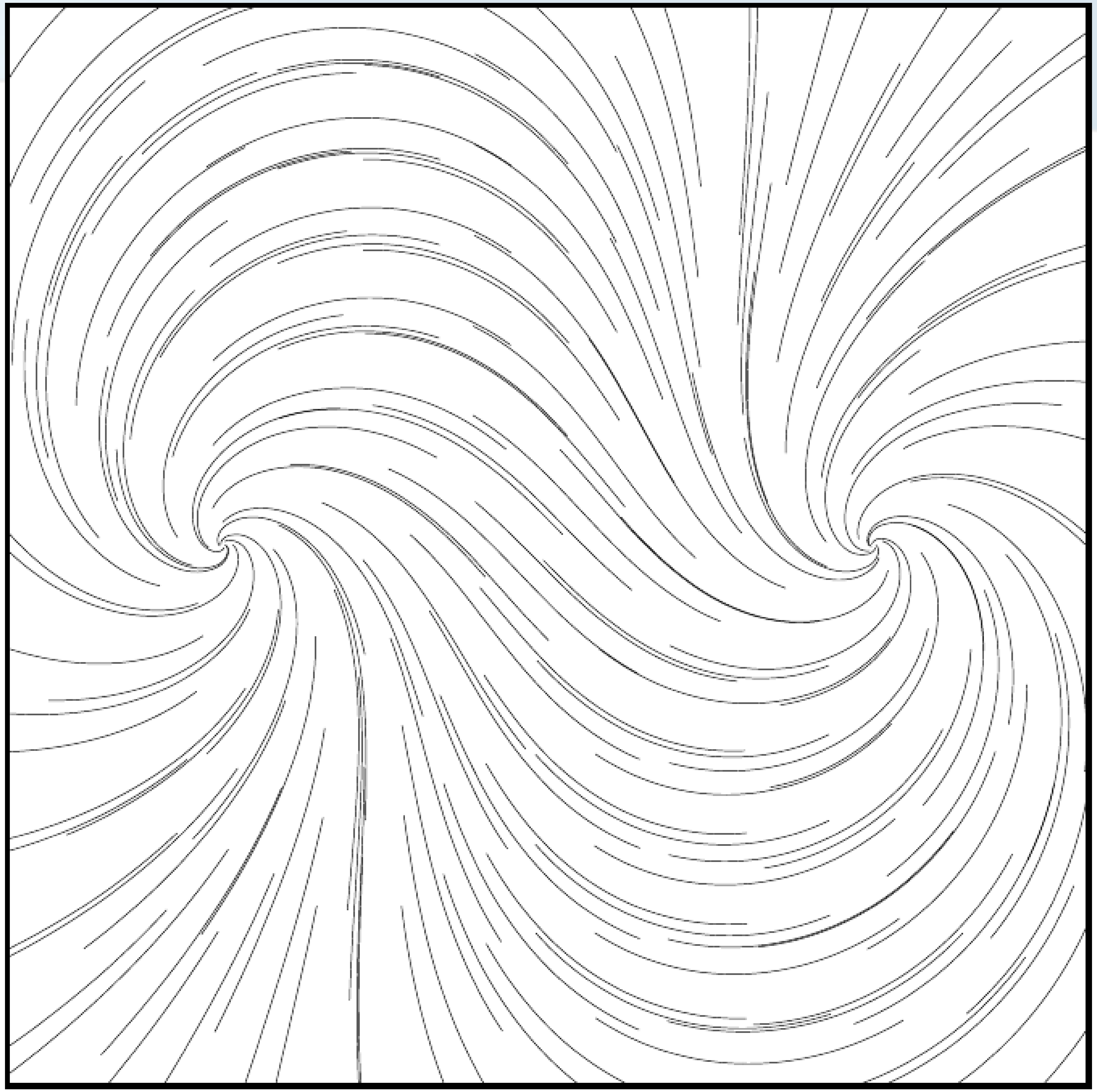
  - Not evenly distributed streamlines



[Jobard et al. 97]

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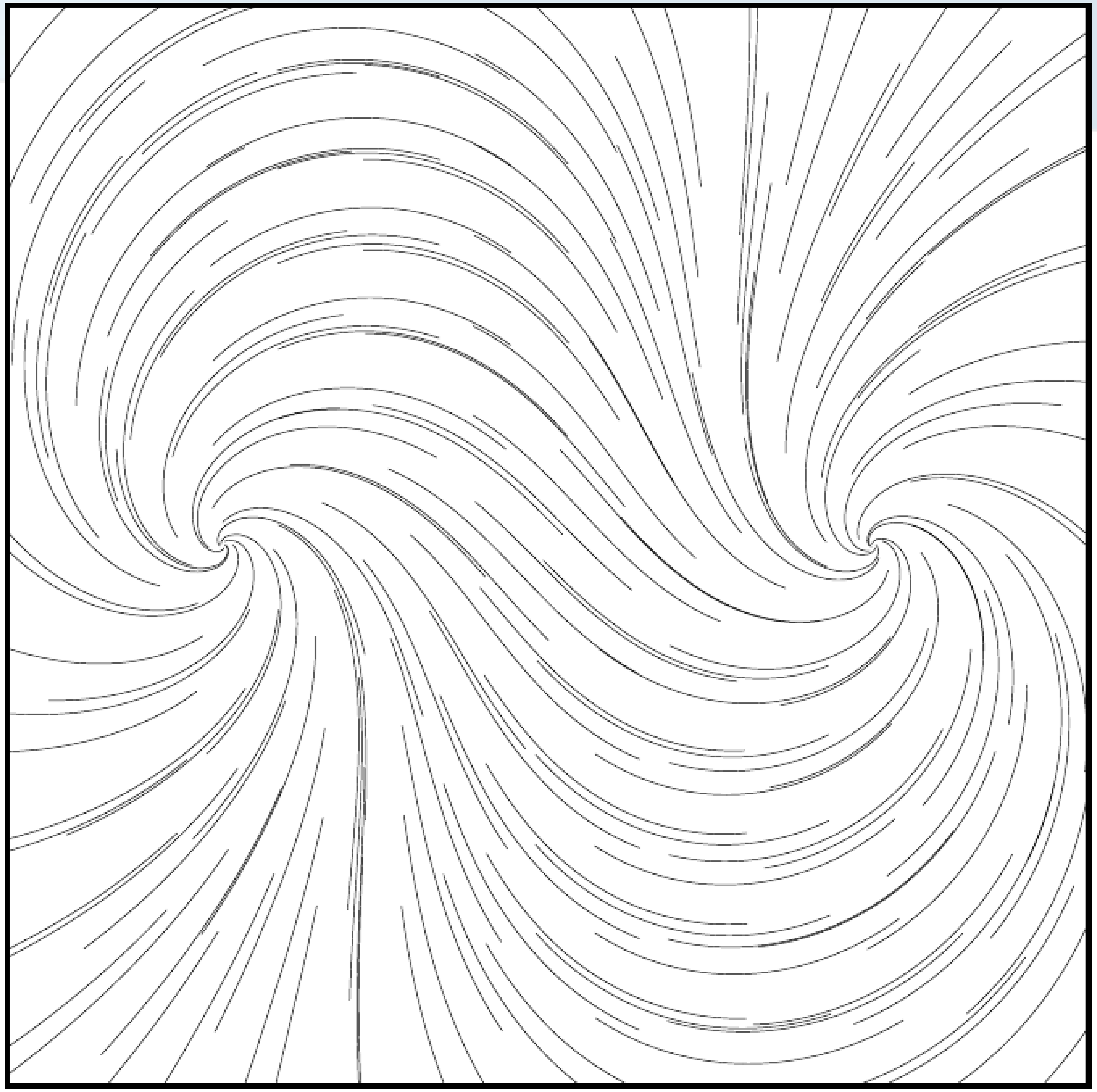

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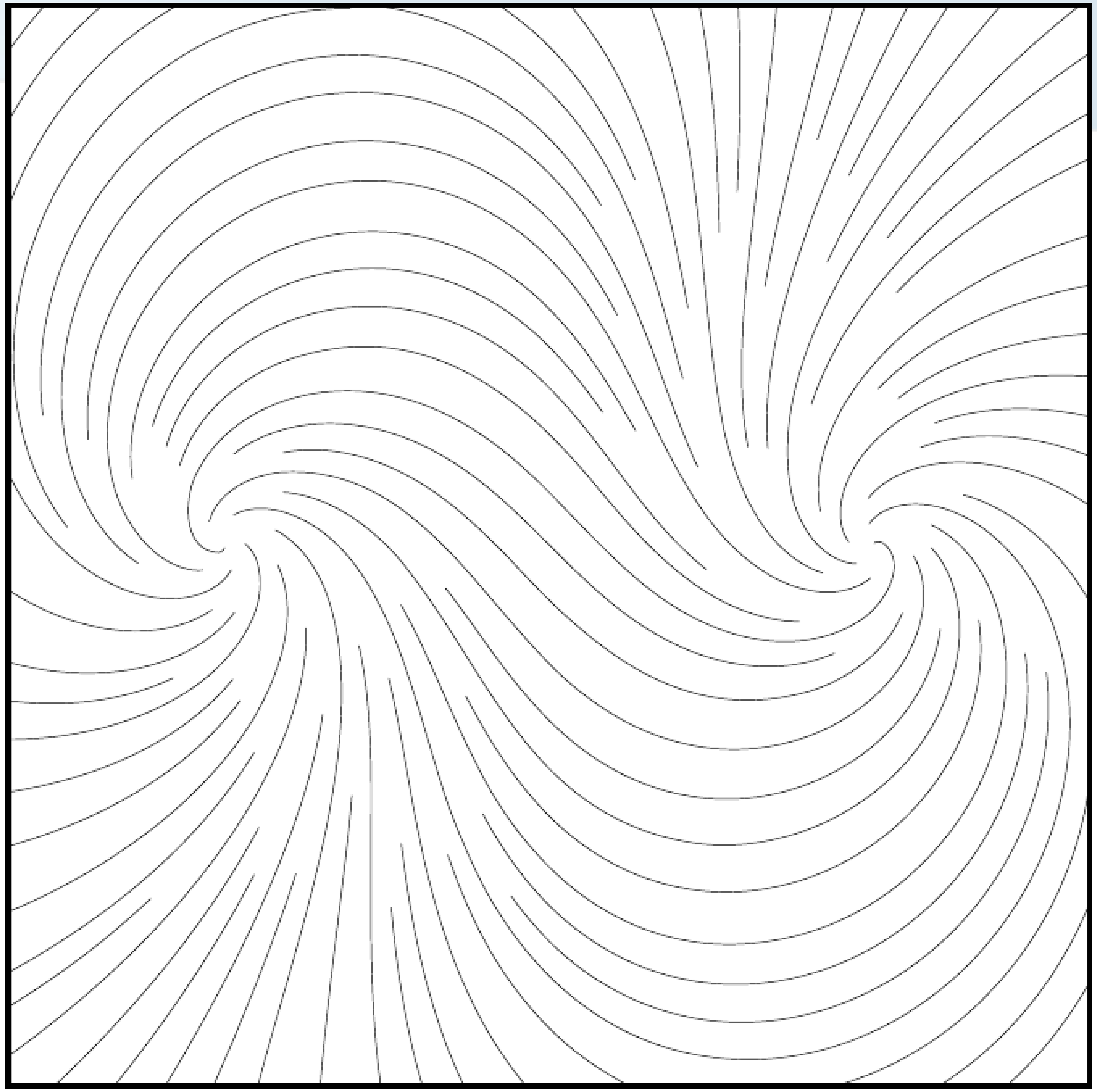

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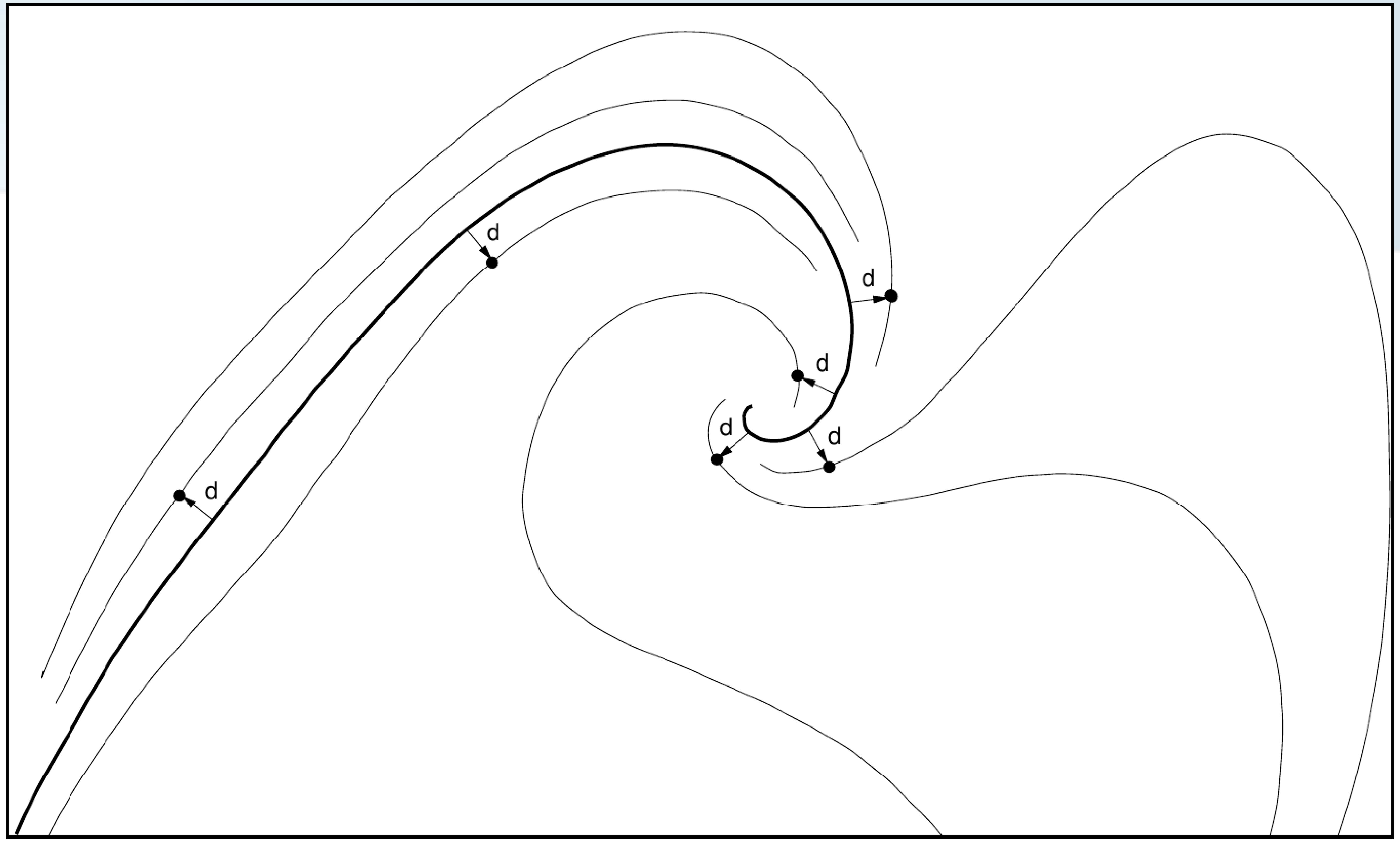

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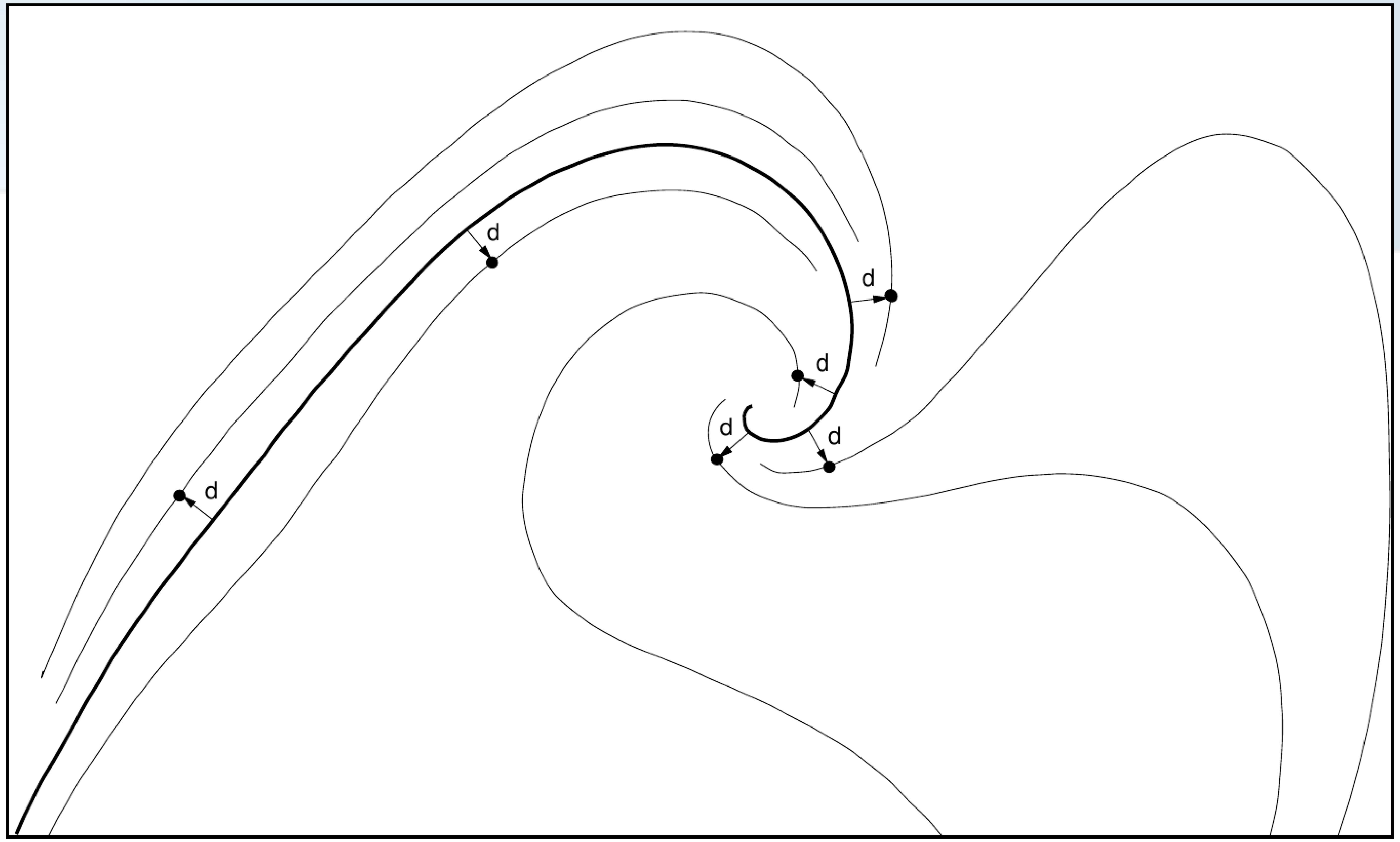

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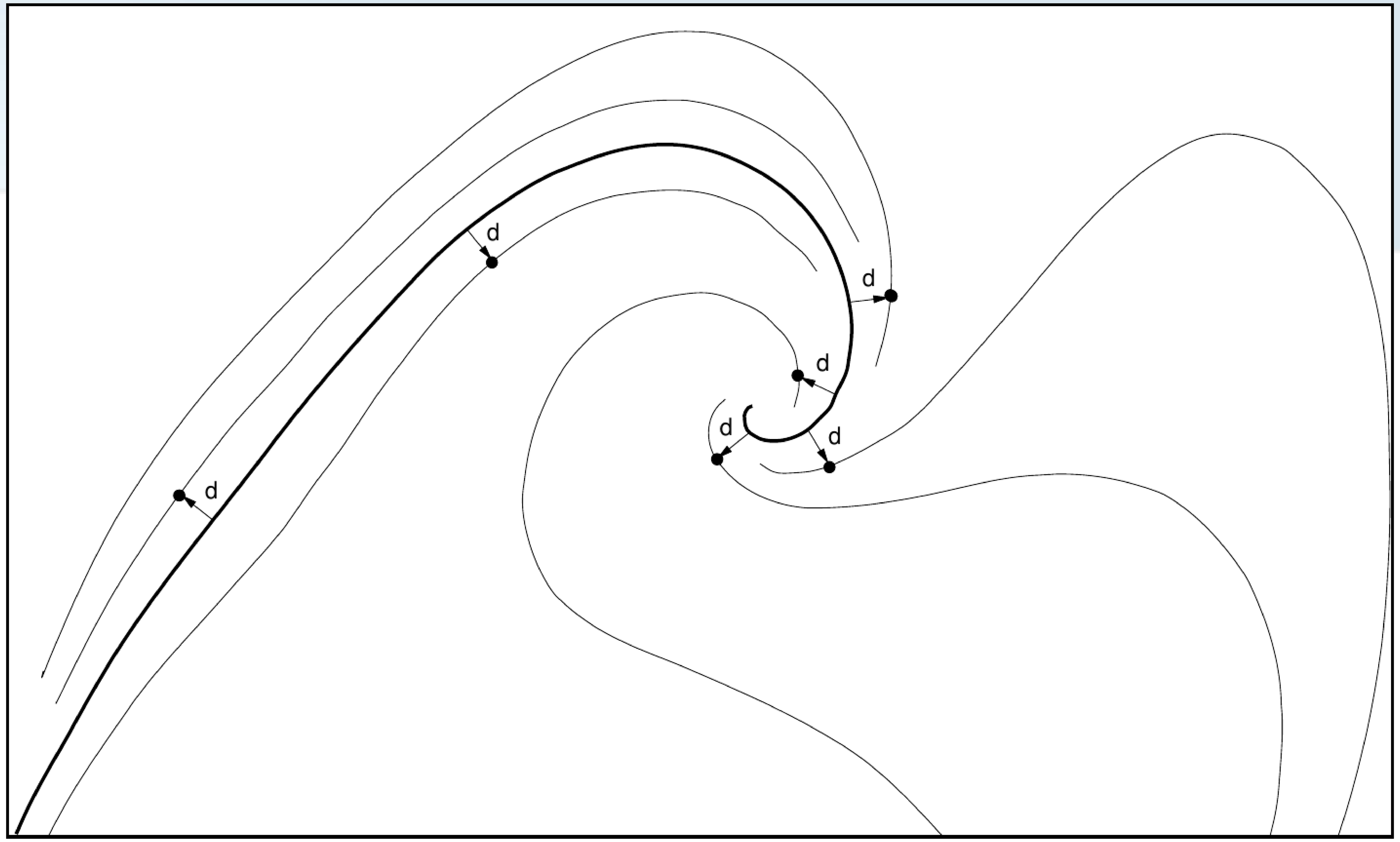

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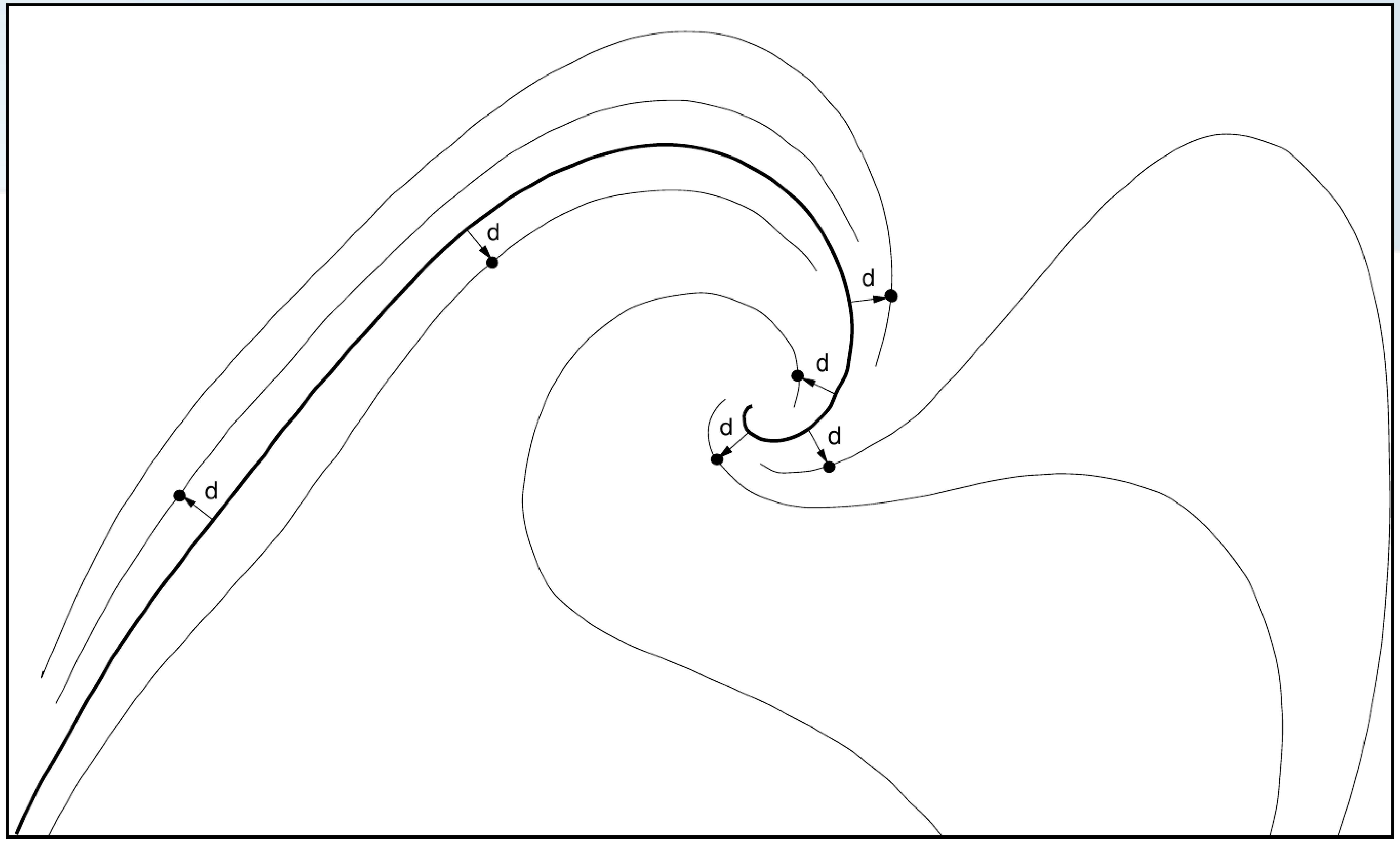

[Jobard et al. 97]

Seeding streamlines

- Evenly distributed streamlines
  - Choose a seed at distance d_{start} from existing streamlines
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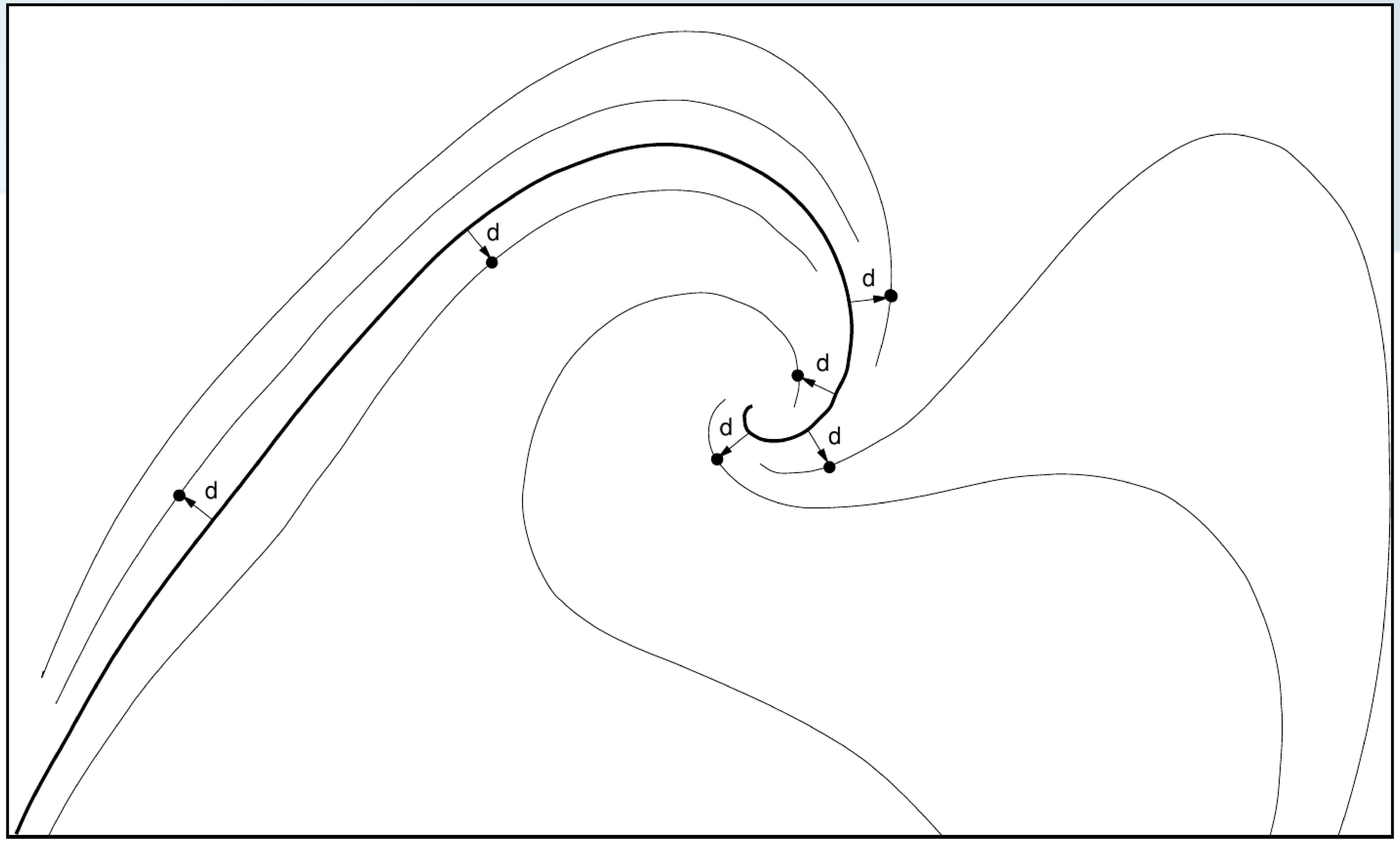
    - Until distance d_{end} is reached
  - Repeat the process
- Several termination criteria
  - Boundary of the domain, self distance, etc



[Jobard et al. 97]

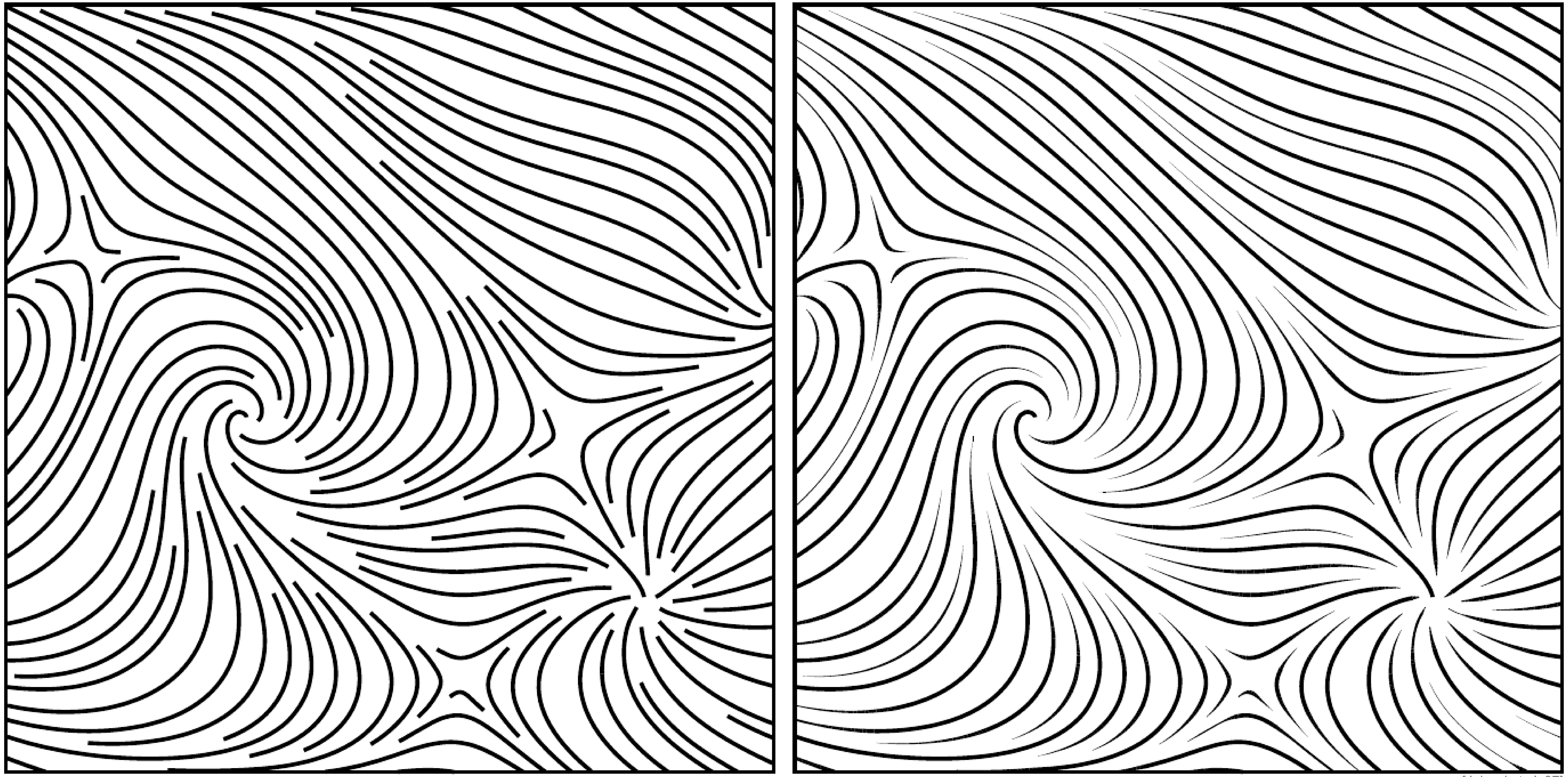
Seeding streamlines

- Effect of d_{start}

[Jobard et al. 97]

Seeding streamlines

- Adaptive width



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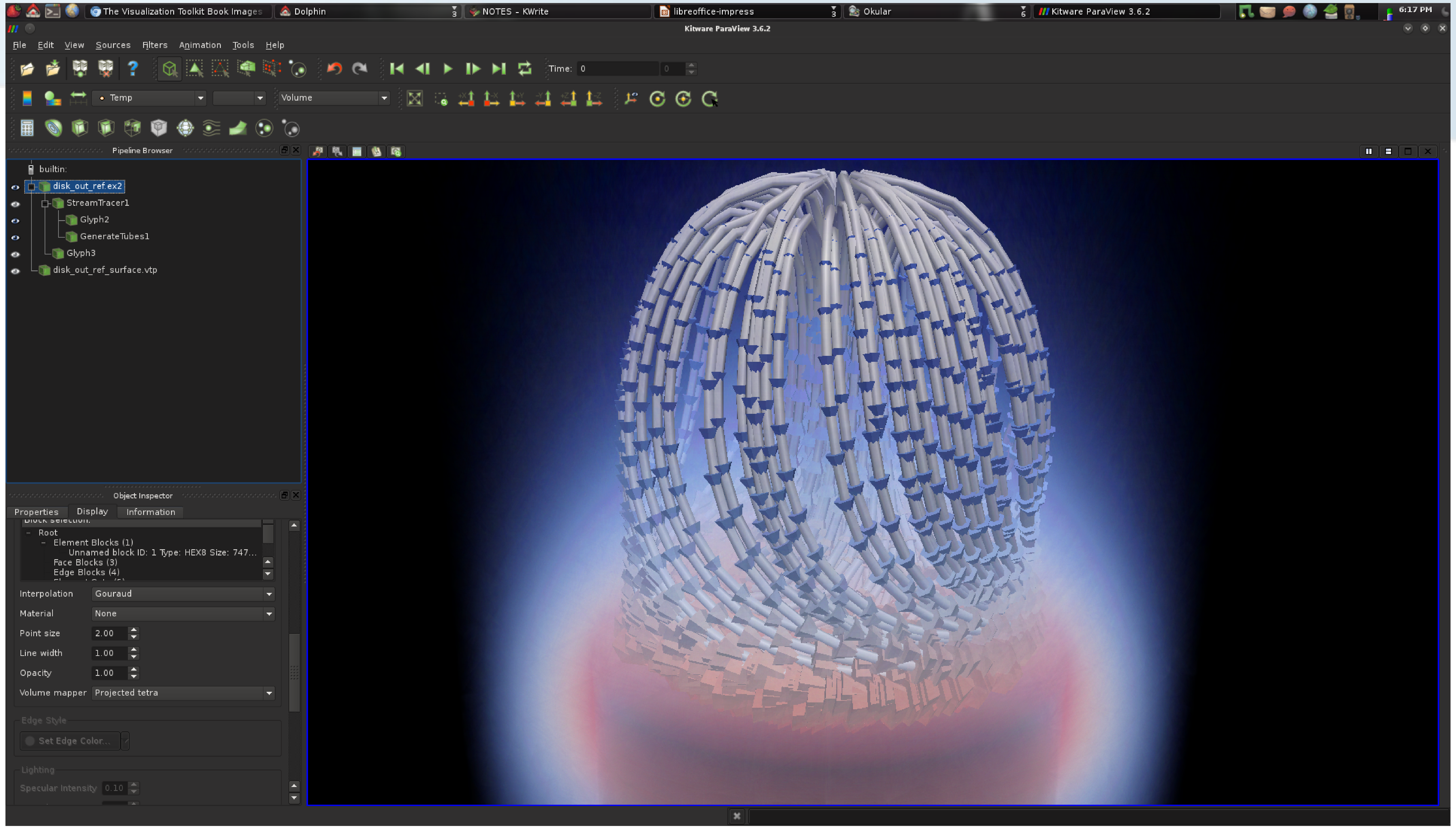
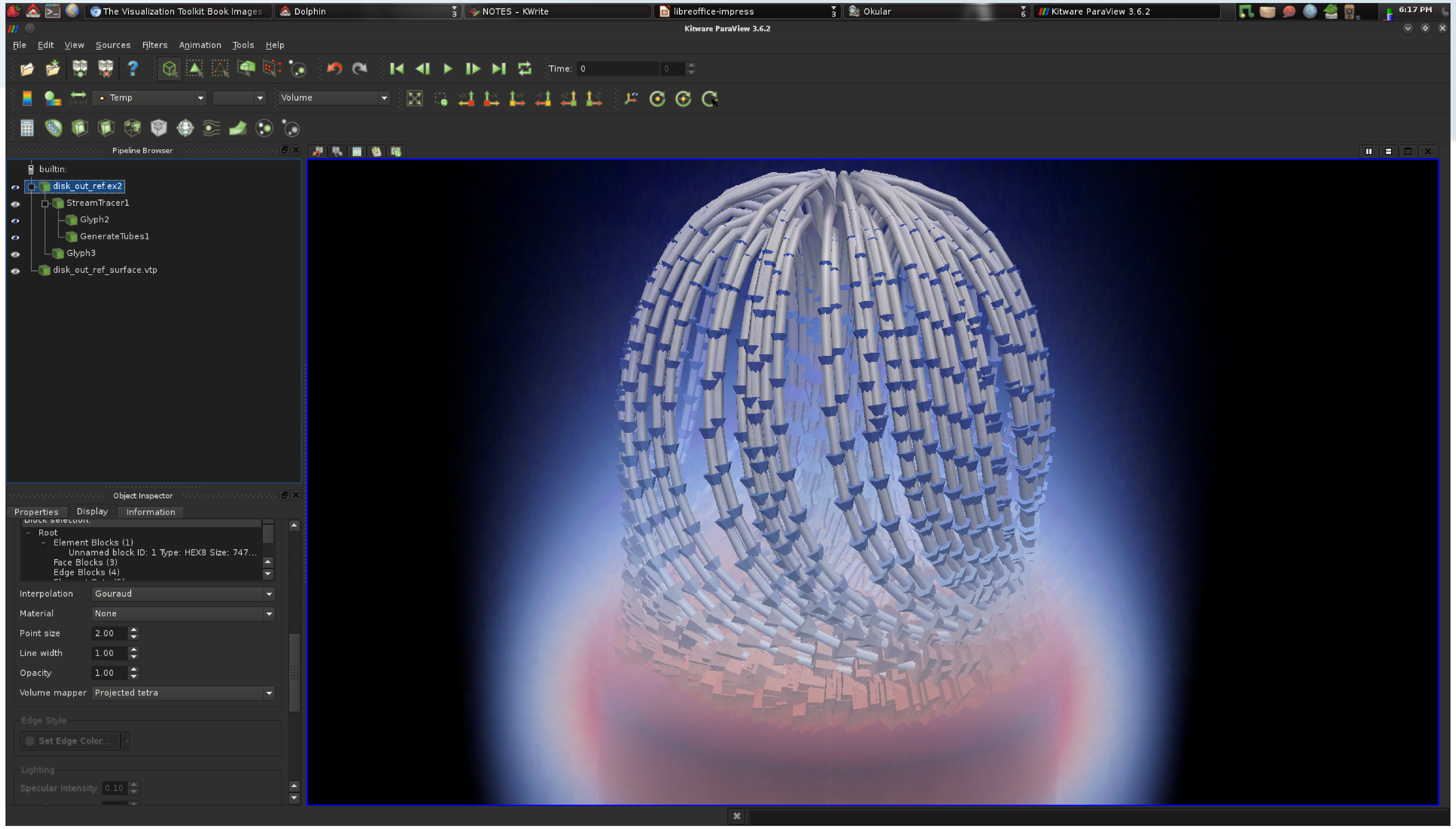
Volumetric domains



Volumetric domains

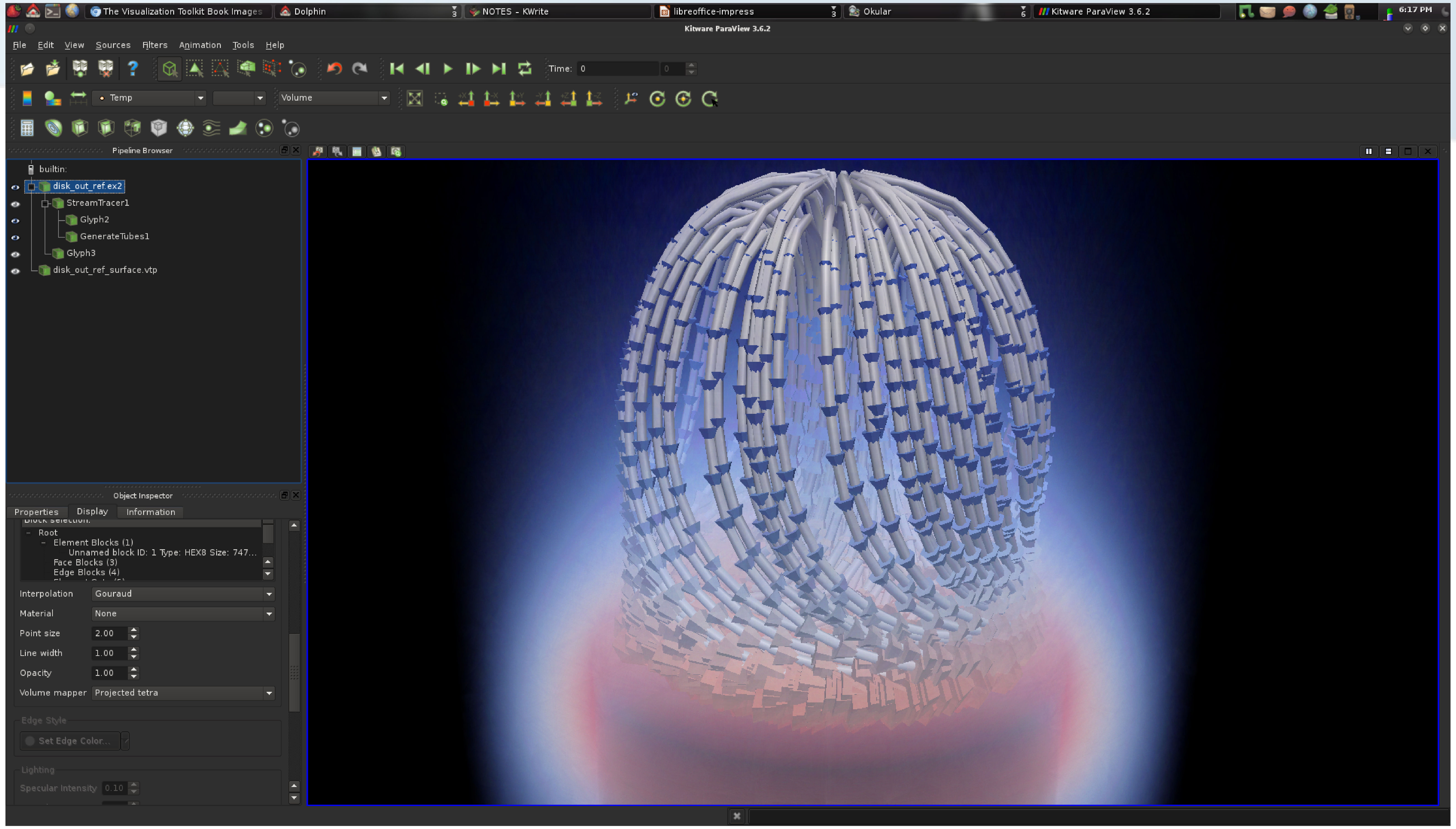
- Trivial extensions
  - Streamline computation
  - Streamline seeding



Volumetric domains

- Trivial extensions
  - Streamline computation
  - Streamline seeding

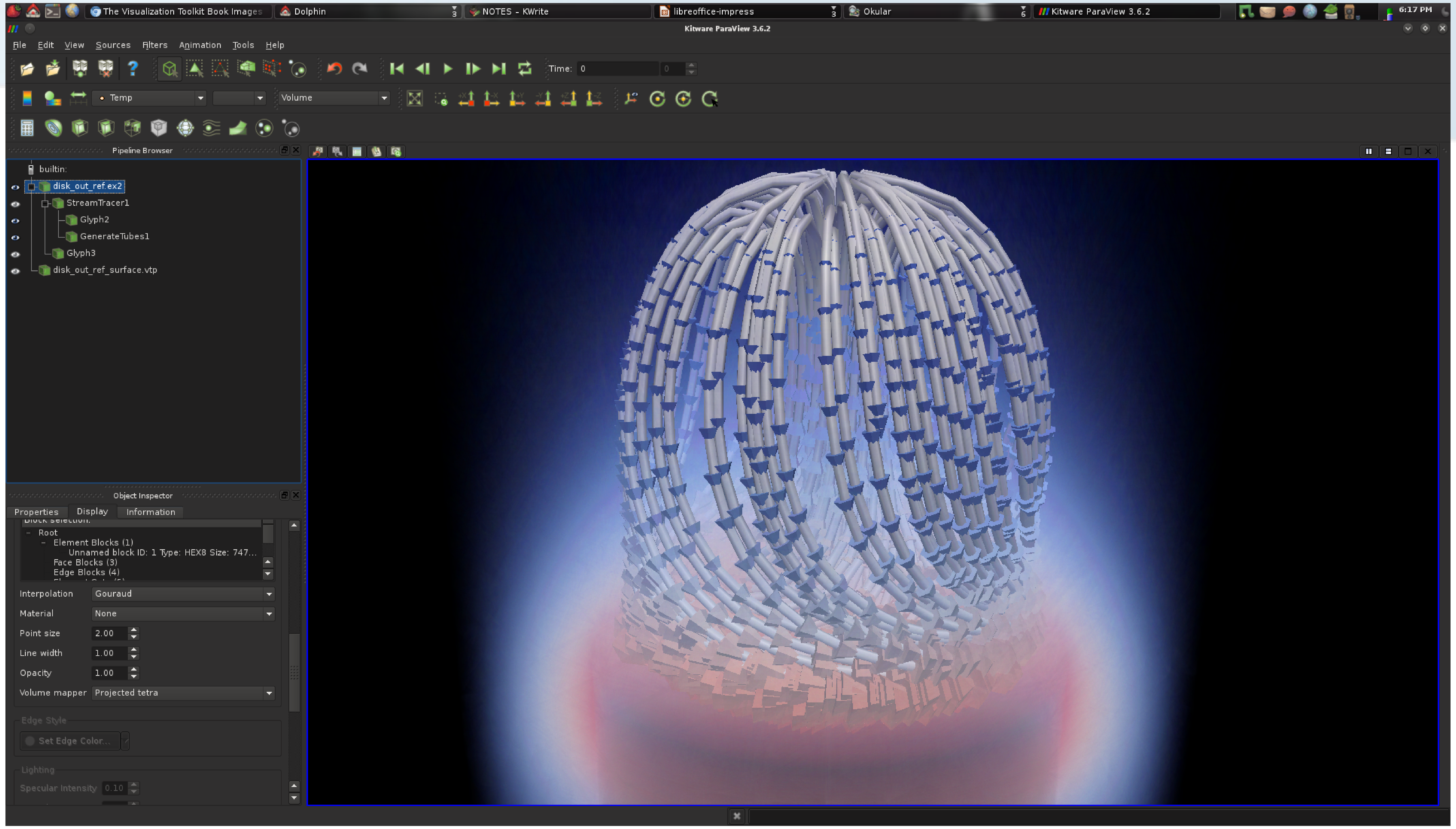
- Notion of *streamsurface*



Volumetric domains

- Trivial extensions
  - Streamline computation
  - Streamline seeding

- Notion of *streamsurface*
  - Given a PL 1-manifold $\mathcal{C}$



Volumetric domains

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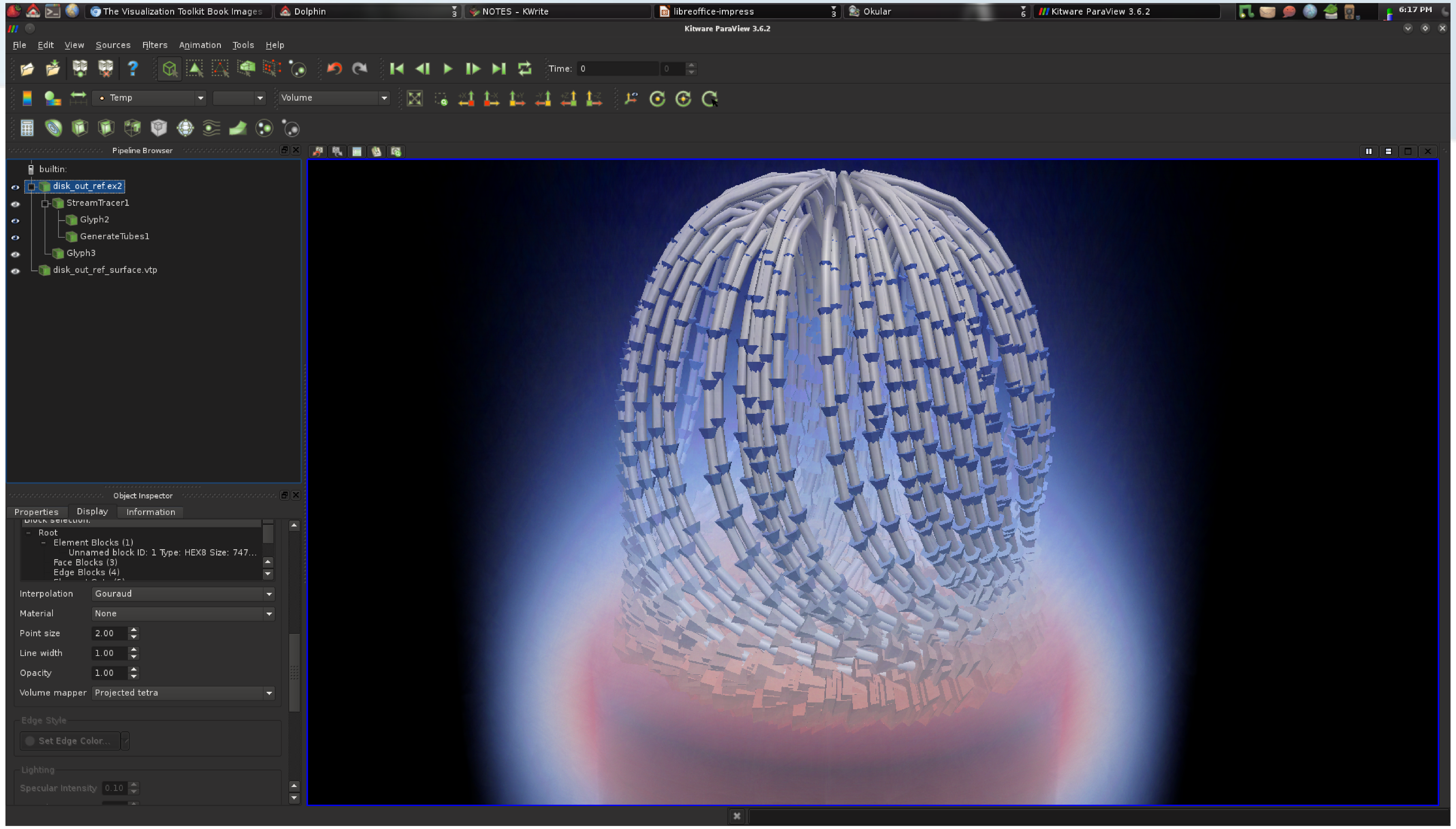

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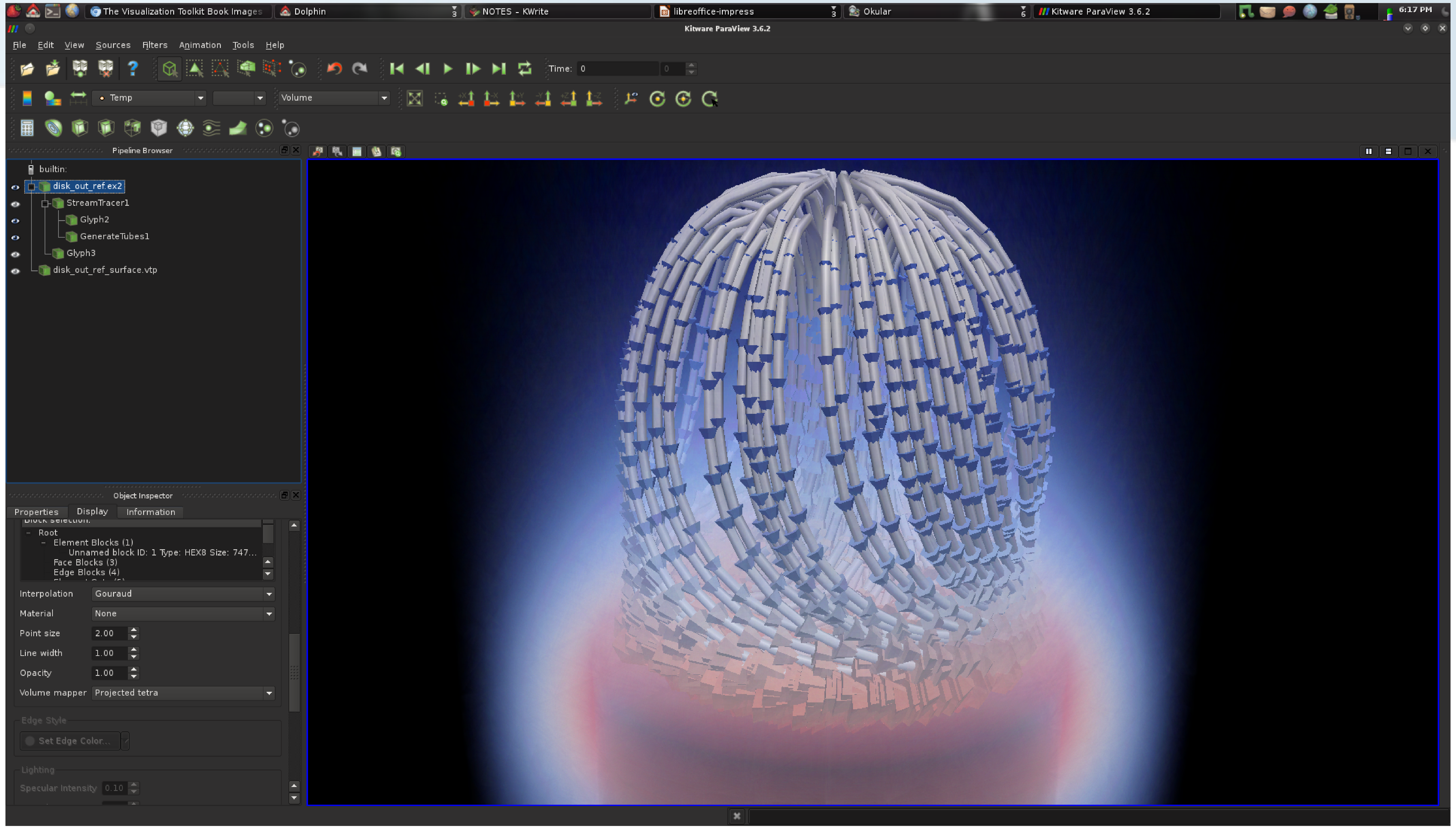
Volumetric domains



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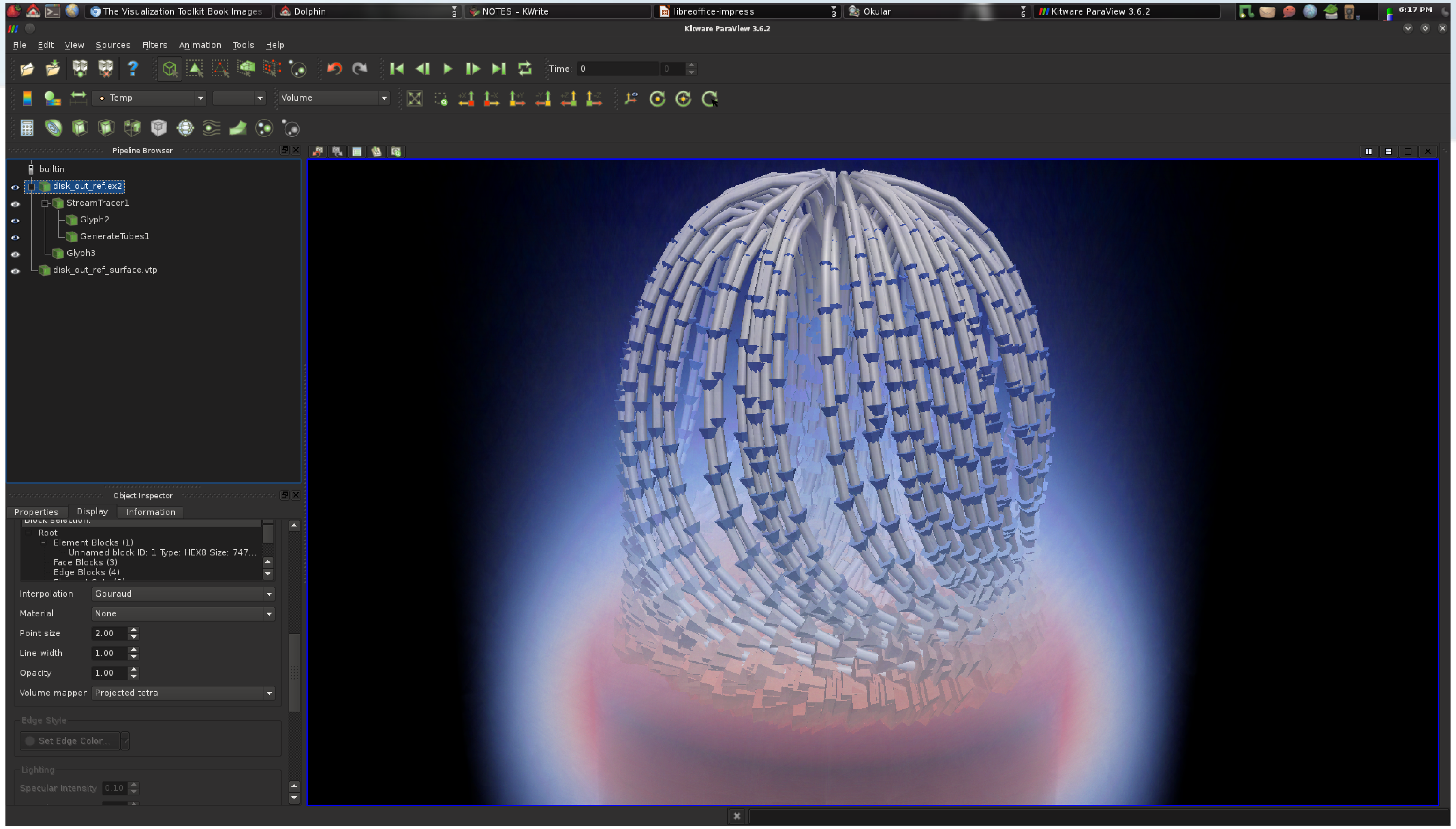
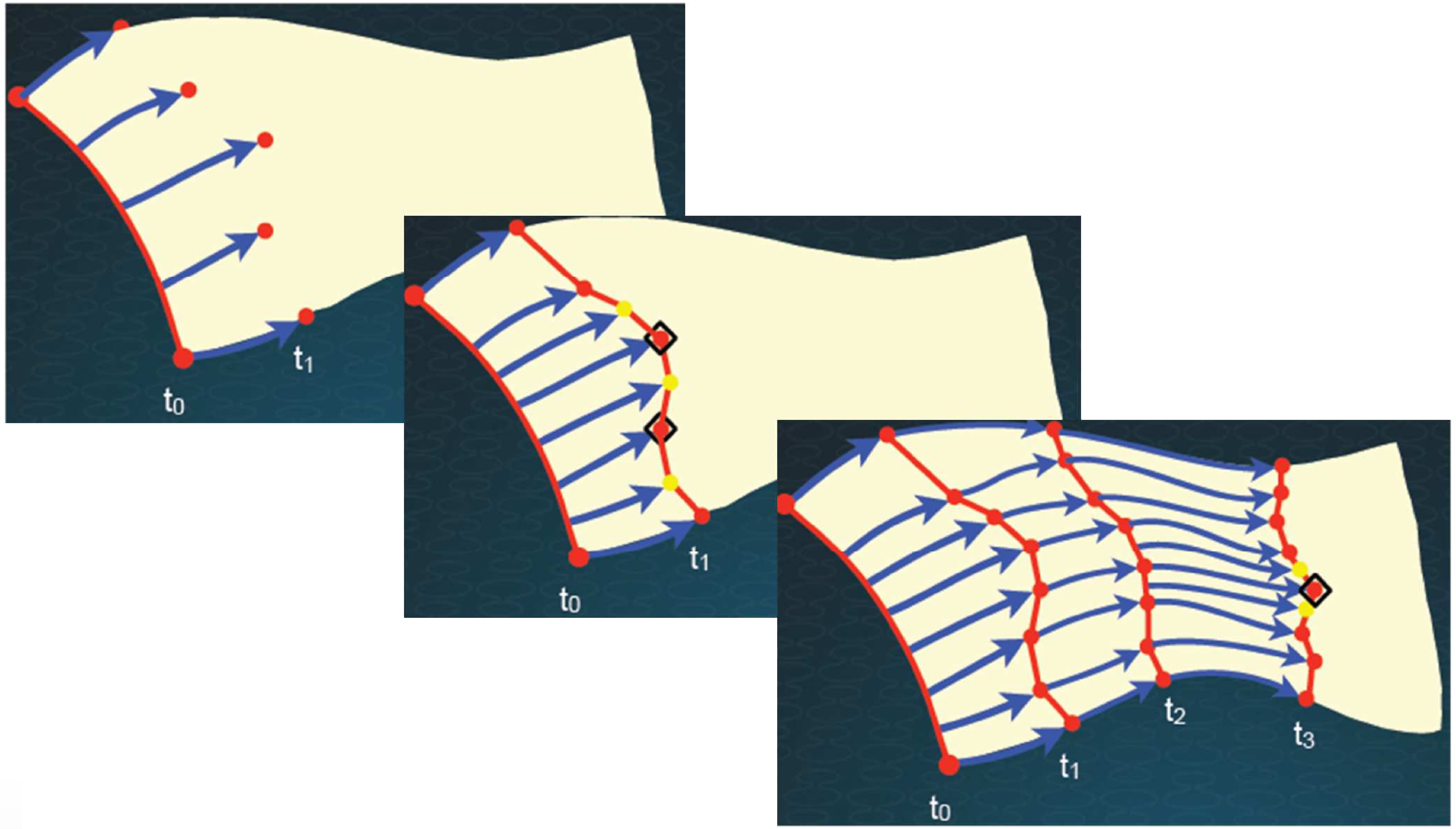


- Notion of *streamsurface*
  - Given a PL 1-manifold $\mathcal{C}$
    - Consider each vertex as a seed
    - Consider the image of $\mathcal{C}$ at several steps of integration
  - Surface everywhere tangential to the flow

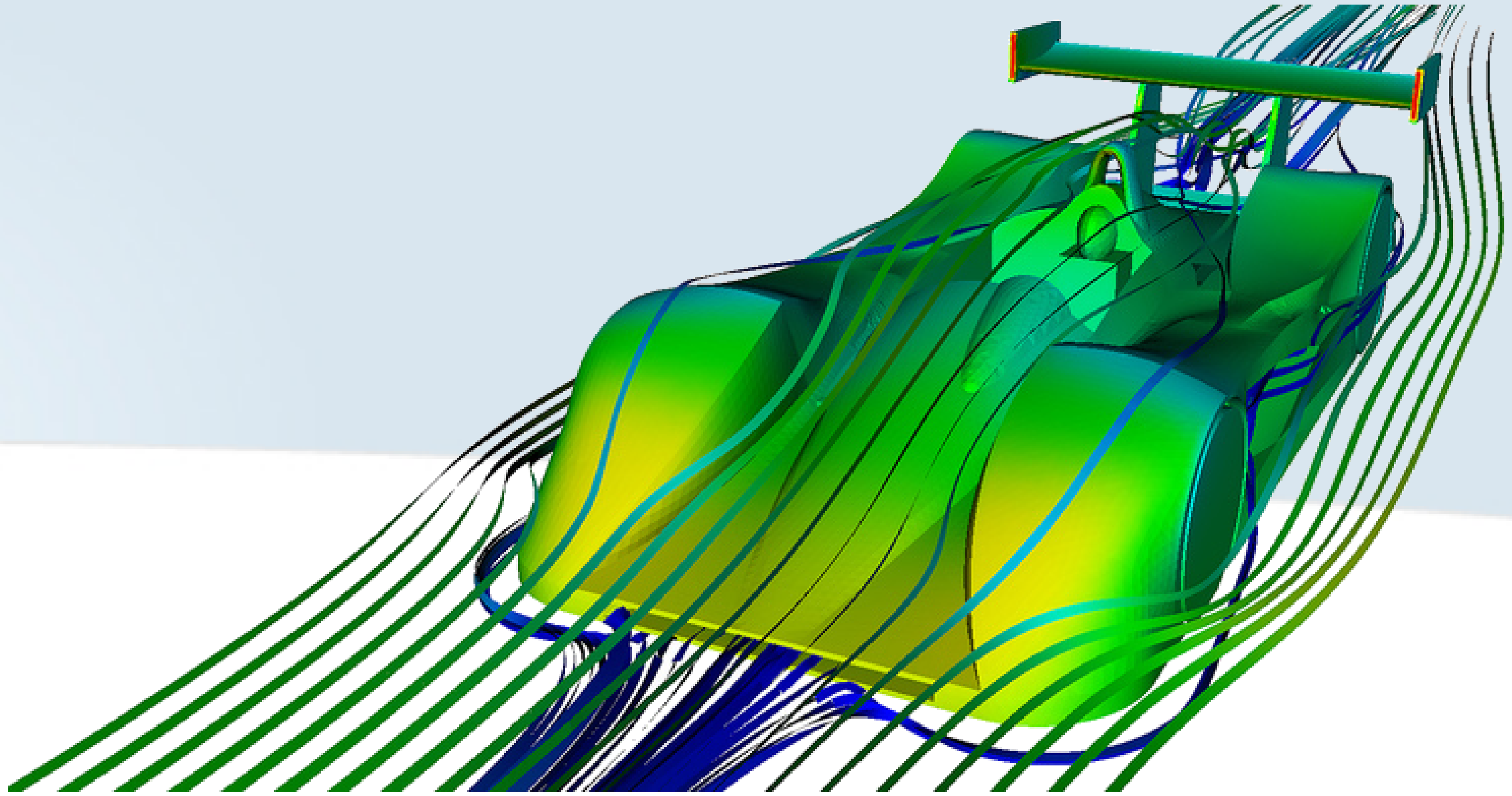
Streamsurfaces

- Adaptive sampling depending on the curvature of the seed curve

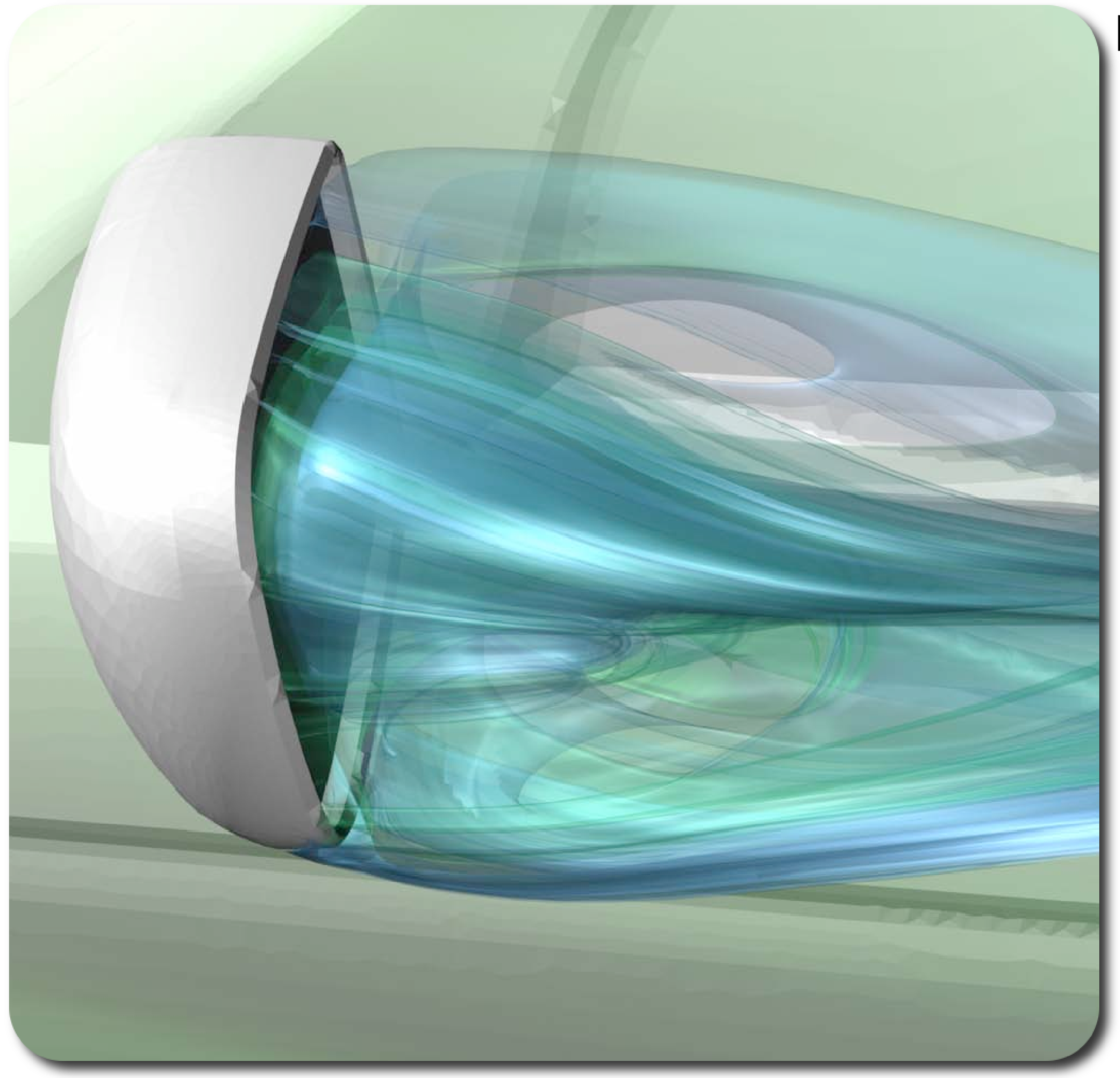


[Garth 09]

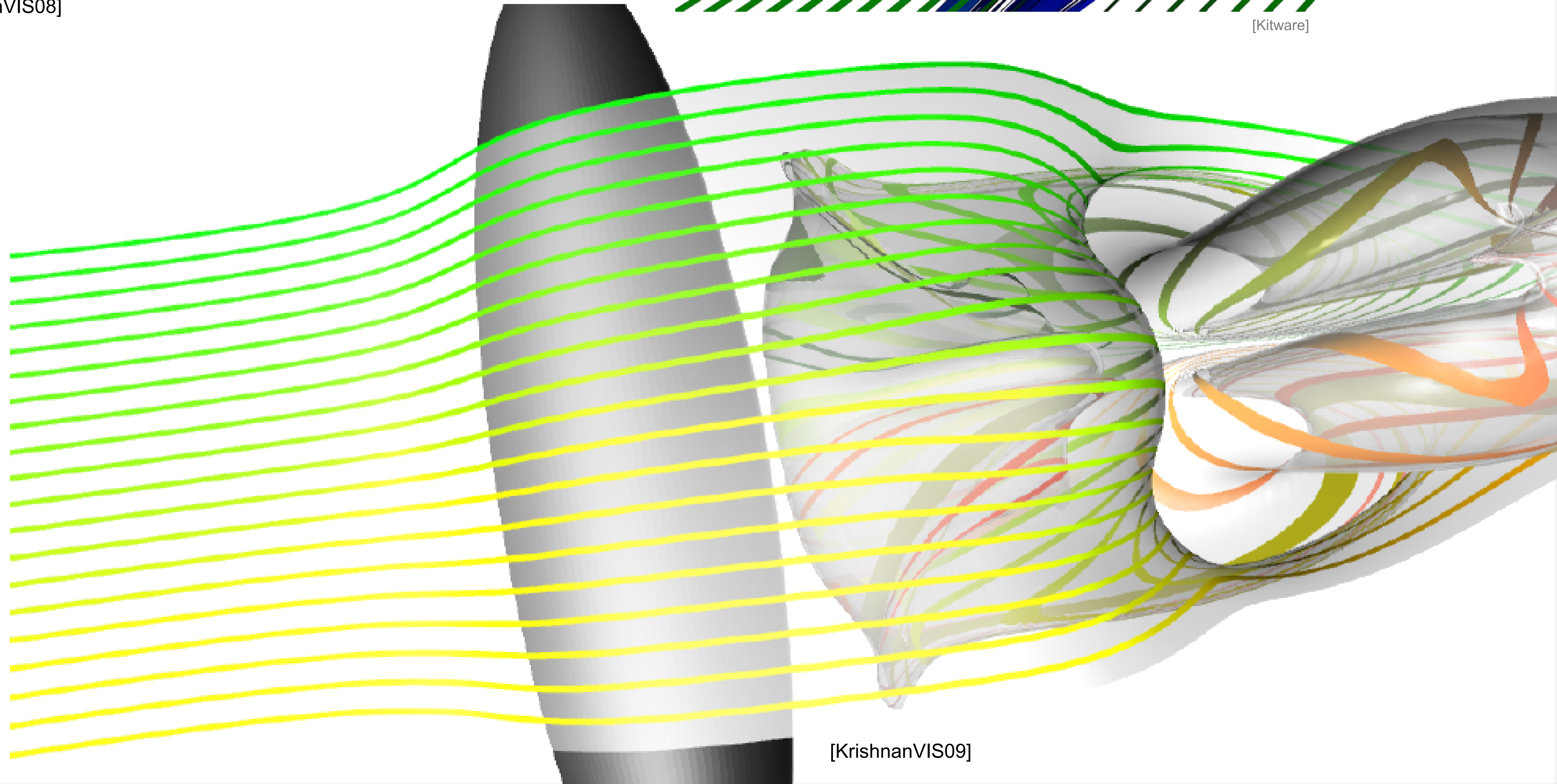
Examples



[Kitware]



[GarthVIS08]



[KrishnanVIS09]

Steady vector fields

- What else is there to visualize?

Steady vector fields

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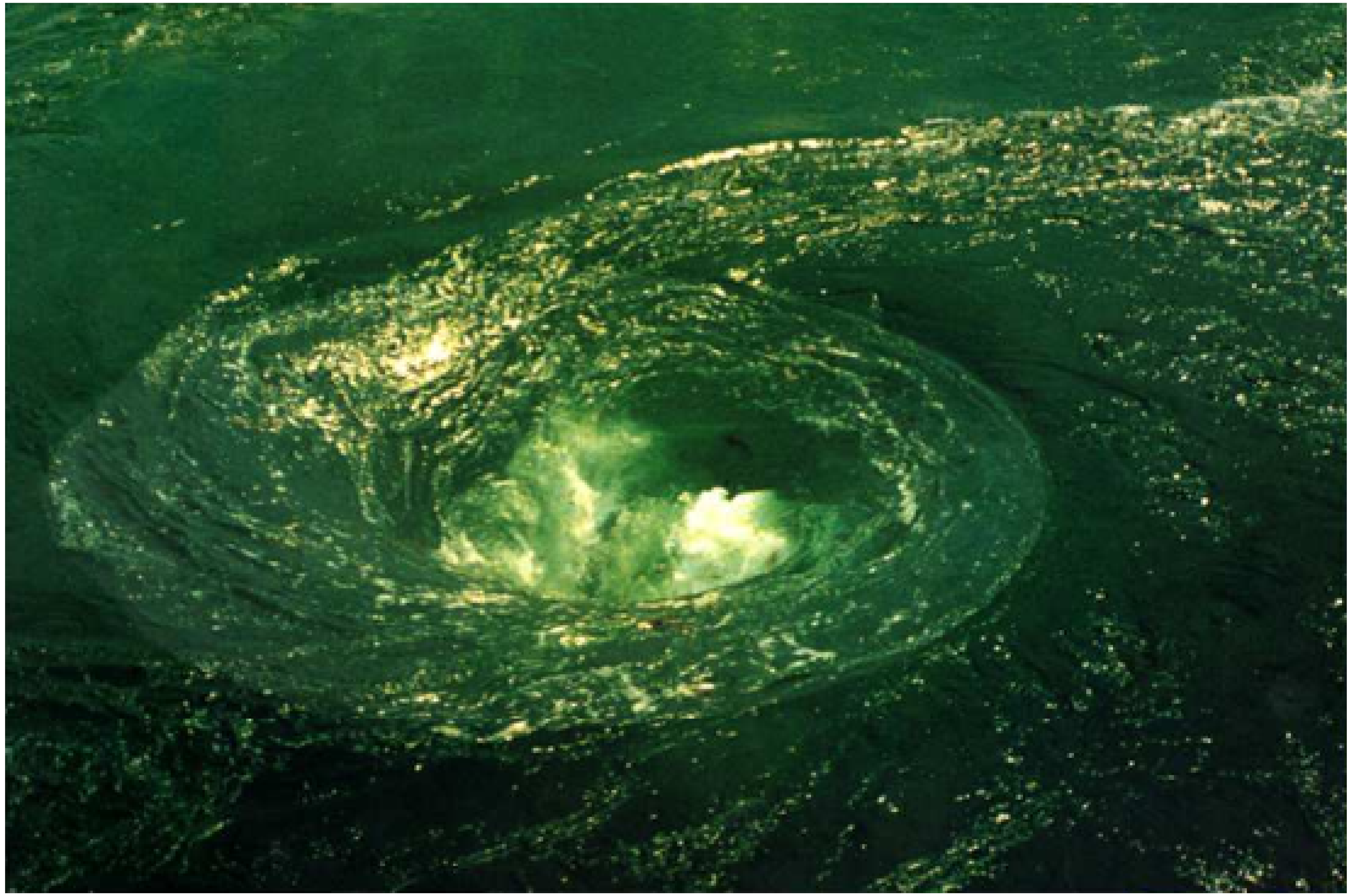
  - Get inspiration from nature



[Post et al. 2003]

Steady vector fields

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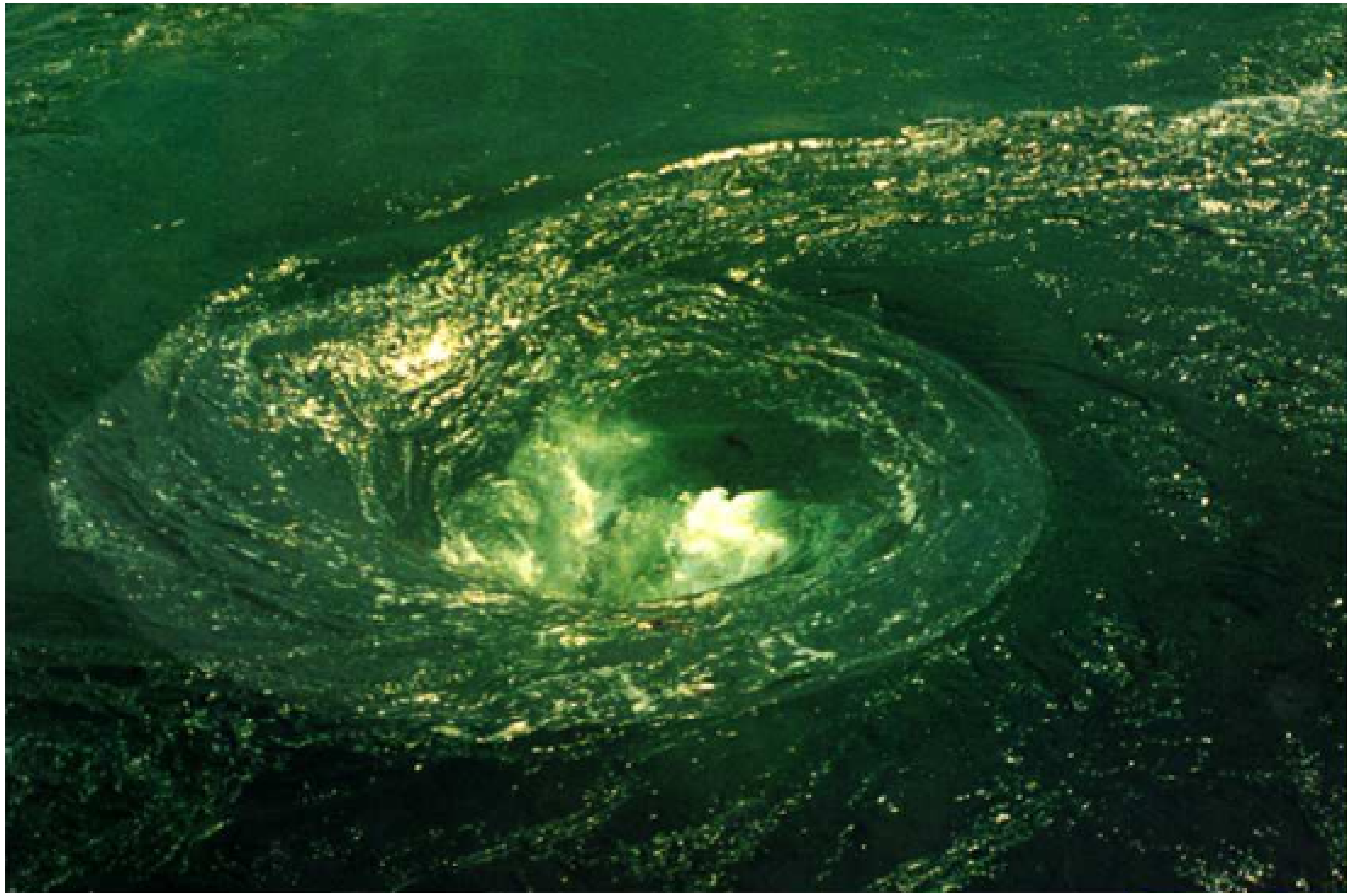
  - Visualize the flow **globally**
  - For each point of the domain



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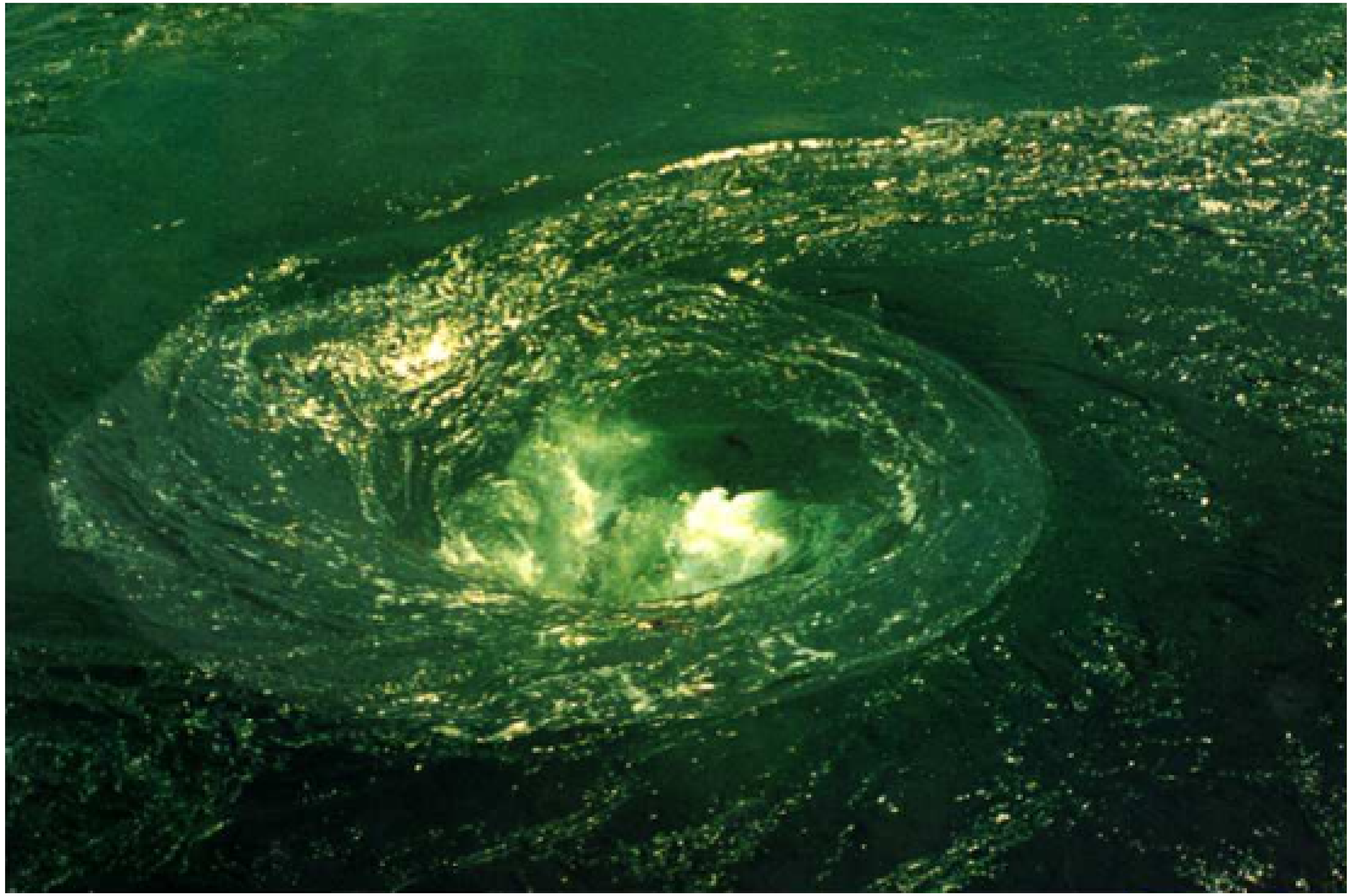

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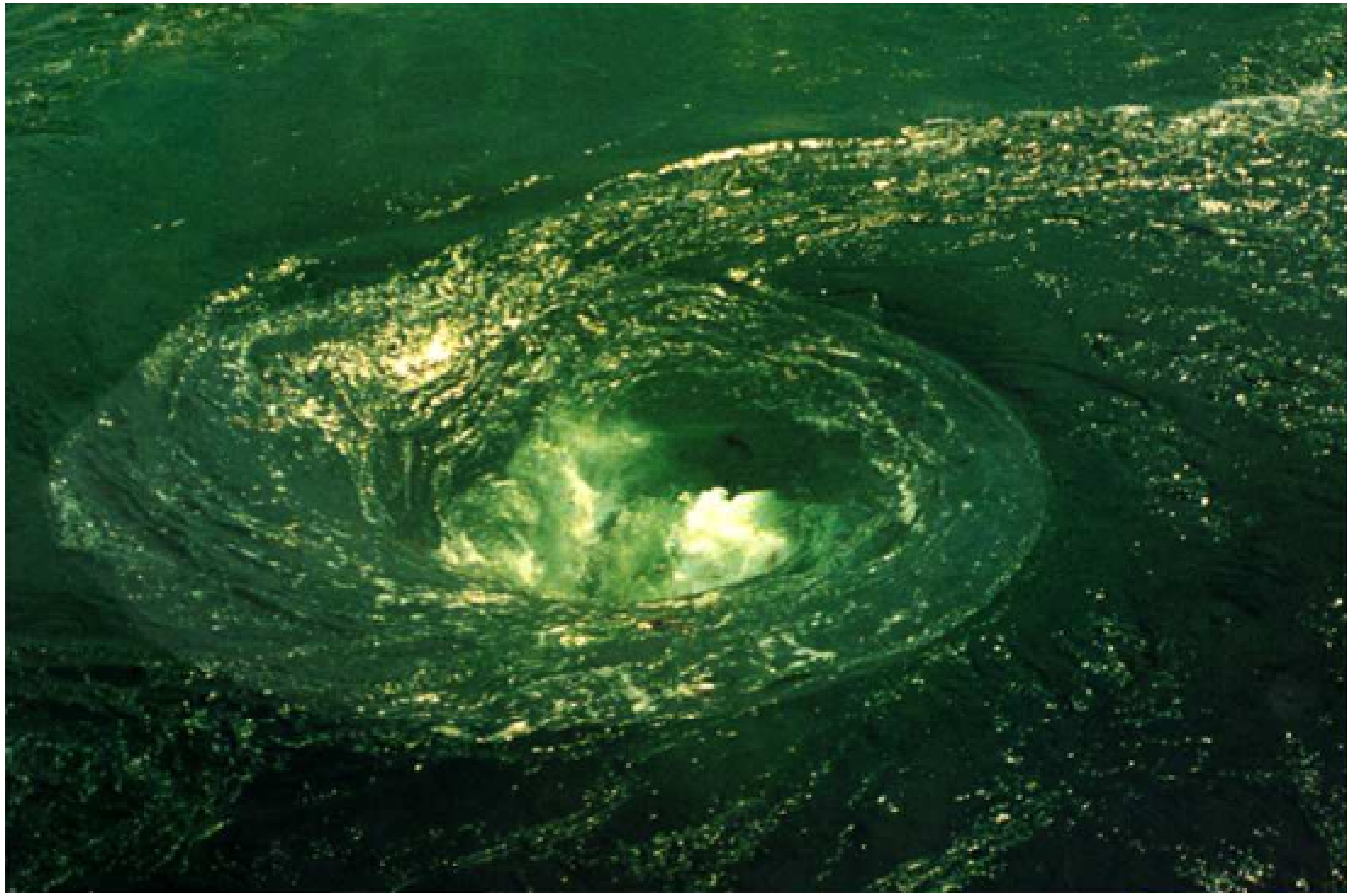
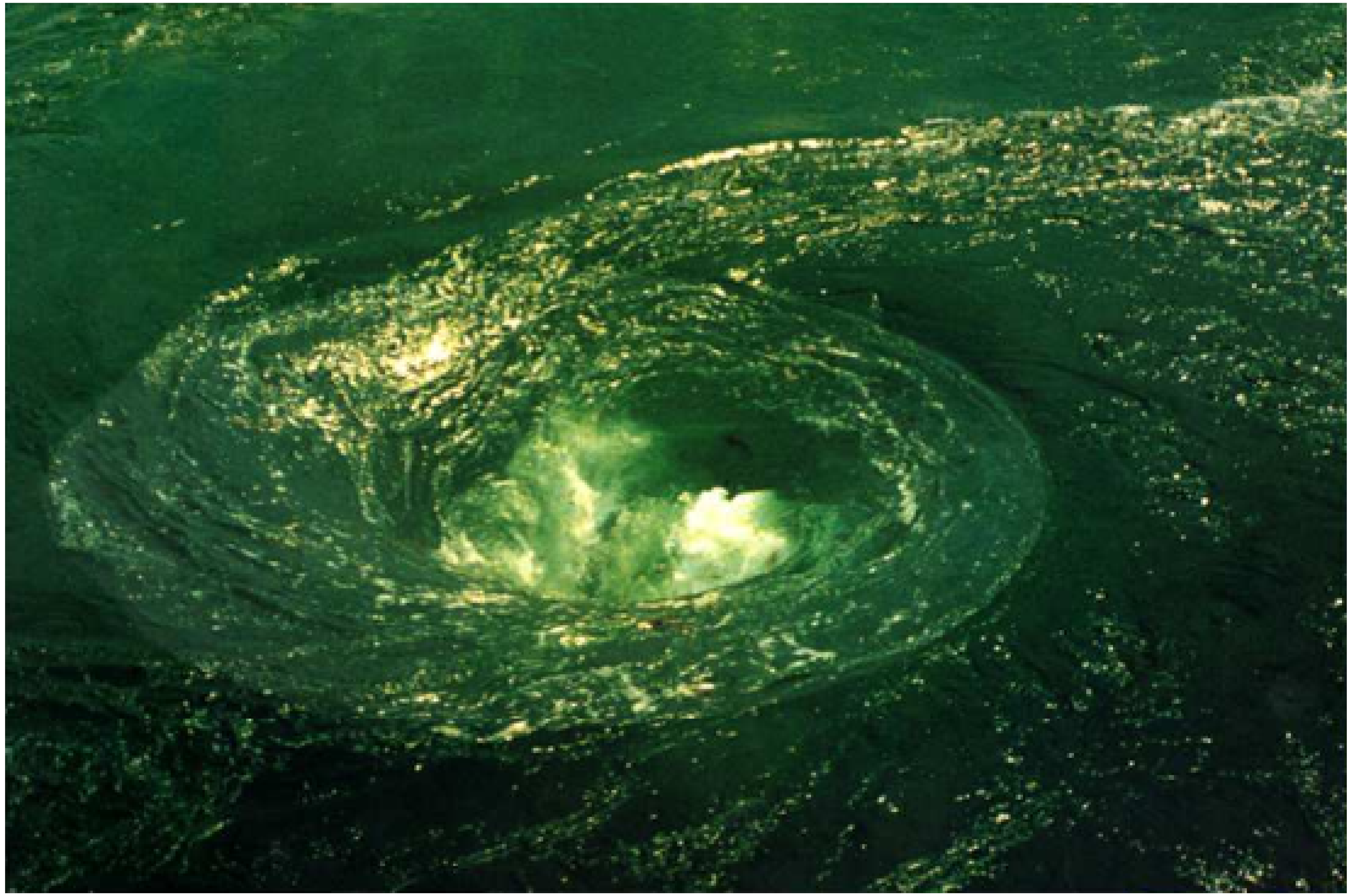
- Line Integral Convolution



[Post et al. 2003]

Line Integral Convolution

- Basic idea



[Post et al. 2003]

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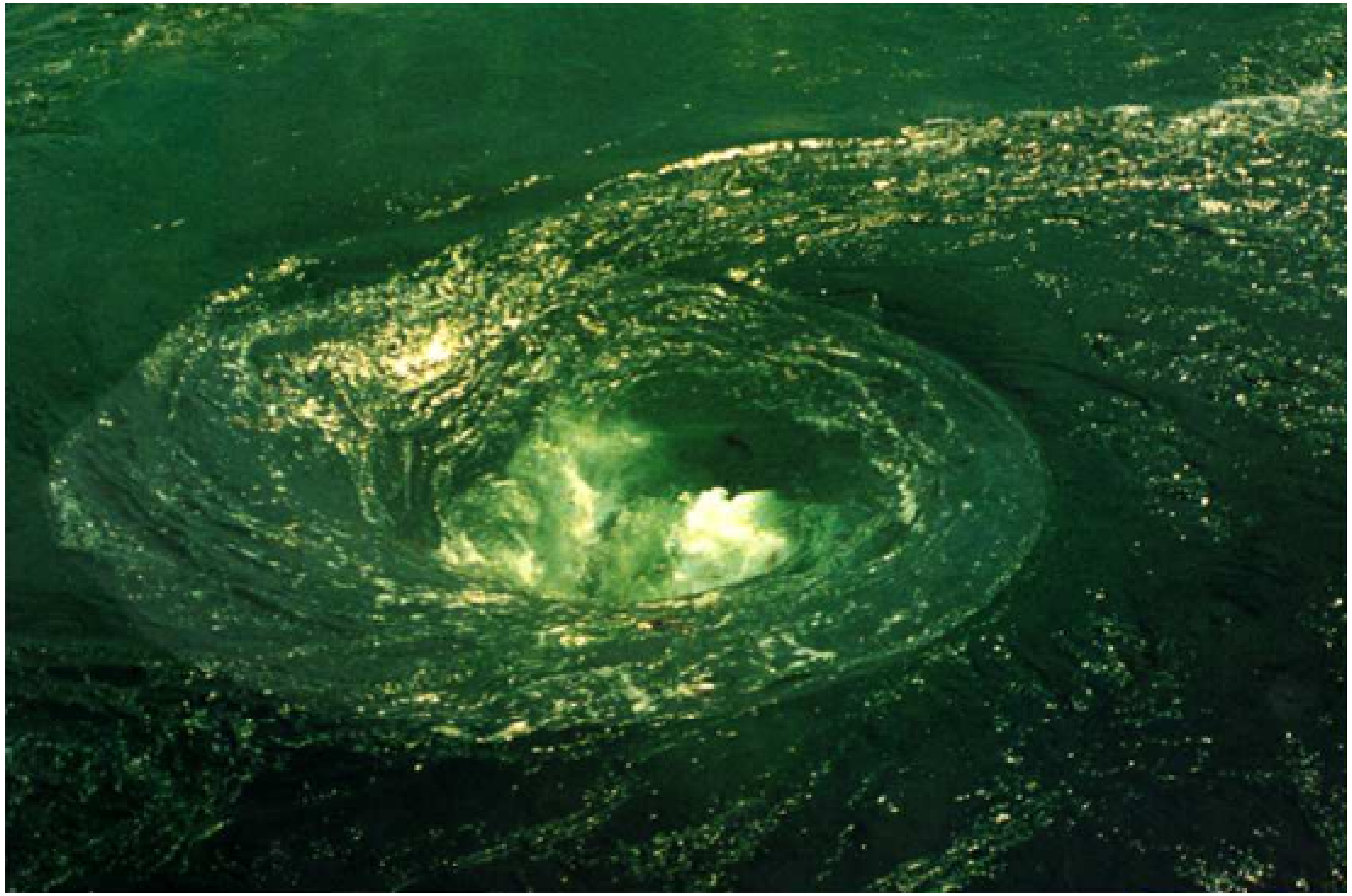
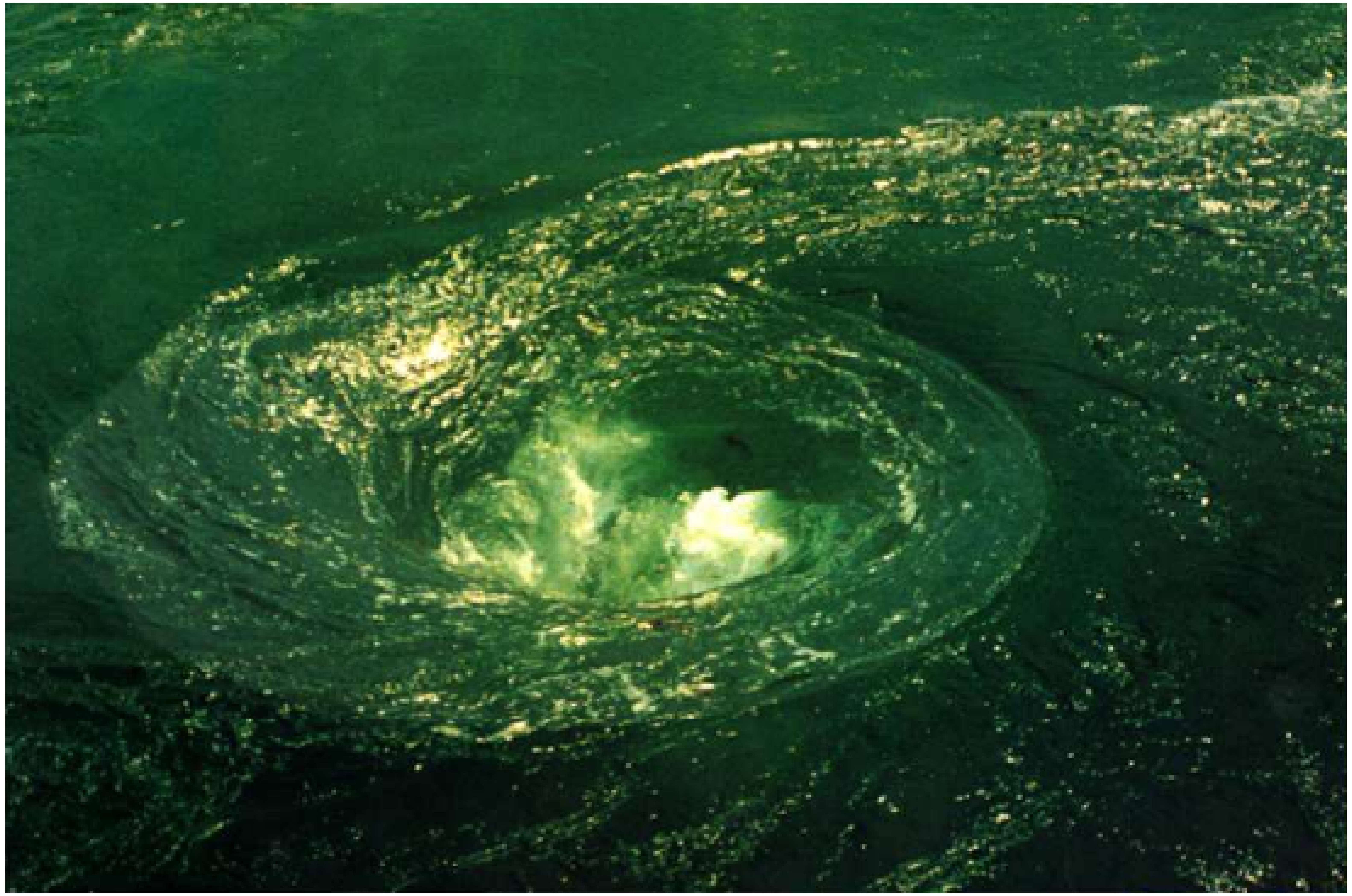


[Post et al. 2003]

Line Integral Convolution

- Basic idea
  - Global visualization
    - Compute an integral curve for each point of the domain



[Post et al. 2003]

Line Integral Convolution

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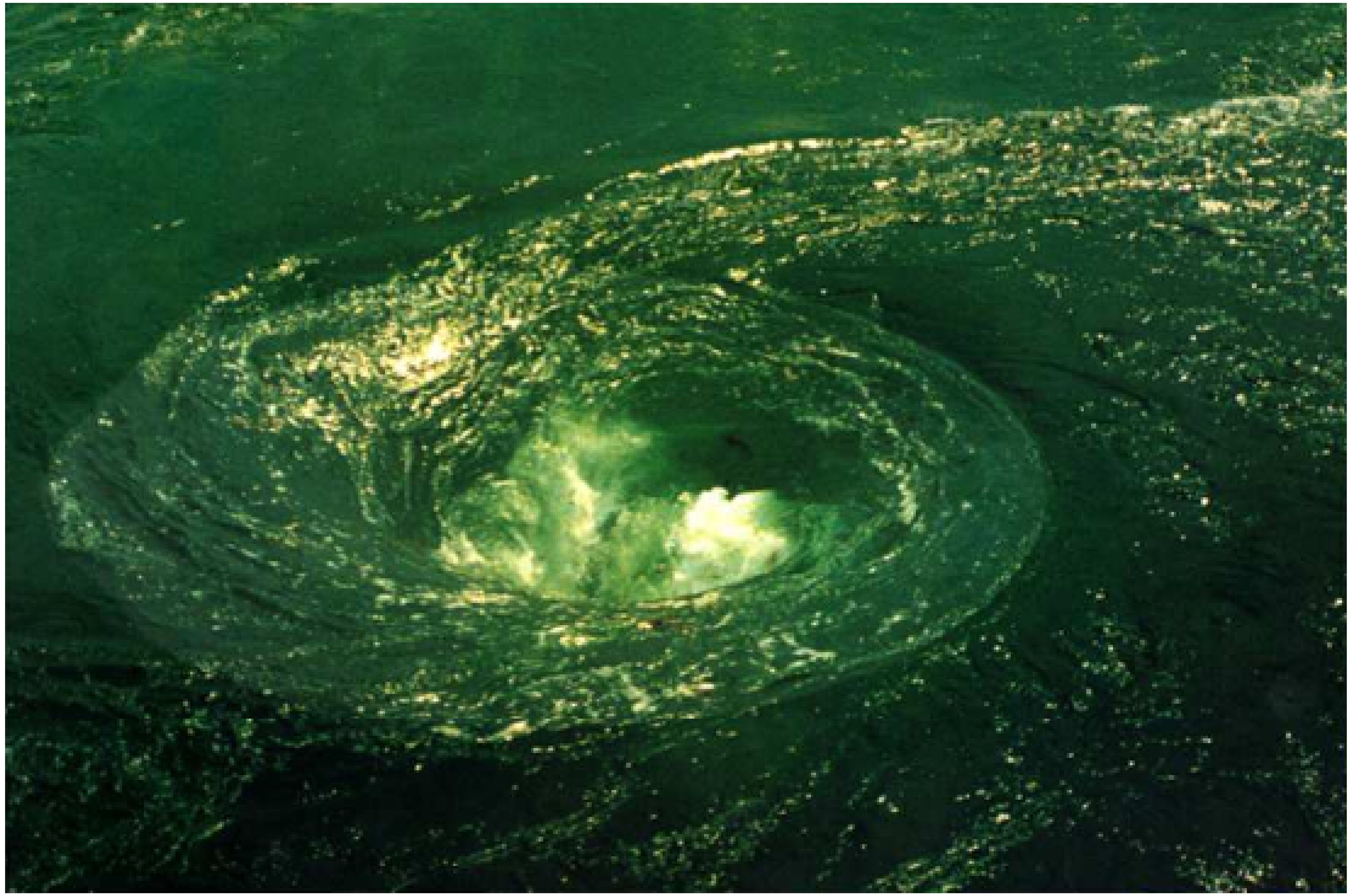
    - Hard to see anything



[Post et al. 2003]

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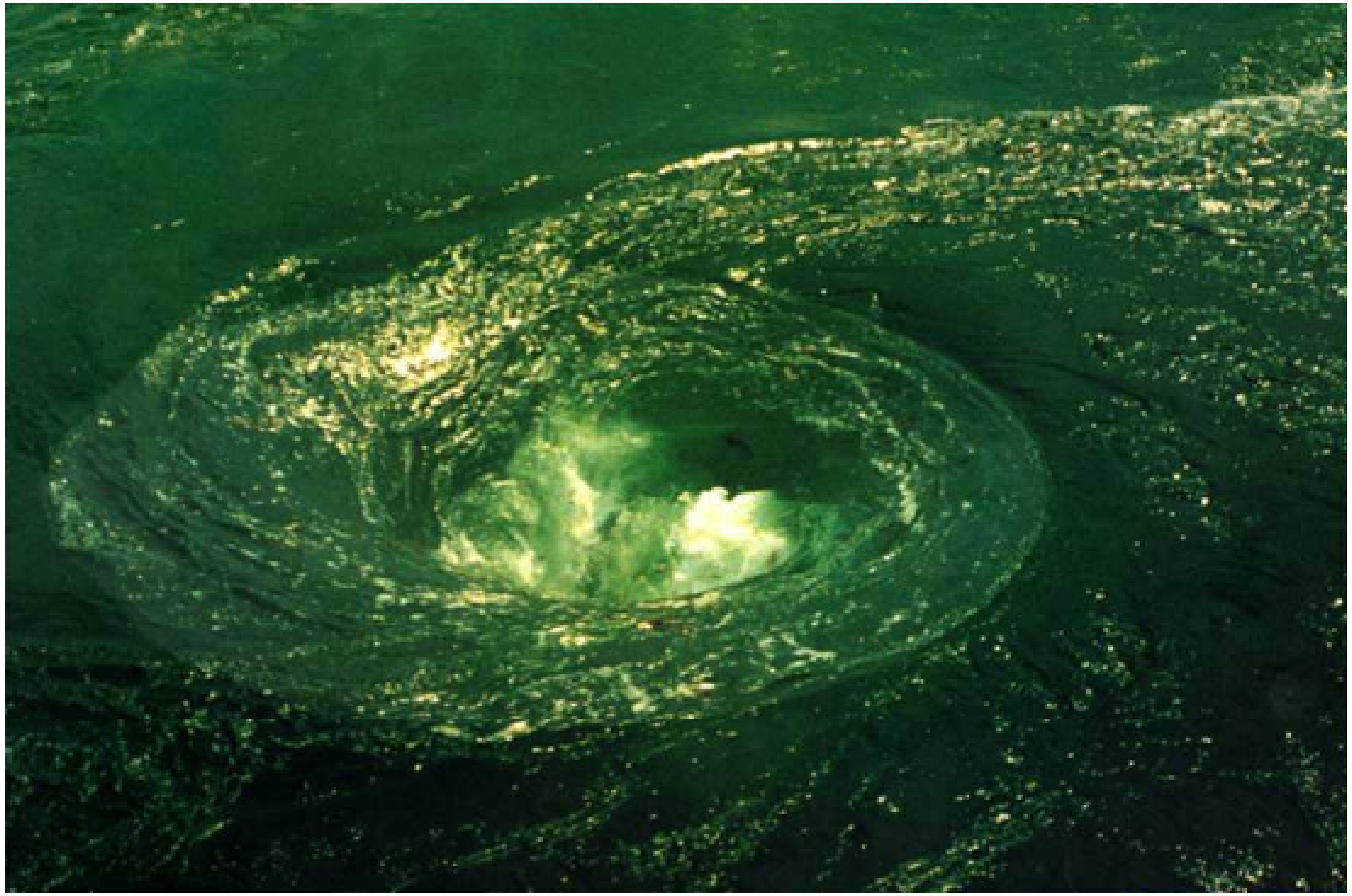
    - Mimic light variation (noise)



[Post et al. 2003]

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  - Key idea
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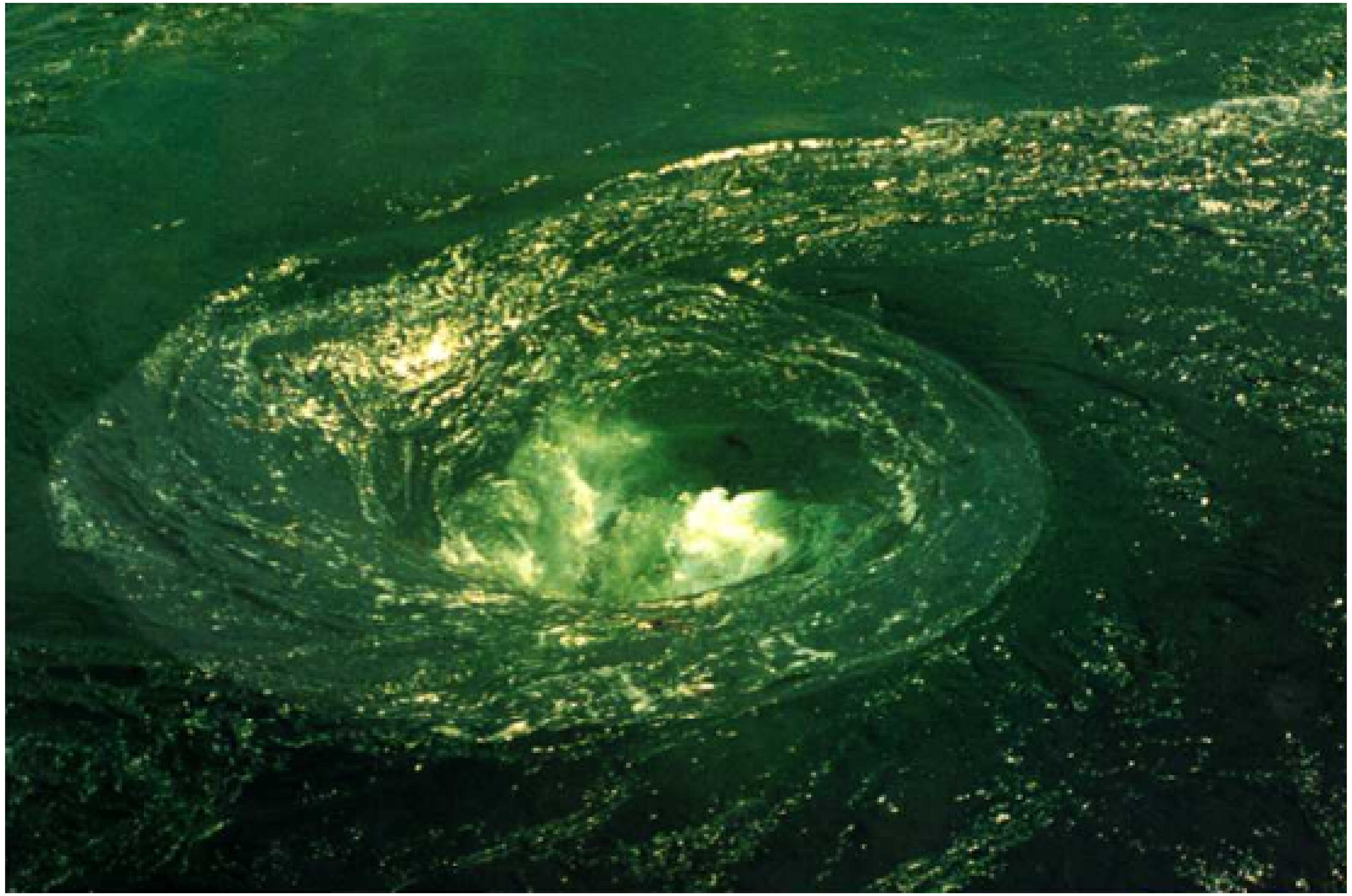
    - Blend curves with noise



[Post et al. 2003]

Line Integral Convolution

- Algorithm

Line Integral Convolution

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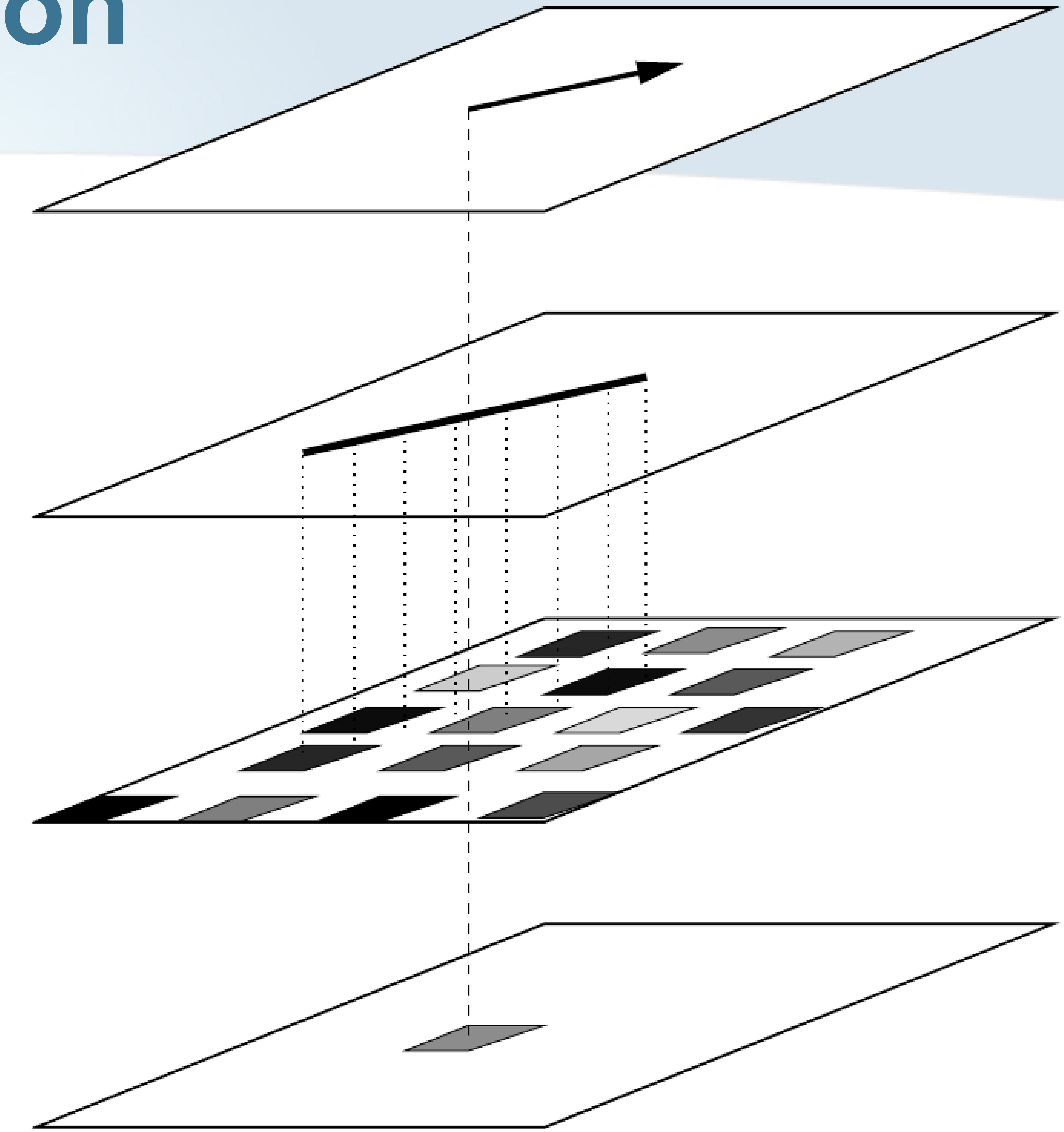
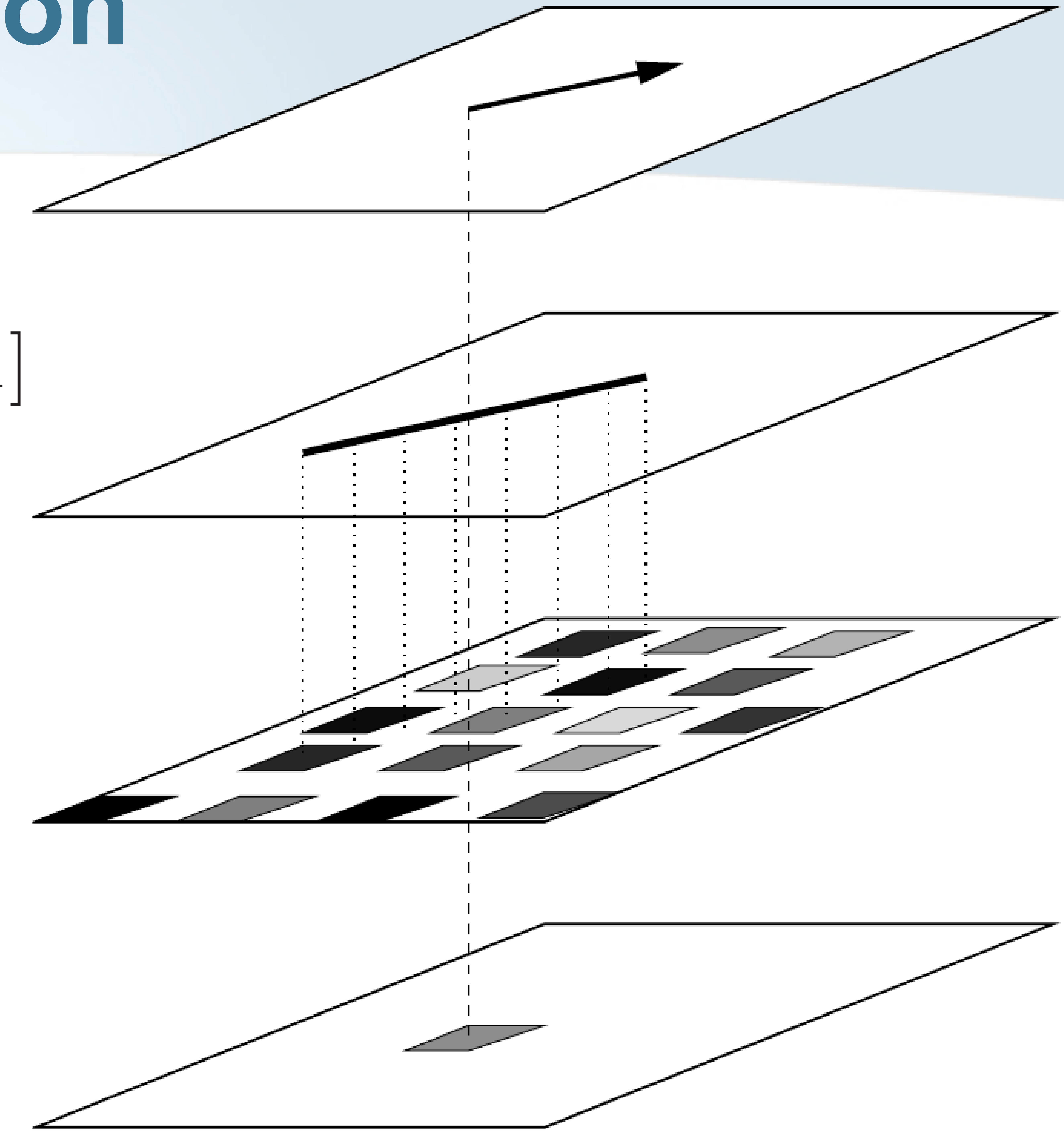


[Helgeland]

Line Integral Convolution

- Algorithm
  - Compute a noise field $\mathcal{N} : \mathcal{D} \to [0, 1]$



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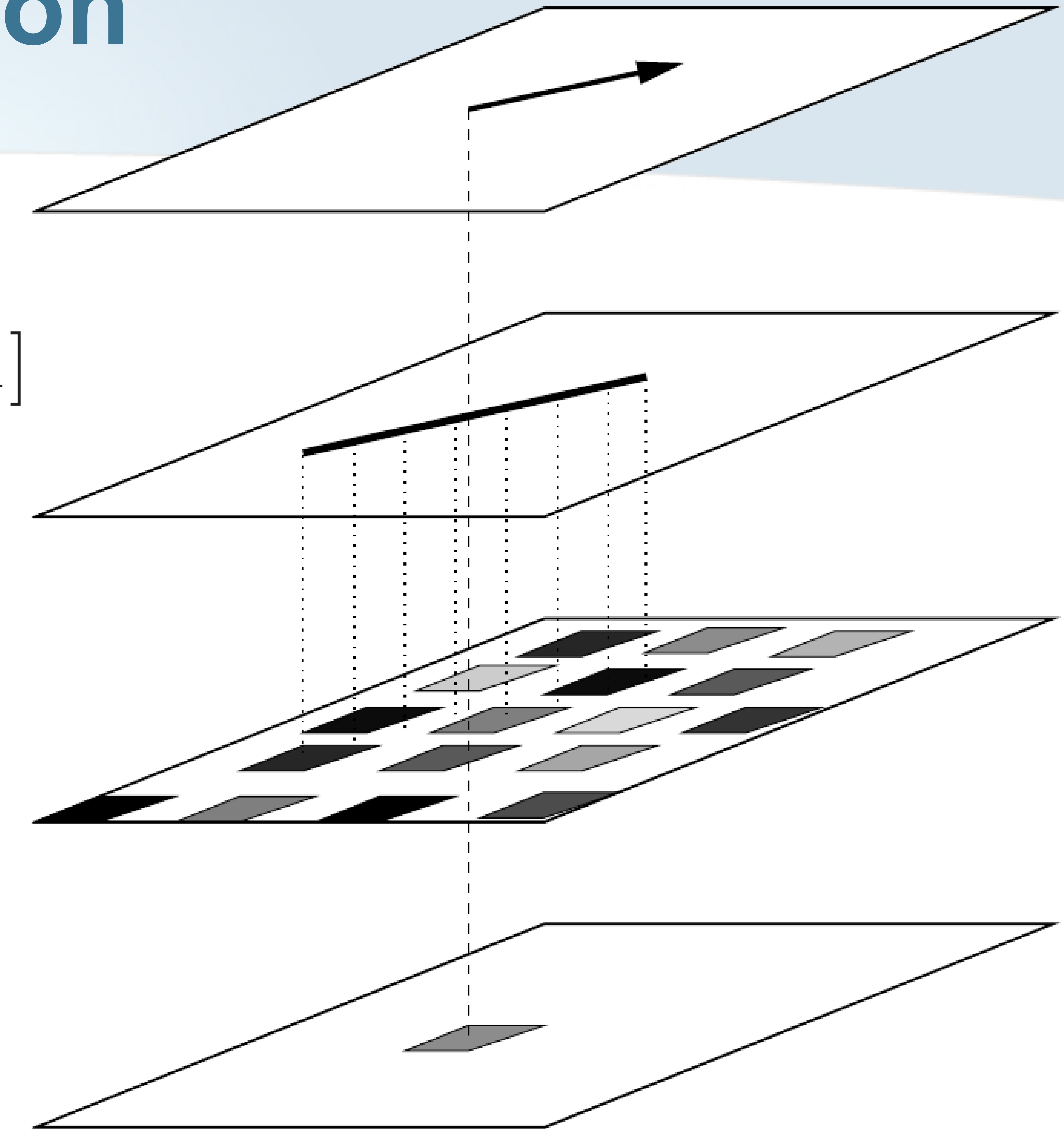

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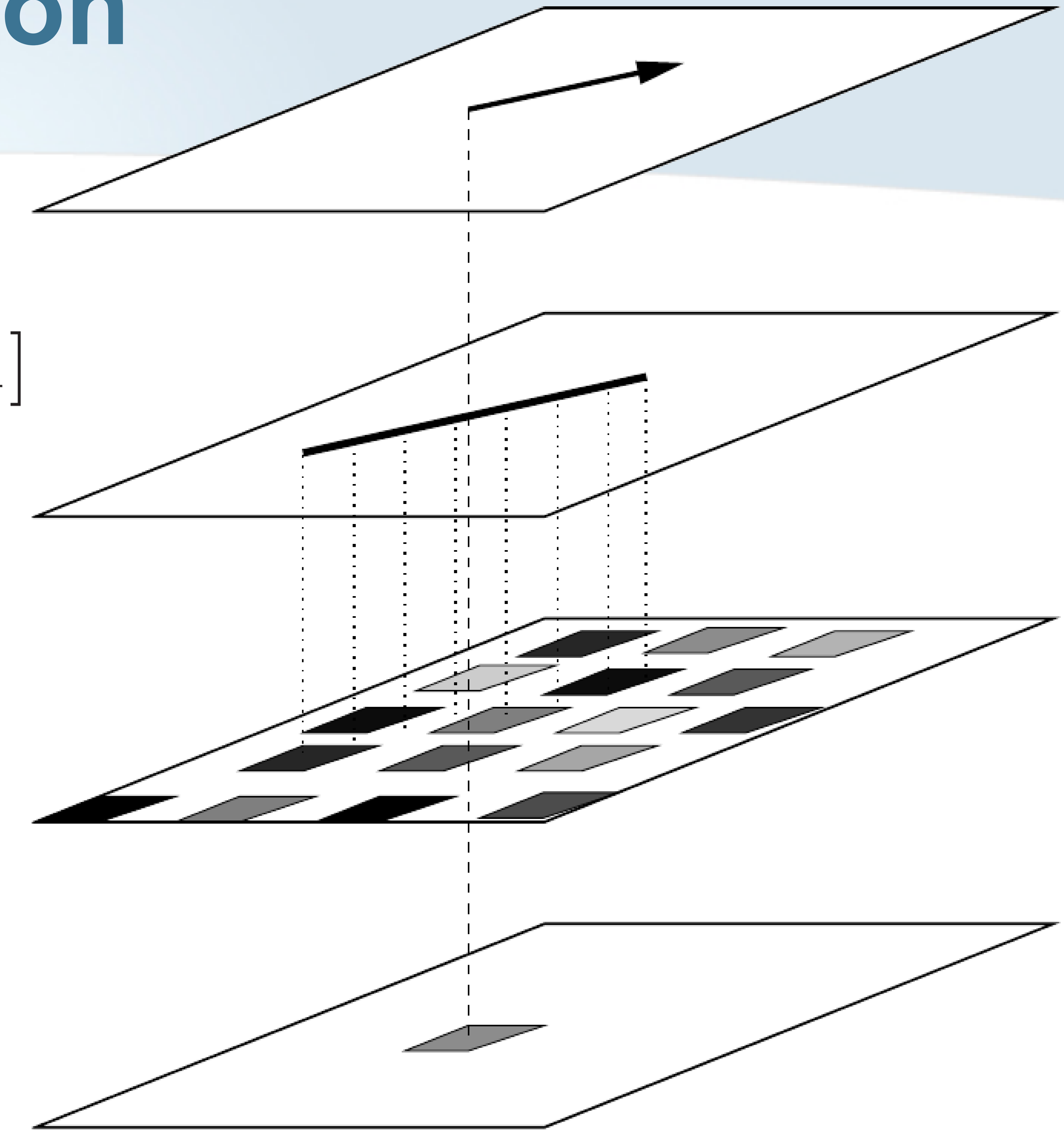
      - Backwards and forwards



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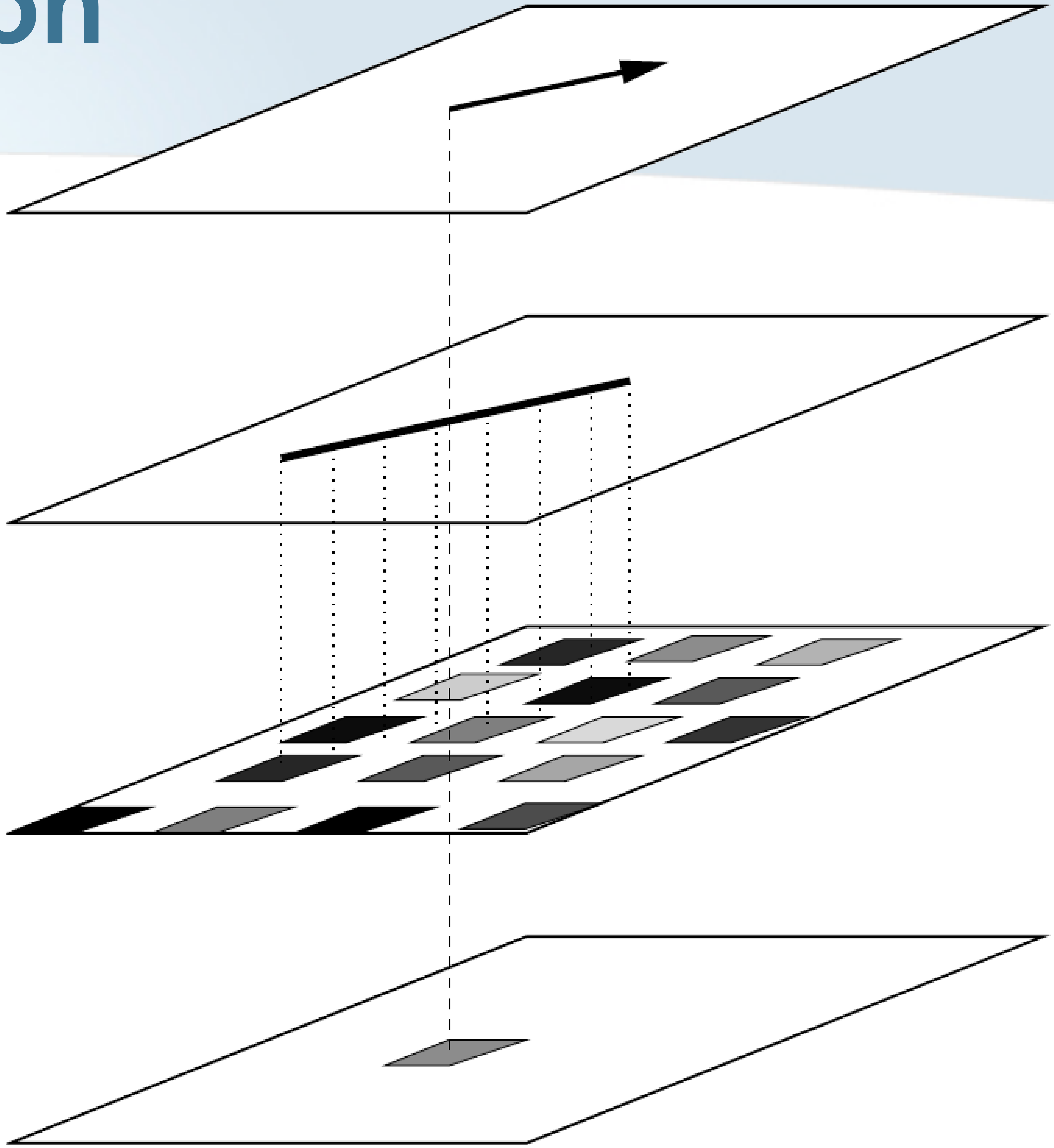

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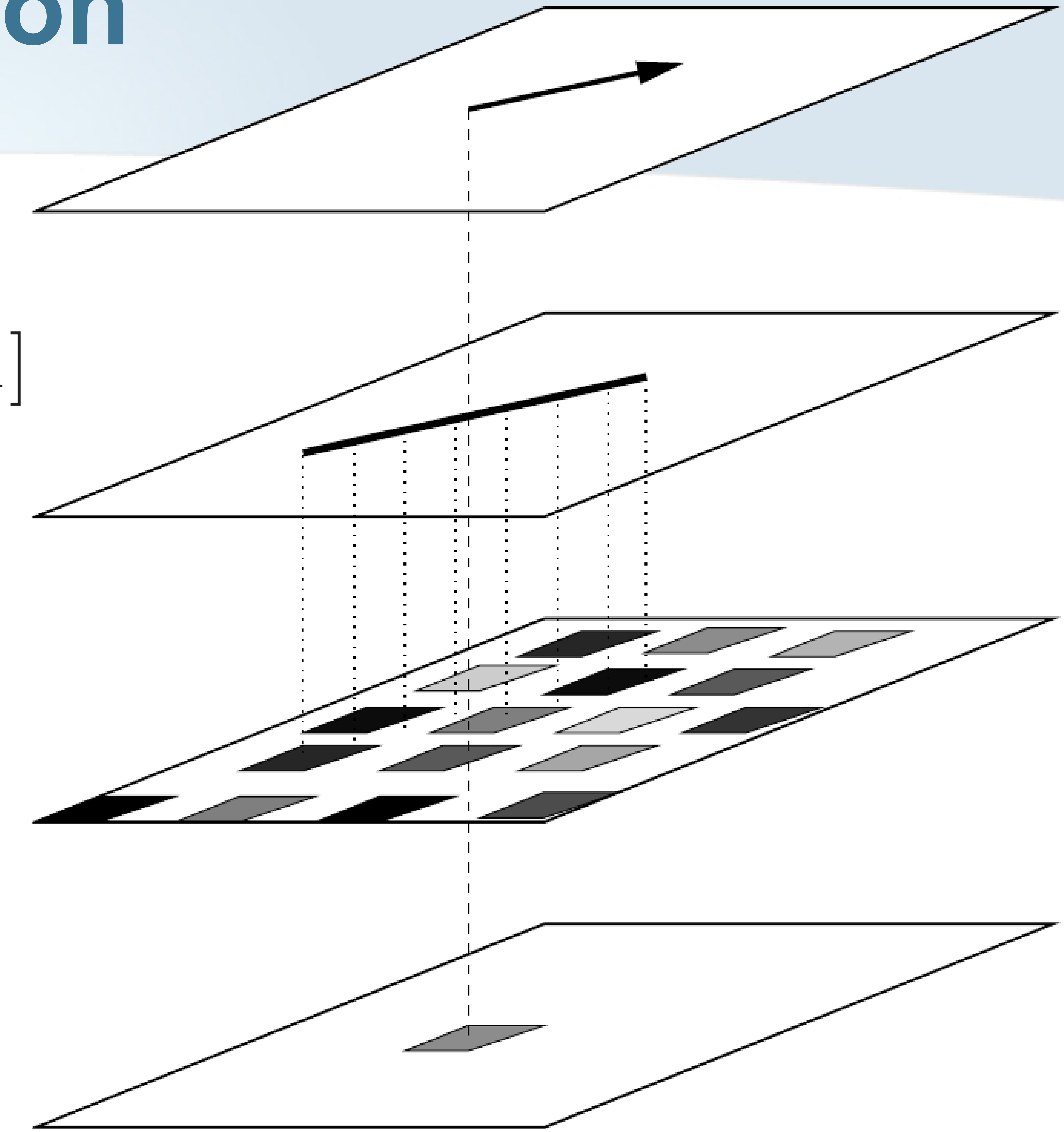
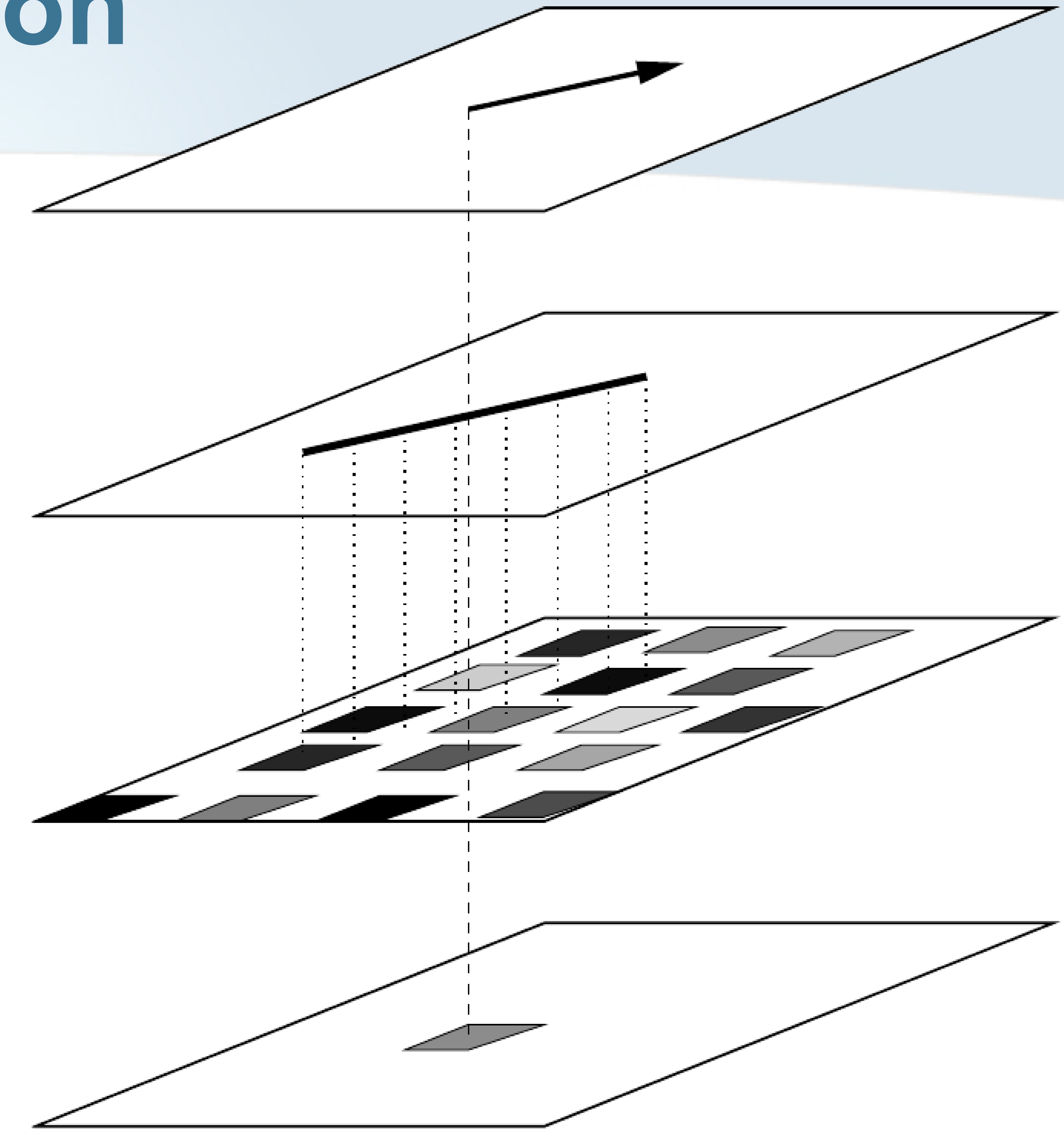
$$LIC(p) = \int_{c(p)-L}^{c(p)+L} k(u) \,.\, \mathcal{N}(c^{-1}(u))du$$



[Helgeland]

Line Integral Convolution

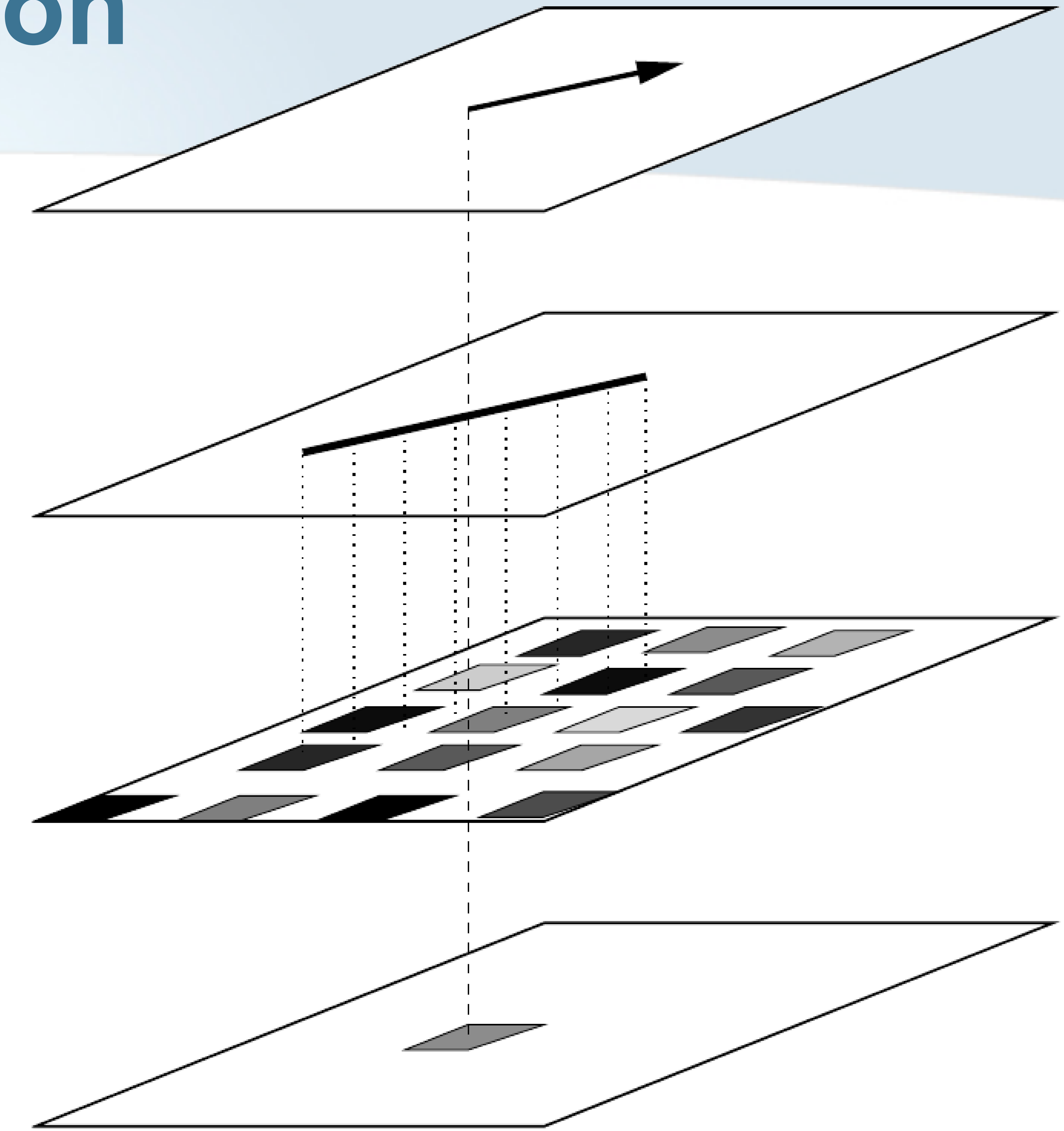
- Convolution kernel $k(u)$



[Helgeland]

Line Integral Convolution

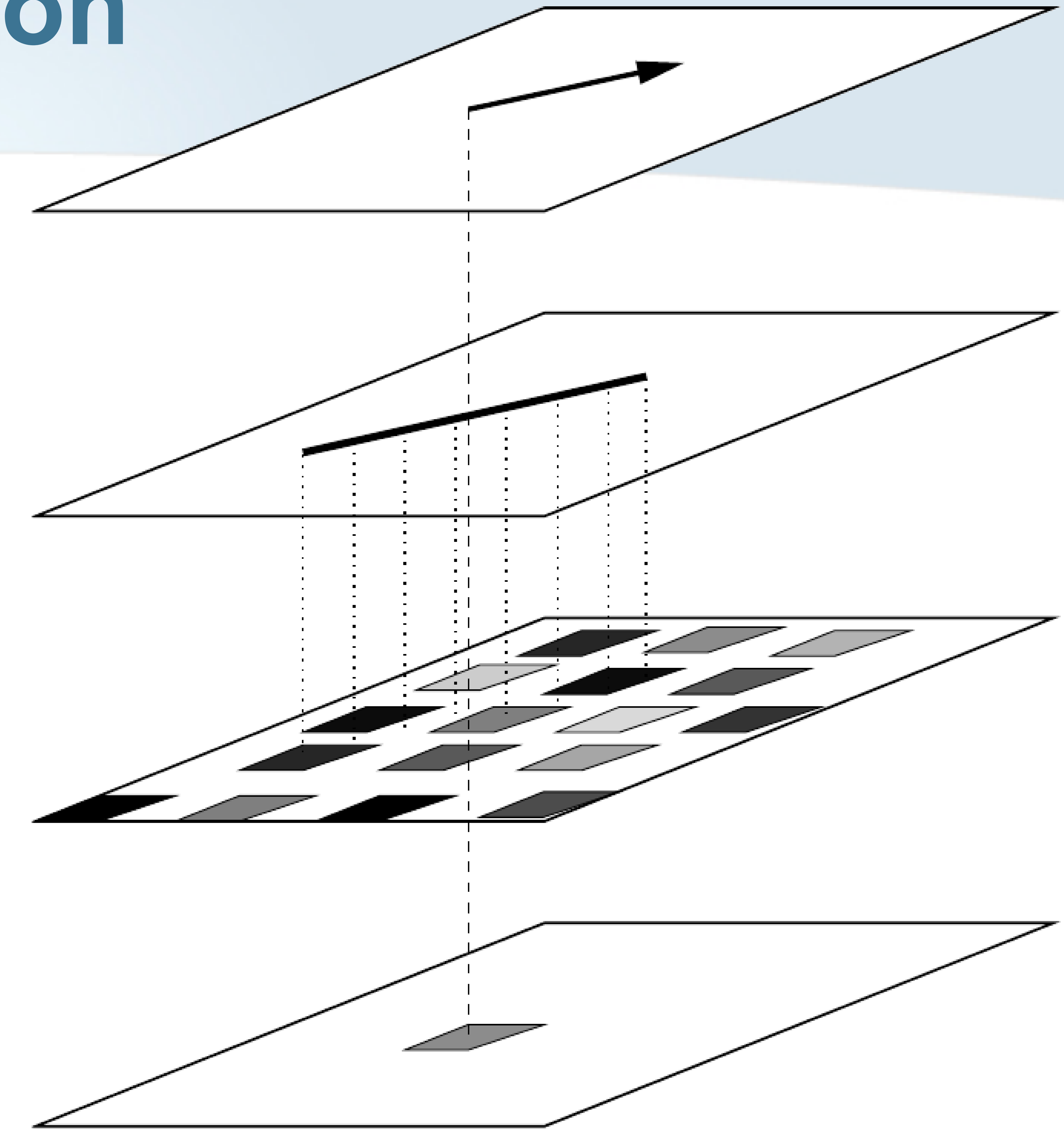
- Convolution kernel $k(u)$
  - Gaussian kernel



[Helgeland]

Line Integral Convolution

- Convolution kernel $k(u)$
  - Gaussian kernel
  - Finite support
    $[c(p) - L, c(p) + L]$



[Helgeland]

Line Integral Convolution

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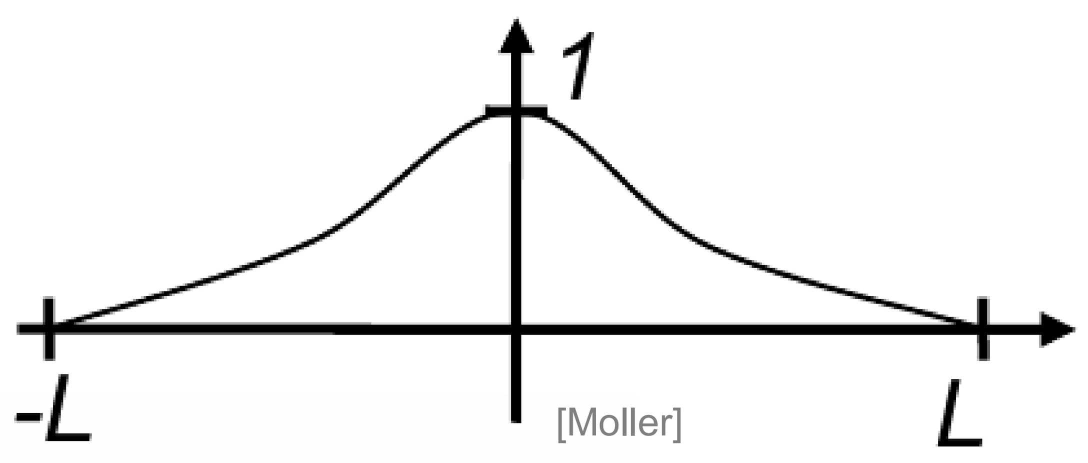
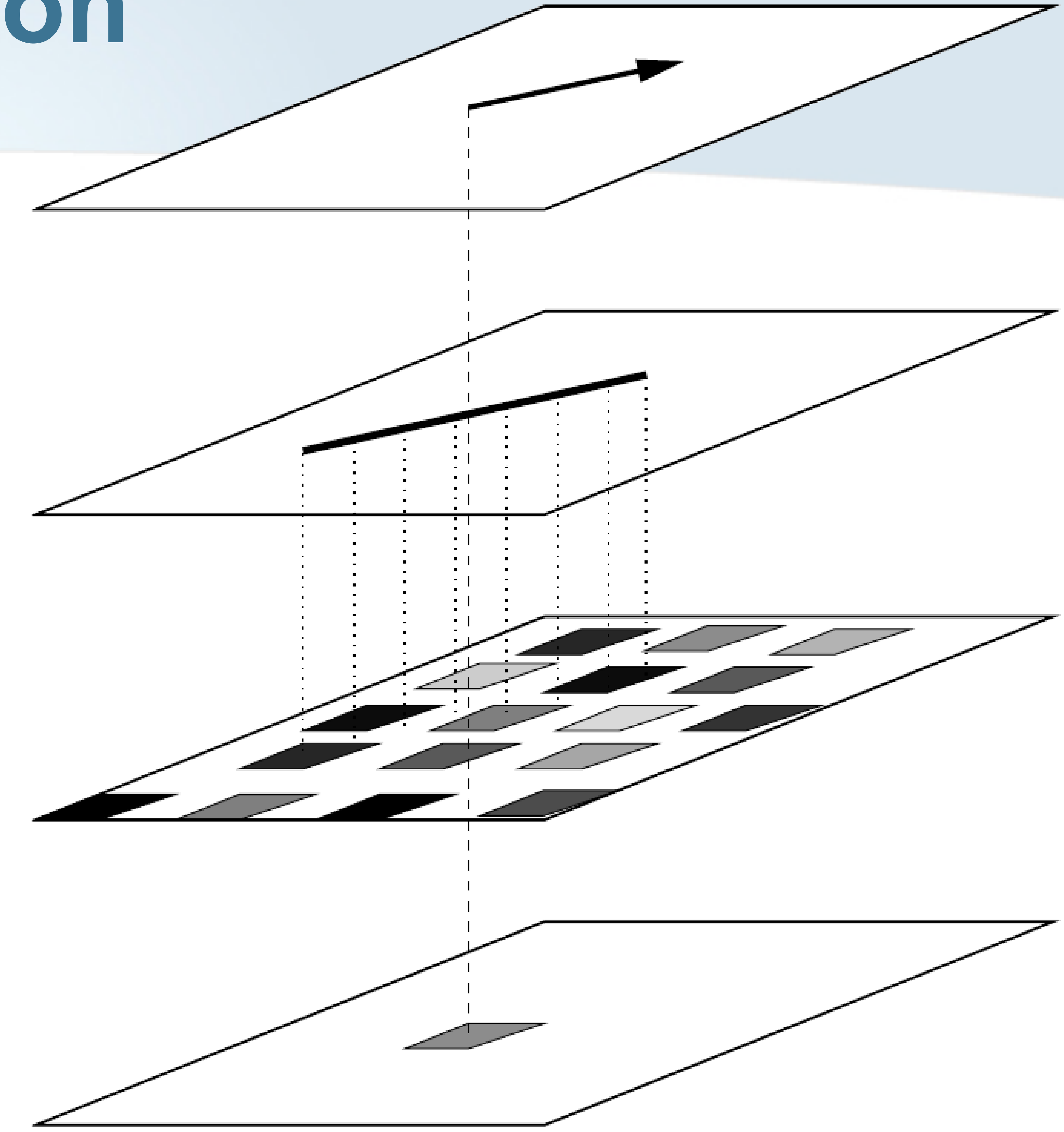
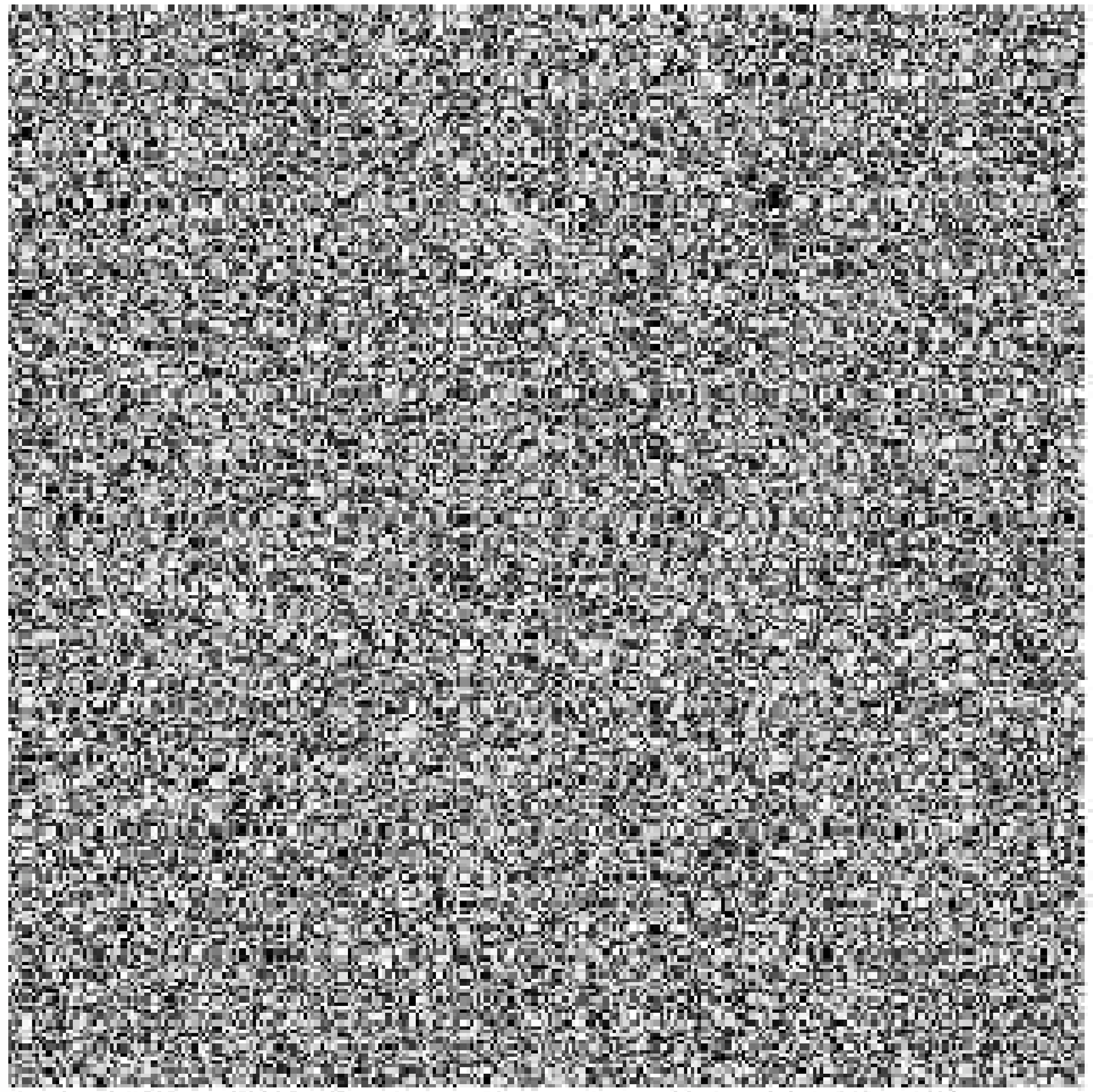
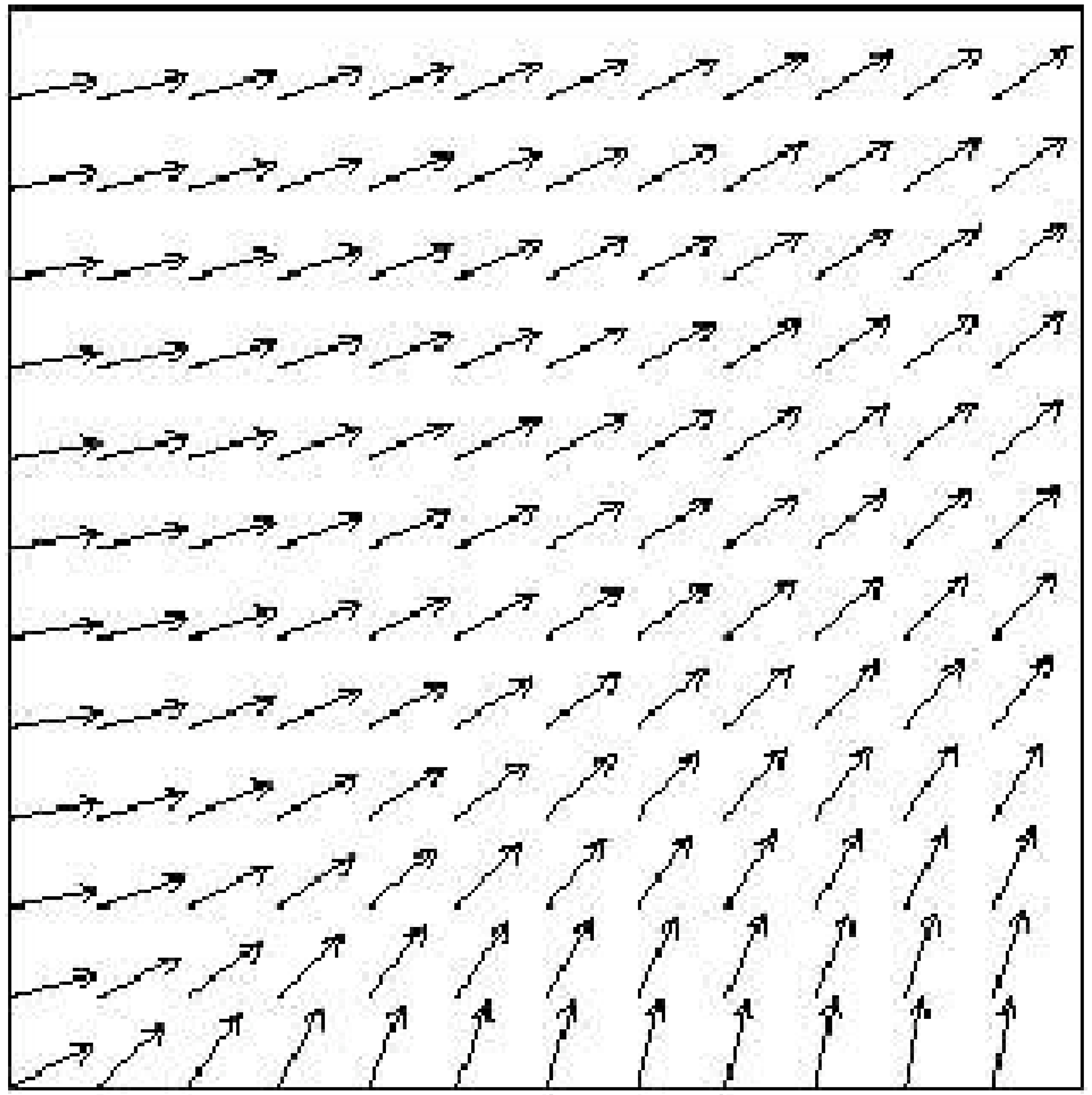
  - Gaussian kernel
  - Finite support $[c(p) - L, c(p) + L]$
  - Normalized $$\int_{c(p)-L}^{c(p)+L} k(u)du = 1$$





Line Integral Convolution





[Helgeland]

LIC for volumetric domains

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- Easy way
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LIC for volumetric domains

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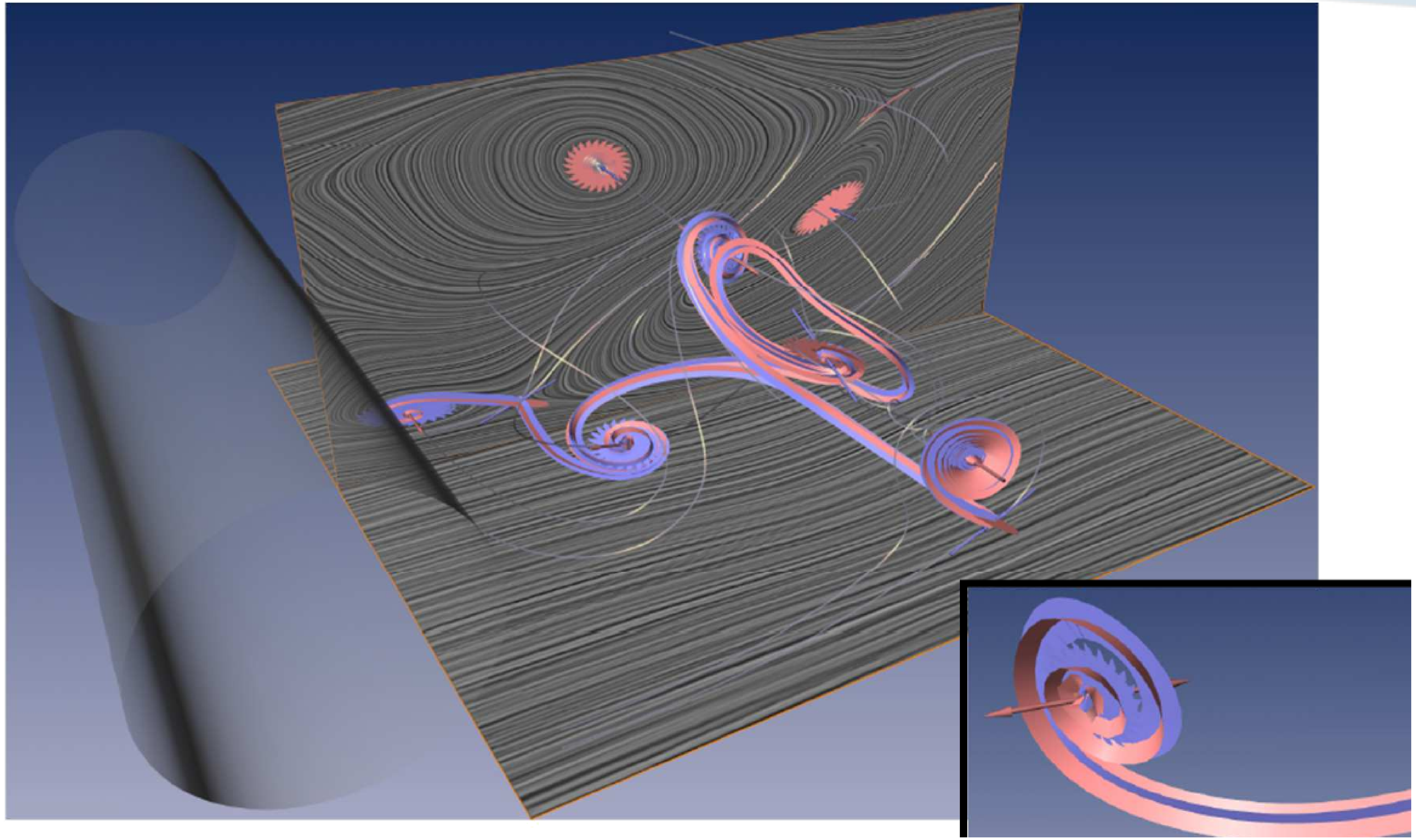

[Theisel 03]

LIC for volumetric domains

- Easy way
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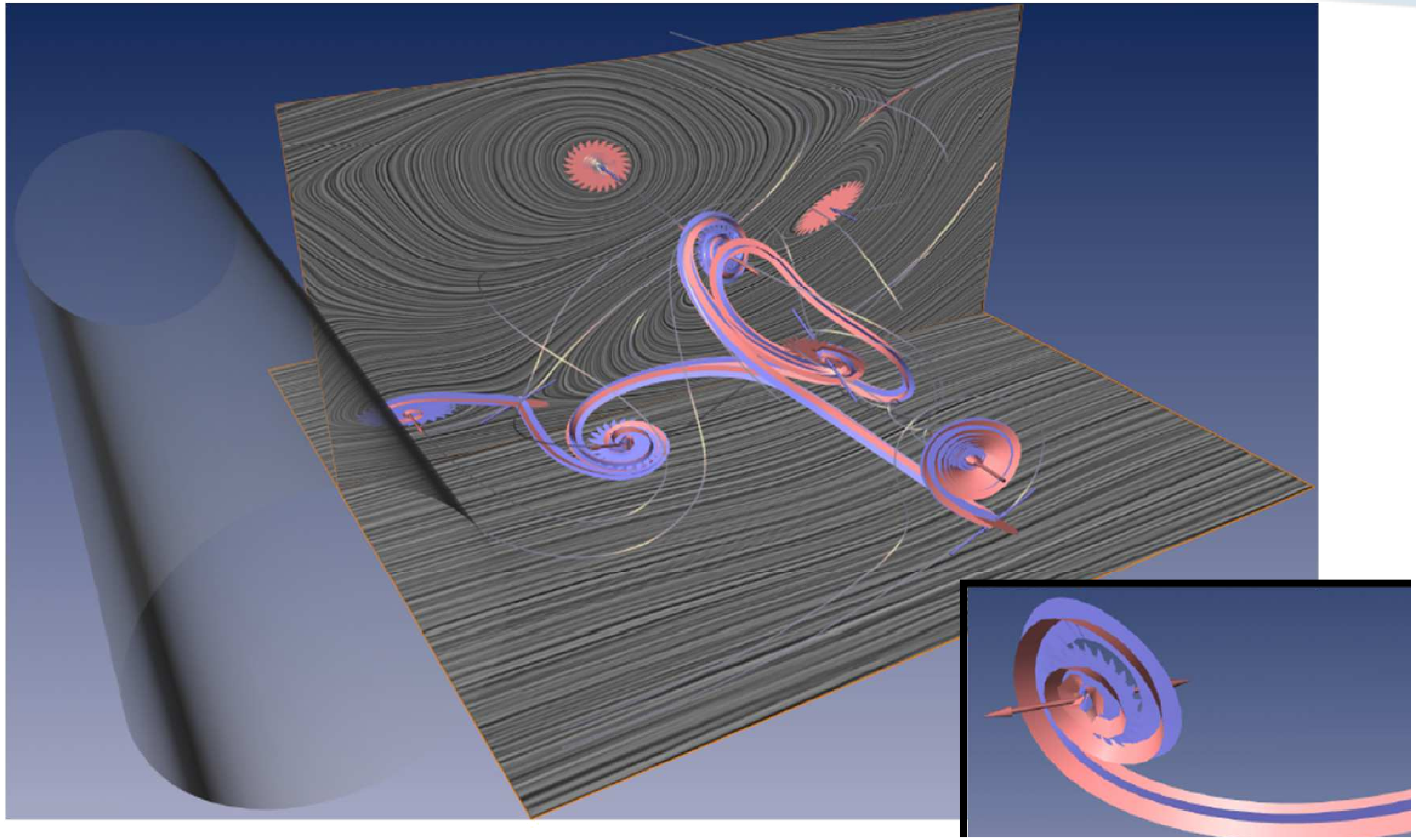
- Also,
  - What is LIC in the end?



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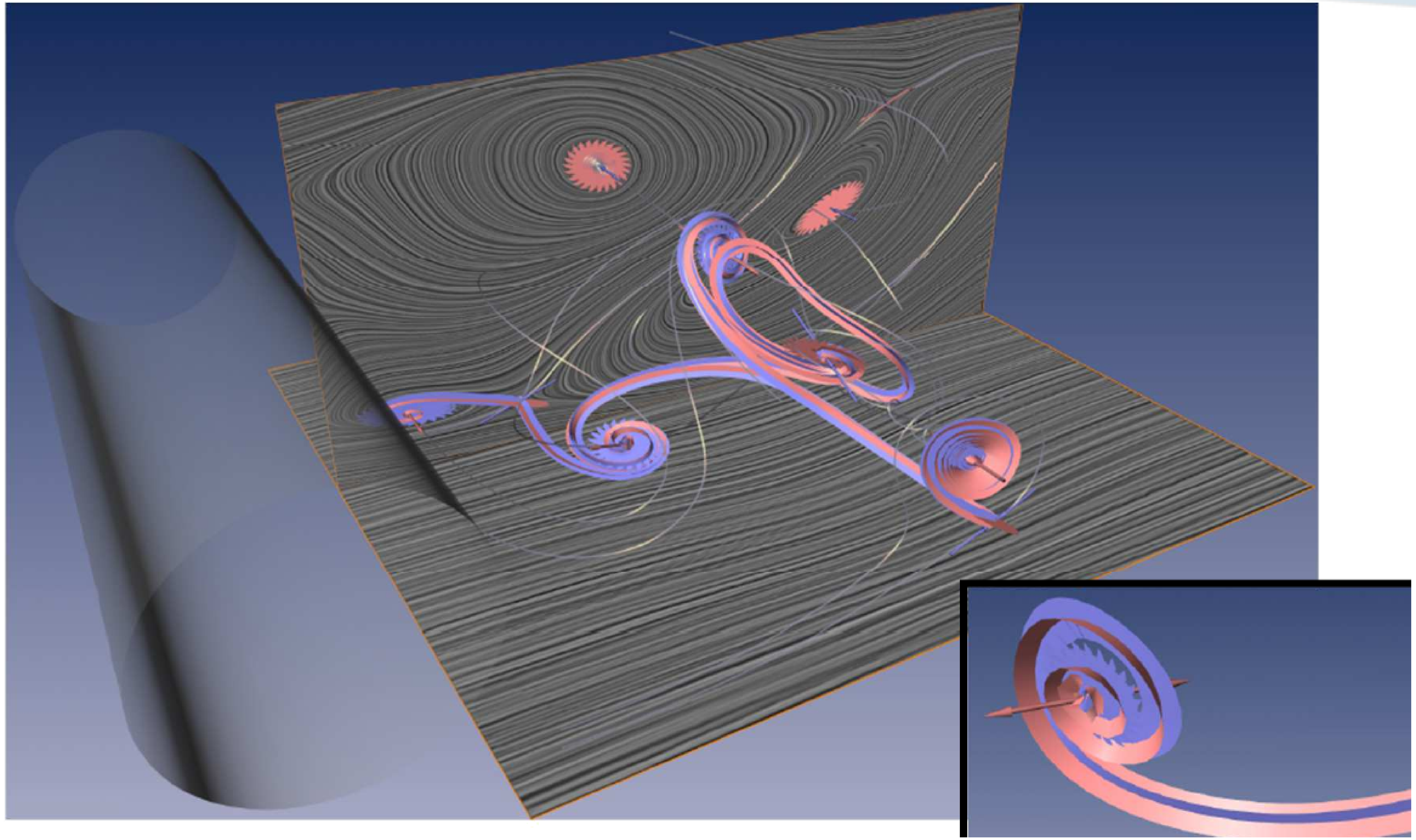

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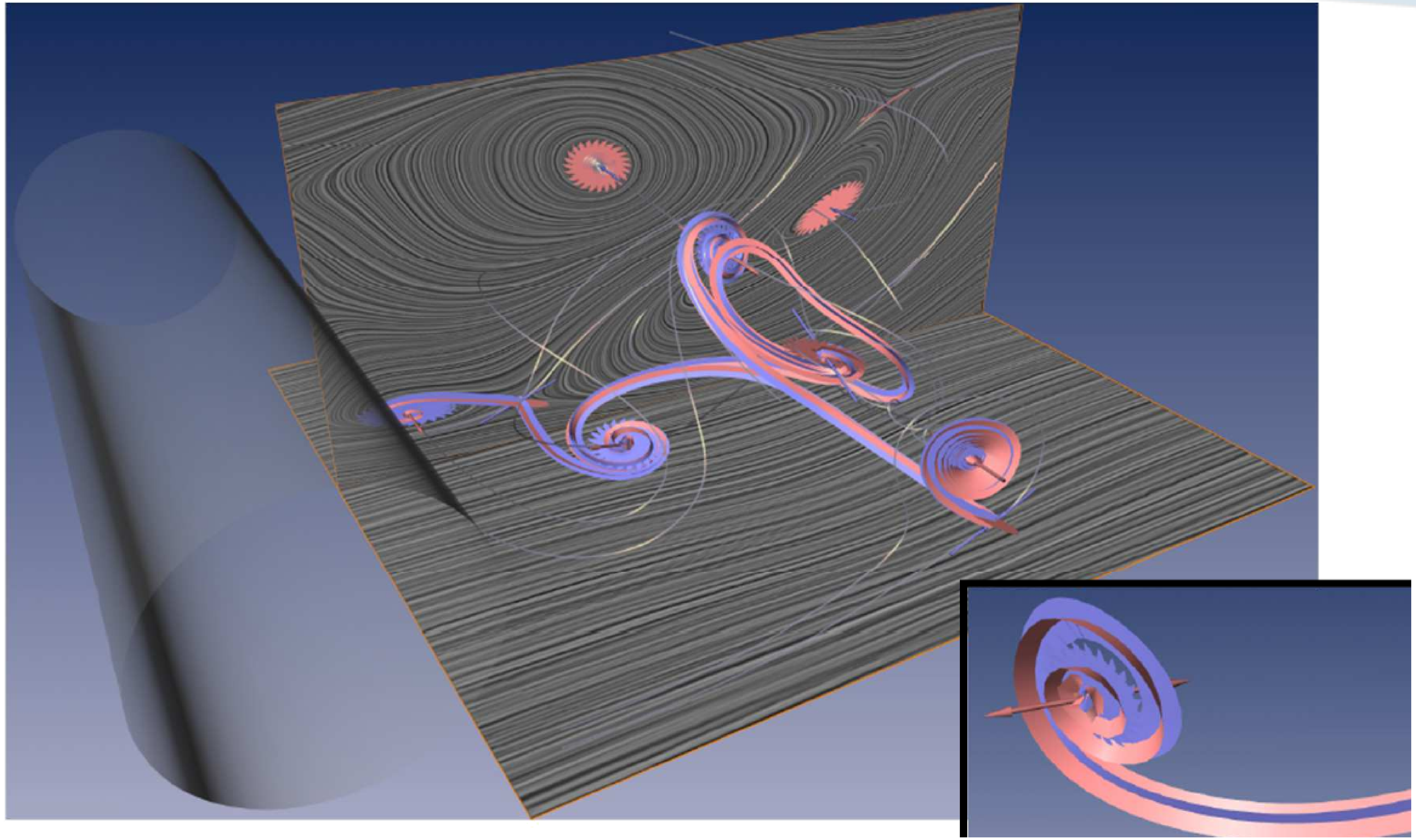
    - Volume rendering!



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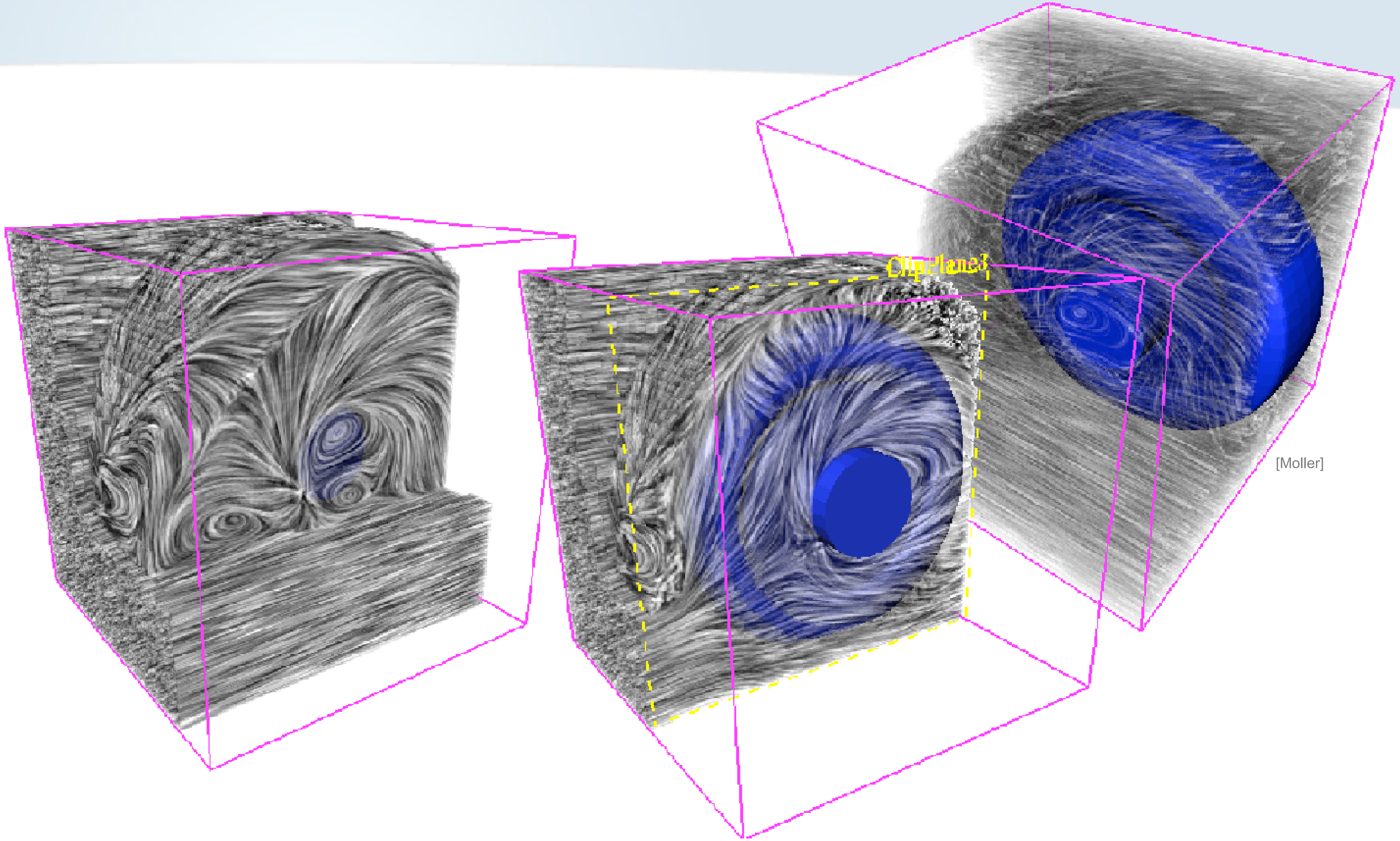

[Moller]

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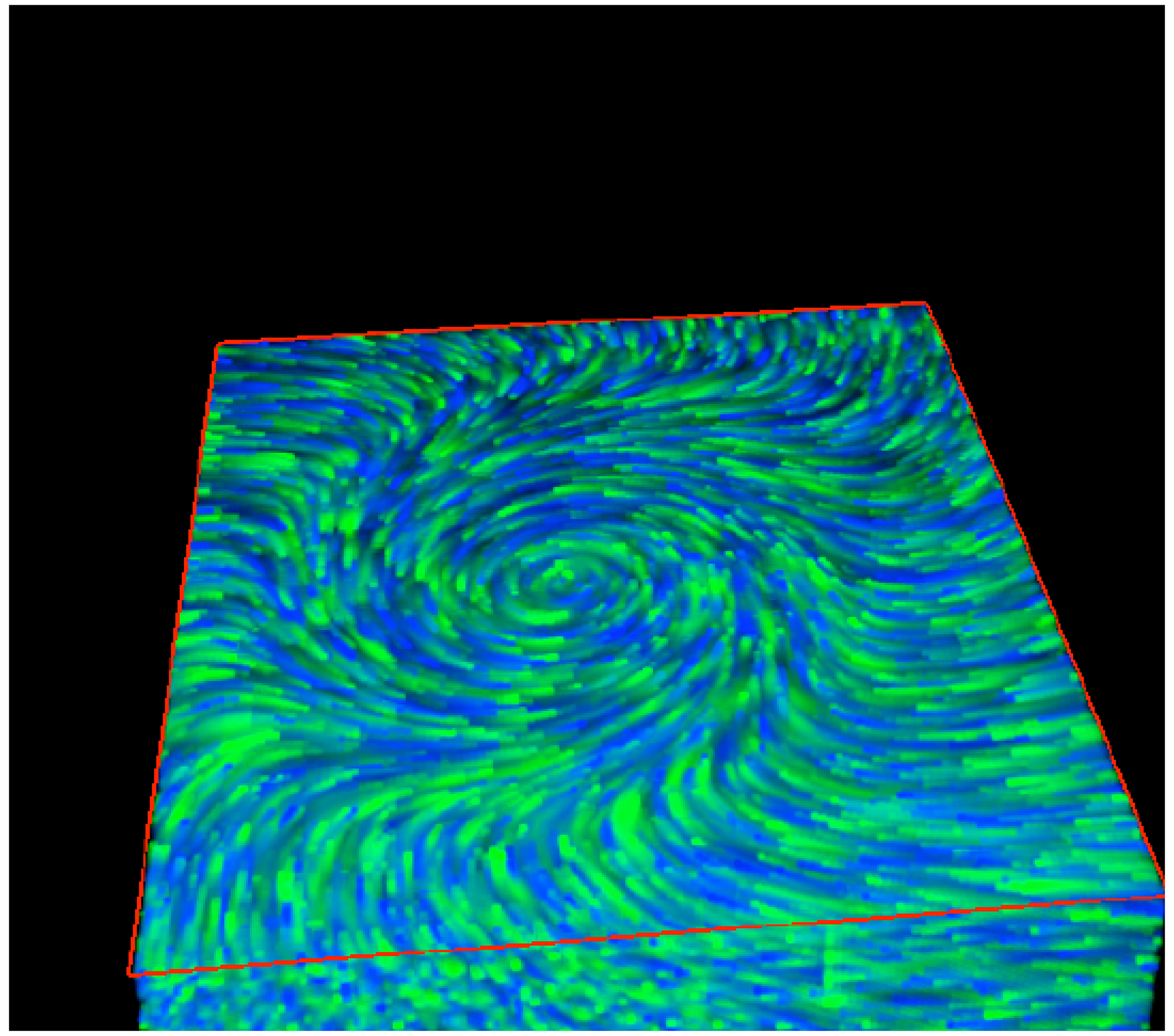
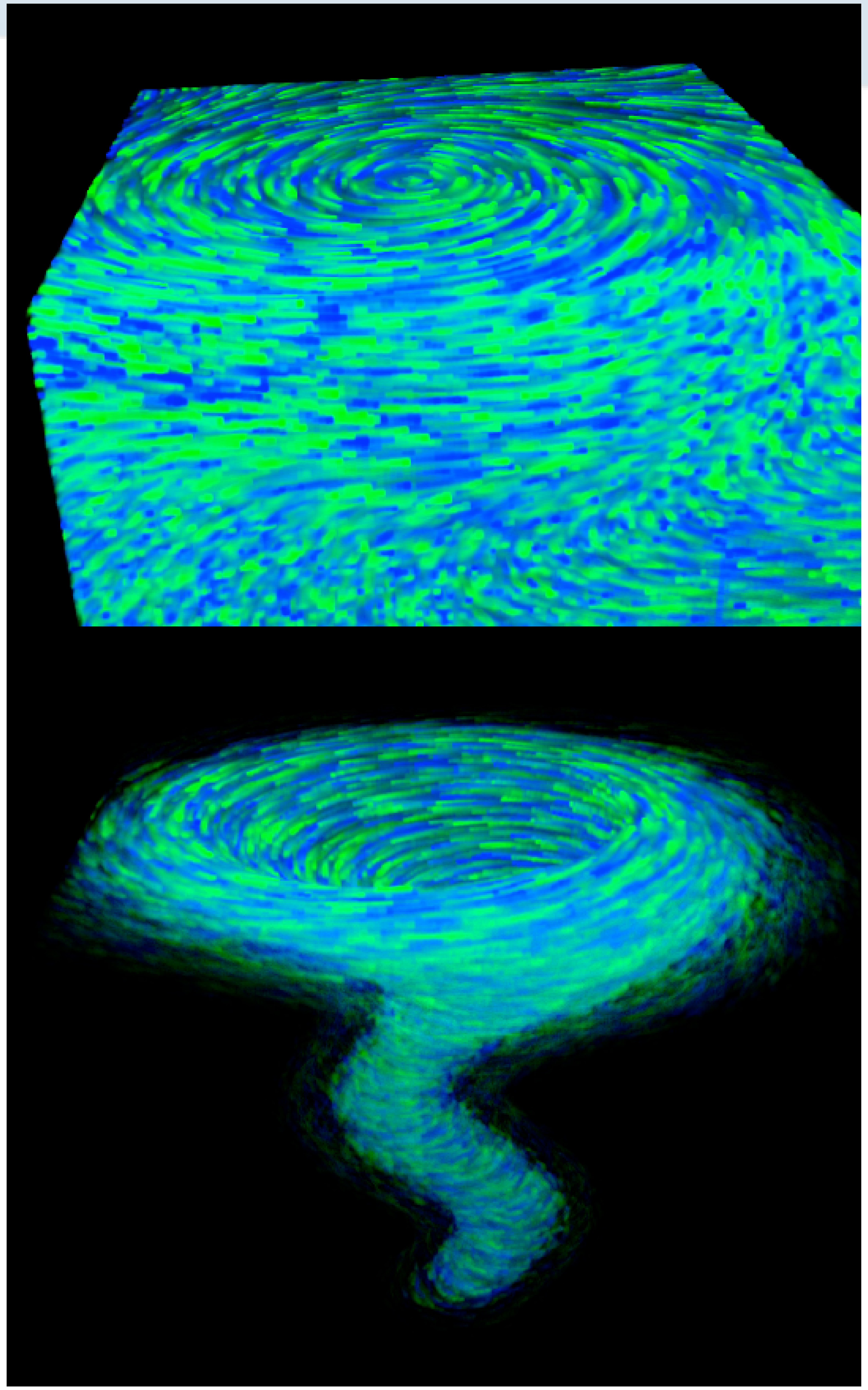
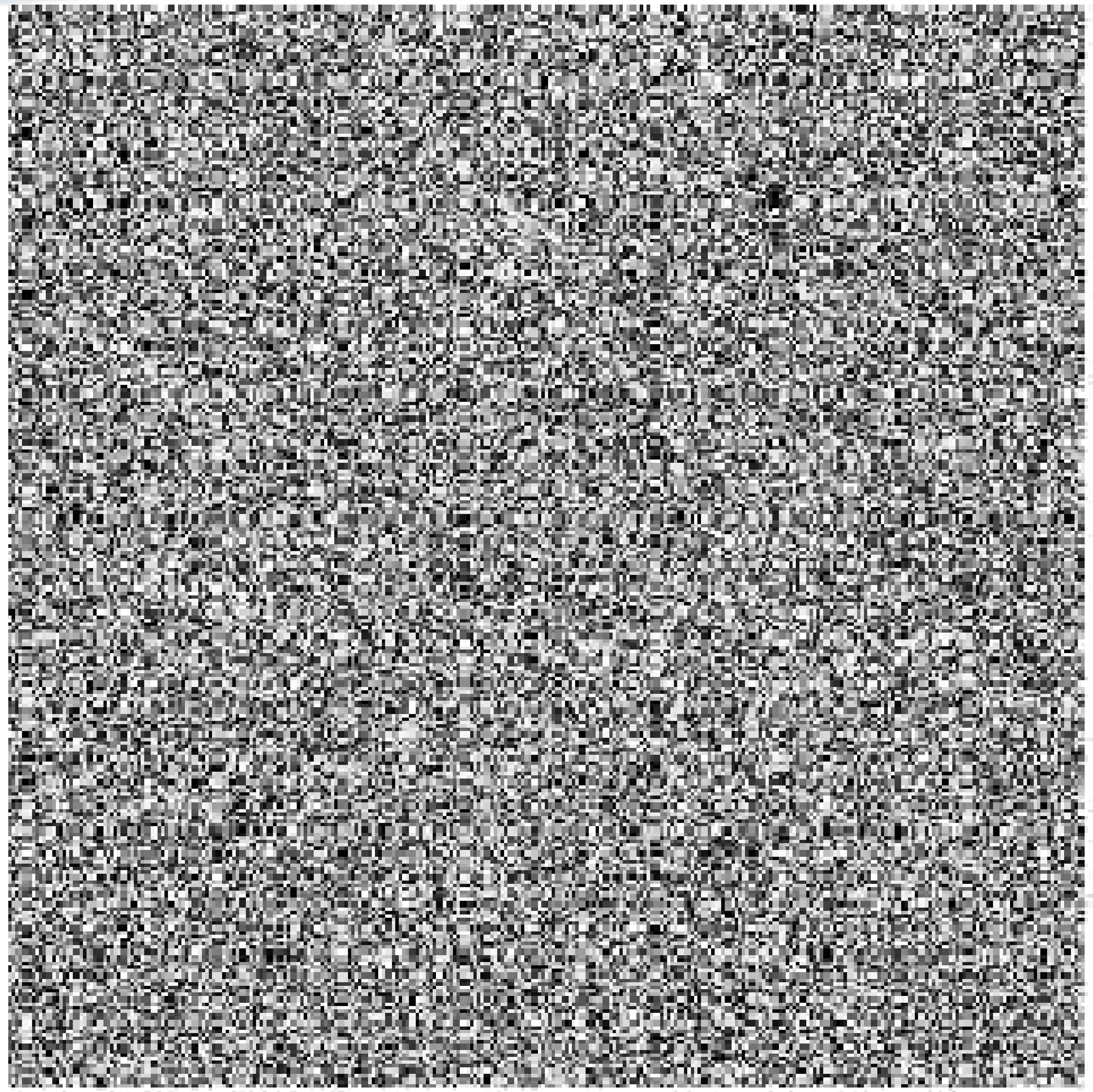
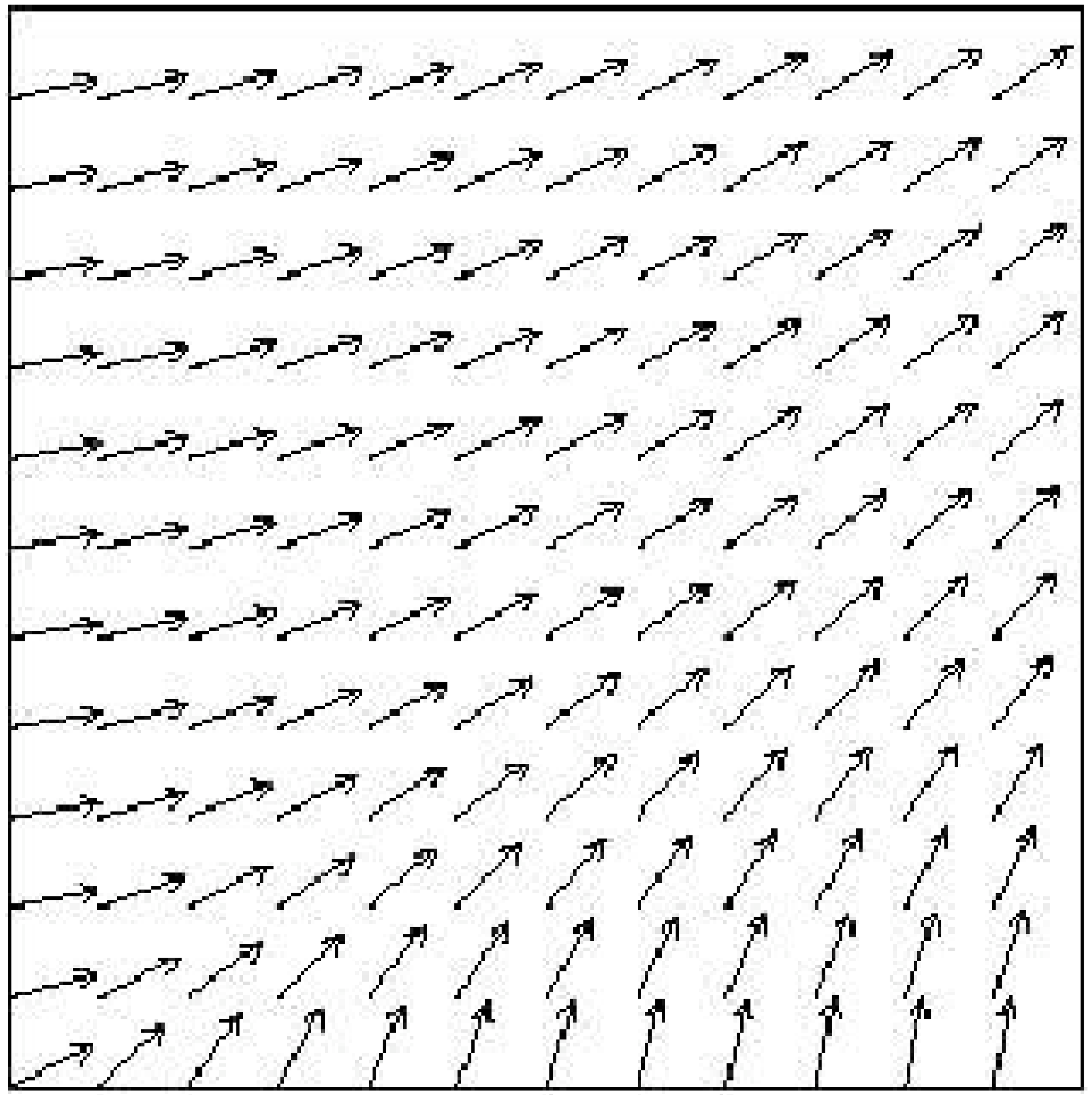




[Moller]

Line Integral Convolution





[Helgeland]

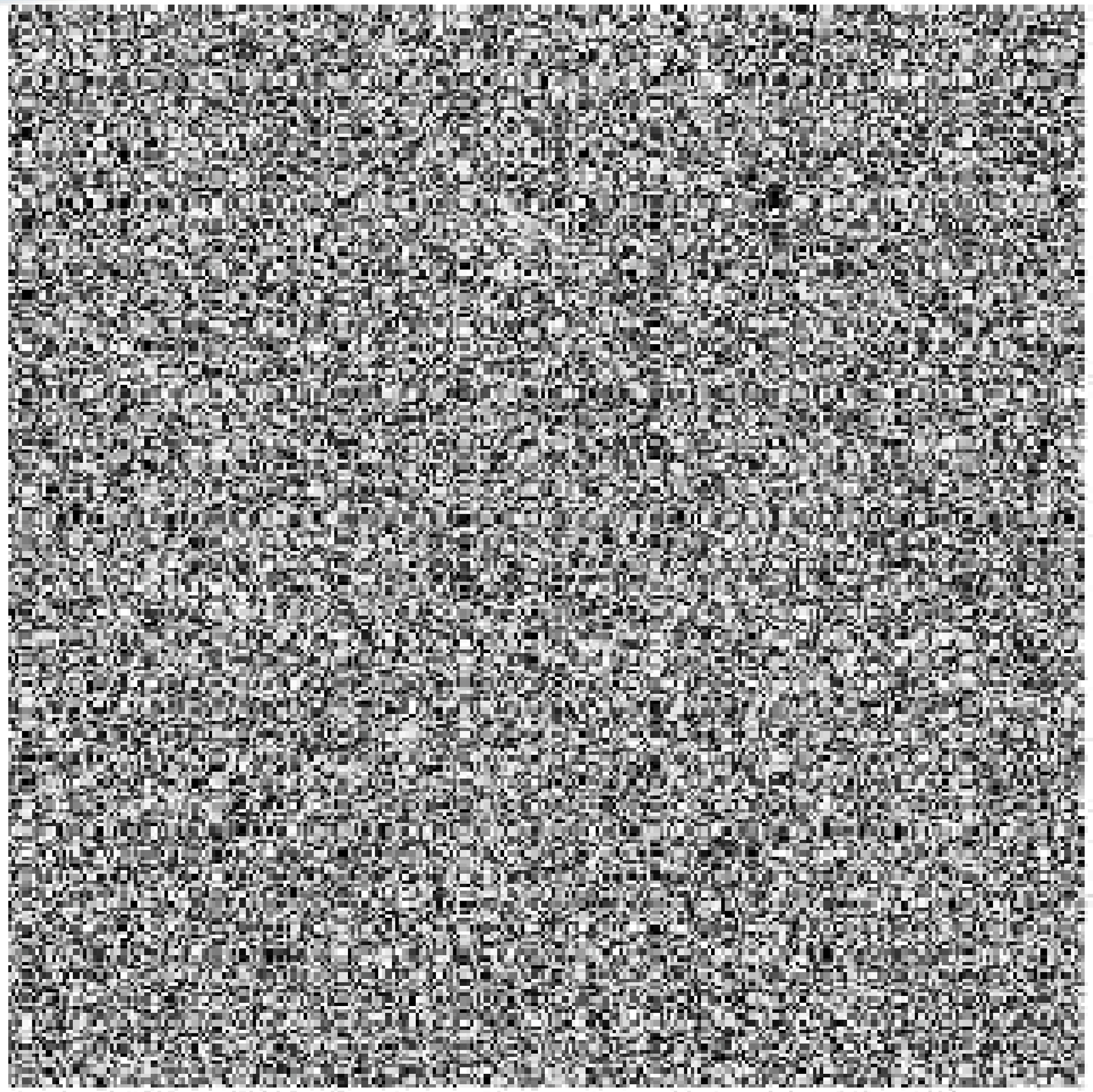
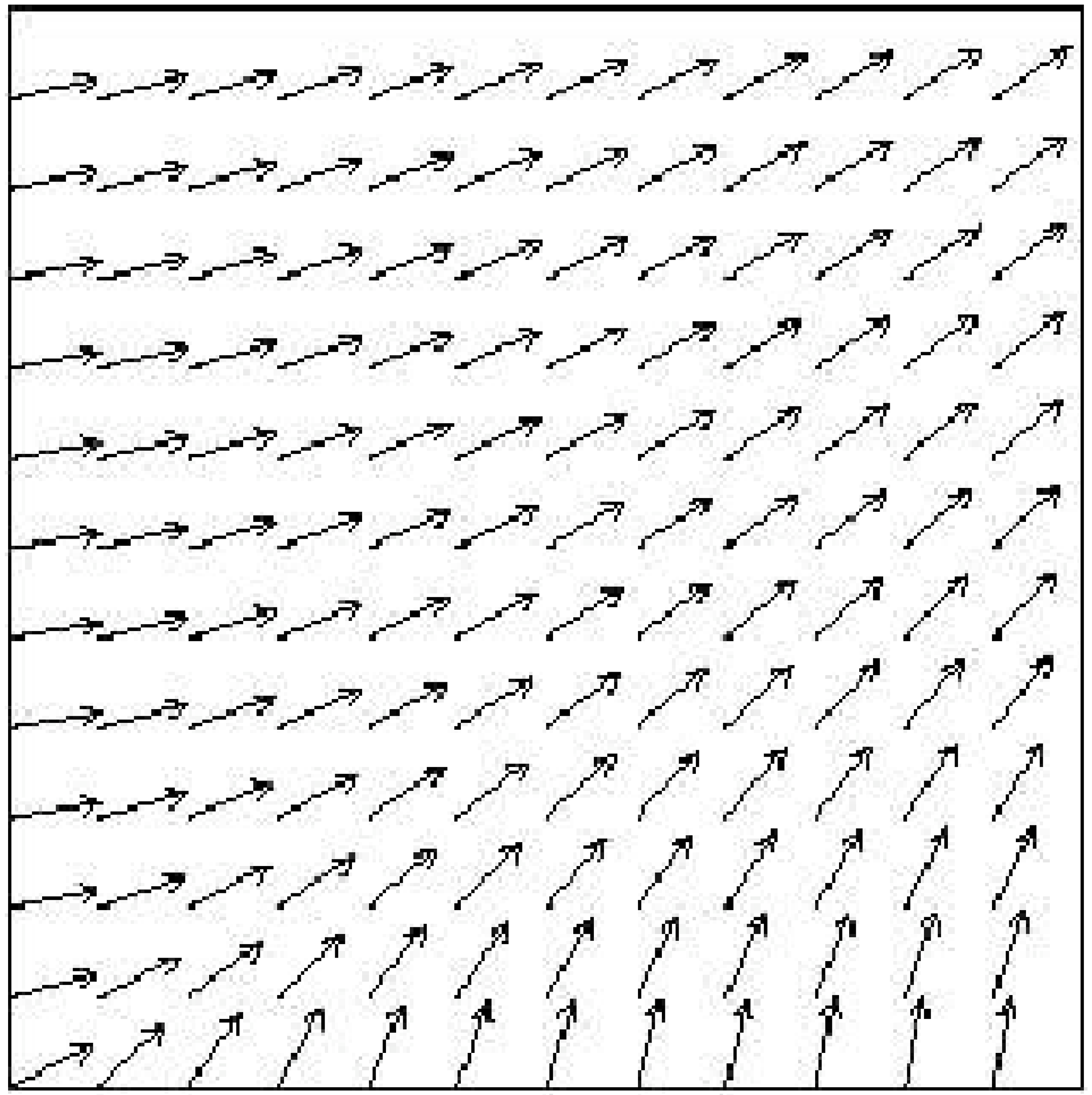
- Global visualization of the direction of the flow

Line Integral Convolution

[Helgeland]

- Global visualization of the direction of the flow
  - What about its magnitude?

Line Integral Convolution

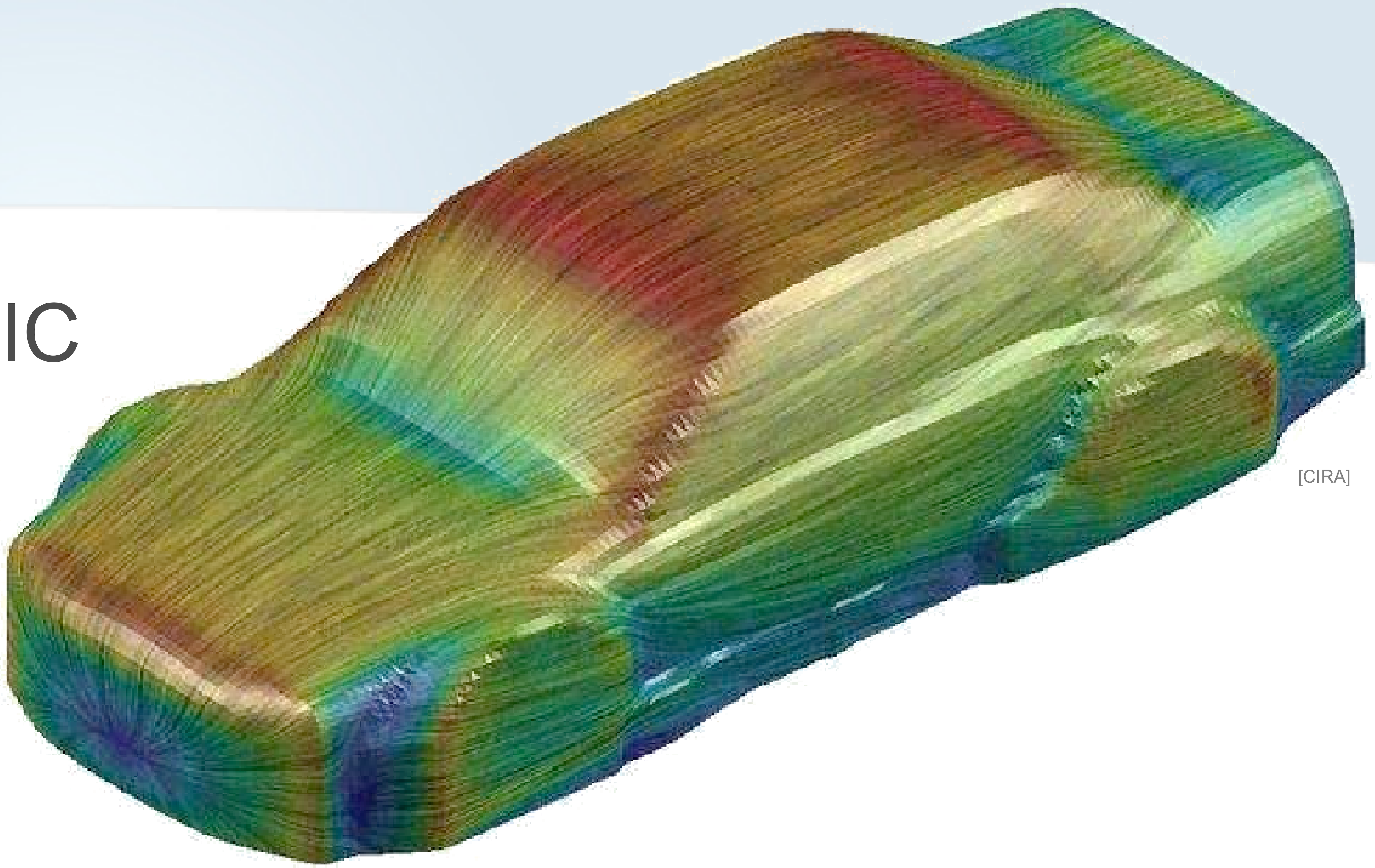




[Helgeland]

- Global visualization of the direction of the flow
  - What about its magnitude? Its orientation?

Derived scalar fields

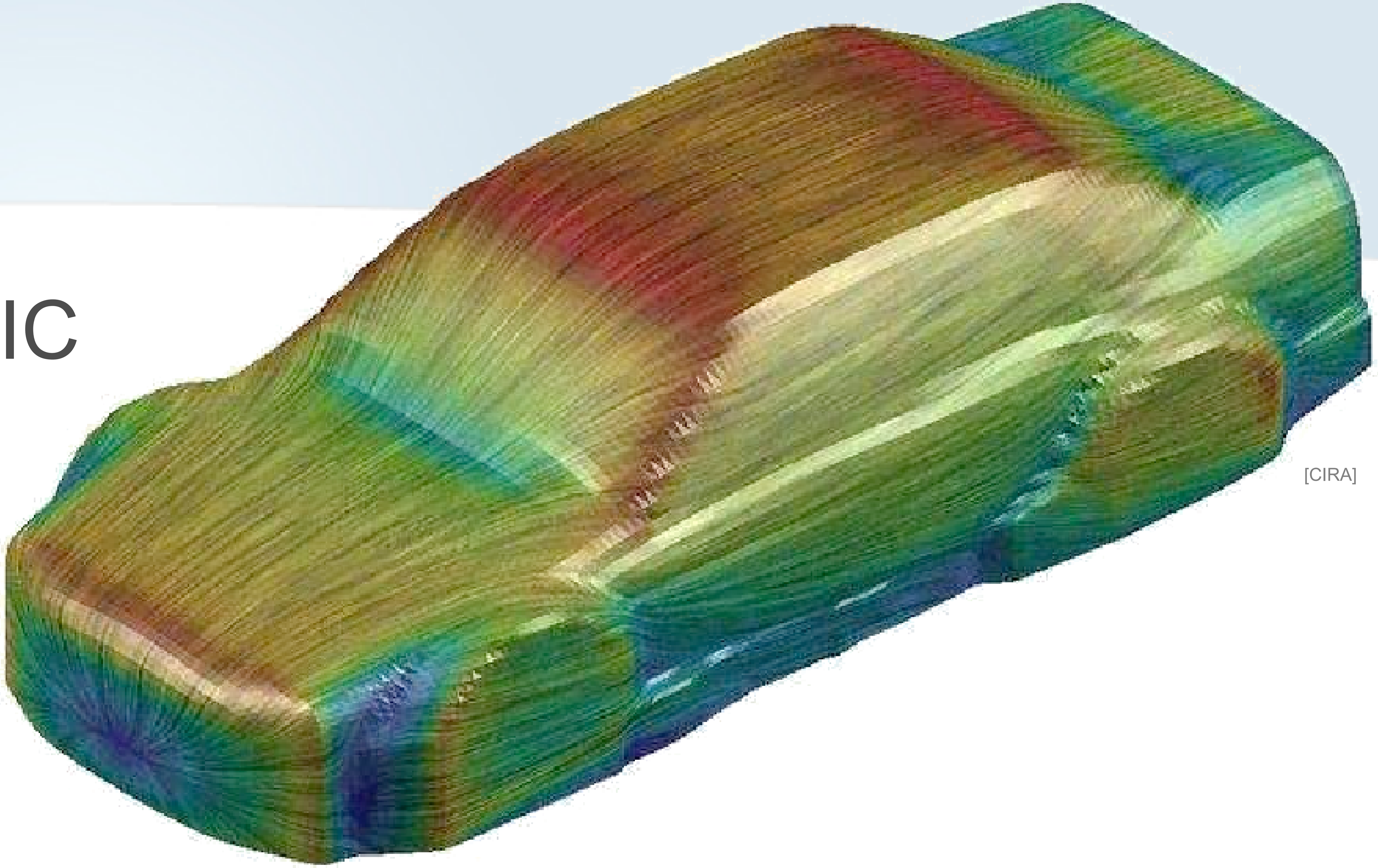
- Combine the magnitude with the LIC



[CIRA]

Derived scalar fields

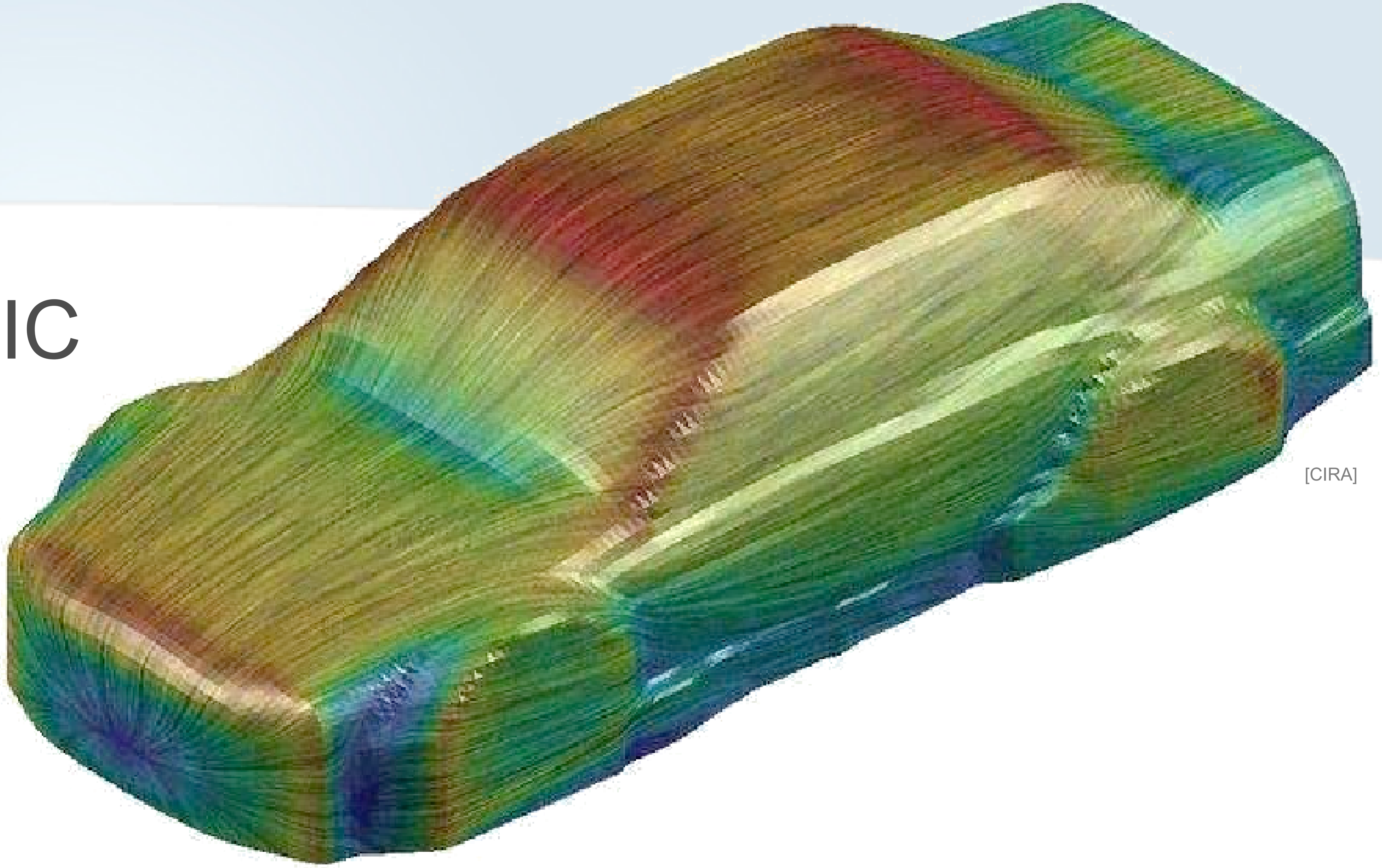
- Combine the magnitude with the LIC
  - Color map of the magnitude



[CIRA]

Derived scalar fields

- Combine the magnitude with the LIC
  - Color map of the magnitude
  - For each channel (RGB)
    - Multiply by the LIC value



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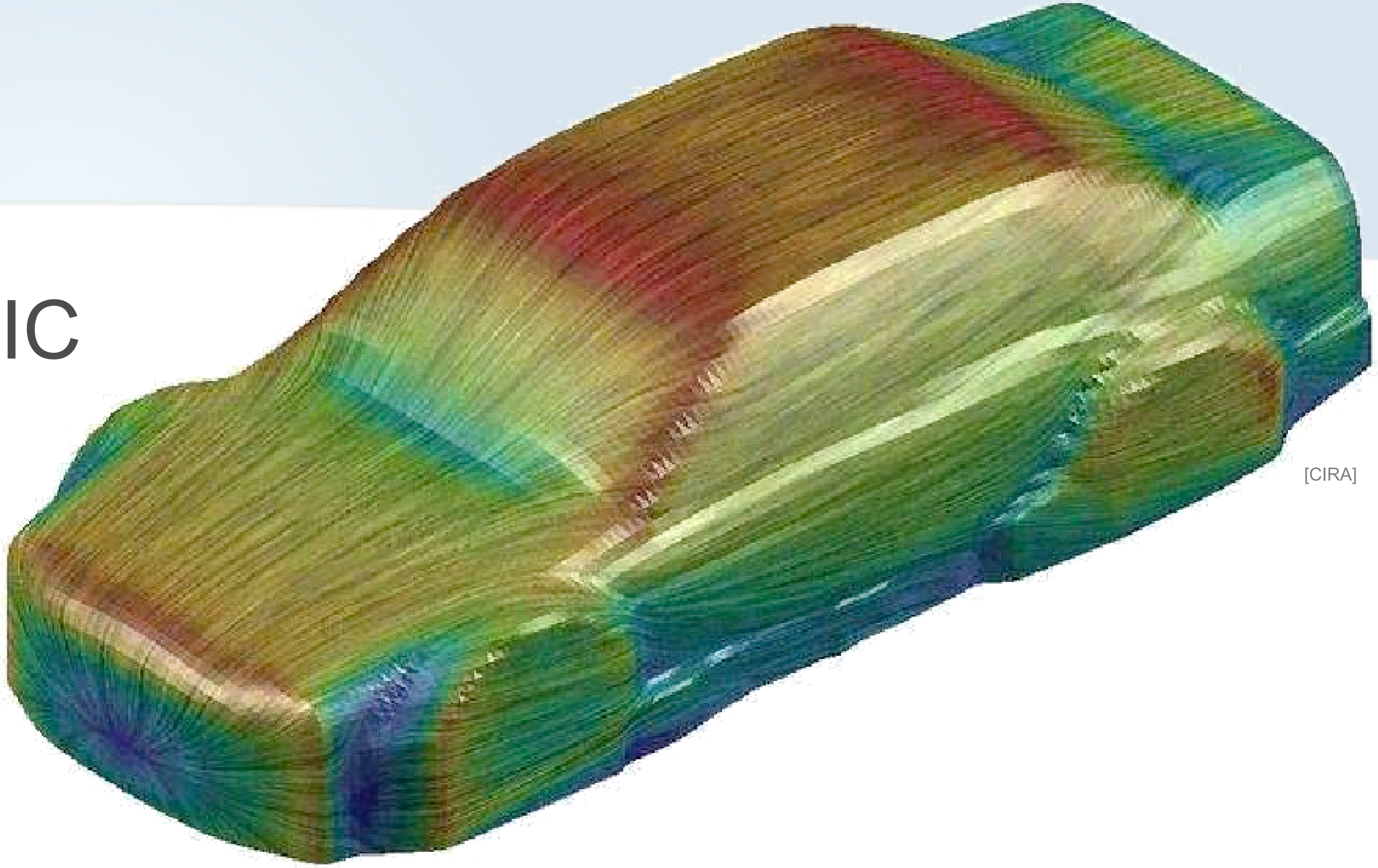

[CIRA]

- What other derived scalar fields would be interesting?

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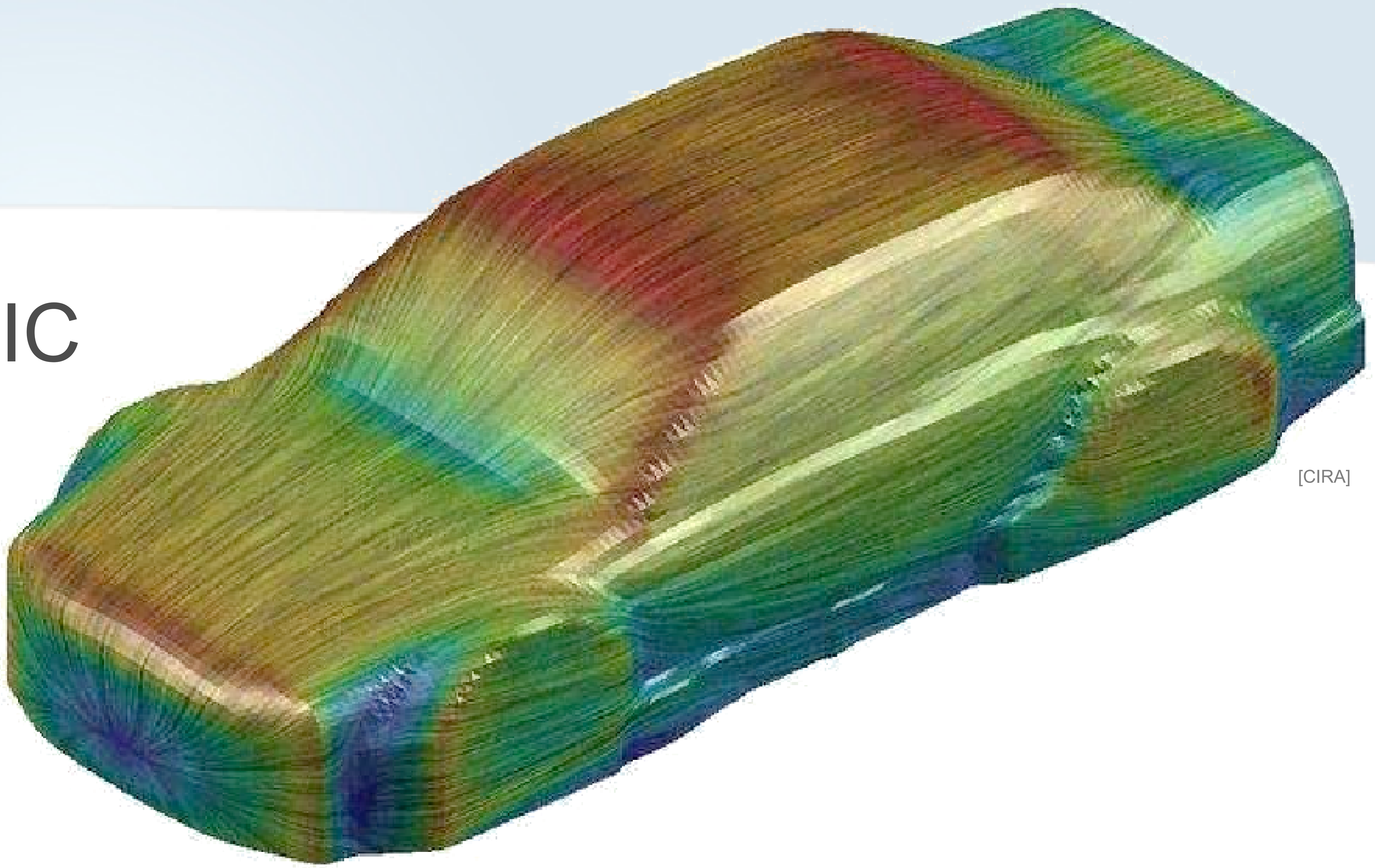
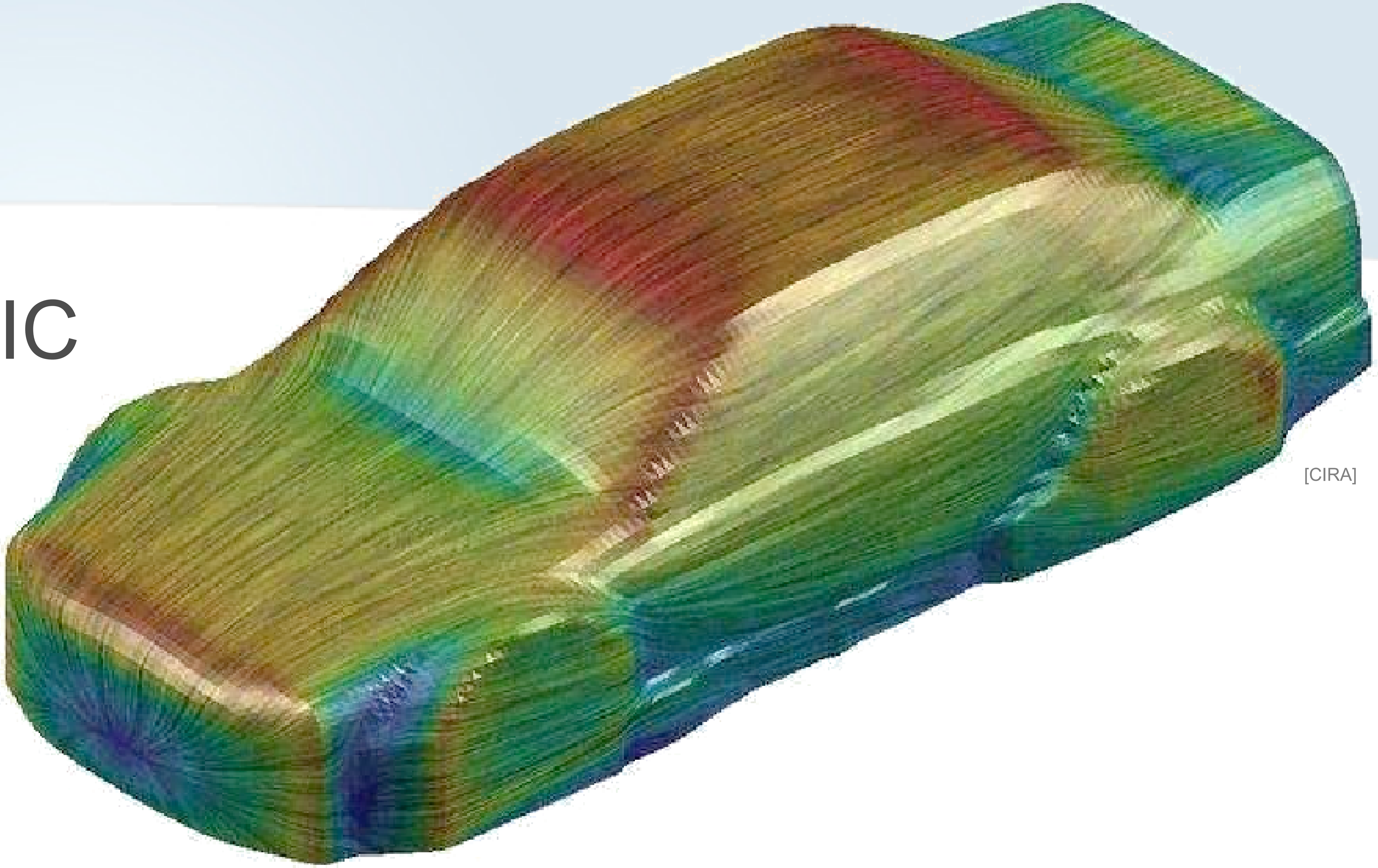


[CIRA]

- What other derived scalar fields would be interesting?
  - Flow orientation

Derived scalar fields


[CIRA]

- Combine the magnitude with the LIC
  - Color map of the magnitude
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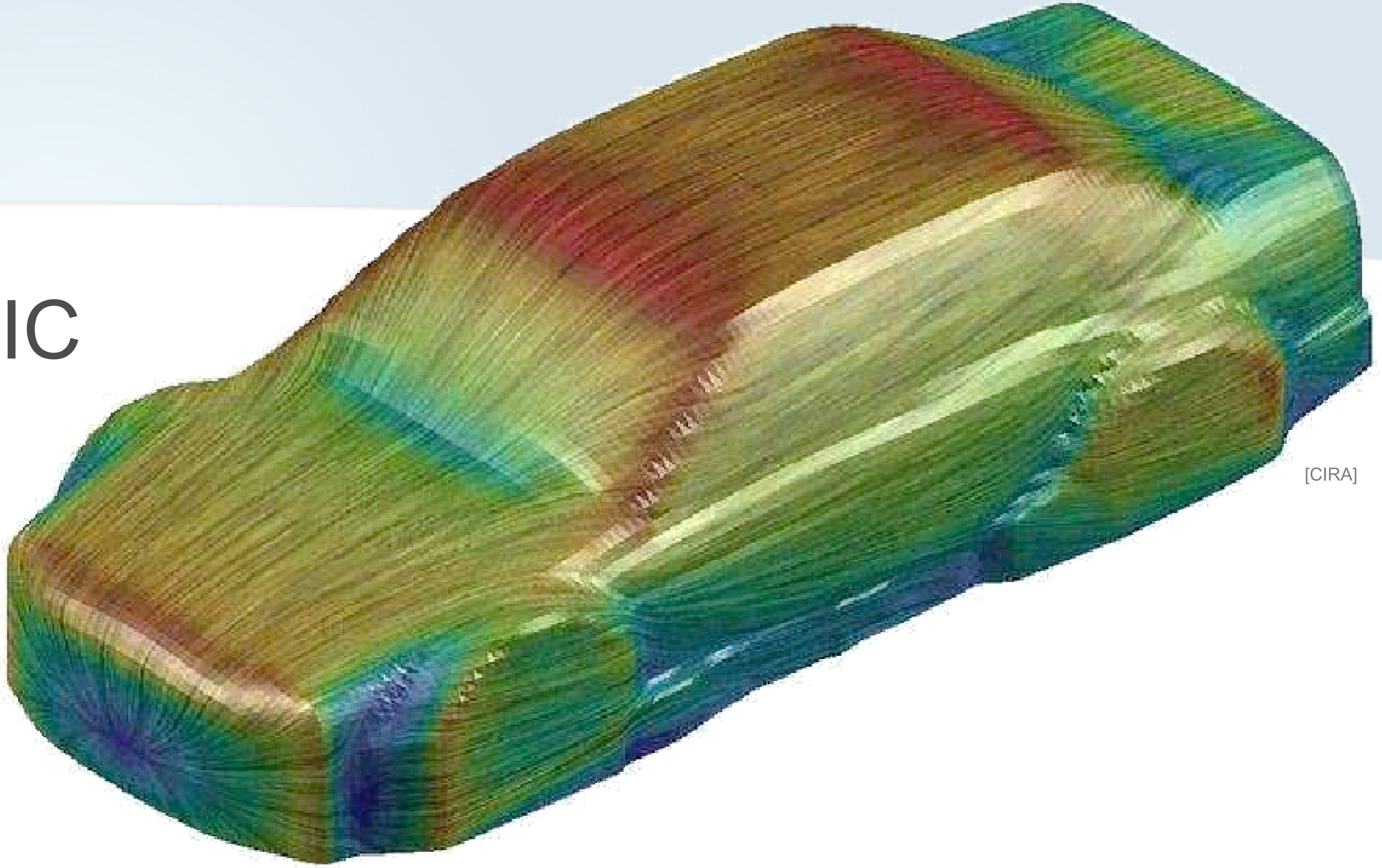
    - Multiply by the LIC value

- What other derived scalar fields would be interesting?
  - Flow orientation: divergence

Derived scalar fields

- Combine the magnitude with the LIC
  - Color map of the magnitude
  - For each channel (RGB)
    - Multiply by the LIC value



[CIRA]

- What other derived scalar fields would be interesting?
  - Flow orientation: divergence
  - Angular speed

Derived scalar fields

- Combine the magnitude with the LIC
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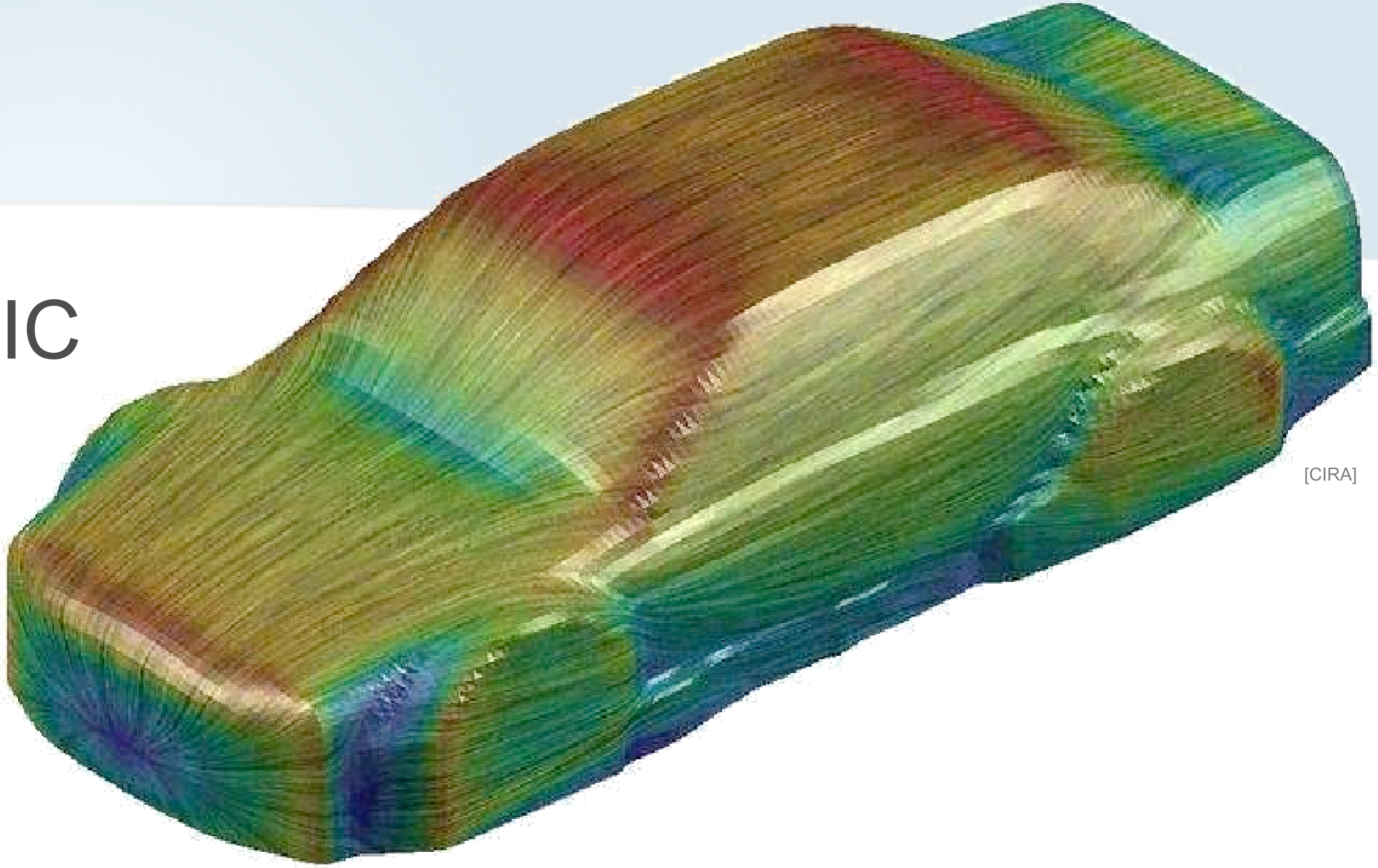

[CIRA]

- What other derived scalar fields would be interesting?
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  - Angular speed: magnitude of the curl

Derived scalar fields

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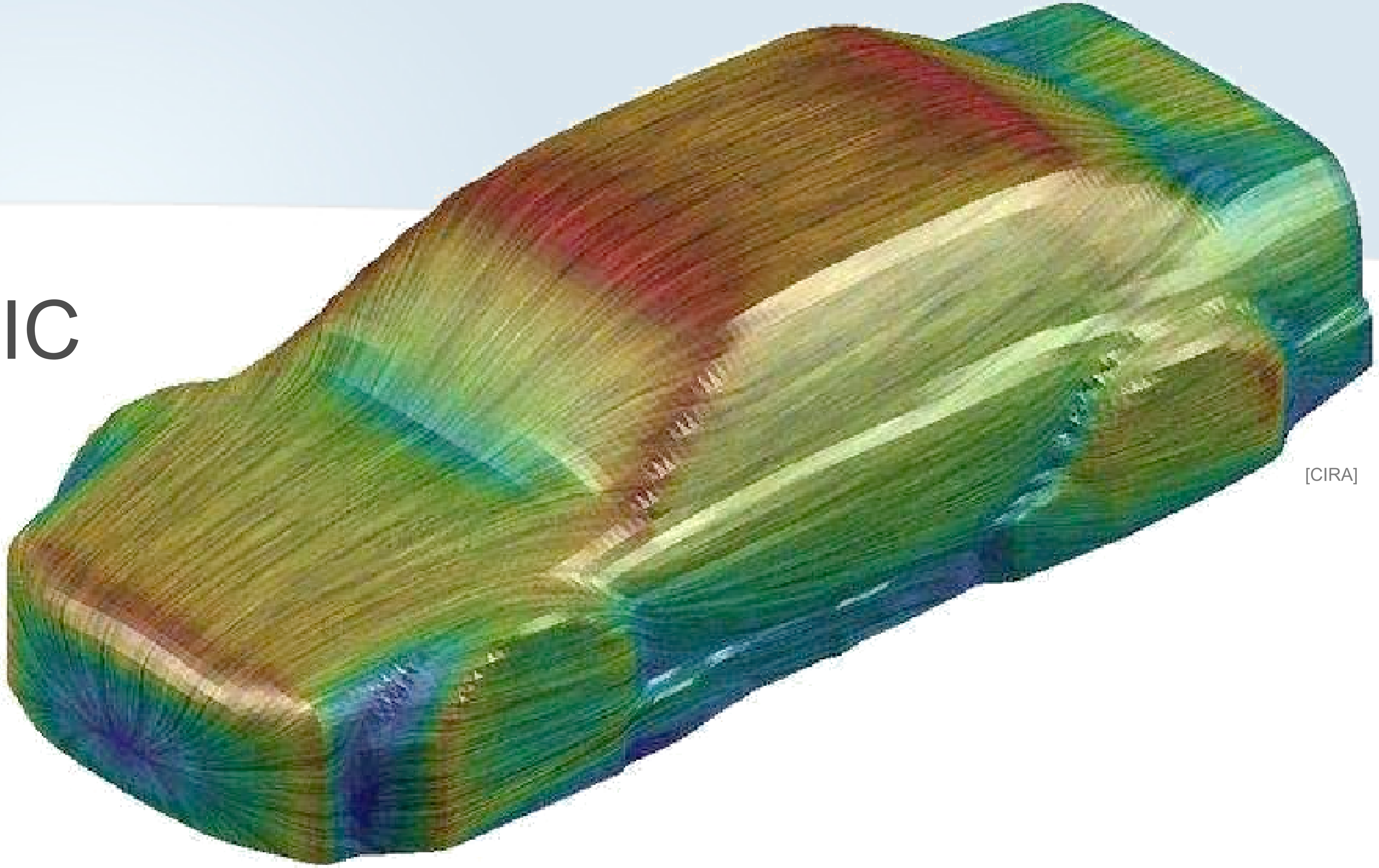

[CIRA]

- What other derived scalar fields would be interesting?
  - Flow orientation: divergence
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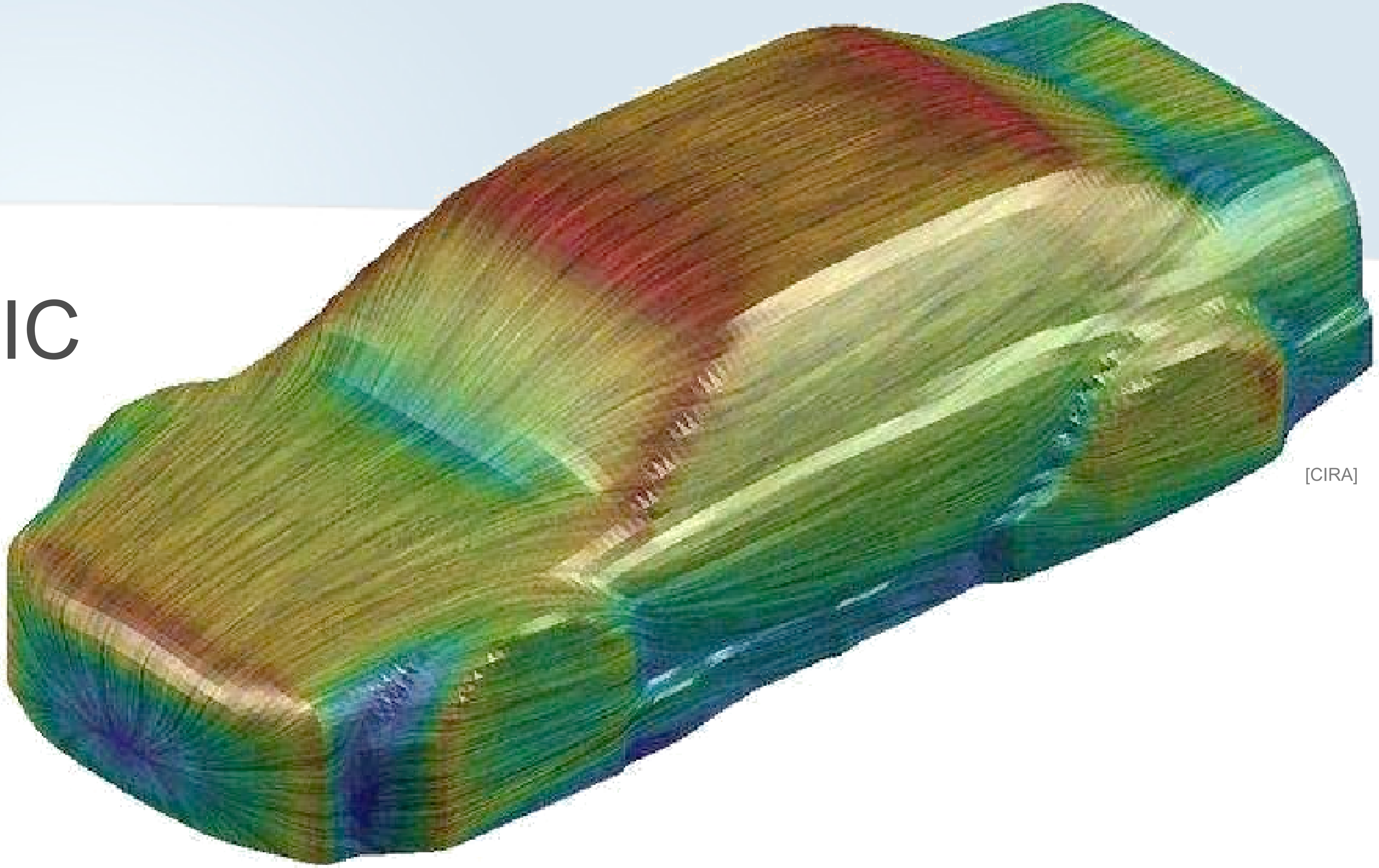


[CIRA]

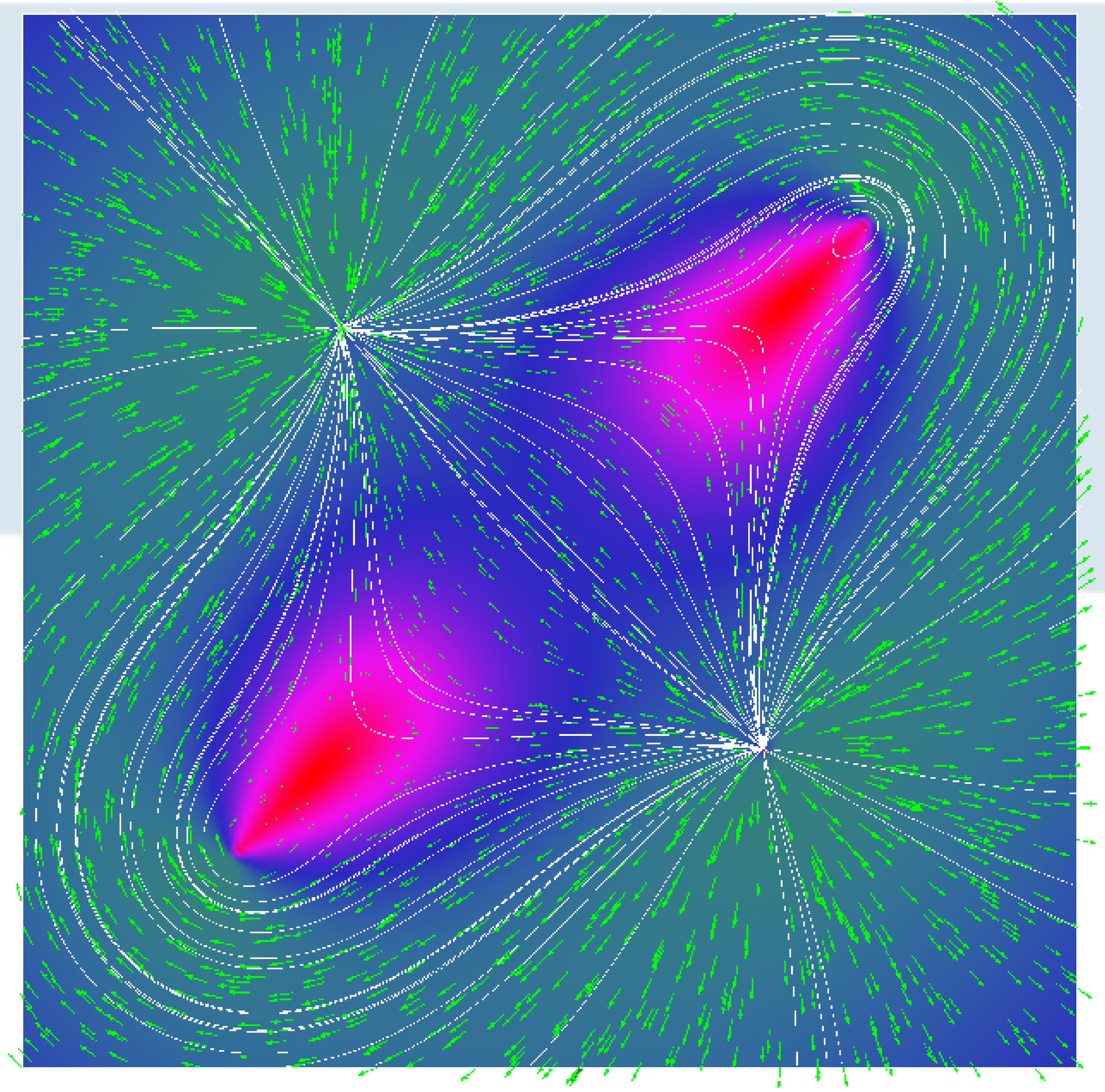
- What other derived scalar fields would be interesting?
  - Flow orientation: divergence
  - Angular speed: magnitude of the curl
  - Flow distortion: Finite Time Lyaponov Exponent

Flow orientation

Flow orientation

- Divergence of the flow, for instance
  - $f : \mathbb{R}^3 \to \mathbb{R}^3$

Flow orientation



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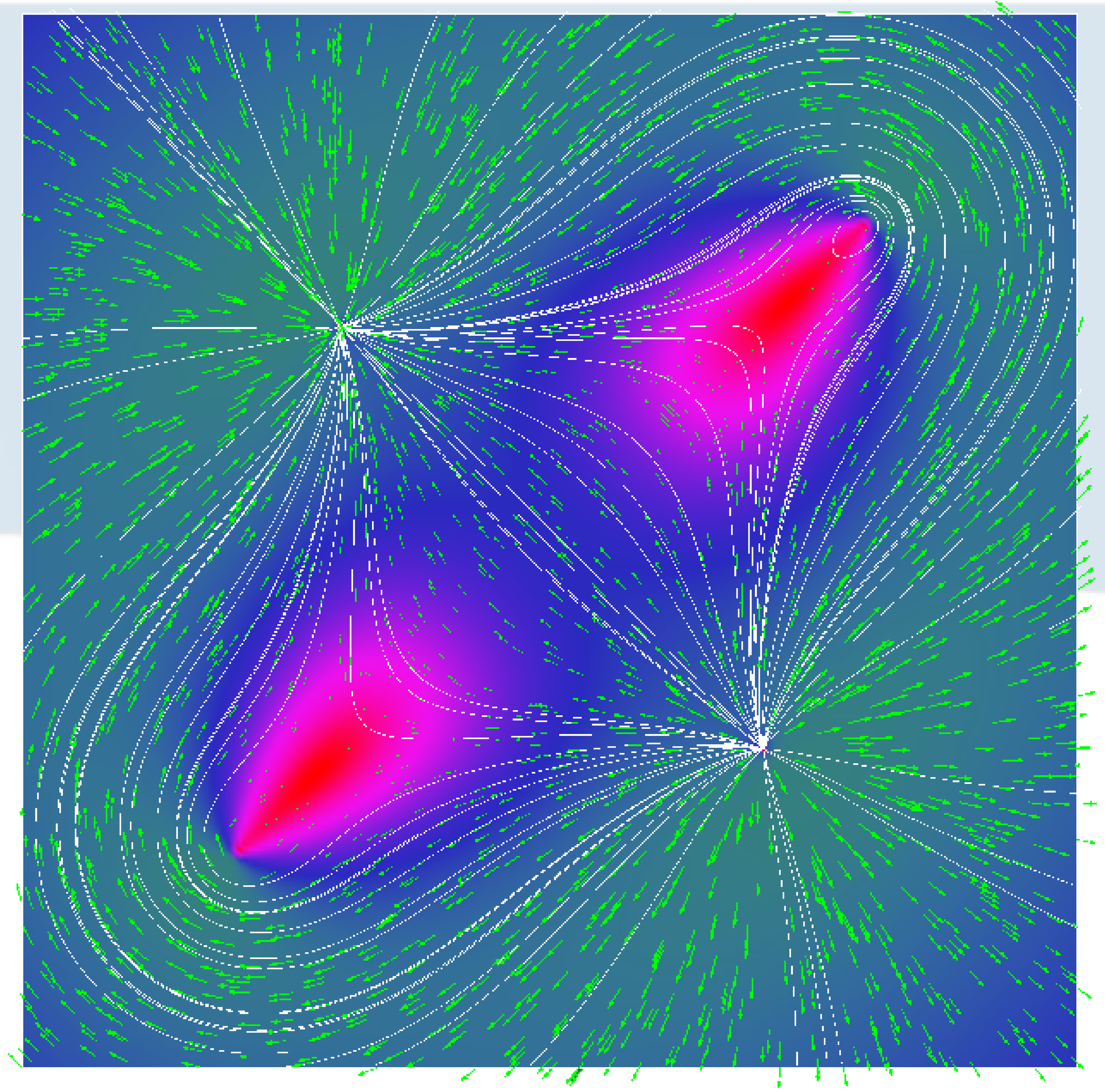
[http://www.deepakantony.com/]

Flow orientation



- Divergence of the flow, for instance
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$$div f = \nabla . f = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z}$$

[http://www.deepakantony.com/]

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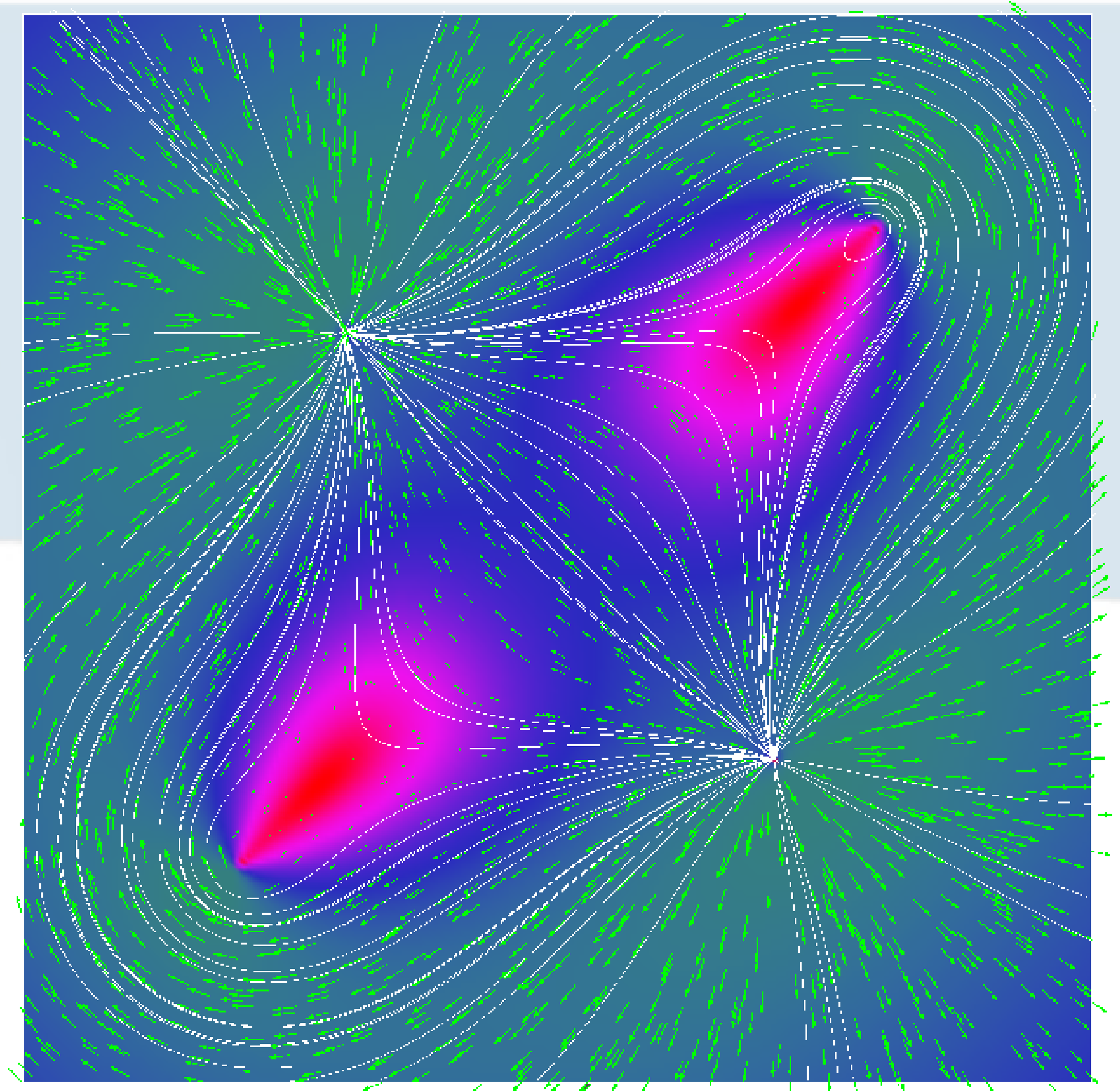

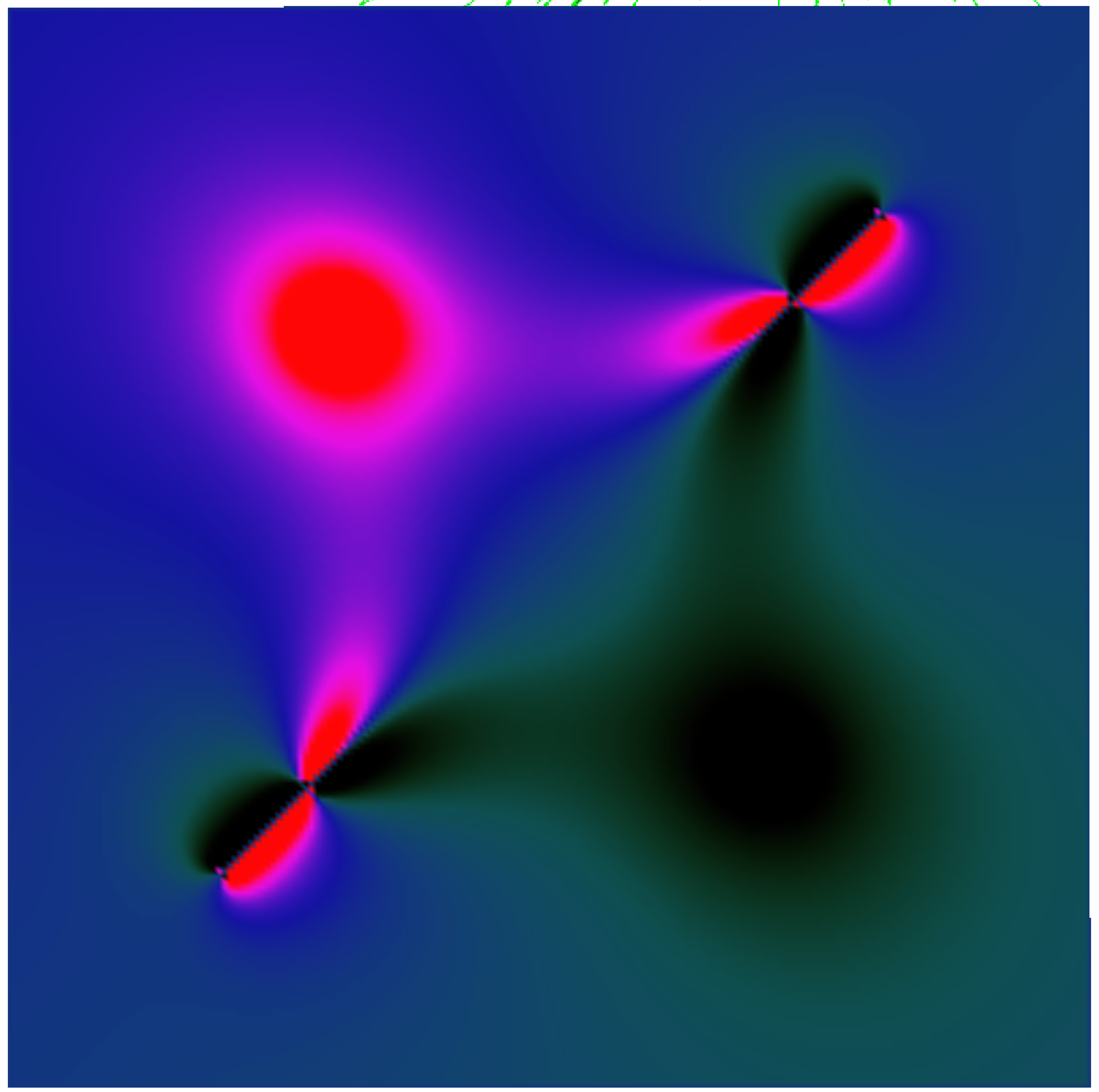

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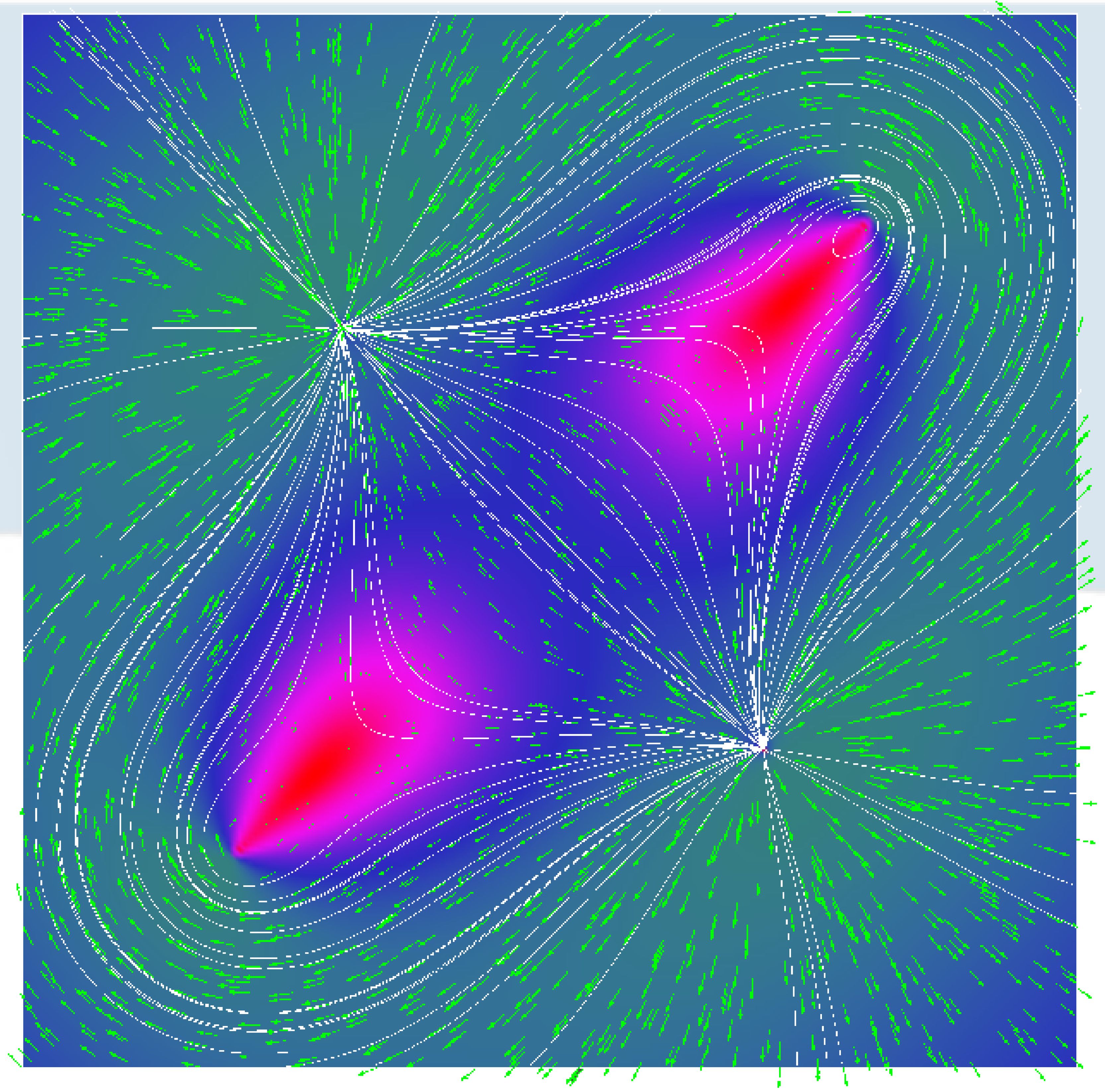
Angular speed



[http://www.deepakantony.com/]

Angular speed

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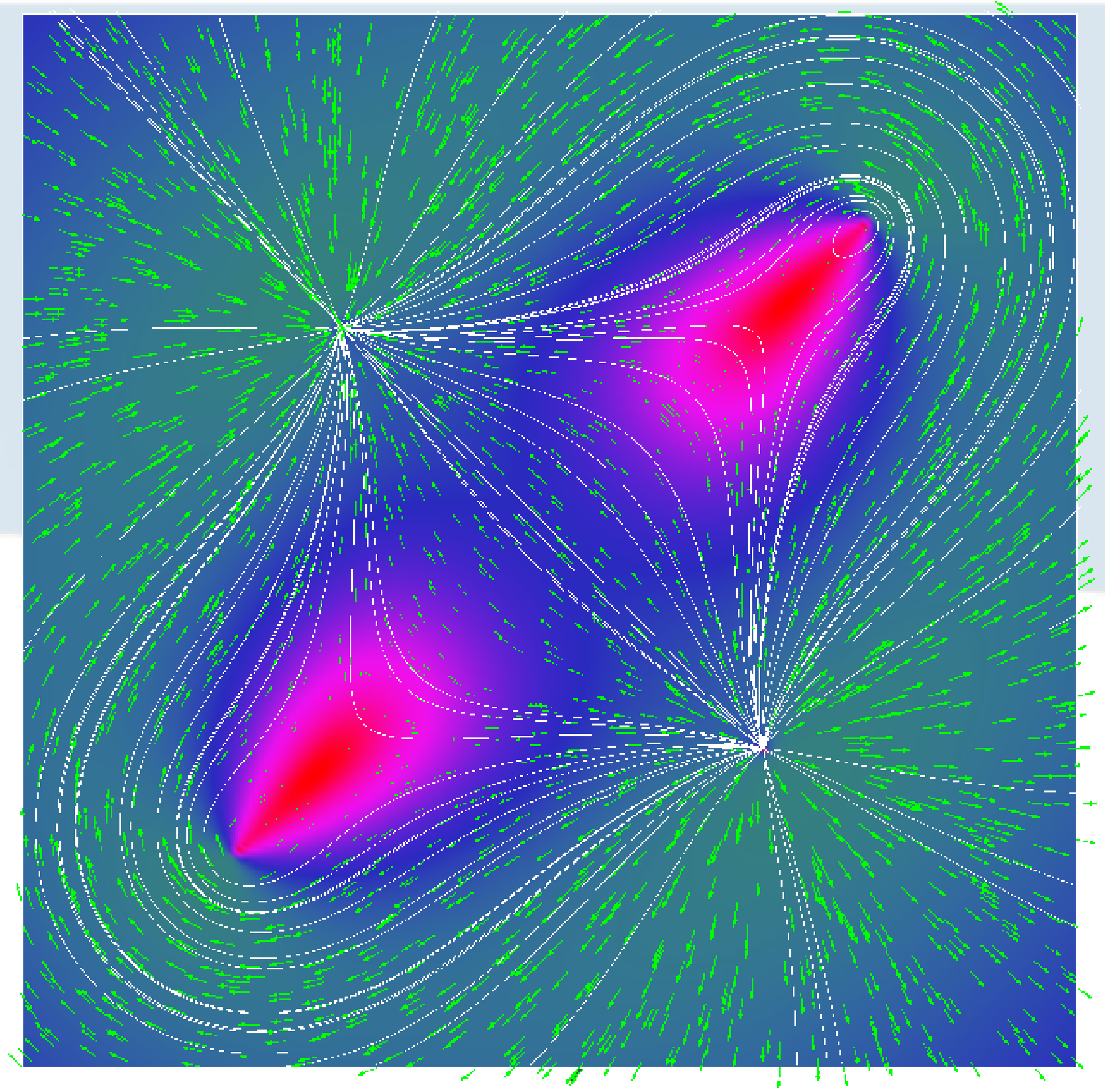

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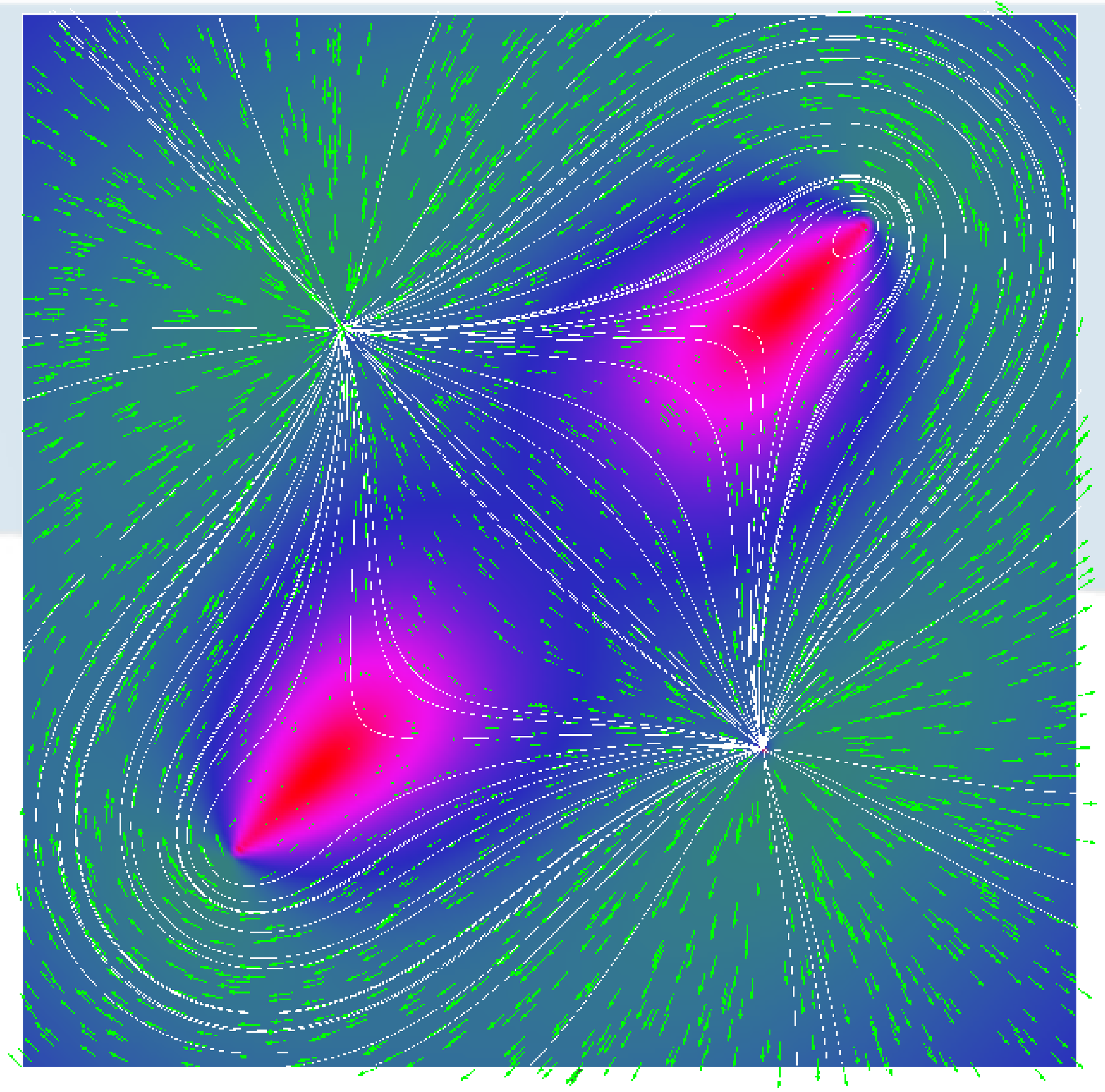
Angular speed



- Magnitude of the curl, for instance
  - $f : \mathbb{R}^3 \to \mathbb{R}^3$

$$\nabla \times f =$$

$$(\frac{\partial f_z}{\partial y} - \frac{\partial f_y}{\partial z})\vec{i} + (\frac{\partial f_x}{\partial z} - \frac{\partial f_z}{\partial x})\vec{j} + (\frac{\partial f_y}{\partial x} - \frac{\partial f_x}{\partial y})\vec{k}$$

[http://www.deepakantony.com/]

Angular speed

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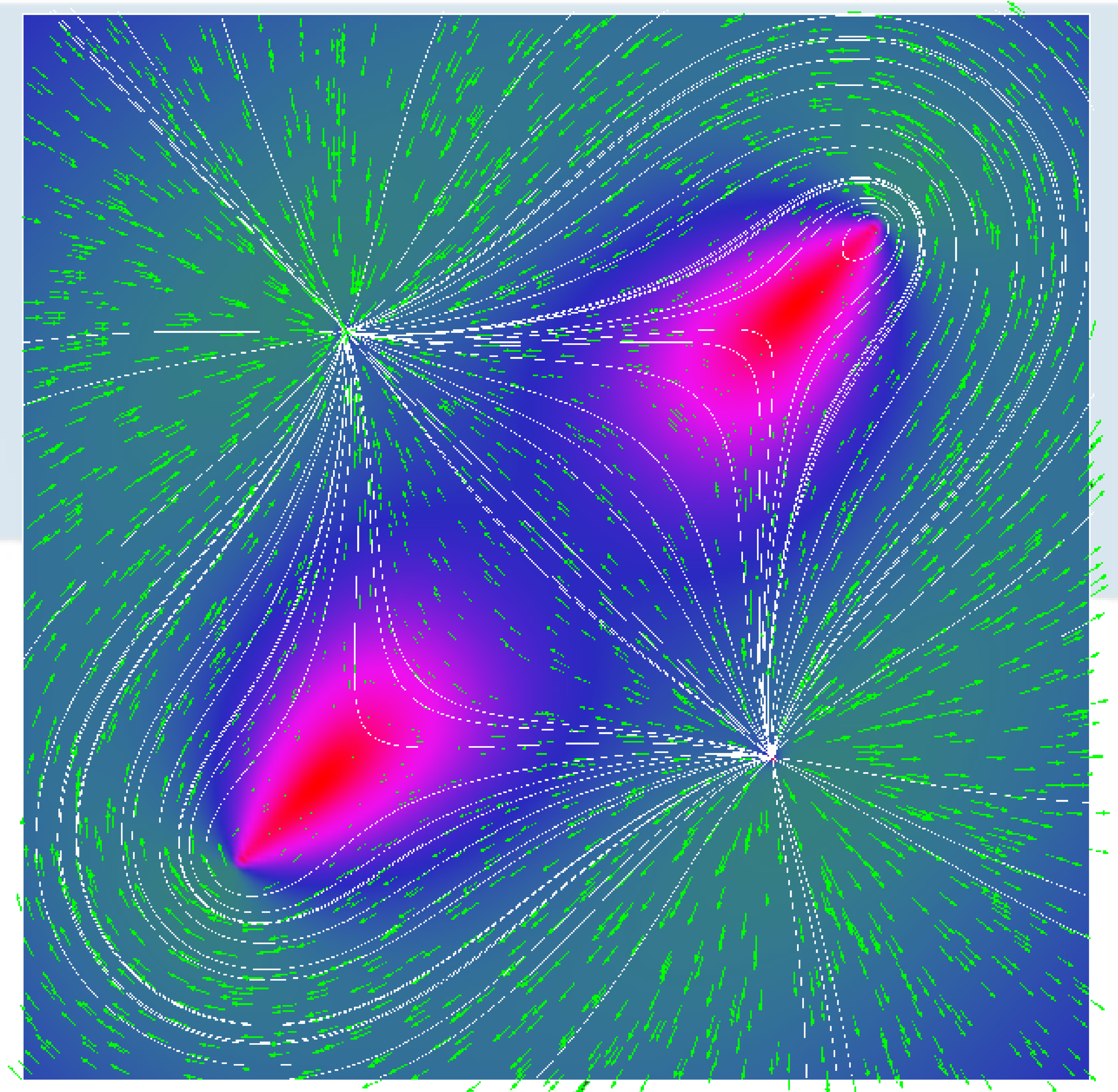

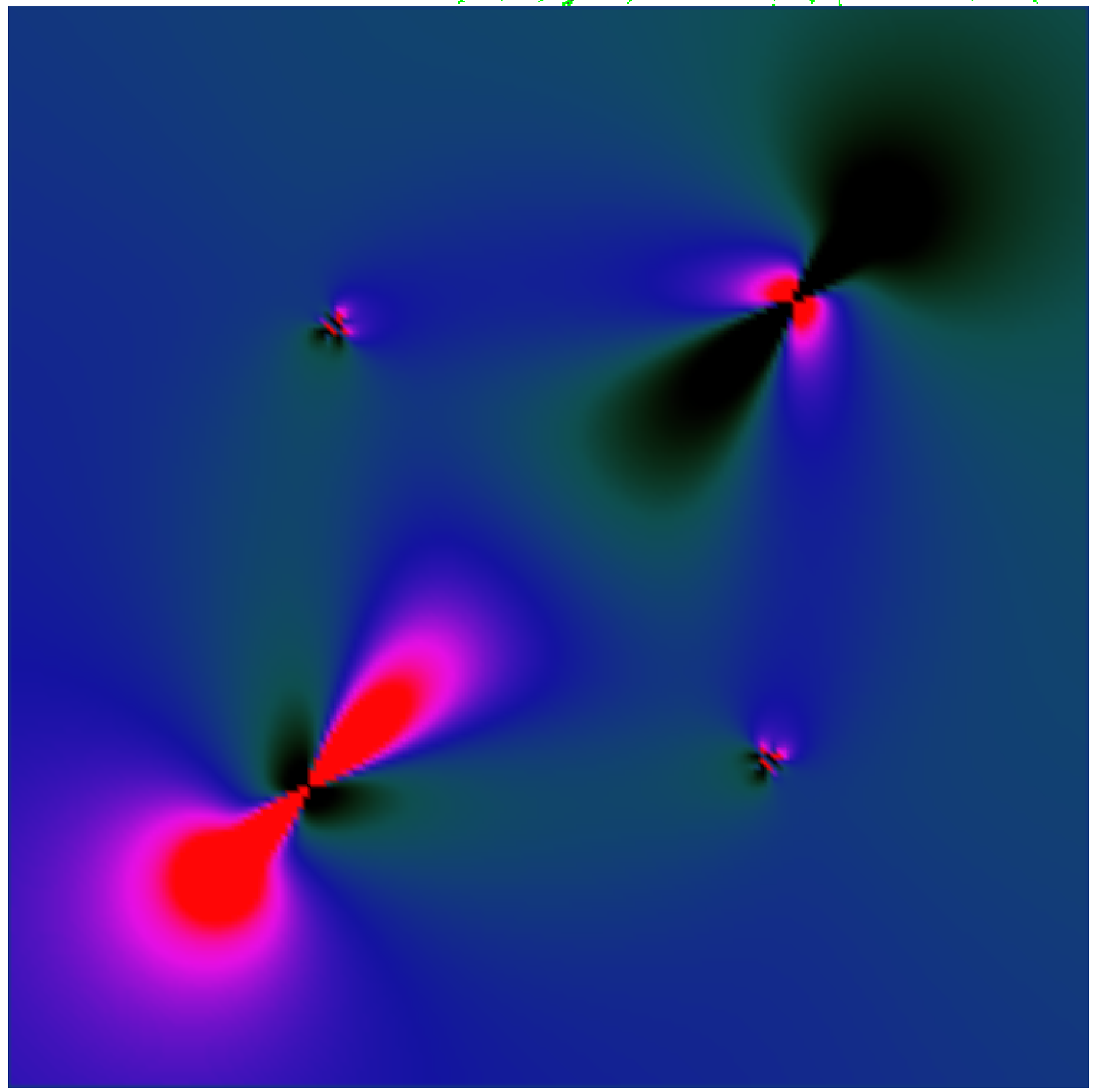

[http://www.deepakantony.com/]

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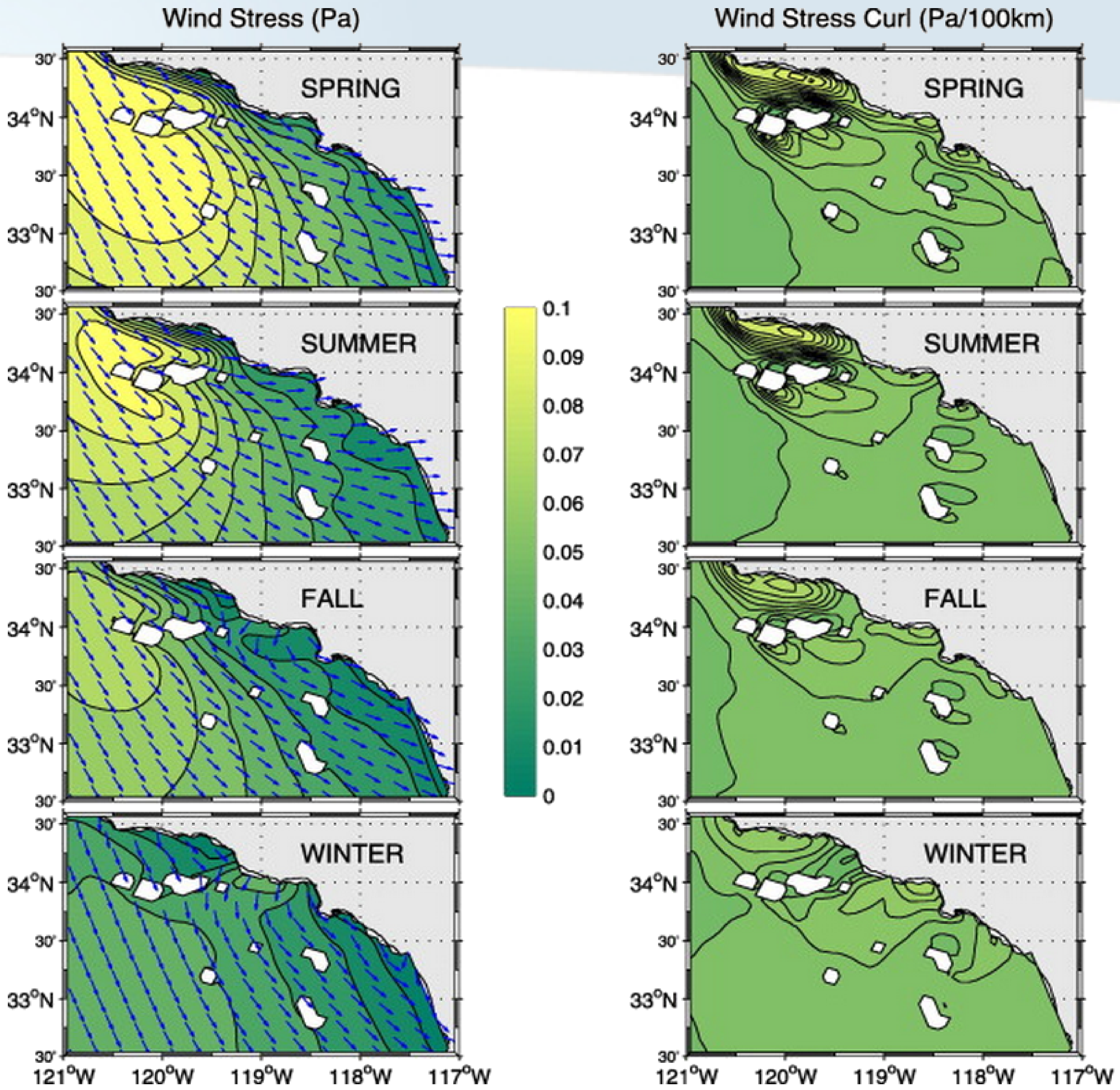

[Dong09]

Angular speed

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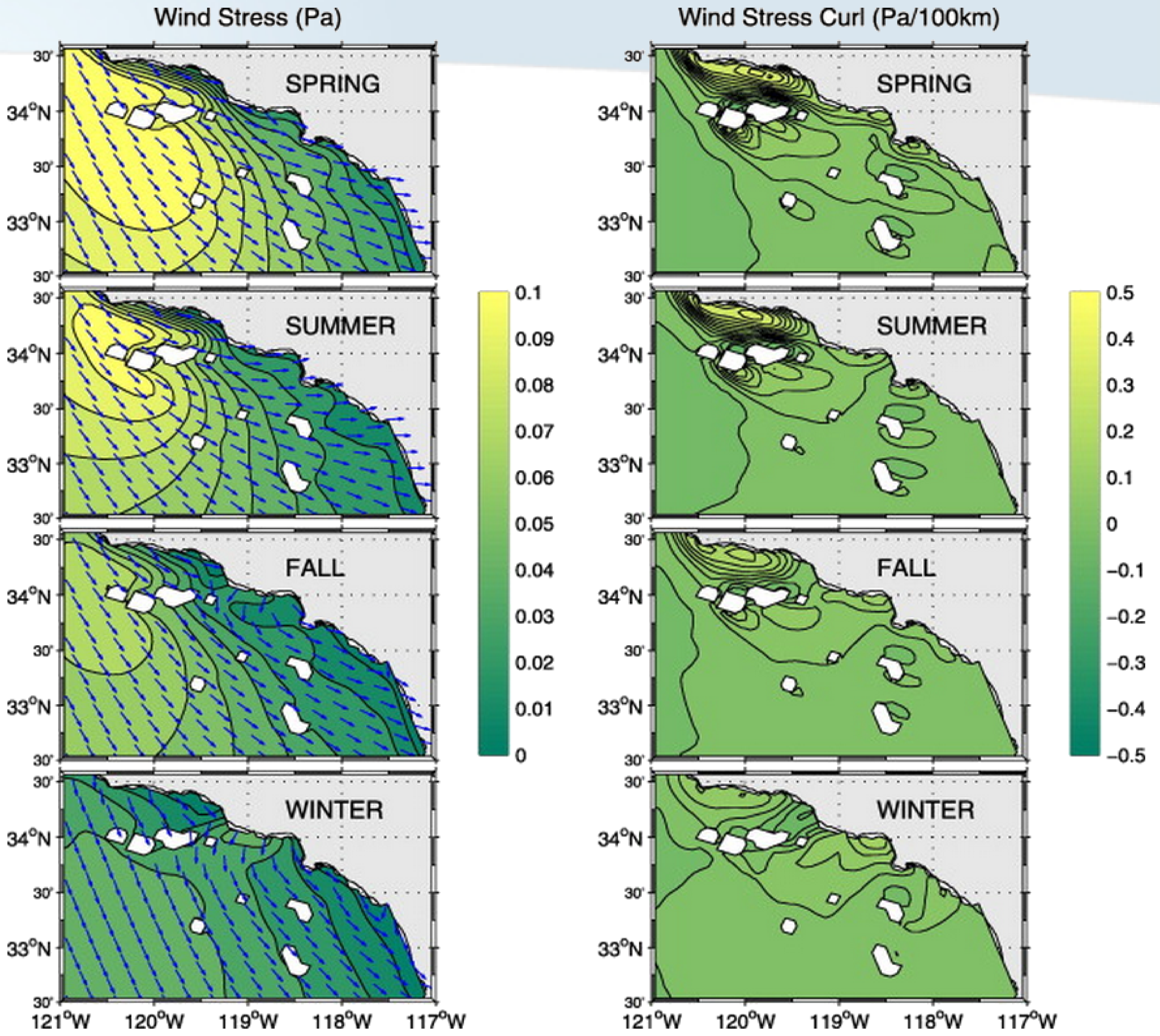
$$\left(\frac{\partial f_z}{\partial y} - \frac{\partial f_y}{\partial z}\right)\vec{i} + \left(\frac{\partial f_x}{\partial z} - \frac{\partial f_z}{\partial x}\right)\vec{j} + \left(\frac{\partial f_y}{\partial x} - \frac{\partial f_x}{\partial y}\right)\vec{k}$$

  - On PL-manifolds?



[Dong09]

Flow distortion

[www.publicaffairs.water.ca.gov/swp/swptoday.cfm]

Flow distortion

- Finite Time Lyaponov Exponent
  - Analysis of unsteady flows

[www.publicaffairs.water.ca.gov/swp/swptoday.cfm]

Flow distortion

- Finite Time Lyaponov Exponent
  - Analysis of unsteady flows
  - For steady flows
    - Streamline distortion

[www.publicaffairs.water.ca.gov/swp/swptoday.cfm]

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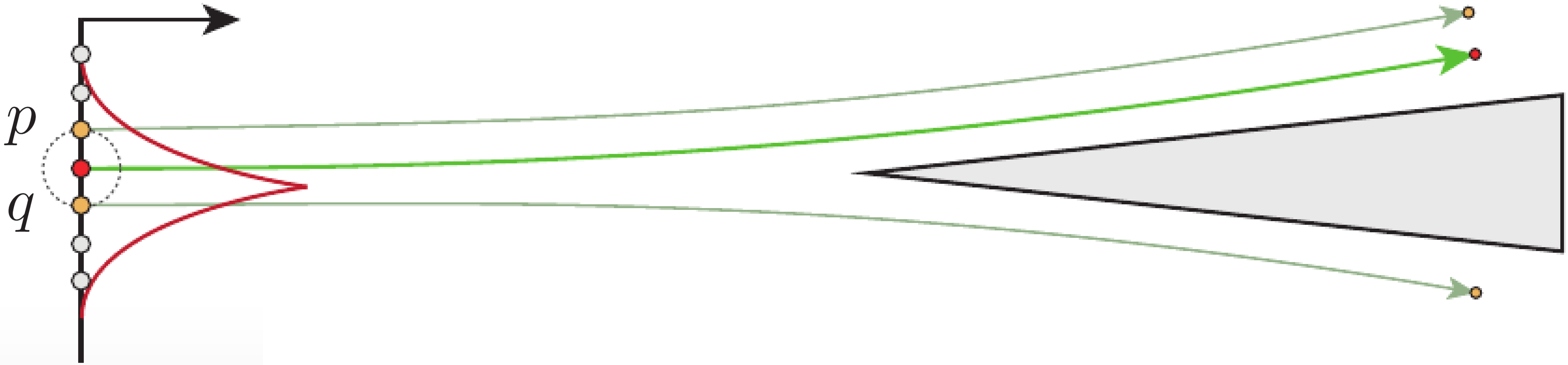
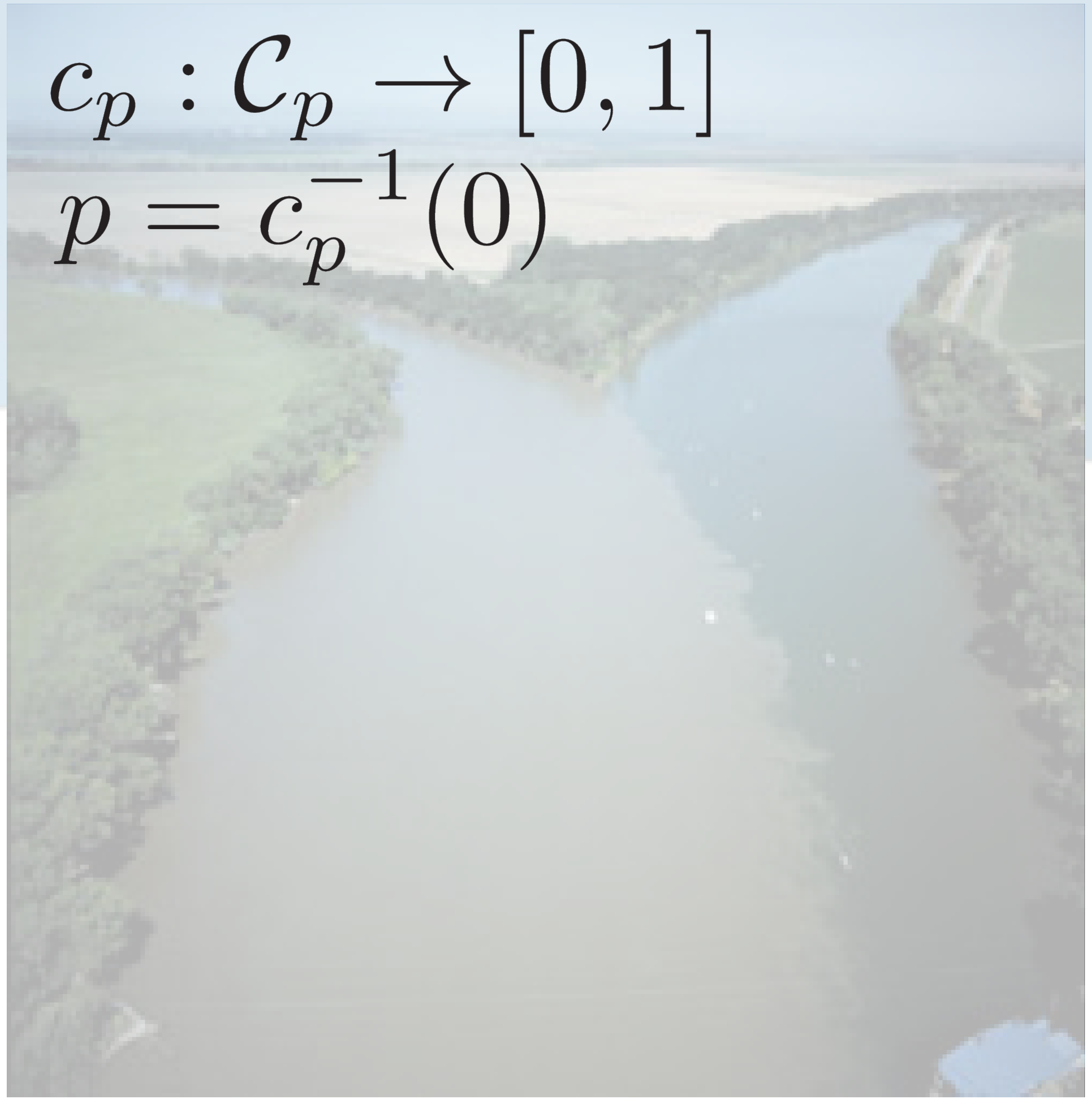


[Uffinger12]

Flow distortion

- Finite Time Lyaponov Exponent
  - Analysis of unsteady flows
  - For steady flows
    - Streamline distortion



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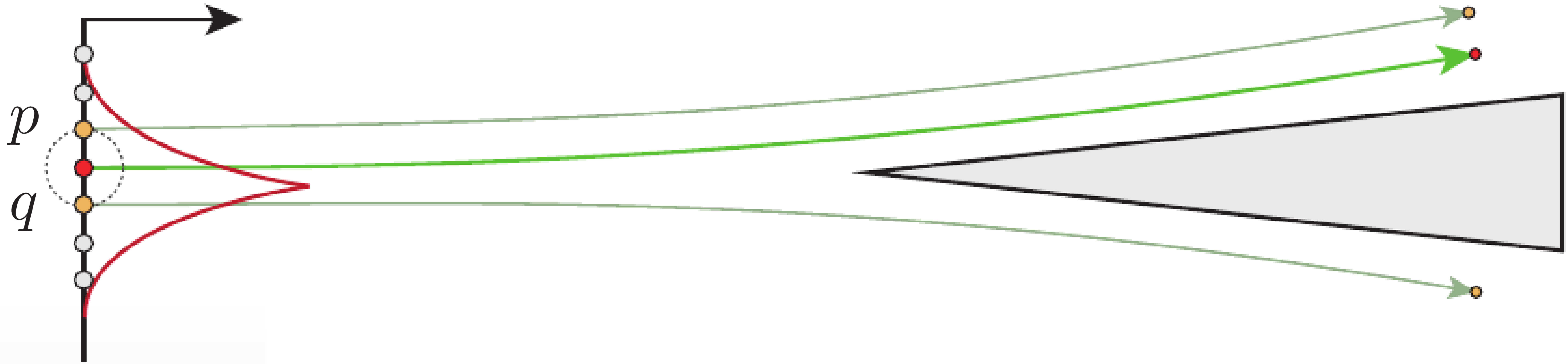

[Uffinger12]

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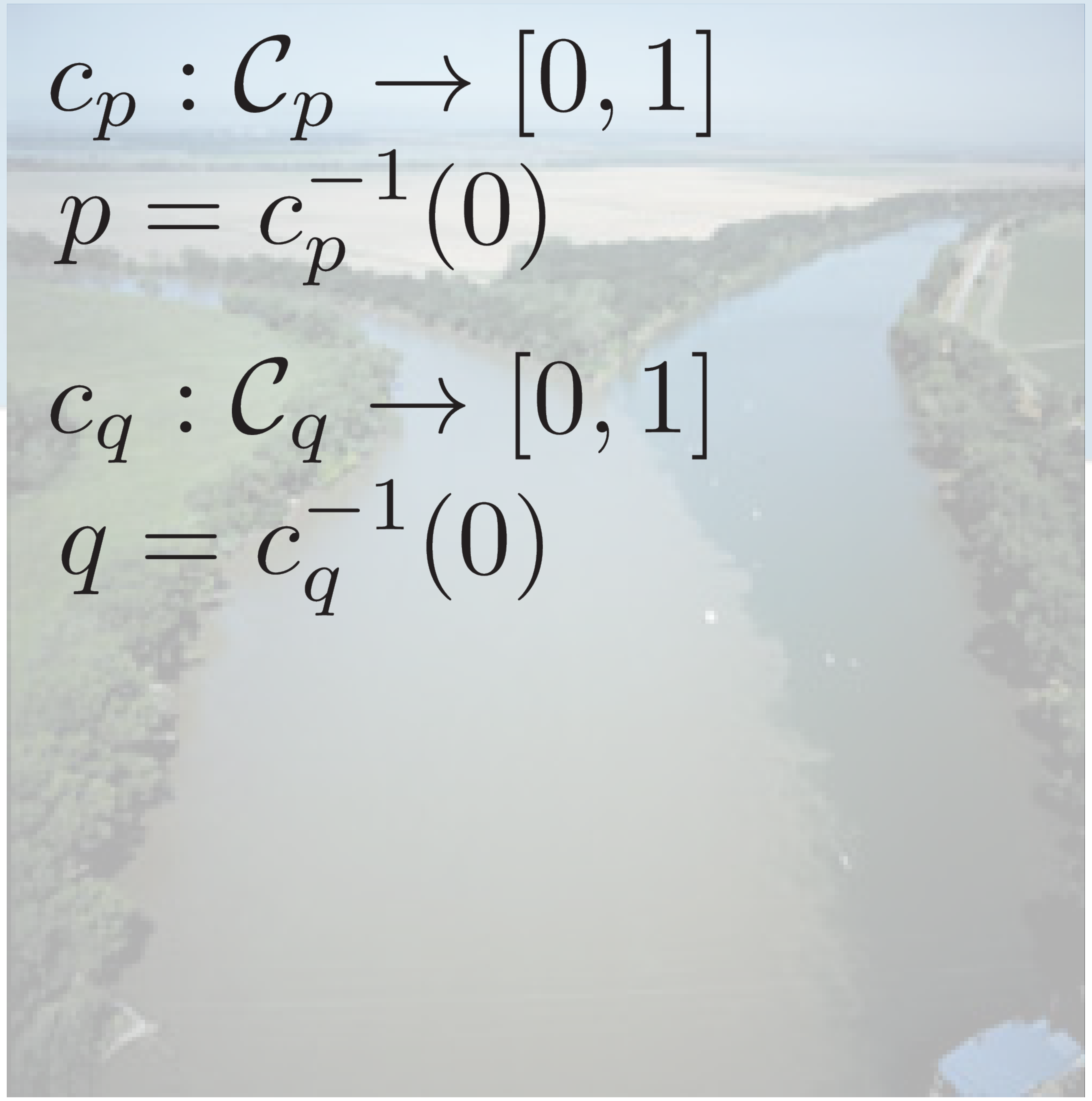
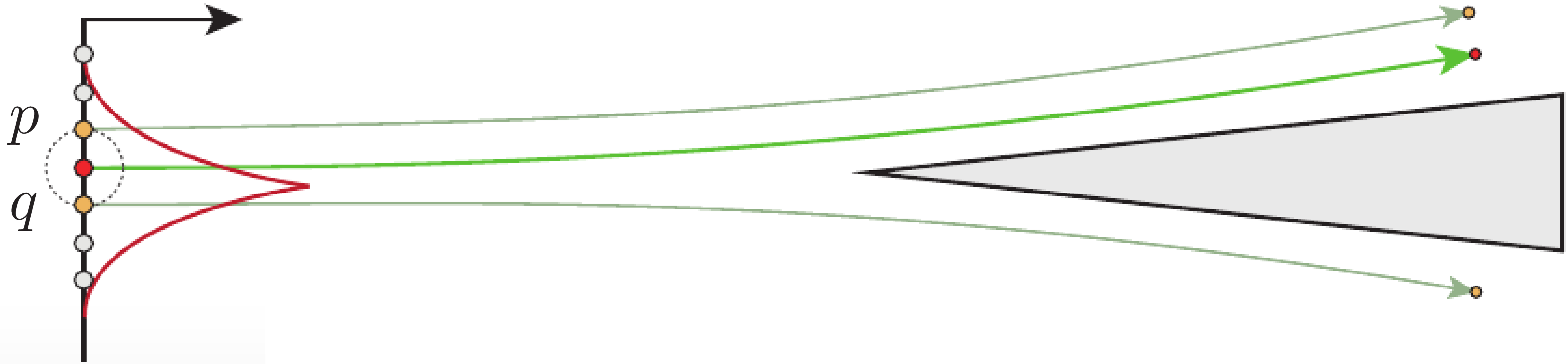


$$c_p : \mathcal{C}_p \to [0, 1]$$
$$p = c_p^{-1}(0)$$
$$c_q : \mathcal{C}_q \to [0, 1]$$
$$q = c_q^{-1}(0)$$

[www.publicaffairs.water.ca.gov/swp/swptoday.cfm]



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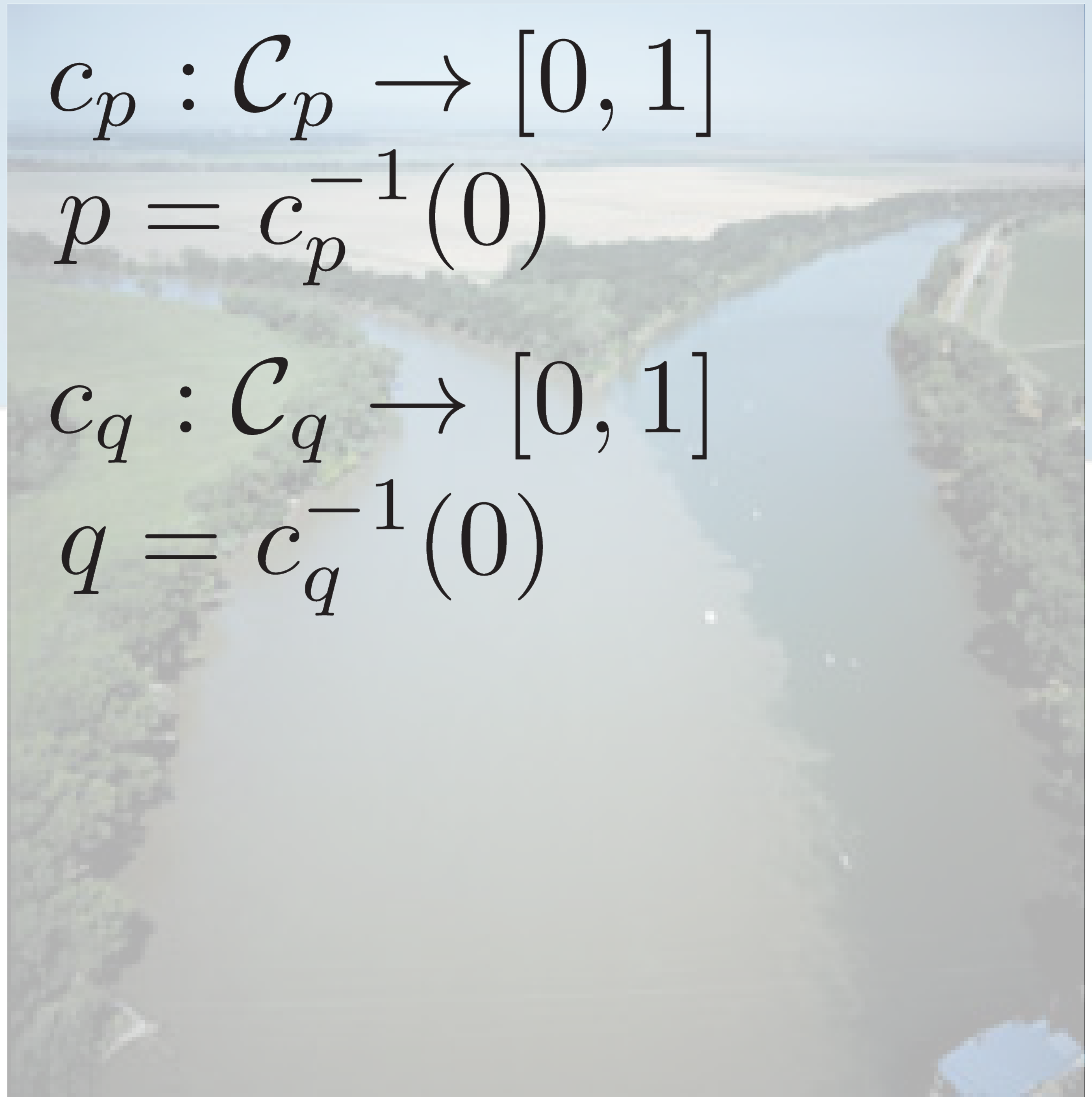

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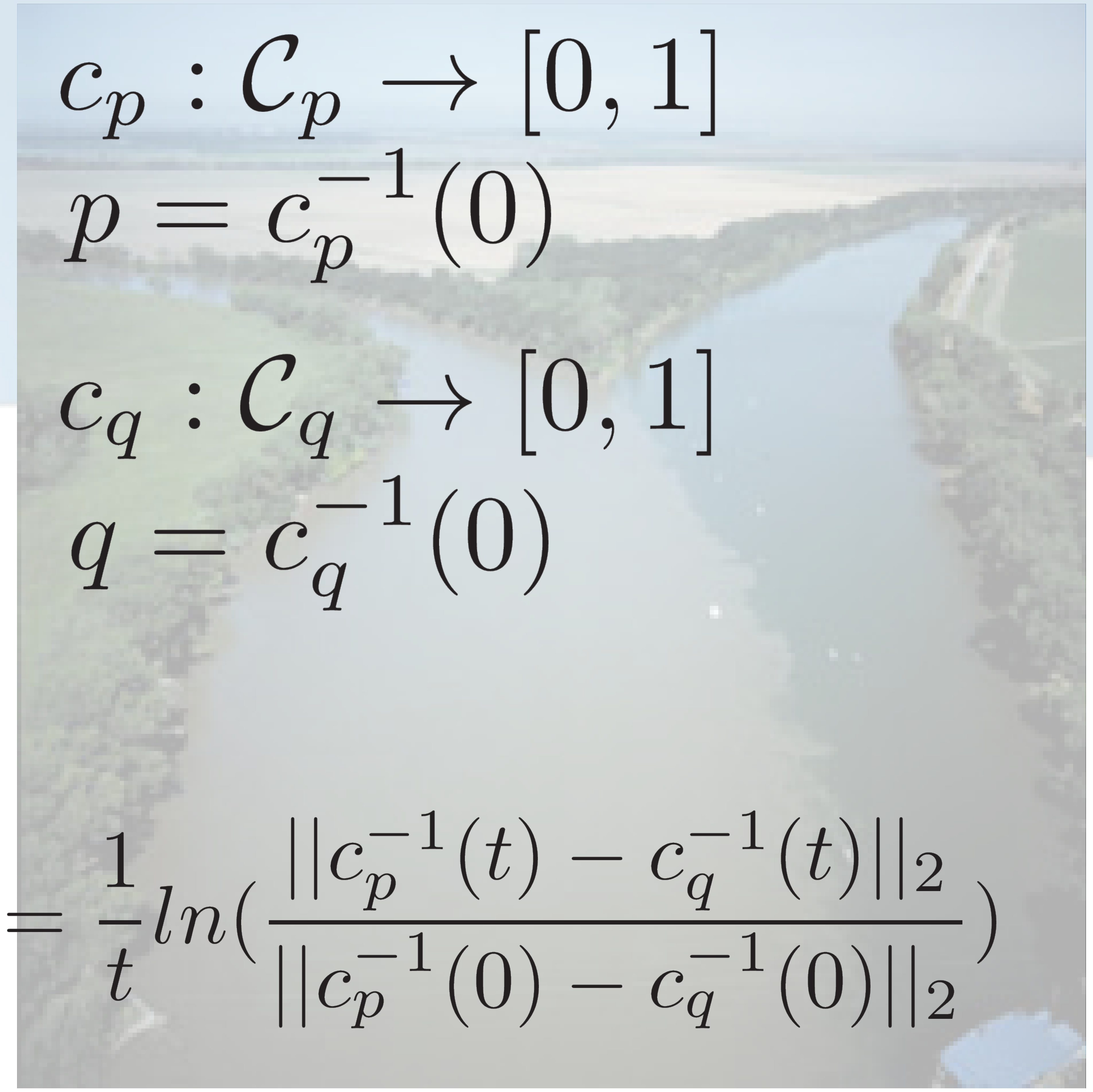

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$$q = c_q^{-1}(0)$$

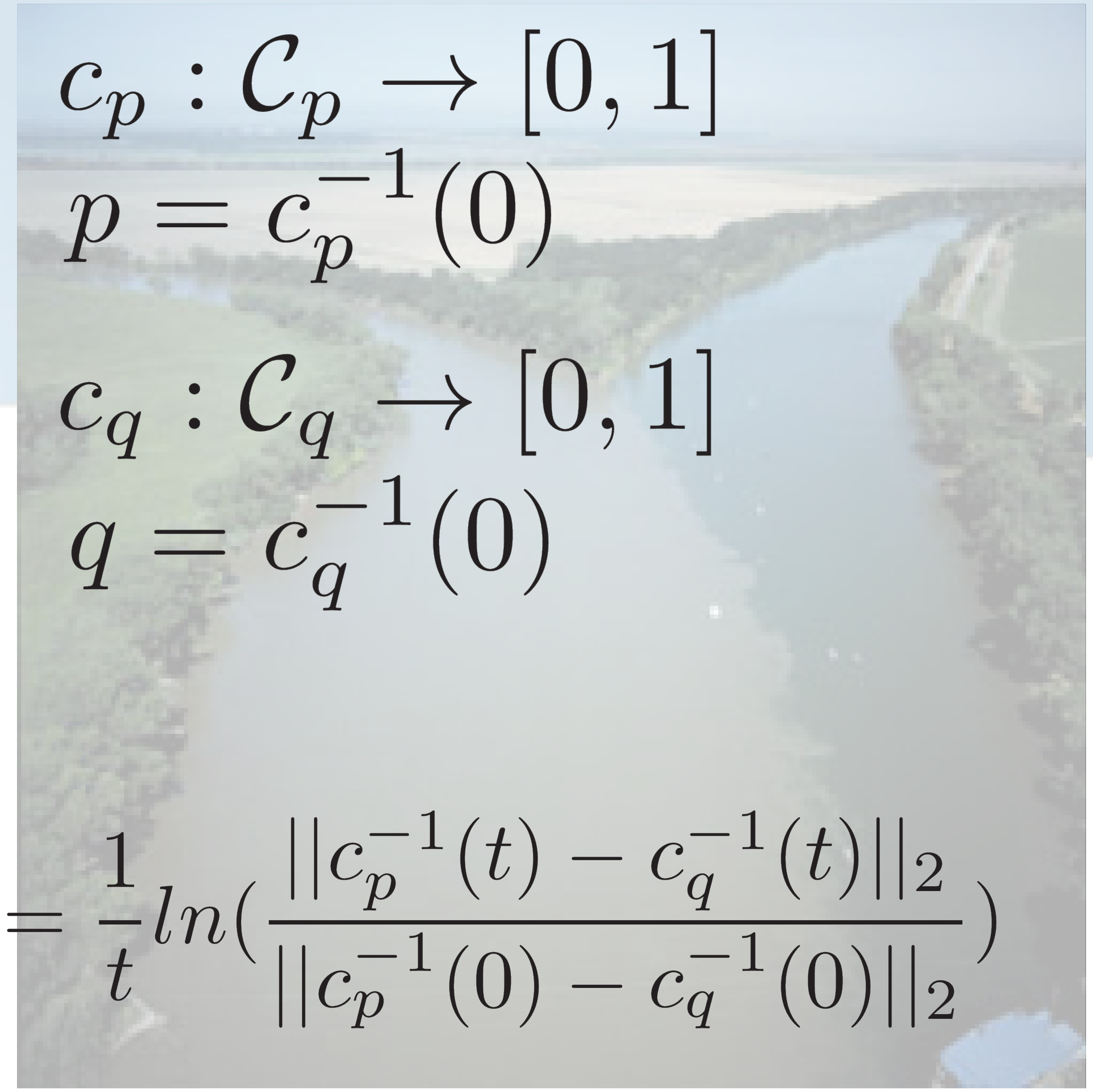
$$FTLE(x,t) = \frac{1}{t} ln(\frac{||c_p^{-1}(t) - c_q^{-1}(t)||_2}{||c_p^{-1}(0) - c_q^{-1}(0)||_2})$$

[www.publicaffairs.water.ca.gov/swp/swptoday.cfm]

[Uffinger12]

Flow distortion

- Finite Time Lyaponov Exponent
  - Analysis of unsteady flows
  - For steady flows
    - Streamline distortion
    - Multi-scale measure (t)



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[Uffinger12]

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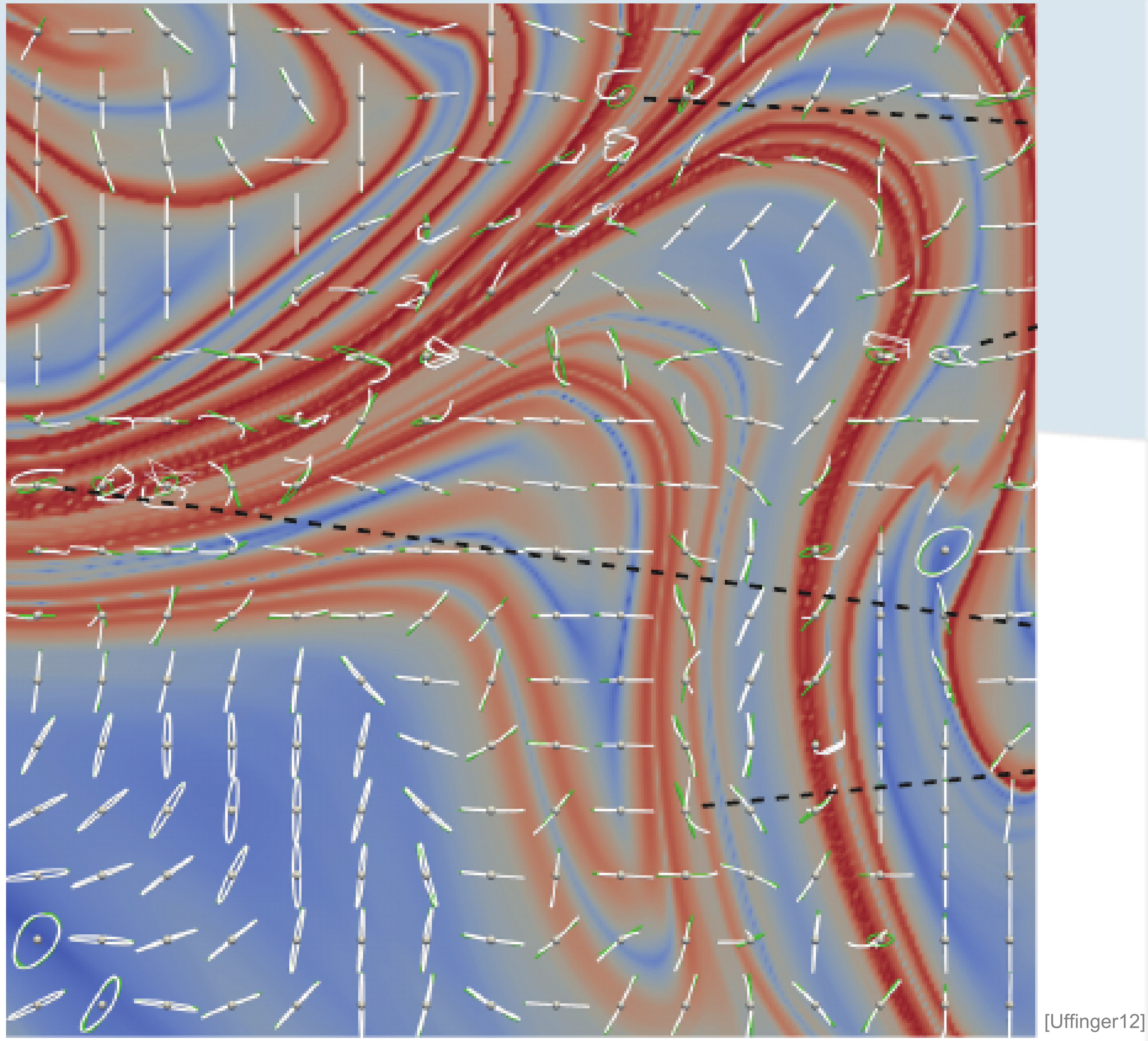

So far

- Extract geometrical features
  - Streamlines

So far

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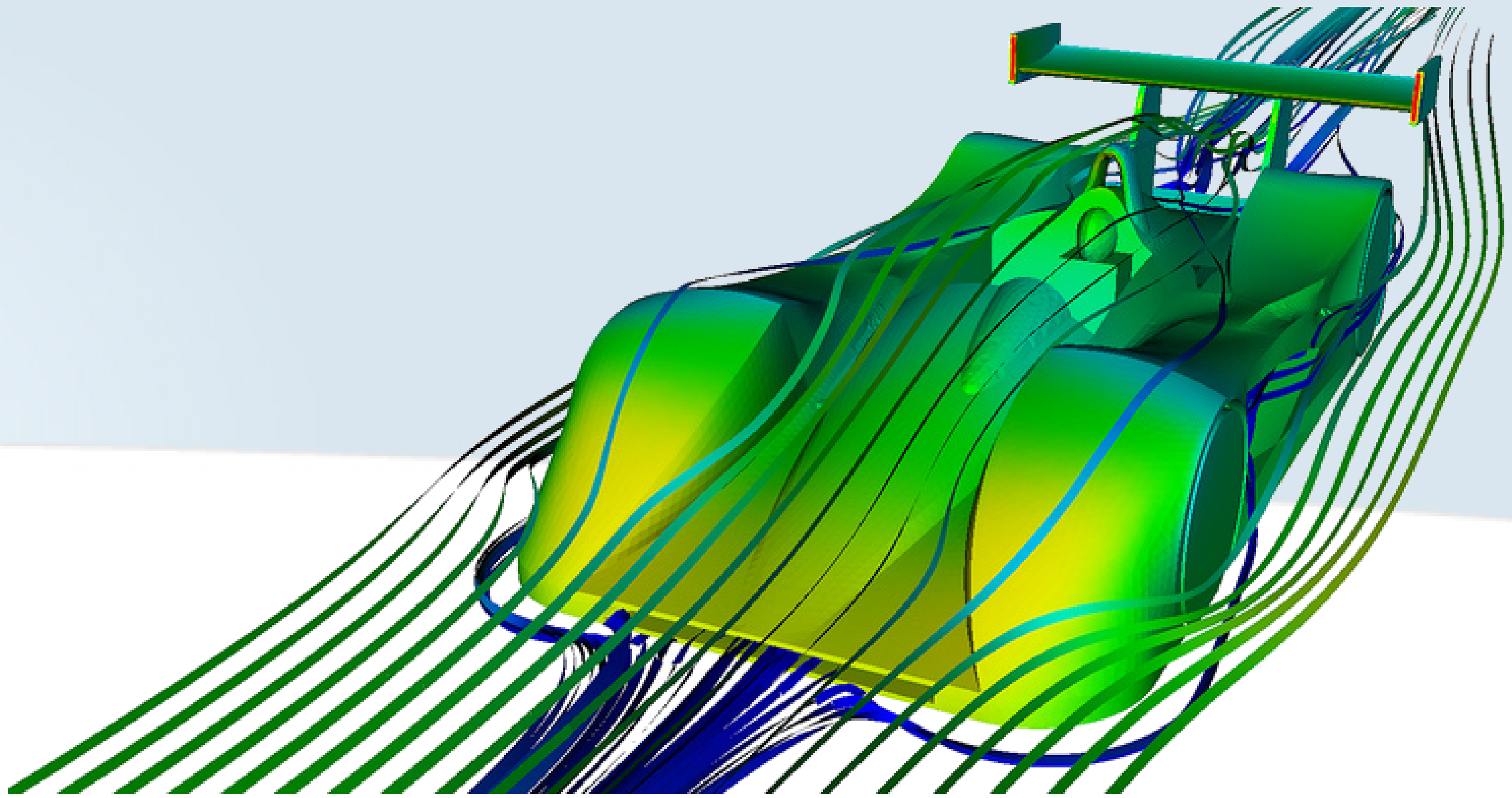
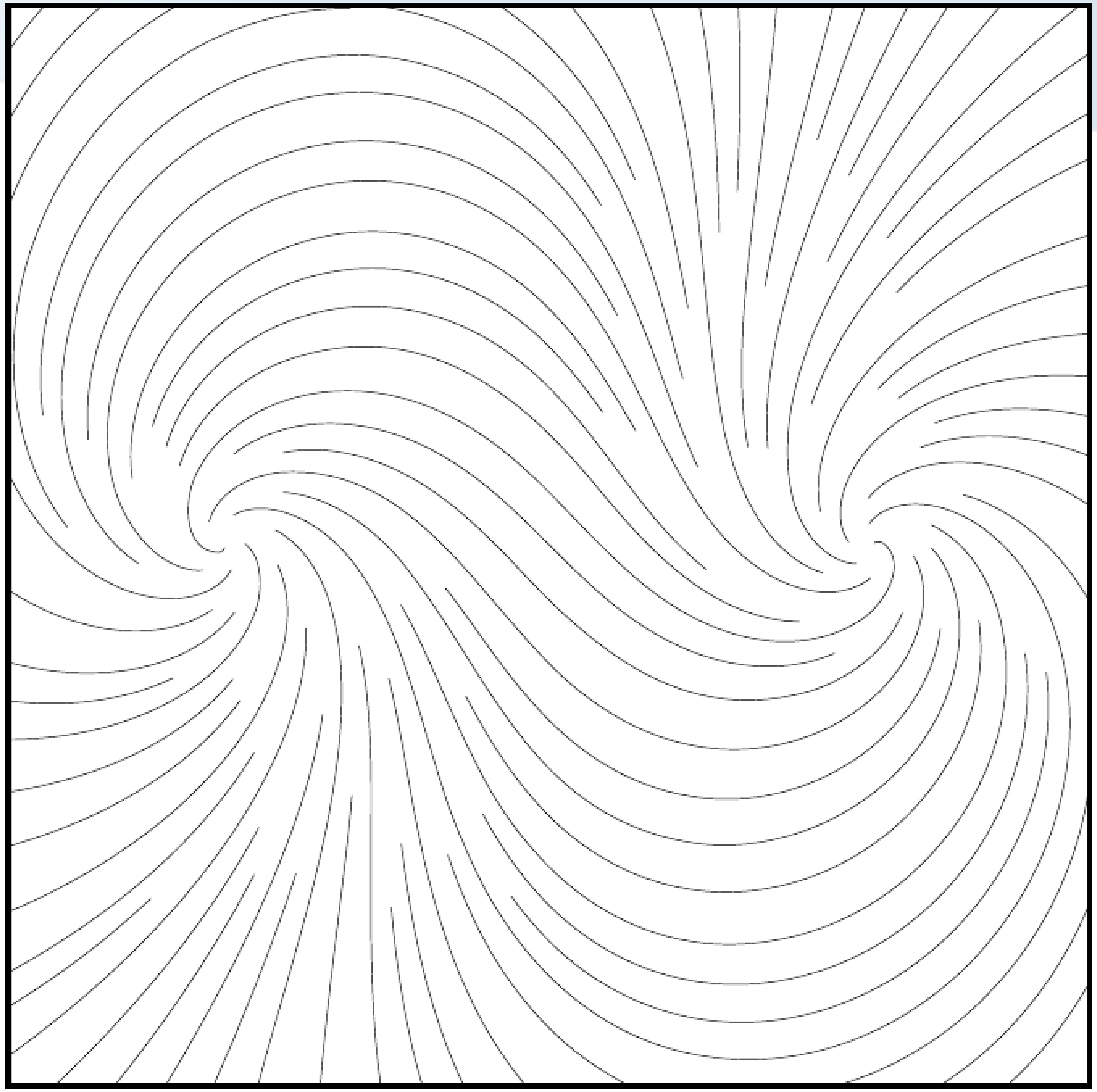


[Kitware]

So far

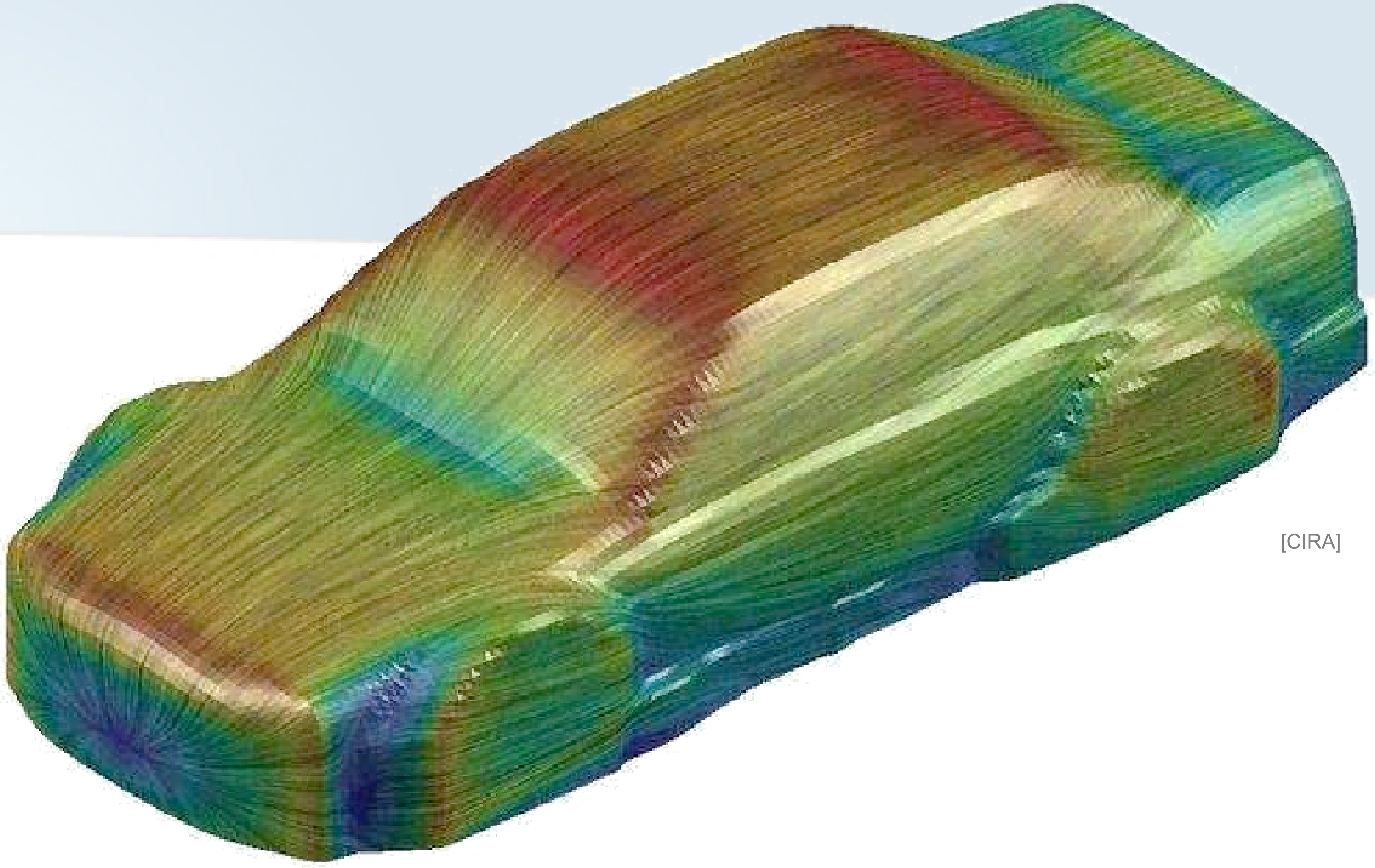
- Extract geometrical features
  - Streamlines
    - Seeding



[Jobard et al. 97]

So far

- Extract geometrical features
  - Streamlines
    - Seeding, Line Integral Convolution



[CIRA]

So far

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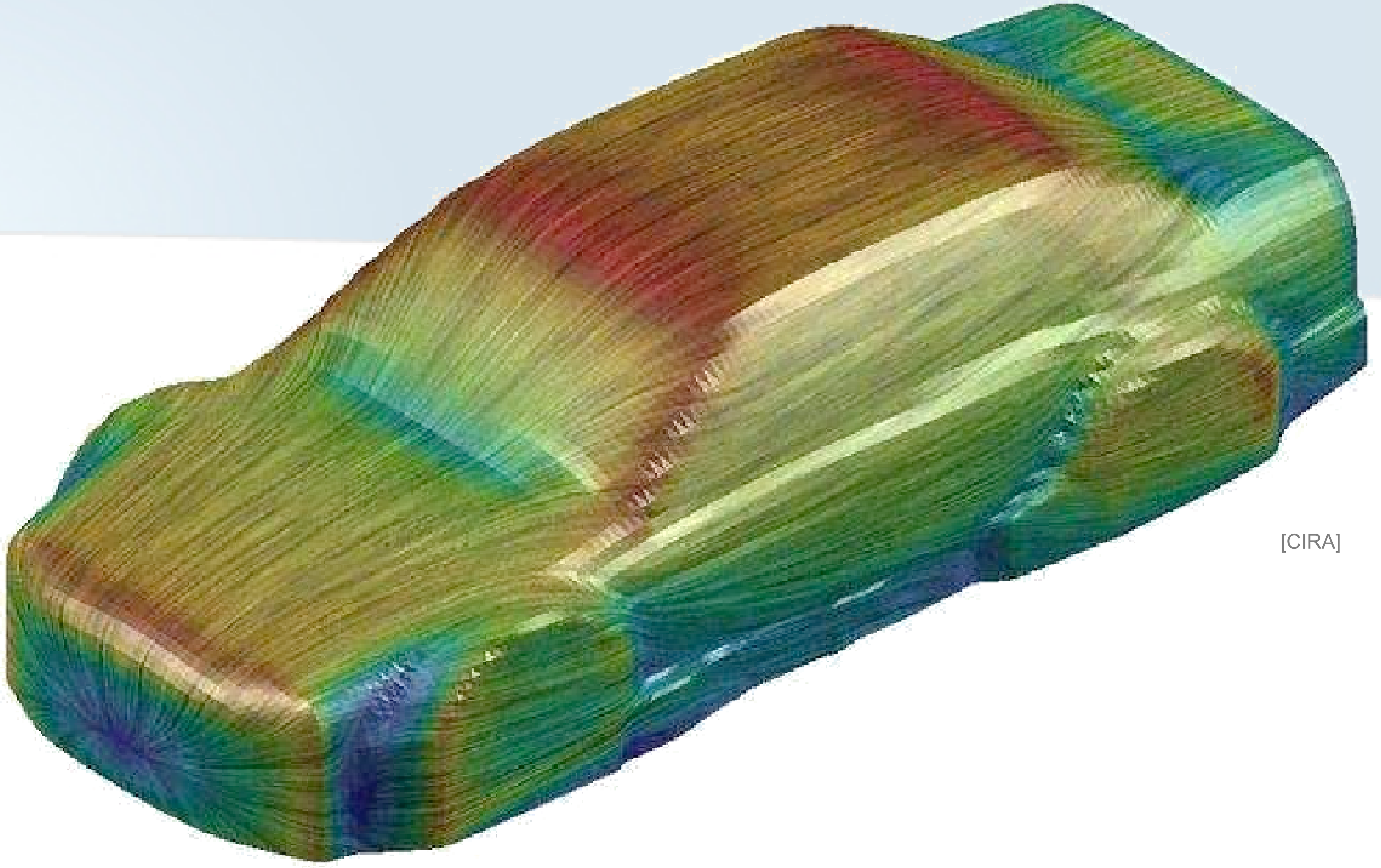
    - Seeding, Line Integral Convolution



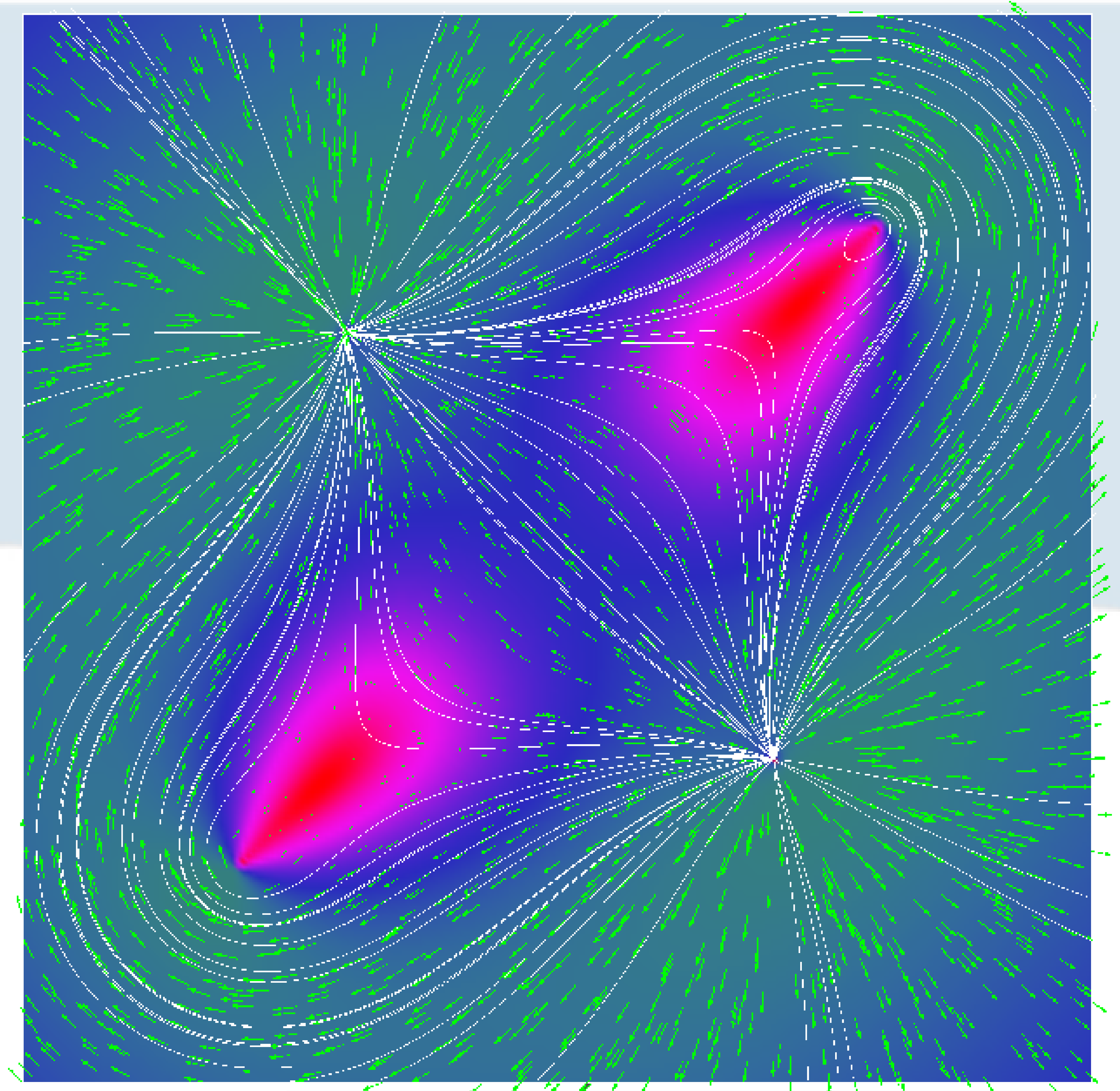
[CIRA]

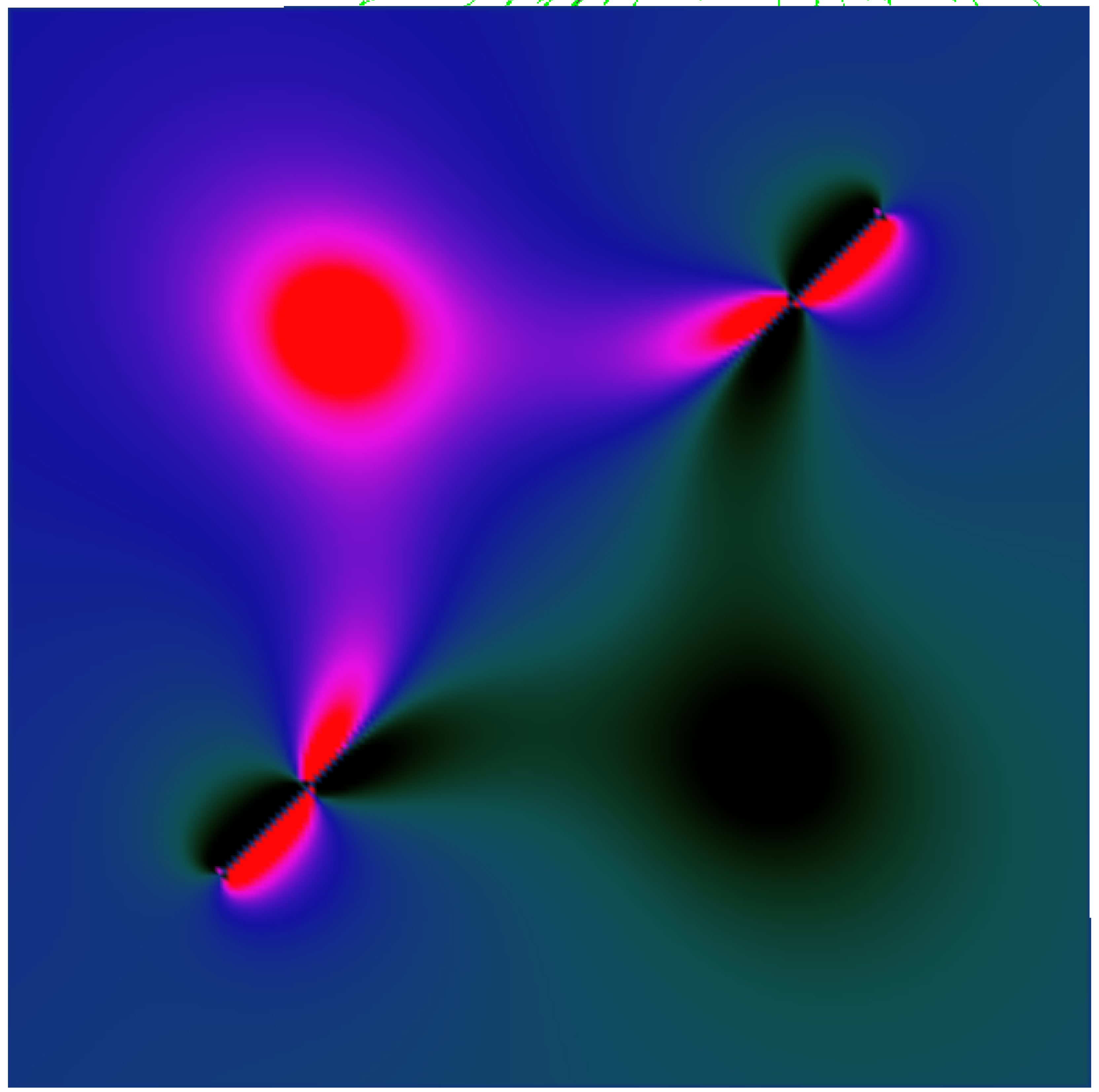
- Extract geometrical measures

So far

- Extract geometrical features
  - Streamlines
    - Seeding, Line Integral Convolution

- Extract geometrical measures
  - Magnitude, orientation, angular speed, distortion

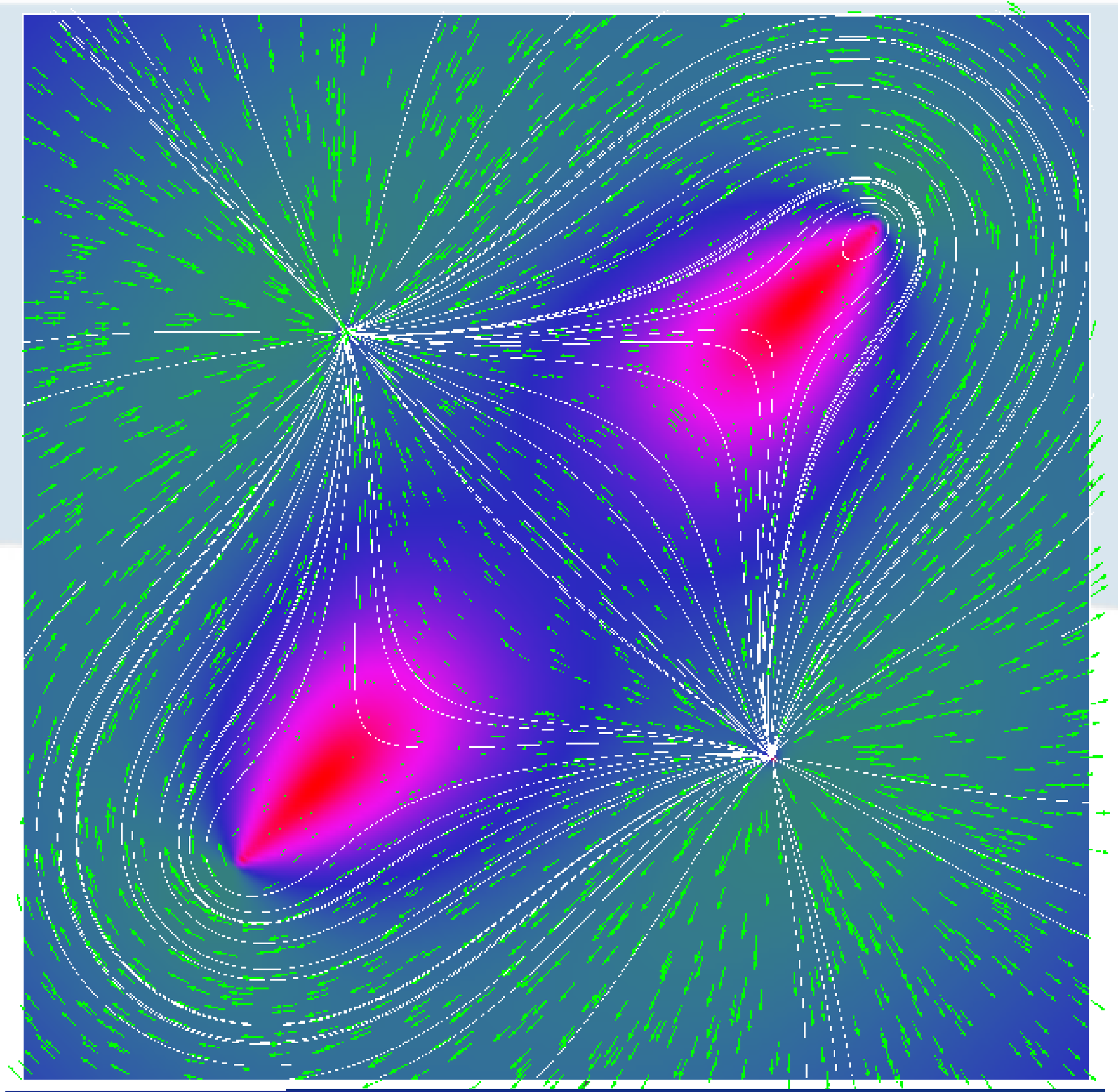


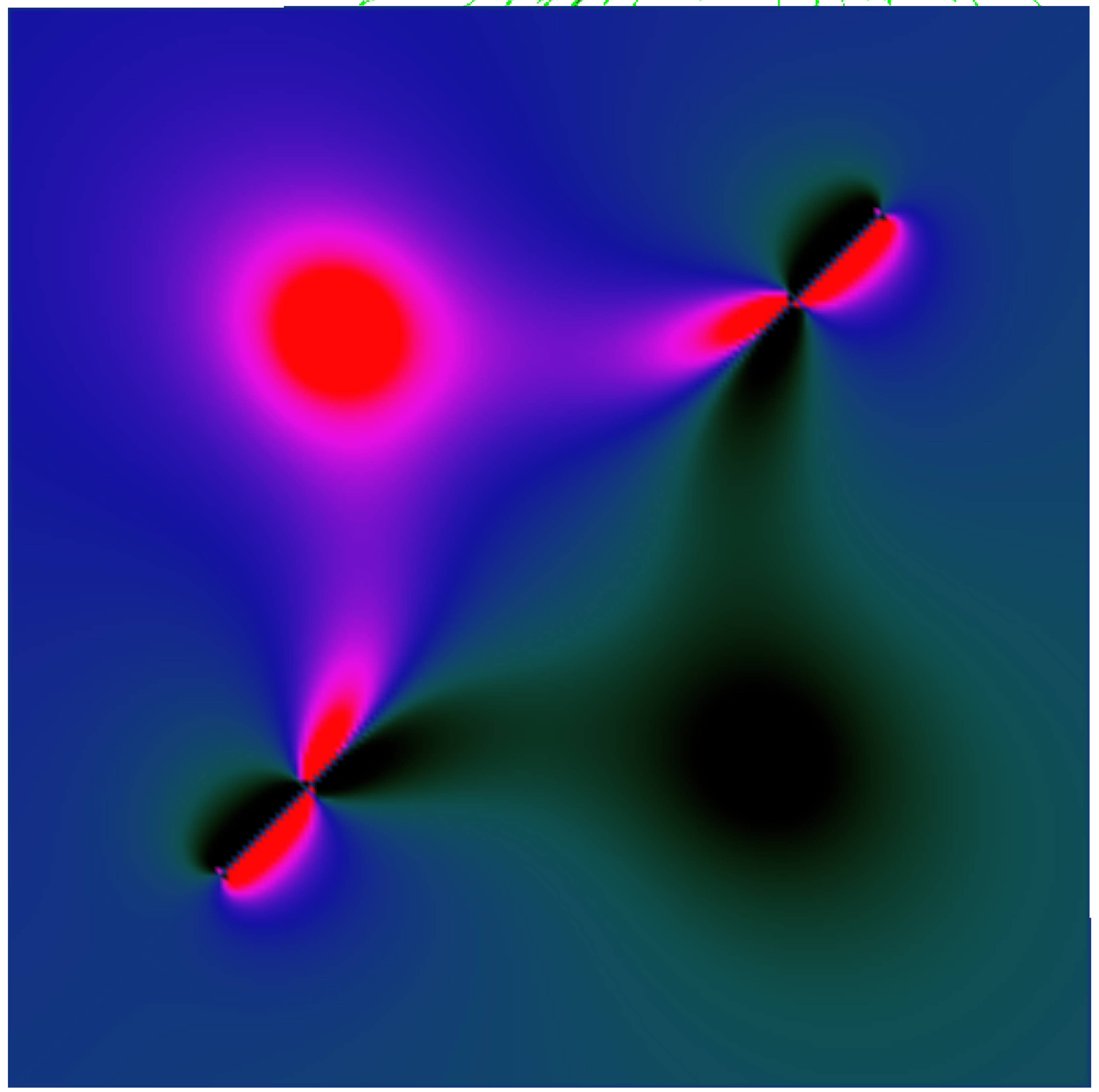


[http://www.deepakantony.com/]

So far

- Extract geometrical features
  - Streamlines
    - Seeding, Line Integral Convolution
    - **Where do they end/start?**
- Extract geometrical measures
  - Magnitude, orientation, angular speed, distortion



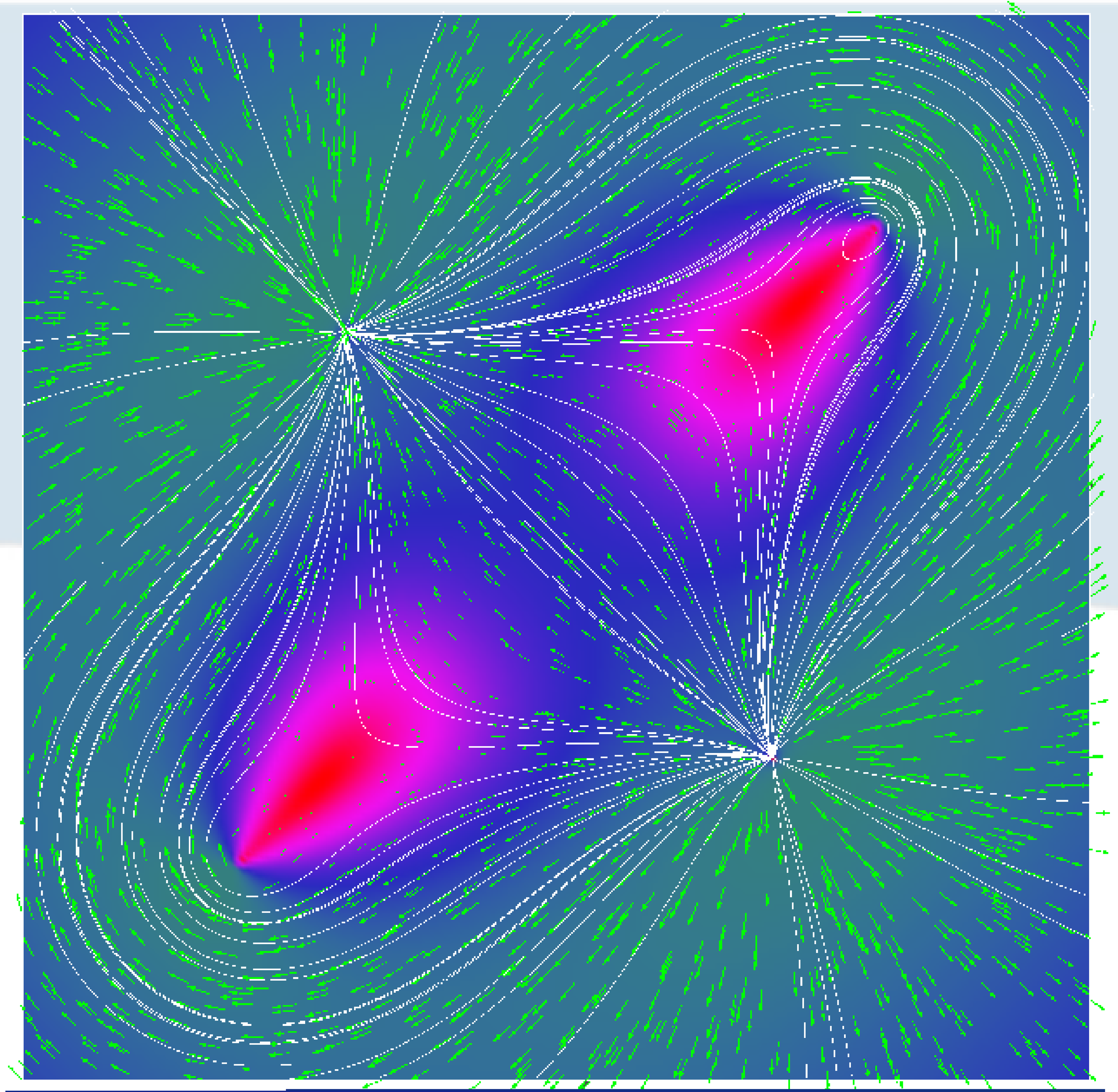


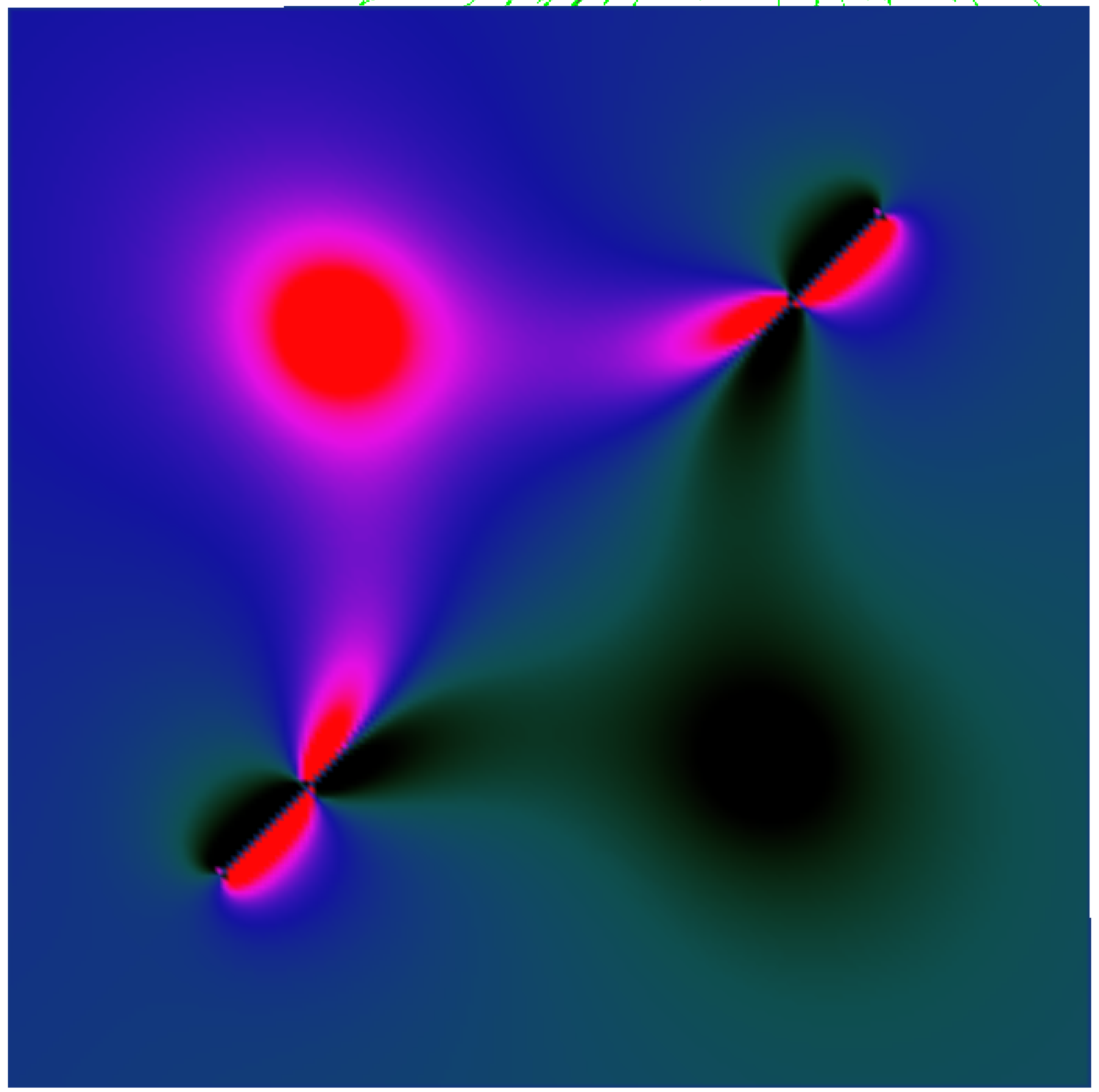
[http://www.deepakantony.com/]

So far

- Extract geometrical features
  - Streamlines
    - Seeding, Line Integral Convolution
    - **Where do they end/start?**
- Extract geometrical measures
  - Magnitude, orientation, angular speed, distortion

- Vector field topology



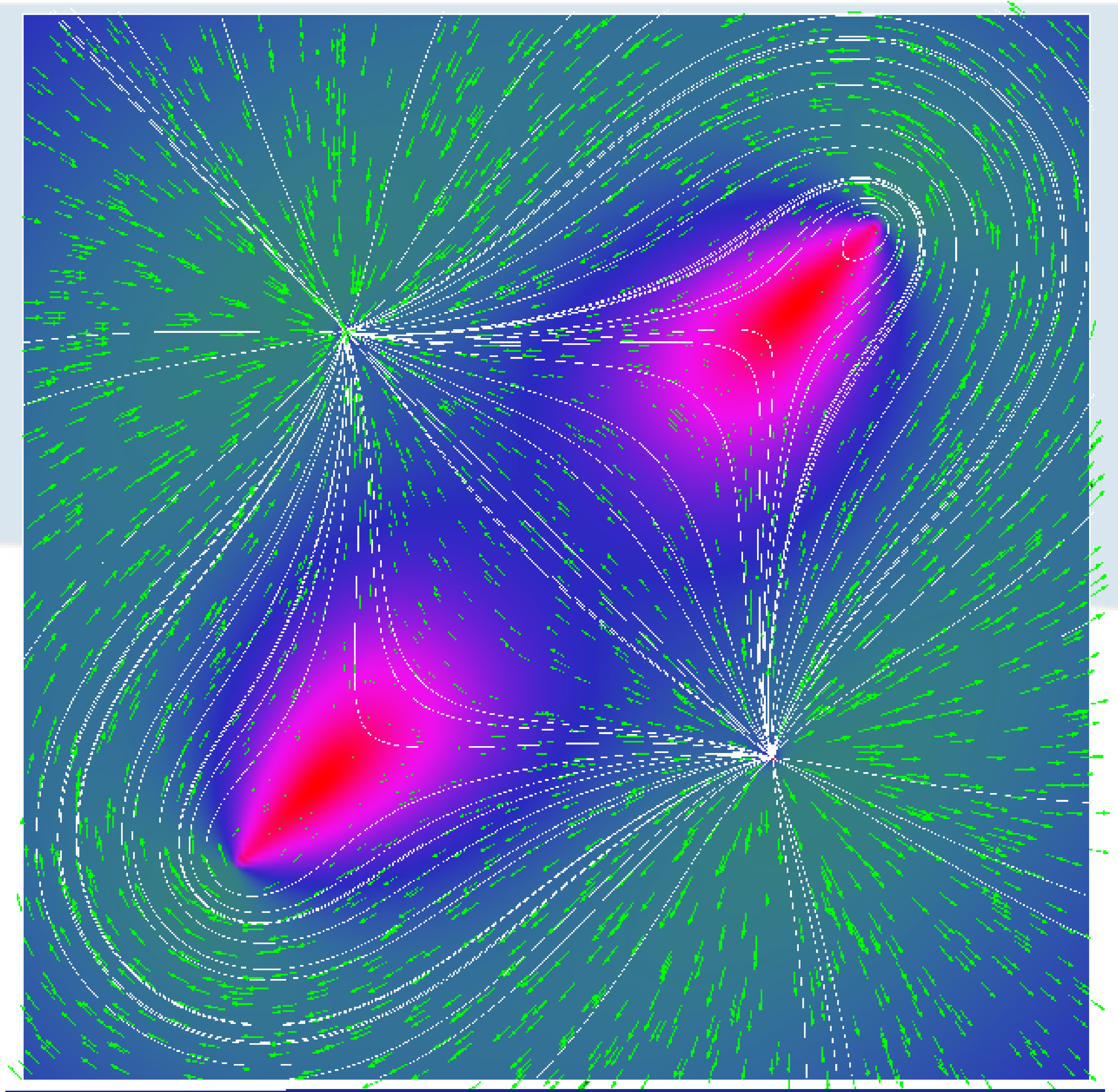


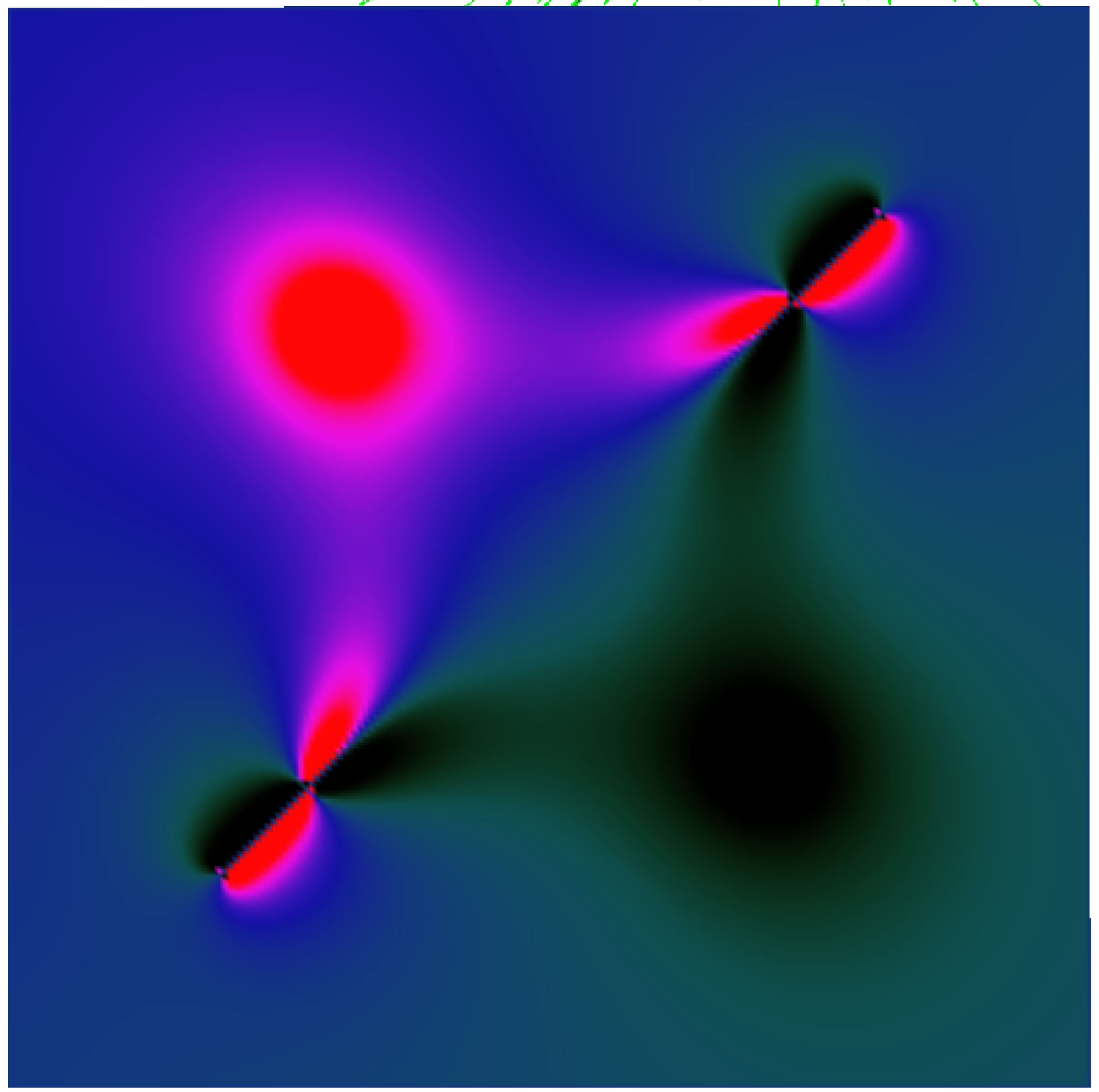
[http://www.deepakantony.com/]

So far

- Extract geometrical features
  - Streamlines
    - Seeding, Line Integral Convolution
    - **Where do they end/start?**
- Extract geometrical measures
  - Magnitude, orientation, angular speed, distortion

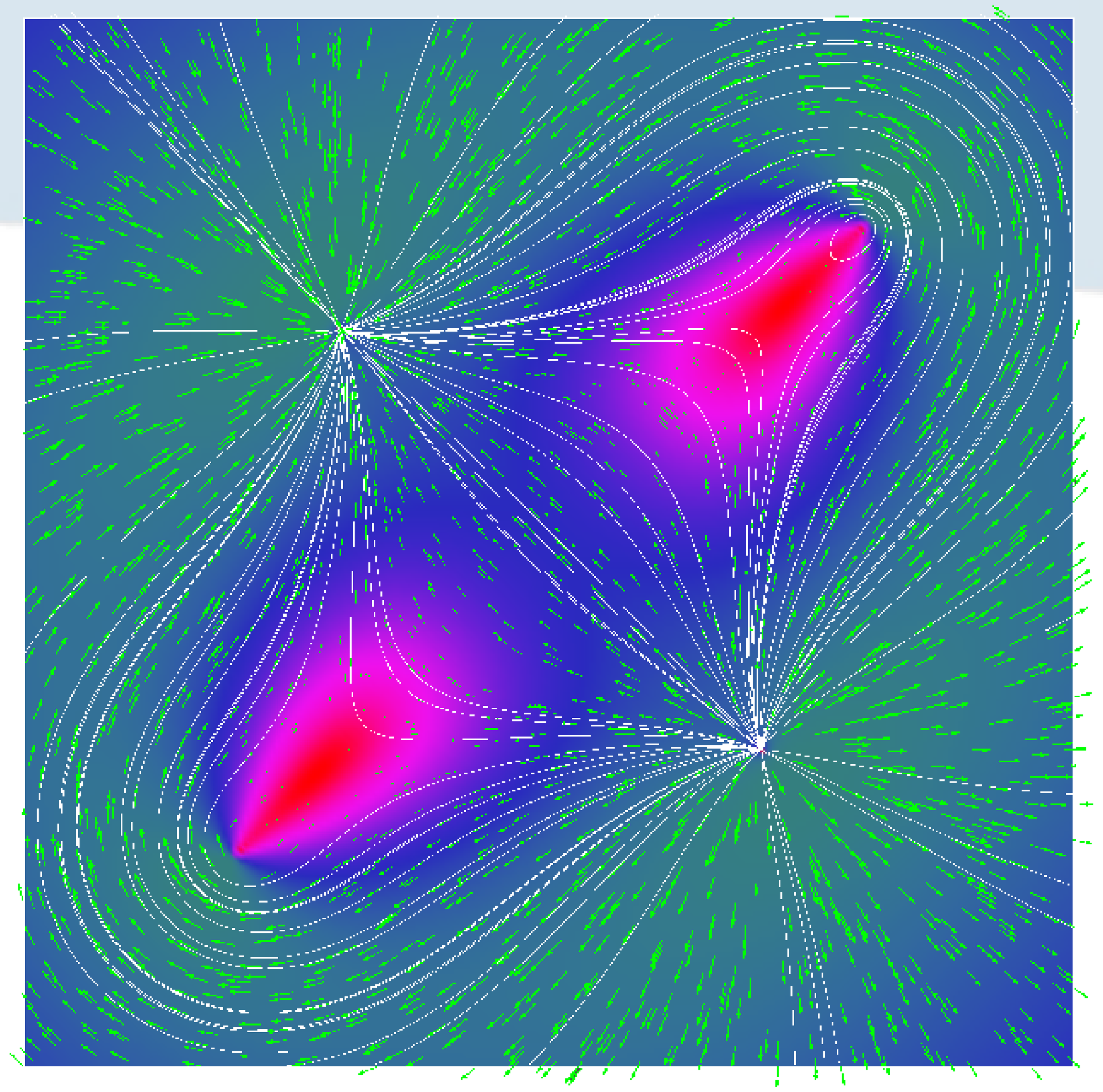
- Vector field topology
  - Summarizes all this information

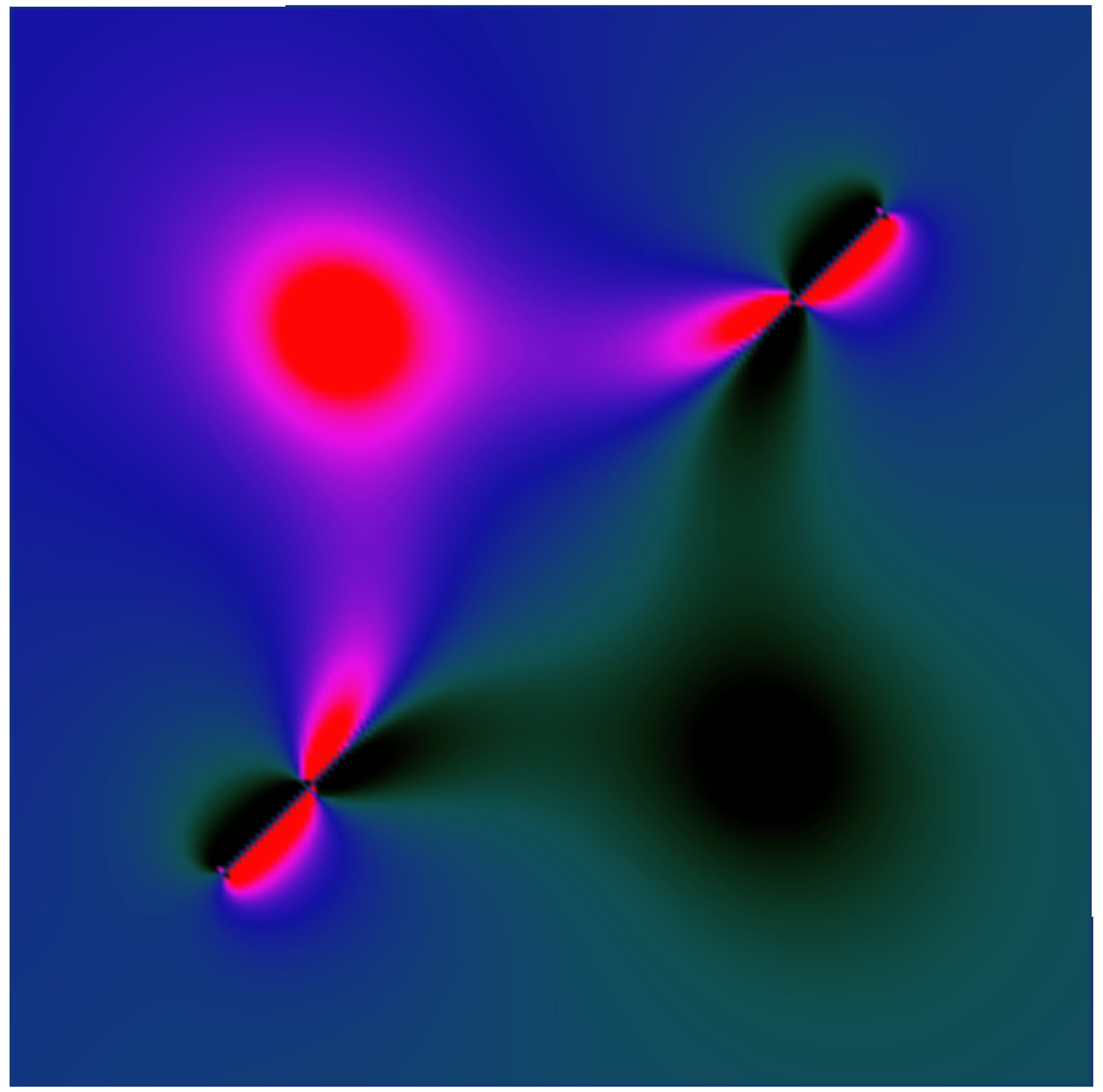




[http://www.deepakantony.com/]

Vector field topology for dummies

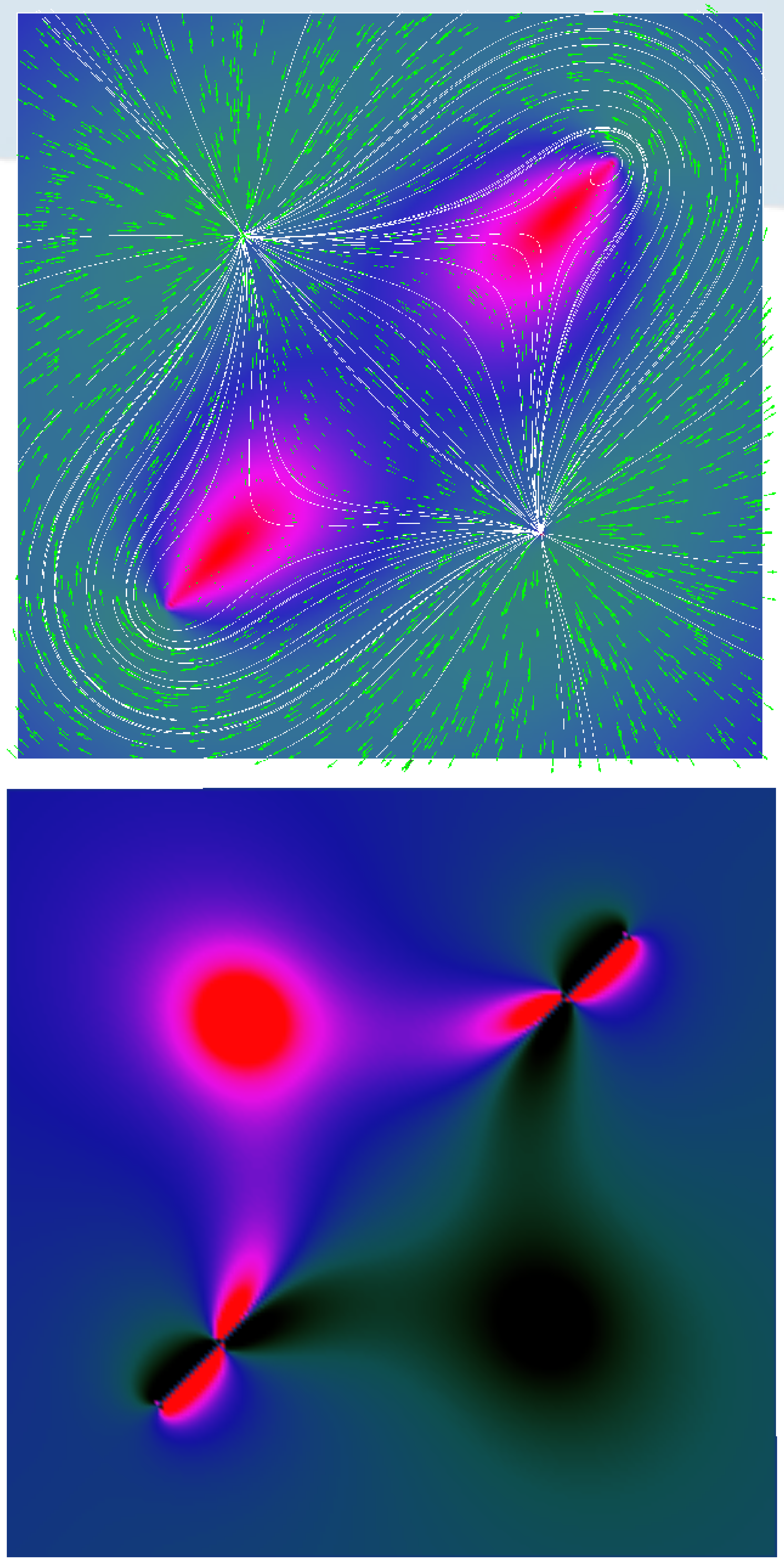




[http://www.deepakantony.com/]

Vector field topology for dummies

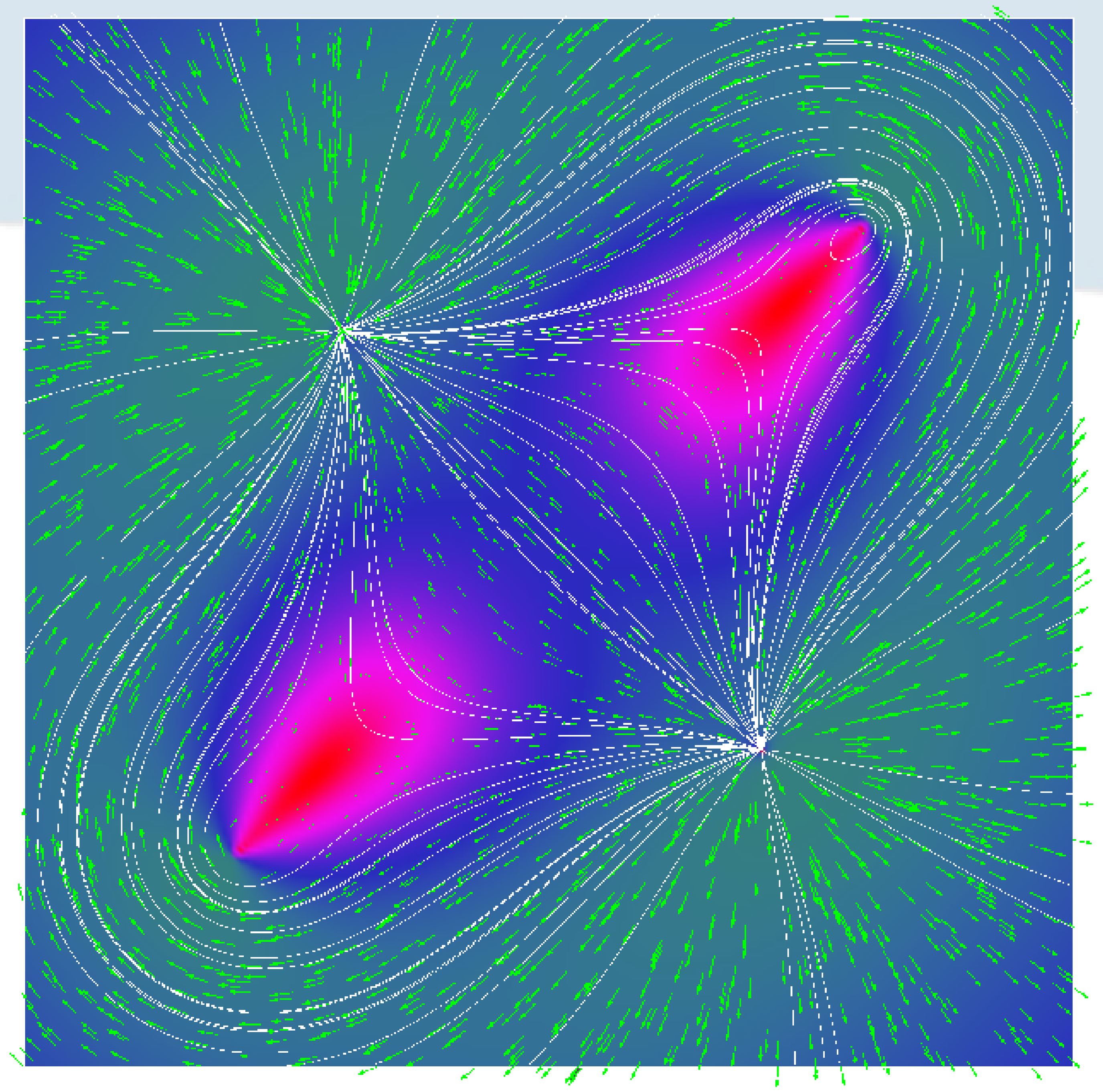
- What is the correspondence between

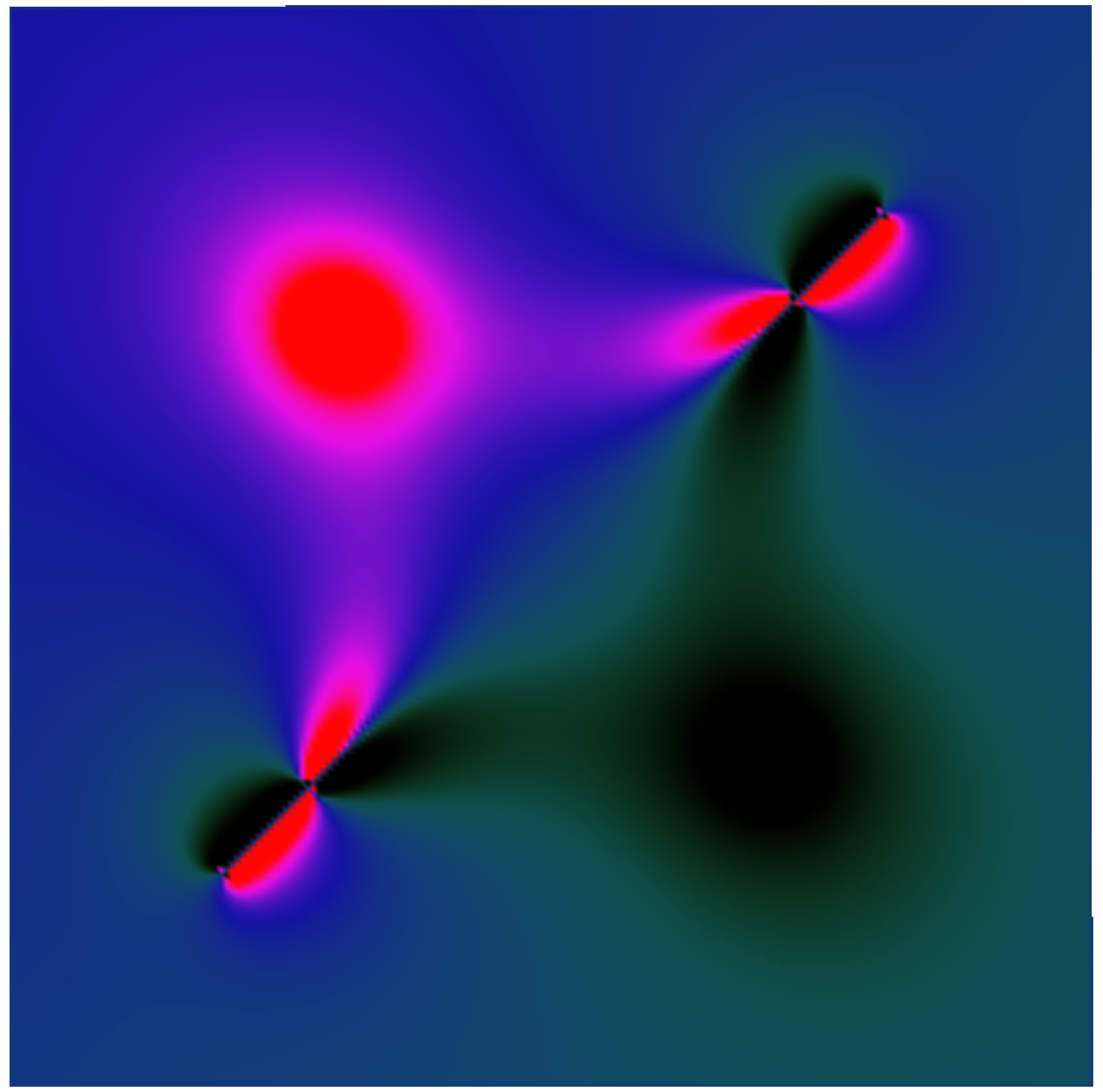


[http://www.deepakantony.com/]

Vector field topology for dummies

- What is the correspondence between
  - The geometrical features we introduced
  - The critical points of the field



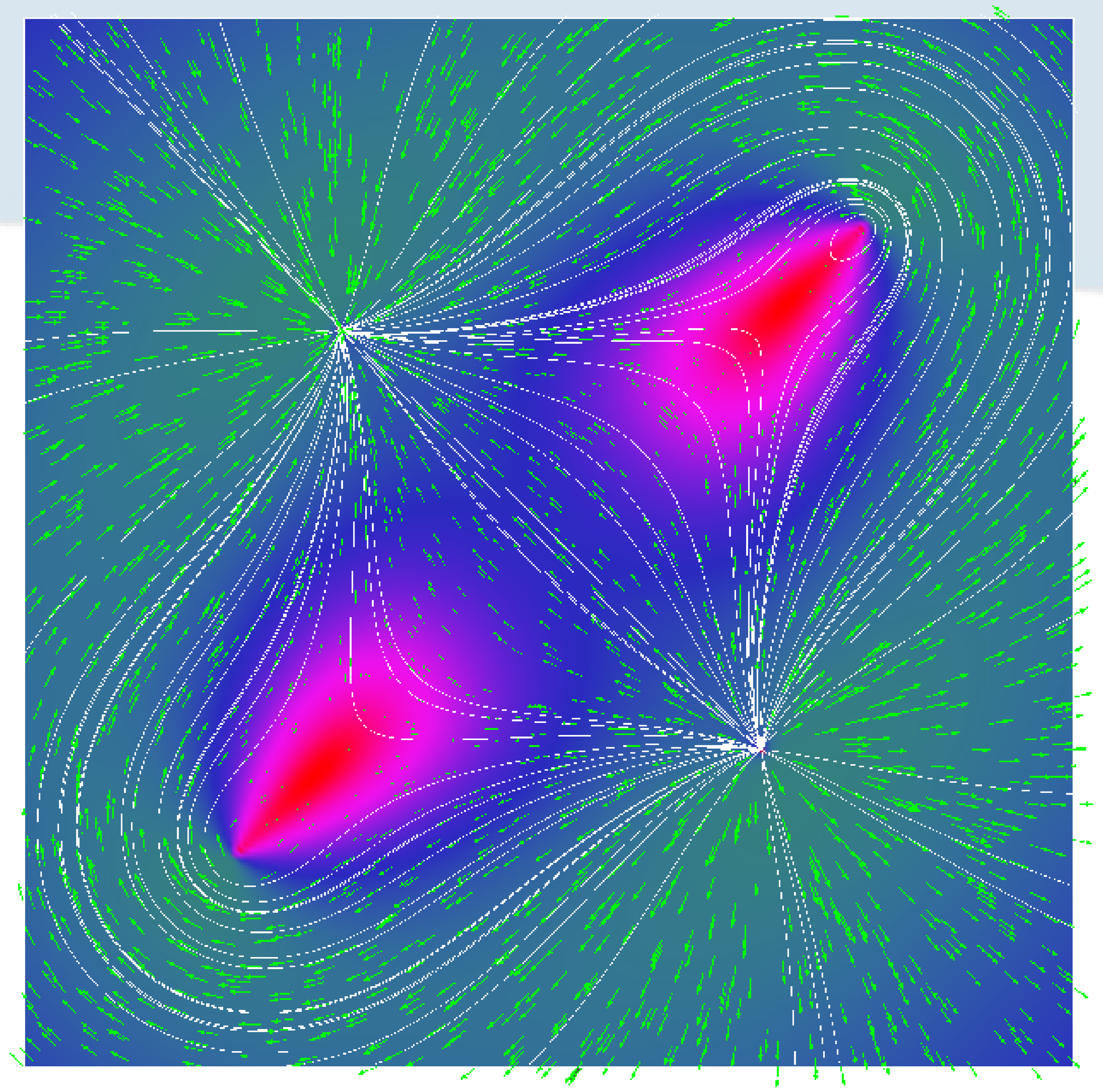


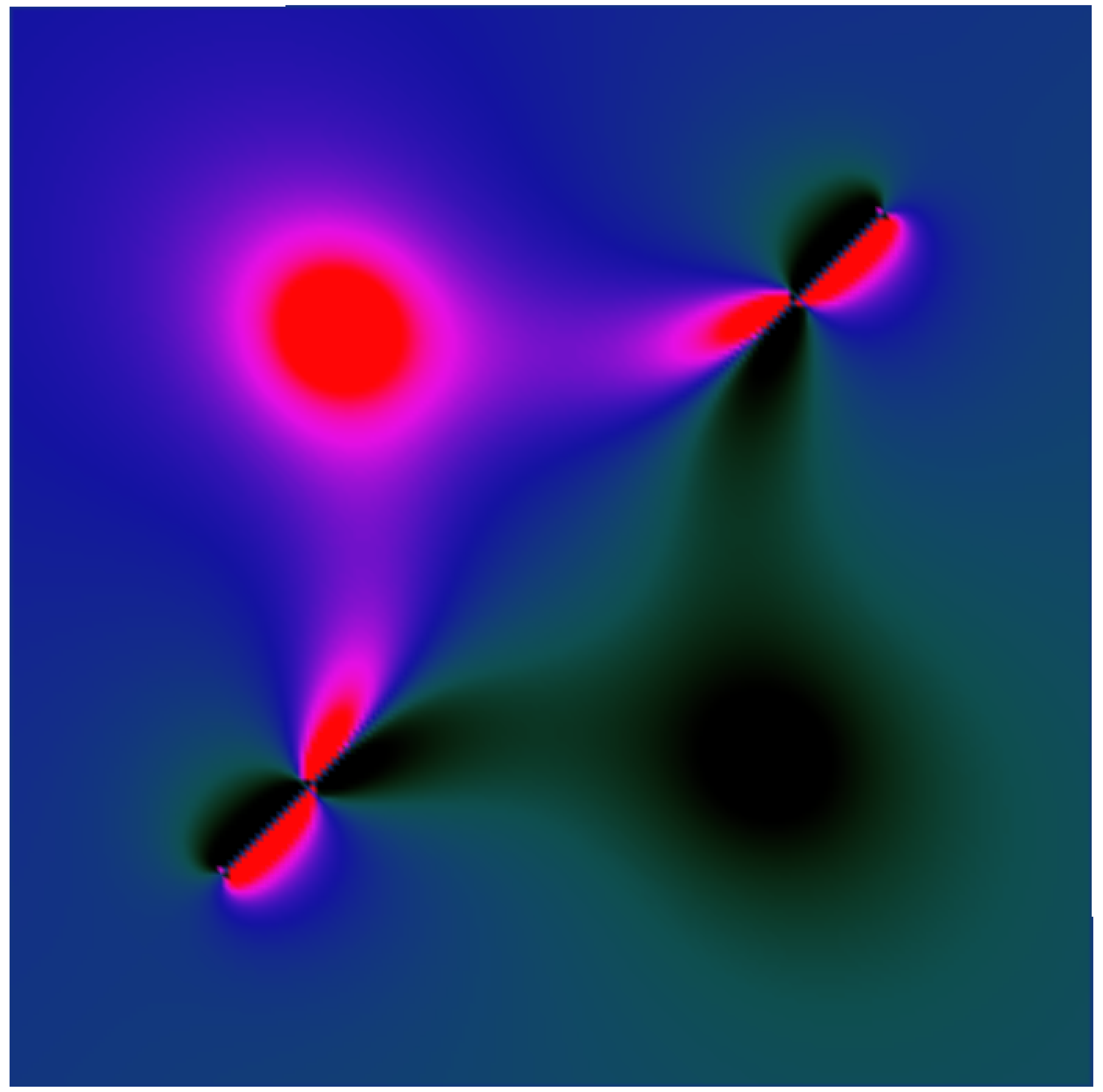
[http://www.deepakantony.com/]

Vector field topology for dummies

- What is the correspondence between
  - The geometrical features we introduced
  - The critical points of the field

- Regarding critical points



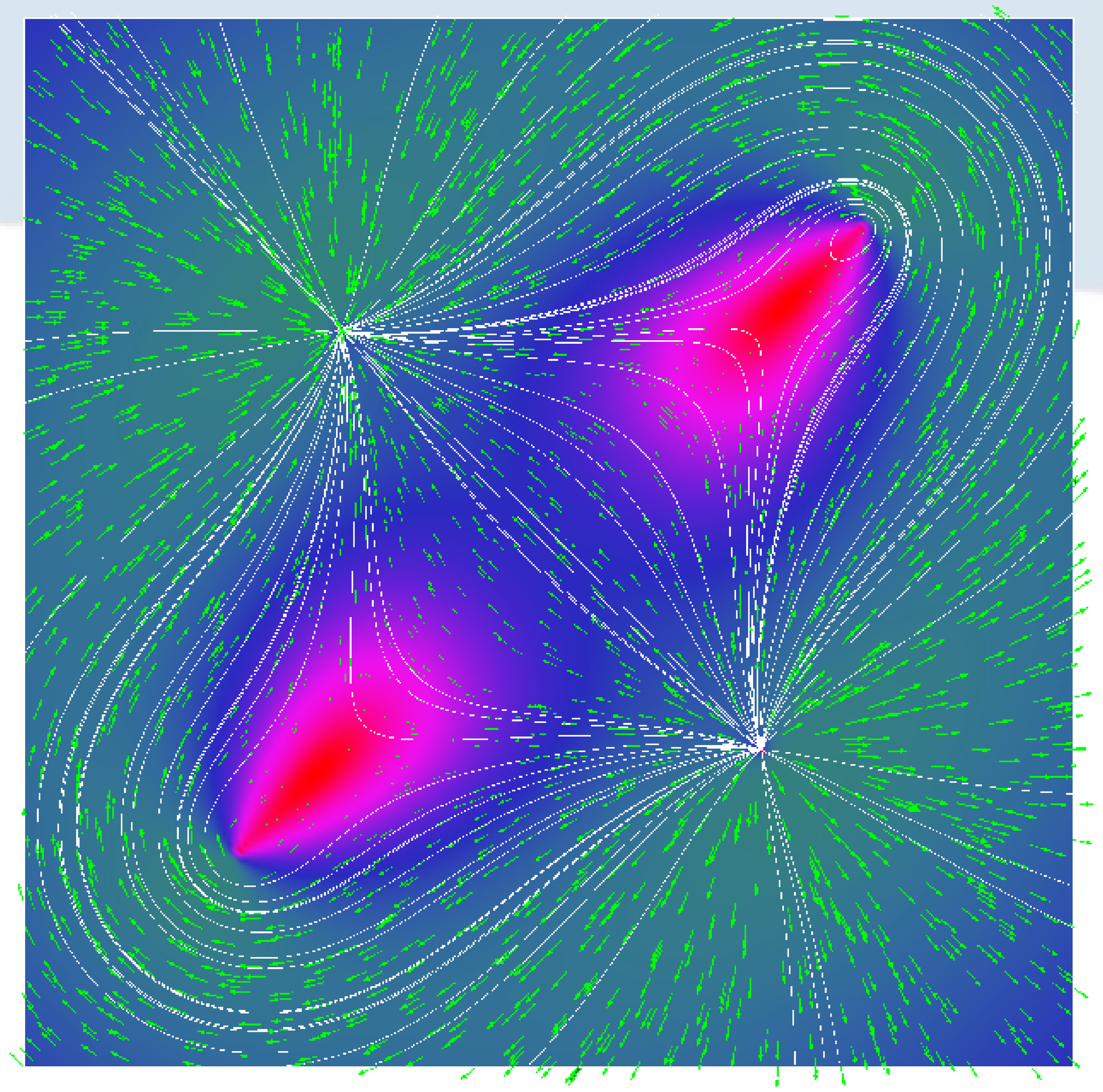


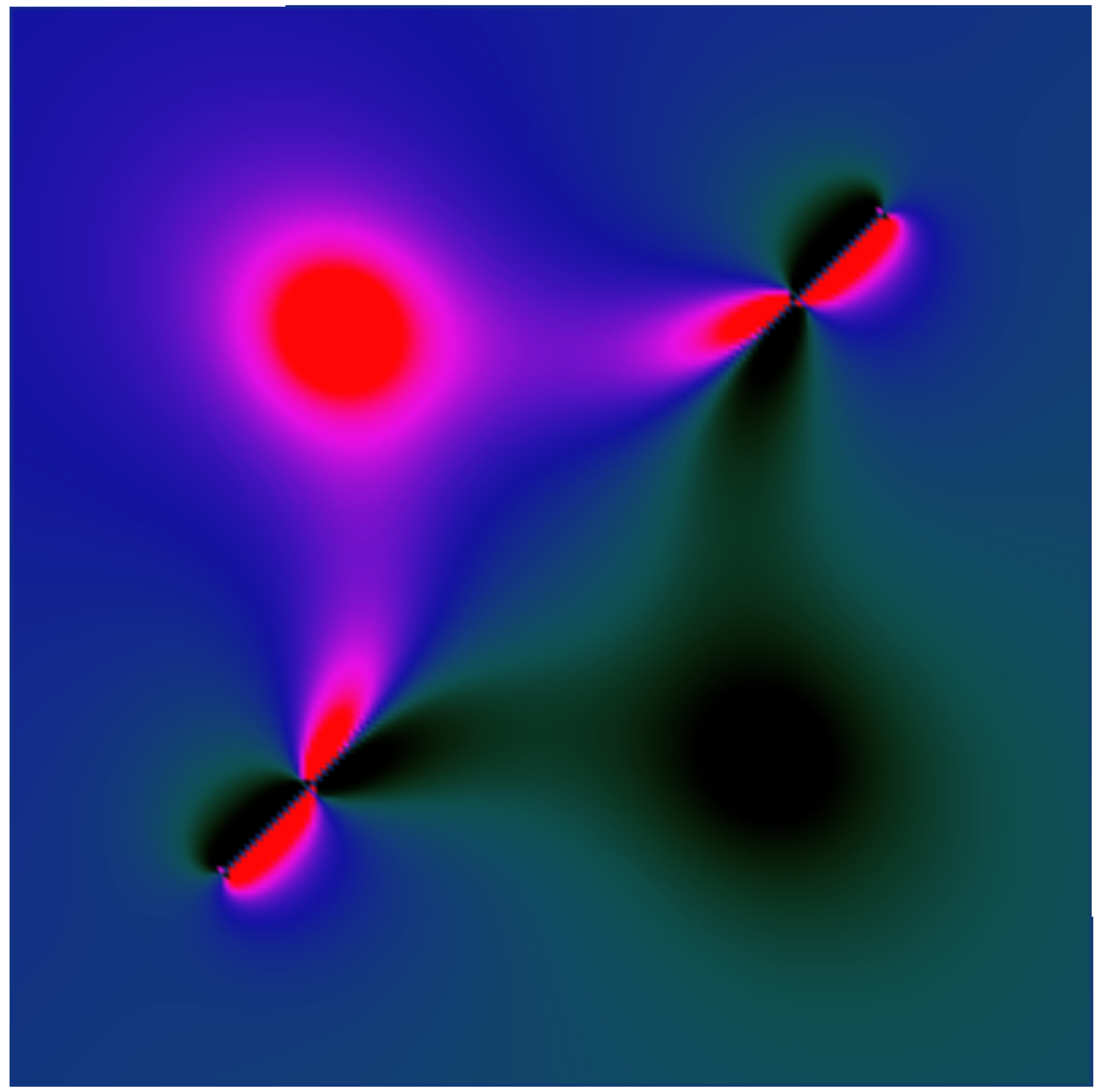
[http://www.deepakantony.com/]

Vector field topology for dummies

- What is the correspondence between
  - The geometrical features we introduced
  - The critical points of the field

- Regarding critical points
  - What is the connection between them?
  - What does it imply for the features we introduced?

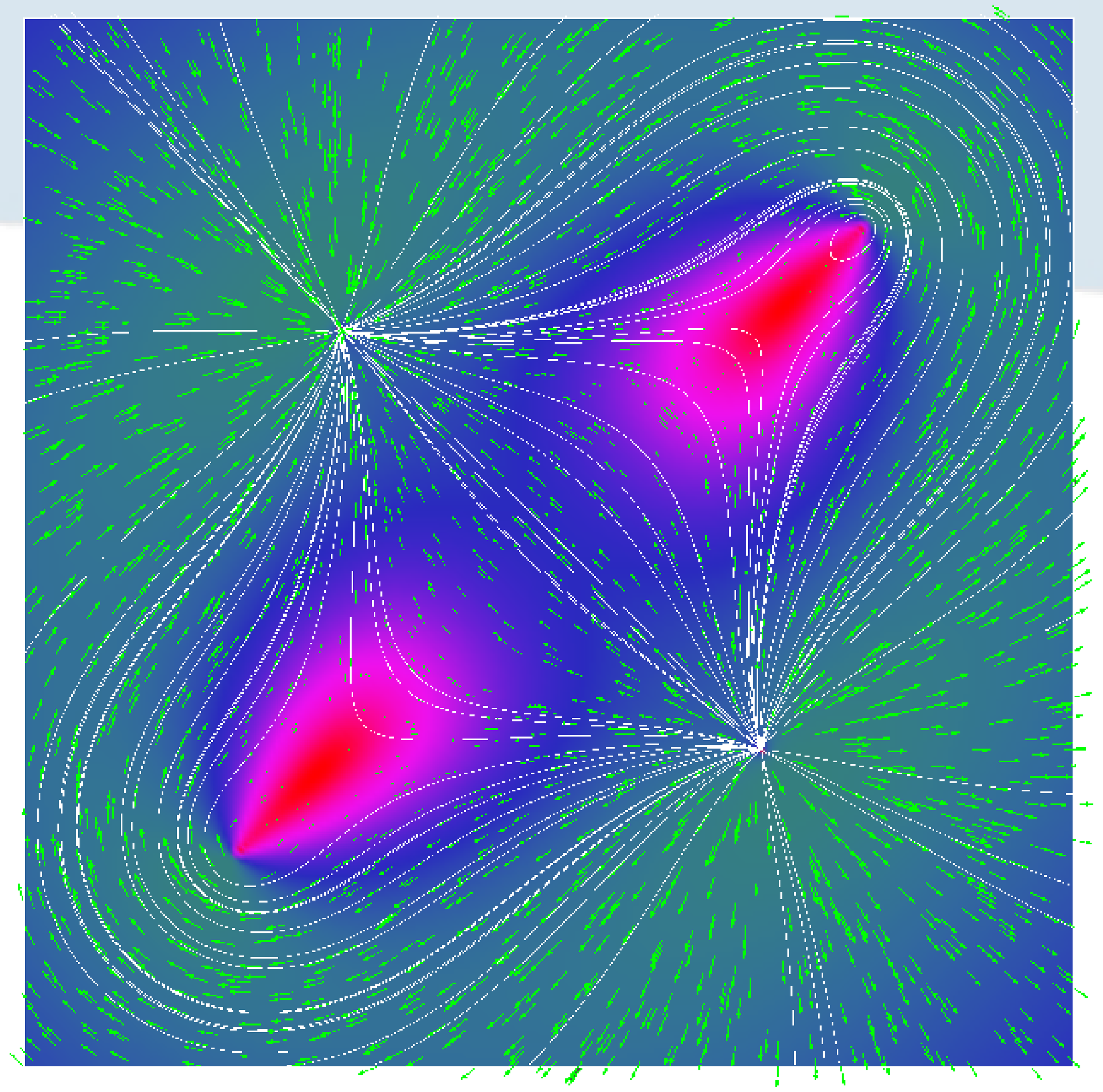


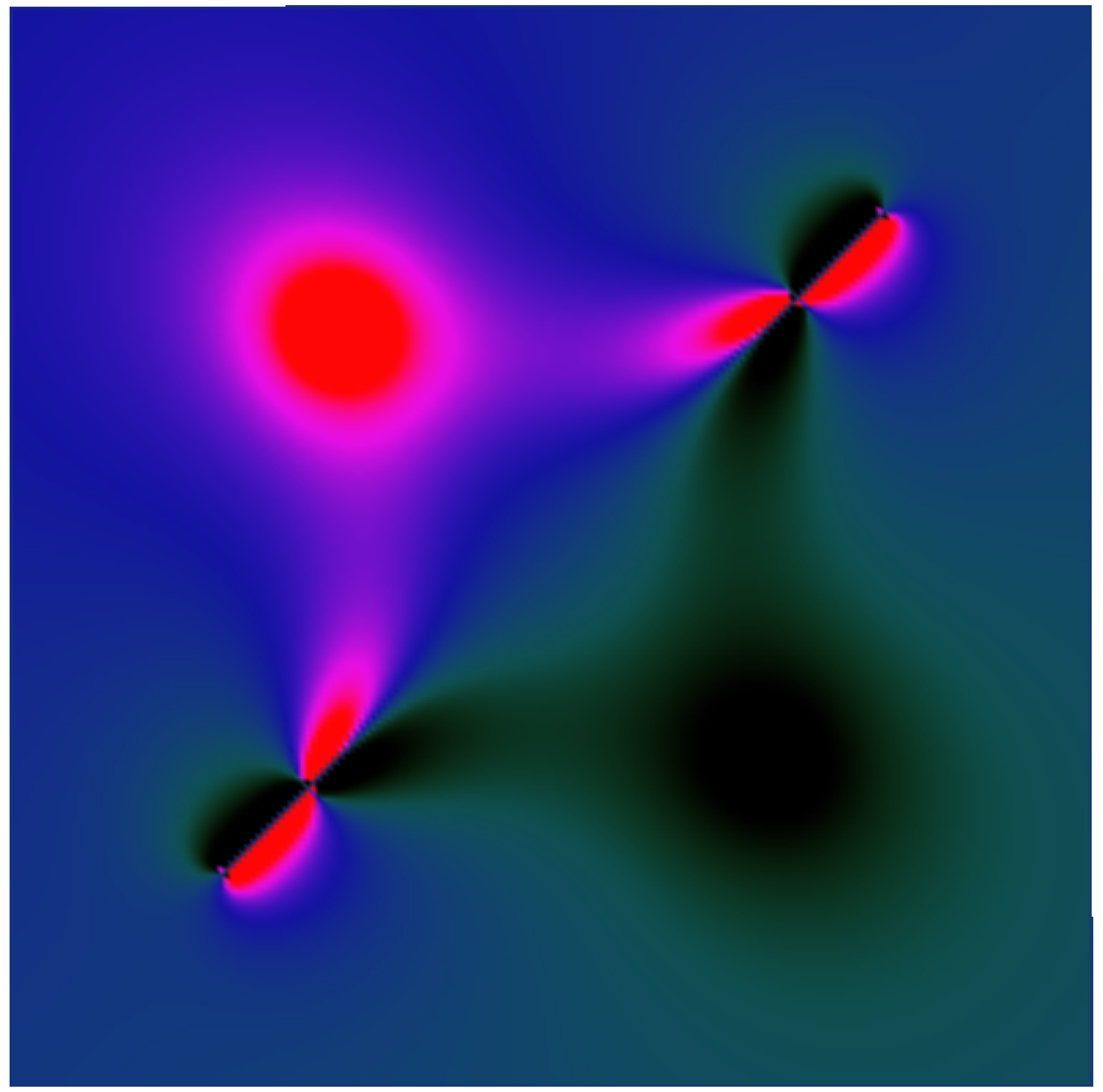


[http://www.deepakantony.com/]

Vector field topology for dummies

- What is the correspondence between
  - The geometrical features we introduced
  - The critical points of the field
- Regarding critical points
  - What is the connection between them?
  - What does it imply for the features we introduced?
- Analogy to scalar field topology





[http://www.deepakantony.com/]

Gradient field topology

Gradient field topology

- For instance
  - $\mathcal{D} \subset \mathbb{R}^n$
    - $f : \mathbb{R}^n \to \mathbb{R}^n$

Gradient field topology

- For instance
  - $\mathcal{D} \subset \mathbb{R}^n$
    - $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$
  - Gradient field, example
    - $g : \mathbb{R}^n \rightarrow \mathbb{R}$
    - $f = \nabla g = (\frac{\partial g}{\partial x_1}, \frac{\partial g}{\partial x_2}, \ldots, \frac{\partial g}{\partial x_n})$

Gradient field topology

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  - $\mathcal{D} \subset \mathbb{R}^n$
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  - Gradient field, example
    - $g : \mathbb{R}^n \to \mathbb{R}$
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- PL scalar fields
  - Piecewise constant gradient field

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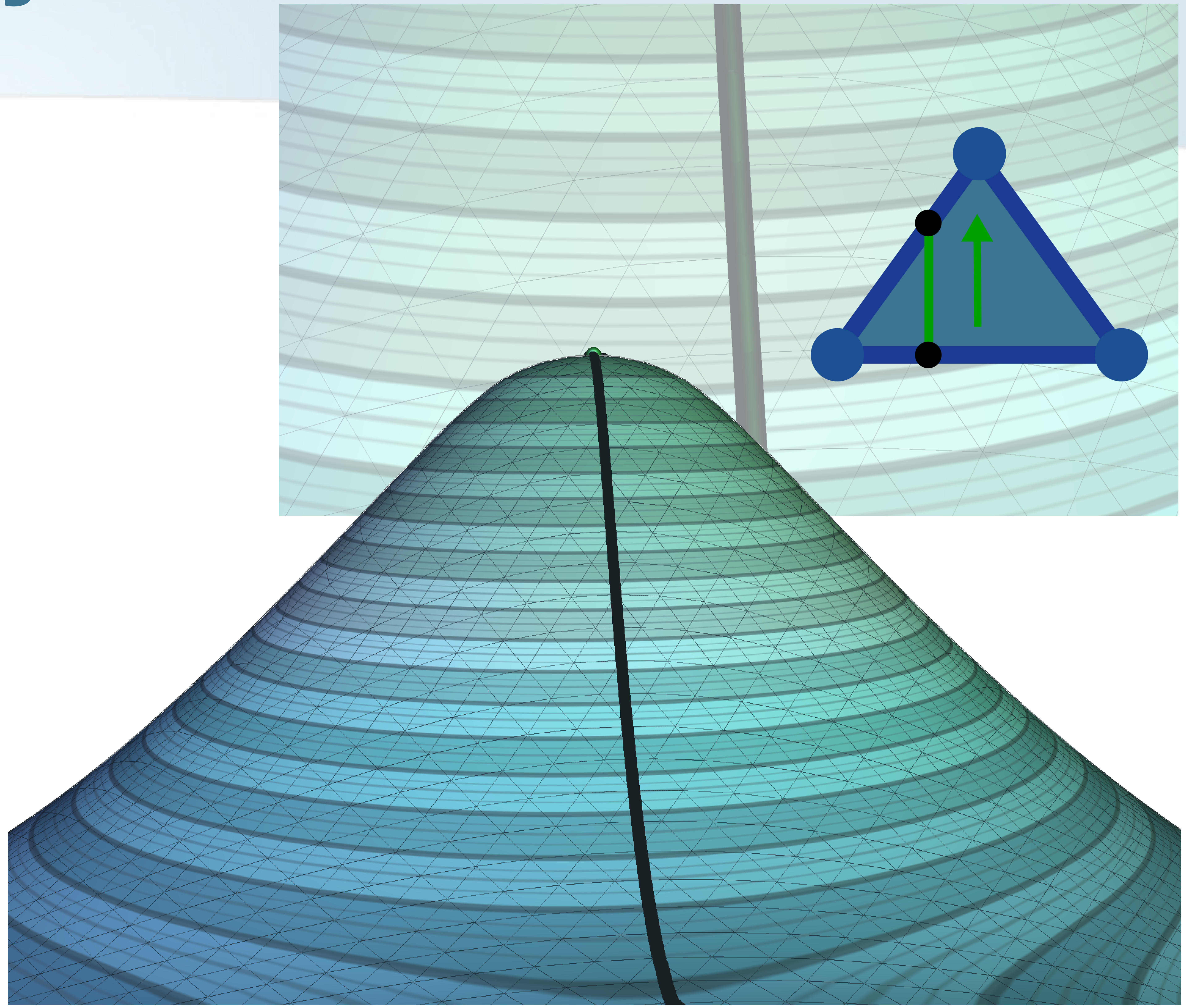
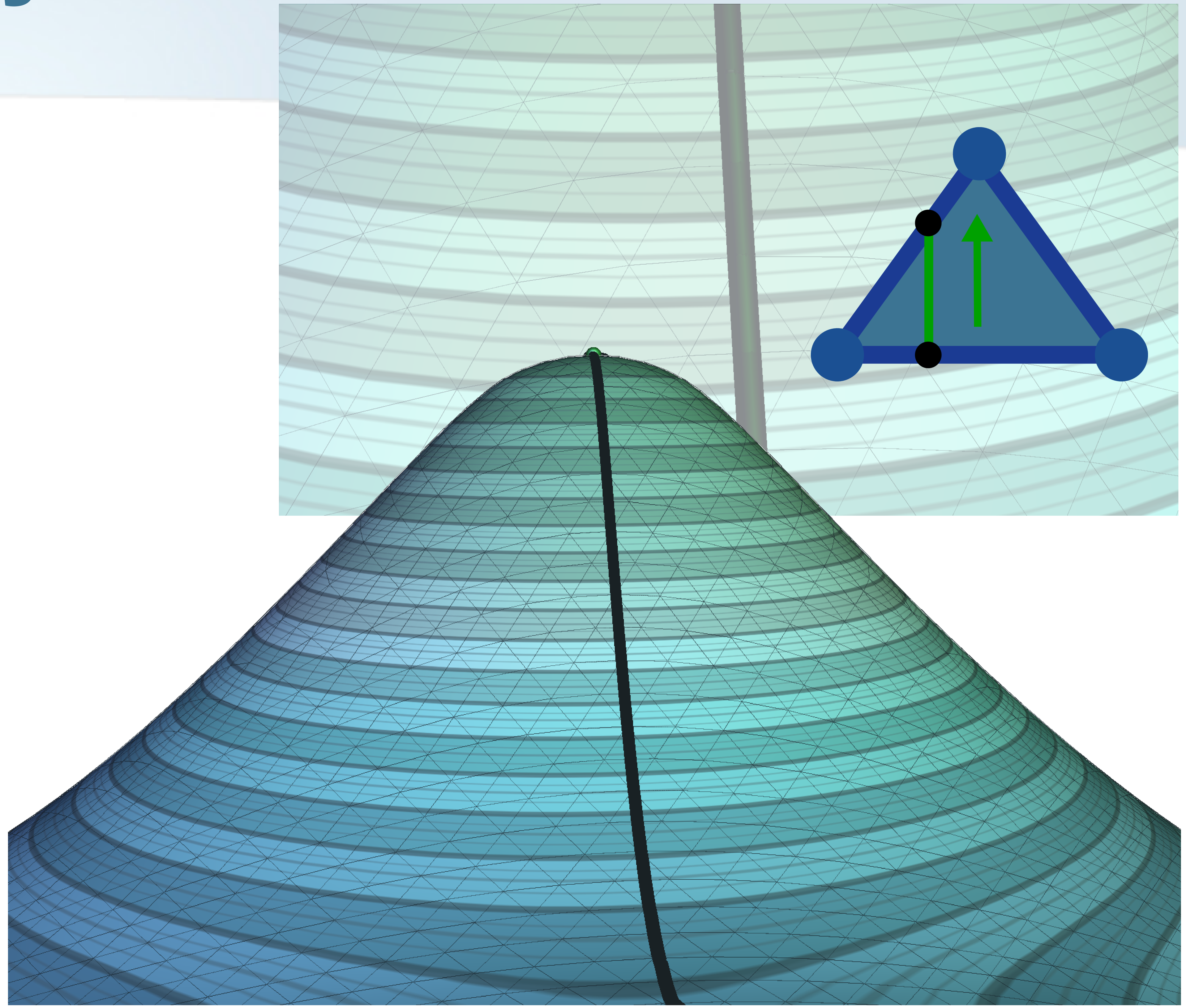


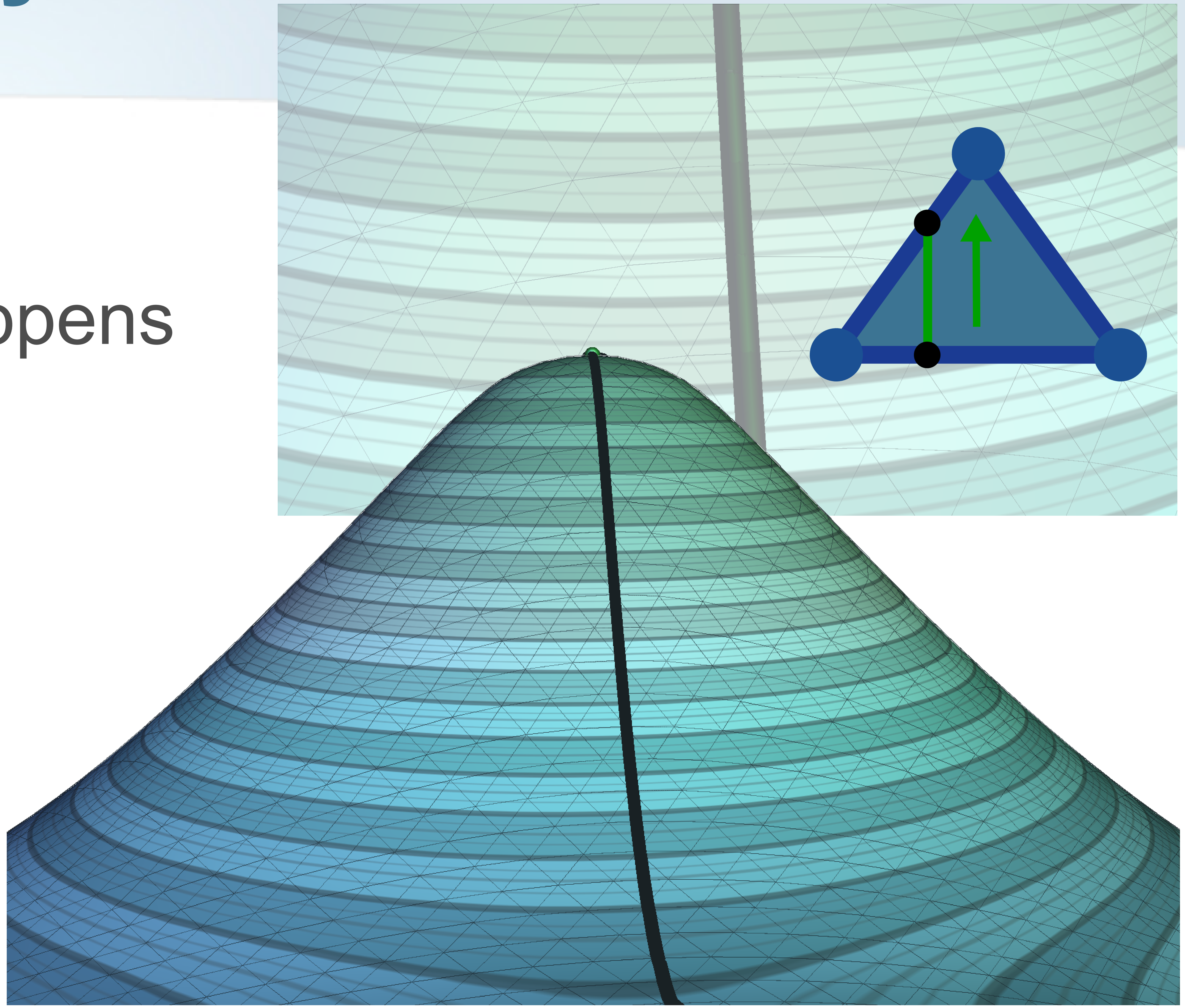
Gradient field topology

- Intuition of critical points



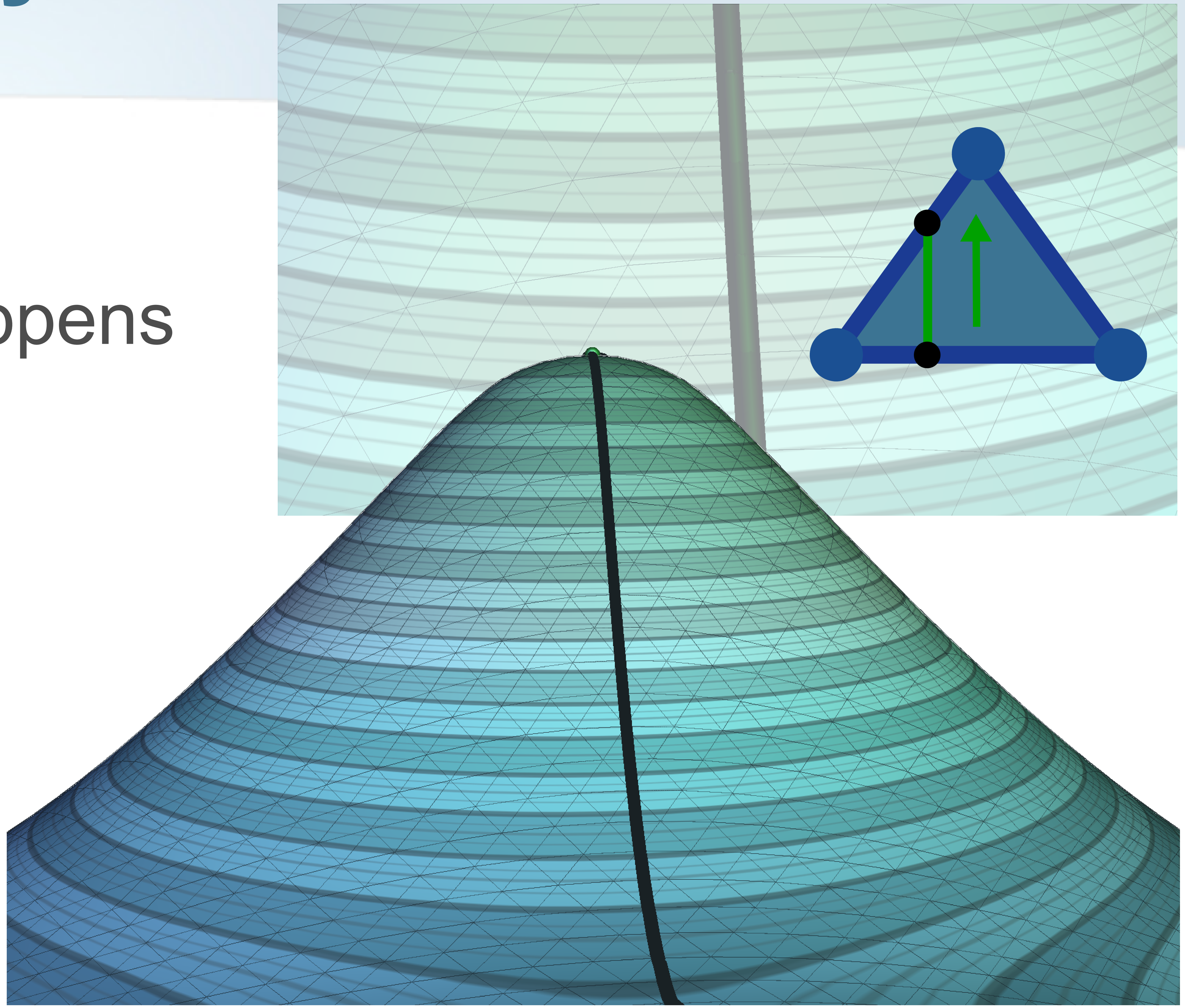
Gradient field topology

- Intuition of critical points
  - Points where something critical happens



Gradient field topology

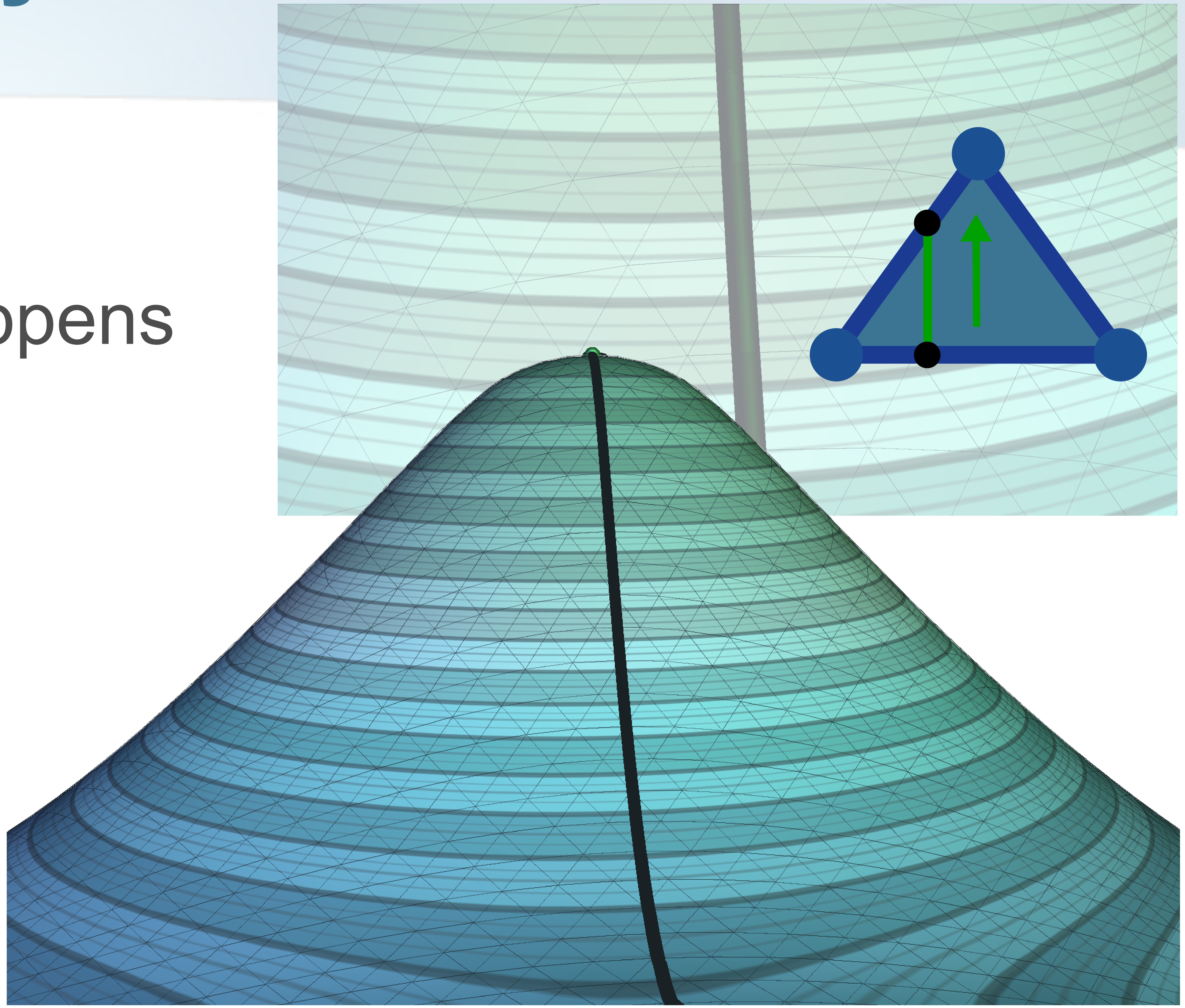
- Intuition of critical points
  - Points where something critical happens
  - Where the flow stops



Gradient field topology

- Intuition of critical points
  - Points where something critical happens
  - Where the flow stops

- Where does the gradient stops?



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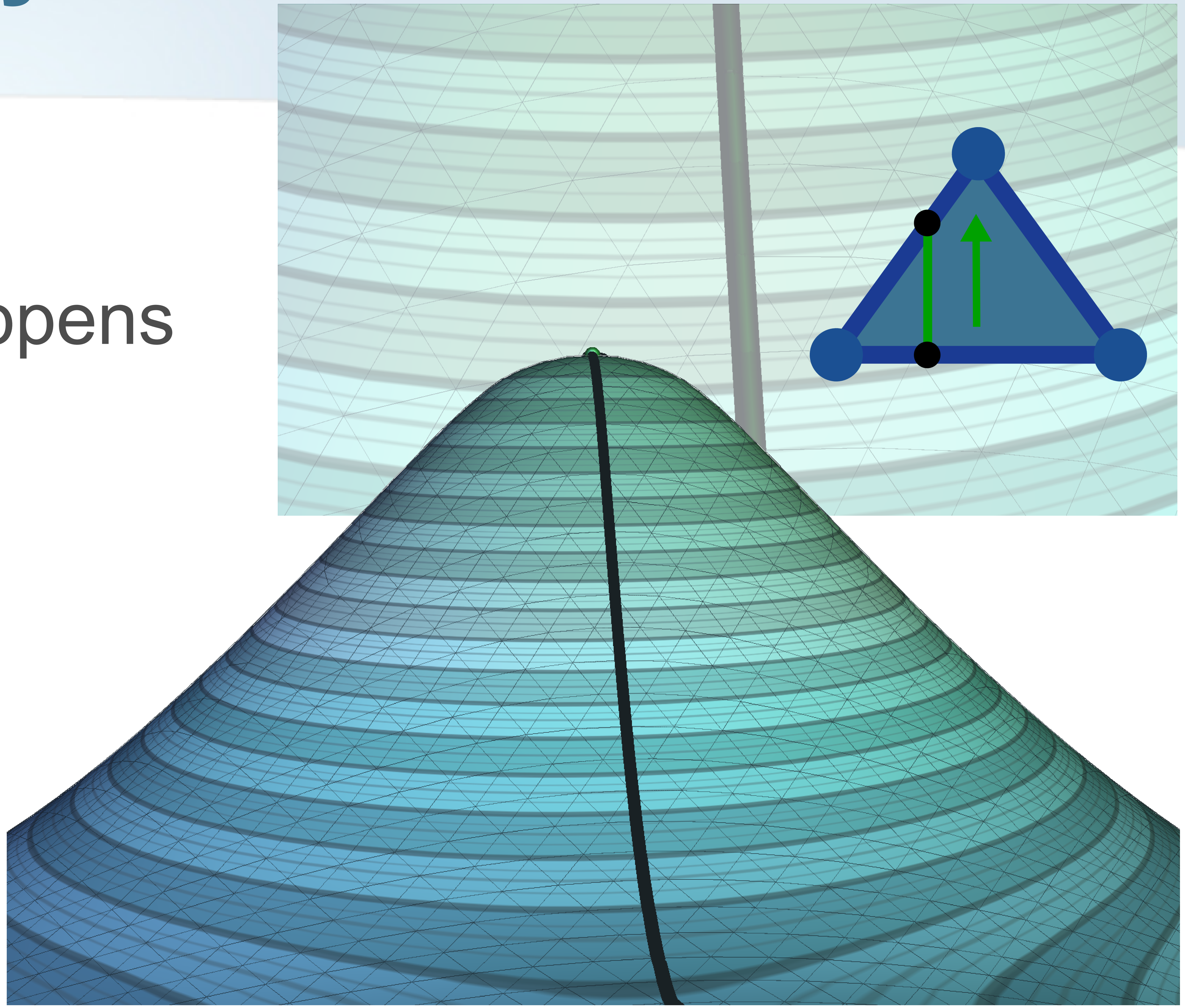

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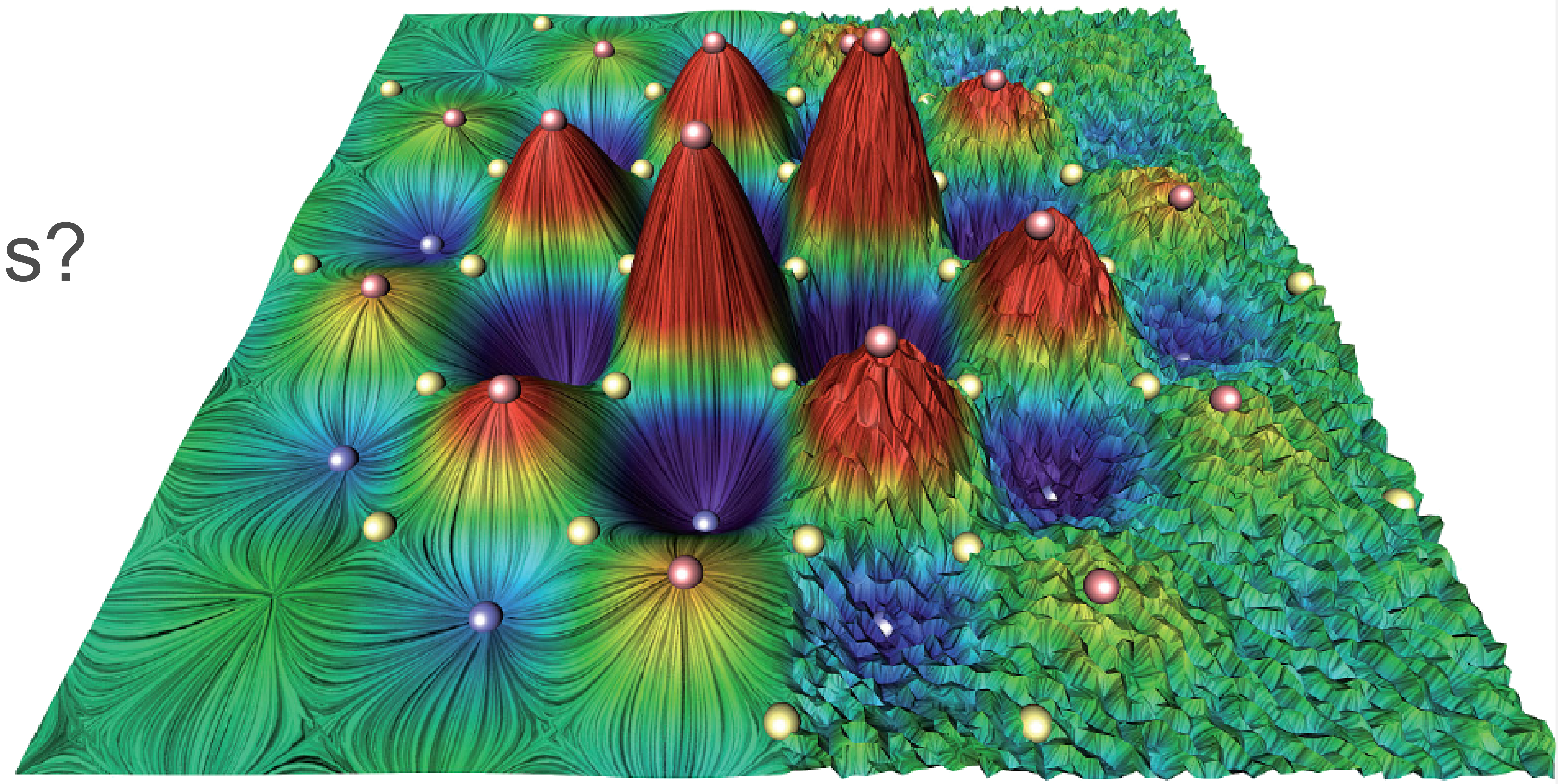

[Reininghaus11]

Gradient field topology

- Intuition of critical points
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- Where does the gradient stops?
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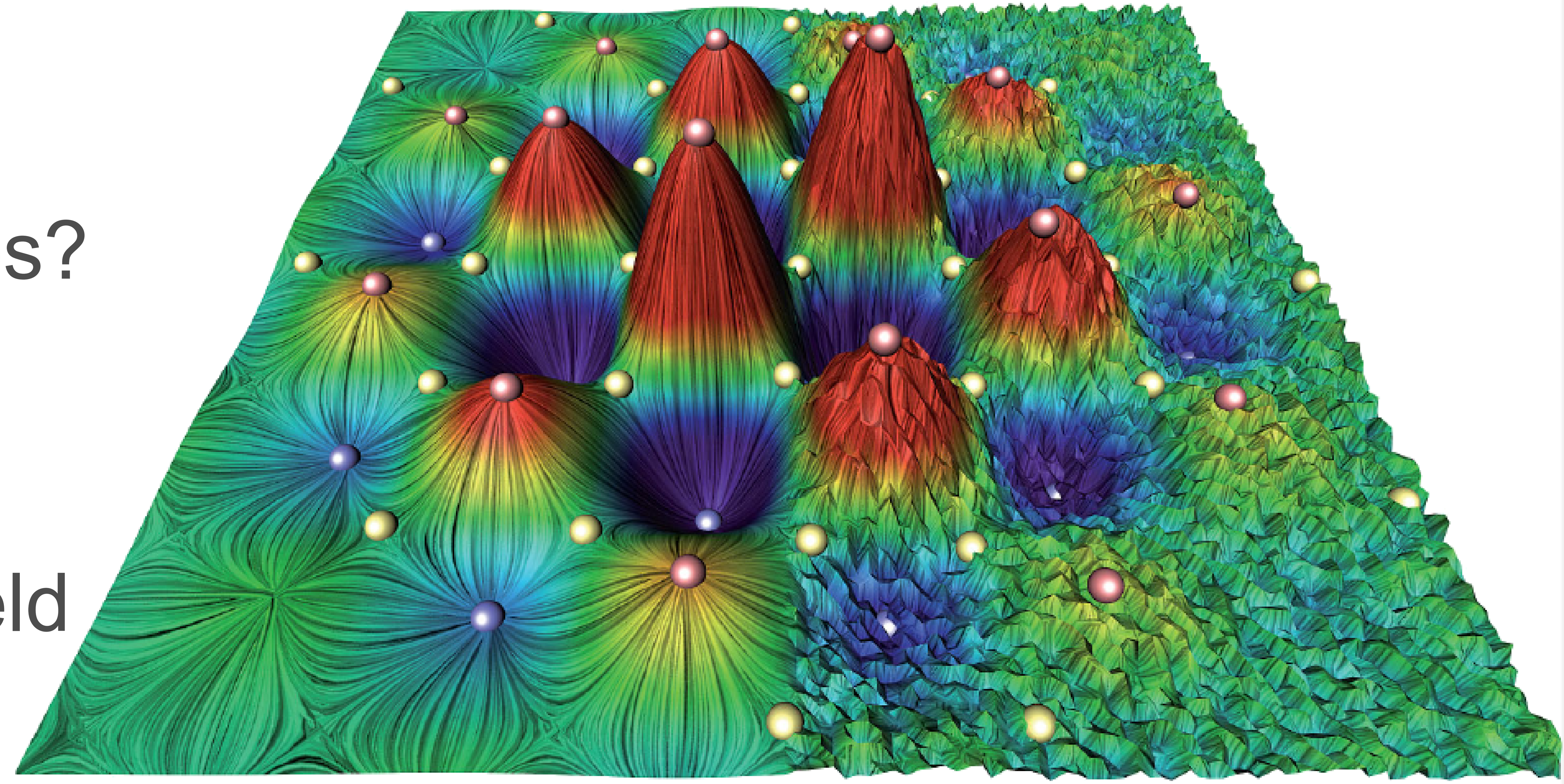
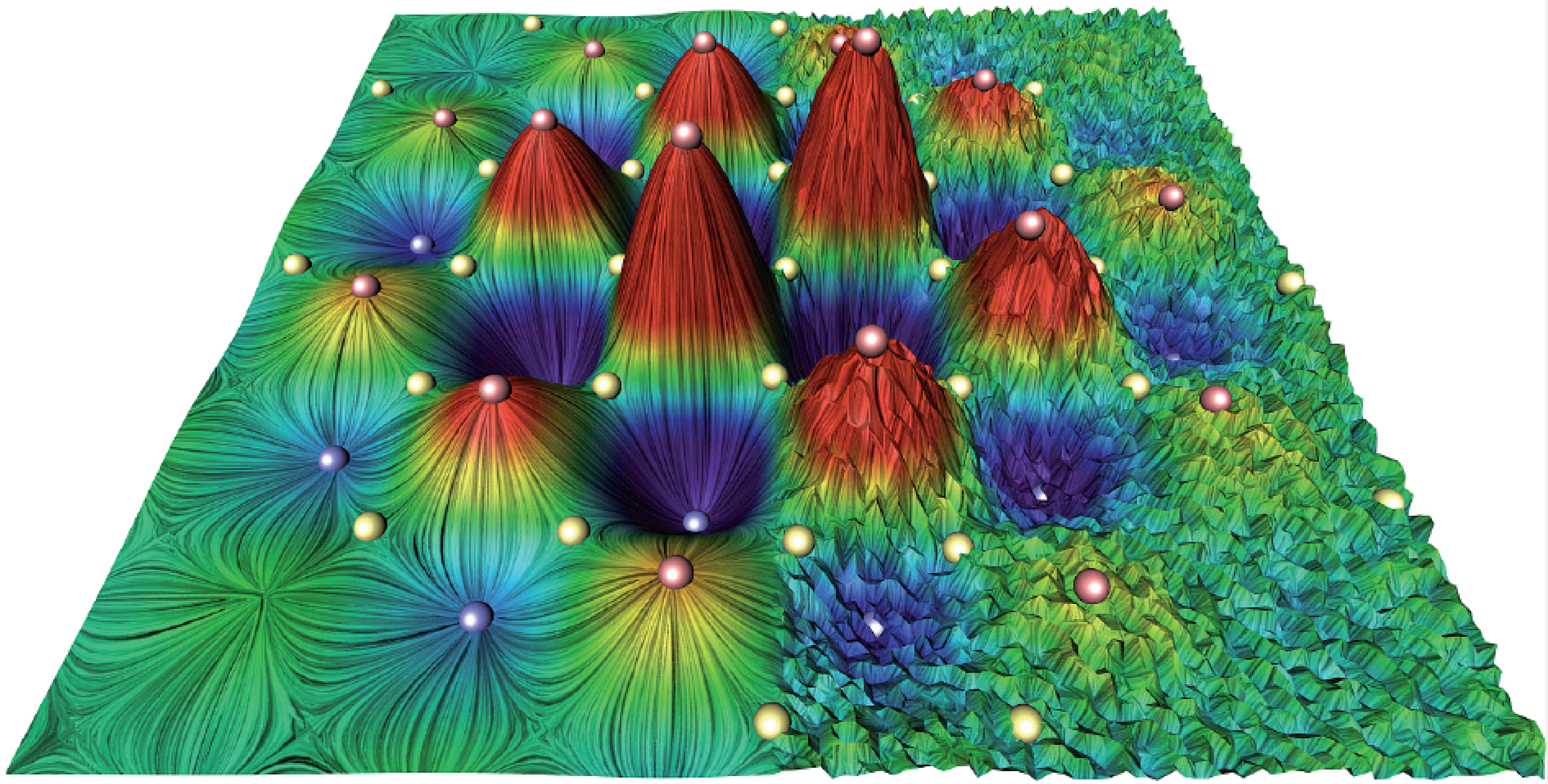
- Critical points of a gradient field
  - Same as for scalar fields



[Reininghaus11]

Gradient field topology

- Critical points of a vector field
  - Points where the magnitude vanishes



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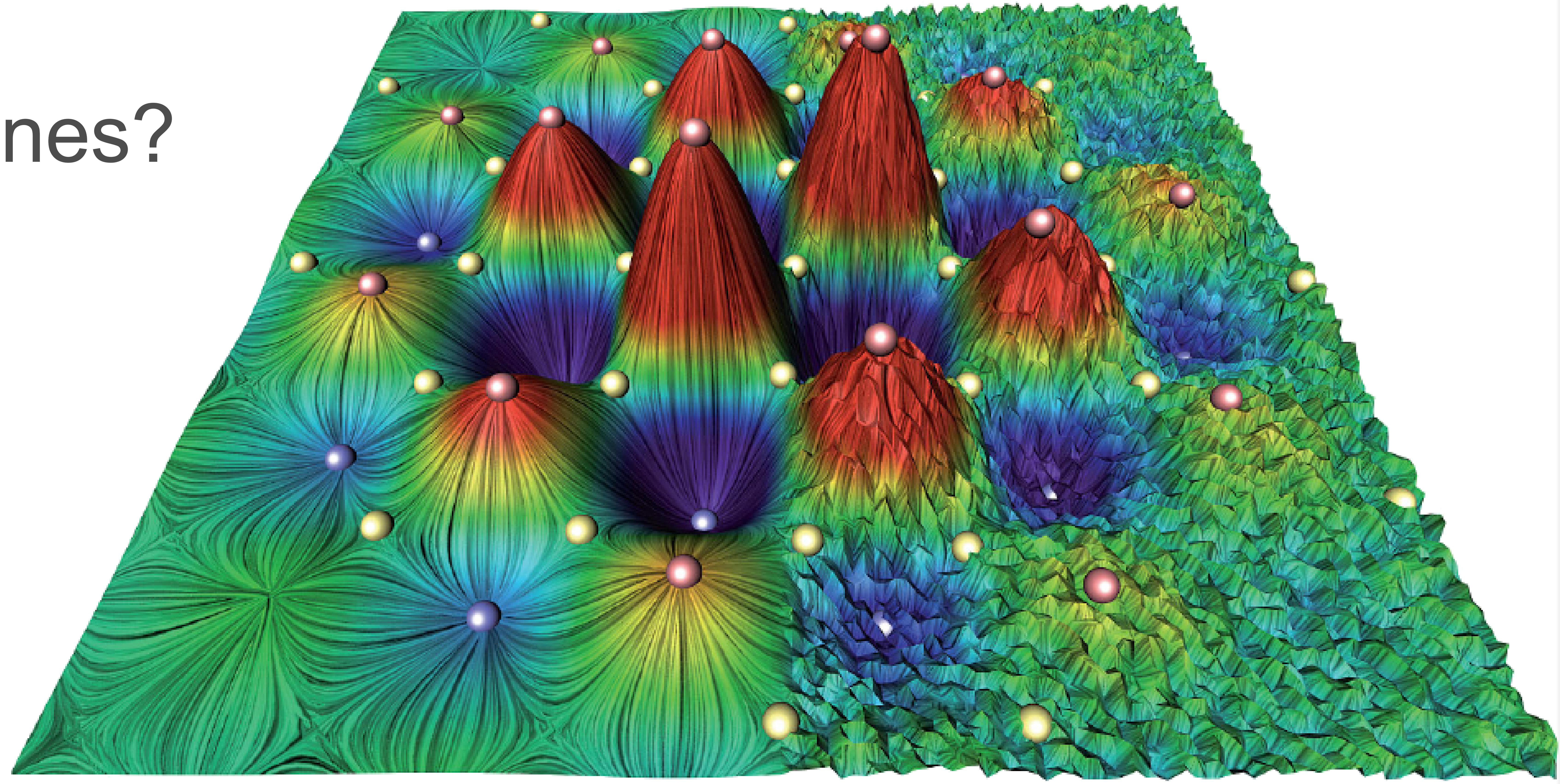

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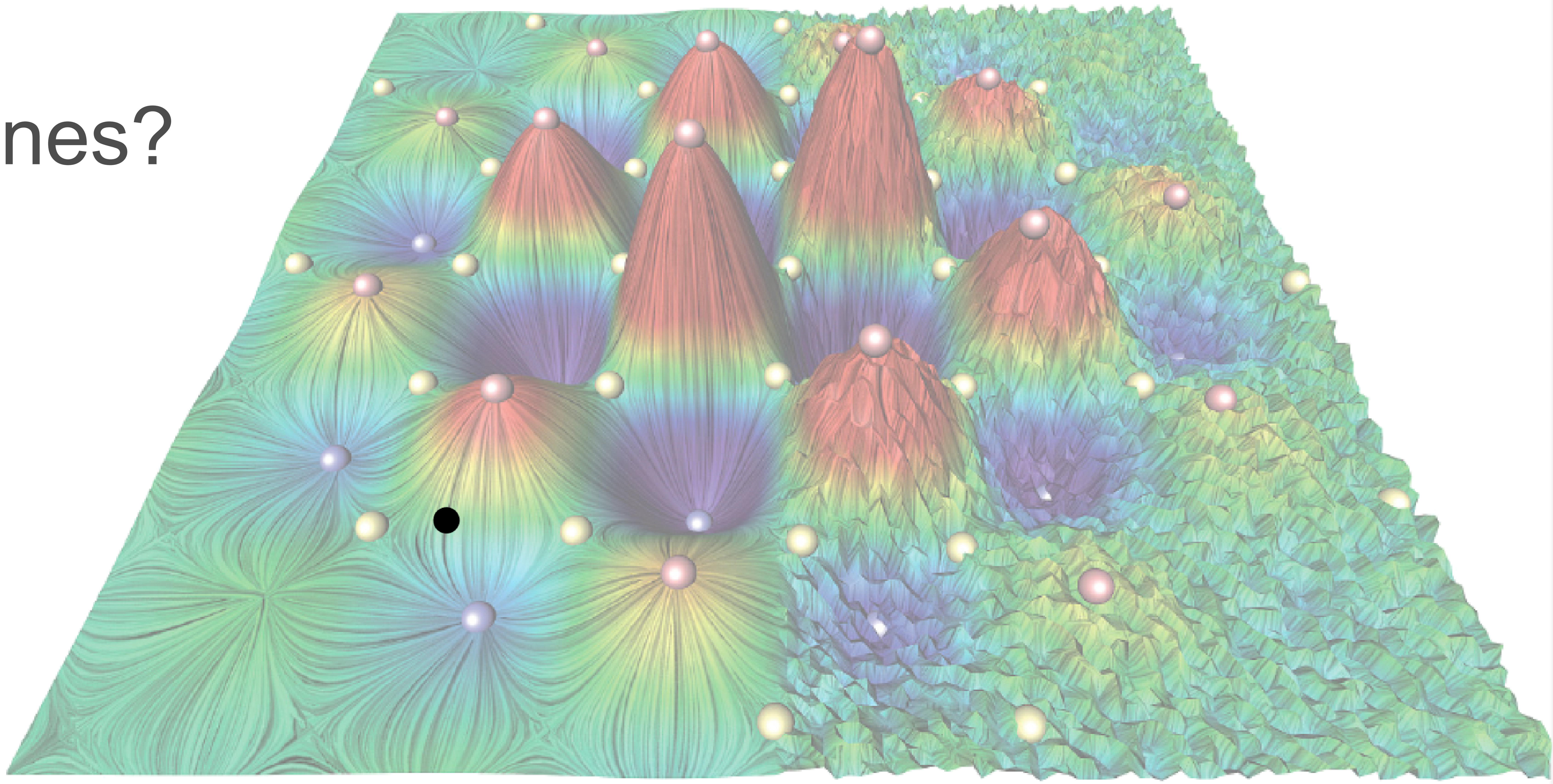

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- Critical points of a vector field
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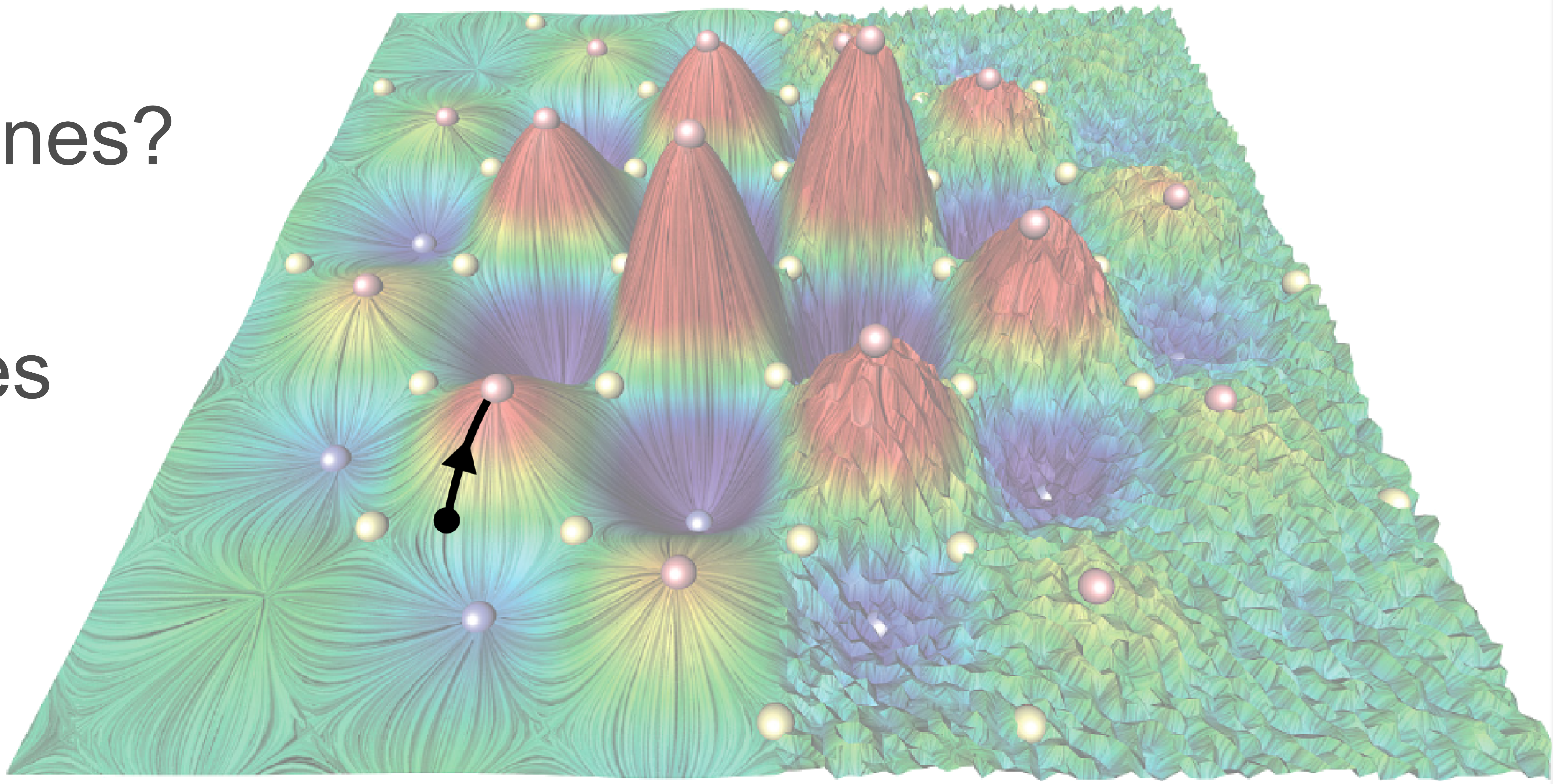
- What's the relation to streamlines?
  - On closed domains
  - Critical points are streamlines extremities



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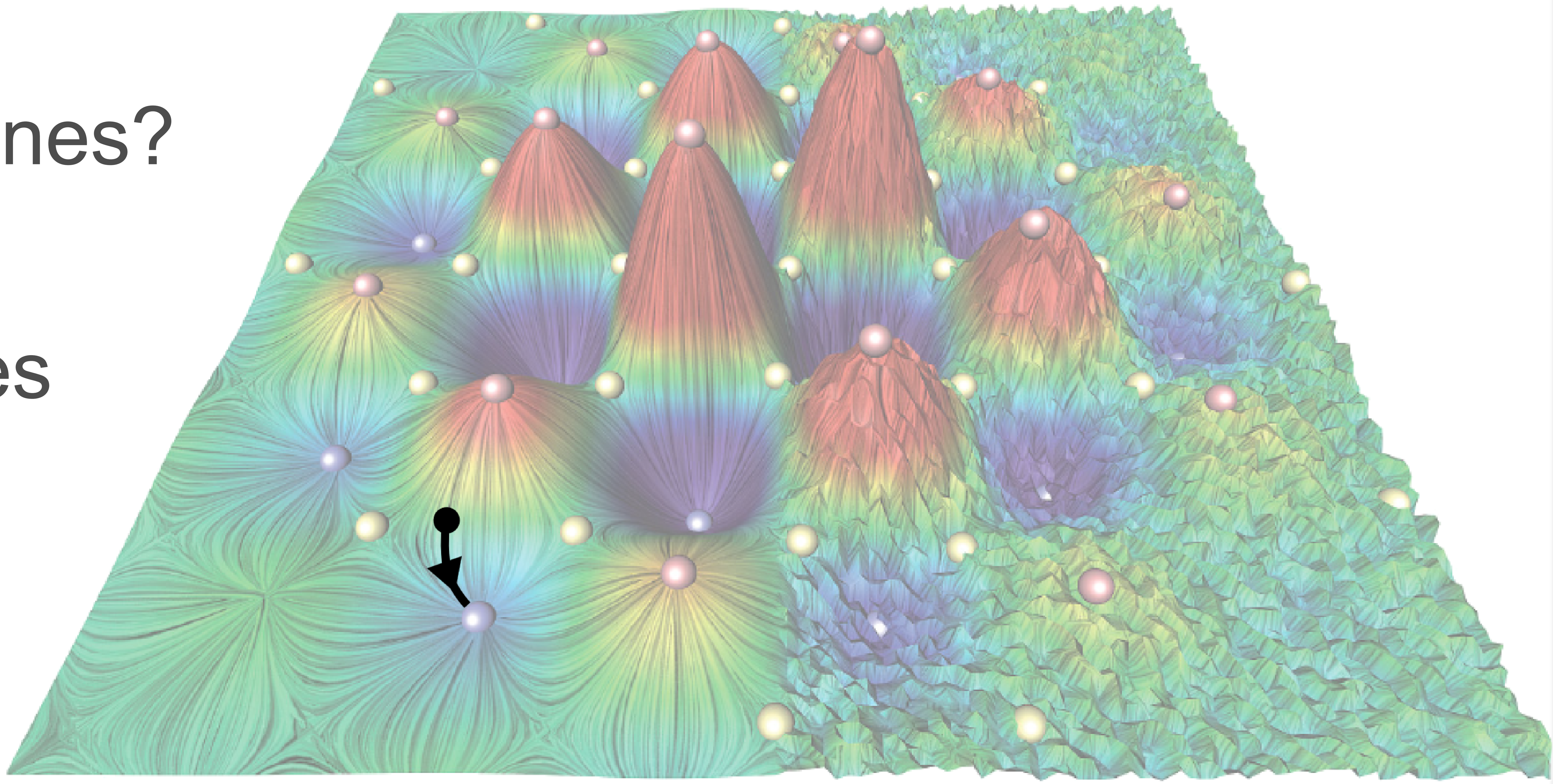

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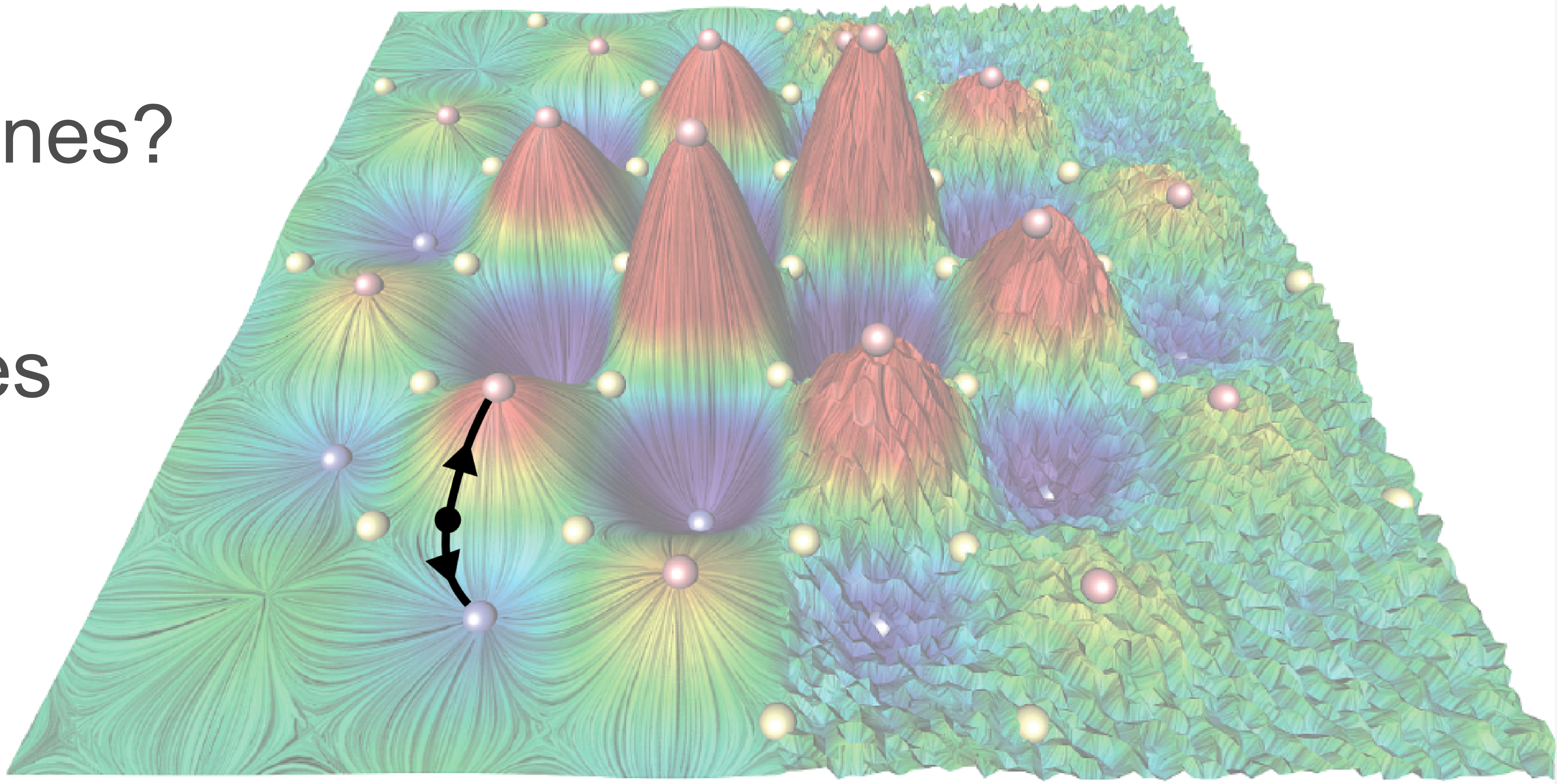

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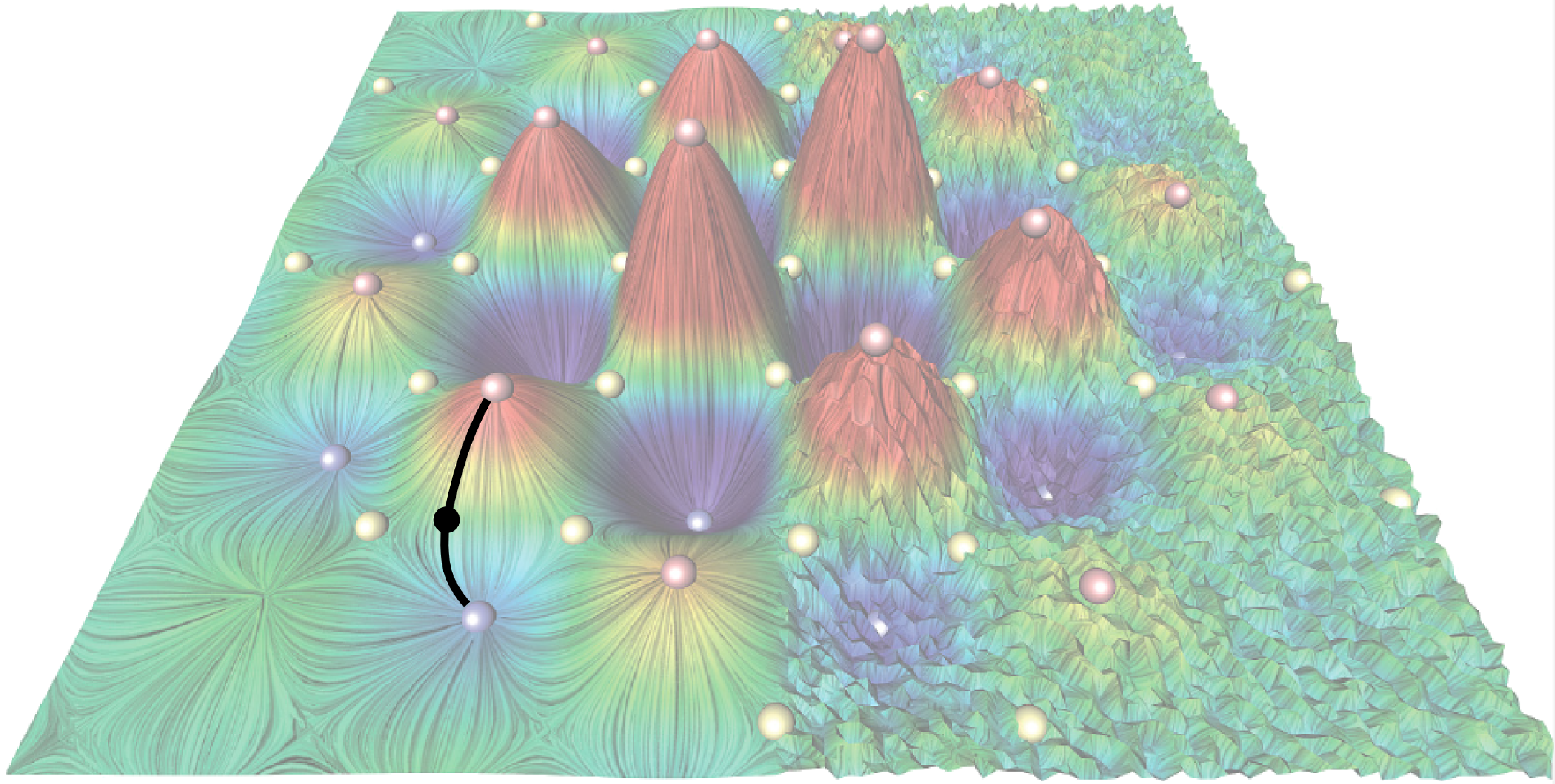
Gradient field topology

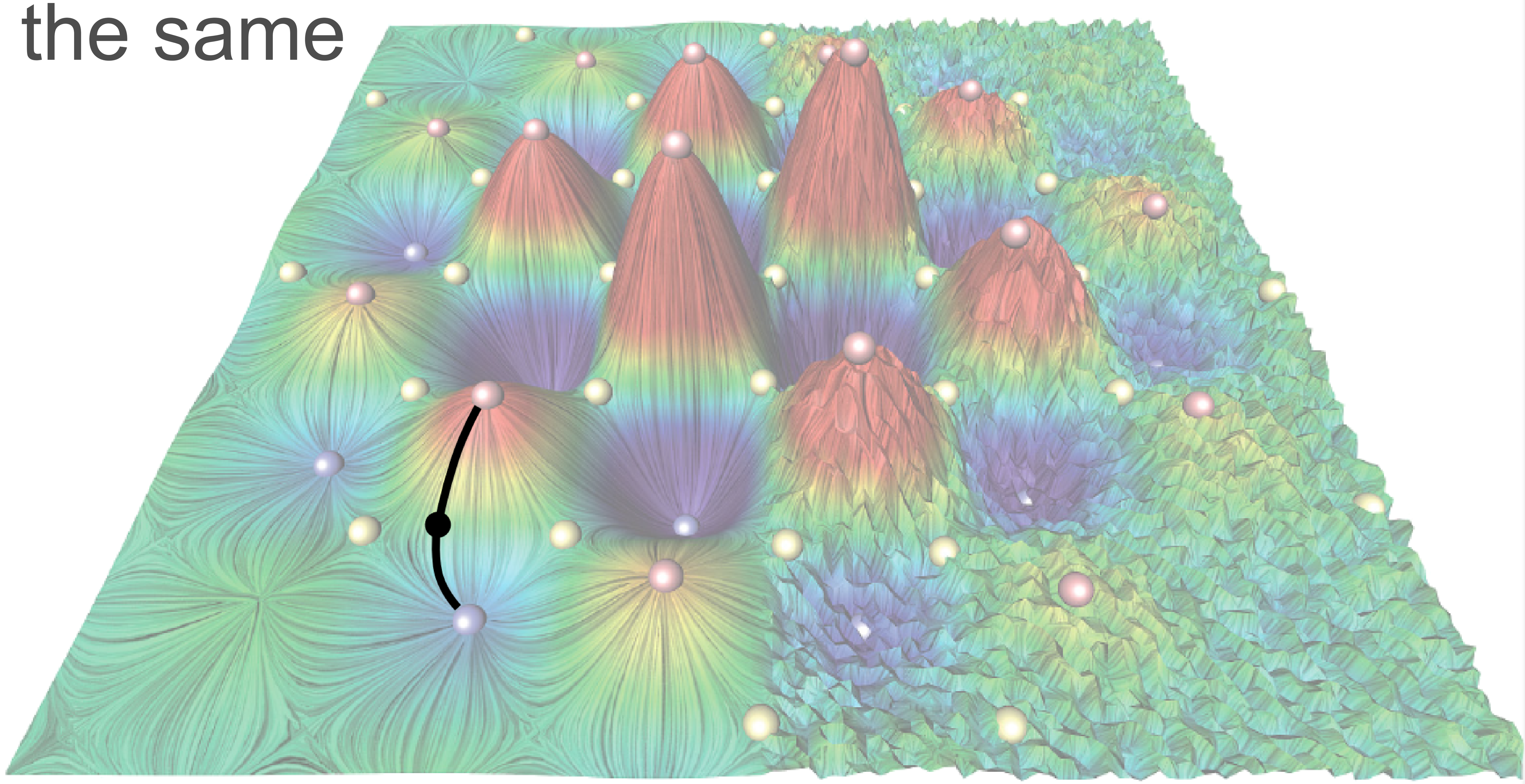
- Understanding the structure of the critical points



[Reininghaus11]

Gradient field topology

- Understanding the structure of the critical points
- Several streamlines can have the same extremities



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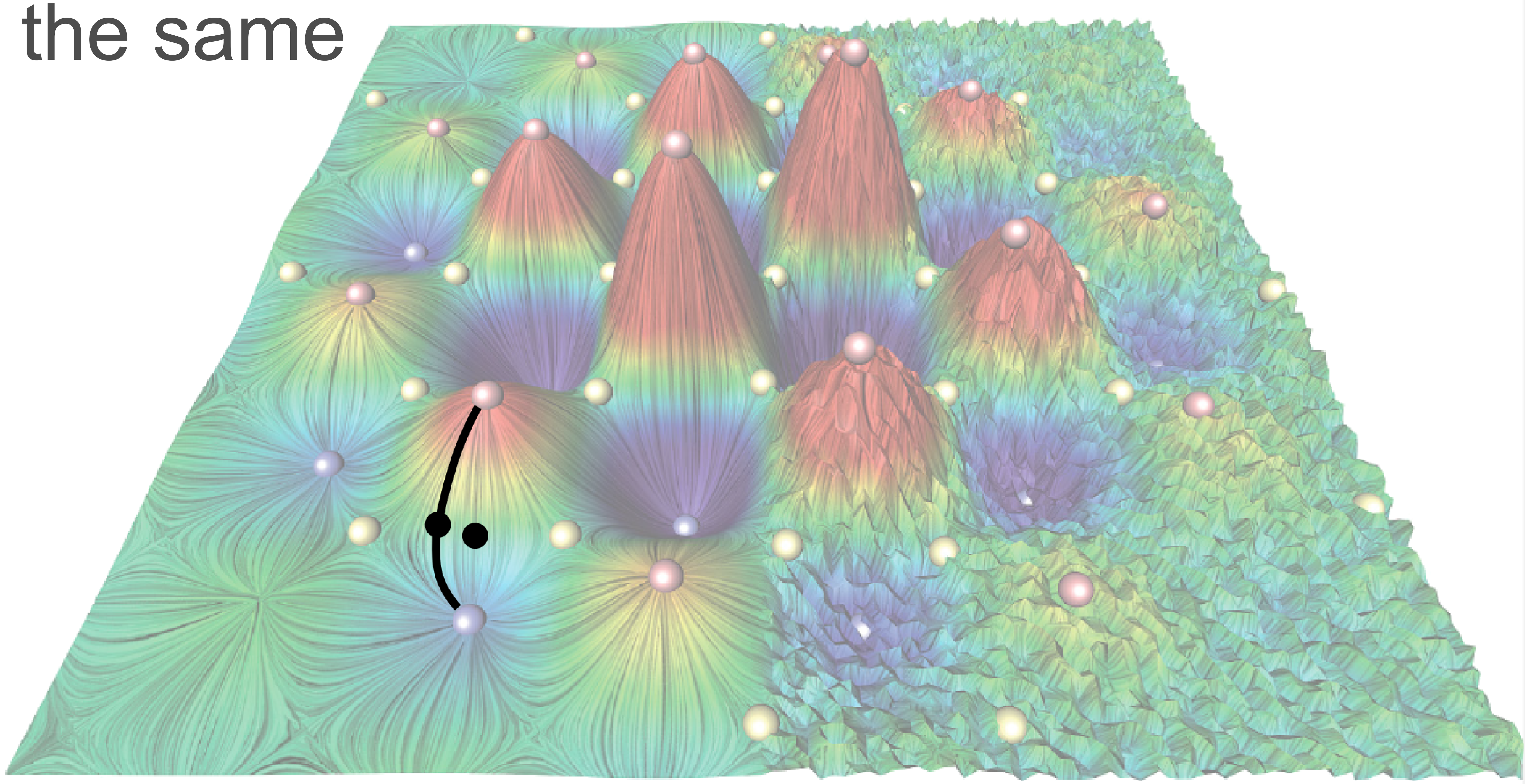

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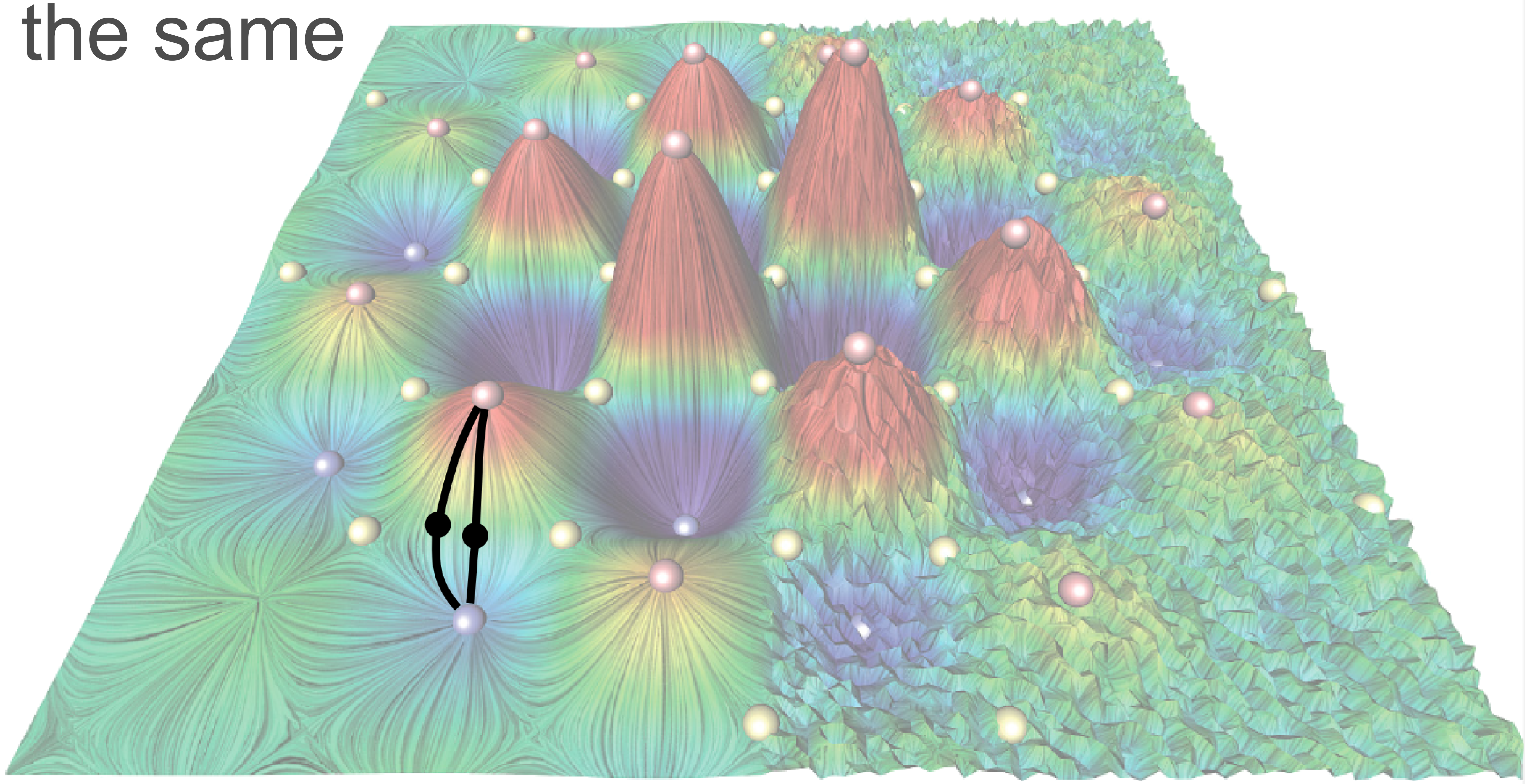

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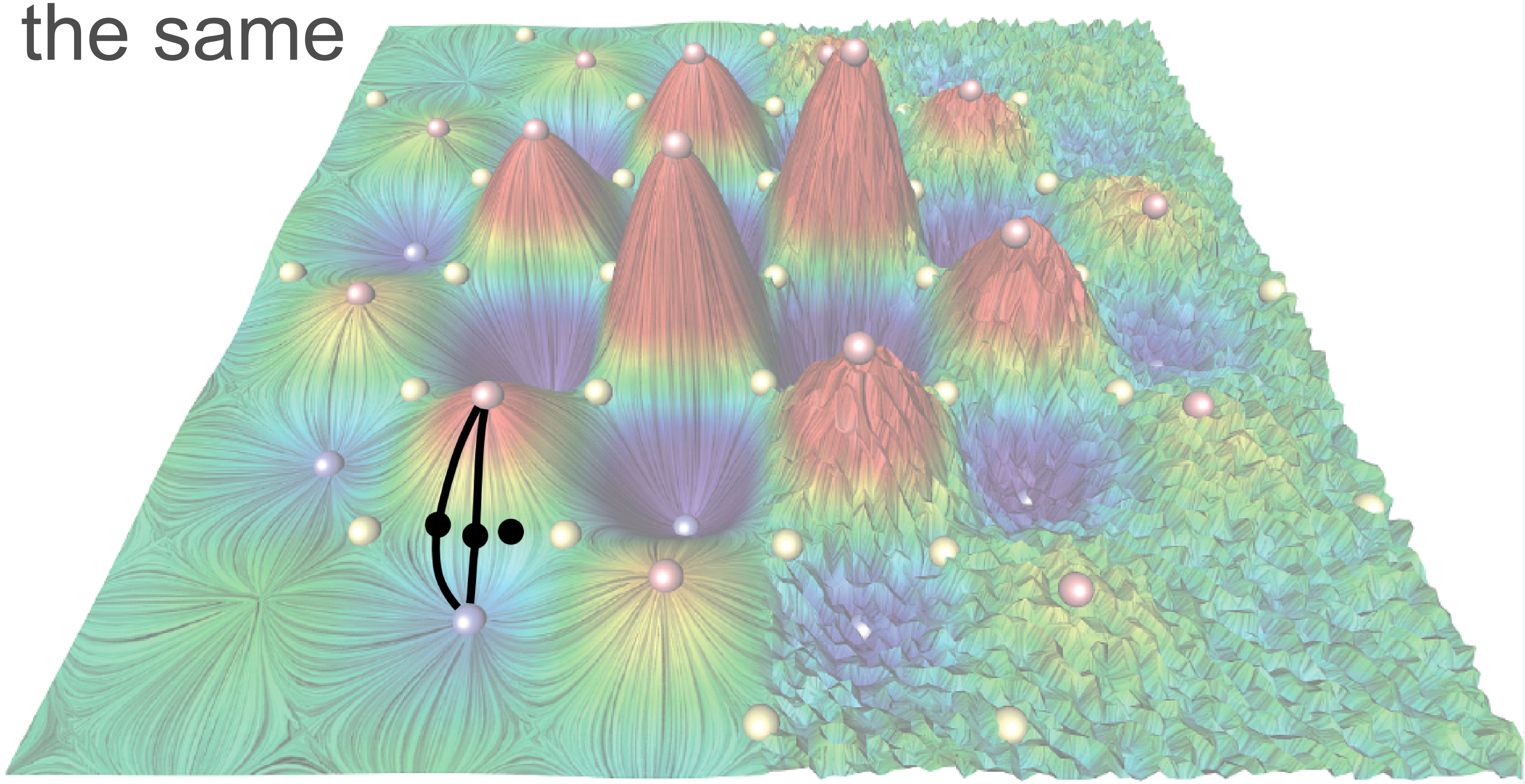

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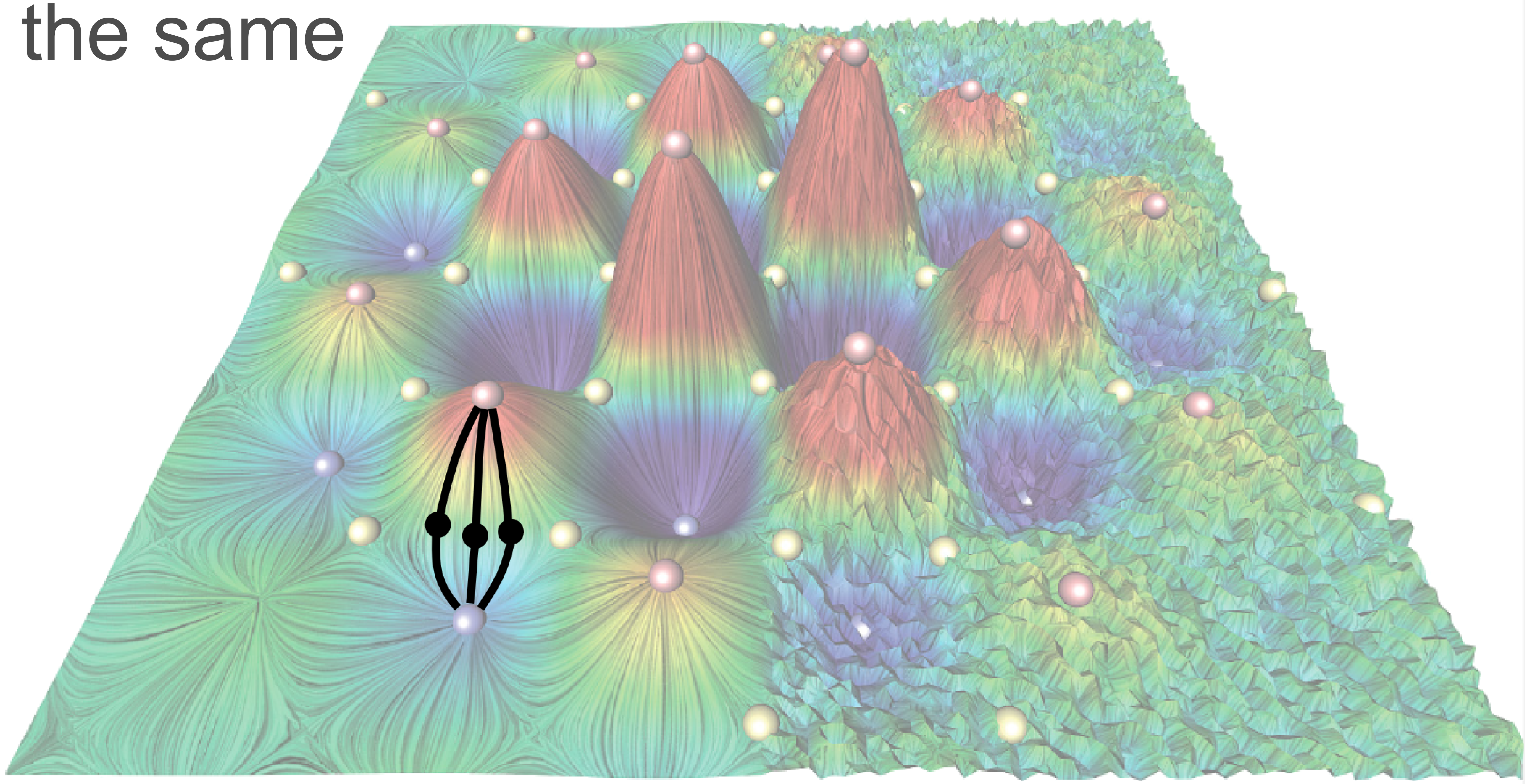
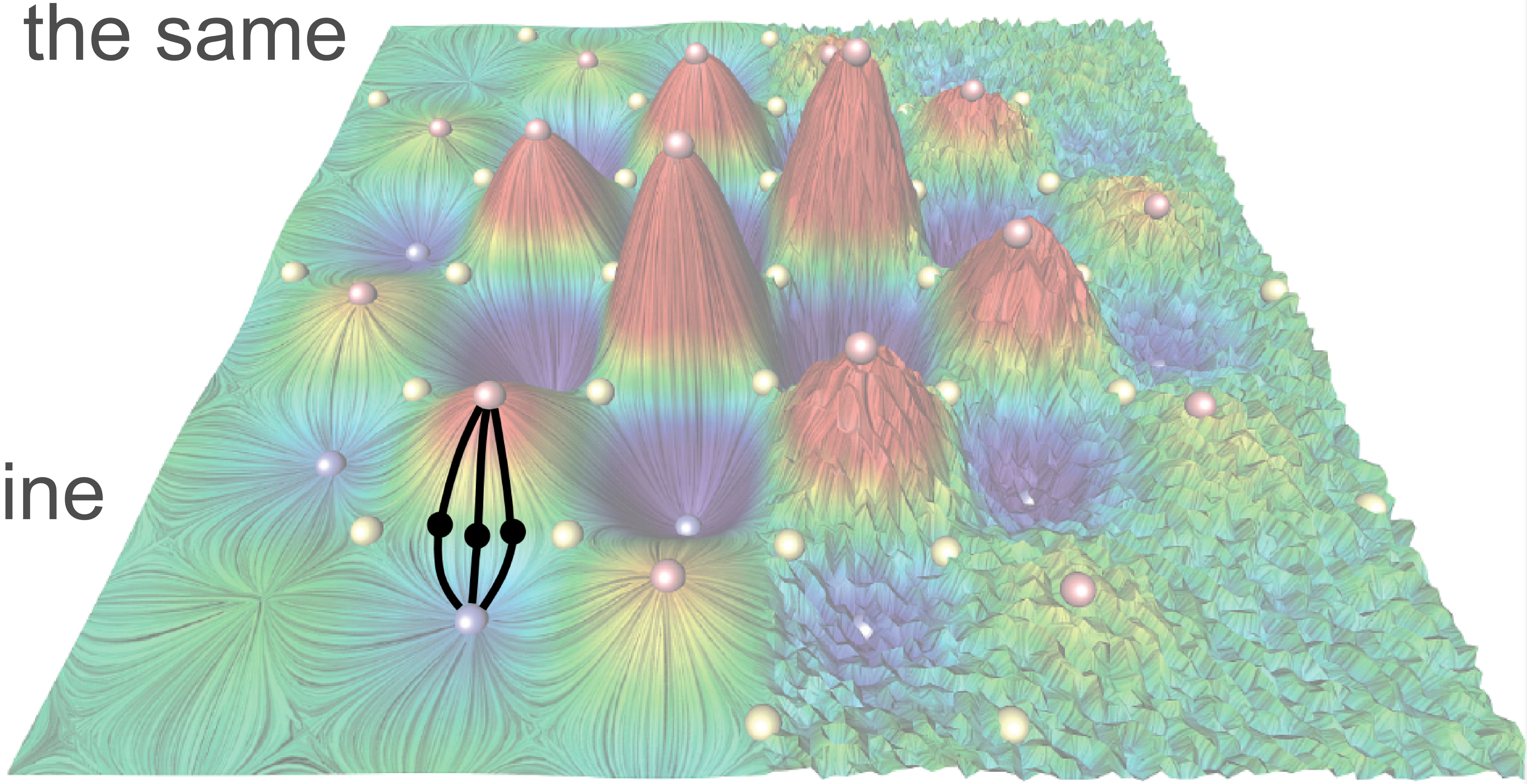


[Reininghaus11]

Gradient field topology

- Understanding the structure of the critical points
- Several streamlines can have the same extremities
- Equivalence relation
  - All the points whose streamline shares identical extremities



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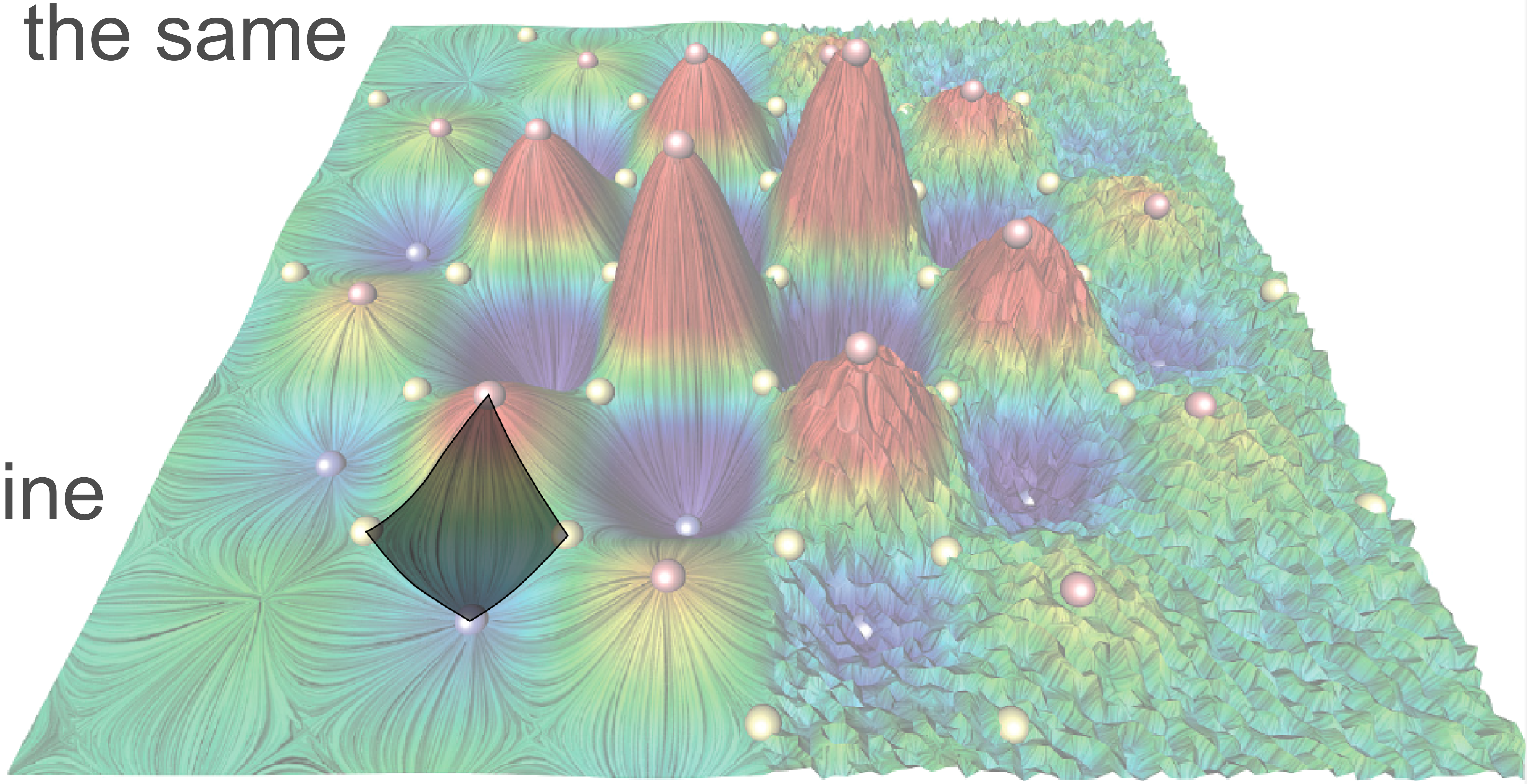
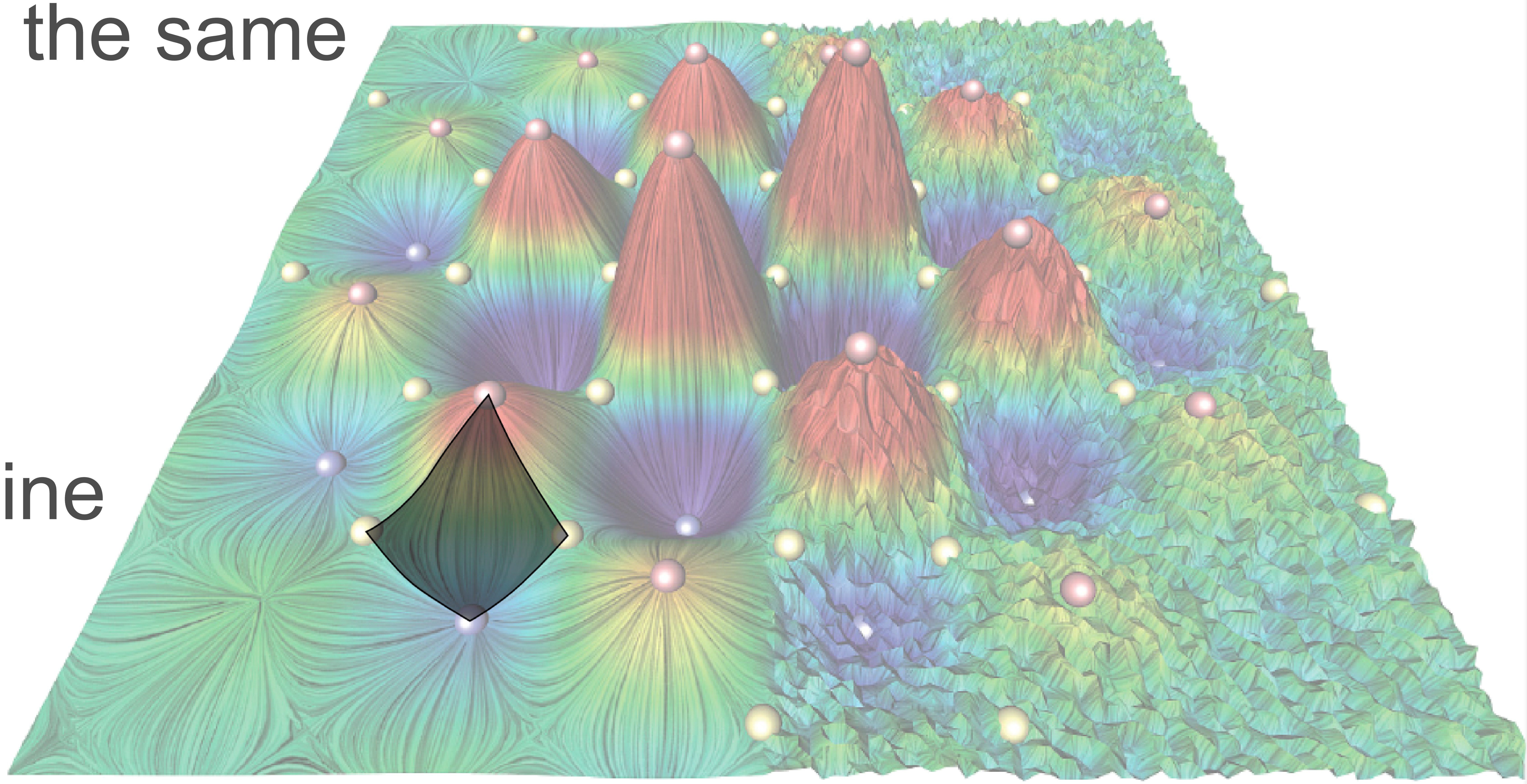


[Reininghaus11]

Gradient field topology

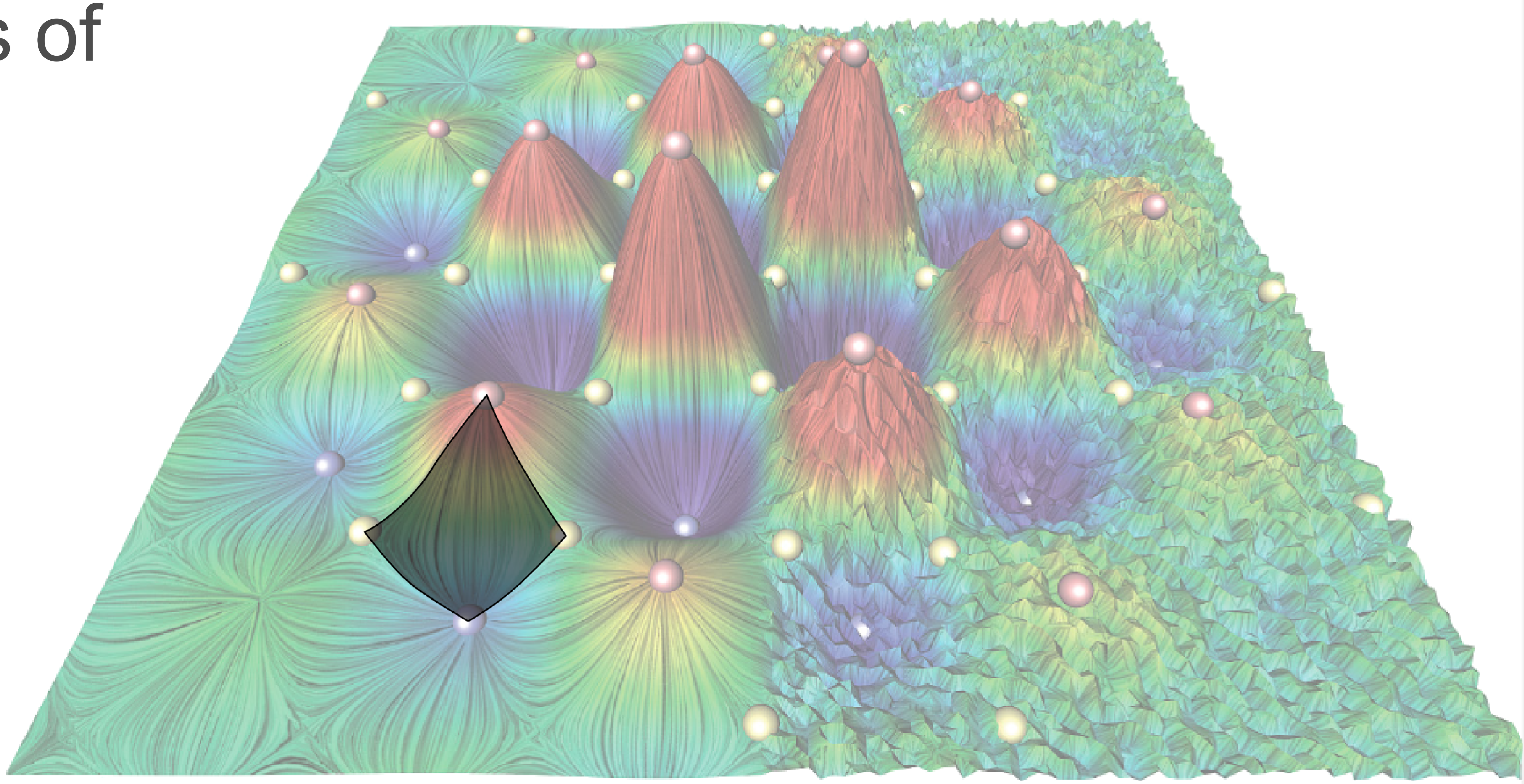
- Understanding the structure of the critical points
- Several streamlines can have the same extremities
- Equivalence relation
  - All the points whose streamline shares identical extremities
  - **Notion of flow cell**



[Reininghaus11]

Gradient field topology

- Understanding the structure of the critical points
- Now, what are the boundaries of the flow cells?



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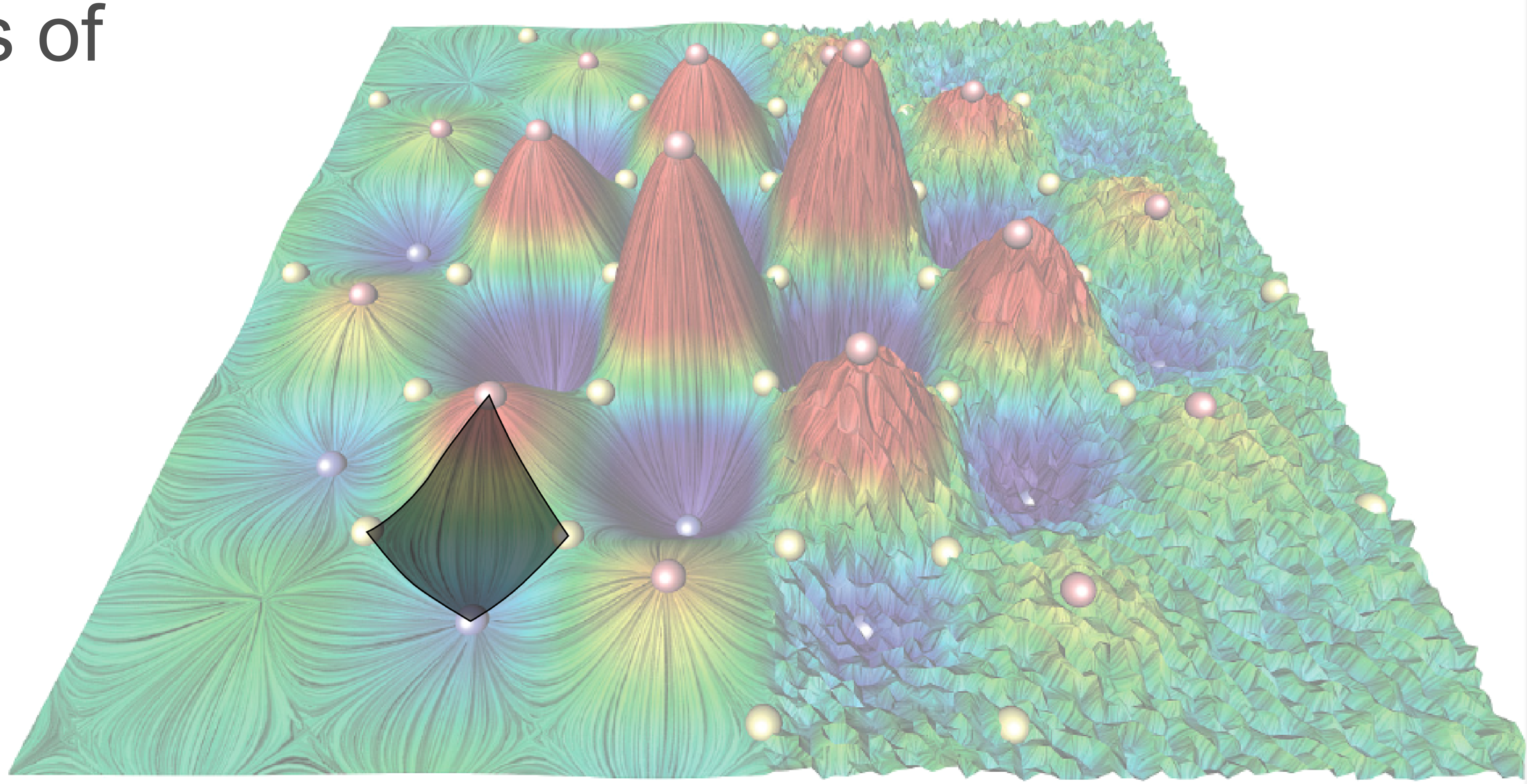

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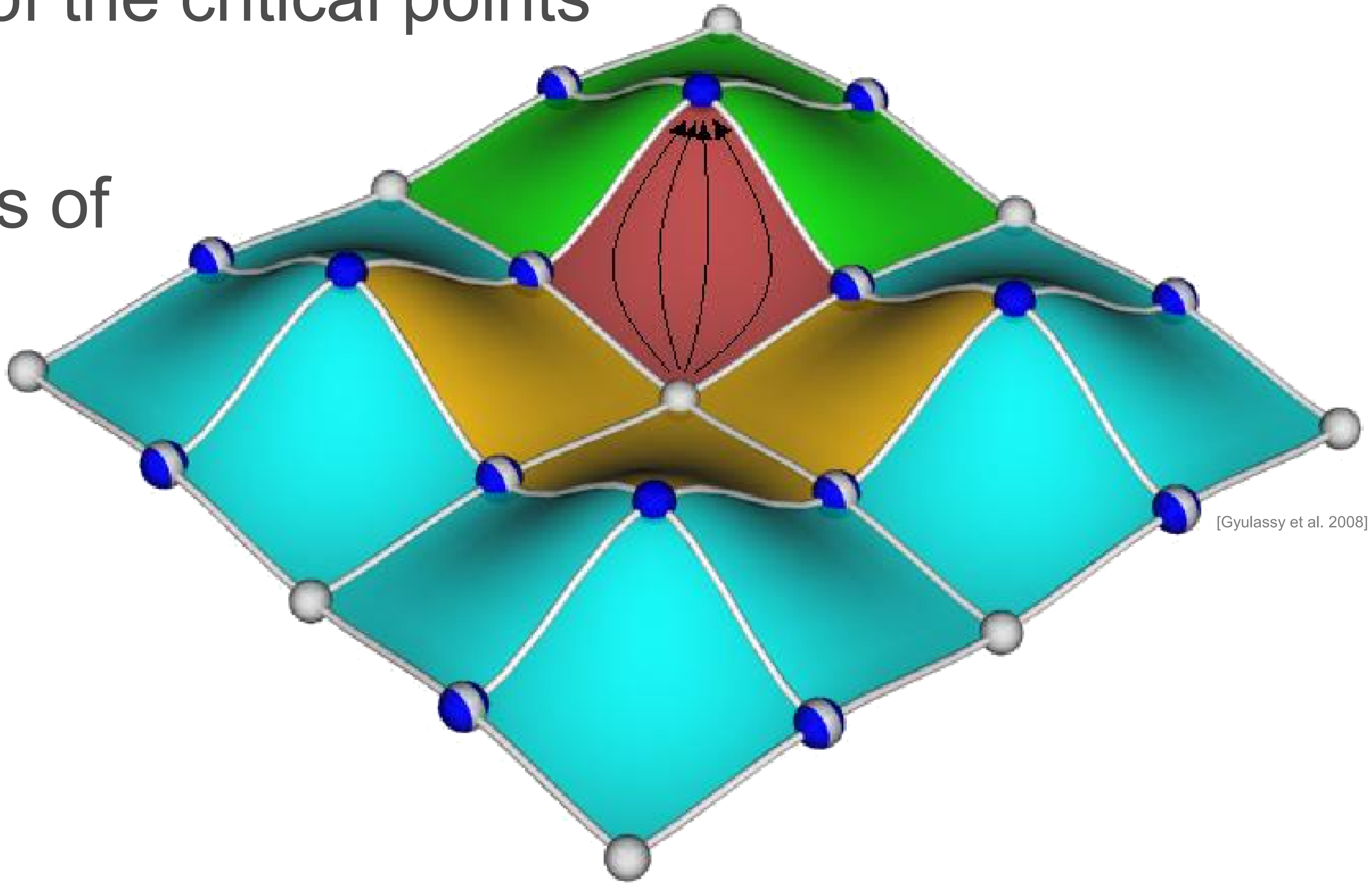
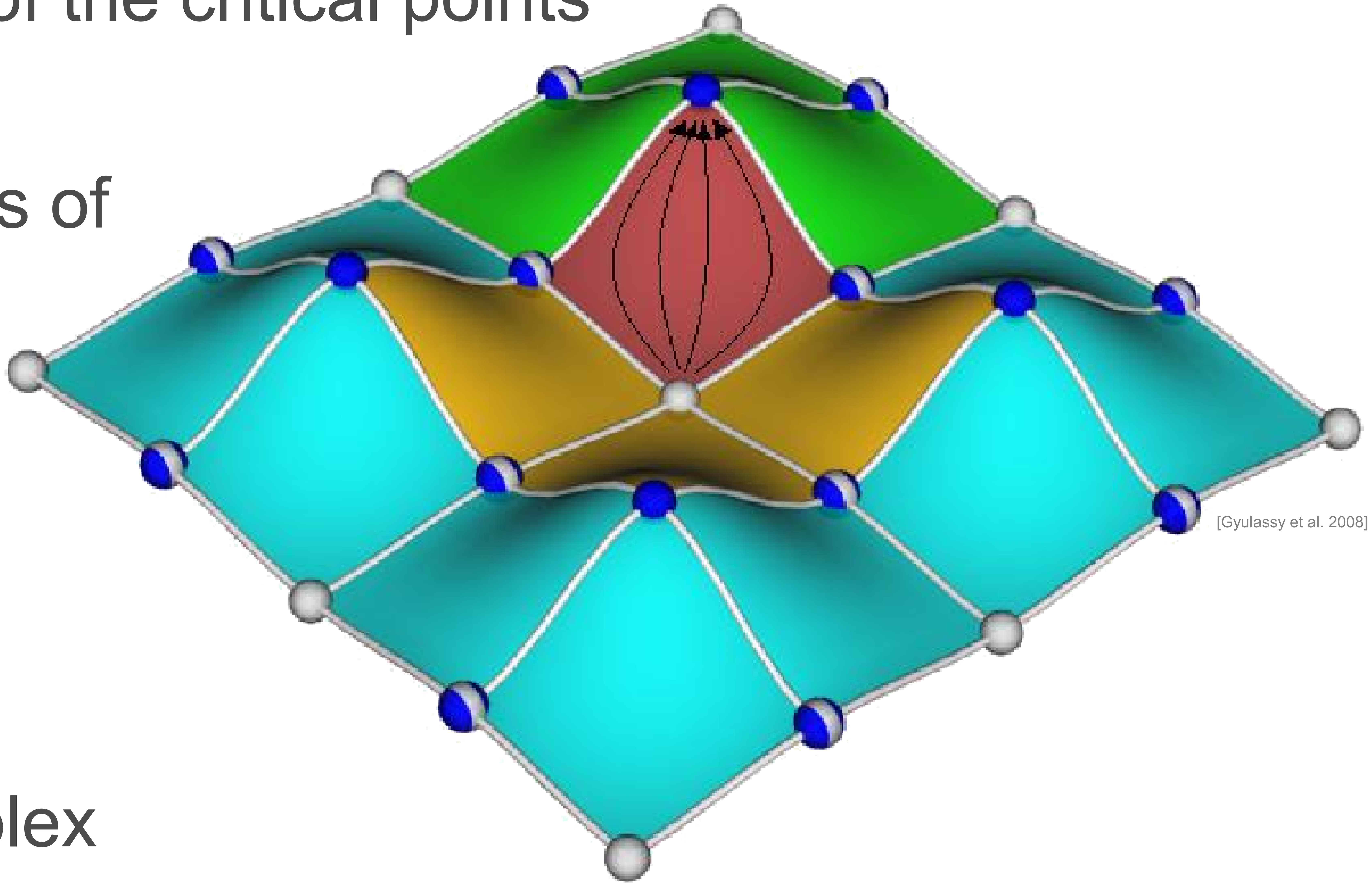


[Gyulassy et al. 2008]

Gradient field topology

- Understanding the structure of the critical points
- Now, what are the boundaries of the flow cells?
- Streamlines between critical points
- Notion of Morse-Smale complex



[Gyulassy et al. 2008]

Morse-Smale complex

- Notion of ascending manifold of a critical point p

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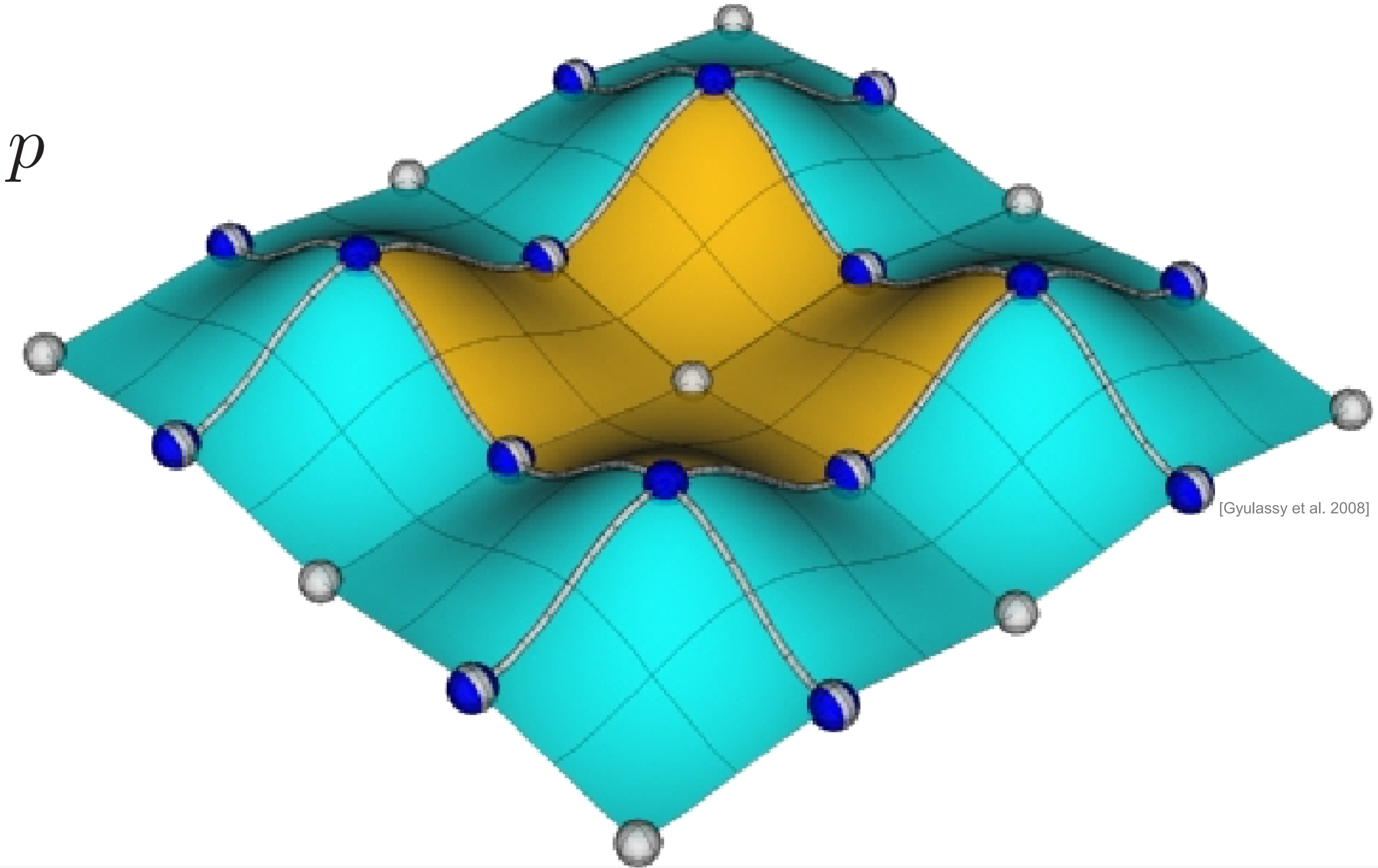

[Gyulassy et al. 2008]

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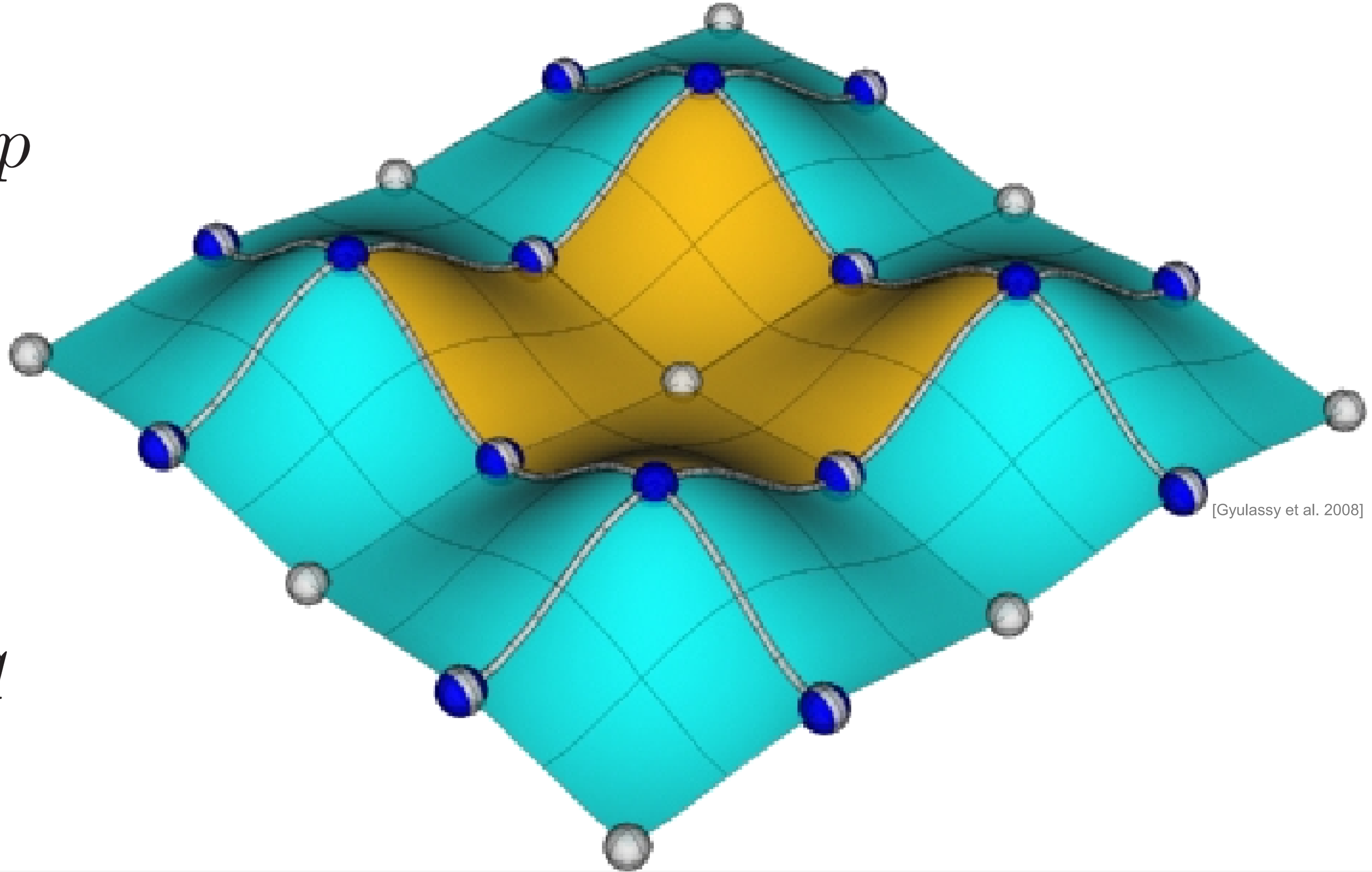

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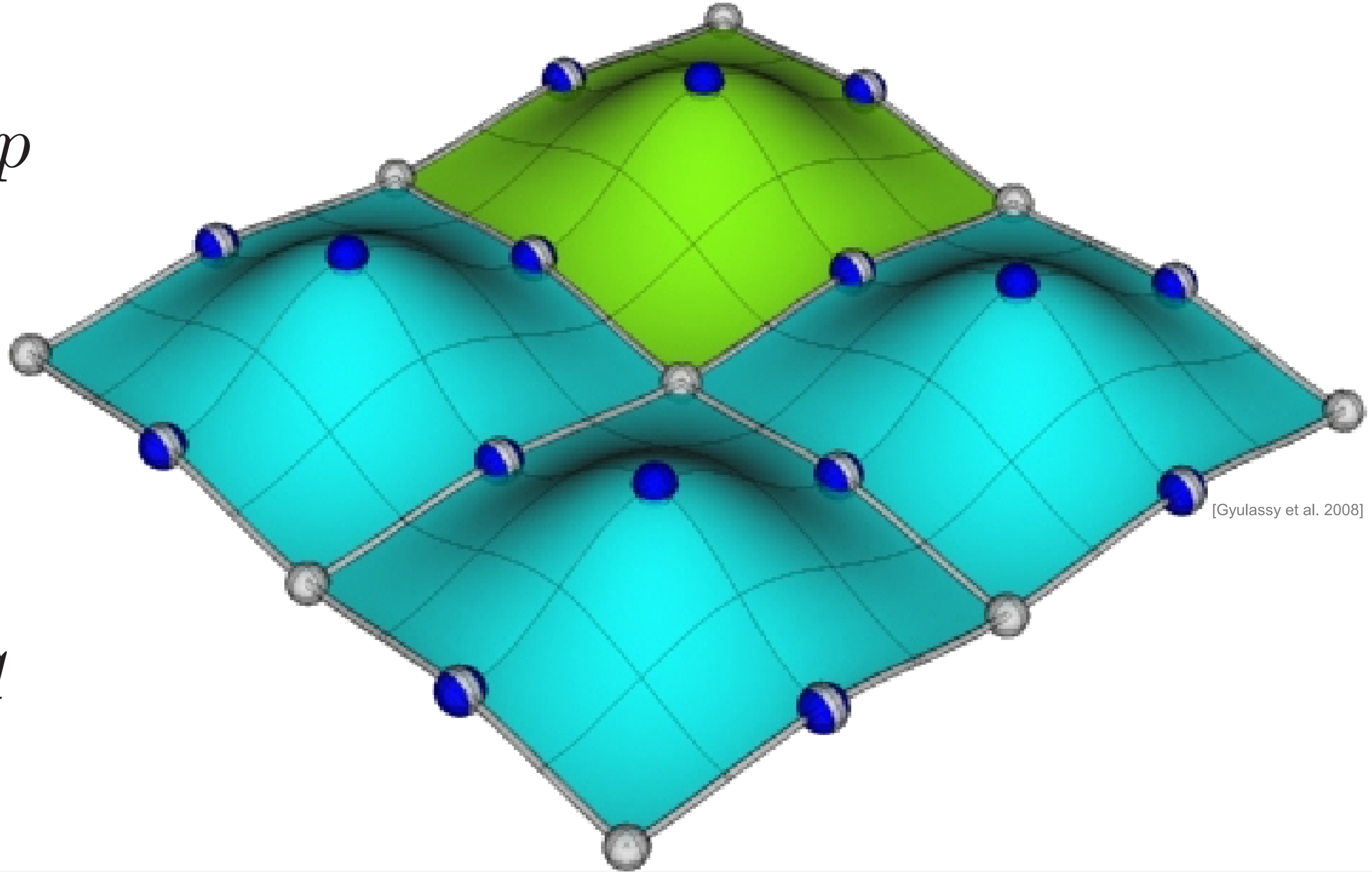

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Morse-Smale complex

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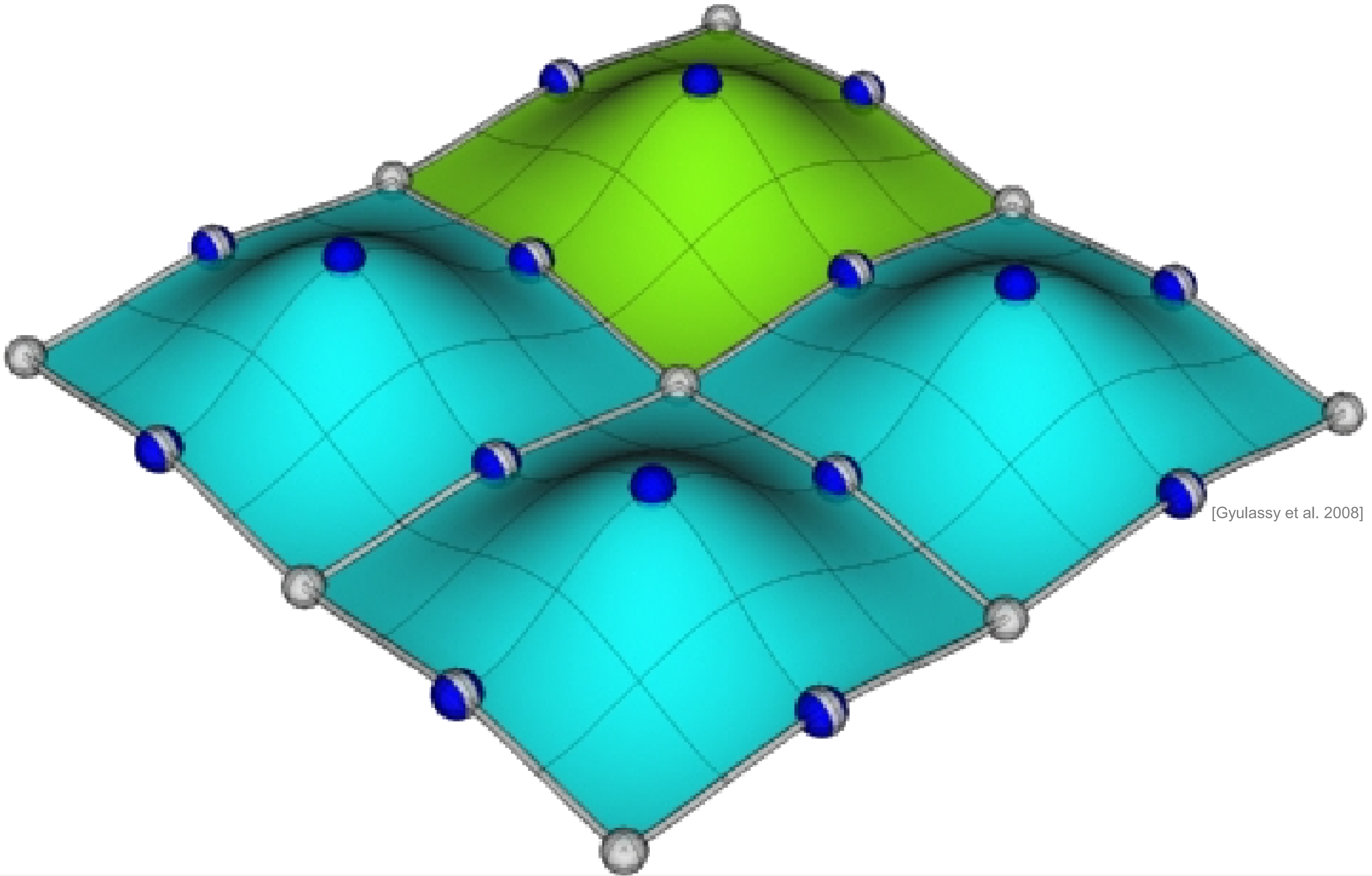
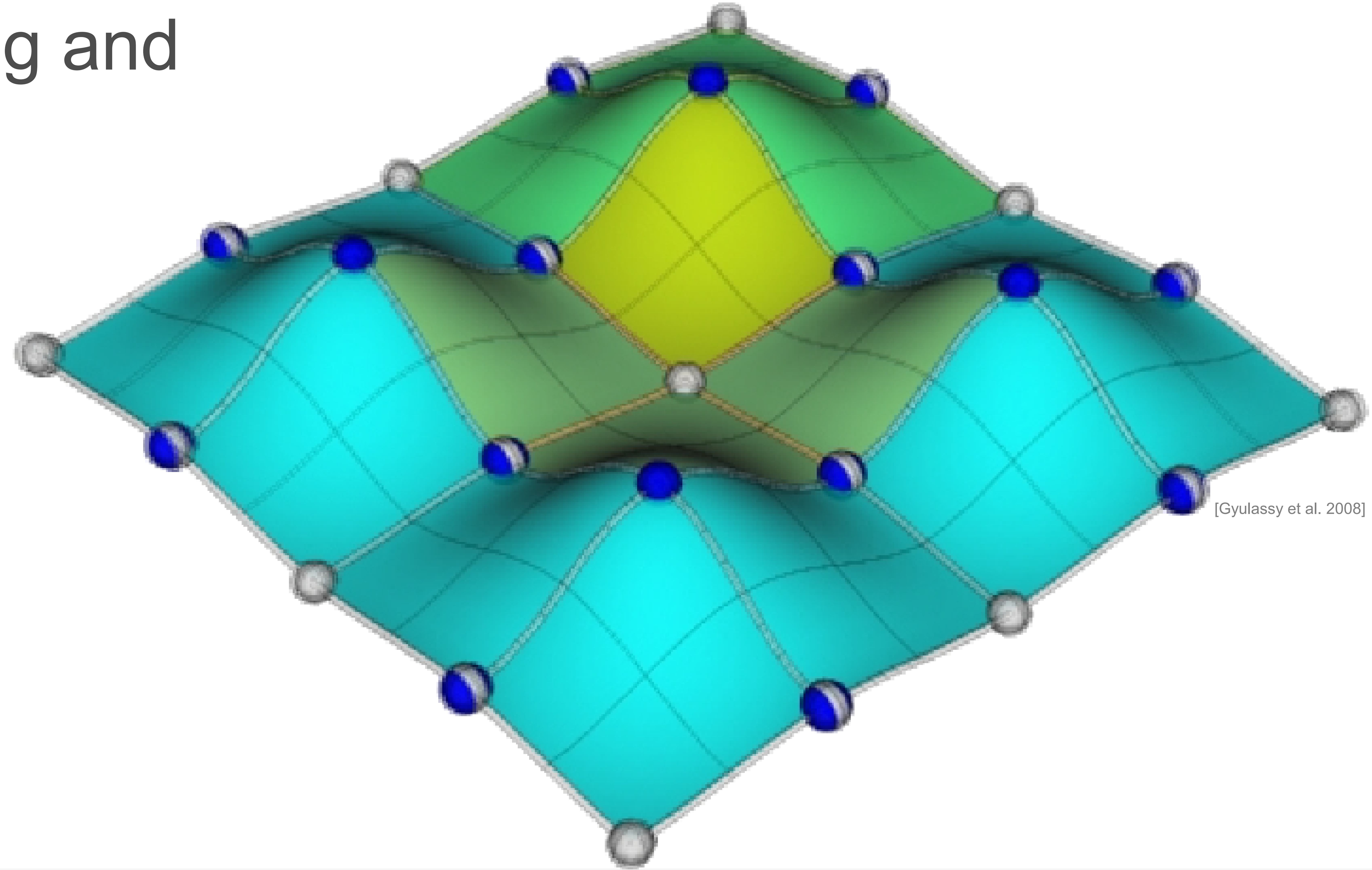


[Gyulassy et al. 2008]

Morse-Smale complex

- Notion of Morse-Smale Complex
  - Intersection of the ascending and descending manifolds



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Morse-Smale complex

- Notion of Morse-Smale Complex
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- Composed of:
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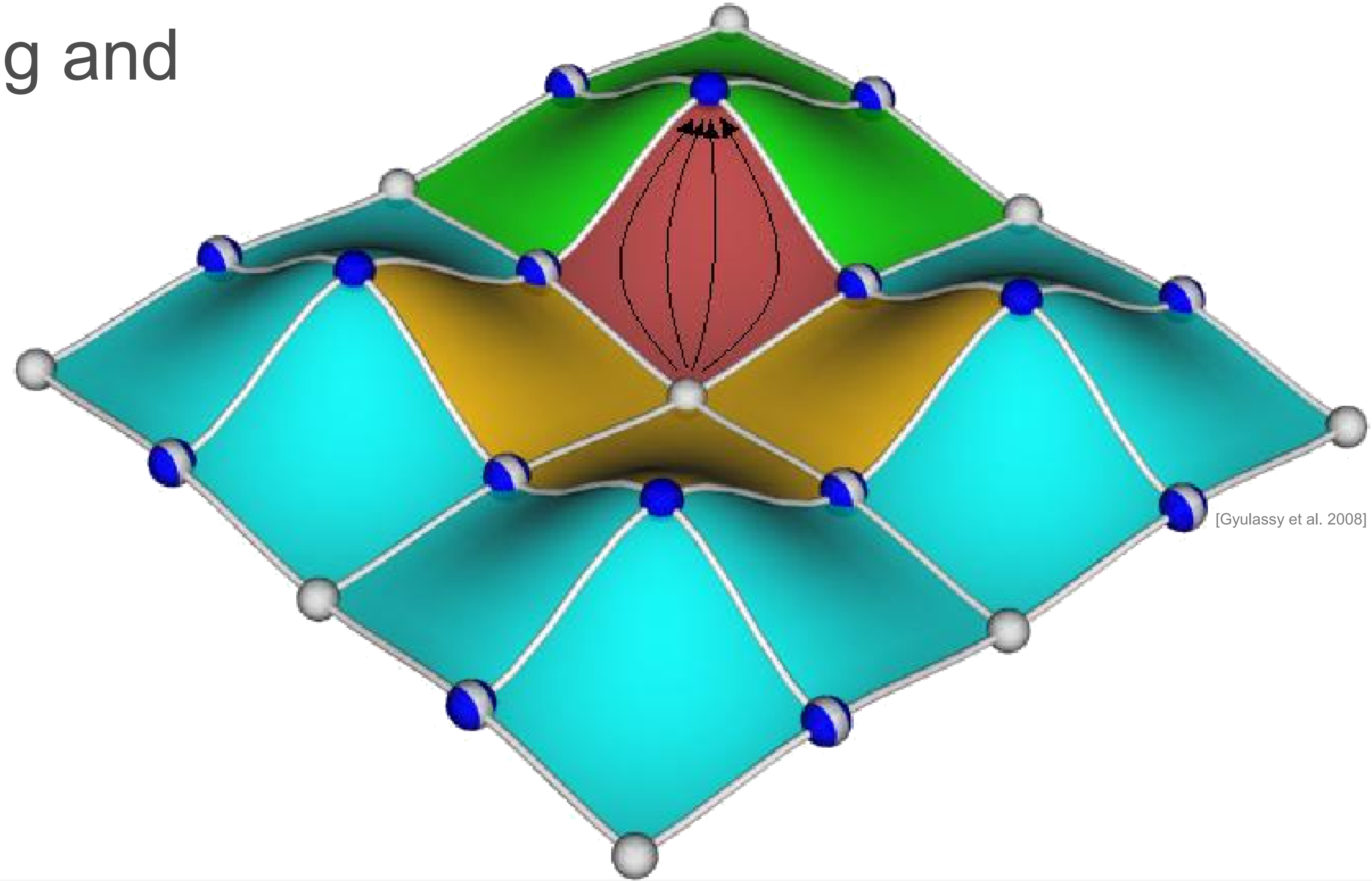

[Gyulassy et al. 2008]

Morse-Smale complex

- Notion of Morse-Smale Complex
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- Composed of:
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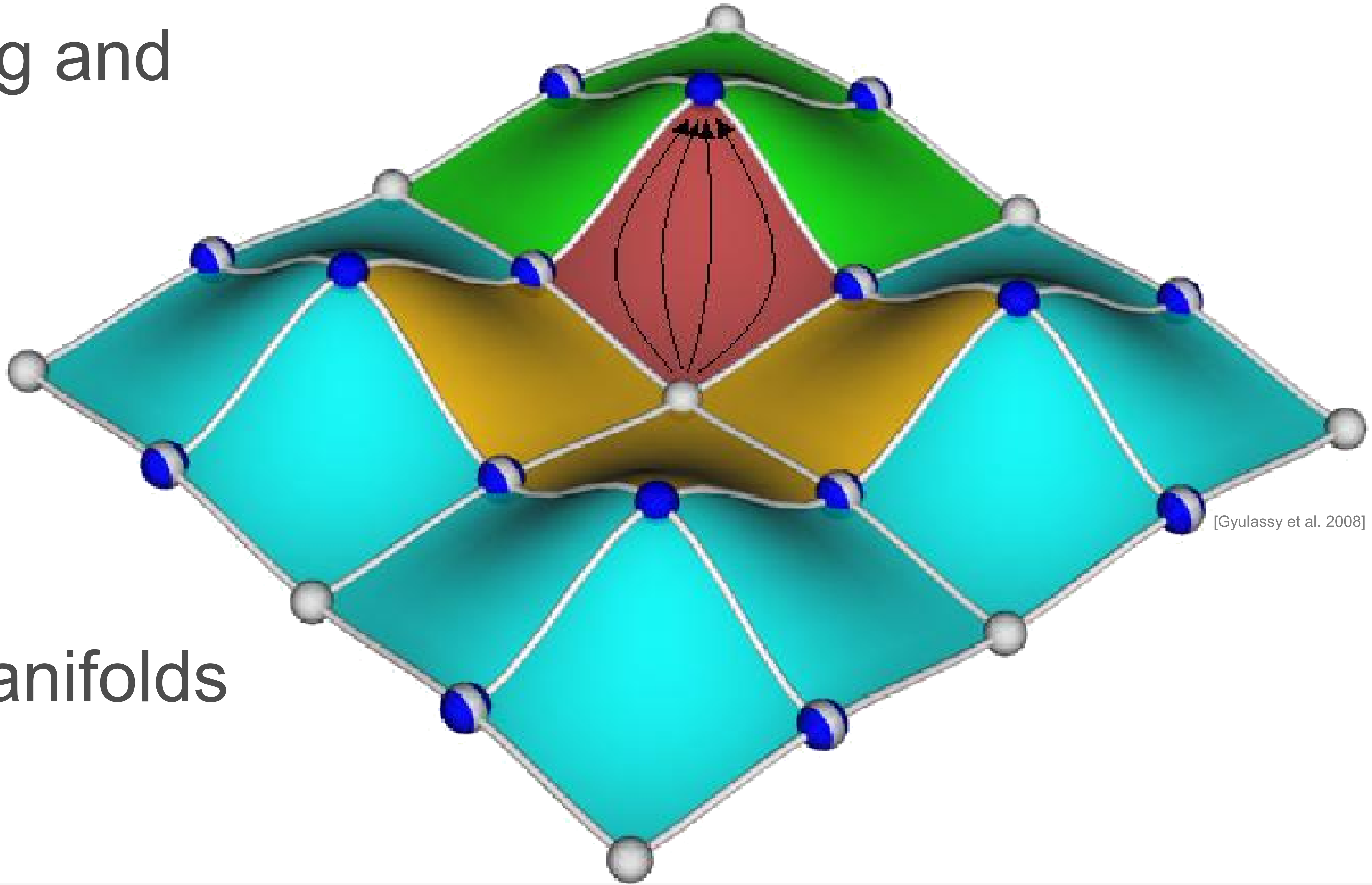
  - Ascending/descending 2-manifolds



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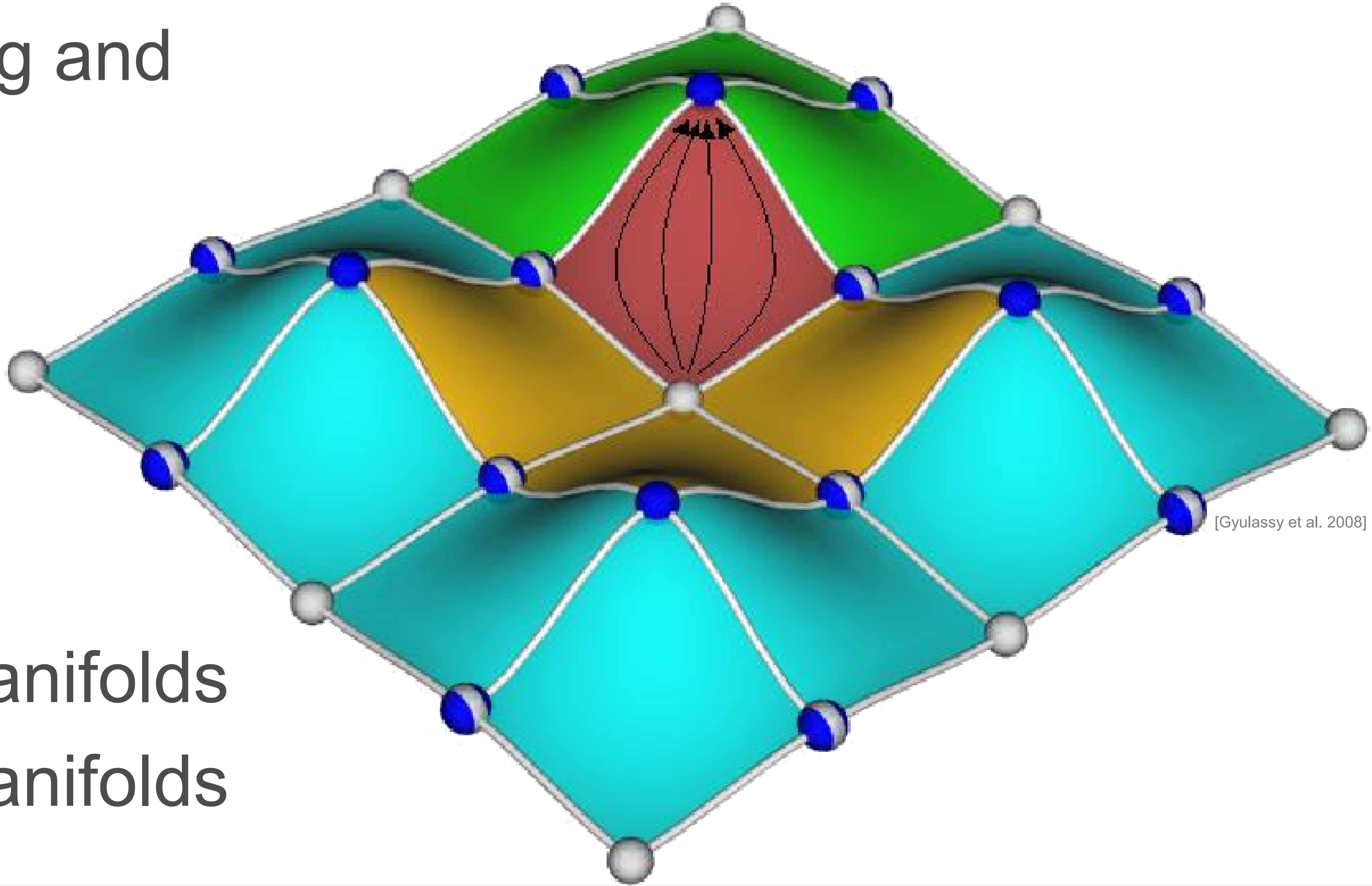
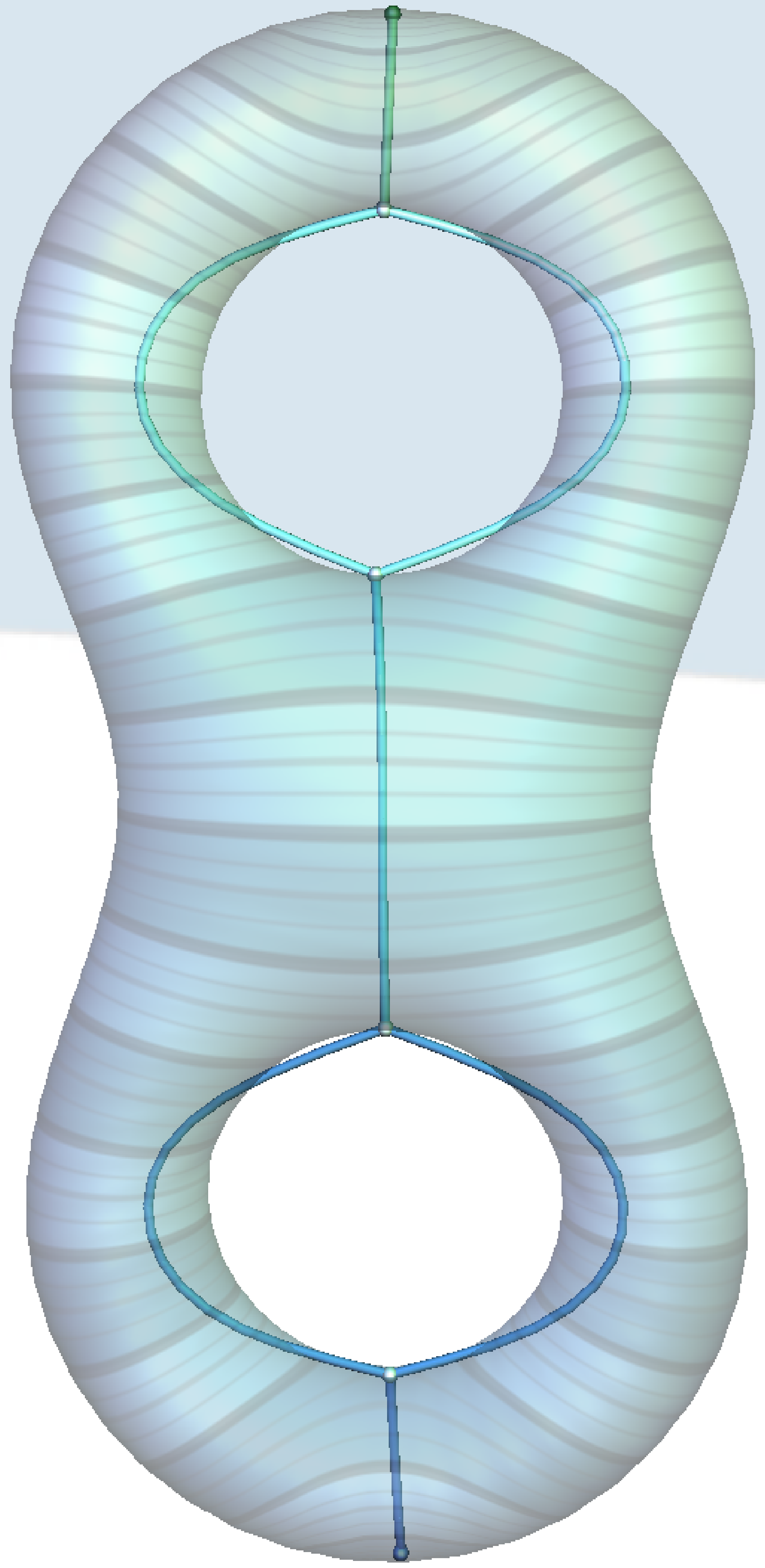


[Gyulassy et al. 2008]

Analogy to scalar fields

- Scalar fields

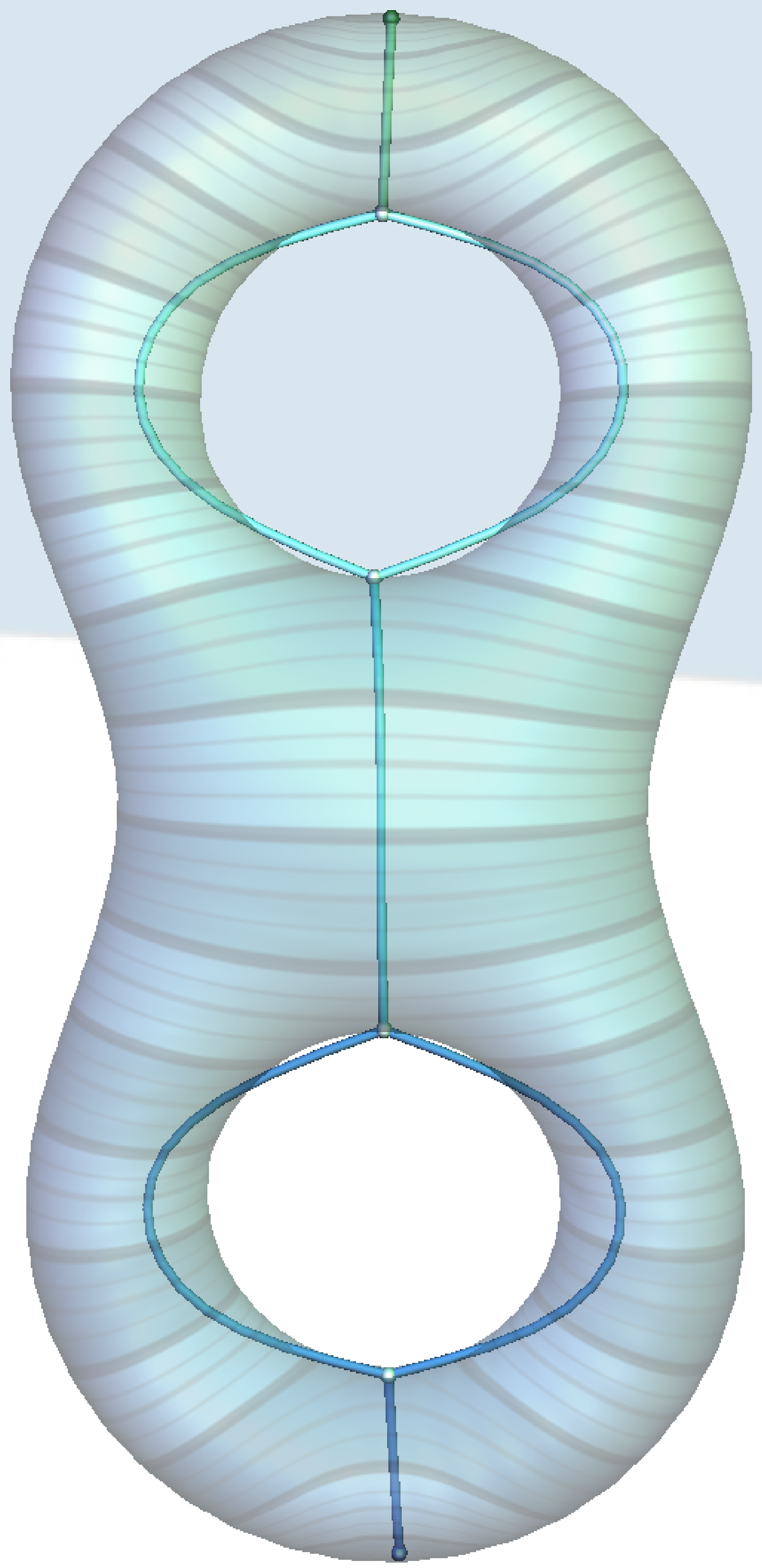
Analogy to scalar fields



- Scalar fields
  - Reeb graphs

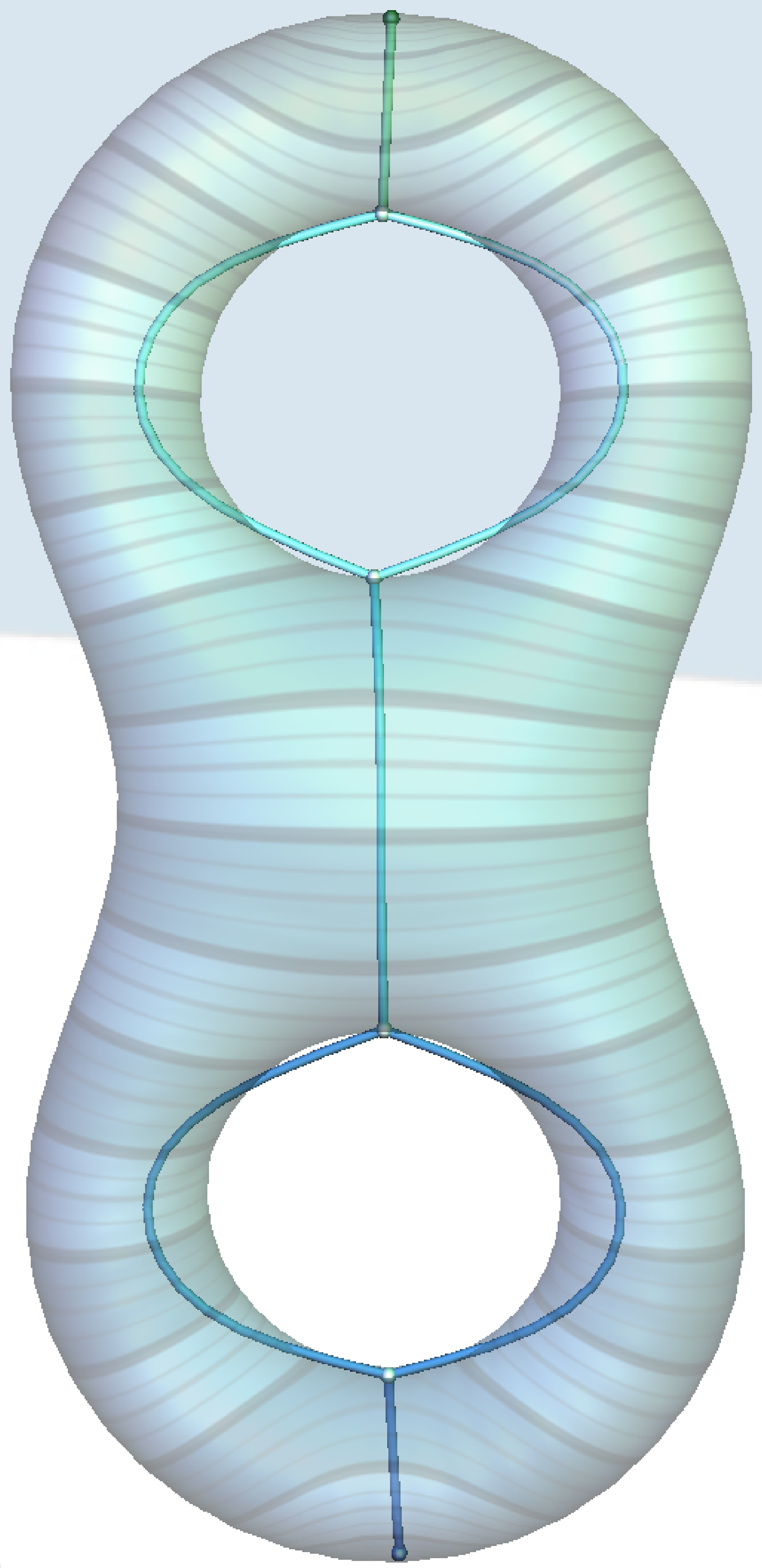
Analogy to scalar fields

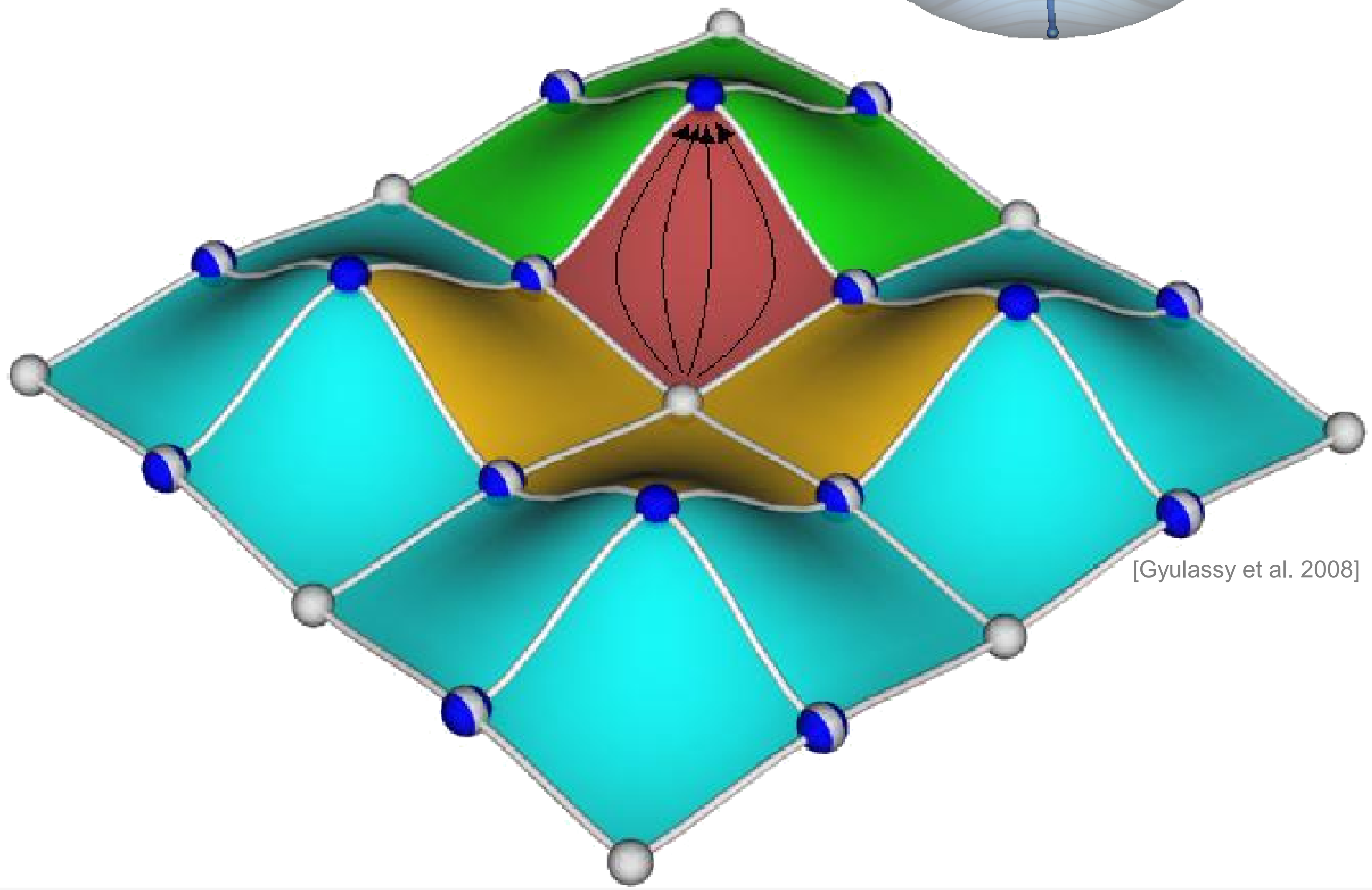
- Scalar fields
  - Reeb graphs
    - Summarize the connectivity of **all** level sets



Analogy to scalar fields

- Scalar fields
  - Reeb graphs
    - Summarize the connectivity of **all** level sets
- Gradient fields
  - Morse-Smale complexes





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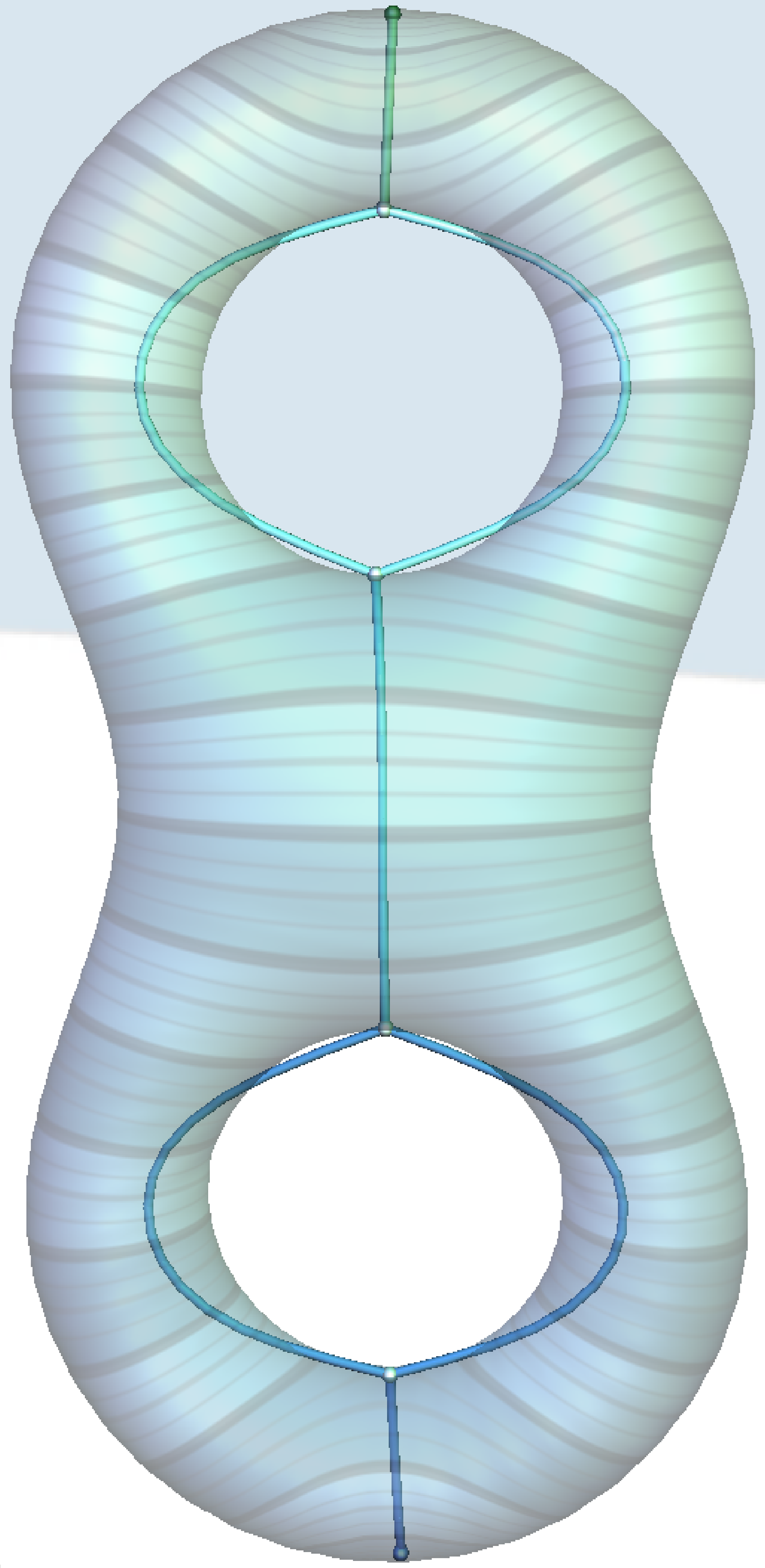
Analogy to scalar fields



- Scalar fields
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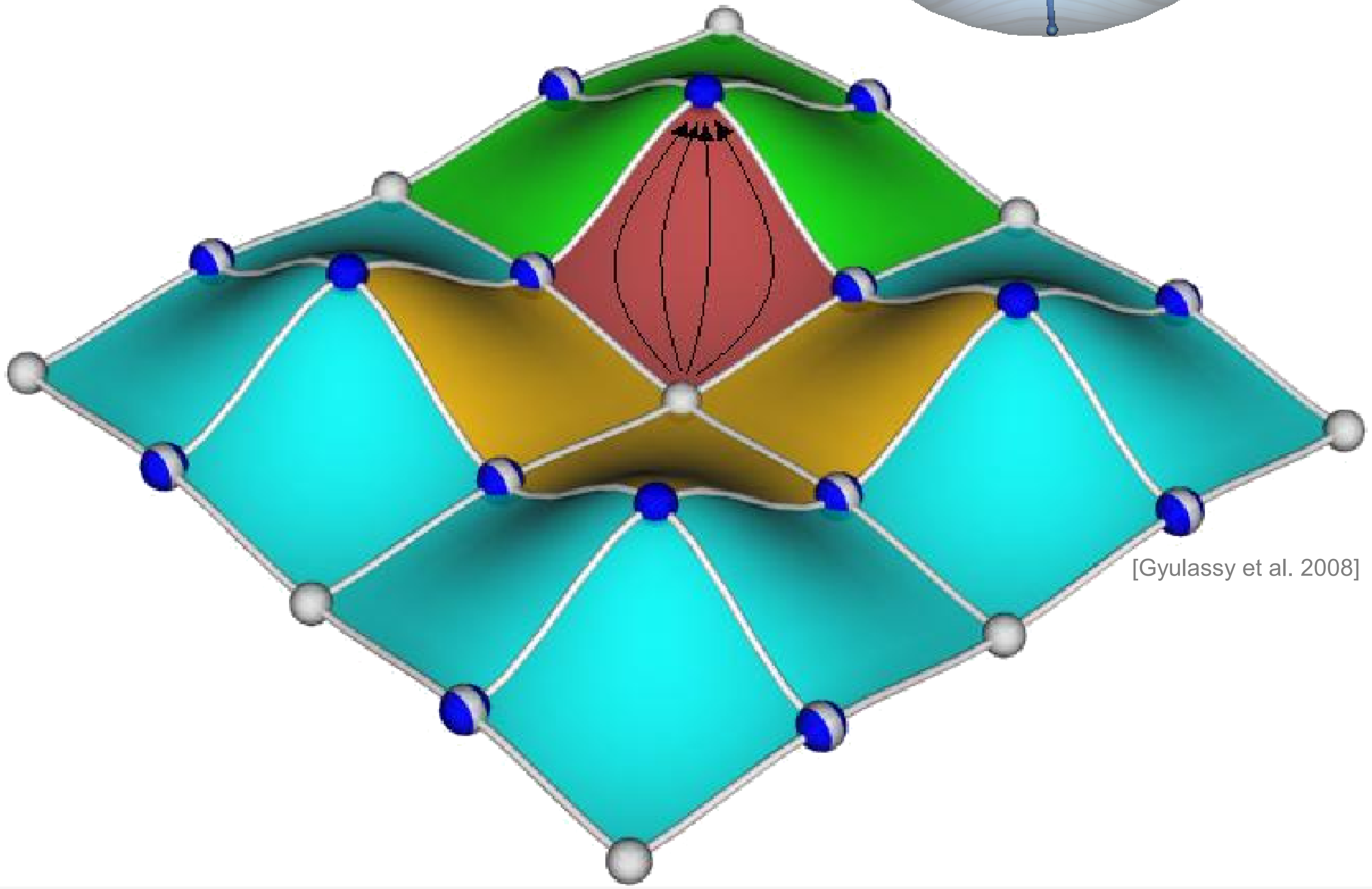
    - Summarize **all** the streamlines



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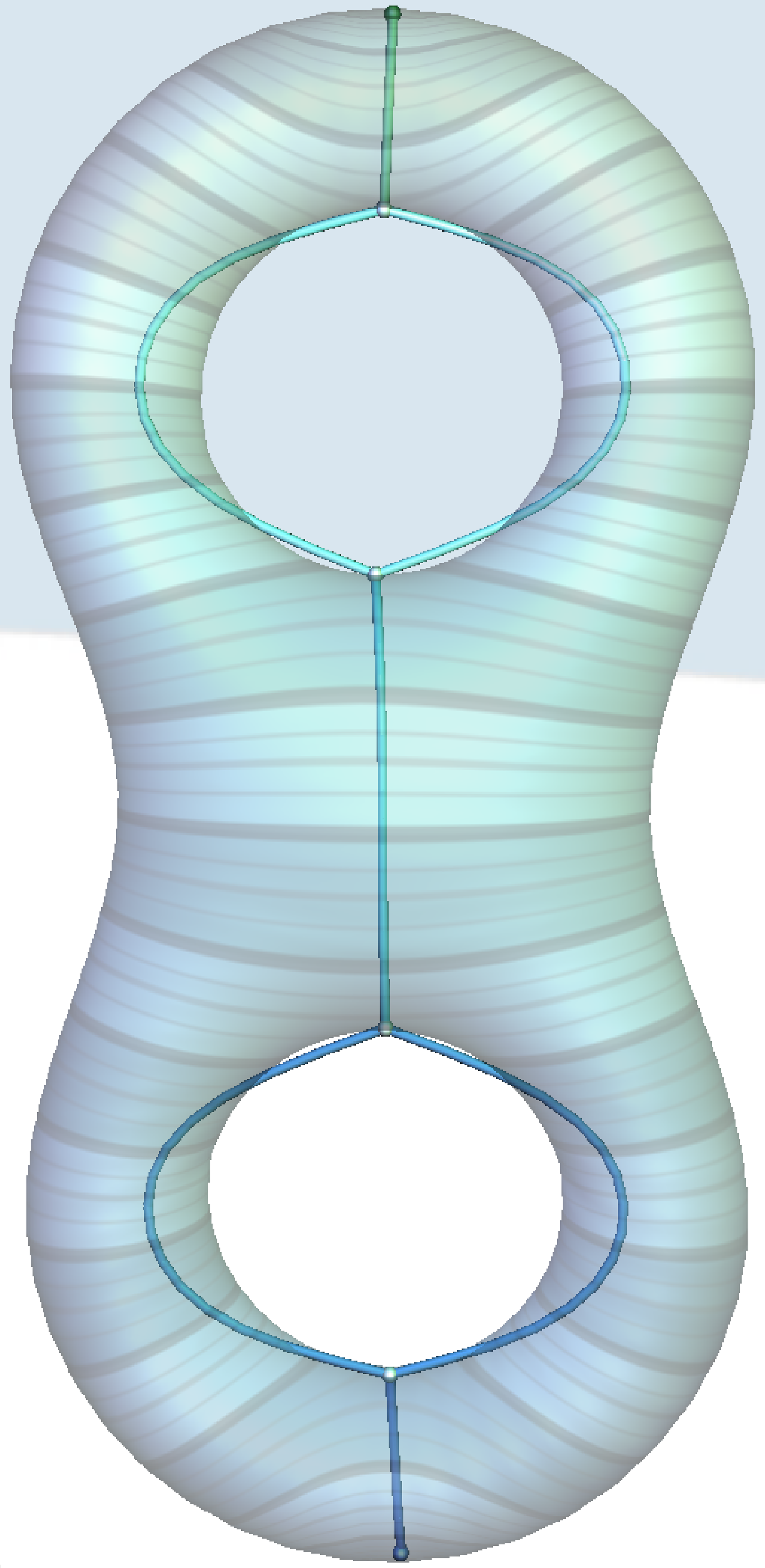
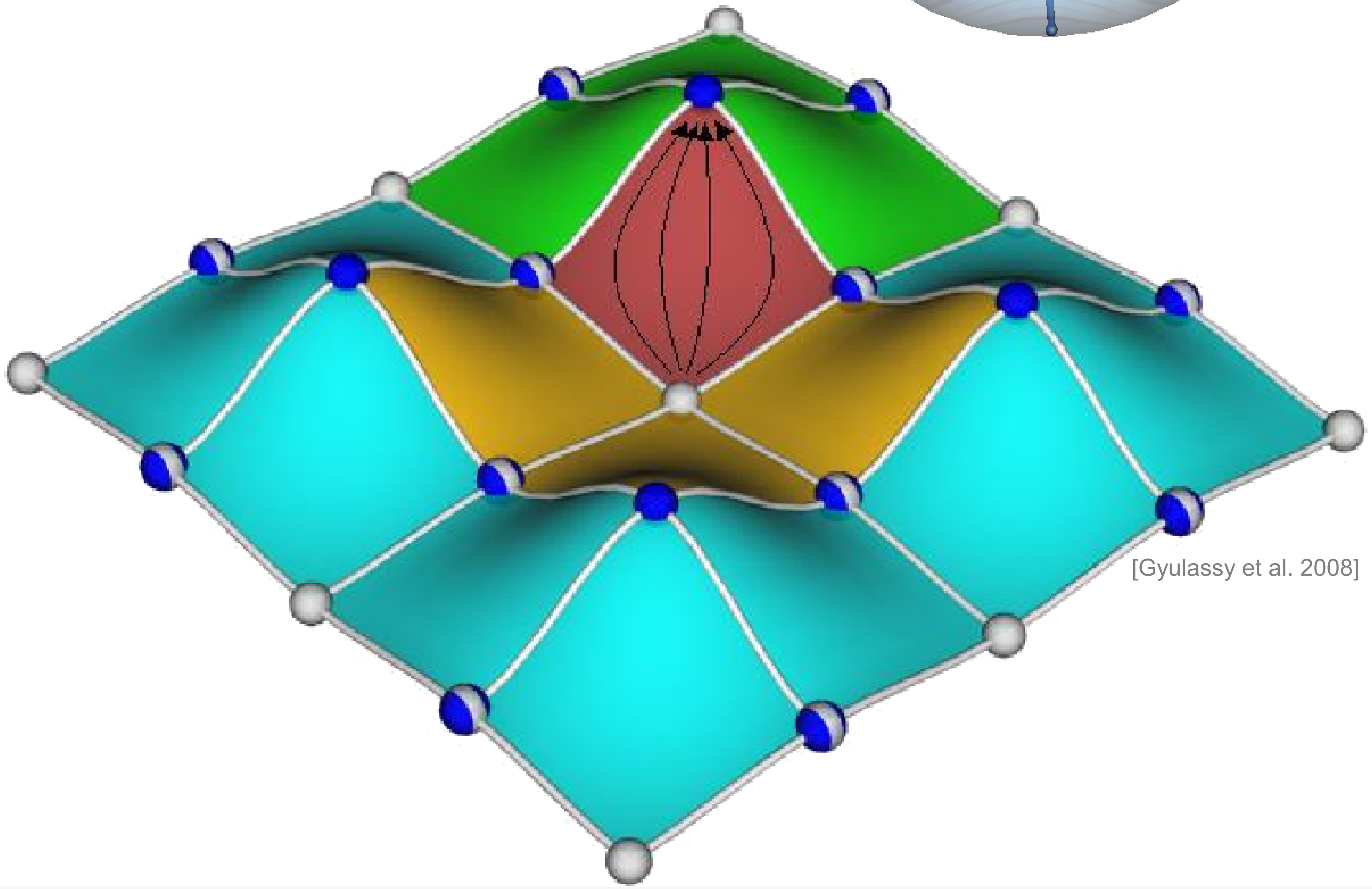


- Gradient fields
  - Morse-Smale complexes
    - Summarize **all** the streamlines
      – Source, sinks (divergence)



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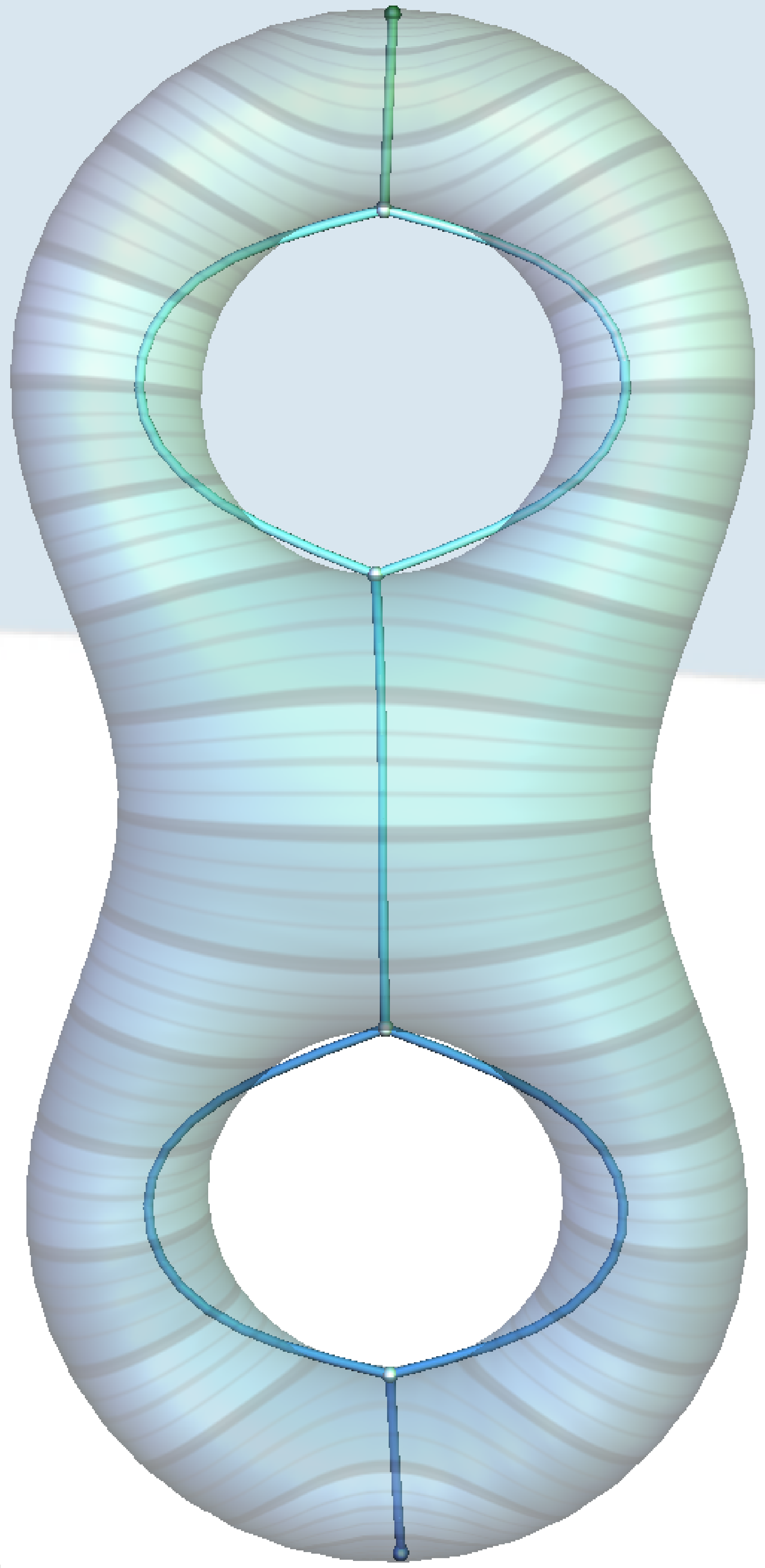
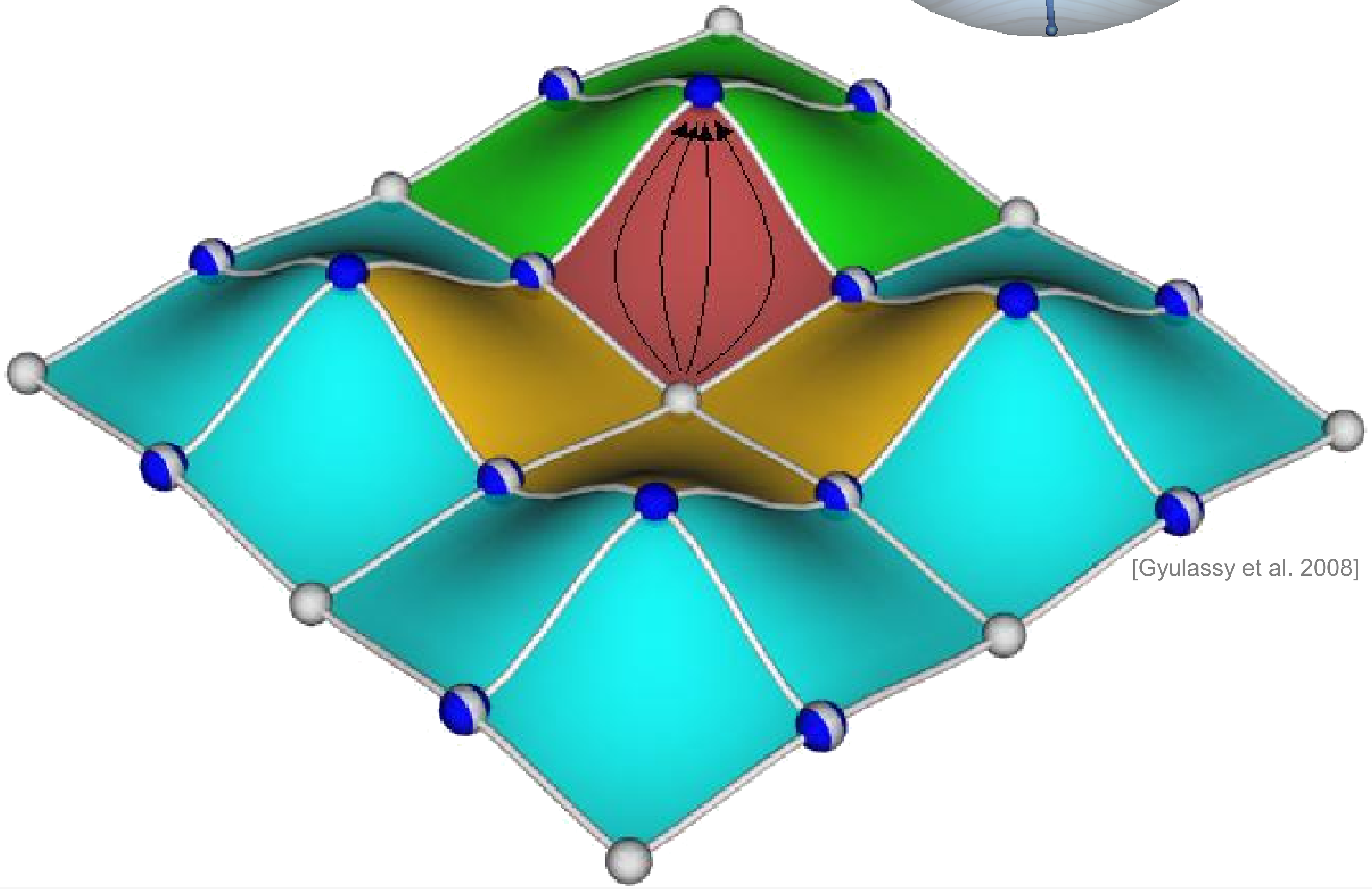
Analogy to scalar fields



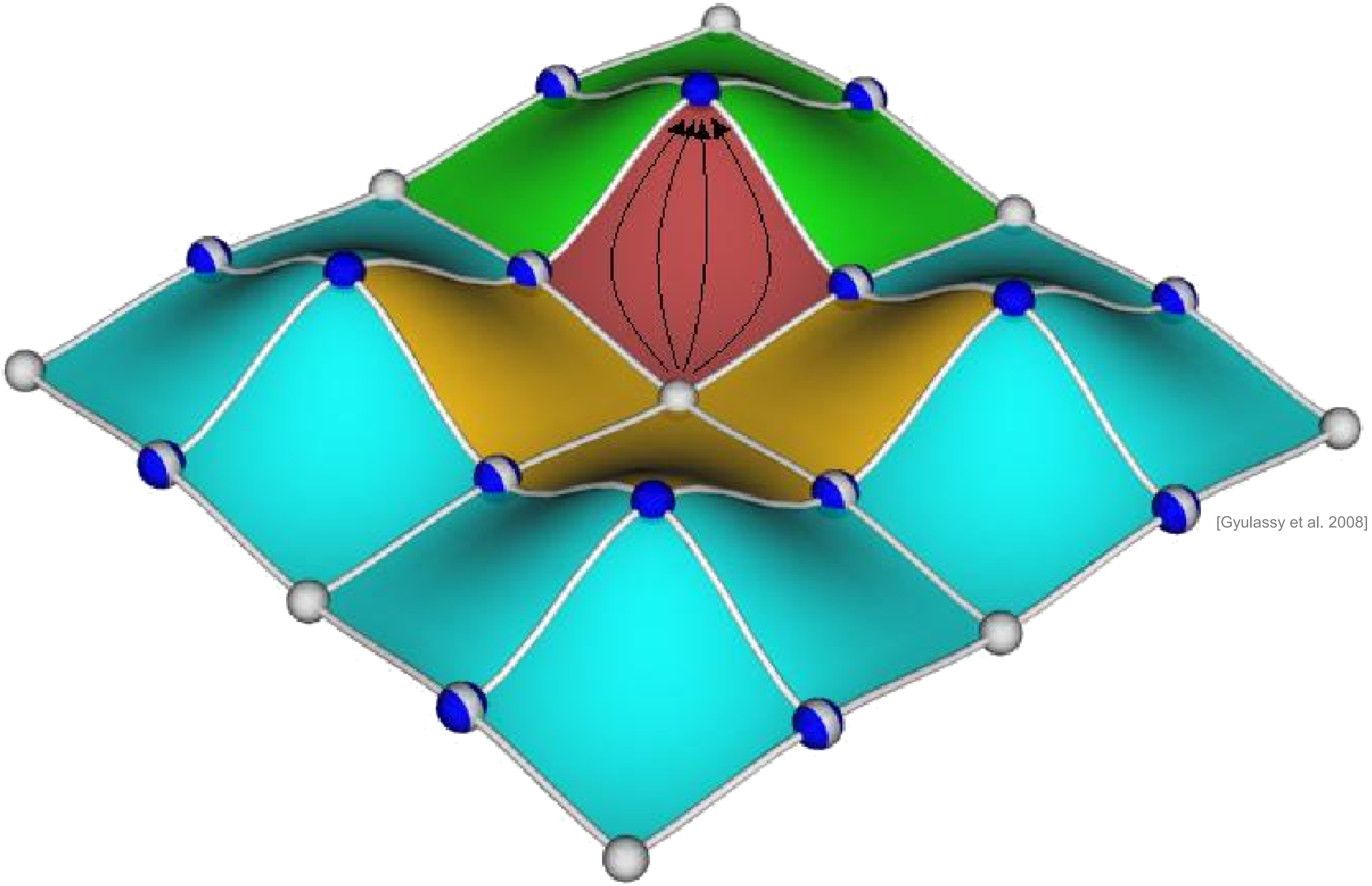
- Scalar fields
  - Reeb graphs
    - Summarize the connectivity of **all** level sets

- Gradient fields
  - Morse-Smale complexes
    - Summarize **all** the streamlines
      - – Source, sinks (divergence)
      - – Separatrices (distortion)



[Gyulassy et al. 2008]

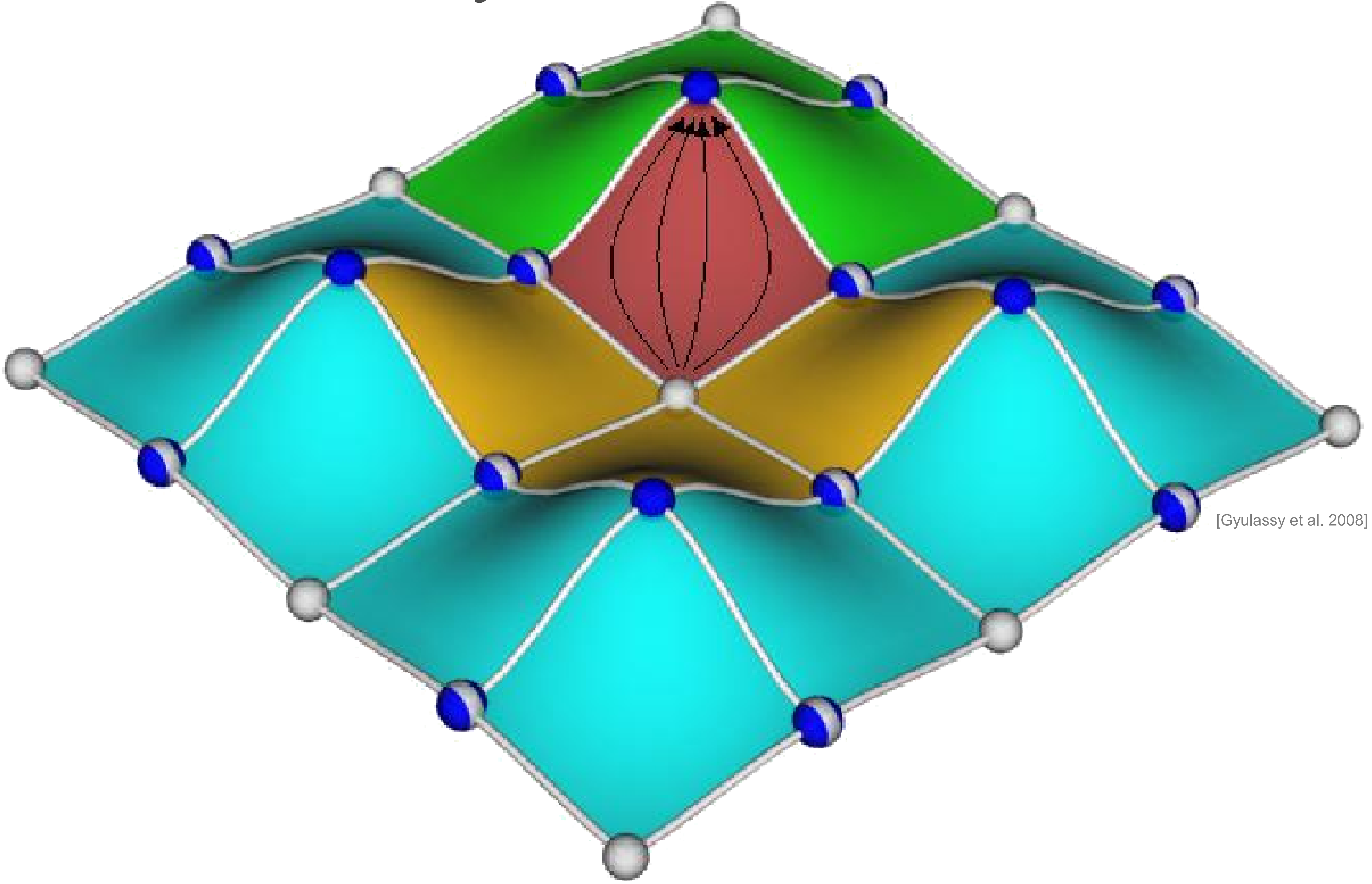
Vector field topology



[Gyulassy et al. 2008]

Vector field topology

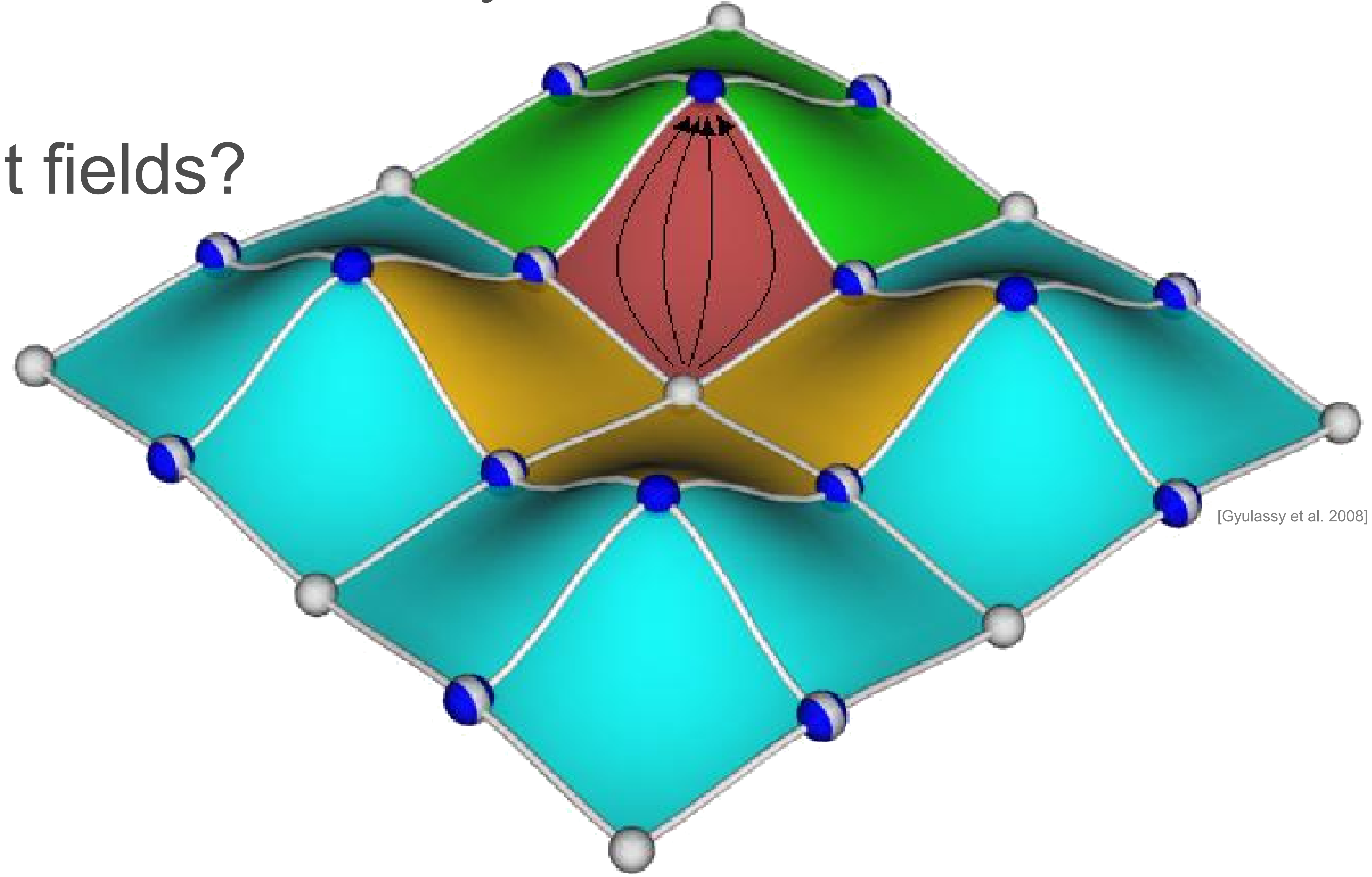
- How to extend this decomposition to arbitrary vector fields?



[Gyulassy et al. 2008]

Vector field topology

- How to extend this decomposition to arbitrary vector fields?
- What's special about gradient fields?

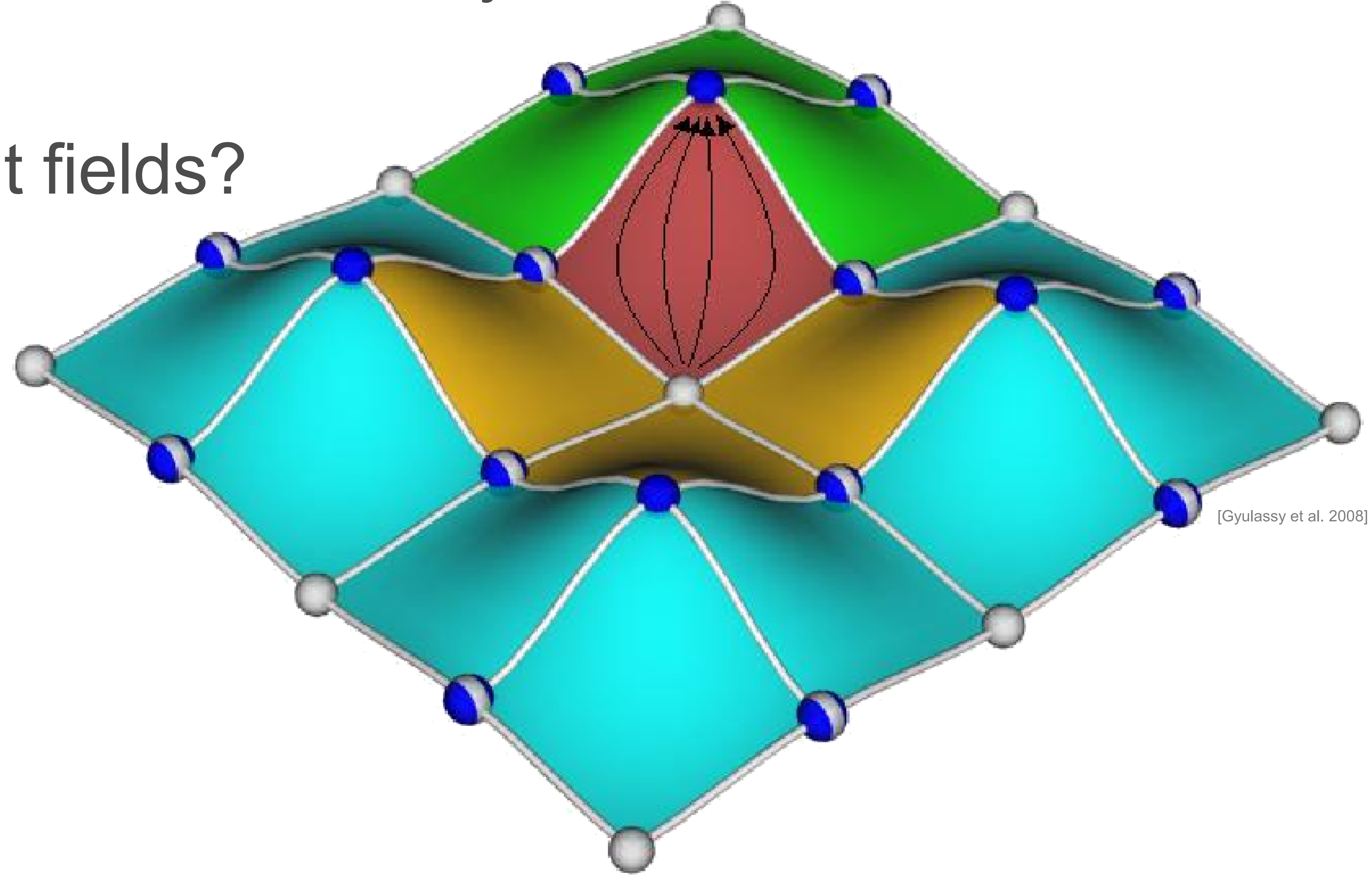


[Gyulassy et al. 2008]

Vector field topology

- How to extend this decomposition to arbitrary vector fields?

- What's special about gradient fields?
  - No curl

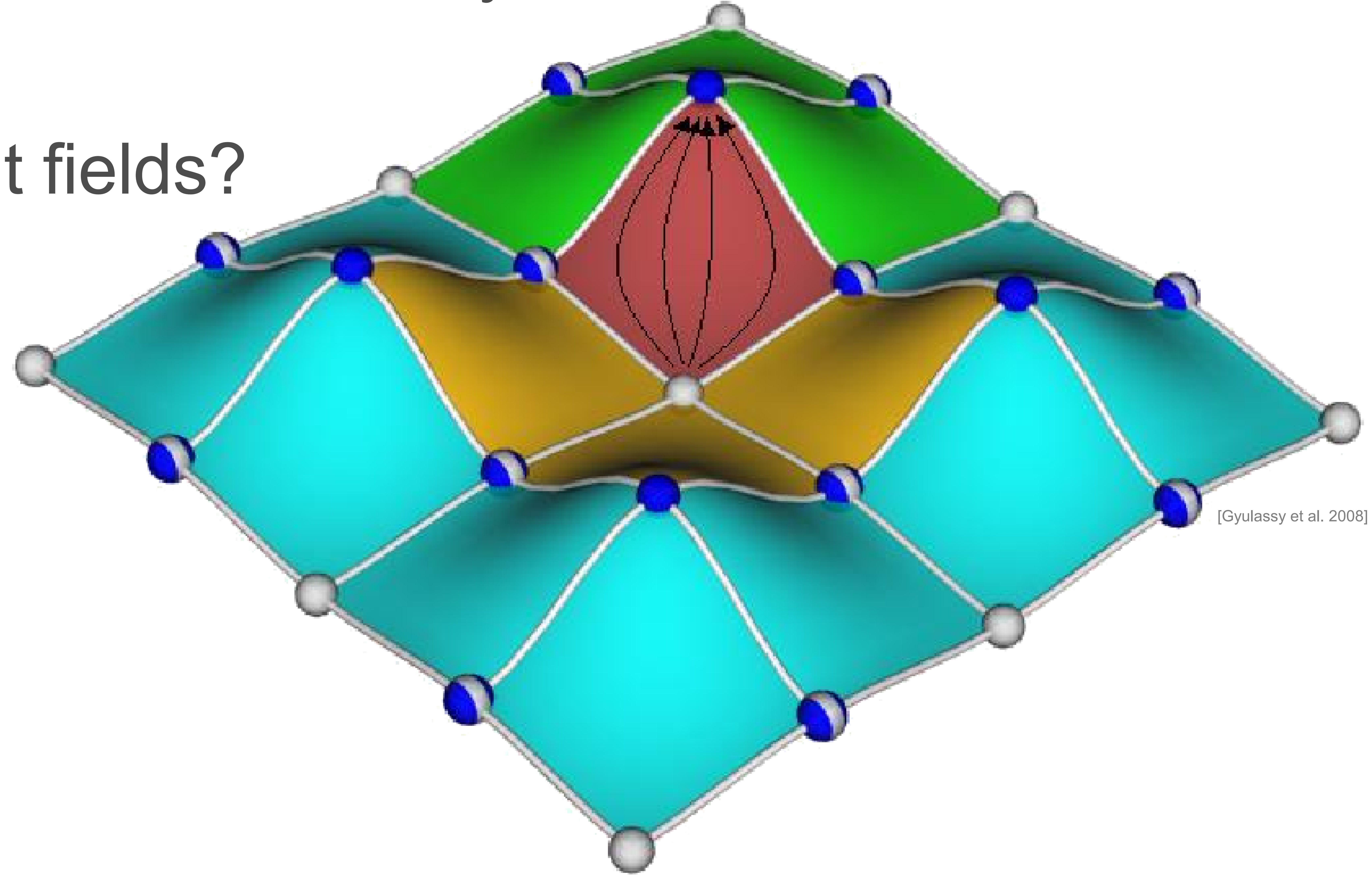


[Gyulassy et al. 2008]

Vector field topology

- How to extend this decomposition to arbitrary vector fields?

- What's special about gradient fields?
  - No curl
  - What does it imply?



[Gyulassy et al. 2008]

Vector field topology

- How to extend this decomposition to arbitrary vector fields?

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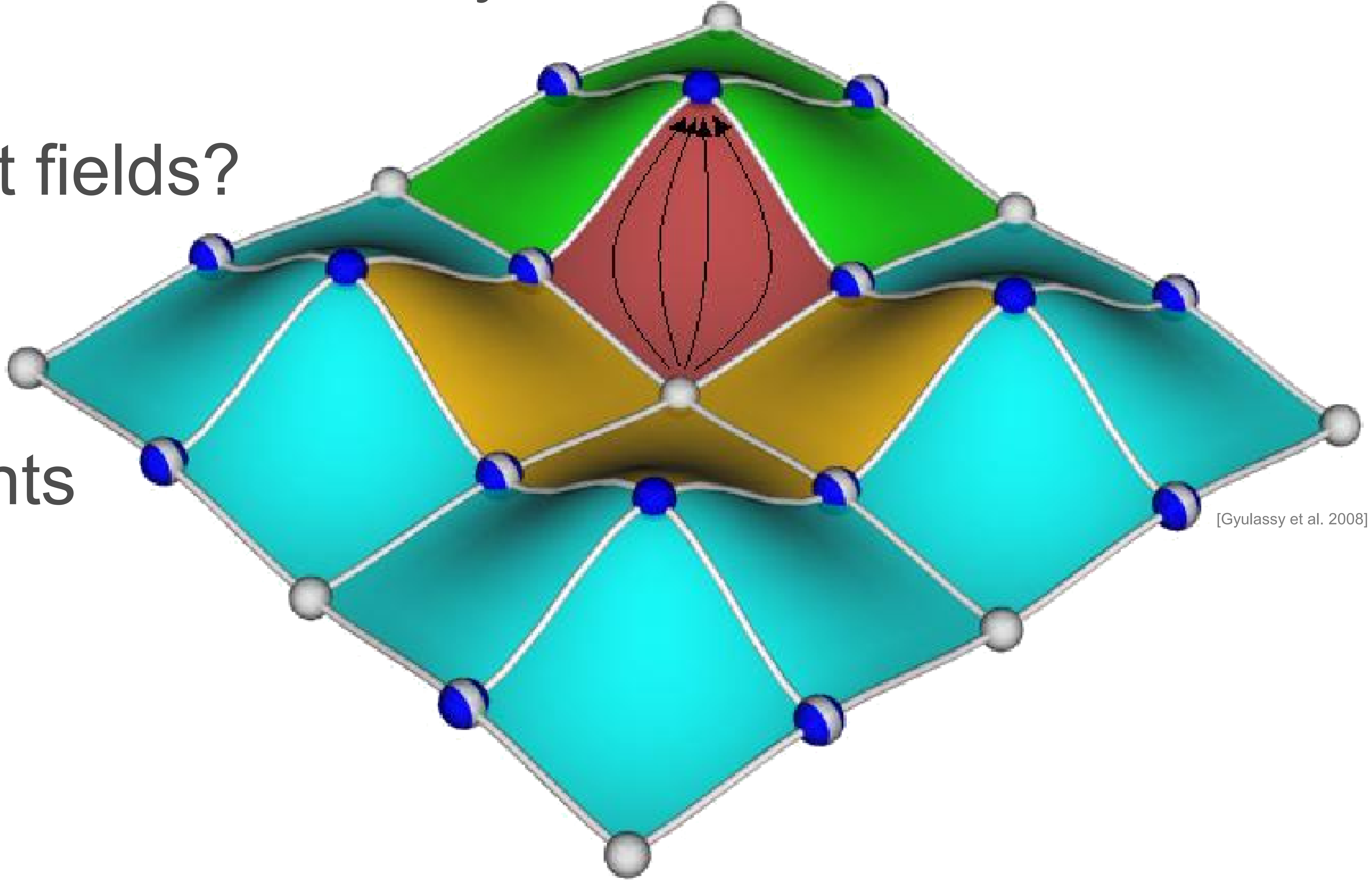
  - No curl
  - What does it imply?
    - Other types of critical points

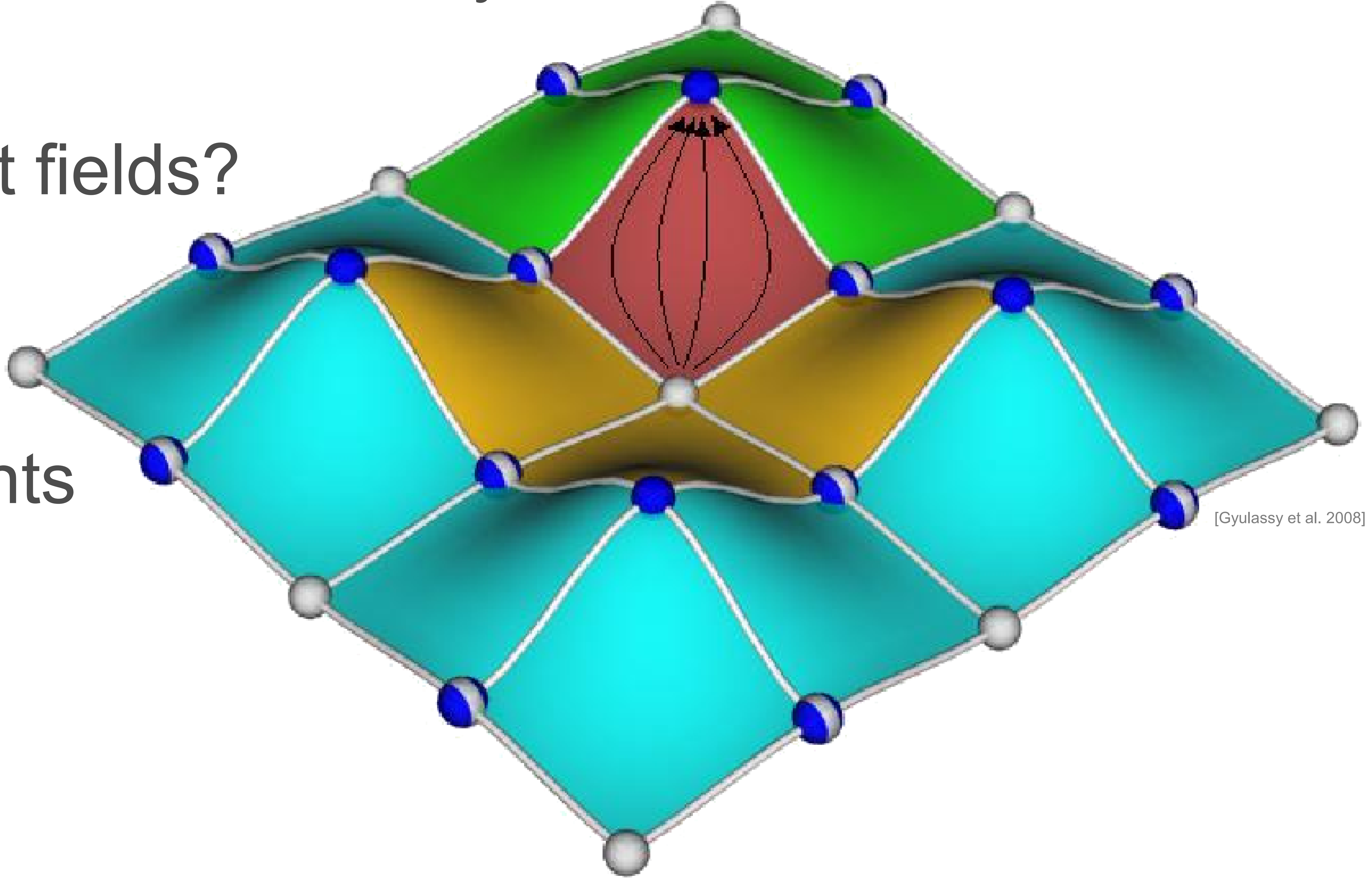


[Gyulassy et al. 2008]

Vector field topology

- How to extend this decomposition to arbitrary vector fields?

- What's special about gradient fields?
  - No curl
  - What does it imply?
    - Other types of critical points
    - Particular streamlines



[Gyulassy et al. 2008]

Critical points of a vector field

- For example
  - $f : \mathbb{R}^2 \to \mathbb{R}^2$

Critical points of a vector field

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  - $f(p) = p$

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- For example
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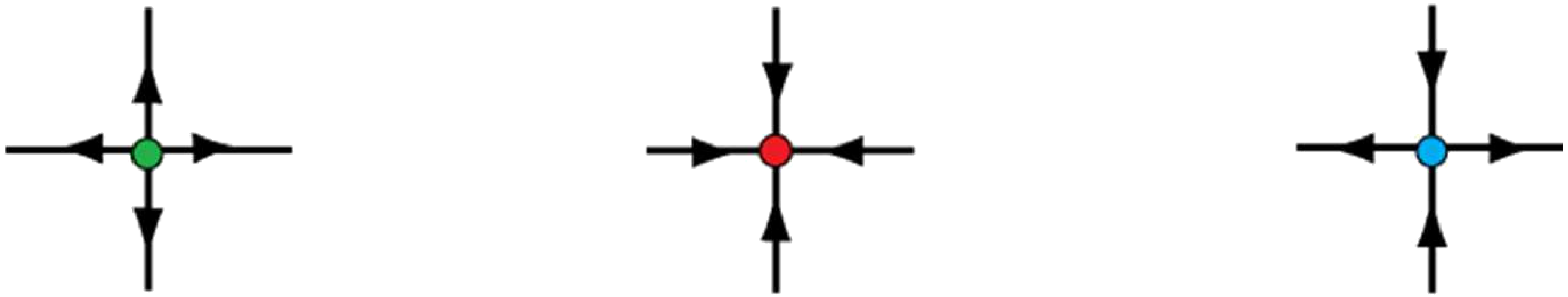
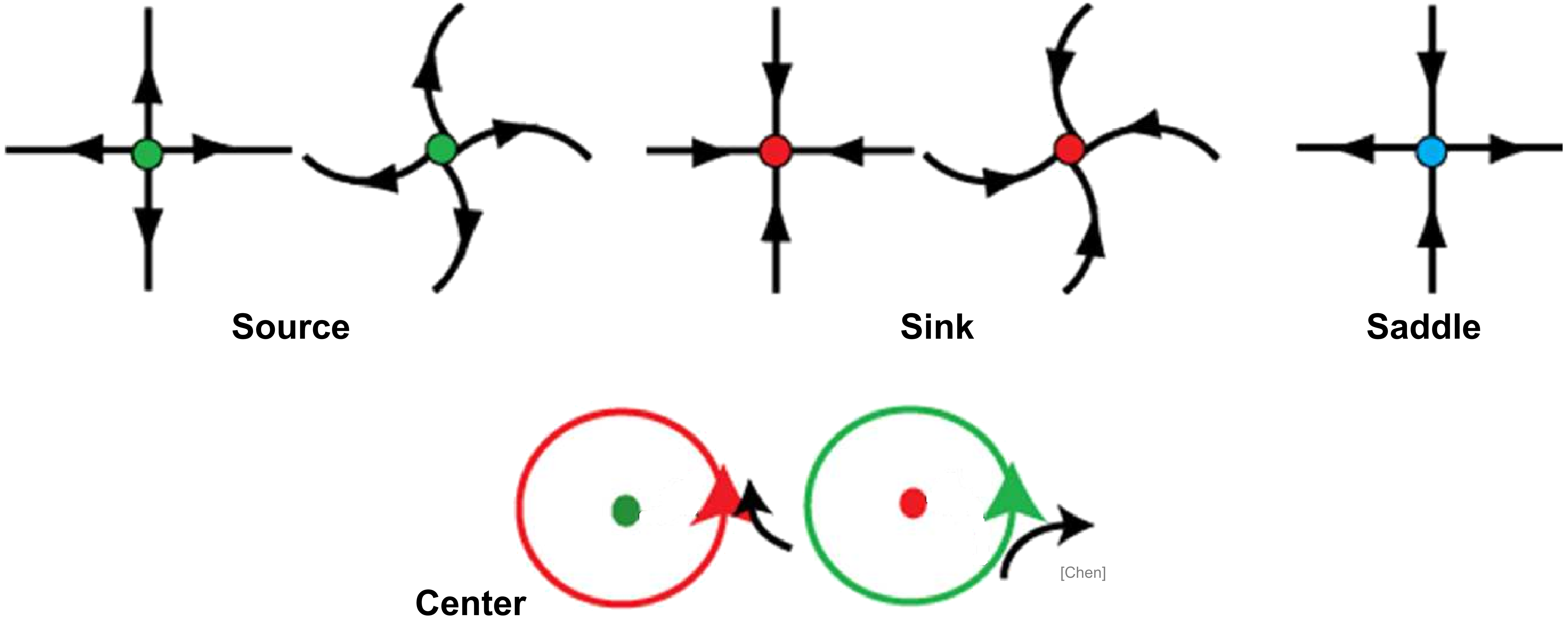


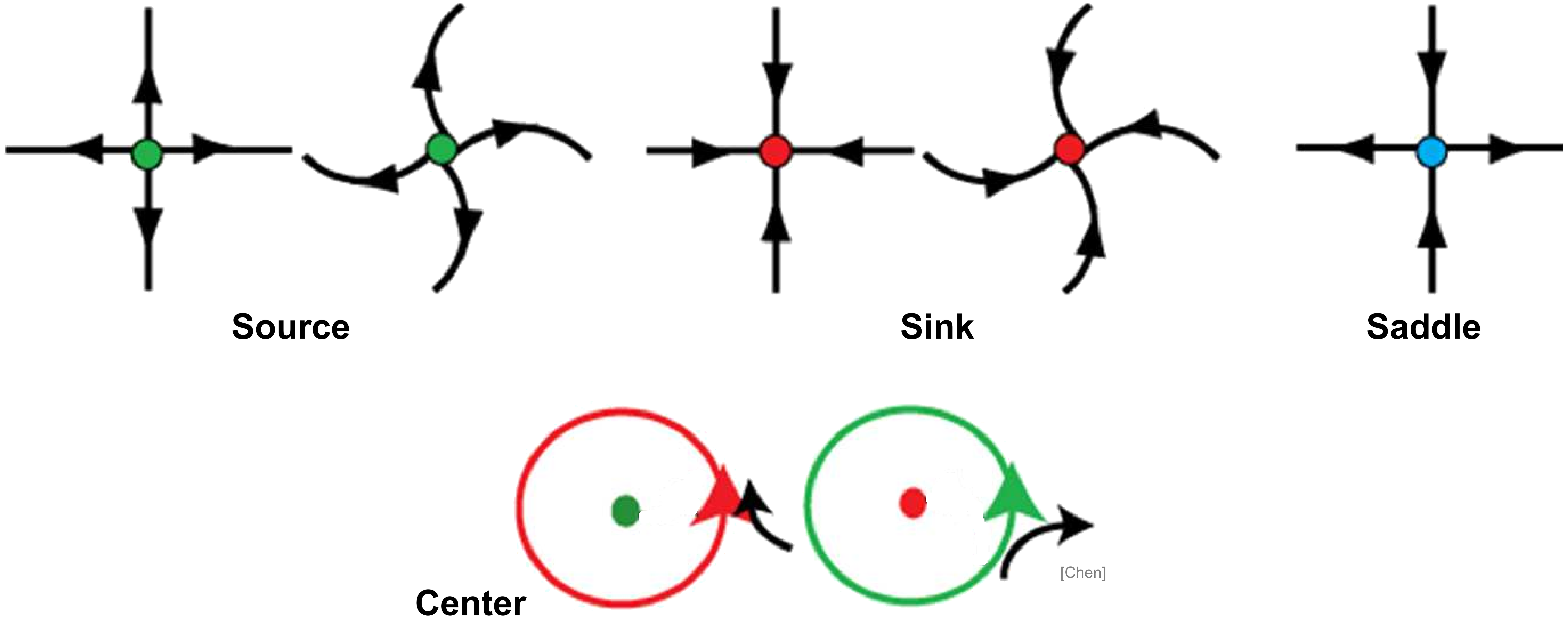
[Chen]

Critical points of a vector field

- For example
  - $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$
- Points where the magnitude vanishes
  - $f(p) = p$
- With curl
  - More critical points

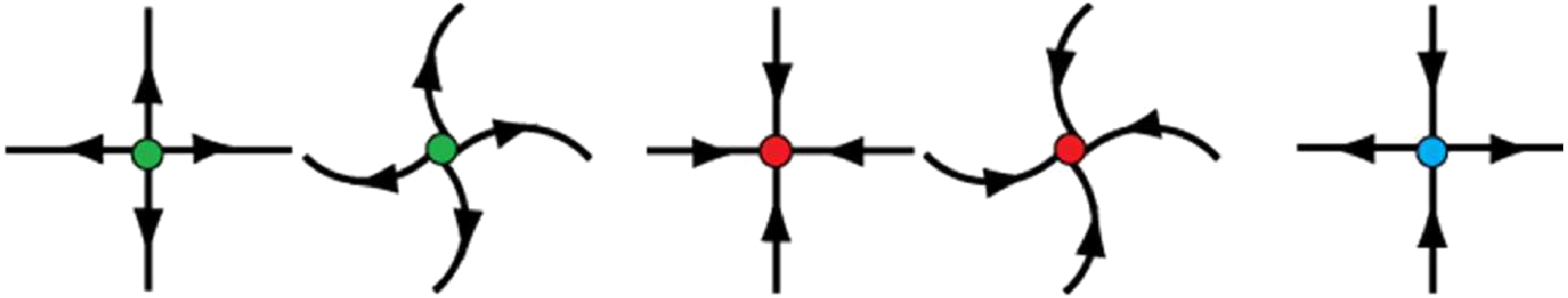


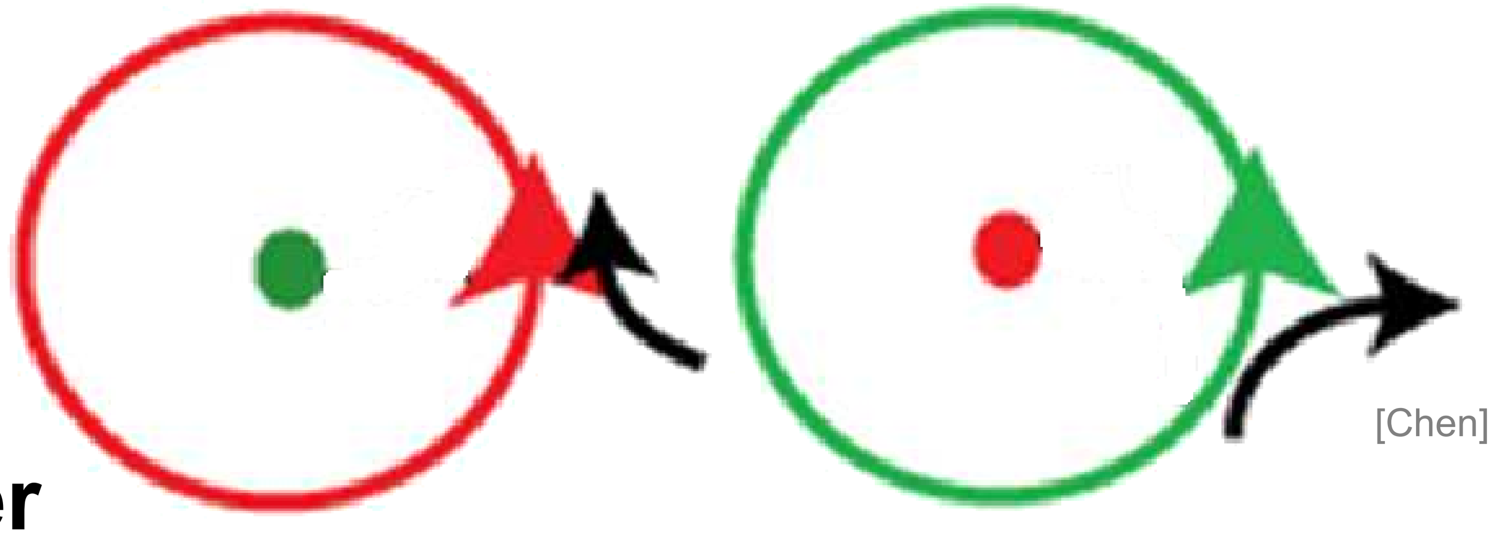
Classifying critical points



Classifying critical points

- Jacobian of the vector field

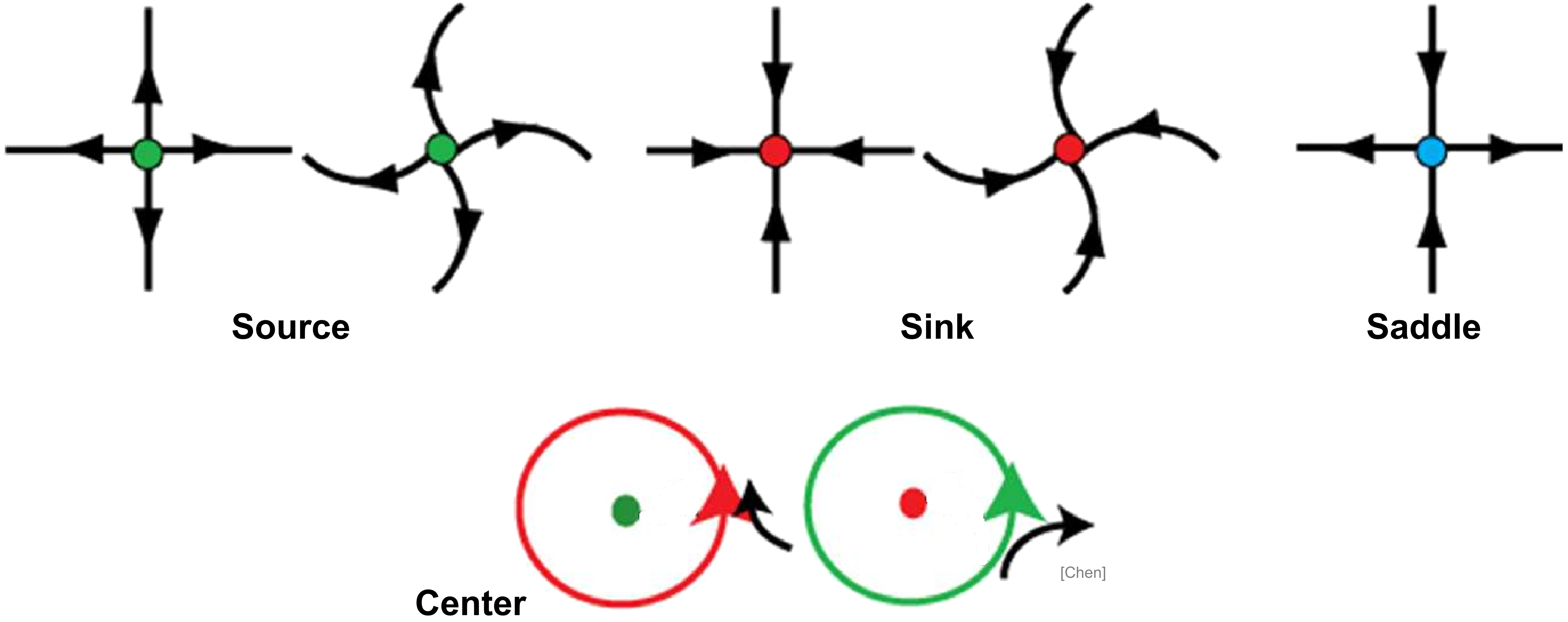




Classifying critical points

- Jacobian of the vector field

$$J = \begin{bmatrix} \frac{\partial f_x}{\partial x} & \frac{\partial f_x}{\partial y} \\ \frac{\partial f_y}{\partial x} & \frac{\partial f_y}{\partial y} \end{bmatrix}$$



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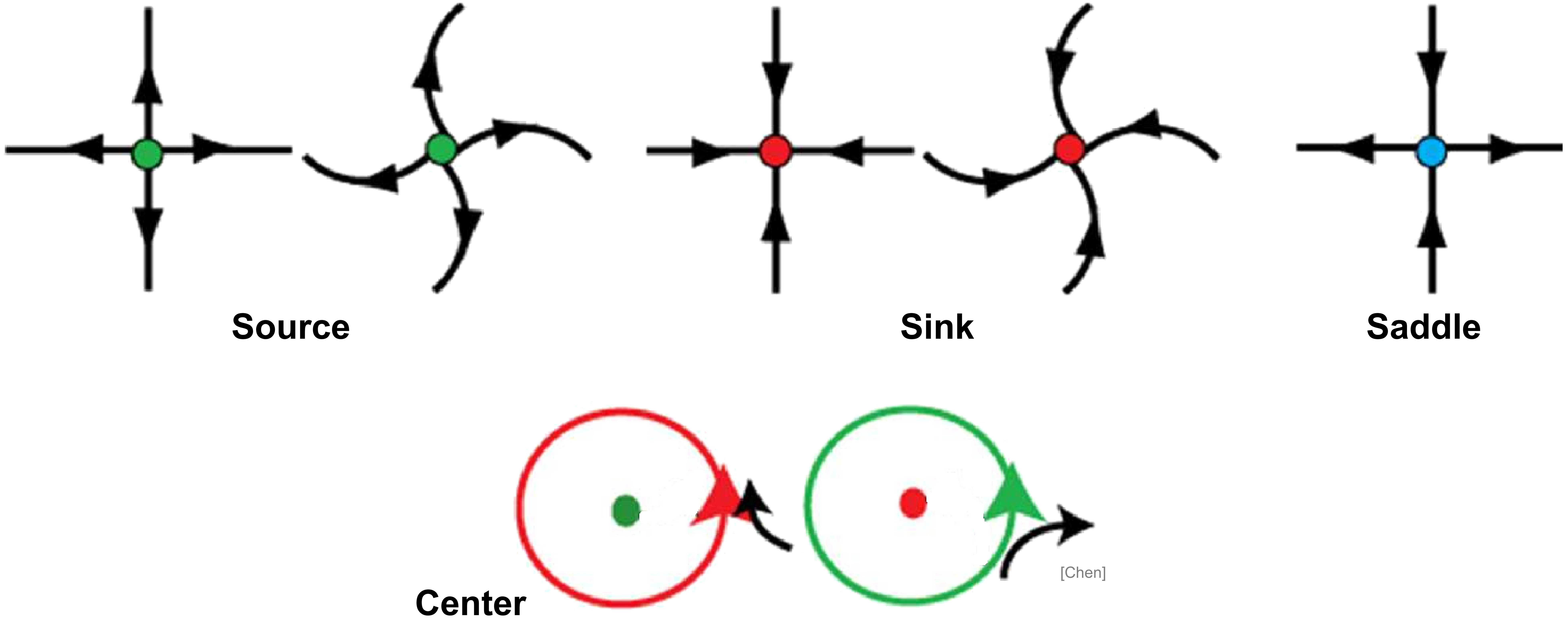
  - 2 eigenvalues



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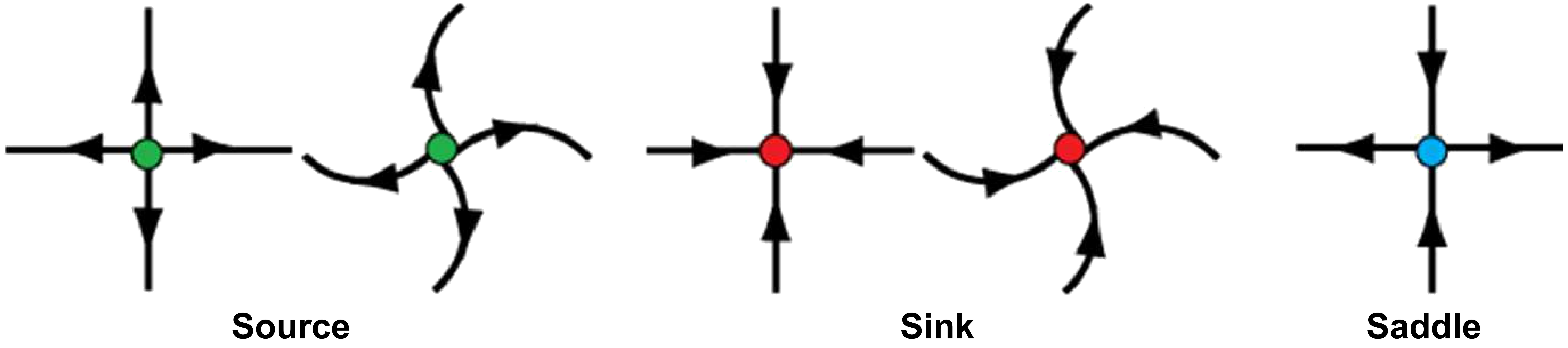

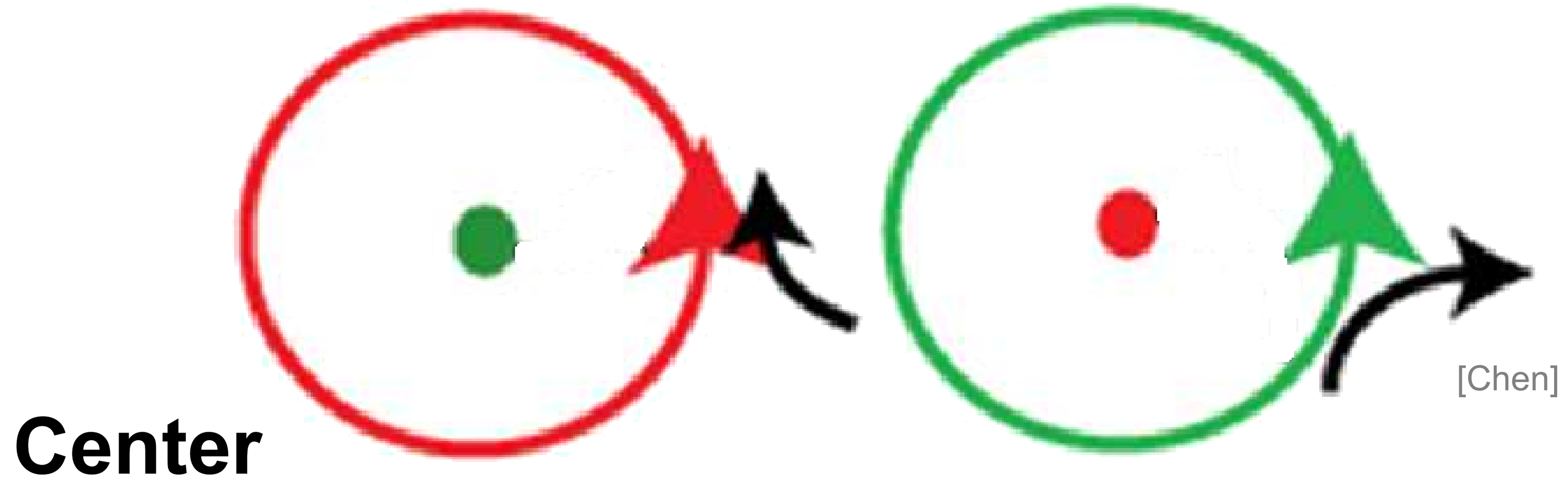

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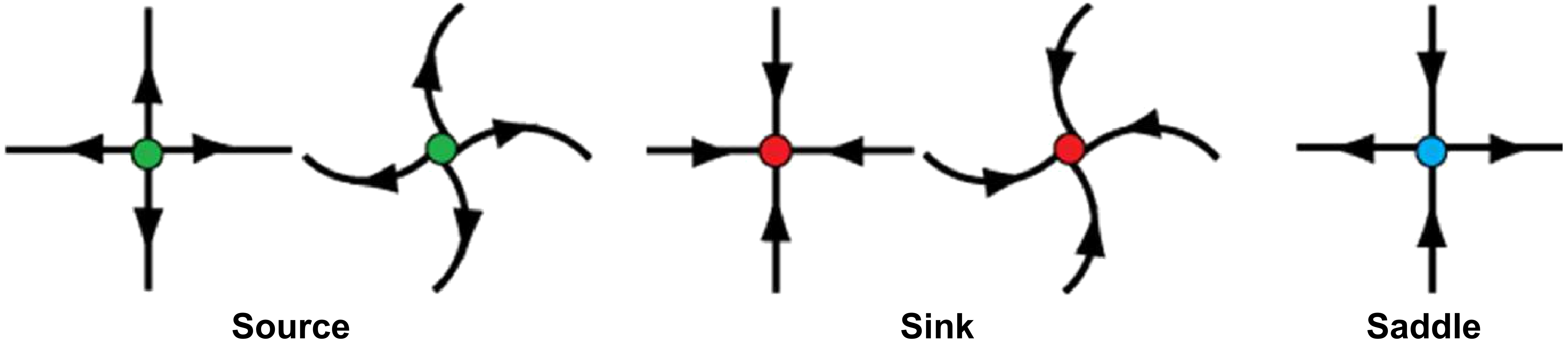

Center

Classifying critical points

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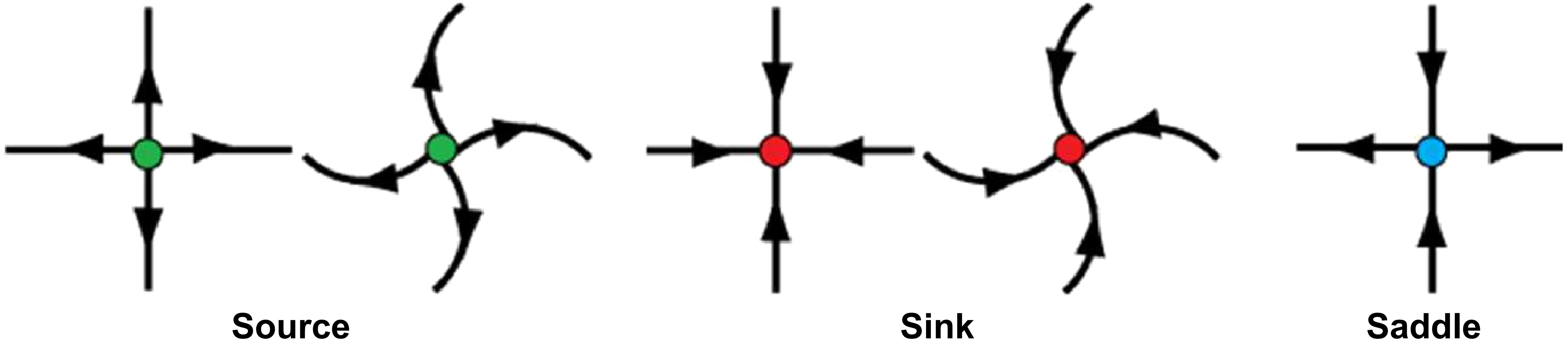

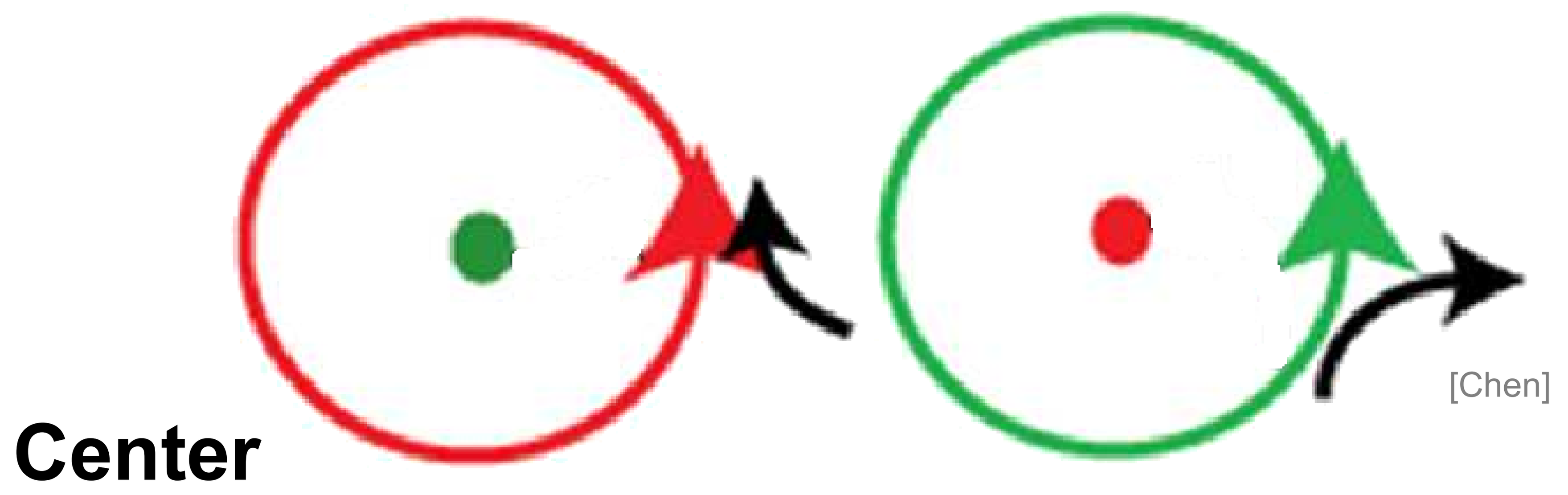

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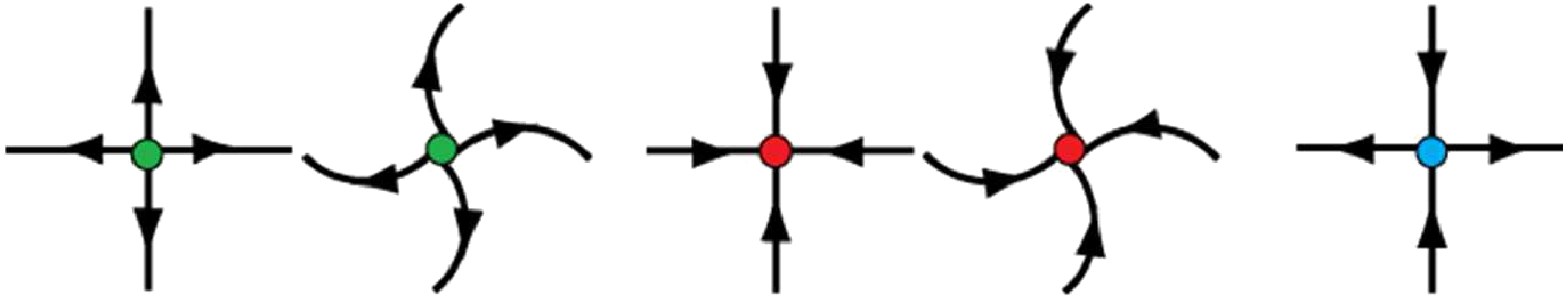
$$R_1 > 0, R_2 > 0$$
$$I_1 = 0, I_2 = 0$$



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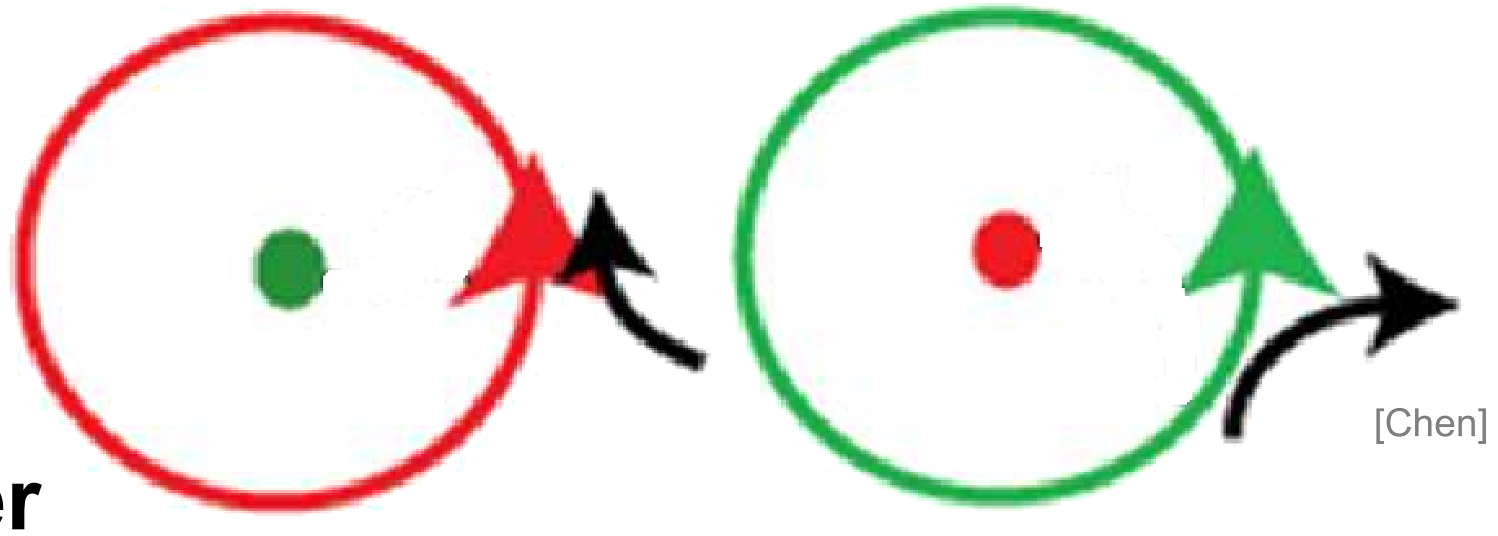

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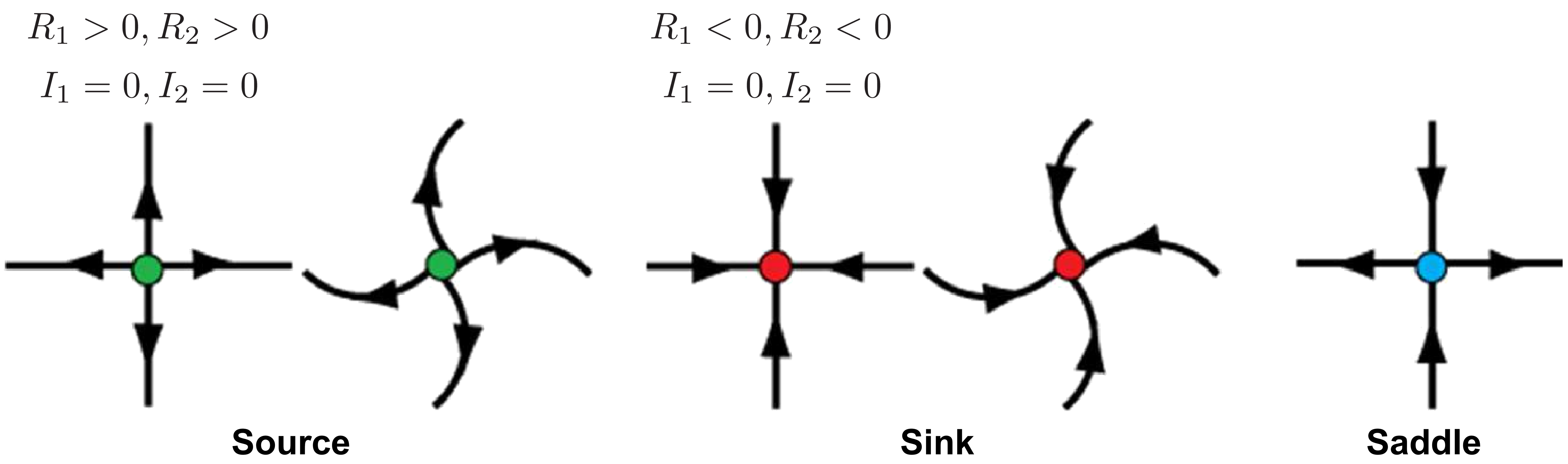

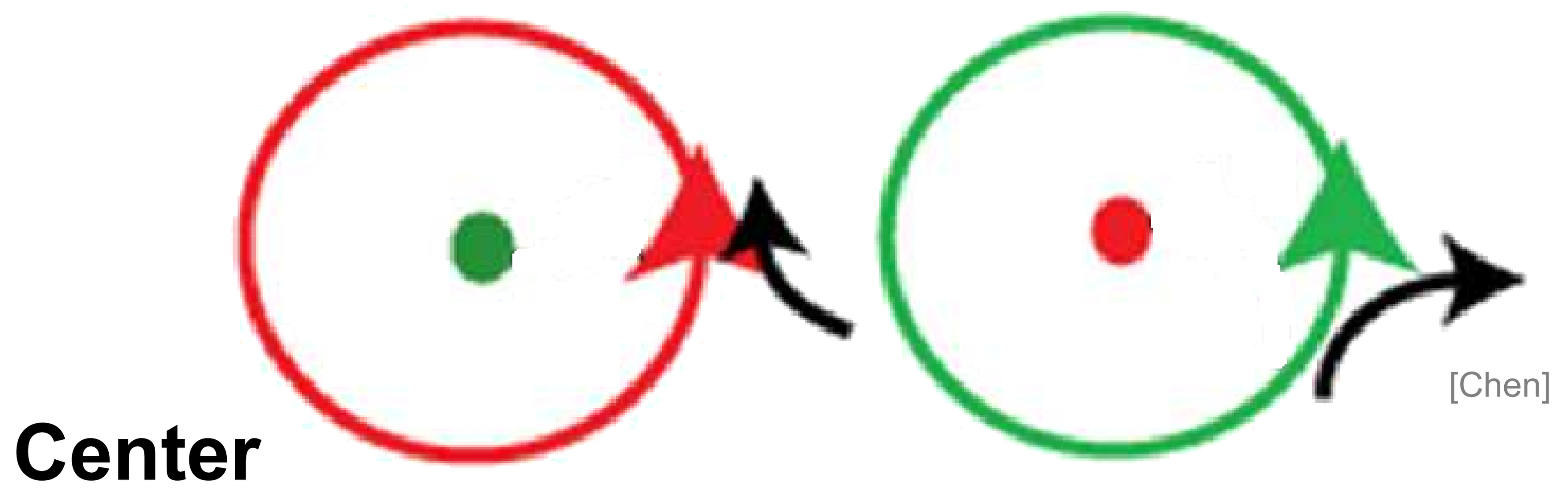

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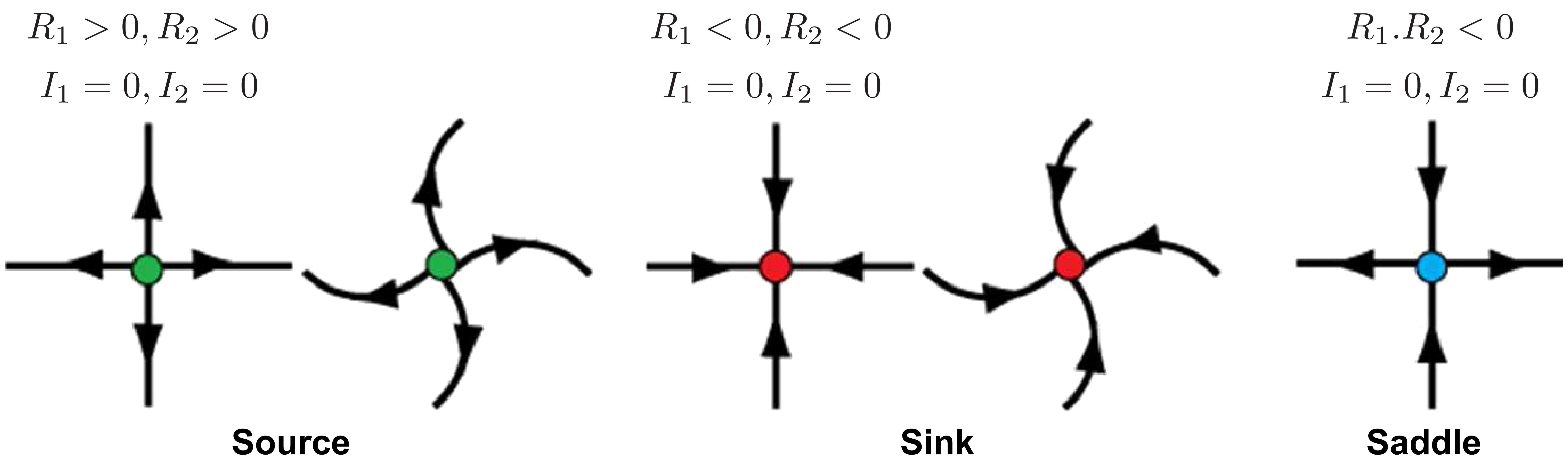

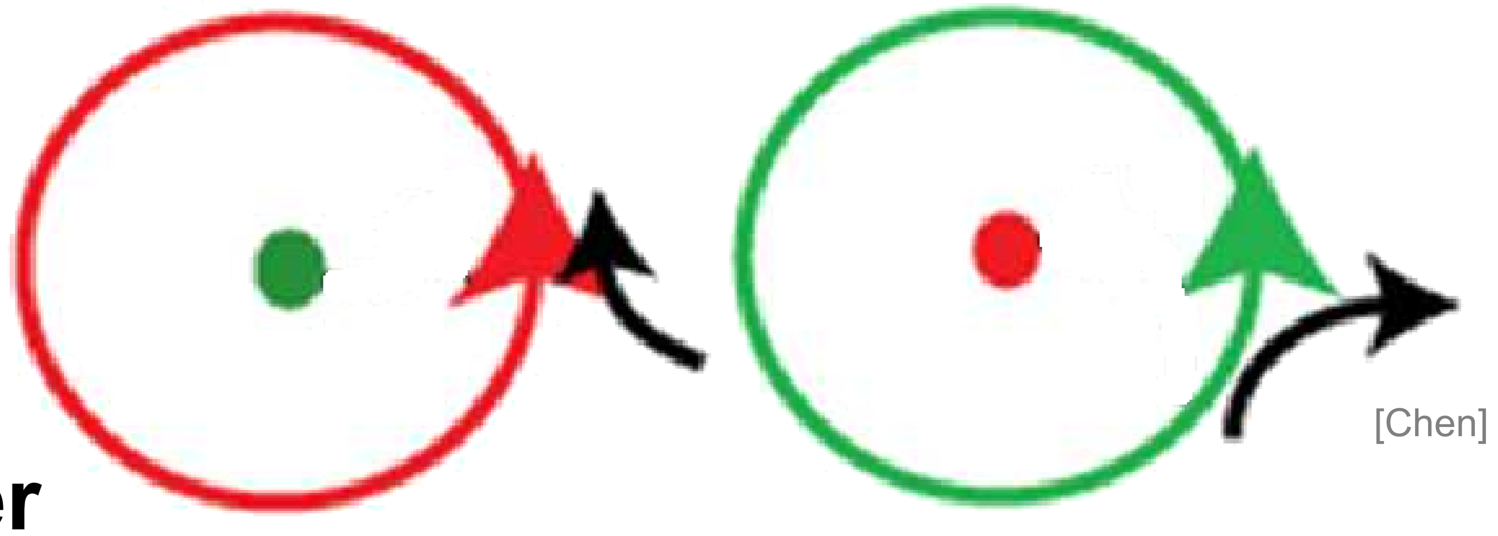

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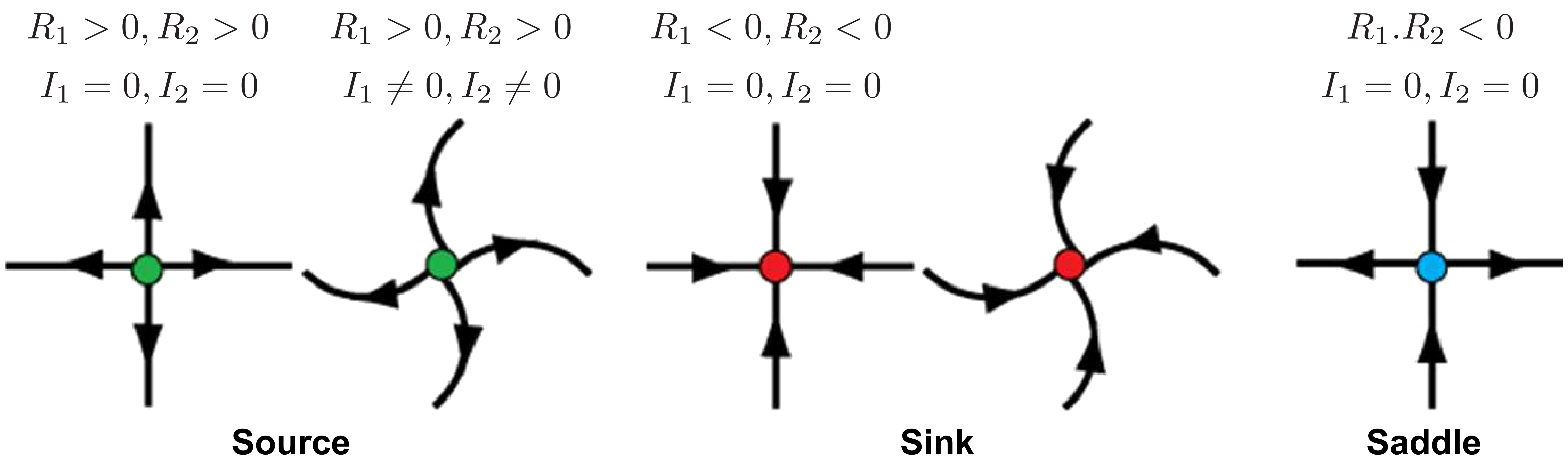

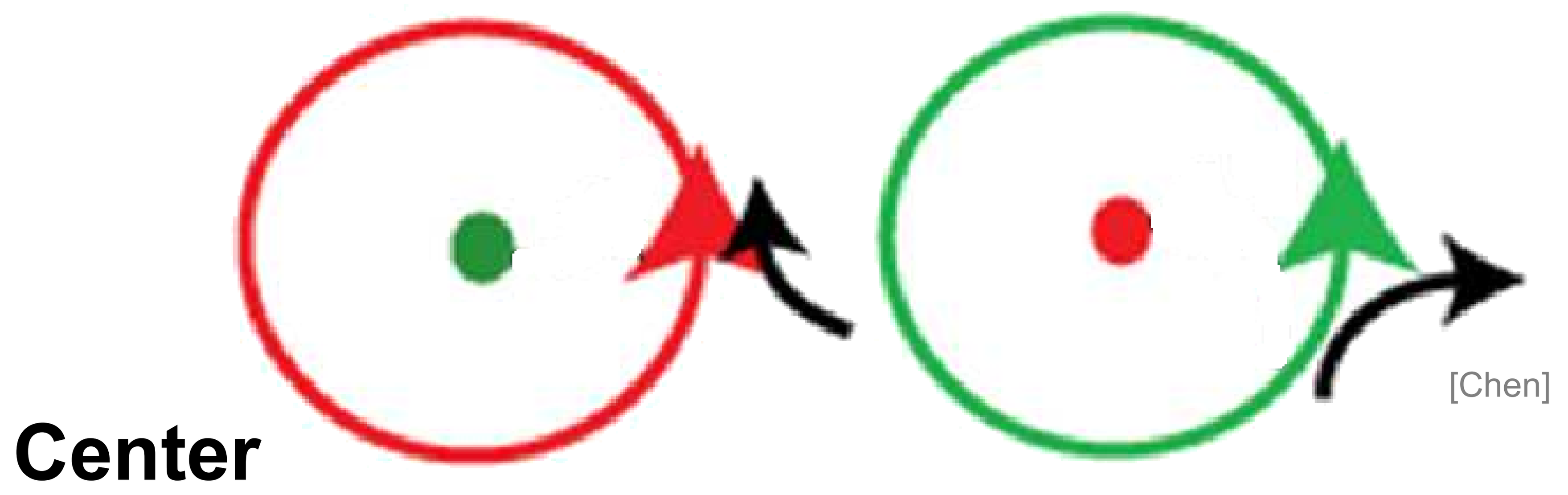

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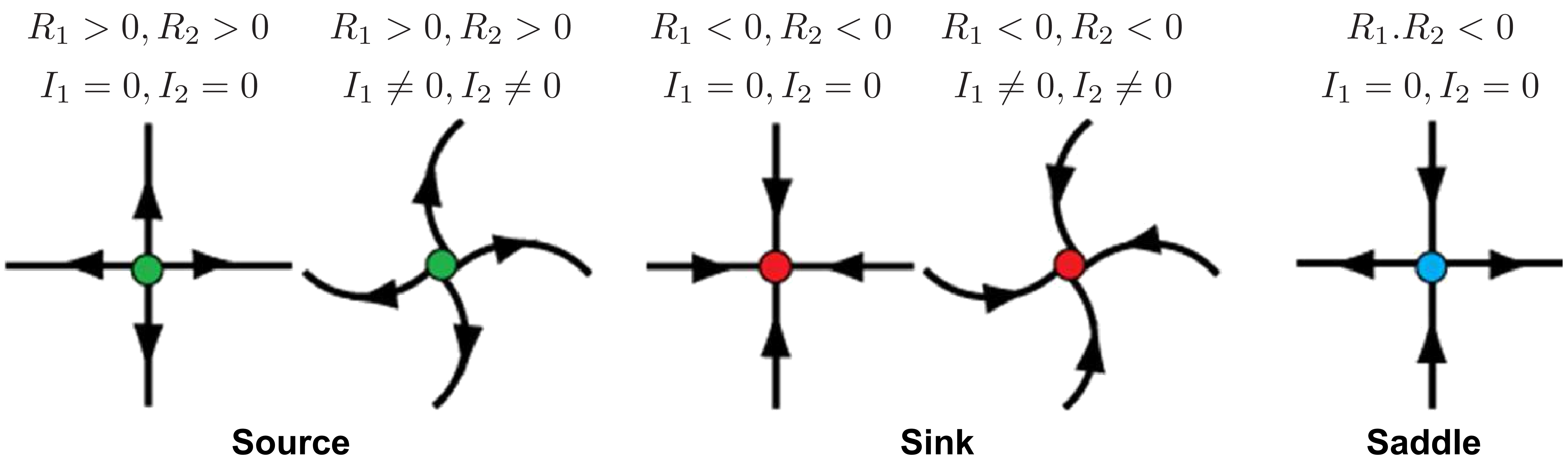

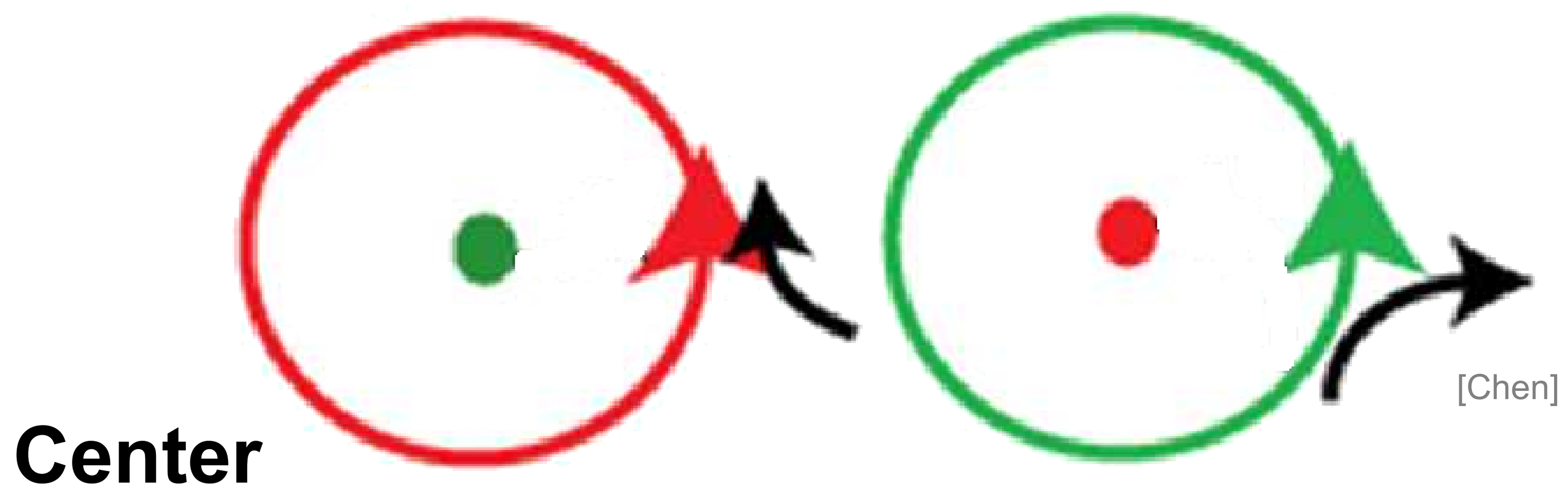

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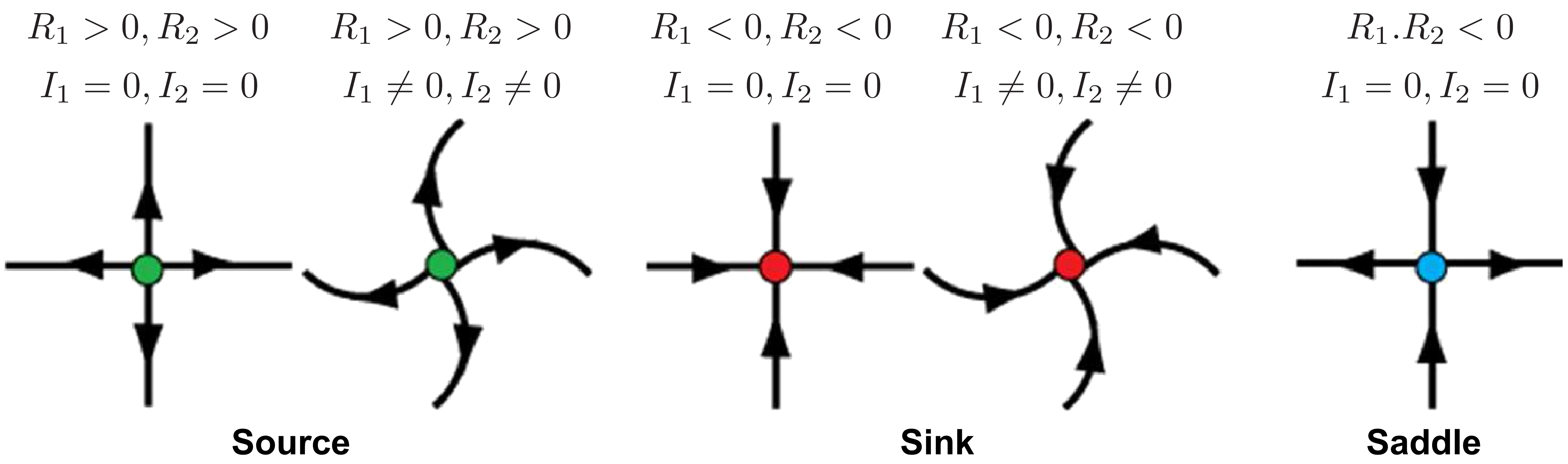

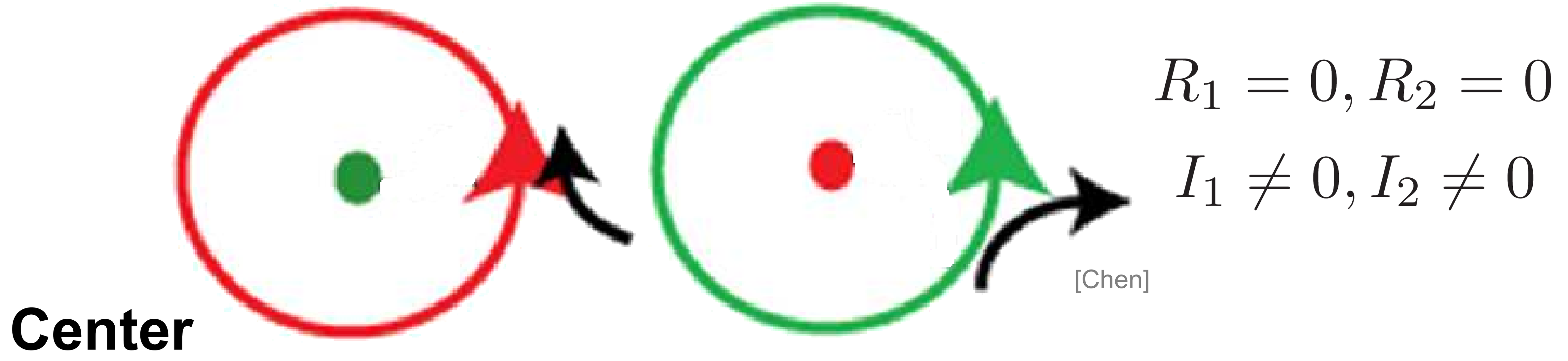

Vector field decomposition

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- Critical point extraction
  - Identify the cells of the domain containing critical points

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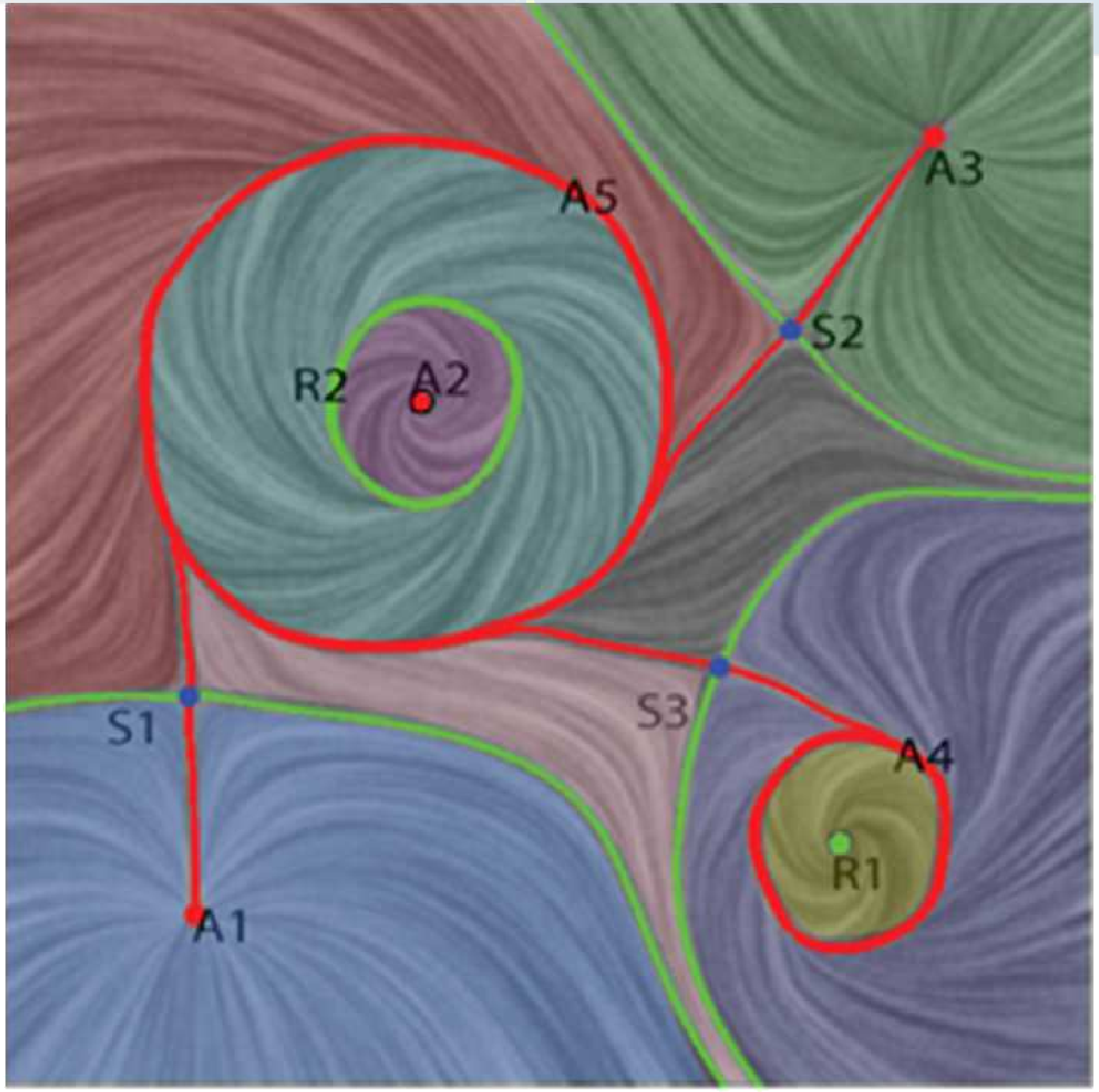
- Decomposition

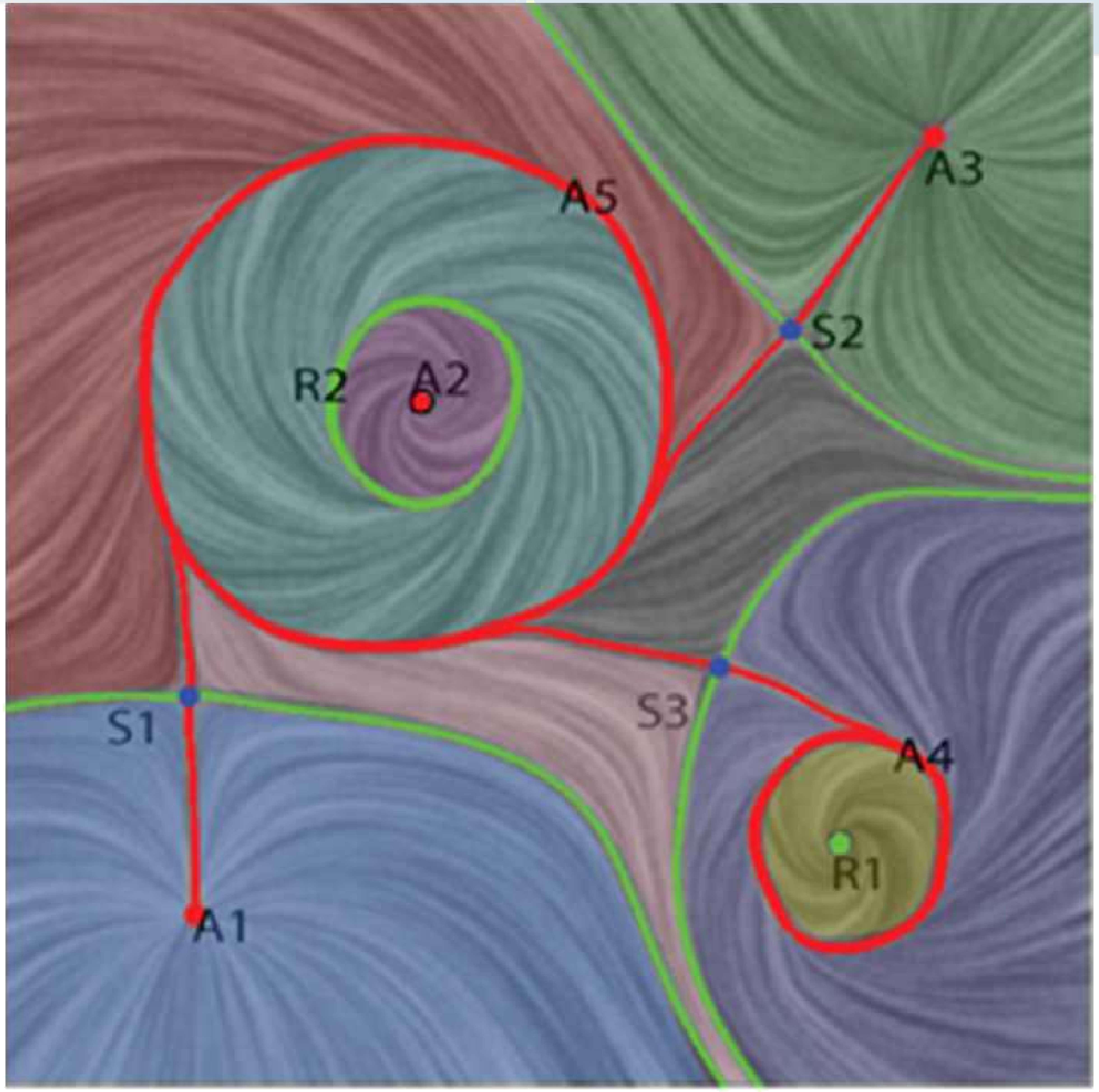


[Chen]

Vector field decomposition

- Critical point extraction
  - Identify the cells of the domain containing critical points
  - Sub-sampling for accurate locations

- Decomposition
  - Backward and forward streamlines from the critical points



[Chen]

Vector field decomposition

- Critical point extraction
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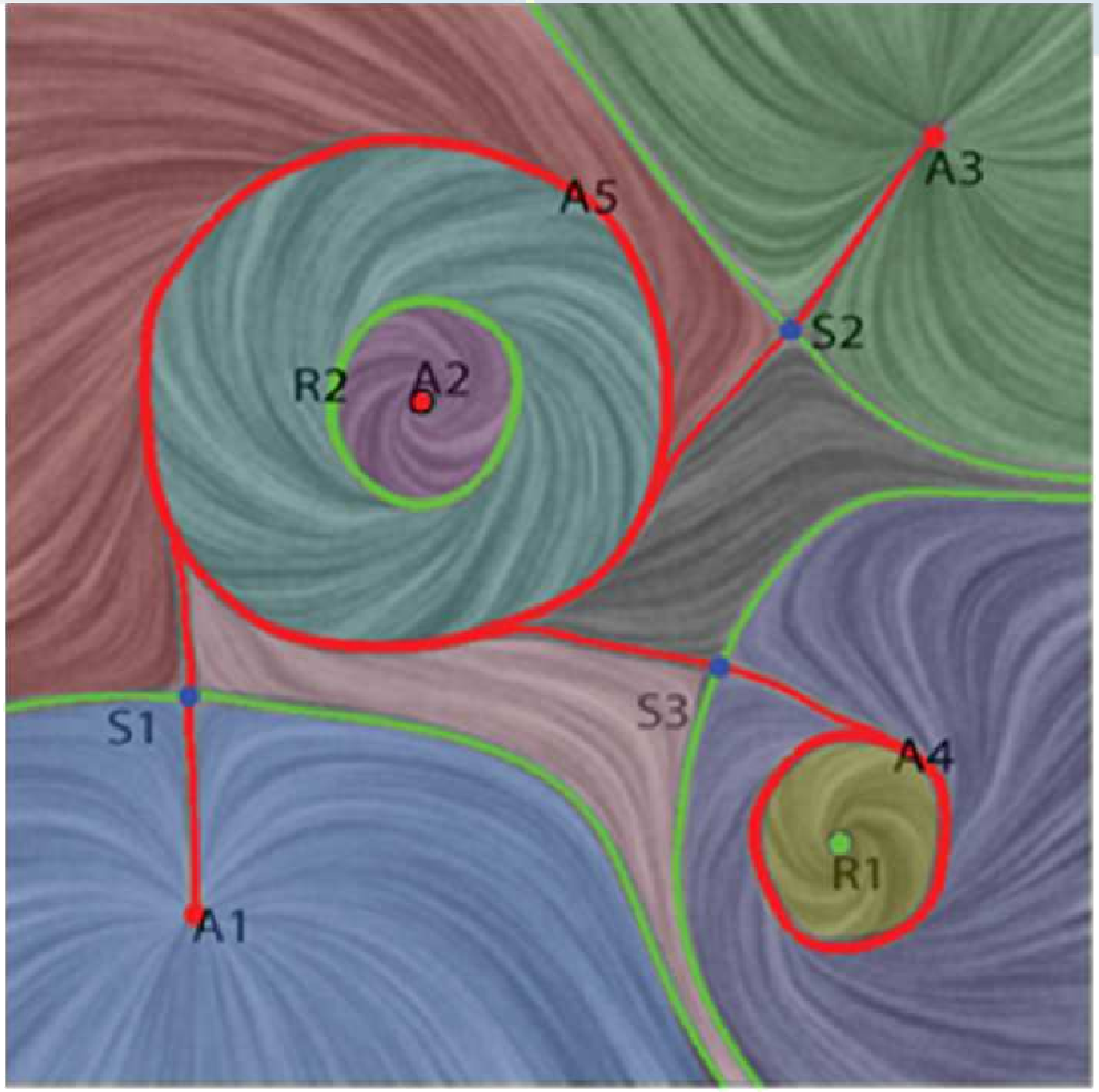
  - Sub-sampling for accurate locations

- Decomposition
  - Backward and forward streamlines from the critical points
  - Periodic orbits!



[Chen]

Non planar domains

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Non planar domains

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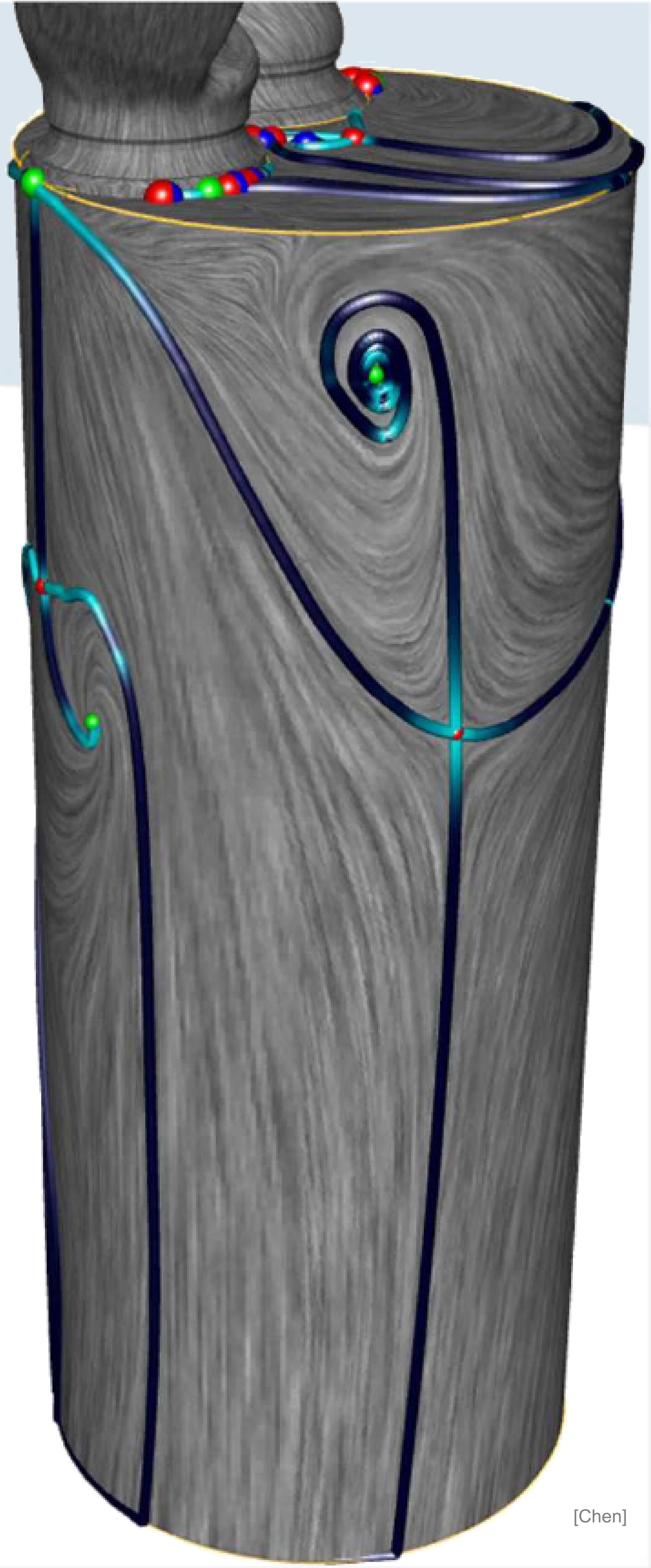

[Chen]

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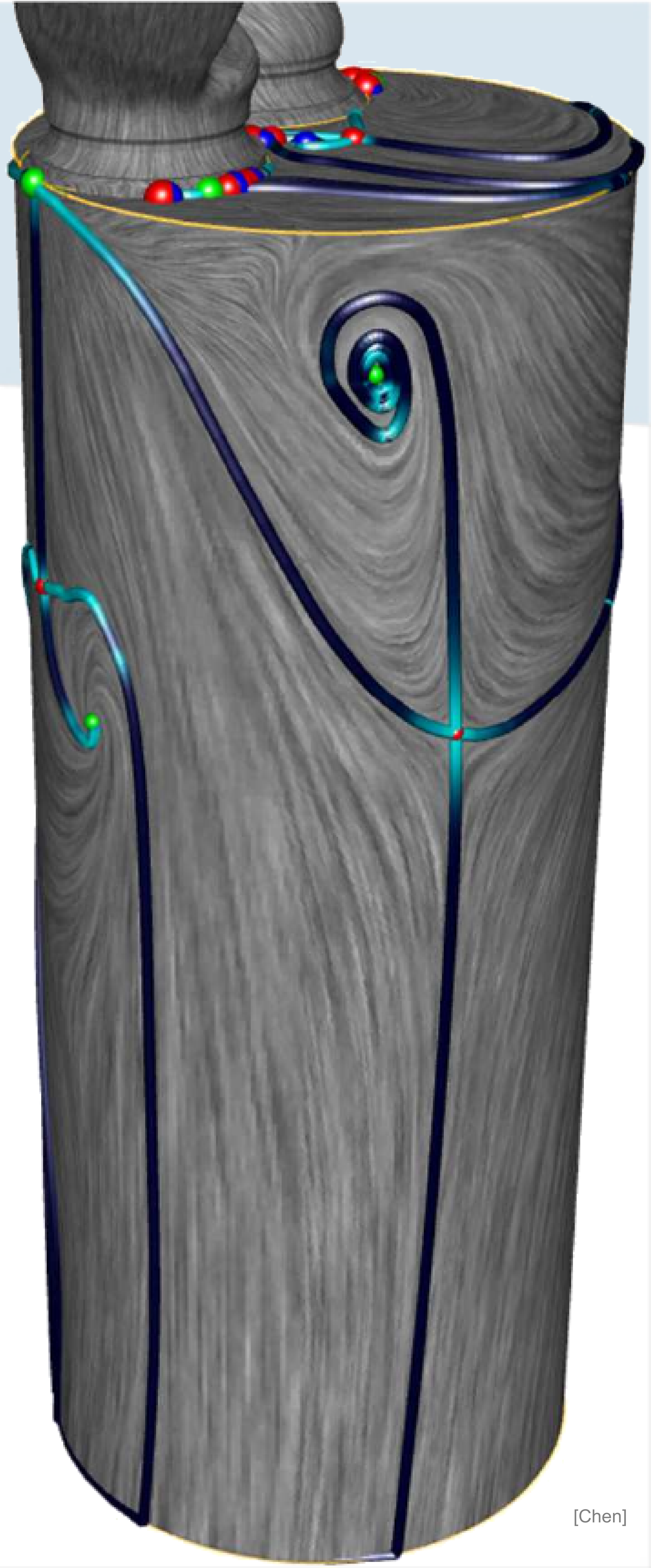
  - Numerical evaluations slightly more involved
- Volumetric domains
  - Similar process



[Chen]

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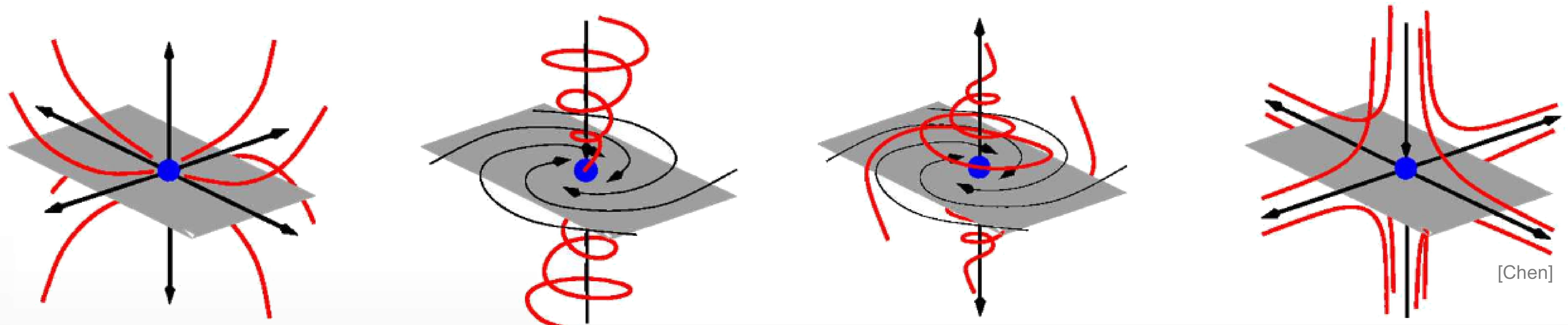
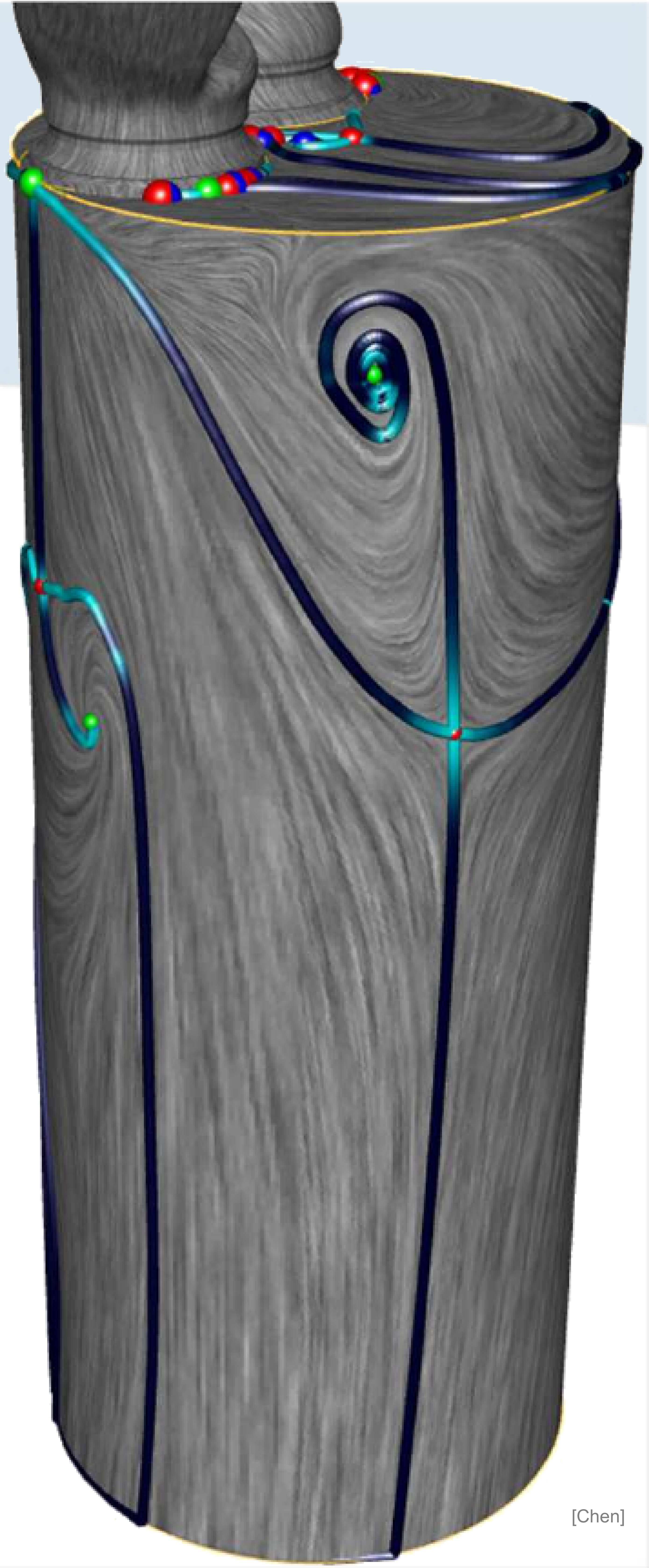
    - Critical points: spiral effects





Non planar domains

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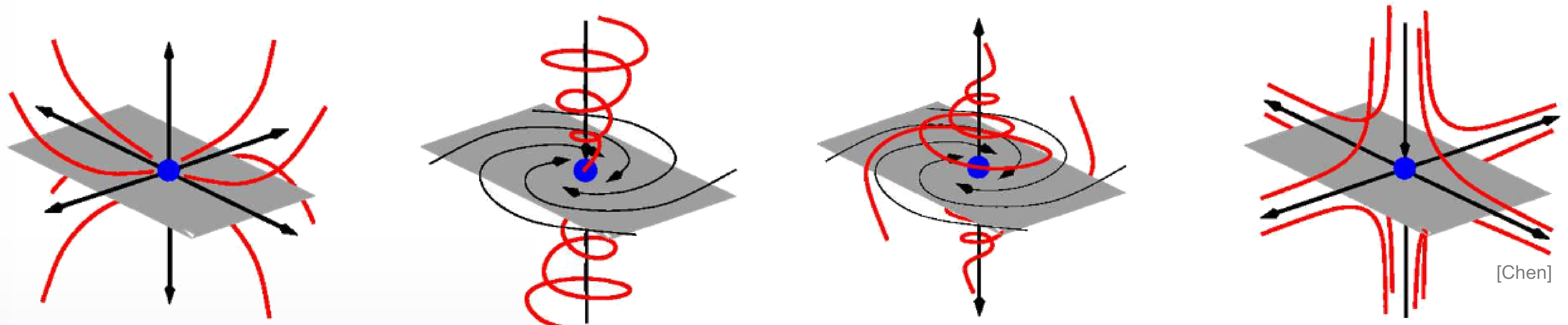
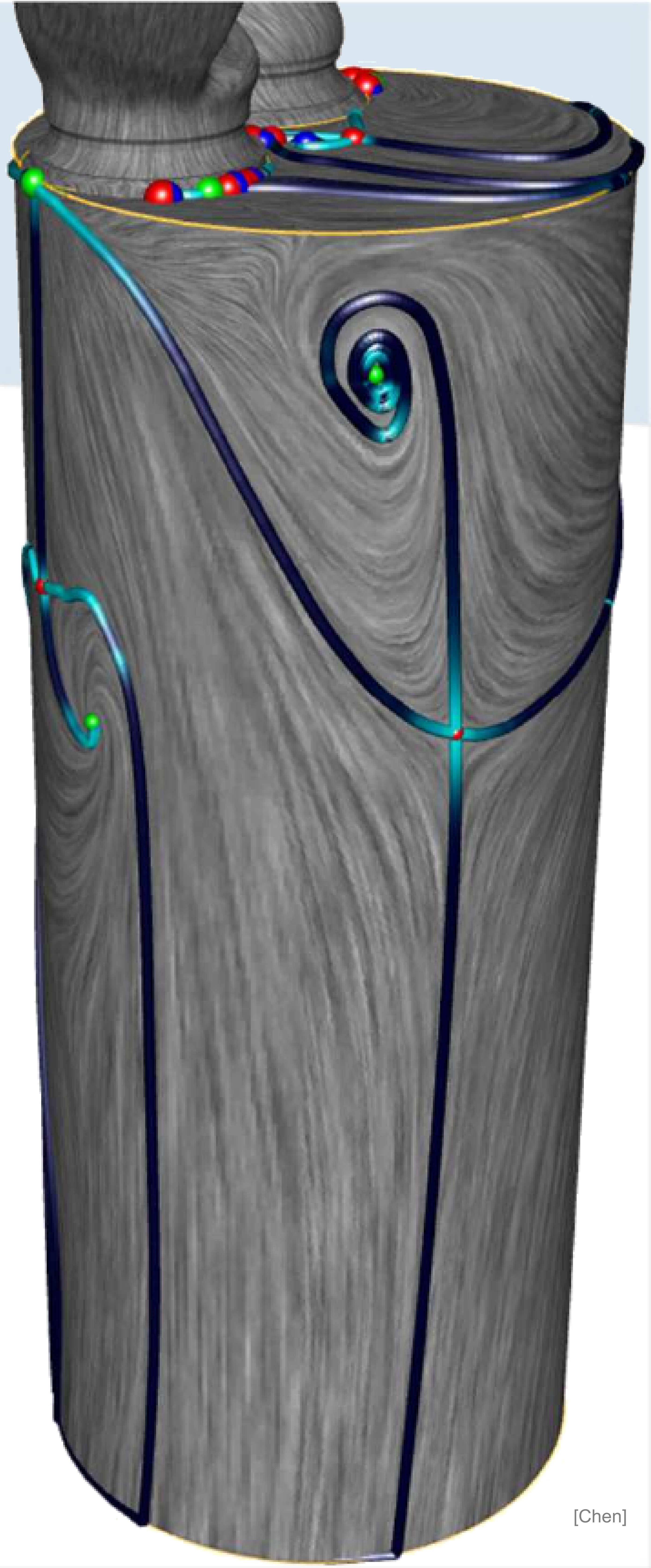
  - Similar process
    - Critical points: spiral effects
    - Streamlines and streamsurfaces as separatrices



[Chen]



[Chen]

Vector field topology 2.0

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Vector field topology 2.0

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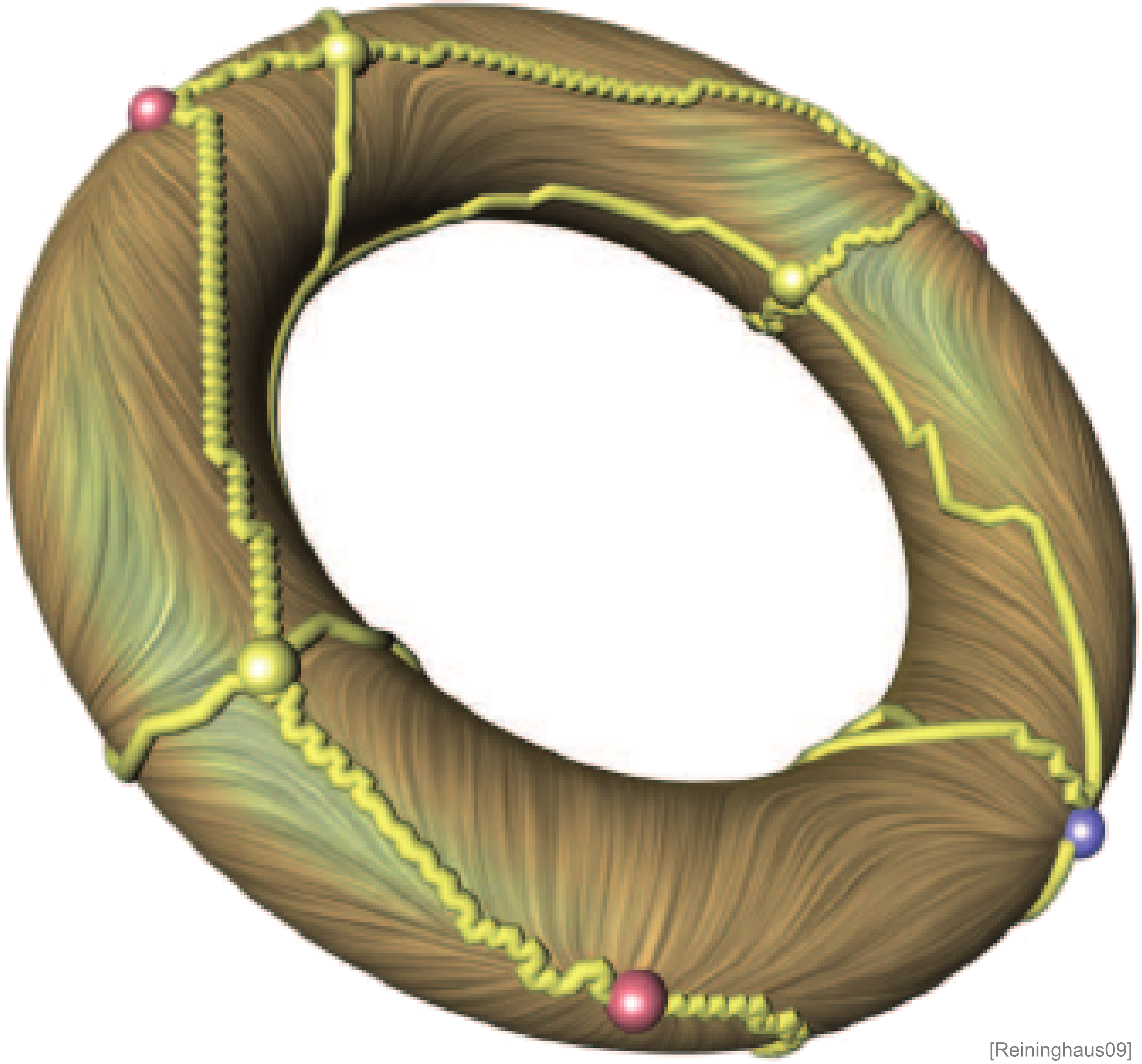
- Vector field topology

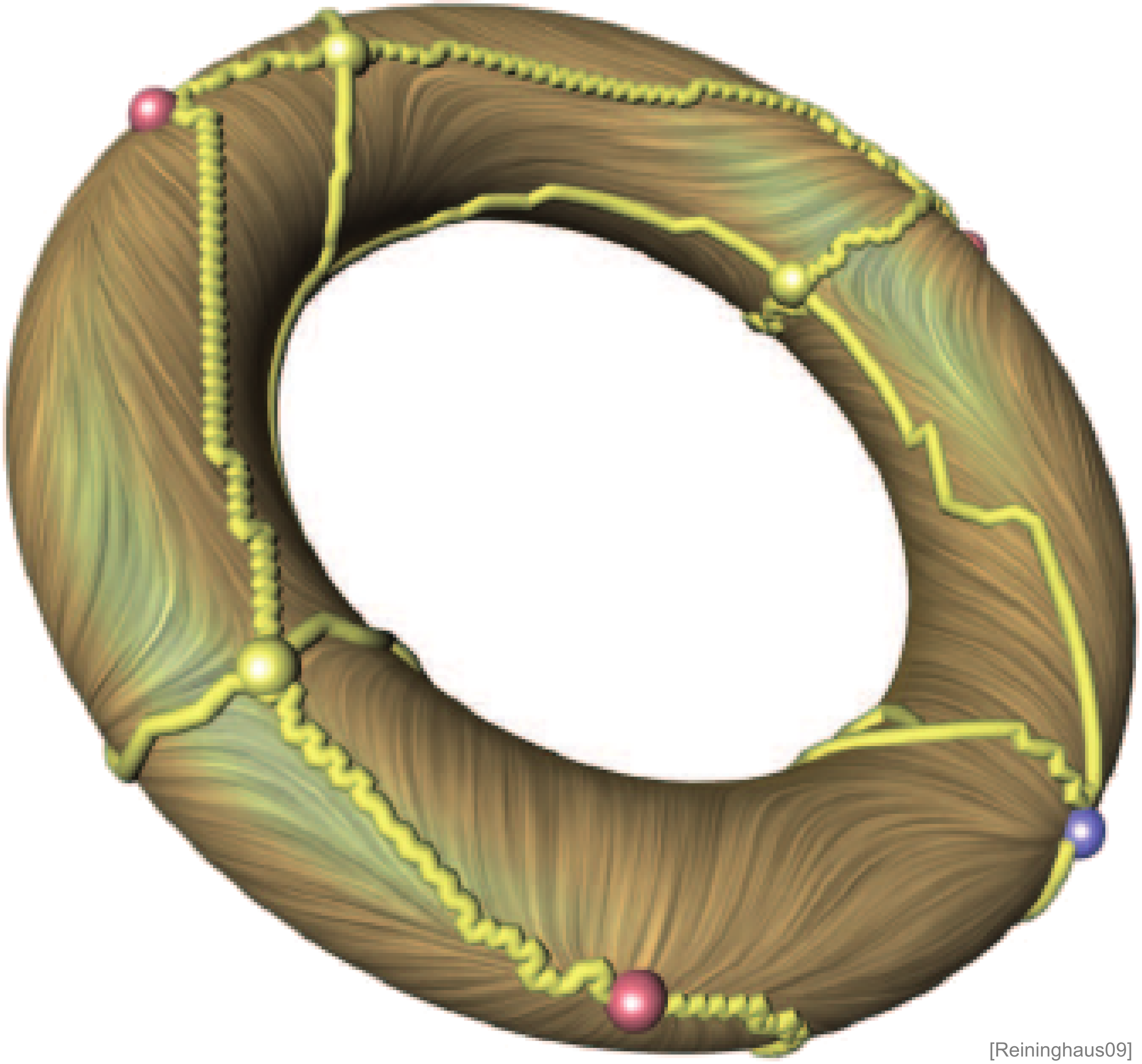


[Reininghaus09]

Vector field topology 2.0

- Scalar field topology
  - Combinatorial critical point identification

- Vector field topology
  - Critical points
    - Derivative estimation



[Reininghaus09]

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  - Streamlines
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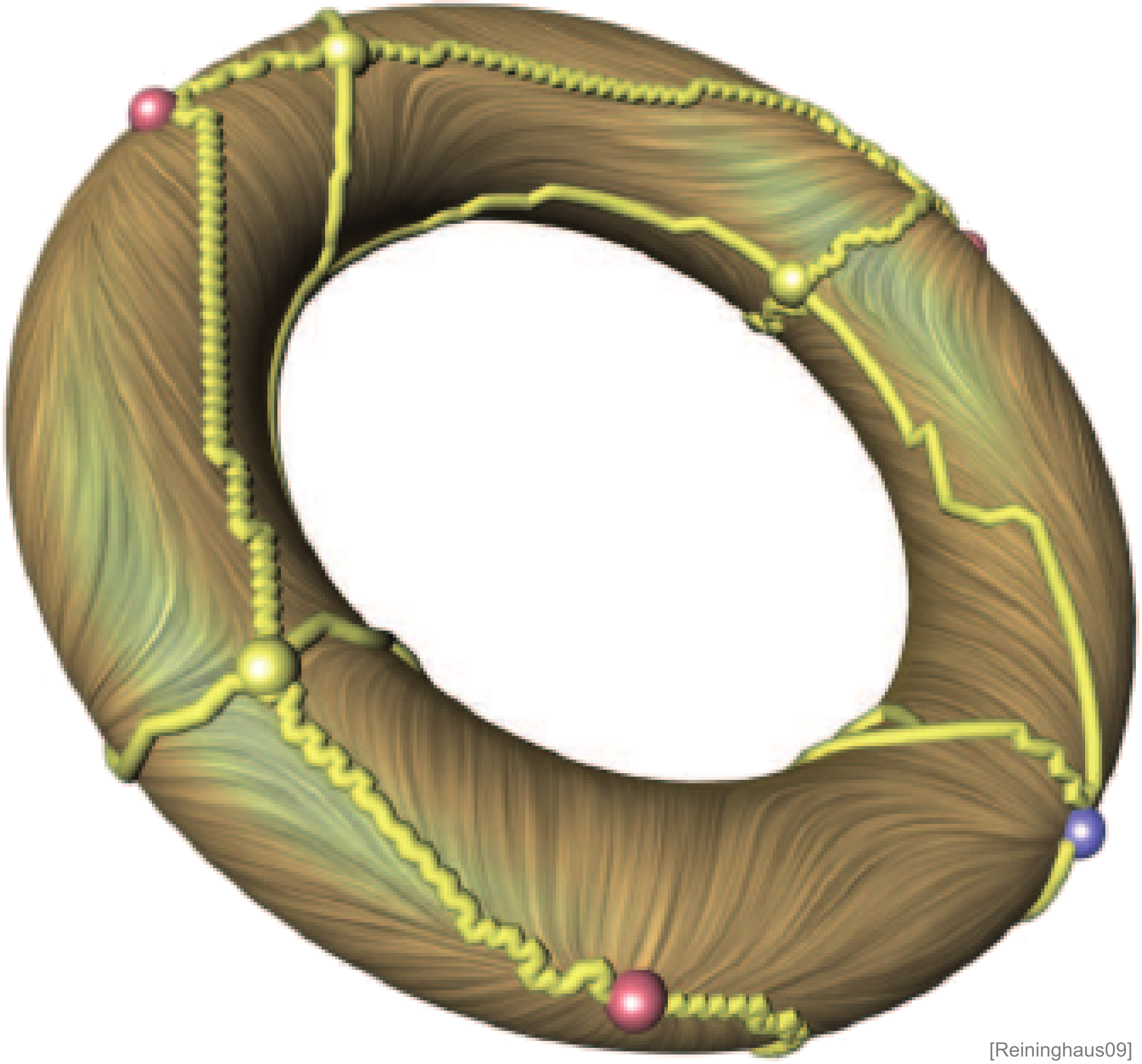

[Reininghaus09]

Vector field topology 2.0

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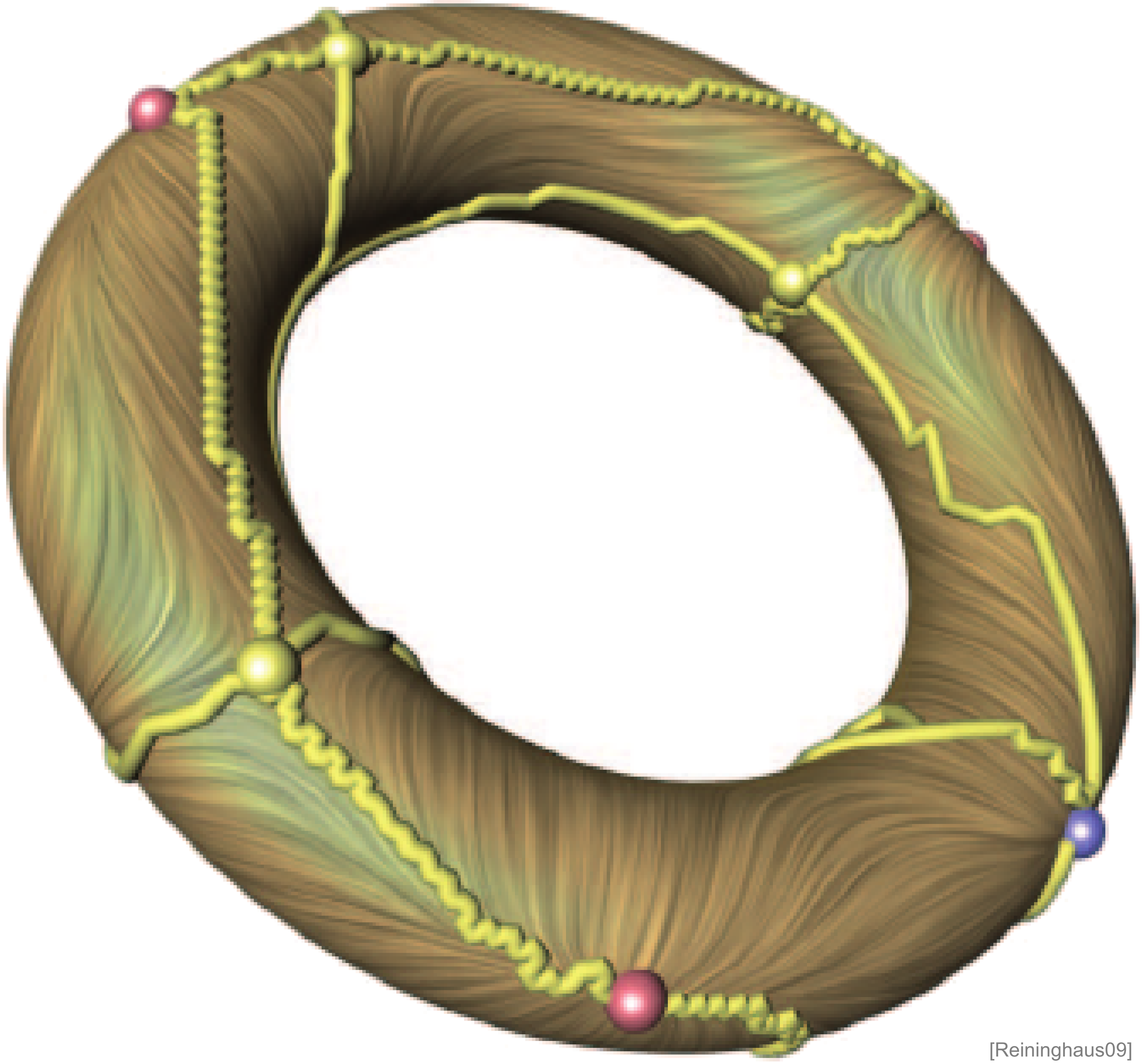
- Vector field topology
  - Critical points
    - Derivative estimation
  - Streamlines
    - Numerical integration
  - Prone to global inconsistencies

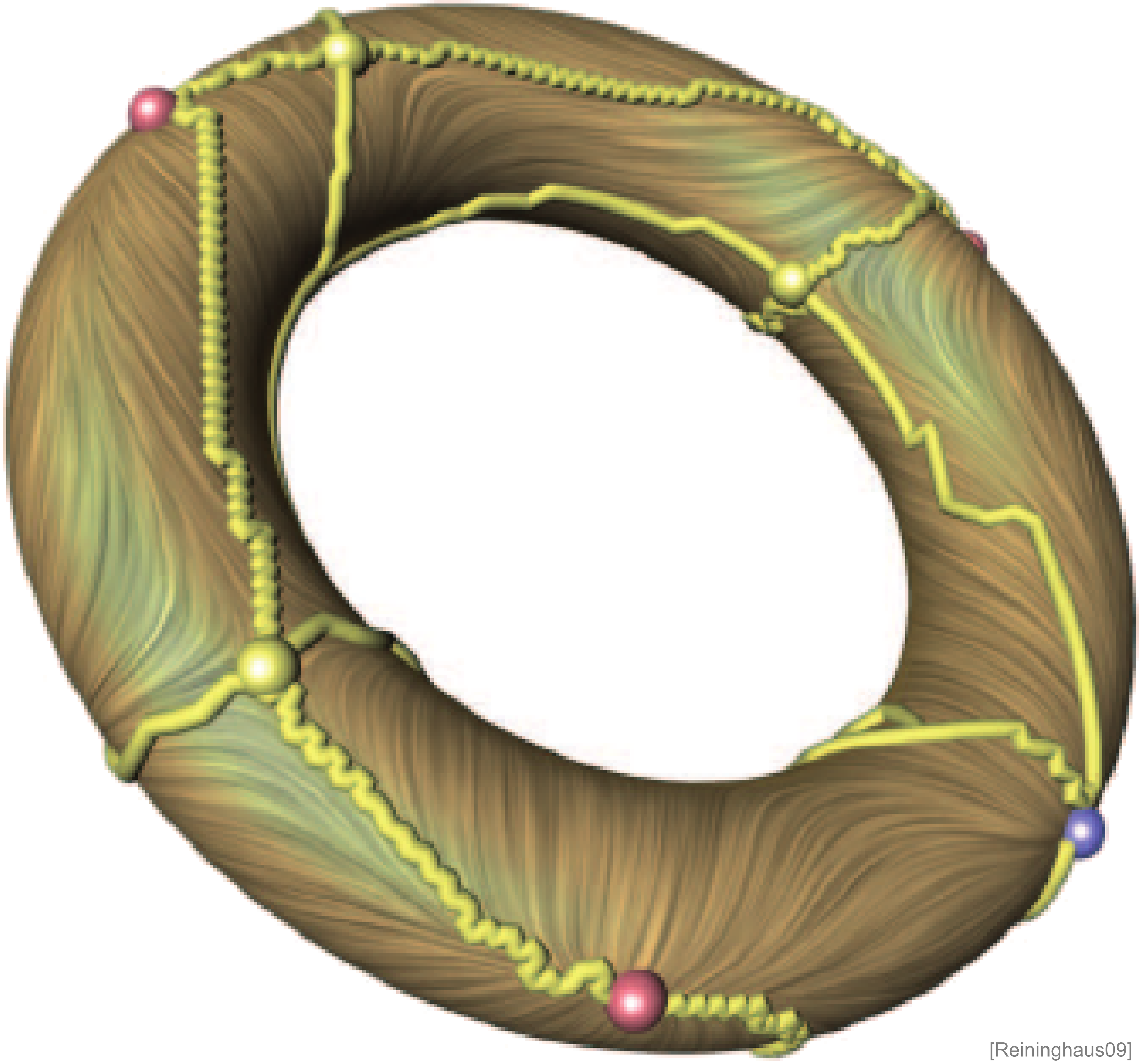


[Reininghaus09]

Vector field topology 2.0

- Scalar field topology

$$\chi(\mathcal{D}) = \sum_{i=0}^{d} (-1)^i \mu_f^i$$



[Reininghaus09]

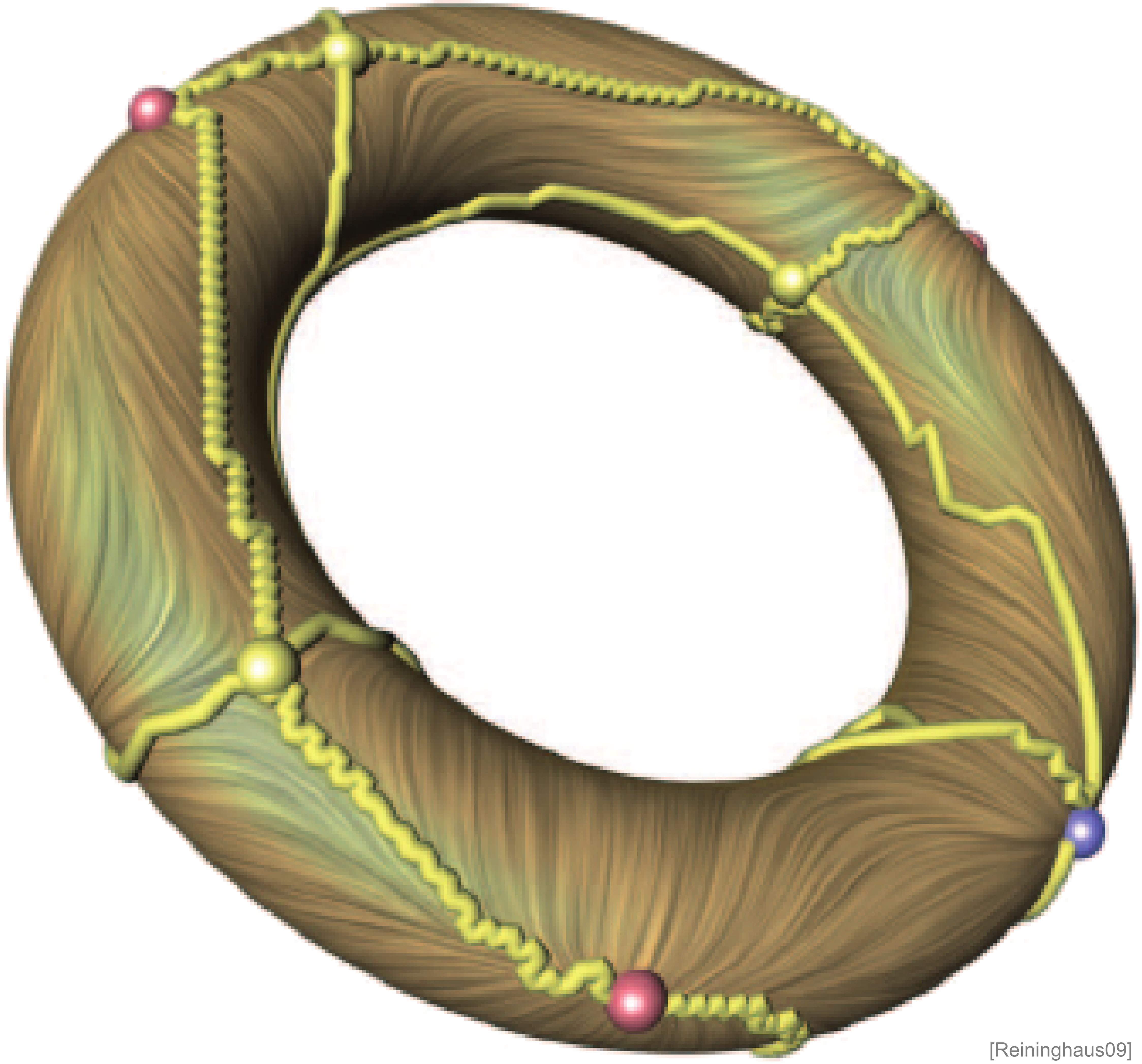
Vector field topology 2.0

- Scalar field topology

$$\chi(\mathcal{D}) = \sum_{i=0}^{d} (-1)^i \mu_f^i$$

- Vector field topology

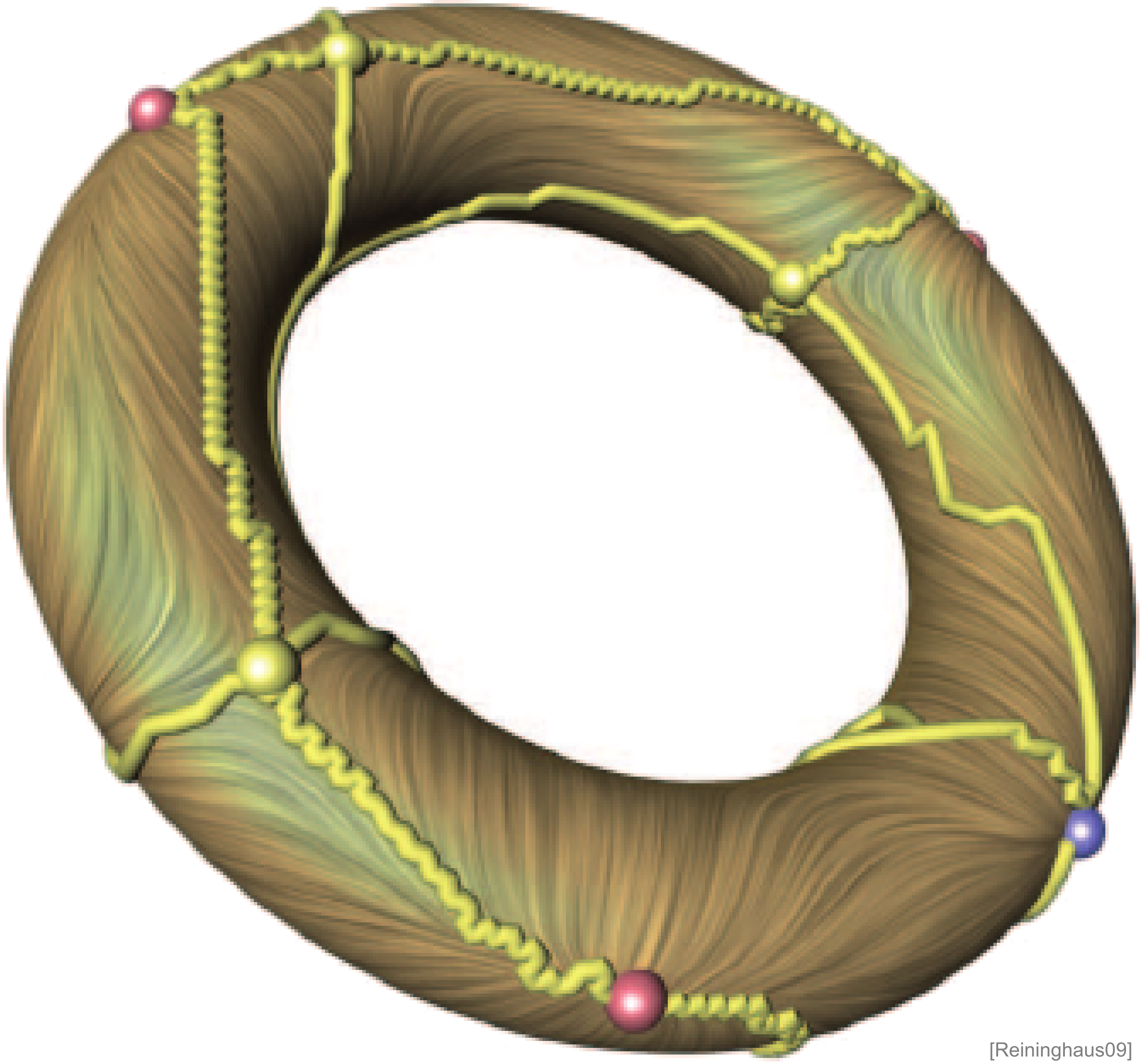
$$\chi(\mathcal{D}) = \sum_{j=1}^{n} index(p_j)$$



[Reininghaus09]

Vector field topology 2.0

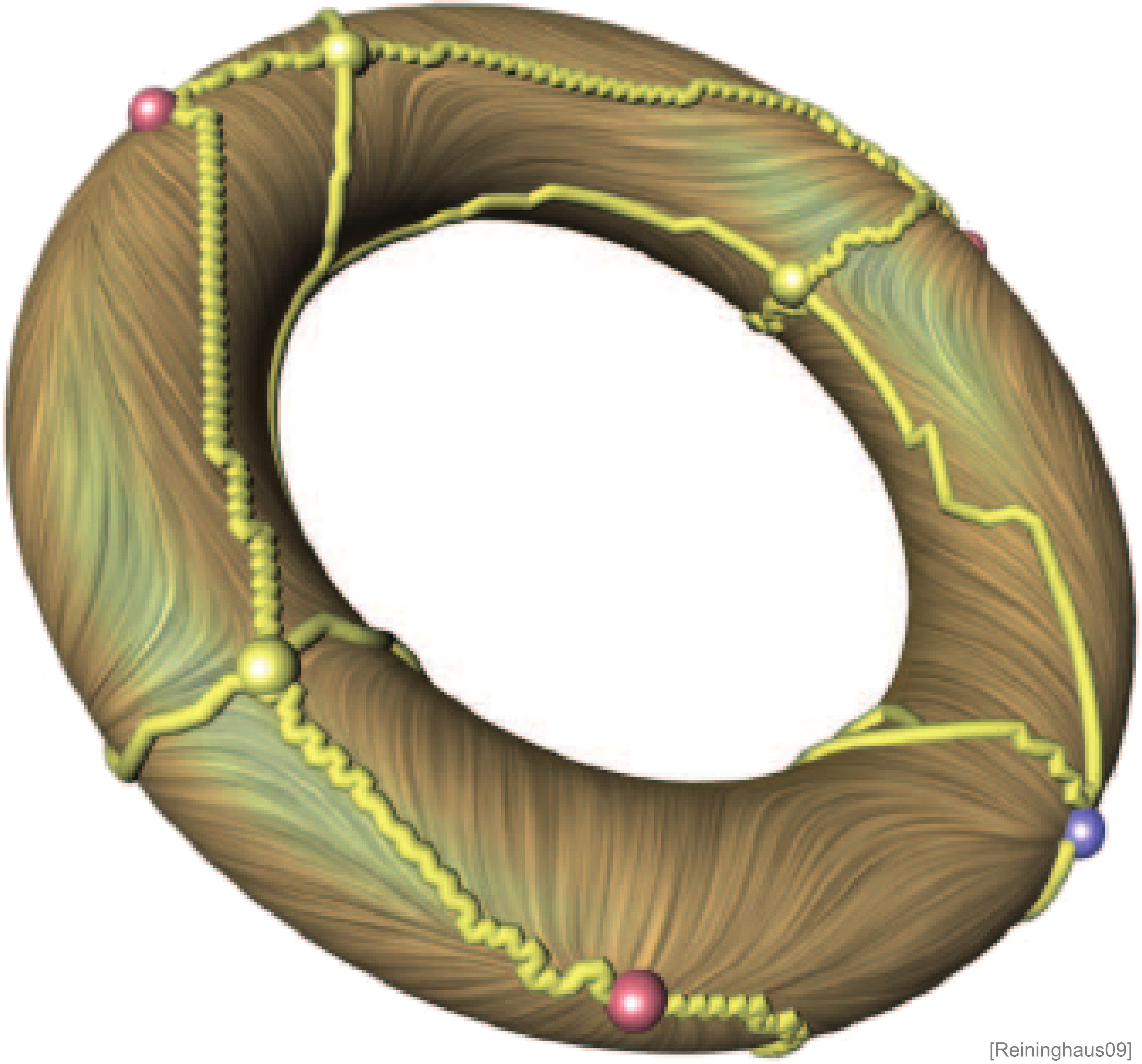
- Combinatorial vector field topology



[Reininghaus09]

Vector field topology 2.0

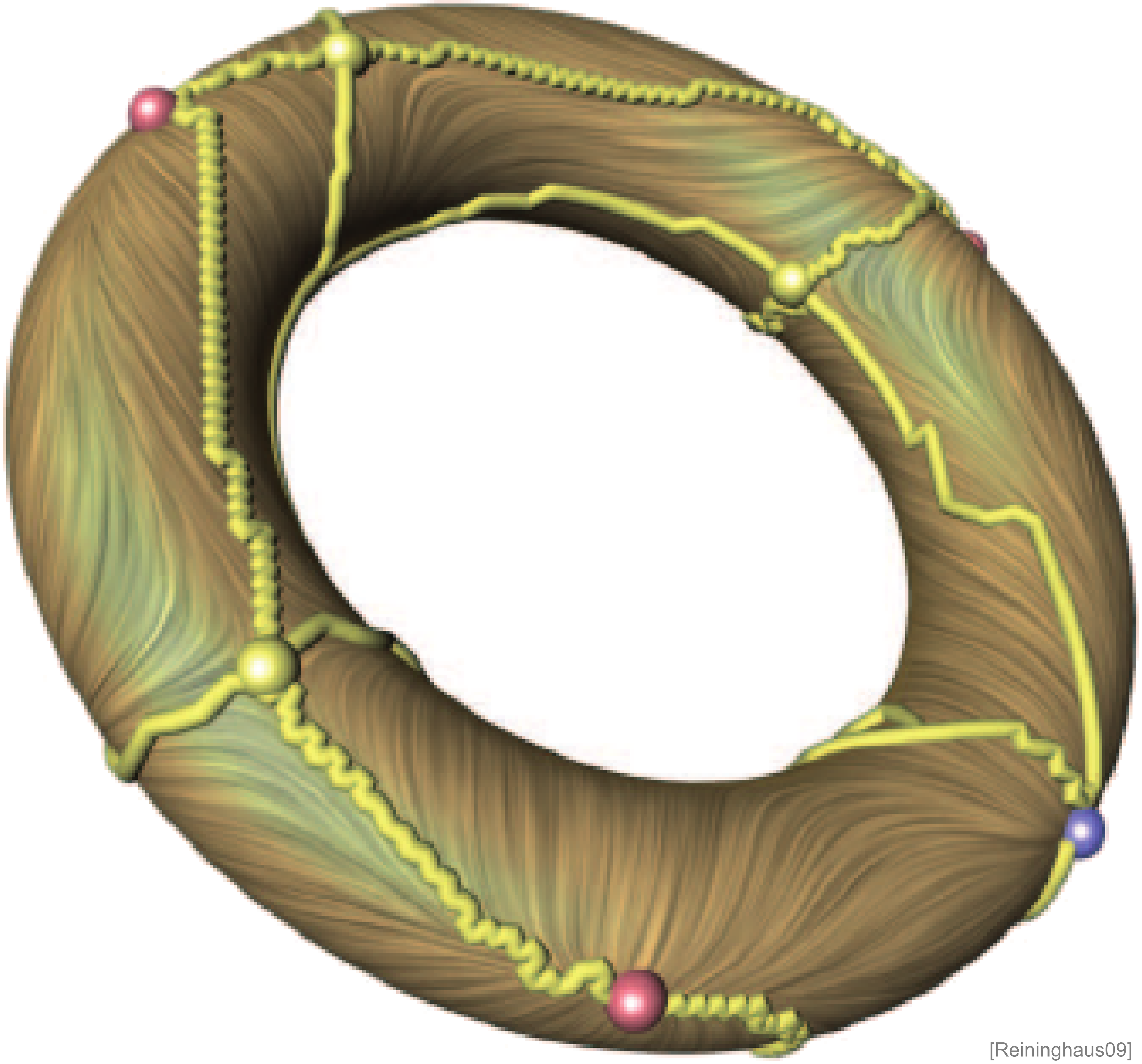
- Combinatorial vector field topology
  - Try to characterize the field



[Reininghaus09]

Vector field topology 2.0

- Combinatorial vector field topology
  - Try to characterize the field
  - With no numerical approximation



[Reininghaus09]

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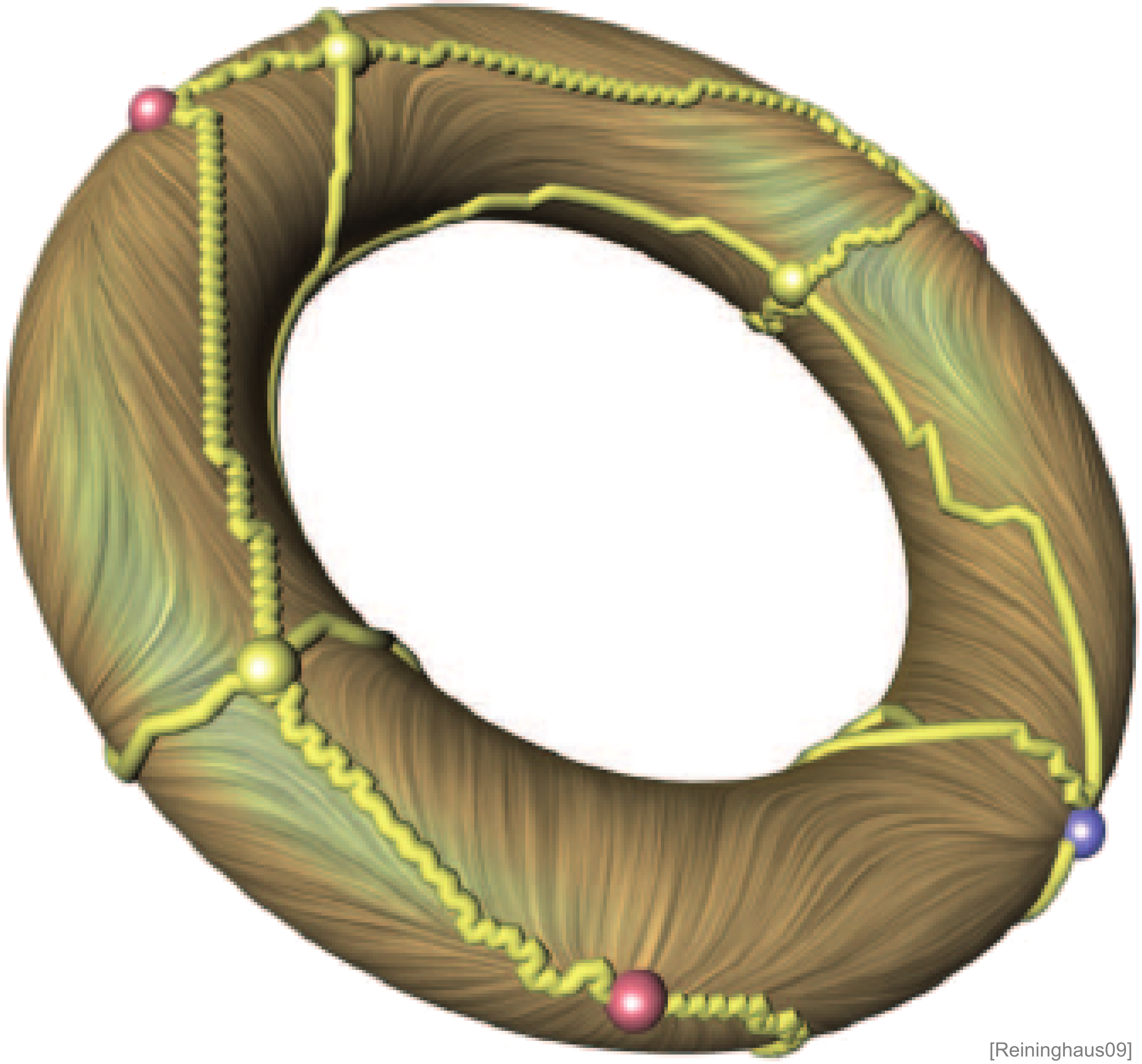
  - Try to characterize the field
  - With no numerical approximation
    - Combinatorial characterization



[Reininghaus09]

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  - Try to characterize the field
  - With no numerical approximation
    - Combinatorial characterization
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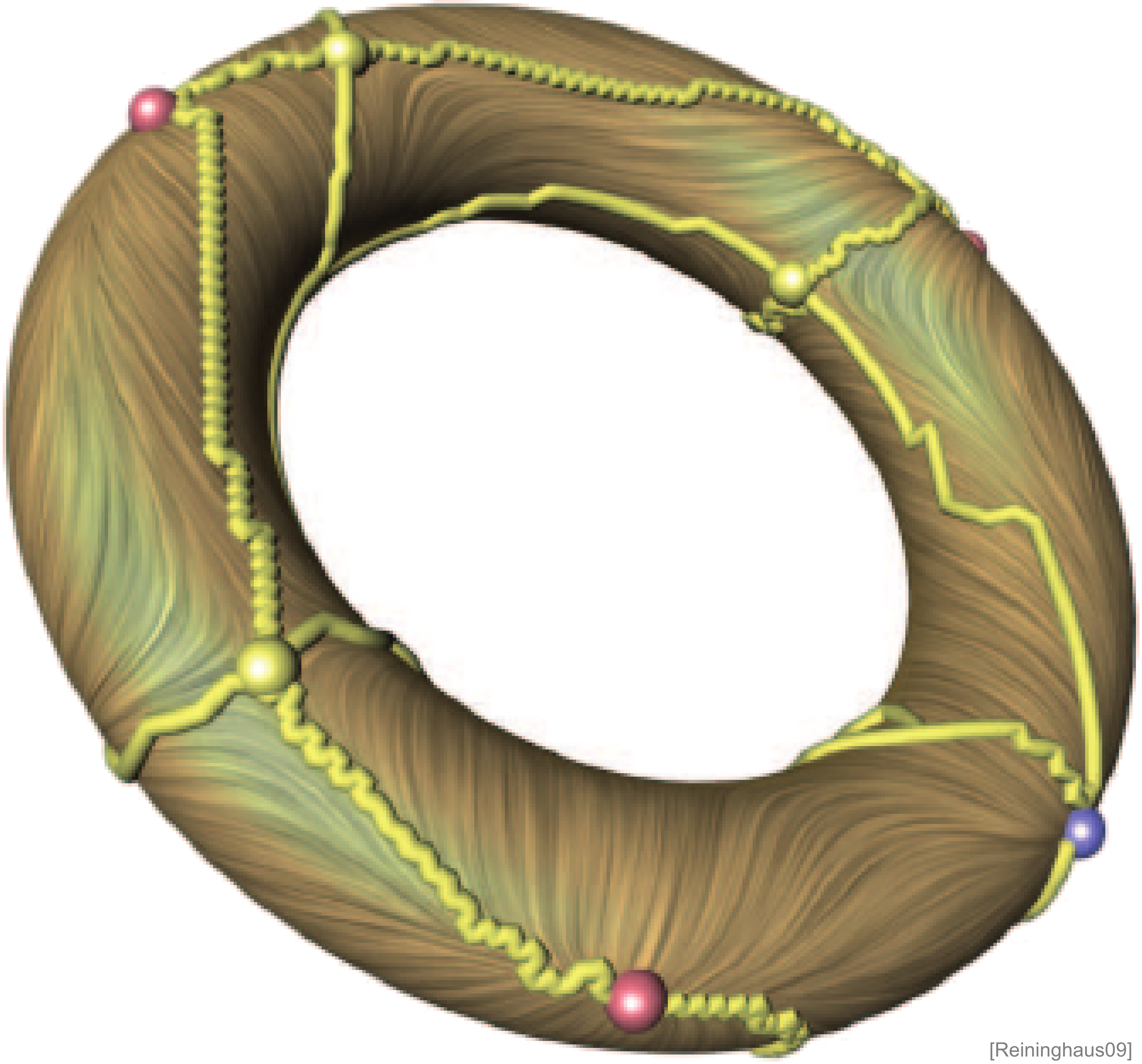
      - Separatrices



[Reininghaus09]

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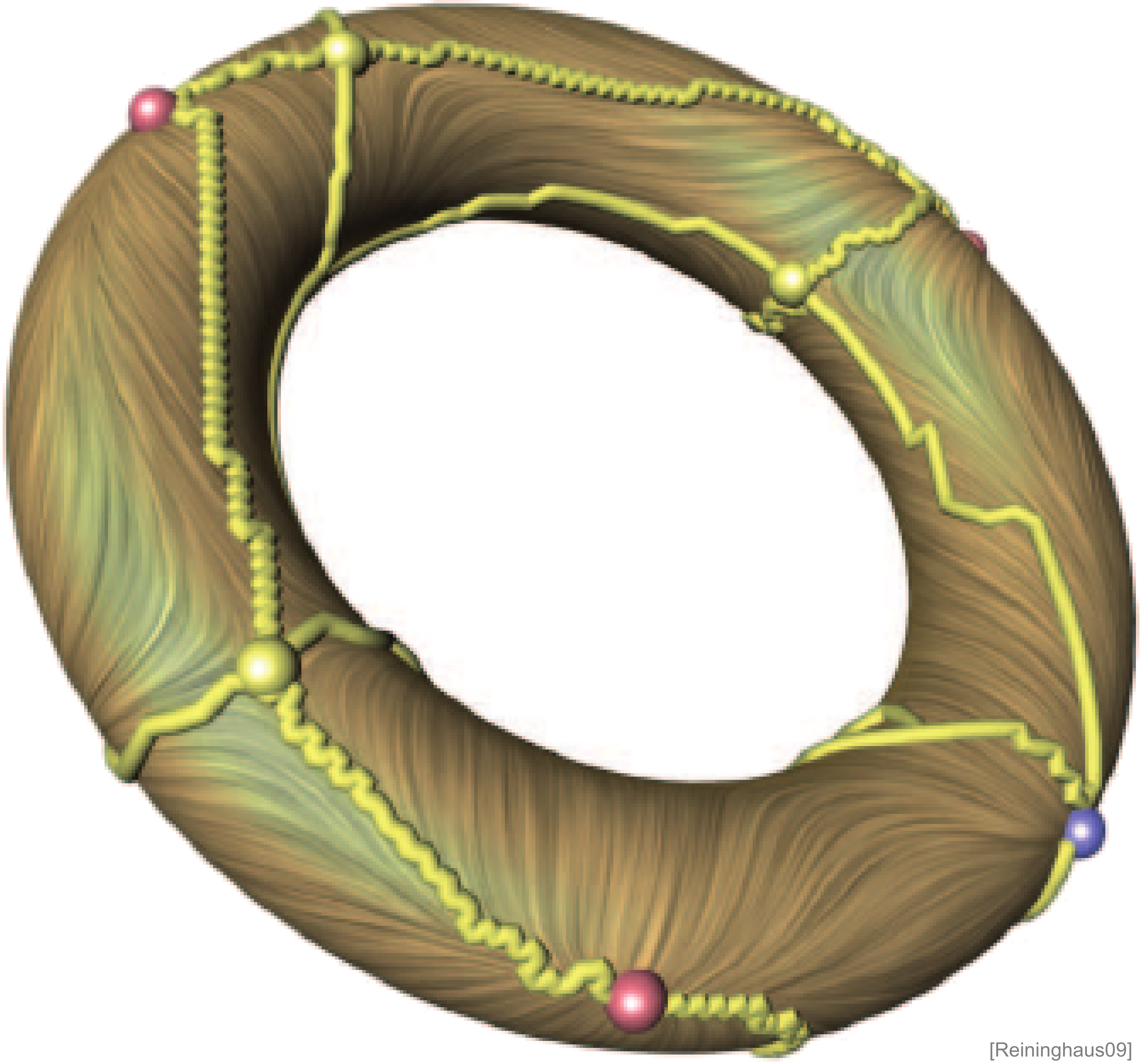
- At the expense of geometrical accuracy
  - Work in progress



[Reininghaus09]

In conclusion

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Acknowledgments

- Many thanks to
  - Guoning Chen, University of Houston
  - Josh Levine, Clemson University